Real Effects of Bank Capital Structure

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Abstract

I present a theory of bank capital structure based on a governance problem between the banker, outside equity investors, and households. The banker determines both the investment level in a project and its financing. A unique mix of equity capital and short-term debt maximizes the project's surplus for the investors. However, if the rents in banking are high and the banker's internal funding is scarce, the equilibrium features lower equity financing than the social optimal, implying a higher likelihood of bank runs and under-investment in the project. In this setting a minimum equity capital requirement simultaneously reduces the risk of bank runs and increases the investment level, while the banker is unambiguously worse-off. However, an excessively high requirement leads to the banker's unwillingness to terminate bad projects, which again results in under-investment. (JEL G21, G28, D86)

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1 Introduction

The level of equity capital in banking is under intense scrutiny following the substantial impact of the 2008 financial crisis on the real economy. The large U.S. financial institutions, bank holding companies and investment banks, had $3-5\%$ book equity-to-asset ratios at the beginning of 2007. To limit the damaging effects of a similar systemic bank run in the future, Dodd-Frank Act of 2010 requires that the quantity and the quality of capital held by banks conform to Basel III capital standards. For example, the minimum equity capital requirement as a percentage of risk-weighted assets is raised from 2% to 4.5%. With the additional requirements for large institutions, the minimum book equity-to-asset ratio can reach 9.5%.

Critiques of the new requirements warn that the regulation reduces the cost and the likelihood of bank runs at the expense of lowering bank lending to the real economy[1]. Calomiris and Kahn (1991) and Diamond and Rajan (2000, 2001) point out that in order to limit the moral hazard problem of bankers diverting cash flows away from investors, who are unable to act as effective monitors, it is optimal for the bank to be financed with short-term debt so that the banker is subject to the market discipline of runs. This strand of the literature argues that an equity capital requirement can reduce the associated bankruptcy costs but the investors demand a higher return on equity to compensate for the greater moral hazard problem, which diminishes the bank's ability to raise funds. In another strand of the literature, DeAngelo and Stulz (2014) and Gorton and Winton (2014) focus on the liquidity premium households are willing to pay to hold the bank's short-term debt. A higher equity capital requirement reduces the supply of liquid deposits. Finally, the belief that the equity is the costliest form of financing and that higher equity capital requirements increase the overall cost of funding is pervasive in the banking industry.

What capital structure for banks best serves the interests of the real economy? To address the potential trade-offs between bank lending and the riskiness of the capital structure, I introduce a model in which the two are jointly determined. In the model, an owner/manager banker with limited internal funds supplies credit to a long-term project with uncertain returns. The banker receives a signal of future returns and can liquidate the project early. He faces two agency problems in raising external funds: one with the households and another with the outside equity investors. At the equilibrium, the households hold short-term debt and bank runs occur if there is a bad expectation of cash flows. I characterize the investment level and the short-term debt/equity mix that maximize the project's surplus, and then analyze the equilibrium in which they are chosen by the utility-maximizing banker.

The paper has two main results. First, there is a unique benchmark investment level and a

[1]For example, Larry Summers voiced this concern in the IMF Conference, November 8, 2013 that "most of what might be done under the aegis of preventing a future crisis would be counterproductive, because it would, in one way or another, raise the cost of financial intermediation."

corresponding equity-to-investment ratio that lead to the highest surplus from the project. At this benchmark there is just enough equity in the bank to absorb the anticipated losses from liquidation so that the short-term debt is risk-free. Second, if the banker's internal funds are scarce, the equilibrium features both a riskier capital structure with too many bank runs and lower investment in the project. Therefore, a minimum equity capital requirement to force the banker to issue outside equity can reduce the likelihood of bank runs and increase investment at the same time.

Capital structure decisions are concerned with giving the banker the right incentives. The banker and the outside equity investors are better-skilled at monitoring the cash flows from the project than the households are. The banker cares about both monetary returns and non-monetary private benefits from allowing projects to continue. He may want to avoid liquidating bad projects to protect the private benefits. In this framework short-term debt has a dual disciplining effect on the banker. First, the banker truthfully reveals his information about the future cash flow because trying to divert cash flows by announcing them to be low leads to mass withdrawal resulting in the liquidation of the project and the loss of private benefits. Second, forced liquidations are in accordance with the outside equity investors' interest if the cash flow from continuing these projects is less than their liquidation value.

The uniqueness of benchmark capital structure follows from a hump-shaped relation between the project's surplus and the equity-to-investment ratio. If the banker overly relies on short-term debt to finance the investment, the households suffer a loss whenever the project is liquidated, because the liquidation recovers only a fraction of the investment. For the households to break even in expectation, the banker pays a compensating premium whenever the project is completed. This premium determines a threshold below which the banker is insolvent and runs may occur. The larger the investment and the lower the banker's equity cushion, the greater the likelihood that the banker ends up liquidating a high cash flow project at a loss. The anticipated loss from bank runs depresses the expected return on investment and lowers the willingness to invest ex ante. Likewise, over-reliance on equity capital is undesirable because the banker lowers the expected quality of completed projects by allowing some bad ones to survive for their private benefit. The anticipated drop in the marginally completed project's quality also lowers the willingness to invest in the project ex ante.

The banker treats inside and outside equity differently and his preferred capital structure follows a pecking order: internal funds are preferred to any other external funding and short-term debt is preferred to outside equity. The banker and the outside equity investors share the project's surplus so they both capture rents at the equilibrium. This feature makes outside equity privately costly for the banker because the households earn zero rent on their short-term debt. When the internal funds are scarce, the banker compares the marginal increase in the project's surplus to the marginal decrease in the claim on the new surplus. I show that if the rent is large enough, the banker is better-off leveraging the internal funds by short-term debt and capturing the entire surplus as the only residual claimant. This insight remains valid even as the outside equity investors become perfectly competitive and earn zero rent because the banker's rent includes private benefits that are reduced when the controlling interest is diluted with outside equity.

There are no gains from an equity capital requirement if the banker's internal funds are abundant. In this case there is no need for outside equity to build the necessary buffer against liquidation losses. The banker issues risk-free, short-term debt and only the low cash flow projects are liquidated at the equilibrium. If the private benefits are small, the unregulated equilibrium obtains the benchmark outcomes. Requiring an increase in the equity-to-investment ratio leads to excessive continuation of bad projects, which depresses the investment level.

In my model the motivation for the banker to choose low equity levels come from the loss of rents on issuing new equity. Much of the recent literature focuses instead on the role of regulatory frictions leading to excessively risky capital structure. Admati and Hellwig (2013) argue that the banks perceive equity as costly because bailouts, deposit insurance, and tax shields subsidize short-term debt. Van den Heuvel (2008); Begenau (2014); Harris, Opp and Opp (2014); and Nguyen (2014) analyze the optimal bank capital ratios that mitigate the excessive risk-taking incentives created by mispriced government guarantees. In contrast, I identify a role for capital requirements even if no regulatory friction exists. An insurance/bailout scheme can be introduced in my model as a policy alternative but it cannot improve on a correctly set minimum equity capital requirement, because the policymaker does not observe the banker's cash flow signal and has to design an incentive-compatible mechanism[2] since the households do not run and discipline the banker once their claim is insured.

The paper is organized as follows: Section 2 introduces the model, Section 3 studies the liquidation rule induced by the capital structure and investment, Section 4 analyzes the effect of capital structure on optimal investment level and Section 5 presents the unregulated equilibrium outcome. All proofs are relegated to the Appendix.

2 The Model

This section lays out the model economy. After the main assumptions, I determine the benchmark outcomes. Then, I introduce informational and contractual structure of the model. Last, I provide an intermediate result on short-term debt and bank runs.

The model has three types of agents: a banker, a continuum of outside investors, and another

[2]The bailout literature suggests additional constraints that the policymaker might face, e.g. the time-inconsistent preferences as in Chari and Kehoe (2013), so the second-best surplus of the policymaker's mechanism might be strictly lower.

continuum of households. There are three periods $t \in \{0, 1, 2\}$. All agents are risk-neutral, protected by limited liability, and do not discount future gains for simplicity. Opportunity cost of supplying a unit of capital is the same for all agents. The analysis is concerned with a representative banking relationship between the three agents. The regulator's role is suppressed in the baseline model and all taxes on capital are assumed to be zero.

The real economy in this model is passive. The banker has access to a project with investment outlay I subject to diminishing returns and a costly real option to liquidate early. The level of investment is chosen at $t = 0$ and cannot be altered at later dates. The project yields cash flow $z\phi(I)$ at $t = 2$ for its investors. Here $\phi(I)$ denotes the baseline production function and z is the realization of continuous random variable Z. At $t = 1$ the banker observes a non-verifiable signal of z and this signal perfectly reveals the cash flow at $t = 2$. At this point the banker can decide to liquidate the project. Early liquidation at $t = 1$ recovers a fraction $\alpha < 1$ of the initial investment I. There are no profitable reinvestment opportunities if the project's capital is sold at $t = 1$. Figure 1 illustrates the time line of events.

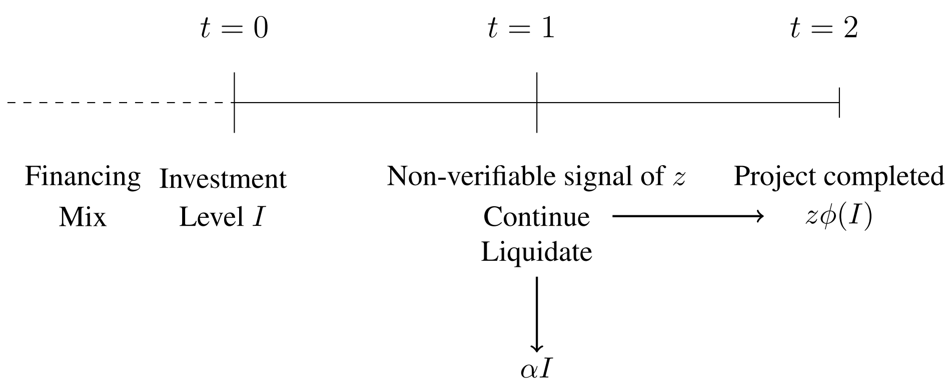


Figure 1: The Time Line of Events

I make a standard assumption regarding the production function:

Assumption 1 *$\phi : R_+ \to R_+$ is strictly concave, strictly increasing and $\phi(0) = 0$. ϕ' is convex and satisfies:*

$$\lim_{I \to 0} \phi'(I) = \infty, \lim_{I \to \infty} \phi'(I) = 0.$$

Z has a continuous and differentiable probability density function f on $[0, \infty)$ and F denotes the distribution function. Z satisfies the following condition:

Assumption 2 *Z is a log-concave random variable i.e. $\ln f(z)$ is concave.*

The log-concave family includes commonly used distributions such as Normal, Exponential, Uniform, Logistic and certain classes of Gamma and Beta. Bagnoli and Bergstorm (2005) provides a comprehensive list of distributions that satisfy various log-concavity properties.

2.1 The Benchmark

As a benchmark, I show there is a unique investment level and a liquidation rule that maximize the project's surplus. For any given $I > 0$, it is optimal to liquidate the project at $t = 1$ if the signal $z < z^*(I)$ where z^* is determined by

$$z^*\phi(I) = \alpha I \tag{2.1}$$

The function z^* gives a liquidation rule which is increasing in I by Assumption 1. Taking this liquidation rule as given, denote I^* as a solution to:

$$\max_I \int_{z^*}^{\infty} z\phi(I)dF(z) + F(z^*)\alpha I - I \tag{2.2}$$

satisfying the first-order condition:

$$\int_{z^*}^{\infty} z\phi'(I)dF(z) + \alpha F(z^*) - 1 = 0 \tag{2.3}$$

Proposition 1 *There exists a unique benchmark investment level $I^* \in (0, \infty)$.*

Absent Assumption 2, (2.2) is not a standard problem. For an incremental rise in investment level, any completed project has lower marginal return by concavity of $\phi(I)$. On the other hand, the expected payoff to completed projects rises because better projects are completed. Log-concavity of the distribution resolves this ambiguity always in favor of diminishing marginal returns. The proof of Proposition 1 establishes that (2.2) is quasi-concave in I under Assumption 1 and 2. In fact it is sufficient for Z to have increasing hazard rates, $h(z) = f(z)/1 - F(z)$ to be an increasing function for all $z \in Z$ which is weaker than Assumption 2, for this result to hold.

