Network Hazard and Bailouts

Selman Erol

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Job market paper

Abstract

I introduce a model of contagion with endogenous network formation and strategic default, in which a government intervenes to stop contagion. The anticipation of government bailouts introduces a novel channel for moral hazard via its effect on network architecture. In the absence of bailouts, the network formed consists of small clusters that are sparsely connected. When bailouts are anticipated, firms in my model do not make riskier individual choices. Instead, they form networks that are more interconnected, exhibiting a core-periphery structure (wherein many firms are connected to a smaller number of central firms). Interconnectedness within the periphery increases spillovers. Core firms serve as a buffer when solvent and an amplifier when insolvent. Thus, in my model, ex-post time-consistent intervention by the government improves ex-ante welfare but it increases systemic risk and volatility through its effect on network formation. This paper can be seen as a first attempt at introducing a theory of mechanism design with endogenous network externalities.

Keywords: Network Formation, Bailouts, Moral Hazard, Systemic Risk, Volatility, Interconnectedness, Core-periphery, Contagion, Rationalizability, Strong stability, Phase transition.

JEL classification numbers: D85, G01, H81.

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ii University of Pennsylvania; erols@sas.upenn.edu.
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1 Introduction

The financial crisis of 2008 alerted many to the risk that the failure of a few individual financial institutions might, through the interconnectedness of the financial system, damage the economy as a whole. Such systemic risk can be ameliorated ex-ante using regulatory tools, yet the inability of government to credibly commit to not intervening suggests that an ex-post response, in the form of bailouts, is unavoidable. Bailouts of failing institutions are criticized because they encourage excessive risk taking by individual institutions. Excessive risk taking may trigger cascading failures, but explanations of what generates the underlying interconnectedness are lacking. In this paper, I argue that the anticipation of bailouts influences the formation of networks among financial institutions, creating a novel form of moral hazard: ‘network hazard.’ I exhibit a model in which the anticipation of bailouts has two main effects. First, it loosens the market discipline and generates more interconnectedness. Second, it leads to the emergence of systemically important financial institutions, and this produces core-periphery networks.\(^1\) As a consequence, systemic risk, volatility, and ex-ante welfare all increase.

My model has three stages. Stage one is the network formation stage. Firms\(^2\) form links with each other by mutual consent. A pair of firms that have a link are called counterparties of each other. A link in its most general form represents a mutually beneficial trading opportunity.\(^3\) However, benefits from trade are realized only if neither party subsequently reneges; in the model doing so is called default. Stage two is the intervention stage. Each firm receives an idiosyncratic exogenous shock, good or bad. Shocks capture the fundamental productivity of firms: a firm that experiences a good shock is called a good firm, while one that experiences a bad shock is called a bad firm. When shocks occur the government intervenes. Stage three is the contagion stage. A firm can take one of two actions: continue or default. A firm that defaults receives a payoff, an outside option, independent of the actions of its counterparties. A firm that continues receives a payoff contingent upon the actions of its counterparties, its own action, and its shock. More links yield more potential benefits, but these are offset by the costs imposed by defaulting counterparties. (For a visualization of timing of events, see Figure 1 for the benchmark model with the absence of intervention and Figure 12 for the full model with the presence of intervention.)

In the model, a bad firm’s dominant action is default and receipt of the outside option.

\(^1\)Core-periphery architecture is widely observed in practice. See, among many others, Vuillemey and Breton (2014), and Craig and Von Peter (2014).

\(^2\)To emphasize the wide number of interpretations of the model I refer to agents as firms rather than as financial institutions. See Section 7.1.

\(^3\)Other interpretations of the network are discussed in Section 7.1.
Given that a defaulting firm imposes costs on its counterparties, a good firm with sufficiently many defaulting counterparties may find it iteratively dominant to default so as to enjoy the outside option. Thus, default decisions triggered by bad shocks might propagate through the entire network. Foreseeing this “contagion,” the government intervenes at the end of stage two. Specifically, the government commits to a transfer policy that is conditional on the actions taken by firms in stage three, and this transfer policy maximizes ex-post welfare. Government cannot commit to a transfer policy prior to stage two. The difference between the absence and presence of intervention in the network formed can be explained as follows.

In the absence of the anticipation of intervention, a firm prefers that its counterparties are counterparties only of each other. This is so because in the model benefits come from links with immediate counterparties, and counterparties of counterparties typically harm a firm in expectation. A firm that does not have a “second-order counterparty” limits its exposure to second-order counterparty risk: the risk that it incurs losses due to defaults by good counterparties that default because of their own defaulting counterparties. This force generates a market discipline that leads uniquely to the formation of dense clusters that are isolated from each other, and this network structure eliminates second-order counterparty risk.

To illuminate the main effects of the anticipation of intervention on the network formed, consider as a starting point a baseline case of government intervention. In this case, good firms contribute to welfare by continuing and bad firms reduce welfare by continuing. Bailouts are costless and the government is not restricted by a budget or by any other form of ex-ante commitment. Therefore, the government optimally induces good firms to continue and bad firms to default. In return, each firm knows that its good counterparties are going to continue even if these good counterparties have many bad counterparties of their own. Thus, second-order counterparty risk is eliminated as a byproduct of optimal intervention. This loosens the market discipline because firms no longer concern themselves with the counterparties of their counterparties. The significant effect of bailouts on the network topology emerges because each firm anticipates that its counterparties can get bailed-out and not because each anticipates that itself will be bailed-out. The elimination of second-order counterparty risk has two main effects on the induced network topology and systemic risk.