2.2 The Governance Problem

I formalize the conflicts of interests between the banker, outside investors, and households by introducing an information asymmetry and private benefits. The banker can finance up to a fraction $\eta < 1$ of the investment I with internal funding and the remainder fraction $(1-\eta)$ must be financed by issuing claims (external funding). Here the banker is an owner/manager; I assume his incentives are perfectly aligned with bank insiders who supply the inside equity for the banker's use by a managerial compensation contract that is left out of the analysis.

Households represent the general public, who cannot monitor the banker's cash flow. I assume that the banker has superior information about the cash flow at both $t = 1, 2$. The banker can divert cash flows away from the households at $t = 2$ when they accrue. Therefore, any incentive-compatible contract would require the banker to reveal his signal at $t = 1$ and commit to a repayment based on this announcement. Cash flow announcements are publicly verifiable and the banker is held accountable if he reneges on his obligations based on what he reported[3].

Outside investors represent any other financial institution interested in buying long-term, non-controlling claims on cash flows. The simplest of such claims is equity[4]. They observe and enforce their claim on cash flows without friction. Outside investors are passive shareholders in the sense that they value the monetary returns to their equity taking the banker's decisions as given. As they are more skilled investors than the households, I assume that they are in short supply in the economy and capture some of the project's surplus as rent, whereas the uninformed households are perfectly competitive and earn zero rent.

Table 1 illustrates a typical balance sheet. Let κ denote the fraction of the investment financed by equity and a fraction η/κ of this equity is held internally by the banker. In the baseline model, the inside and outside equity earn the same payoff per share. That is, the outside equity investors own a $(1 - \eta/\kappa)$ fraction of the bank's equity and they claim $(1 - \eta/\kappa)$ of the cash flows. Section 5.1 analyzes the limiting case in which the banker can sell outside equity without leaving any rent to the outside investors.

Table 1: Balance Sheet of the Bank

Assets	Liabilities	
	ηI	Inside Equity
I	$(\kappa - \eta)I$	Outside Equity
	$(1 - \kappa)I$	Household Claims

A conflict of interest between the banker and the outside investors exists because they have different preferences. The banker cares about the monetary returns to his inside equity and derives utility from control rights in the bank. To capture this idea, the banker receives a non-monetary private benefit proportional to his claim on cash flows. The smaller the banker's ownership, the

[3]One might argue why the regulator does not monitor the banker on behalf of the households. Although it is true that the banks disclose more substantive information to the regulators, nothing can be disclosed to the public unless the banker is charged with fraud. Not all forms of diverting cash flows are fraudulent e.g. reinvesting cash flows through a subsidiary outside the regulator's control. I abstract away from legal considerations of what is a fraud by assuming that the discount rate after $t = 2$ is infinity so that suing the bank to collect cash flows is not a worthwhile option for the households. Implementation of the contract requires a minimal regulatory role. Since the households cannot learn how much the banker actually has, earnings can be overstated at $t = 1$ and the banker claims insolvency later at $t = 2$. All that is necessary is to freeze the asset and impose a penalty to offset the private benefit.

[4]I do not consider additional compensation schemes because the outside investor, by assumption, is not interested in controlling the bank at $t = 0$.

larger the outside investors' influence in decision-making after $t = 2$. I take it as an axiom that the banker dislikes outside interference. For example, the outside investors may impose a different long-run agenda than the banker wants, alter his compensation contract, or replace him altogether. Private benefit is lost if the project is liquidated and the banker is fired. Therefore, the banker has an incentive to complete projects with cash flows lower than the liquidation value. I formalize these ideas as follows[5]:

$$\text{Private Benefit} = \begin{cases} \frac{\eta}{\kappa} \times B > 0 & \text{if the project survives} \\ 0 & \text{otherwise} \end{cases} \tag{2.4}$$

This is well-defined for $0 < \eta \leq \kappa$ and set the private benefit equal to B if $\eta = \kappa = 0$. Section 5.2 analyzes the case without private benefits which includes $\eta = 0 < \kappa$.

2.3 Demandable Debt and Bank Runs

I conclude the model section by proving that the only incentive-compatible claim the banker can issue to raise funds from uninformed households is demandable debt and that bank runs can occur at $t = 1$.

The problem between the banker and the households is mechanically similar to Townsend (1979) and Gale and Hellwig (1985) and conceptually similar to Calomiris and Kahn (1991) and Diamond and Rajan (2001). Consider a contract that requires the banker to announce the future cash flow at $t = 1$ and gives the right to withdraw at $t = 1$. The banker needs to liquidate the project early to meet the demand for withdrawals. The project cannot be partially liquidated and so let $\mathbb{1}_{\mathcal{B}}(y)$ be a function taking value 1 when the announced state y is in the set $\mathcal{B}$ in which some households withdraw funds. A bank run is defined as an event in which households exercise the withdraw option en masse. The set $\mathcal{B}$ is referred to as *bank run* states.

The report y at $t = 1$ is contractible so let $R(y)$ denote the repayment to the households if a non-bank run state $y \notin \mathcal{B}$ is announced. The equity of any kind is junior to the claim issued to the households. For a given $(I; \kappa)$, the households' claim is a triplet $\{(1 - \kappa)I, R(z; \kappa), \mathcal{B}(\kappa)\}$, the amount borrowed, repayment function, and a set of enforced liquidations, respectively.

Lemma 1 *The unique incentive-compatible claim the banker can issue to the households is demandable debt. Households receive a constant repayment $\bar{R}(\kappa)$ whenever the project is completed. If the cash flow is not enough to repay $\bar{R}(\kappa)$, a bank run occurs and the project is liquidated.*

[5]In the Appendix, I use a general functional form to illustrate that the insights behind the results are robust to many other specifications.The proportionality is also desirable for tractability. The banker's monetary return to inside equity in the event of completion and liquidation are $z\phi(I)$ and αI times his fractional claim on these cash flows, which is η/κ in the baseline model. If the private benefit is also proportional to η/κ then the decision is distorted by a constant B.

A flat repayment removes the banker's incentive to misreport his type as long as he can repay $\bar{R}(\kappa)$. For the banker to reveal the truth when $z \notin \mathcal{B}(\kappa)$, first he must expect to pay the same amount had he announced a different $y \notin \mathcal{B}(\kappa)$. Second, the repayment has to be feasible with the project's cash flow $z\phi(I) \geq R(z;\kappa)$. These two assertions lead to a constant repayment on the complement set of $\mathcal{B}(\kappa)$ – non-bank run states.

Demandable claims attract uninformed capital by giving an option to threaten the banker with liquidation. For simplicity of illustration, suppose that $\eta = \kappa = 0$ and the banker is the only residual claimant without private benefits. The banker is always tempted to offer the liquidation value αI. If the households commit to withdrawing whenever αI is announced, it would force the banker's hand to liquidate. Liquidation makes him worse off even if the project he had were $z^*(I)$ because he would also lose the private benefit. Pursuing this logic, the threshold below which households withdraw can be pushed up to a high enough level $\bar{R}(\kappa)$ to make it individually rational to buy the claim.

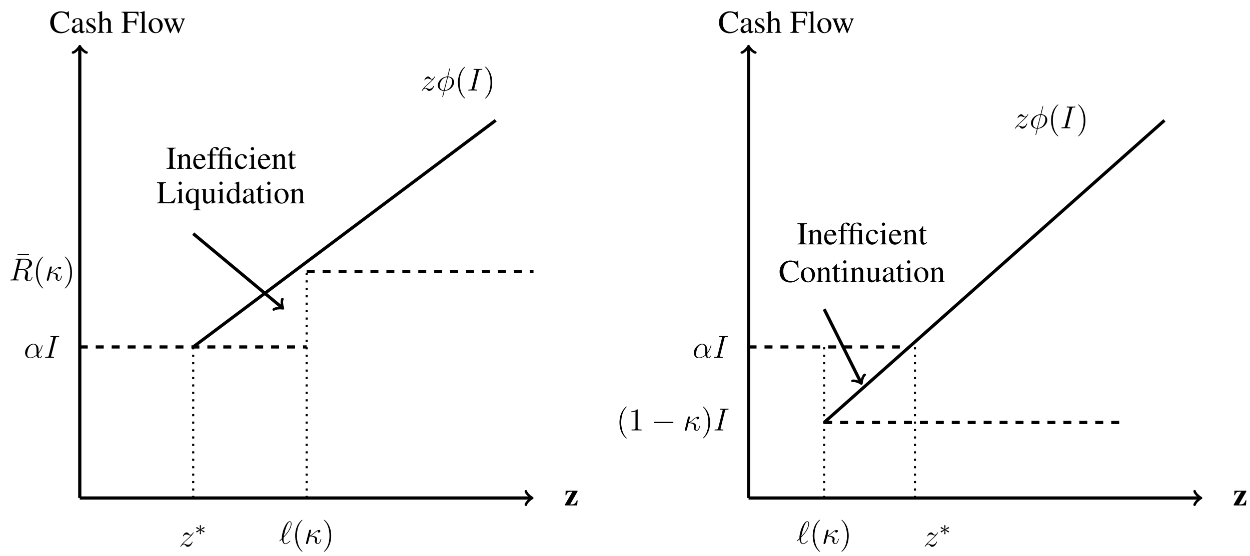
Determining $\mathcal{B}$ at $t = 0$ might appear as if assuming mass pre-commitment to withdrawal decisions at $t = 1$. Following Diamond and Rajan (2001), this issue can be easily resolved by augmenting the claim with a first-come first-served clause on $\mathcal{B}(\kappa)$ to encourage everyone to be among the first $\alpha < 1$ to withdraw and redeem I if the banker announces that he cannot meet the promised repayment $\bar{R}(\kappa)$. When strictly less than α fraction withdraws, they disproportionately reduce the liquidation value αI to be shared among those who wait. When more than α fraction withdraws simultaneously, each take a random place in the queue to share the liquidation value among fewer people and those who wait receive nothing. It is easy to see that withdrawing is the dominant strategy in this game. It is to nobody's benefit to withdraw if the banker announces that he can repay $\bar{R}(\kappa)$ so I assume the first-come first-serve clause can be restricted to the set $\mathcal{B}(\kappa)$.

3 Equilibrium Liquidation Rule

Taking the investment I and the capital structure decision κ as given, I solve for the liquidation rule $\ell(I;\kappa)$. This rule determines the marginally completed project which is comparable to the benchmark $z^*(I)$ in (2.1). Figure 3 graphically summarizes the key result of this section. The higher the leverage, the more liquidation takes place at $t = 1$.

Incentive-compatibility constrains the banker to liquidate at least the projects in the set $\mathcal{B}(\kappa)$. Yet the statement of Lemma 1 leaves it open that not every project that can repay the debt is necessarily completed. When the project is liquidated, the households have seniority in claiming the liquidation value αI. So the critical κ^* to consider is

$$\alpha I \equiv (1 - \kappa^*)I \tag{3.1}$$



(a) Under-capitalized $\kappa < 1 - \alpha$

(b) Over-capitalized $\kappa > 1 - \alpha$

Figure 2: Payoffs for a given $I > 0$

or simply $1 - \alpha$. If $\kappa > 1 - \alpha$, then the bank has so much equity that it can repay the households even in liquidation states. If $\kappa < 1 - \alpha$, then the converse is true; the bank has too little equity to spare the households from liquidation loss. Within the model's context, the cases $\kappa \lesseqgtr 1 - \alpha$ are referred to as under- and over- capitalized bank, respectively.

3.1 Excessive Liquidation When $\kappa < 1 - \alpha$

Fix $I > 0$ and let $\ell(\kappa)$ denote the marginal project that can repay $\bar{R}(\kappa)$

$$\bar{R}(\kappa) \equiv \ell(\kappa)\phi(I) \tag{3.2}$$

The banker strictly prefers the marginal project to be completed because even though the payoff to equity is zero, he gets a positive private benefit. When the banker completes any project that is not forced into liquidation by a bank run, the liquidation rule is implicitly given by the households' participation constraint:

$$F(\ell(\kappa))\, \alpha I + (1 - F(\ell(\kappa)))\bar{R}(\kappa) \geq (1 - \kappa)I \tag{3.3}$$

with equality for each I in a competitive capital market.