The first effect arises across ex-ante identical firms. Because firms no longer concern them-
selves with second-order counterparty risk, the isolated clusters that form in the absence of intervention dissolve, and an interconnected network emerges (See Figure 14 for a visualization). But bad firms do not get bailed-out under the optimal policy. Consequently, each good firm still incurs losses because bad counterparties still default. If a good firm has too many bad counterparties and thus is forced into default, the government steps in and bails-out the good firm. However, to induce the good firm to continue the government offers it the smallest transfers possible, and so the good firm is indifferent between defaulting or not. In other words, a firm gains nothing when it is bailed-out, and the risk that it incurs losses due to defaults by bad counterparties (first-order counterparty risk) remains unaltered. As a consequence, during network formation, firms do not overconnect or underconnect: instead, each firm has the exact same number of counterparties whether or not intervention occurs. That said, the network becomes more interconnected when clusters dissolve. As the network becomes more interconnected, and when each firm has the same number of counterparties, the extent of potential contagion increases. The threat of system wide default also increases, but the government intervenes to stop contagion. Compared to no intervention, ex-ante welfare is higher under interventions that involve bailouts, and with bailouts systemic risk increases. In other words, the number of firms that face indirect default and get bailed-out under intervention is larger than the number of firms that face indirect default and do, in fact, default in the absence of intervention.

The second effect arises across firms that are not identical ex-ante. Such differences arise because some firms have less equity than others or some firms specialize in different sectors. Under heterogeneity, some firms typically have a greater appetite for counterparties than others. In the absence of bailouts, such high demanding firms are unable to convince low demanding firms to become counterparties. This occurs because high demanding firms would have too many counterparties, which would increase second-order counterparty risk for their low demanding counterparties. When bailouts eliminate second-order counterparty risk, high demanding firms are welcome to become counterparts with all firms. In this manner, high demanding firms become central to the network. The network exhibits a core-periphery structure because of bailouts: high demanding central firms make up the core of the network

6Rochet and Tirole (1996b), too, discuss this type of direct assistance to good firms that face failure due to counterparties (as opposed to indirect assistance rendered through bad counterparties). In Sections 5 and 6 I examine in detail why factors such as bailout costs and budget constraints that render indirect assistance to good firms through bad counterparties are optimal.

7An indirect default refers to a default decision by a good firm which suffers sufficiently many counterparty losses.

8As explained in Section 7.1, firms can have ex-ante differences for many reasons. For example, some banks are located at money centers that have access to many investors while others are small deposit-collecting banks.
and low demanding firms make up the periphery (See Figure 19 for a visualization). Because of the firms at the core of the network the counterparty risks faced by peripheral firms are correlated. In return, when a sufficient number of core firms default after experiencing bad shocks, peripheral firms become less resilient; that is, a few bad shocks to the peripheral counterparties of a peripheral good firm will cause the latter to default. Thus, the core serves as an amplifier of contagion across the periphery. When a sufficient number of core firms experience good shocks and then either continue or get bailed-out, peripheral firms become more resilient. Only a large number of bad shocks to the peripheral counterparties of a peripheral good firm will force the latter to default. Thus, the core serves as a buffer against contagion. The formation of a core-periphery structure makes very bad and very good outcomes more likely, and this generates volatility.

The force most responsible for network hazard is the elimination of second-order counterparty risk. Network hazard is a genuine source of moral hazard. Consider a scenario in which, during network formation, each firm can individually choose between two risk levels: safe investments or high risk/high return investments. Firms exploit this choice, and when they do it affects how the network is formed because it alters the number of each firms’ counterparties. However, firms make the identical risk choices whether or not intervention is available. Moreover, firms that anticipate bailouts do not overconnect or underconnect; instead, their networks become more interconnected. When bailouts are available heterogeneous firms continue to form core-periphery networks. This is so because in the model, firms offered bailouts are indifferent about whether or not to default. Accordingly, firms do not benefit from overconnection or underconnection, nor will they benefit from choosing riskier investments in face of bailouts. Network hazard is a genuine form of moral hazard that emerges only when a network is formed. This is true even when firms are not incentivized to choose riskier individual investments.

Other extensions of the baseline case are worth mentioning. Under some extensions, incentives to form a core-periphery network are particularly strong. Consider a core-periphery network that has high demanding firms at the core and low demanding firms at the periphery. If a sufficient number of core firms suffer bad shocks, many peripheral firms will be forced into default. If bailouts are costly, if there is a budget constraint on the government, or if the government is committed to bailing-out only systemically important firms, the government in some cases might bailout the bad core firms in order to indirectly support troubled good firms at the periphery. This would be an alternative to bailing out an excessive number of peripheral firms. As a consequence, the ex-ante payoff to a peripheral firm would increase because it would now have counterparties (core firms) that would be bailed-out even if they suffered bad shocks. In a core-periphery structure this possibility reduces the first-order
counterparty risk of peripheral firms, and this in turn increases their incentives to maintain a core-periphery network. Note that these arguments do not require ex-ante heterogeneity of firm types. Indeed, when bailouts are costly, even ex-ante identical firms that have the same demand for counterparties can in response to bailouts form a core-periphery network. That is, core-periphery is not an artifact of firm heterogeneity.

Under these same extensions, incentives to form an interconnected network across identical firms also become stronger. If the network is interconnected (if it does not consist of isolated clusters), some good firms might benefit from the bailouts of bad counterparties that are optimally executed for the sake of other good firms. This indirect assistance to a good firm that is not facing default increases its payoff. Accordingly, firms have higher incentives to make their network more interconnected in order to force the government to execute more bailouts and benefit from such indirect support even when the support is not needed to avoid default.

Related literature: A voluminous literature examines moral hazard in a variety of contexts, including banking (Chari and Kehoe (2013), Cordella and Yeyati (2003), Freixas (1999), Holmstrom and Tirole (1997), Keister (2010), Mailath and Mester (1994), and many others), yet this literature contains very little discussion of network formation. In examinations of bailouts and systemic risk, authors such as Caballero and Simsek (2013), Elliott, Golub and Jackson (2014), Freixas, Parigi and Rochet (2000), Gaballo and Zetlin-Jones (2015), Leitner (2005), and Rochet and Tirole (1996b) analyze networks of bilateral exposures. But in these analyses, which contrast with mine, network architecture is not endogenously formed by firms and moral hazard arises from individual choices about excessive risk taking, bankers’ decision to shirk, lack of monitoring among banks, etc.

A few recent papers such as Acharya and Yorulmazer (2007), Acharya (2009), and Farhi and Tirole (2012) propose, on the basis of correlations of investment risks, that moral hazard arises from collective behavior. This important form of interconnection does not constitute a network of bilateral relationships formed through mutual consent. Such a correlation of risk generates systemic risk, but it does not do so via contagion through a network of bilateral linkages. I examine how bailouts affect both incentives for forming bilateral links and collective incentives that shape interconnections, and how this process affects systemic risk and welfare.

There is also a large and growing literature that examines systemic risk and networks. Early contributors include Allen and Gale (2000), Eisenberg and Noe (2001), Kiyotaki and Moore (1997), Rochet and Tirole (1996b), and in more recent years, Acemoglu, Ozdaglar and Tahbaz-Salehi (2015b), Elliott, Golub and Jackson (2014), Glasserman and Young (2014),
and others.\textsuperscript{9} These papers examine contagion within fixed networks. Other scholars, among them Drakopoulos, Ozdaglar and Tsitsiklis (2015a), Freixas, Parigi and Rochet (2000), and Minca and Sulem (2014) examine the problem of how to stop contagion in exogenous networks.\textsuperscript{10} Acemoglu, Ozdaglar and Tahbaz-Salehi (2015c), Cabrales, Gottardi and Vega-Redondo (2014), Elliott and Hazell (2015), Erol and Vohra (2014), Farboodi (2015), Goldstein and Pauzner (2004) and others\textsuperscript{11} study the formation of networks by agents who take systemic risk into account. In contrast to mine, these studies do not consider the possibility that the anticipation of ex-post government intervention might affect the network. My paper contributes to this literature by investigating how the anticipation of ex-post bailouts affects endogenous networks and systemic risk.

My model most closely resembles one proposed by Erol and Vohra (2014). Indeed, I substantially generalize their model to allow for arbitrary levels of exposures (strength of contagion), more general payoff functions, heterogenous firms, incomplete information, and government intervention. In technical terms, my network formation theorem examines the formation of strongly stable networks\textsuperscript{12}, wherein the payoffs to agents within each network are derived from a semi-anonymous graphical game with complementarities.\textsuperscript{13} Moreover, the structure of the network formed and the extent of systemic risk on the resulting network features a phase transition property in the number of firms (See Figure 9 for a visualization). To the best of my knowledge, this is the first phase transition result in the number of players for endogenously formed networks. As for the case of intervention, the model can be seen as a first attempt towards developing a theory of mechanism design that has endogenously determined network externalities at an ex-ante stage.


\textsuperscript{10}Other similar papers are Amin, Minca and Sulem (2014), Drakopoulos, Ozdaglar and Tsitsiklis (2015b), and Motter (2004). There is also another less related branch of papers examining mitigation of systemic risk by ex-ante regulation. Rochet and Tirole (1996a) can be seen as an example, comparing the efficacy of different payment systems.


\textsuperscript{13}See Jackson (2010) for definitions of these technical terms.
Structure: Section 2 introduces the benchmark model. Section 3 studies networks formed in the absence of intervention. Section 4 examines the baseline case of government intervention, and it introduces the concepts of induced interconnectedness and core-periphery. Section 5 examines the robustness of the induced architecture and Section 6 examines extensions. Section 7 discusses various interpretations of the model as well as future research, and Section 8 concludes. Each section ends with remarks that summarize its core messages.

2 Benchmark model

I introduce the benchmark model with complete information and no government intervention.

2.1 Environment

Let $N = \{n_1, n_2, ..., n_k\}$ be a set of $k$ firms. Each firm $n_i \in N$ has a type $\gamma_i \in \Gamma$, where $\Gamma$ is a finite set. There are three stages.