The left panel of Figure 2 illustrates the payoffs for $\kappa < 1 - \alpha$. The areas under the two dashed lines corresponds to the first and second terms in (3.3). For lending to the bank to be individually rational, households should be paid a premium in the completion states to compensate the loss

$(\alpha-(1-\kappa))I < 0$ they make in liquidation states. The banker pays the premium only if failing to do so is more costly. By mere observation $\ell(\kappa) > z^*$ is necessary to satisfy (3.3). Hence, some of the projects with $z\phi(I) > z^*\phi(I) = \alpha I$ must be liquidated at $t = 1$. The intuition is that the bank runs burn surplus to force the banker to pay an interest rate that he would otherwise never pay.

The marked triangle in Figure 2a is the deadweight loss from bank runs. Not all liquidations are wasteful; bank runs liquidate all projects $z < z^*(I)$ that would be liquidated in the benchmark. Excessive liquidations $z^* < z < \ell(\kappa)$ can be viewed as an externality. Had the banker financed the same investment I with more equity, the project had more cash flow available to the households in the liquidation states. Thus, the repayment $\bar{R}(\kappa)$ for households to break even could be reduced. Less likely the banker is insolvent and hit by a run, more projects are completed and the deadweight loss triangle in Figure 2a gets smaller.

3.2 Excessive Continuation When $\kappa > 1-\alpha$

With enough equity, the banker can guarantee households a fixed payoff for all states of the world without jeopardizing incentive-compatibility. Despite this enticing outcome, the liquidation rule is given by:

$$\ell(\kappa)\phi(I) \equiv \max\Big((1-\kappa)I, \alpha I - B\Big) \tag{3.4}$$

and it is always lower than the benchmark z^*.

Figure 2b illustrates why there is a deviation from the benchmark. Consider the project $z\phi(I) = (1-\kappa)I < \alpha I = z^*\phi(I)$. Outside investors are better served if this project is liquidated because their payoff would be $(1-\eta/\kappa)(\alpha-(1-\kappa))I$, the share in profits after the debt is repaid, positive. Yet the banker might have a large private benefit $(\eta/\kappa)B > (\eta/\kappa)(\alpha-(1-\kappa))I$ such that completing the project is appealing even though it makes nothing for the equity. The earlier discussion insinuates a benign view of bank runs in terms of forcing bad projects into liquidation. Given that the private benefit distorts the banker's incentive to be lenient towards bad projects, the bank runs prevent excessive continuation.

Proposition 2 generalizes the observations made so far and gives a full characterization of the equilibrium liquidation function.

Proposition 2 *The liquidation rule $\ell(I;\kappa)$ is:*

1. *non-increasing in κ for each I: $\ell_\kappa \leq 0$*
2. *singly crossing the benchmark at $1-\alpha$ for each I: $\ell(I;1-\alpha) \equiv z^*(I)$*
3. *increasing in I for each κ: $\ell_I > 0$*
4. *submodular in $(I;\kappa)$: $\ell_{I\kappa} \leq 0$*

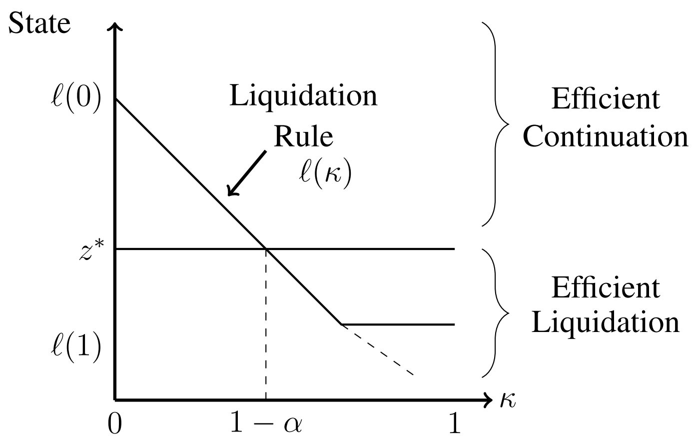


Figure 3: Liquidation Rule for a Given I

The first two items are visualized in Figure 3. Dewatripont and Tirole (1994) obtain a similar result whereby the debt/equity mix can lead to excessive liquidation or continuation. The third item is analogous to $z_I^* > 0$, better projects survive for a higher level of investment. For the fourth item, take $\kappa^L < \kappa^H < 1 - \alpha$ and compute the range of projects that be spared from excessive liquidation: $\ell(I; \kappa^L) - \ell(I; \kappa^H)$. Submodularity implies that the difference is rising in I. As the size of the bank grows, the lack of equity cushion leads to more excessive liquidations. A parallel statement can be made for the benefit of issuing demandable debt when κ is too high.

4 Investment in the Project

In this section I analyze the investment level and the surplus from the project for any given capital structure the banker may choose. I show that whenever $\ell(\kappa)$ deviates from the benchmark z^*, both the investment and the surplus from the project are distorted compared to the benchmark.

Given the liquidation rule $\ell(I; \kappa)$, the surplus from the project captured by the equity-holders is:

$$
\begin{aligned}
S(I;\kappa) &\equiv \int_{\ell(\kappa)}^{\infty} (z\phi(I) - \bar{R}(\kappa))\, dF(z) + F(\ell(\kappa)) \max(\alpha I - (1-\kappa)I, 0) - \kappa I \\
&\equiv \int_{\ell(\kappa)}^{\infty} z\phi(I)\, dF(z) + F(\ell(\kappa))\alpha I - I \qquad (4.1)
\end{aligned}
$$

The foremost two terms in the first line represent the cash flow after debt and the last term is the opportunity cost of equity capital. The second line substitutes the value of debt using households' participation constraint (3.3). After the substitution (4.1) is directly comparable to the benchmark (2.2).

Two features of $S(I;\kappa)$ need to be emphasized. First, the banker's private benefit is excluded. The aim of this section is to isolate how a given capital structure affects the willingness to invest in the project and the way the benchmark is defined implicitly assumes that the social purpose of banking is to maximize the surplus in the real economy[6]. Second, since the payoff to the outside equity investors is proportional to $S(I;\kappa)$ by their fractional ownership $(1-\eta/\kappa)$, they capture rents at any equilibrium while the households break even.

The surplus-maximizing investment is denoted $I^S(\kappa)$:

$$I^S(\kappa) \equiv \arg\max_I S(I;\kappa) \tag{4.2}$$

The surplus $S(I;\kappa)$ coincides with the benchmark only if the liquidation rule $\ell(\kappa)$ corresponds to z^* which occurs at $\kappa = 1-\alpha$ in Figure 3. For any $\kappa \neq 1-\alpha$, there will be an additional term that shifts willingness-to-invest. I study under- and over-capitalized cases separately.

4.1 Under-capitalized Case $\kappa < 1-\alpha$

Decompose S such that $I^S(\kappa)$ solves:

$$\max_I \int_{z^*}^{\infty} z\phi(I)\, dF(z) + F(z^*)\alpha I - I - \int_{z^*}^{\ell(\kappa)} (z\phi(I) - \alpha I)\, dF(z) \tag{4.3}$$

The last term is the deadweight loss triangle in Figure 2a and I^* maximizes the rest of (4.3). $I^S(\kappa)$ deviates from I^* if the marginal deadweight loss

$$\ell_I(\kappa)\Big(\ell(\kappa)\phi(I) - \alpha I\Big) f(\ell(\kappa)) + \int_{z^*}^{\ell(\kappa)} (z\phi'(I) - \alpha) dF(z) \tag{4.4}$$

is non-zero evaluated at I^*.

The first term of (4.4) is unambiguously positive for all I. When investment rises from I' to I'', the type $\ell(I',\kappa) < \ell(I'',\kappa)$ is inefficiently liquidated at $t=1$. This is a disincentive to invest. The sign of the second term is hard to predict. The ambiguity stems from diminishing marginal returns, because while payoff levels are the same $z^*\phi(I) = \alpha I$, the marginals favor liquidation $z^*\phi'(I) < \alpha$. This ambiguity makes it uncertain whether switching from completing the project z to liquidation, $z\phi'(I) - \alpha$, would yield to a total marginal gain or loss. It is possible, in principle, that the banker is content to swap low marginal return projects with α even though his payoff level is going down. Despite this interesting twist, disincentive to invest dominates the second effect for

[6]The benchmark is not defined as a problem of a benevolent social planner who cares about the banker's private benefit. It is possible to take a firmer stance that the banker's private benefit is an uncompensated transfer from the agents in the unmodeled broader economy and not a positive spillover the policymaker/regulator should internalize.

all I regardless of its sign.

Proposition 3 *An under-capitalized bank under-invests in the project:* $I^S(\kappa) < I^*$.

Proposition 3 proves that the deadweight loss triangle in Figure 2a grows with the investment level. The bigger the size of an under-capitalized bank, the more wasteful bank runs become. To see why under-investment occurs in a simple example, consider the following:

Example 1 $Z \sim UNIF(0, p)$ *with* $f(z) = 1/p$ *is log-concave and* $F(z) = z/p$ *is linear.*

I claim that the first positive term in (4.4), the disincentive to invest, is greater than the largest negative value of the integral term. To show that, replace the integral in (4.4) with its lower bound when the integrand is evaluated at z^* for all z on its support. Since $z^*\phi(I) = \alpha I$ by definition and $z_I^*\phi(I) + z^*\phi'(I) = \alpha$ by chain rule, the marginal deadweight loss is bounded below by:

$$\begin{aligned} &\phi(I)\Big[\ell_I(\kappa)(\ell(\kappa) - z^*)f(\ell(\kappa)) - z_I^*\Big(F(\ell(\kappa)) - F(z^*)\Big)\Big] \\ &= \phi(I)\Big(\frac{\ell(\kappa) - z^*}{p}\Big)(\ell_I(\kappa) - z_I^*) > 0 \end{aligned} \tag{4.5}$$

The sign is positive because for $\kappa < 1 - \alpha$, Proposition 2 proves $\ell(\kappa) > z^*$ and $\ell_{I\kappa} < 0$ implies $\ell_I(\kappa) > \ell_I(1 - \alpha) = z_I^*$.

Underinvestment is a typical outcome in asymmetric information models in which the principal forgoes some surplus to make it incentive-compatible for the agent. Here the forgone surplus is the high cash flow projects liquidated at a discount. An under-capitalized bank invests less than the benchmark because the optimal investment has to trade off maximizing the project's surplus with the growing cost of excessive liquidations. Proposition 3 does not imply a stronger prediction that investment always rises in κ. However, since $I^S(\kappa)$ is continuous in κ and bounded above by I^* which is obtained as the limit $I^S(\kappa) \to I^*$ as $\kappa \to 1-\alpha$, the comparative statics must hold locally. A strong comparative static can be made for the surplus from the project.

Corollary 1 *Optimal investment and the resulting surplus from the project are positively correlated to the bank's equity-to-investment ratio.*

1. *There exists a* $\delta > 0$ *such that*

$$\forall \kappa \in [1 - \alpha - \delta, 1 - \alpha] : I^S(\kappa) \nearrow \kappa$$

2. *For all* $\kappa < 1 - \alpha$

$$S(I^S(\kappa); \kappa) \nearrow \kappa$$

4.2 Over-capitalized Case $\kappa > 1 - \alpha$

Throughout this subsection, I make the following assumption:

Assumption 3 *The private benefit satisfies:* $E[z\phi'(B/\alpha)] > 1$

This assumption rules out an unedifying situation in which the private benefit is so large that the banker completes every project and liquidation never takes place at the optimum.