In stage one, the network formation stage, firms form bilateral relationships, called links, by mutual consent. The details can be found in Section 2.4. If two firms $n_i$ and $n_j$ decide to form a link, the link formed is denoted $e_{ij} = e_{ji} = \{n_i, n_j\}$, and the resulting set of links is denoted $E \subset [N]^2$. $(N, E)$ is the realized network. If $e_{ij} \in E$, $n_i$ and $n_j$ are called counterparties. Given $(N, E)$, $N_i = \{n_j : e_{ij} \in E\}$ denotes the set of counterparties of $n_i$, and $d_i = |N_i|$ the degree of $n_i$.

In stage two, firms receive shocks. Each firm independently gets a good shock $G$ with probability $\alpha \in (0, 1)$, or a bad shock $B$ with probability $1 - \alpha$. $\theta_i \in \{G, B\}$ denotes the realized shock to firm $n_i$.

In stage three, the contagion stage, each firm can choose to continue business and fulfill all obligations, or not continue via a default option. The decision to continue is denoted $C$ and the decision to default is denoted $D$. Firm $n_i$’s action in stage three is denoted $a_i \in \{C, D\}$.

Upon termination of stage three, each firm $n_i$ receives a payoff depending on its type $\gamma_i$, its degree in the realized network $d_i$, its shock $\theta_i$, its action $a_i$, and the number of its coun-

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14 In the remainder of the paper, definitions are inline and boldfaced.
15 Types determine the payoff function of each firm. These differences can arise due to many reasons including equity level, specialization, access to investment opportunities, access to depositors, business model, geographic location, location specific regulatory restrictions,...
16 For ease of notation I drop the $E$ subscript from $N_i$ and $d_i$. 

terparties that default (or fail) $f_i = | \{ j \in N_i : a_j = D \}|$.\(^{17}\) Formally, the payoff of firm $n_i$ is denoted $U_i$ and is given by

$$U_i(\bar{a}, \bar{\theta}, E, \bar{\gamma}) = P(a_i, f_i, d_i, \theta_i, \gamma_i)$$

where $P(a, f, d, \theta, \gamma) : \{C, D\} \times \mathbb{N} \times \mathbb{N} \times \{G, B\} \times \Gamma \rightarrow \mathbb{R}$.

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\(^{17}\)In fact, the payoff depends on the action profile of counterparties. The names and types of counterparties do not matter so that the payoff can be written as a function of $d_i$ and $f_i$ only instead of the whole action profile of counterparties.

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\(^{18}\)Throughout the paper, assumptions that are maintained from the point they are stated have numbers. Assumptions that are invoked as needed have names rather than numbers. This is to make it easier to recall the meaning and effect of each particular assumption. Other inline assumptions are particular to the subsection they appear in.

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2.2 Assumptions

**Assumption 1.** For any $d, \theta, \gamma; P(C, f, d, \theta, \gamma)$ is strictly decreasing in $f$, and $P(D, f, d, \theta, \gamma)$ is constant in $f$.\(^{18}\)

$P$ being decreasing in $f$ for $a = C$ captures the idea that a defaulting firm causes costs to its counterparties that continue business. The costs need not be additive. On the other hand, $P$ being constant in $f$ for $a = D$ means that default can be seen as walking away from obligations with an outside option which does not depend on the number of one’s counterparties that default.
Under Assumption 1, for any \((d, \theta, \gamma)\), \(P(a, f, d, \theta, \gamma)\) is submodular in \((a, f)\).\(^{19}\) In return, for any \((\bar{\theta}, E, \bar{\gamma})\), \(U_i(\bar{a}, \bar{\theta}, E, \bar{\gamma})\) is supermodular in \(\bar{a} = (a_1, a_2, ..., a_k)\). Therefore, the game in stage three is a supermodular game.

**Assumption 2.** For any \(d, \gamma\); \(P(C, 0, d, B, \gamma) < P(D, -, d, B, \gamma)\) and \(P(C, 0, d, G, \gamma) > P(D, -, d, G, \gamma)\).

Assumption 2 allows one to interpret \(B\) as a large bad shock and \(G\) as a good shock. The first condition in Assumption 2 ensures that it is strictly dominant for any firm with a bad shock to default. Otherwise, “contagion” never starts. The second condition in Assumption 2 ensures that a firm with a good shock continues if all of its counterparties continue. Otherwise every firm always default in any equilibrium. Note that there is no assumption on how many defaulting counterparties will force a firm into default. That is, any level for the “strength of contagion” is allowed.

An example of a function \(P\) that satisfies both Assumptions 1 and 2 is given as follows.
\[
P(C, f, d, G, \gamma) = (d + 1) - c_\gamma \times f, \quad P(C, f, d, B, \gamma) = -(d + 1) - c_\gamma \times f, \quad P(D, f, d, \theta, \gamma) = 0
\]
where \(c_\gamma > 0\) for all \(\gamma \in \Gamma\).

### 2.3 Interpretation of the model

Each link, in its most general form, represents a mutually beneficial trading opportunity, such as a joint project, between the counterparties involved. However, benefits realize in full only if neither party reneges, called default in the model. Moreover, firms cannot selectively default on their counterparties. That is, a firm either maintains all its obligations to all counterparties, or breaks all obligations. In the following, this assumption is without loss of generality. A firm optimally chooses to default on all or none even if allowed to selectively default.