Decompose S such that $I^S(\kappa)$ solves:

$$\max_I \int_{z^*}^{\infty} z\phi(I)\, dF(z) + F(z^*)\alpha I - I - \int_{\ell(\kappa)}^{z^*} (\alpha I - z\phi(I))\, dF(z) \tag{4.6}$$

In this case the last term is the cost of excessive continuation to the outside investors, which is the triangle in Figure 2b. To see how the size of the triangle changes with investment, differentiate it in I to get:

$$-\ell_I(\kappa)\Big(\alpha I - \ell(\kappa)\phi(I)\Big) f(\ell(\kappa)) + \int_{\ell(\kappa)}^{z^*} (\alpha - z\phi'(I))\, dF(z) \tag{4.7}$$

Excessive continuation affects the willingness-to-invest through two channels but contrary to the under-capitalized analogue (4.4), they unambiguously go in opposite directions. The first term of (4.7) captures the level effect that encourages more investment. For an increase from I' to I'', the type $\ell(I', \kappa)$ which is inefficiently completed at $t = 1$ is instead liquidated with the new and higher threshold $\ell(I'', \kappa)$. When the banker is reducing the quality of completed projects, the first response is to invest more to improve the quality of the marginally completed project. The second term of (4.7) captures the disincentive to invest. For an incremental rise in the investment level, the outside investors forgo $\alpha - z\phi'(I) > 0$ on the margin for each $z \in [\ell(\kappa), z^*]$. Figure 4 illustrates the two channels for two distributions.

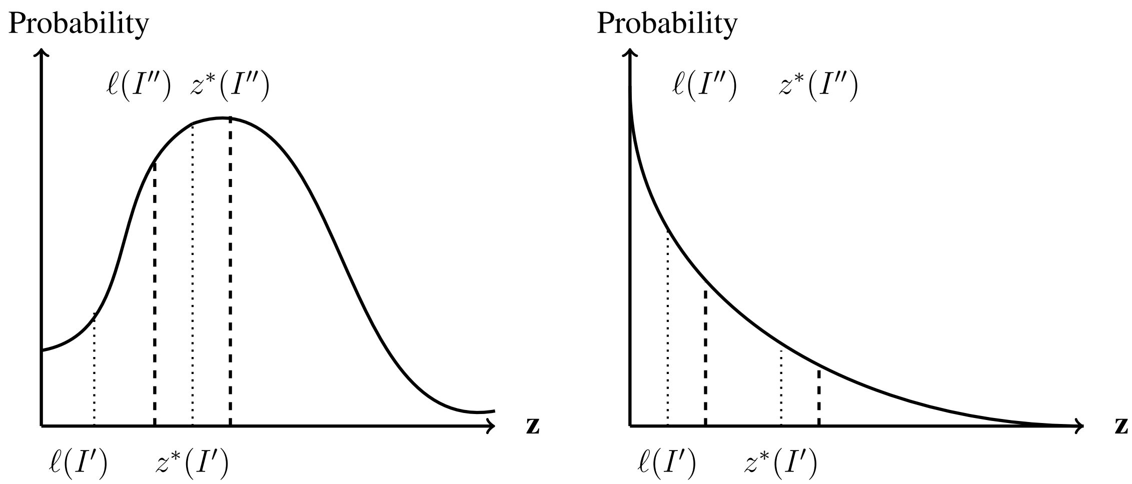
Which effect dominates at the optimum? Unlike the under-capitalized case where the dead-weight loss always grows with investment, here the answer depends on both the distribution of Z and the level of I. I prove in the next proposition that the cost of excessive continuation is increasing [decreasing] in I for small [large] I .

Proposition 4 *For each $\kappa > 1 - \alpha$ there exists a threshold $T(\kappa)$ such that*

1. *If $T(\kappa) > I^*$, there is under-investment: $I^S(\kappa) < I^*$*

2. *If $T(\kappa) < I^*$, there is over-investment: $I^S(\kappa) > I^*$*

It is possible to make accurate predictions for specific choices of Z. Recall $Z \sim UNIF(0, p)$ in Example 1. Following exactly the same steps described before leads to an increasing cost of



(a) Truncated Gaussian (b) Exponential

Figure 4: Excessive Continuation and Investment $I' < I''$

excessive continuation. Therefore $T(\kappa) \to \infty$ and there is under-investment for any κ. Consider a second example:

Example 2 *$Z \sim EXP(p)$ with $f(z) = pe^{-pz}$ decreasing and log-concave, $F(z) = 1 - e^{-pz}$ increasing and concave.*

I claim that as $\kappa \to 1$ so that the bank is almost exclusively financed by equity, there is over-investment. Weigh the largest disincentive to invest to the positive level effect by replacing the integral in (4.7) by the integrand evaluated at $\ell(1)$ for all z on its support. The upper bound on (4.7) is

$$\ell_I(1)\phi(I)\Big[\Big(F(z^*) - F(\ell(1))\Big) - (z^* - \ell(1))f(\ell(1))\Big] \leq 0 \tag{4.8}$$

Since F is concave, the value at a higher point is bounded above by its first-order Taylor approximation at a lower point, $F(z^*) \leq F(\ell(1)) + (z^* - \ell(1))f(\ell(1))$, which proves the cost of excessive continuation is shrinking in I. This example corresponds to $T(\kappa) = B/\alpha < I^*$ by Assumption 3 for large enough κ.

It is not possible to accurately predict under- and over-investment when the probability density of Z is non-monotone, such as Truncated Gaussian in Figure 4a, and κ is relatively low so that the optimum occurs on the falling segment of Figure 3. The only unambiguous comparative static in κ is the following:

Corollary 2 *The surplus from the project $S(I^S(\kappa);\kappa)$ decreases in κ.*

The intuition behind Corollary 2 is that when the demandable debt is risk-free, it plays a desirable disciplining role on the banker to cap excessive continuation. Therefore, limiting debt issuance does not increase the surplus in any way because every project liquidated due to insolvency is a low cash flow project that is also liquidated at the benchmark.

5 The Banker's Optimal Capital Structure

In this last section I characterize the investment level and the capital structure of an unregulated equilibrium taking the limit on the banker's internal funding η as given. I show that if $\eta < 1-\alpha$, the banker's internal funding is less than necessary to maximize the surplus from the project, the banker does not issue enough outside equity to close the gap. Therefore, both the investment and the surplus from the project increases if the policymaker requires an equity-to-investment ratio of $1-\alpha$.

The banker's utility for a given $(I;\eta,\kappa)$ is

$$U(I;\eta,\kappa) \equiv \frac{\eta}{\kappa}\Big(S(I;\kappa) + (1-F(\ell(\kappa))B\Big) = \frac{\eta}{\kappa}V(I;\kappa) \tag{5.1}$$

I have computed the surplus all equity-holders capture in (4.1) denoted by $S(I;\kappa)$. The monetary payoff to the banker's inside equity is proportional to his fractional ownership η/κ. The second term in (5.1) is the expected private benefits which are also proportional to the ownership by (2.4).

I refer to V as the banker's rent. The banker's utility is decomposed in a way that the investment maximizes $V(I;\kappa)$ independent of η. Denote the utility-maximizing investment by $I^U(\kappa)$.

Proposition 5 *The banker invests less than the level that maximizes the surplus from the project* $\forall\kappa : I^U(\kappa) < I^S(\kappa)$

Higher investment increases the marginally completed project, $\ell_I > 0$ for every κ, making it less likely to capture the private benefit ex ante. This intuition would be true even if the private benefit had positive returns to scale in investment. In that case, the expected loss of private benefit from higher investment dominates whatever the marginal increase in private benefit might be for large I. I explore this case in the Appendix.

Evaluate the banker's utility (5.1) at the optimal investment $I^U(\kappa)$ so that the only choice variable is κ:

$$U(I^U(\kappa);\eta,\kappa) = \frac{\eta}{\kappa}V(I^U(\kappa);\kappa) \equiv \frac{\eta}{\kappa}\hat{V}(\kappa) \tag{5.2}$$

I assume $\hat{V}(\kappa) \geq 0$ for all κ so that the banker's decision is non-trivial. I provide a key property of $\hat{V}(\kappa)$ to facilitate the analysis.

Lemma 2 *The banker's rent $\hat{V}(\kappa)$ is weakly increasing in κ. There exists a $K \in (1-\alpha, 1)$ with*

$$\hat{V}_\kappa(K) = 0 \tag{5.3}$$

such that $\hat{V}(\kappa)$ is constant for all $\kappa \geq K$.

Lemma 2 yields three intermediate results. First, if the banker had no limits on internal funding, $\eta = 1$, then he would never issue outside equity but he might issue some demandable debt. Let $K \in (1-\alpha, 1)$ denote the threshold above which the liquidation rule is constant in Figure 3. The debt does not interfere with the banker's liquidation decision beyond a threshold because even if the banker is insolvent, the cash flow from the project is so low that the banker is willing to liquidate the project despite the private benefit. Second, when the banker's inside equity is constrained $\eta \leq K$, he would use all of it before considering external funding. Third, if the banker issues outside equity when $\eta < K$, the total equity never exceeds K because the rent is constant but the ownership is reduced.

Using the implications of Lemma 6 the banker chooses $\kappa^U \in [\eta, K]$ to maximize (5.1). Choosing $\kappa^U = \eta$ means that the banker does not issue outside equity. The choice critically depends on the size of the maximal rent. The banker's rent is *high* if $\hat{V}(K) > K$ or *low* if $\hat{V}(K) \leq K$. Note that B can be chosen large enough to guarantee that the banker's rent is high.

Proposition 6 *If the banker's rent is high, $\hat{V}(K) > K$, then the optimal capital structure follows a pecking order. Inside equity is preferred to any external funding. Then demandable debt is preferred to outside equity if external financing is used.*

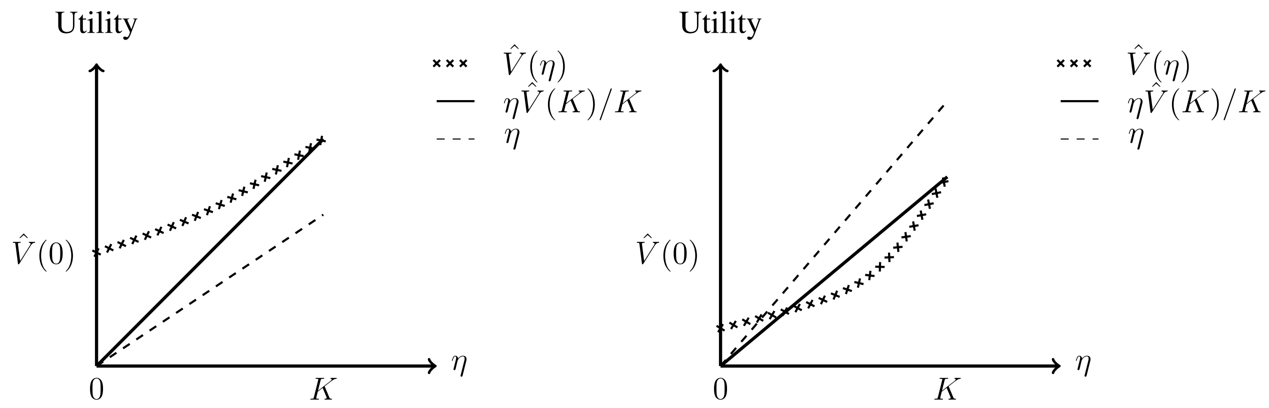
The proof of this Proposition is simple and instructive so it is presented here. The banker compares three options: (i) at the lower bound the banker leverages the inside equity by demandable debt; (ii) at the upper bound the banker capitalizes the bank up to K to capture the maximum rent; and (iii) a potential interior solution. The utility from the options (i) and (ii) are respectively $\hat{V}(\eta)$ and $\eta\hat{V}(K)/K$. For the third option, differentiate the left-hand side of (5.1) in κ:

$$\left(\frac{\eta}{\kappa}\hat{V}(\kappa)\right)' = \frac{\eta}{\kappa}\left(\hat{V}_\kappa - \frac{\hat{V}(\kappa)}{\kappa}\right) \tag{5.4}$$

The banker's utility is eventually declining since $\hat{V}_\kappa(K) = 0$. Suppose there exists a local interior maximum k' such that

$$\hat{V}_\kappa(k') = \frac{\hat{V}(k')}{k'} \iff (\ln \hat{V}(\kappa))'\Big|_{\kappa=k'} = (\ln \kappa)'\Big|_{\kappa=k'} \tag{5.5}$$

or simply $\hat{V}(k') = k'$. The banker's utility at the local maximum should be η. By construction this



(a) The banker's rent is high: $\hat{V}(K) > K$ (b) The banker's rent is low: $\hat{V}(K) \leq K$

Figure 5: The Banker's Utility for $\eta < K$

utility should be larger than the utility at K where it is negatively sloped. However, $\frac{\eta}{K}\hat{V}(K) > \eta$ is a contradiction. Therefore, (5.4) must be negative for all κ and the unique solution is the lower bound $\kappa^U = \eta$.