There are various interpretations the model which I elaborate on in Section 7.1. Here I present a lead example for the reader who wishes a concrete setting to keep in mind.

**Lead example:** Each firm has a specialization.\(^{20}\) Each link is a joint project that requires the expertise and effort of both counterparties to succeed. Kickstarting each project initially costs each counterparty 1 unit. Each firm borrows these initial funds from outside the system in stage one, with interest rate \(r\) due in stage three.\(^{21,22}\) These loans are directly invested

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\(^{19}\)The order on \(\{C, D\}\) is one in which \(C\) is the higher action and \(D\) is the lower action. The order for \(f \in \mathbb{N}\) is the regular increasing order on \(\mathbb{N}\).

\(^{20}\)The specialization is not necessarily the type \(\gamma\).

\(^{21}\)For example, a bank borrows from depositors, a real sector firm borrows from banking sector.

\(^{22}\)\(r\) can also be thought of as payments to employees due the returns from projects in stage three.
into the projects. Each project requires costly supervision by both counterparties. In stage two, each firm receives an idiosyncratic shock that determines their cost of supervision.\textsuperscript{23} A firm with a bad shock has cost per project $\tilde{c}(B, \gamma)$, and a firm with a good shock has cost per project $\tilde{c}(G, \gamma)$. Upon observing the shocks, each firm decides to continue or default. Projects which have both counterparties continuing yields safe return $R$ to each counterparty. Projects which have at least one defaulting counterparty fails.\textsuperscript{24} Assume that $\tilde{c}(B, \gamma) > R > r \geq 1$ and $R - r > \tilde{c}(G, \gamma) \geq 0$.

This way, a firm that continues, which has $d$ projects (hence counterparties) out of which $f$ many has failed, receives $R \times (d - f)$ from projects and incurs $\tilde{c} \times (d - f)$ cost of effort. It further pays its loans $rd$. Thus, its payoff is $P(C, f, d, \theta, \gamma) = (R - \tilde{c}(\theta, \gamma) - r) \times d - (R - \tilde{c}(\theta, \gamma)) \times f$. On the other hand, a firm that defaults has no return from projects, cannot pay its loans back, and gets $P(D, -, d, \theta, \gamma) = -\varepsilon$.\textsuperscript{25,26}

Now I describe a diversification example. The reader may skip directly to Section 2.4 without loss of understanding.

**Diversification example:** Each firm has one proprietary project and one non-proprietary project. A proprietary project has high management costs, so that other firms do not buy parts of the proprietary project due to its high moral hazard costs. On the other hand, non-proprietary projects have low management costs, so that other firms may find it beneficial to buy shares of non-proprietary projects.

The uncertainty regarding a proprietary project is resolved in stage two. The uncertainty regarding a non-proprietary project is resolved at the end of stage three. Once two firms sell each other shares of their non-proprietary projects, a link is formed between the two firms. The rationale for this exchange is diversification against the risk in stage three. If the non-proprietary projects of a firm yield low returns, it may be unable to pay for its liabilities. Accordingly, it may have to liquidate some other assets at discounted prices, leading to liquidation costs. By selling each other shares of their non-proprietary projects, firms increase the likelihood that their liquid assets (returns from projects) remain above

\textsuperscript{23}This can directly be a cost of effort, or some change in the prices of the inputs that the firm buys for producing its specialized product that is needed for the project to succeed.

\textsuperscript{24}This is also without loss of generality. A continuing counterparty of a defaulting firm, by incurring an extra cost $c^* > R$, can finish the project and get $2R$. For simplicity I assume that the project fails since it needs a specialized input of the counterparty that cannot be replaced.

\textsuperscript{25}$\varepsilon > 0$ is an arbitrarily small number to ensure that firms with degree 0 continue. It is not essential for anything in the model, and $\varepsilon = 0$ is equally fine in technical terms.

\textsuperscript{26}Consider the case in which firms are allowed to selectively default. Clearly, each firm defaults on projects in which the counterparty defaults. Suppose that a firm continues with $d^+$ projects, and defaults on $d - d^+$, where $0 \leq d^+ \leq d - f$. $P(C, f, d, \theta, \gamma) = (R - \tilde{c}(\theta, \gamma)) \times d^+ - rd$, so the firm always chooses $d^* = d - f$ or 0. $d^+ = d - f$ corresponds to action $C$ and $d^+ = 0$ corresponds to action $D$. 

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their liabilities. This, in expectation, reduces liquidation costs. Below are examples of balance sheets that illustrate this situation.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proprietary project</td>
<td>Liabilities</td>
</tr>
<tr>
<td>Non-proprietary project of n₁</td>
<td>Net worth</td>
</tr>
<tr>
<td>Illiquid assets</td>
<td>No links</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proprietary project</td>
<td>Liabilities</td>
</tr>
<tr>
<td>Shares left</td>
<td>Shares from n₂</td>
</tr>
<tr>
<td>Shares from n₃</td>
<td>Net worth</td>
</tr>
<tr>
<td>Illiquid assets</td>
<td>2 links</td>
</tr>
</tbody>
</table>

Figure 2: Balance sheet of firm n₁ in stage one

Consider a firm n₁. Each project of n₁ returns the value depicted in the first balance sheet if it is a successful project. If unsuccessful, a project returns 0. If firm n₁’s proprietary project is unsuccessful (θ₁ = B) and returns 0, n₁’s net worth is negative and it defaults. Suppose that n₁’s proprietary project was successful (θ₁ = G).

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-proprietary project of n₁</td>
<td>Net worth</td>
</tr>
<tr>
<td>Illiquid assets</td>
<td>No links</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shares left</td>
<td>Shares from n₂</td>
</tr>
<tr>
<td>Shares from n₃</td>
<td>Net worth</td>
</tr>
<tr>
<td>Illiquid assets</td>
<td>2 links</td>
</tr>
</tbody>
</table>

Figure 3: Balance sheet of firm n₁ in stage three conditional on θ₁ = G

If n₁ has no links, as illustrated in the first balance sheet, and if firm n₁’s non-proprietary project fails and returns 0 at the end of stage three, n₁ must liquidate the illiquid assets at a cost. If n₁’s non-proprietary project succeeds, n₁ can pay for its liabilities. The expectation of this final payoff over the returns from the non-proprietary investment gives n₁’s payoff \( P(C, 0, 0, G, \gamma) \). However if firm n₁ has two links, with firms n₂ and n₃ as depicted in the second balance sheet, unless all three projects of n₁, n₂, and n₃ fail, firm n₁ does not incur the liquidation cost of illiquid assets. The expectation of the final payoff over the returns from the non-proprietary investment is now \( P(C, 2, 0, G, \gamma) \), which is larger than \( P(C, 0, 0, G, \gamma) \) due to
reduced expected losses from liquidation. This way firms diversify against the risk of getting low returns from non-proprietary projects and having to liquidate illiquid assets at discounted prices in order to pay for liabilities. Accordingly, each link brings some diversification benefit to a firm.

However, links also bring some potential costs to a firm depending on the default decisions of its counterparties. If a firm defaults, the projects it originated fail. Therefore, if a firm continues and some of its counterparties default, the shares of the defaulting counterparties’ non-proprietary projects return 0 for sure. Accordingly, the continuing firm incurs losses since it now has only some portion of the returns from the project it originated. In the first balance sheet, $n_1$, in expectation over the returns from its non-proprietary project, has payoff $P(C, 0, 0, G, \gamma)$. In the second balance sheet, if firms $n_2$ and $n_3$ default, their projects fail and $n_1$ receives nothing back from the corresponding shares. Therefore, $n_1$ incurs some direct costs. If $n_1$ continues, it can get at most half of the full value of its non-proprietary project. Its payoff in expectation over returns from its non-proprietary project is then $P(C, 2, 2, G, \gamma)$. Now $n_1$ may find an orderly default in stage two optimal for early liquidation of illiquid assets instead of risking fire sales in stage three. This example is further elaborated in Section 7.1.

2.4 Solution concepts

In stage three, firms play a supermodular game given the realized network and shocks. The solution concept is the cooperating equilibrium: it is the Nash equilibrium in which, any firm which can play $C$ in at least one Nash equilibrium, plays $C$. Due to supermodularity of the game in stage three, this equilibrium notion is well-defined. Supermodularity of $\{U_i\}_{i \in N}$, via Topkis’ Theorem, implies that the best-responses are increasing in others’ actions. In return, by Tarski’s Theorem, the set of Nash equilibria is a complete lattice. The cooperating equilibrium is the unique highest element of the lattice of Nash equilibria.\(^{27}\)

The cooperating equilibrium can be obtained in two ways. The first is by iterating the myopic best-response dynamics starting with the ‘everyone plays $C$’ action profile.\(^{28}\) The second way, which is subtly different, is to apply iterated elimination of strictly dominated strategies.\(^{29}\) In both cases, the constructed sequence of action profiles reaches and stops at the cooperating equilibrium. Following the latter, an alternative definition of the cooperating equilibrium

\(^{27}\)See Vives (1990) for more on how complementarities generate a lattice structure on the set of Nash equilibria. See Milgrom and Shannon (1994) for more on supermodular games.

\(^{28}\)This is standard. A similar algorithm is considered in Vives (1990), Eisenberg and Noe (2001), Elliott et al. (2014), Morris (2000), Goyal and Vega-Redondo (2005), and others.

\(^{29}\)This link between rationalizability and the extreme points of the lattice is introduced in Milgrom and Roberts (1990).
can be given via iterated elimination of strictly dominated strategies. ‘The rationalizable strategy profile in which all firms play the highest action they can rationalize’ is identical to cooperating equilibrium. This has a natural contagion interpretation. Call firms that receive a bad shock, bad firms and firms that receive a good shock, good firms. Bad firms, are insolvent and find it strictly dominant to default on their obligations. Call these direct defaults. After some bad firms default in this way, some good firms who are counterparties with sufficiently many defaulting firms also become ‘troubled’, and find continuing iteratively strictly dominated, and so on. Call these indirect defaults. Contagion stops when no further firm finds it iteratively strictly dominated to continue business. Iterated elimination resembles contagion black-boxed into a single period. Below is an illustration of how contagion works.