Figure 5 plots the banker's utility for each of the three options as a function of η. The left panel covers the high-rent case in Proposition 6. I have proven the inequality $\hat{V}(\eta)/\eta > \hat{V}(K)/K$ for any $\eta < K$ provided that $\hat{V}(K)/K > 1$. Therefore, the banker's options can be ranked and not issuing outside equity dominates all options. I illustrate the low-rent case in Figure 5b. Even if the equilibrium surplus from the project and the private benefit are both small, the banker still does not issue outside equity if η is low enough.

There are two reasons behind the banker's preference against issuing outside equity even if it increases the investment in the project and likelihood of its survival. One is because the outside investors capture some of the new surplus and the other is because the private benefit is reduced. I isolate the role played by each in the next two subsections. A simple way to illustrate the wedge between what is socially desirable and what is privately optimal is by writing the term that determines the sign of (5.4) as

$$\epsilon(\kappa) \equiv \frac{\hat{V}_\kappa(\kappa)\kappa}{\hat{V}(\kappa)} \tag{5.6}$$

I interpret $\epsilon(\kappa)$ as the point elasticity of the banker's rent to new equity capital. The banker computes whether the increase in his rent is more than enough to compensate the loss of the claim on cash flows. If (5.4) is negative, the elasticity $\epsilon(\kappa)$ is less than unity. When $0 < \epsilon(\kappa) < 1$, outside equity is privately costly for the banker and he does not want to issue any. The policymaker wants more equity capital because he is concerned only with the surplus from the project and indifferent about how the surplus is distributed between the banker and the outside equity investors.

5.1 Perfect Competition among Outside Equity Investors

In this subsection I relax the assumption that the outside equity earn a higher return than the demandable debt. I show that the equilibrium can still exhibit under-capitalization and under-investment even though the pecking order result in Proposition 6 does not necessarily hold.

In the baseline model if the banker owns η/κ of the bank's equity at $t = 0$, he claims η/κ fraction of the cash flows after repaying the debt. This way the banker offers the same excess return per share that the inside equity earns to the outside equity investors. Suppose instead that the outside equity investors are perfectly competitive and the banker offers a new fractional claim on cash flows $\mathbb{f}$ for the outside investors to break-even:

$$\mathbb{f} \times \Big(\int_{\ell(\kappa)}^{\infty} z\phi(I) dF(z) + F(\ell(\kappa))\alpha I - (1-\kappa)I \Big) = (\kappa - \eta)I \tag{5.7}$$

The right-hand side of (5.7) is the opportunity cost and the parenthetical term is expected cash flows after debt computed earlier in (4.1). I retain the assumption that the private benefit is proportional to the banker's claim on cash flows, therefore $1 - \mathbb{f}$ replaces η/κ in (2.4). With the new specification the banker's utility maximization differs from (5.1):

$$\max_{I,\kappa} U(I;\eta,\kappa) \equiv S(I;\kappa) + (1-\mathbb{f})(1-F(\ell(\kappa))B \tag{5.8}$$

If all the capital markets are competitive, the banker captures the entire surplus from the project $S(I;\kappa)$ for himself. The only reason a different $(I;\kappa)$ is chosen is because of the private benefits. Although $\mathbb{f}$ is a complicated function of $(I;\kappa)$, it is possible to prove an analogue of Proposition 5 that the banker wants to invest less than the surplus-maximizing level $I^S(\kappa)$ for each κ. However, the optimal κ^U is no longer tractable so I provide a sufficient condition instead.

Lemma 3 *If the banker's internal funds are scarce $\eta < 1-\alpha$, both the investment level I^U and the equity-to-investment ratio κ^U that solve (5.8) are less than the benchmark $(I^*, 1-\alpha)$ if evaluated at $I^U(\kappa)$, the outside investors' claim on cash flows is not too small:*

$$\lim_{\kappa \to 1-\alpha} \mathbb{f} > \lim_{\kappa \to 1-\alpha} \Big(-\ell_\kappa \frac{f(\ell(\kappa))}{1-F(\ell(\kappa))}(\kappa - \eta) \Big) \tag{5.9}$$

Since the banker extracts more surplus from the project by issuing outside equity, the only downside is how much ownership he has to give up that reduces the private benefits. The right-hand side of (5.9) is the threshold above which the private benefit is more valuable to the banker than the increase in the project's surplus. Therefore, even if the banker has some incentive to issue outside equity, he does not enough have incentive to capitalize up to $1-\alpha$ provided (5.9) is satisfied.

5.2 No Private Benefits

In this subsection I analyze the baseline model without the private benefits by setting $B=0$. The private benefits create a friction at both $t=0$ and $t=1$. Without the private benefits, there is no longer excessive continuation at $t=1$ when $\kappa > 1-\alpha$. Therefore, 100% equity capital gets the same benchmark outcome as $1-\alpha$. Then the $\eta \geq 1-\alpha$ case is uninteresting because the banker's preferences are perfectly aligned with the policymaker and he does not face an external funding constraint. I analyze $\eta < 1-\alpha$ case and show that the equilibrium again features an under-capitalized bank with low level of investment.

The banker's utility (5.1) is modified to

$$U(I;\eta,\kappa) \equiv \frac{\eta}{\kappa} S(I;\kappa) \tag{5.10}$$

The banker receives a monetary payoff from the project's surplus proportional to his ownership $t=0$ and the outside equity investors capture the rest. Since $B=0$, Proposition 5 does not apply and the banker's choice of investment corresponds to $I^S(\kappa)$ modeled in Section 4. Evaluated at $I^S(\kappa)$, Corollary 1 shows that the equilibrium surplus $S(I^S(\kappa);\kappa)$ is rising in κ until $1-\alpha$ and constant thereafter, as argued in the opening paragraph of this subsection. Therefore, the banker chooses $\kappa \in [\eta, 1-\alpha]$ to maximize (5.10) evaluated at $I^S(\kappa)$. I get an analogue of Proposition 6:

Lemma 4 *If the banker's internal funds are scarce $\eta < 1-\alpha$, then both the investment level and the equity-to-investment ratio that solve (5.10) are less than the benchmark* $(I^*, 1-\alpha)$.

The derivative of (5.10) in κ evaluated at $I^S(\kappa)$ using the Envelope Theorem is

$$\frac{\eta}{\kappa}\Big(S_\kappa(I^S(\kappa);\kappa) - \frac{S(I^S(\kappa);\kappa)}{\kappa}\Big) \tag{5.11}$$

It suffices to show that as $\kappa \to 1-\alpha$, the marginal surplus is zero at the maximum so that the optimal choice κ^U must satisfy $\eta \leq \kappa^U < 1-\alpha$. Proposition 3 proves $I^S(\kappa^U) < I^*$ and completes the proof. It is possible to predict pecking order, $\kappa^U = \eta$ and no outside equity, if the maximum surplus from the project $S(I^*, 1-\alpha)$ is larger than $1-\alpha$. This is the analogue of $\hat{V}(K) > K$ in the statement of Proposition 6.

6 Conclusion

I have developed a model of bank capital structure and investment to evaluate the commonly held view that reducing the risk of bank runs with more equity capital trades off the volume of lending to the real economy. The main results of this paper suggest the opposite conclusion: a minimum equity capital requirement can make the bank safer and at the same time, create an incentive to invest more.

In his survey of empirical evidence, Thakor (2014) writes that "in the cross-section of banks, higher capital is associated with higher lending, higher liquidity creation, higher bank values and higher probabilities of surviving crisis". For example, Berrospide and Edge (2010) and Kapan and Minoui (2013) find that well-capitalized banks with a stronger ability to buffer losses cut lending less in response to a negative shock. My model is consistent with these findings, except for liquidity creation left outside the model, and can explain why the banks do not increase their equity capital by themselves.

How should the policymaker set the requirement? This paper offers two insights. The requirement depends only on α, the recovery rate of investment but not size-related parameters η or I, or the distribution F. If the parameter α is interpreted as a measure of asset liquidity, then the more liquid a bank's portfolio is, the higher the leverage the bank can sustain. Recently Brunnermeier et al. (2014) develop a measure of mismatch between the asset liquidity and the funding liquidity as an alternative prudential tool. My model suggests that this is a more promising approach than the standard risk-weights. Second, the policymaker can monitor the riskiness of the bank's short-term debt instead of tracking α. At the benchmark of my model, the debt is risk-free. Market signals such as CDS spread on the individual bank's debt might reveal how far that bank is away from the benchmark.

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A Appendix

Lemma 5 $\forall \bar{z} : \partial E(Z - \bar{z}|Z > \bar{z})/\partial \bar{z} = h(\bar{z})E(Z - \bar{z}|Z > \bar{z}) - 1 \leq 0$ *where* h *is the hazard function* $f/(1-F)$ *of* Z*.*

Proof. $E(Z - \bar{z}|z > \bar{z})$ is known as the mean residual lifetime function. Bagnoli and Bergstorm (2005) Theorem 6 proves that a random variable with an increasing hazard rate has a decreasing mean residual lifetime. Assumption 2 guarantees that the hazard rate is increasing. ∎

Proof of Proposition 1. Divide the left hand side of (2.3) by $1 - F(z^*)$; the probability that the project is completed. Rearrange terms to get:

$$\phi'(I)E(Z|Z > z^*) - 1 - \frac{1}{1 - F(z^*)}(1 - \alpha)F(z^*) \tag{A.1}$$

(A.1) has the same sign as the left-hand side of (2.3) and I prove that (A.1) singly crosses 0 once from above at an interior I^* which proves that (2.2) is quasi-concave I with a single peak at I^*. (A.1) is interesting on its own. The term $\phi'(I)E(Z|Z > z^*) - 1$ is the marginal return from investing a dollar in the project at $t = 0$ conditional on the project being completed. The remainder term is the conditional marginal cost.

Note that $z^* = \alpha I/\phi(I)$ is an increasing function of I by Assumption 1 with limits $z^* \to 0$ as $I \to 0$ and ∞ as $I \to \infty$. Evaluate (A.1) as $I \to 0$. Marginal return goes to infinity and the marginal cost goes to zero and thus (A.1) goes to infinity as $I \to 0$. To see the limit $I \to \infty$, use the inequality in Lemma 5 evaluated at z^* and multiply both sides by $\phi'(I) > 0$. Using $z^*\phi'(I) < \alpha < 1$, obtain an upper bound on the marginal return

$$\phi'(I)E(z|z > z^*) - 1 < \frac{\phi'(I)}{h(z^*)} \tag{A.2}$$

where h is the hazard function.

This upper bound is a decreasing function that converges to 0 as $I \to \infty$. The marginal cost goes to ∞ as F goes to 1. So (A.1) goes to $-\infty$ as $I \to \infty$. All terms in the left-hand side of (A.1) are continuous in I so (A.1) crosses 0 at least once. Let I^* denote that point. To establish the uniqueness of I^*, I prove that both marginal return and cost are monotone.

The marginal cost $F(z^*)/(1 - F(z^*))$ is increasing in I: by chain rule F is increasing in z^* and z^* is increasing in I. The slope of marginal return is given by

$$\phi''(I)E(Z|Z > z^*) + \phi'(I)\frac{\partial E(Z|Z > z^*)}{\partial z^*}z^*_I \tag{A.3}$$

The first term is negative and the second is positive so the sign is ambiguous. However, Lemma 5 implies that $0 \leq \partial E(Z|Z > z^*)/\partial z^* \leq 1$ and by chain rule $z_I^* = (\alpha - z^*\phi'(I))/\phi(I) > 0$. Using these two inequalities, (A.3) is bounded from above by

$$
\begin{aligned}
&\phi''(I)E(Z|Z > z^*) - z^* \frac{\phi'(I)^2}{\phi(I)} + \alpha \frac{\phi'(I)}{\phi(I)} \\
&< z^* \left(\phi''(I) - \frac{\phi'(I)^2}{\phi(I)}\right) + \alpha \frac{\phi'(I)}{\phi(I)} \\
&= z^*\phi(I) \left(\frac{\phi'(I)}{\phi(I)}\right)' + \alpha \frac{\phi'(I)}{\phi(I)} = \alpha \left\{ \left(\frac{\phi'(I)}{\phi(I)}\right)' I + \frac{\phi'(I)}{\phi(I)} \right\} \qquad \text{(A.4)}
\end{aligned}
$$

By Assumption 1 $\phi'(I)$ and $1/\phi(I)$ are decreasing convex functions so their product is also decreasing convex. Then $\phi'(I)/\phi(I)$ satisfies

$$
\left| \left(\frac{\phi'(I)}{\phi(I)}\right)' \right| \geq \frac{1}{I} \left(\frac{\phi'(I)}{\phi(I)}\right)
$$

That is, the slope is larger than the average in absolute value. Using this inequality the upper bound (A.4) is non-positive and therefore, the conditional marginal return is decreasing. This suffices to conclude that (A.1) single crosses 0 from above at I^*. Since their signs are the same, (2.3) is uniquely satisfied at I^* as well. ■

Proof of Lemma 1. Fix any $I > 0$, the dependence of functions on I is suppressed throughout. The proof is similar to Townsend (1979) and Gale and Hellwig (1985). Incentive-compatibility can be stated in its most general form as for any z

$$
\begin{aligned}
z = \frac{\eta}{\kappa} \times \arg\max_y \Big[&\mathbb{1}_{\mathcal{B}(\kappa)}(y) \max\big((1 - \alpha - \kappa)I, 0\big) \\
&+ (1 - \mathbb{1}_{\mathcal{B}(\kappa)}(y)) \max\big(z\phi(I) - R(y;\kappa) + B, (1 - \alpha - \kappa)I, 0\big)\Big] \qquad \text{(A.5)}
\end{aligned}
$$

The first term captures the liquidation payoff to the banker and the second term says the banker receives the maximum of completion and liquidation payoffs if there is no run.

Suppose that $\kappa < 1 - \alpha$ so the equity gets nothing in liquidation. Since $B > 0$ and $z\phi(I) \geq R(z;\kappa)$, (A.5) can be simplified to

$$
z = \frac{\eta}{\kappa} \times \arg\max_y (1 - \mathbb{1}_{\mathcal{B}(\kappa)}(y))(z\phi(I) - R(y;\kappa) + B)
$$

For any $z, z' \notin \mathcal{B}(\kappa)$, it must be that $R(z;\kappa) = R(z';\kappa) = \bar{R}(\kappa) \leq \inf_{y \notin \mathcal{B}(\kappa)} y\phi(I)$. There is no incentive for $z \notin \mathcal{B}(\kappa)$ to pretend to be $y \in \mathcal{B}(\kappa)$ or for a $z \in \mathcal{B}(\kappa)$ to report another $y \in \mathcal{B}(\kappa)$.

However, $z \in \mathcal{B}(\kappa)$ may pretend to be $y \notin \mathcal{B}(\kappa)$. As Footnote 3 explains, there is a non-pecuniary penalty at least as large as B in case the banker reports y which he later fails to repay $R(y;\kappa)$ at $t = 2$. So for this deviation to be unprofitable, it has to be that $\bar{R}_\kappa \geq \sup_{y \in \mathcal{B}(\kappa)} y\phi(I)$. Putting all together, $\bar{R}(\kappa) \equiv \ell(\kappa)\phi(I)$ such that $\mathcal{B}(\kappa) = [0, \ell(\kappa)]$.

Consider now $\alpha I > (1-\kappa)I$ or $\kappa > 1-\alpha$. Here there are two sub-cases. First suppose that $\alpha I > (1-\kappa)I > \alpha I - B$. The private benefit is so large that whenever $z \notin \mathcal{B}(\kappa)$, the banker is better off completing the project. In this sub-case the proof is identical to the one above; the only difference is that the liquidation payoff is shifted from zero to $(1-\alpha-\kappa)I > 0$. Last, suppose that $\alpha I > \alpha I - B > (1-\kappa)I$. The steps behind constant repayment $\bar{R}(\kappa)$ are identical but in this sub-case there are projects $z \notin \mathcal{B}(\kappa)$ that the banker does not want to complete. The run threshold is lower than the liquidation threshold $\ell(\kappa)$ as the latter is defined $\ell(\kappa)\phi(I) - \bar{R}(\kappa) + B \equiv \alpha I - (1-\kappa)I$ and therefore $\ell(\kappa)\phi(I) > \bar{R}(\kappa)$.

Pinning down $\ell(\kappa)$ is simpler in either sub-case of $\kappa > 1-\alpha$. The equation

$$F(\ell(\kappa))(1-\kappa)I + (1 - F(\ell(\kappa)))\bar{R}(\kappa) = (1-\kappa)I$$

holds when $\bar{R}(\kappa) = (1-\kappa)I$. If $(1-\kappa)I > \alpha I - B$, then $\ell(\kappa)\phi(I) = (1-\kappa)I$. Otherwise $\ell(\kappa)\phi(I) = \alpha I - B$ for all κ. Put together $\ell(\kappa)\phi(I) = \max((1-\kappa)I, \alpha I - B)$.

A Note on the General Functional Form for Private Benefits

Suppose instead the private benefit has a general functional form $B(I, \eta, \kappa)$. As long as B function is positive, the proof is identical for $\kappa \leq 1-\alpha$. In $\kappa > 1-\alpha$ case the $\bar{R}(\kappa) = (1-\kappa)I$ result is also identical; if the banker can repay the face value in both liquidation and completion, he never compensates the households more regardless of how his private benefits work. The only change would be the liquidation rule is replaced by

$$\kappa > 1-\alpha : \ell(\kappa) = \max\left((1-\kappa)I, \alpha I - \frac{\kappa}{\eta}B(I;\kappa,\eta)\right) \tag{A.6}$$

■

Proof of Proposition 2. I prove $\ell_\kappa \leq 0, \ell_I > 0, \ell_{I\kappa} \leq 0$ for $\kappa < 1-\alpha$ first and then repeat the proofs for $\kappa > 1-\alpha$. The single crossing at $1-\alpha$ follows immediately from the first assertion together with $\ell(0) > z^* > \ell(1)$ which is proved in the text.

$\bar{R}(\kappa)$ and $\ell(\kappa)$ are determined by households' participation constraint (3.3) in the text. Define $H(\ell) : [z^*, \infty) \to \mathcal{R}$ by

$$H(\ell) \equiv F(\ell)\alpha I + (1 - F(\ell))\ell\phi(I) - (1-\kappa)I \tag{A.7}$$

and $H(\ell(\kappa)) = 0$ corresponds to the equilibrium solution. The lower bound is z^* because the lowest the banker can pay out to the households is αI which is less than what he owes $(1-\kappa)I$. So at the lower bound $H(z^*) < 0$. Differentiate $H(\ell)$ to get

$$H'(\ell) = (\phi(I) - (\ell\phi(I) - \alpha I)h(\ell))(1 - F(\ell)) \tag{A.8}$$

The sign of $H'(\ell)$ is determined by $\phi(I) - (\ell\phi(I) - \alpha I)h(\ell)$. Since $z^*\phi(I) = \alpha I$, $\lim_{\ell \to z^*} H'(\ell) > 0$. The hazard rate $h(\ell)$ is increasing by Assumption 2, so $\phi(I) - (\ell\phi(I) - \alpha I)h(\ell)$ is a decreasing function of ℓ with limit $-\infty$ as ℓ goes to infinity. Therefore there is a unique interior maximum of H at $\ell^{\max} > z^*$ such that $H'(\ell^{\max}) = 0$. The banker can never raise capital from households if $H(\ell^{\max}) \leq 0$. Otherwise there exists a unique $\ell(\kappa) < \ell^{max}$ such that $H(\ell(\kappa)) = 0$ at which H is positively sloped $H'(\ell(\kappa)) > 0$.

Use the implicit function theorem to get

$$\ell_\kappa = -\frac{I}{H'(\ell(\kappa))} < 0 \tag{A.9}$$

and

$$\ell_I = -\frac{F(\ell(\kappa))\alpha + (1 - F(\ell(\kappa)))\ell(\kappa)\phi'(I) - (1-\kappa)}{H'(\ell(\kappa))} \tag{A.10}$$

where $H'(\ell(\kappa)) > 0$. To sign the nominator of (A.10), first divide both sides of (3.3) by $I > 0$ to write $F(\ell)\alpha + (1 - F(\ell))\ell\phi(I)/I = (1-\kappa)$. By strict concavity of ϕ, $\phi(I)/I > \phi'(I)$. This suffices to show that the nominator is negative and thus the sign of (A.10) positive.

To prove submodularity, differentiate (A.9) to get:

$$\ell_{I\kappa} = -\Big[\frac{1}{H'(\ell(\kappa))} - \frac{H''(\ell(\kappa))\ell_I I}{H'^2(\ell(\kappa))}\Big] \tag{A.11}$$

The only previously unstudied term in (A.11) is $H''(\ell(\kappa))$. $H'(\ell)$ is defined in (A.8) as a product of two decreasing functions. The second is always positive, the first decreasing function alternates sign but I have proved that it is positive at the optimum $\ell(\kappa)$. This suffices to argue that $H''(\ell(\kappa)) < 0$. All other terms in the square bracket of (A.11) are positive and therefore the overall sign is negative.

Proving these assertions for $\kappa > 1 - \alpha$ is easier since $\ell(\kappa)\phi(I) = \max((1-\kappa)I, \alpha I - B)$ gives an explicit function in $(I;\kappa)$. If $(1-\kappa)I > \alpha I - B$ then $\ell_\kappa = -I/\phi(I) < 0$. In the other sub-case, $\ell(\kappa)\phi(I) = \alpha I - B$ so ℓ is independent of κ and $\ell_\kappa = 0$.

In either sub-case $\ell(\kappa)$ is increasing in I since $I/\phi(I)$ is increasing by strict concavity of $\phi(I)$. Finally if $(1-\kappa)I > \alpha I - B$, then $\ell_{I\kappa} = -\Big(\frac{I}{\phi(I)}\Big)' < 0$ and in the other subcase it is trivially submodular in $(I;\kappa)$ as it does not depend on κ.

A Note on the General Functional Form for Private Benefits

I derived the liquidation rule for a general functional form for private benefits in (A.6). The only change in results is in $\kappa > 1-\alpha$ case. Add a restriction

$$\frac{\partial}{\partial \kappa}\Big(\kappa B(I;\eta,\kappa)\Big) = \kappa B_\kappa + B \geq 0 \tag{A.12}$$

which is an elasticity condition. If in addition $\lim_{\kappa\to 1}(\alpha I - \frac{\kappa}{\eta}B(I;\kappa,\eta)) > 0$, then once more there exists a unique $\kappa' > 1-\alpha$ such that the liquidation rule does not depend on B function for $\kappa \leq \kappa'$ and I can only focus on $\kappa > \kappa'$. Under the premise of (A.12), $\ell_\kappa \leq 0$. The restriction for $\ell_I \geq 0$ is another elasticity condition

$$\frac{\partial}{\partial I}\Big(\frac{B(I;\eta,\kappa)}{\phi(I)}\Big) \leq 0 \Rightarrow \frac{B_I}{B} \leq \frac{\phi'(I)}{\phi(I)} \tag{A.13}$$

For $\ell_{I\kappa} \leq 0$ to go through, a third and the last restriction is

$$B_{I\kappa}B \leq B_\kappa B_I \tag{A.14}$$

The baseline form (2.4) satisfies all the restrictions: (A.12) always holds with equality and $B_I = B_{I\kappa} = 0$. ■

Proof of Proposition 3. The arguments of functions ℓ and z^* are suppressed throughout the proof. The optimality condition is

$$\int_{z^*}^{\infty} z\phi'(I)dF(z) + F(z^*)\,\alpha - 1 - \int_{z^*}^{\ell}(z\phi'(I)-\alpha)dF(z) - \ell_I(\ell\phi(I)-\alpha I)f(\ell) = 0 \tag{A.15}$$

The first three terms correspond to (2.3) and Proposition 1 shows that its limit is ∞ as $I \to 0$. The last two terms are the marginal loss. ℓ is implicitly defined by (3.3). Both ℓ and z^* go to 0 for $I \to 0$ and the marginal loss disappears. Thus the left hand side of (A.27) evaluated as $I \to 0$ is ∞.