Note. For simplicity, examples (not results) in the paper use an additively separable form given by

\[
P(C, f, d, G, \gamma) = u(d, \gamma) - c(f, \gamma); \quad P(C, f, d, B, \gamma) = -r - c(f, \gamma); \quad P(D, \cdot) = 0,
\]

where \( u, c : \mathbb{N}_0 \times \Gamma \to \mathbb{R}_+ \) and \( c \) is strictly increasing in \( f \). A good shock brings revenue given by \( u \). Returns from a bad shock is \(-r < 0\). Counterparty losses are subtracted from revenue. Default gives a safe outside option normalized to \( 0 \). Henceforth I present only the functions \( u \) and \( c \) in the examples, not \( P \) as a whole.

Example 1. \( u(d, \gamma) = d, c(f, \gamma) = 2f \). In this example, a firm with degree \( d \) defaults once it has strictly more than \( d/2 \) defaulting counterparties. The figure below illustrates how defaults propagate through the system, and how cooperating equilibrium can be obtained via iterated elimination of strictly dominated strategies.
Throughout the paper, I refer to losses due to bad counterparties as **first-order counterparty losses**. Losses due to defaulting good counterparties who default due to their bad counterparties is dubbed **second-order counterparty loss**. Higher order counterparty losses are defined analogously. Expected counterparty losses of a specified order is called **counterparty risk** of that order.\(^{30}\) If a firm faces no counterparty risk of order \(t\), then it faces no counterparty risk of order \(t' > t\) either.

In **stage one**, firms evaluate a network according to their expected payoffs in the cooperating equilibrium in stage three. Firms form the network as follows. Consider a candidate network \((N, E)\) and a subset \(N'\) of firms. A **feasible deviation** by \(N'\) from \(E\) is one in which \(N'\) can simultaneously add any missing links within \(N'\), cut any existing links within \(N'\), cut any of the links between \(N'\) and \(N/N'\).

\(^{30}\)Note that since iterated elimination of strictly dominated strategies reaches the set of rationalizable strategy profiles independent of the order of elimination, one needs to be careful about the higher orders in losses. The specific order I employ is that all strategies that can be eliminated in one iteration are eliminated all at once.
A **profitable deviation** by $N'$ from $E$ is a feasible deviation in which the resulting network yields strictly higher expected payoff to every member of $N'$. A **Pareto profitable deviation** by $N'$ from $E$ is a feasible deviation in which the resulting network yields weakly higher expected payoff to every member of $N'$, and strictly higher payoff to at least one member of $N'$. A network $(N, E)$ is **strongly stable** if there are no subsets of $N$ with a profitable deviation from $E$. A network $(N, E)$ is **Pareto strongly stable** if there are no subsets of $N$ with a Pareto profitable deviation from $E$.\(^{31}\)

In the model, the advantage of Pareto strong stability is that it gives uniqueness of the network formed, but existence requires some divisibility assumptions on the number of firms, solely to avoid integer problems. Strong stability yields existence without divisibility assumptions on the number of firms, but leaves some small room for multiplicity. Since I aim to compare the absence of government intervention with its presence, I find uniqueness more important. Therefore, I take Pareto strong stability as my main solution concept but provide some results for strong stability as well.

Pairwise stable networks\(^{32}\) while widely used in the literature are abundant in my setup. Moreover, strong stability guarantees *Pareto efficiency* of the network formed, given the behavior in stage three. It is government’s task to fix the inefficiencies in stage three.

\(^{31}\)Strong stability here follows Dutta and Mutuswami (1997). They establish the link of this concept to strong Nash equilibria. Pareto strong stability here is called strong stability in Jackson and Van den Nouweland (2005). They tie this solution concept to core. Farboodi (2015) uses strong stability, under the name group stability. Erol and Vohra (2014) also use strong stability under the name core networks.

Strongly stable networks correspond to strong Nash equilibria of an underlying proposal game. See Erol and Vohra (2014) for details of the proposal game. Therefore, strong stability results that follow can be thought of as *characterizing strong Nash equilibria* of a network formation game. See Dutta and Mutuswami (1997) for more on the relation between strong Nash equilibria and strongly stable networks.

\(^{32}\)Networks that don’t have a profitable deviation by any pairs or singletons of firms.

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**Figure 5:** A feasible deviation

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![Original network](image1.png) ![Deviation by green diamonds](image2.png) ![After deviation](image3.png)
3 Absence of government intervention

In this section, I characterize the networks that are formed in the absence of government intervention, and examine various measures of systemic risk.

3.1 Preliminaries

Consider the difference in payoff from continuing or defaulting for a good firm: \( \Delta P(f, d, \gamma) = P(C, f, d, G, \gamma) - P(D, -d, G, \gamma) \). By Assumptions 1 and 2, \( \Delta P(0, d, \gamma) > 0 \), and \( \Delta P(f, d, \gamma) \) is decreasing in \( f \) for any given \( (d, \gamma) \). Define the resilience of a \( \gamma \)-type good firm with degree \( d \) as \( R(d, \gamma) := \max \{ f \leq d : \Delta P(f, d, \gamma) \geq 0 \} \). \( R(d, \gamma) \) is the maximum number of counterparty defaults that a good firm of type \( \gamma \) can absorb before finding it optimal to default. For example, \( R(d, \gamma) = d \) means that no counterparties can force a good firm with type \( \gamma \) and degree \( d \) into default. The following simple conditions characterize the best response of \( n_i \) in stage three for any given \( (a_{-i}, E, \tilde{\theta}, \tilde{\gamma}) \):

- If \( \theta_i = B \), then \( a_i = D \).
- If \( \theta_i = G \), then:
  - If among \( N_i \), more than or equal to \( R(d_i, \gamma_i) \) many firms play \( D \), then \( a_i = D \).
  - If among \( N_i \), less than or equal to \( R(d_i, \gamma_i) \) many firms play \( D \), then \( a_i = C \).