The last term of the marginal loss is unambiguously positive but the sign of the integral component is unknown in general. I now prove that it is bounded below by zero. A lower bound is proposed in the text (4.5). This lower bound requires a precise relationship between ℓ_I to z^*_I that I obtain from (3.3).

$$\begin{aligned}F(\ell)\alpha I + (1-F(\ell))\ell\phi(I) &= (1-\kappa)I \\ F(\ell)z^* + (1-F(\ell))\ell &= (1-\kappa)\frac{I}{\phi(I)} = \frac{1-\kappa}{\alpha}z^* \\ F(\ell)z_I^* + (1-F(\ell))\ell_I - \ell_I(\ell - z^*)f(\ell) &= \frac{1-\kappa}{\alpha}z_I^* \end{aligned} \tag{A.16}$$

Now rewrite the lower bound as

$$\phi(I)\Big[-z_I^*(F(\ell)-F(z^*)) + \ell_I(\ell-z^*)f(\ell)\Big] \tag{A.17}$$

$$= \phi(I)\Big[(1-F(\ell))\ell_I - \Big(\frac{1-\kappa}{\alpha} - F(z^*)\Big)z_I^*\Big] \tag{A.18}$$

and thus the lower bound is non-negative if

$$\frac{\ell_I}{z_I^*} \geq \frac{(1-\kappa)/\alpha - F(z^*)}{1-F(\ell)} \tag{A.19}$$

Notice that while Proposition 2 proves that the ratio of two derivatives is larger than unity, the right-hand-side of (A.19) is strictly larger than unity. The exact ratio can be inferred from (A.16).

$$\frac{\ell_I}{z_I^*} = \frac{(1-\kappa)/\alpha - F(\ell)}{1-F(\ell)-(\ell-z^*)f(\ell)} = \frac{(1-\kappa)/\alpha - F(z^*) - (F(\ell)-F(z^*))}{1-F(\ell)-(\ell-z^*)f(\ell)} \tag{A.20}$$

To simplify notation, let the letters a through d denote:

$$a = (1-\kappa)/\alpha - F(z^*)\ ,\ b = 1-F(\ell)\ ,\ c = F(\ell)-F(z^*)\ ,\ d = (\ell - z^*)f(\ell)$$

All letters are positive. I now claim that

$$\frac{\ell_I}{z_I^*} = \frac{a-c}{b-d} > \frac{a}{b} \tag{A.21}$$

which is the sufficient condition (A.19) to prove that the lower bound is non-negative. (A.21) is equivalent to $a/b \geq c/d$. There are two cases for F being convex/concave on $[z^*, \ell]$. If F is convex on that interval:

$$\frac{a}{b} = \frac{(1-\kappa)/\alpha - F(z^*)}{1-F(\ell)} > \frac{1-F(z^*)}{1-F(\ell)} > 1 \geq \frac{F(\ell)-F(z^*)}{(\ell-z^*)f(\ell)} = \frac{c}{d} \tag{A.22}$$

As observed at the beginning of the proof, the lower bound is easily signed in the convex portion of F , if any. If F is concave on $[z^*, \ell]$, then

$$\frac{a}{b} = \frac{(1-\kappa)/\alpha - F(z^*)}{1-F(\ell)} > \frac{1-F(z^*)}{1-F(\ell)} \geq \frac{f(z^*)}{f(\ell)} \geq \frac{F(\ell)-F(z^*)}{(\ell - z^*)f(\ell)} = \frac{c}{d} \tag{A.23}$$

The middle inequality is increasing hazard rate and the last one is the concavity of F. Lastly, suppose that F is convex at z^* and concave at ℓ such that $F(\ell)-F(z^*) > (\ell - z^*)\max(f(\ell), f(z^*))$. Then there must exists $\bar{z} \in (z^*, \ell)$ such that $F(\ell) - F(z^*) = (\bar{z} - z^*)f(\bar{z})$ and

$$\frac{a}{b} > \frac{1-F(z^*)}{1-F(\ell)} > \frac{1-F(\bar{z})}{1-F(\ell)} \geq \frac{f(\bar{z})}{f(\ell)} > \frac{(\bar{z}-z^*)f(\bar{z})}{(\ell - z^*)f(\ell)} = \frac{F(\ell)-F(z^*)}{(\ell - z^*)f(\ell)} = \frac{c}{d} \tag{A.24}$$

This exhausts all cases an arbitrary investment level I can map onto a convex-concave distribution function F via functions ℓ and z^* and in all of them the marginal loss is bounded above zero. So the left-hand-side (A.15) evaluated at I^* must be negative and this suffices to argue that (A.15) is satisfied at a lower $I^S(\kappa) < I^*$, concluding the proof. ■

Proof of Corollary 1. The first part of the corollary is proven in the text. The second part is an application of Envelope Theorem. The function $S(I^S(\kappa); \kappa)$ is a well-defined continuous function of κ by Proposition 3. Differentiate in κ to get

$$S_\kappa(I^S(\kappa); \kappa) = \Big(S_I\Big|_{(I^S(\kappa),\kappa)} \frac{\partial I^S}{\partial \kappa} + S_\kappa\Big|_{(I^S(\kappa),\kappa)} \Big) = -\ell_\kappa(\kappa)\Big(\ell(\kappa)\phi(I^S(\kappa)) - \alpha I^S(\kappa)\Big) > 0 \tag{A.25}$$

The first term vanishes at the optimum and the second term is positive by Proposition 2. ■

Lemma 6 *For $i \in \{S, U\}$ there exists $K^i \in (1-\alpha, 1)$ such that at the optimum*

$$\alpha I^i(\kappa) - B \gtreqless (1-\kappa) I^i(\kappa) \textit{ for } \kappa \gtreqless K^i \tag{A.26}$$

Proof of Lemma 6. I present the proof for I^S, the steps are identical for I^U. The critical investment level is $B/(\kappa - (1-\alpha))$. The claim is $I^S(\kappa)$ single crosses $B/(\kappa - (1-\alpha))$ from below. $I^S(\kappa)$ starts from below because as $\kappa \to 1-\alpha$, $I^S = I^* < \lim_{\kappa \to 1-\alpha} B/(\kappa - 1 - \alpha) = \infty$. The critical level $B/(\kappa-(1-\alpha))$ is decreasing and convex in κ with lower limit B/α. Suppose for some $\kappa^L < \kappa^H$, $I^S(\kappa^L) \geq B/(\kappa^L - 1 - \alpha)$. Then for all $I \geq B/(\kappa^L - (1-\alpha)) > B/(\kappa^H - (1-\alpha))$, the thresholds are identical $\ell(I; \kappa^L) = \ell(I; \kappa^H)$ as $\ell\phi(I) = \alpha I - B$ is independent of κ. Therefore the optimal investments must be identical. This implies that if $I^S(\kappa)$ crosses $B/(\kappa-(1-\alpha))$ once, it is constant in κ thereafter.

Take $\kappa = 1$. For $I < B/\alpha$, $\ell(I;1) = 0$ so the left-hand-side of (A.27) collapses to $E[z\phi'(I)] - 1$. By Assumption 3, $E[z\phi'(B/\alpha)] - 1 > 0$ and $\phi'' < 0$ so the optimal investment has to be larger than B/α. Hence there exists $K^S \in (1-\alpha, 1)$ such that

$$\forall \kappa \geq (>) K^S : I^S(\kappa) = \frac{B}{K^S - (1-\alpha)} \geq (>) \frac{B}{\kappa - (1-\alpha)}$$

independent of κ, whereas for all $\kappa < K^S : \ell(\kappa)\phi(I^S(\kappa)) = (1-\kappa)I^S(\kappa)$. ■

Proof of Proposition 4. The arguments of functions ℓ and z^* are suppressed thoughout. The optimality condition is

$$\int_{z^*}^{\infty} z\phi'(I)dF(z) + F(z^*)\,\alpha - 1 - \int_{\ell}^{z^*} (\alpha - z\phi'(I))dF(z) + \ell_I(\alpha I - \ell\phi(I))f(\ell) = 0 \quad \text{(A.27)}$$

Once more the first three terms correspond to the benchmark problem and the last two terms, the first of which is negative and the second is positive, represent the marginal loss. For $\kappa > 1-\alpha$, the liquidation rule is $\ell\phi(I) = \max(\alpha I - B, (1-\kappa)I)$. As $I \to 0$ the marginal loss becomes zero and Proposition 1 proves that the rest of the terms tend to ∞. The aim is to sign the marginal loss

$$-\ell_I(\alpha I - \ell\phi(I))f(\ell) + \int_{\ell}^{z^*} (\alpha - z\phi'(I))\,dF(z) \quad \text{(A.28)}$$

Let ζ be the unique mode of Z by log-concavity. I proceed case-by-case.

Case 1 *$\ell(I) < z^*(I) \leq \zeta$ i.e. F is convex on the interval $[\ell(I), z^*(I)]$.*

(A.28) is bounded below by

$$\begin{aligned} LB(I;\kappa) &\equiv -\ell_I(z^* - \ell)\phi(I)f(\ell) + (\alpha - z^*\phi'(I))(F(z^*) - F(\ell)) \\ &= \phi(I)\Big[z_I^*(F(z^*) - F(\ell)) - \ell_I(z^* - \ell)f(\ell)\Big] \geq 0 \end{aligned} \quad \text{(A.29)}$$

The lower bound $LB(I;\kappa)$ is non-negative as $z_I^* \geq \ell_I$ by Proposition 2 and $F(z^*) - F(\ell) \geq (z^* - \ell)f(\ell)$ by convexity of F.

Case 2 *$z^*(I) > \ell(I) \geq \zeta$ or F is concave on the interval $[\ell(l), z^*(I)]$, and $\kappa \geq K$ defined in Lemma 6.*

Whenever F is concave on an interval, the lower bound (A.29) has an ambiguous sign. Using Lemma 6 (A.28) can be bounded above by

$$UB^{\kappa\geq K}(I;\kappa) = \phi(I)\ell_I\Big[F(z^*) - F(\ell) - (z^* - \ell)f(\ell)\Big] \leq 0 \tag{A.30}$$

It is redundant to analyze $I \leq B/\alpha$ since $I^S(\kappa) > B/\alpha$ so totally differentiate $\ell\phi(I) = \alpha I - B > 0$ and replace $\alpha - \ell\phi'(I)$ to simplify the expression. The inequality follows from concavity of F.

Case 3 *$z^*(I) > \ell(I) \geq \zeta$ or F is concave on the interval $[\ell(l), z^*(I)]$, and $\kappa < K$ defined in Lemma 6.*

Neither the lower nor the upper bound can be unambiguously signed in this case. Define

$$z^*/\ell = \alpha/(1-\kappa) = \lambda > 1 \tag{A.31}$$

and observe that the equality also holds for the ratio of their derivatives z^*_I/ℓ_I. Now the lower bound (A.29) can be rewritten

$$\begin{aligned} LB^{\kappa<K}(I;\kappa) &\equiv \ell_I\phi(I)\Big[\lambda(F(\lambda\ell)) - F(\ell)) - (\lambda-1)\ell f(\ell)\Big] \\ &= \ell_I\phi(I)(1-F(\ell))\Big[\lambda(1 - \frac{1-F(\lambda\ell)}{1-F(\ell)}) - (\lambda-1)\ell h(\ell)\Big] \end{aligned} \tag{A.32}$$

Increasing hazard rate implies that $1-F(\lambda\ell)/1-F(\ell)$ is a decreasing function of its argument, which is itself increasing in I. Therefore the first term in the square brackets of (A.32) is an increasing function of I ranging $(0,\lambda)$. The second term is increasing in I ranging $(0,\infty)$ by the very same assumption. Hence there exists an $\bar{I}_1$ such that $LB^{\kappa<K}(I;\kappa) < 0$ whenever $I > \bar{I}_1$. To prove it starts positive, $F(\lambda\ell) - F(\ell) \geq (\lambda-1)\ell f(\lambda\ell)$ by concavity of F so the lower bound itself satisfies

$$LB^{\kappa<K}(I;\kappa) \geq \ell_I\phi(I)(\lambda-1)\ell\Big[\lambda f(\lambda\ell) - f(\ell\Big] \tag{A.33}$$

Notice that the square bracket term is $\partial\Big(F(\lambda\ell) - F(\ell)\Big)/\partial\ell$. Under the log-concavity assumption, this derivative alternates sign from positive to negative once.

Claim 1 *For any $\lambda > 1$, $F(\lambda z) - F(z)$ is unimodal at $\bar{\zeta} \geq \zeta$.*

The claim proves that the lower bound starts positive, crosses zero at some $\hat{I}_1$ and remains negative thereafter. I conclude without loss of generality that

$$LB^{\kappa<K}(I;\kappa) \gtreqless 0 \Longleftrightarrow I \lesseqgtr \bar{I}_1$$

The same analysis can be repeated for the upper bound:

$$
\begin{aligned}
UB^{\kappa<K}(I;\kappa) &\equiv LB^{\kappa<K}(I;\kappa) + z^*\phi'(I)(F(z^*) - F(\ell))) \\
&= LB^{\kappa<K}(I;\kappa) + \frac{\alpha I\phi'(I)}{\phi(I)}(F(\lambda\ell) - F(\ell))
\end{aligned}
\tag{A.34}
$$

Whenever $LB^{\kappa<K}(I;\kappa) \leq 0$, I have argued that $\partial\Big(F(\lambda\ell) - F(\ell)\Big)/\partial\ell \leq 0$. In addition, $\phi(I)/I \geq \phi'(I)$ by concavity of $\phi(I)$. Therefore the additional term in (A.34) is less than or equal to $\alpha(F(\lambda\ell) - F(\ell))$, a decreasing function that converges to 0 as $I \to \infty$. This suffices to conclude that $\exists \bar{I}_2 > \bar{I}_1$ such that

$$UB^{\kappa<K}(I;\kappa) \gtreqless 0 \iff I \lesseqgtr \bar{I}_2$$

Put together, the marginal loss is positive for small $I \leq \bar{I}_1$ and negative for large $I \geq \bar{I}_2$ so there must exist a threshold $T : \bar{I}_1 < T < \bar{I}_2$ such that (A.28) is zero. I assume without loss of generality that it singly crosses zero.

All cases are covered for $\kappa > 1 - \alpha$, in each case there exists $T(\kappa)$ such that if $I^* < T(\kappa)$, the benchmark is in the increasing segment of marginal loss and therefore $I^S(\kappa) < I^*$. Otherwise if $I^* > T(\kappa)$ so that the benchmark is in the decrasing segment of marginal loss, then $I^S(\kappa) > I^*$. The proof of Claim 1 is presented below.

Proof of Claim 1.

The proof of this claim uses the following result. See Ramos and Diaz (2001) for a proof.

Result 1 (Theorem 1.C.29 in Shaked and Shantikumar (2007)) *Let Z be a non-negative, absolutely continuous random variable with density function $f(z)$ on $(0, \infty)$. λZ likelihood ratio-dominates Z for all $\lambda > 1$ if and only if $f(e^z)$ is log-concave.*

The slope of $F(\lambda z) - F(z)$ is $\lambda f(\lambda z) - f(z)$. Let ζ denote the mode of Z such that $f'(z) \gtreqless 0$ whenever $z \lesseqgtr \zeta$. Consider first $z < \zeta$. Since $F''(z) = f'(z)$ this is the convex portion of F. Now the slope can be unambiguously signed as $f(\lambda z) \geq f(z)$ and $\lambda > 1$.

Suppose $z \geq \zeta$ so that F is concave at z or $f'(z) \leq 0$. Notice that if $f(z)$ is the probability density of Z at point z, then $g(z) = f(z/\lambda)$ is the probability density of λZ. By Result 1 $f(z/\lambda)/f(z)$ is increasing, or equivalently $f(\lambda z)/f(z)$ is decreasing, if and only if $f(e^z)$ is log-concave. I now claim that for $z \geq \zeta$, $f(e^z)$ is log-concave.

$$(\ln f(e^z))'' = e^z\frac{f'(e^z)}{f(e^z)} + e^{2z}\Big(\frac{f'(e^z)}{f(e^z)}\Big)' \leq 0 \tag{A.35}$$

The non-positive sign follows from the fact that $f'(e^z) \leq 0$ as e^z is an increasing function of z

so $e^z > z \geq \zeta$ and the second is log-concavity of $f(z)$. Likelihood ratio order is preserved under integration, therefore

$$\frac{f(\lambda z)}{f(z)} \searrow z \Longrightarrow \frac{F(\lambda z)}{F(z)} \searrow z \Longrightarrow \frac{F(\lambda z)}{F(z)} - 1 > \frac{\lambda f(\lambda z)}{f(z)} - 1 \tag{A.36}$$

The second inequality completes the proof. The limit of left-hand side is 0 since $\lim_{z\to\infty} F(\lambda z) = \lim_{z\to\infty} F(z) = 1$ and therefore $\lambda f(\lambda z)/f(z)$ must be below1 as $z \to \infty$. The ratio of densities is decreasing in z so either it single crosses 1 from above or it is always below 1. In either case $\exists \bar{\zeta} \geq \zeta$ such that $\lambda f(\lambda z) - f(z)$ single crosses 0 from above at $\bar{\zeta}$. This proves that $F(\lambda z) - F(z)$ is increasing for $z \leq \bar{\zeta}$ and decreasing for $z \geq \bar{\zeta}$ concluding the proof. ■ ■

Proof of Corollary 2. The derivative of $S(I^S(\kappa);\kappa)$ in κ using envelope theorem is

$$S_\kappa(I^S(\kappa);\kappa) = \Big(S_I\Big|_{(I^S(\kappa),\kappa)}\frac{\partial I^S}{\partial \kappa} + S_\kappa\Big|_{(I^S(\kappa),\kappa)}\Big) = -\ell_\kappa(\kappa)\Big(\ell(\kappa)\phi(I^S(\kappa)) - \alpha I^S(\kappa)\Big) \tag{A.37}$$

When $\kappa > 1-\alpha$, the parenthetical term is negative. $\ell_\kappa(\kappa) \leq 0$ by Proposition 2 concludes that the derivative is non-negative. ■

Proof of Proposition 5. By definition in (5.1), I^U must satisfy $V_I(I^U;\eta,\kappa) = 0$ and Proposition 3 and 4 proves $\exists\, I^S$ that satisfies $S_I(I^S;\kappa) = 0$. Then

$$V_I(I^S;\eta,\kappa) = \frac{\eta}{\kappa}\Big(S_I(I^S;\kappa) - f(\ell(\kappa))\ell_I(I^S;\kappa)B\Big) = -\frac{\eta}{\kappa}f(\ell(\kappa))\ell_I(I^S;\kappa)B < 0 \tag{A.38}$$

since $\lim_{I\to 0} V_I(I;\kappa,\xi) \to \infty$ as $\lim_{I\to 0} S_I(I;\kappa) \to \infty$, there exists $I^U(\kappa) < I^S(\kappa)$ that satisfies the optimality condition.

Consider a general form for private benefits $B(I;\eta,\kappa)$ such that $B_I \geq 0$ and the condition (A.13) that is sufficient for all earlier results to hold. The marginal effect of I on the expected private benefit would be

$$-f(\ell(\kappa)\ell_I B + (1 - F(\ell(\kappa))B_I = (1 - F(\ell(\kappa))B_I\Big(1 - h(\ell(\kappa))\ell_I\frac{B}{B_I}\Big)$$

The parenthetical term determines the sign and is bounded above by $1 - h(\ell(\kappa))\ell_I\phi(I)/\phi'(I)$. It starts positive as $I \to 0$ and since hazard rate h and ϕ/ϕ' are increasing with limit infinity, it crosses zero and stays negative provided ℓ_I is also well-behaved. Let $\bar{I}$ denote the crossing point.

Proposition 5 is valid as long as $I^S(\kappa) > \bar{I}$. ■

Proof of Lemma 2. Compute the marginal rent in κ using Envelope Theorem:

$$\begin{aligned}\hat{V}_\kappa &= \Big(V_I\Big|_{(I^U(\kappa),\kappa)} + V_\kappa\Big|_{(I^U(\kappa),\kappa)}\Big)\\ &= -\ell_\kappa\Big(\ell(\kappa)\phi(I^U(\kappa)) + B - \alpha I^U(\kappa)\Big) \geq 0\end{aligned} \tag{A.39}$$

In Lemma 6 I proved that there exists a K such that for $\kappa < K$ the optimum satifies

$$\ell(\kappa)\phi(I^U) > \alpha I^U(\kappa) - B$$

and by Proposition 2 $\ell_\kappa < 0$ whenever this is the case. Therefore $\hat{V}_\kappa$ is positive for $\kappa < K$ and zero for $\kappa \geq K$. ■

Proof of Lemma 3. First I show that $I^U(\kappa) < I^S(\kappa)$ for all κ where $I^S(\kappa)$ maximizes $S(I;\kappa)$ and $I^U(\kappa)$ maximizes (5.8). Since $1 - F(\ell(\kappa))$ is decreasing in I with the slope $-f(\ell(\kappa))\ell_I(\kappa) < 0$, I study the sign of $\partial\mathbb{f}/\partial I$ and prove it is positive evaluated at $I^S(\kappa)$. Differentiate $\mathbb{f}$ is I to get

$$\frac{\partial \mathbb{f}}{\partial I} \equiv \frac{\partial}{\partial I}\Big(\frac{(\kappa-\eta)I}{S(I;\kappa)+\kappa I}\Big) = \frac{\kappa-\eta}{S(I;\kappa)+\kappa I}\Big(1 - \frac{(S_I(I;\kappa)+\kappa)I}{S(I;\kappa)+\kappa I}\Big) \tag{A.40}$$

Evaluated at $I^S(\kappa)$ the derivative inside the parenthesis vanishes and the remaining terms are unambiguously positive. Therefore:

$$U_I(I^S(\kappa);\eta,\kappa) = S_I(I^S(\kappa);\kappa) + \frac{\partial}{\partial I}\Big((1-\mathbb{f})(1-F(\ell(\kappa))B\Big)\Big|_{I^S(\kappa)} < 0 \tag{A.41}$$

I have proven in Proposition 3 that $I \to 0$, $S_I(I;\kappa) \to \infty$. Then $U_I(I;\eta,\kappa)$ equals zero for some $I^U(\kappa) < I^S(\kappa)$.

For the second part of the result, evaluate (5.8) at $I^U(\kappa)$ and use Envelope Theorem to get

$$U_\kappa(I^U(\kappa);\eta,\kappa) = -\ell_\kappa\Big(\ell(\kappa)\phi(I^U(\kappa)) - \alpha I^U(\kappa) + (1-\mathbb{f})B\Big)f(\ell(\kappa)) - \mathbb{f}_\kappa(1-F(\ell(\kappa))B \tag{A.42}$$

The term $\mathbb{f}_\kappa$ denotes the derivative of $\mathbb{f}$ in κ evaluated at $I^U(\kappa)$. To compute this derivative, first I get:

$$\frac{\partial \mathbb{f}}{\partial \kappa} \equiv \frac{\partial}{\partial \kappa}\Big(\frac{(\kappa-\eta)I}{S(I;\kappa)+\kappa I}\Big) = \frac{I}{S(I;\kappa)+\kappa I}\Big(1 - \frac{(S_\kappa(I;\kappa)+I)(\kappa-\eta)}{S(I;\kappa)+\kappa I}\Big) \tag{A.43}$$

Consider $\kappa \to 1-\alpha$. Since $\ell(\kappa) \to z^*$ and by definition of z^* in (2.1), $\lim_{\kappa\to 1-\alpha} S_\kappa(I;\kappa) = 0$

for all I. Now $\partial \mathbb{f}/\partial \kappa$ simplifies to

$$\lim_{\kappa \to 1-\alpha} \frac{\partial \mathbb{f}}{\partial \kappa} = \frac{\mathbb{f}}{1-\alpha-\eta}(1-\mathbb{f}) \tag{A.44}$$

for all I. Likewise the limit of U_κ simplifies to

$$\lim_{\kappa \to 1-\alpha} U_\kappa(I^U(\kappa);\eta,\kappa) = \lim_{\kappa \to 1-\alpha}(1-\mathbb{f})B\Big(-\ell(\kappa)f(\ell(\kappa)) - \frac{\mathbb{f}}{\kappa-\eta}(1-F(\ell(\kappa)))\Big) < 0 \tag{A.45}$$

under condition (5.9). Therefore, even if $\lim_{\kappa\to\eta} U_\kappa > 0$ so that the banker has an incentive to issue outside equity, he does not have enough incentive to capitalize up to $1-\alpha$. ■