The exact characterization of the cooperating equilibrium depends on the structure of \( (N, E) \). In order to state the main theorems, it suffices to find the cooperating equilibrium payoffs for a specific configuration. A star-shaped network is one in which one node, called the center, is adjacent to all other nodes, and all other nodes are adjacent to only the center node. Suppose that \( (N, E) \) has a subnetwork disjoint from all other vertices, which is star-shaped. Let \( n_i \) be the center of the star with \( d_i \) leaves.\(^{33}\) If the center firm \( n_i \) gets a good shock, and it has less than or equal to \( R(d_i, \gamma_i) \) many bad counterparties, then in the cooperating equilibrium \( n_i \) continues whereas good counterparties continue and bad counterparties default. If more than \( R(d_i, \gamma_i) \) counterparties get bad shocks, then \( n_i \) defaults. Therefore, the expected payoff of \( n_i \) at the center of a disjoint star subnetwork is given by

\[
V(d, \gamma) = \mathbb{E}_\theta \left[ \max \{ P(C, \{ j \in N_i : \theta_j = B \}|, d_i, \theta_i, \gamma_i), P(D, -, d_i, \theta_i, \gamma_i) \} \right] \\
= \mathbb{E}_{\theta_i} [P(D, -, d_i, \theta_i, \gamma_i)] + \alpha \times \mathbb{E}_{\theta_{-i}} [\max \{ \Delta P(\{ j \in N_i : \theta_j = B \}|, d_i, \gamma_i), 0 \}].
\]

\(^{33}\)The leaves of a star network are all nodes except the center.
Proposition 1. In any network in which a \( \gamma \)-type firm has degree \( d \), its expected payoff is at most \( V(d, \gamma) \).

Proof. Consider any \( E \) and take any firm \( n_i \). The distribution of the number of defaulting counterparties of \( n_i \) in the cooperating equilibrium first-order-stochastically dominates the distribution of the number of directly defaulting counterparties of \( n_i \) due to potential spillovers. The latter equals the distribution of the total number of defaulting counterparties of \( n_i \) if \( n_i \) were at the center of a disjoint star with \( d \) leaves, because there is no second-order counterparty risk for \( n_i \) in the star configuration. The second term in the expression \( V(d_i, \gamma_i) \),

\[
\max \{ P(C, f_i, d_i, G, \gamma_i), P(D, -d_i, G, \gamma_i) \},
\]

is a decreasing function of \( f_i \). Since the expectation of a decreasing function decreases with respect to first order stochastic dominance, \( n_i \) gets at most \( V(d_i, \gamma_i) \).

Therefore, a disjoint star subnetwork is an ‘ideal’ configuration for the center of the star conditional on its degree, in the sense that it cannot achieve a higher expected payoff in any other network in which it has the same degree. Thusly, call \( V(d, \gamma) \) the \( \gamma \)-ideal payoff conditional on degree \( d \). Also consider the best degree conditional on being at the center of a star subnetwork in a network of \( m \) firms:

\[
d^*(m, \gamma) := \arg\max_{d < m} V(d, \gamma).
\]

Call \( d^*(m, \gamma) \) the \( \gamma \)-ideal degree among \( m \) firms, \( d^*(m, \gamma) + 1 \) the \( \gamma \)-ideal order among \( m \) firms, and the expected payoff \( V(d^*(m, \gamma), \gamma) \) the \( \gamma \)-ideal payoff among \( m \) firms. Note that \( d^*(m, \gamma) \) is a weakly increasing function of \( m \).

The next result states that a clique with firms of equal or higher resilience is another ideal configuration for a firm.

Proposition 2. Consider a clique with \( d + 1 \) firms which is not connected to any other vertices. Consider a firm \( n_i \) in this clique. If all firms in the clique have same or higher resilience than \( n_i \), then \( n_i \) achieves the \( \gamma_i \)-ideal payoff conditional on degree \( d \).

Proof. If \( f \leq R(d, \gamma_i) \) many firms are bad in the clique, all the good firms in the clique can rationalize continuing: when they all continue, continuing is a best reply. This is because they have the same or higher resilience. Bad firms cannot rationalize continuing so they

\[\text{III assume that } P \text{ is such that } V \text{ admits no indifferences over integers. I also assume that a good firm is never indifferent between default or continue: } P(C, f, d, G, \gamma) \neq P(D, -d, G, \gamma) \text{ for any } f. \text{ These are already true for generic } P. \text{ The purpose is to rule out some cumbersome and unintuitive cases of indifference that would unnecessarily make the analysis messier.}

In any case, for the sake of completeness, the following makes sure that these are satisfied. For some \( \hat{P} \) taking values in \( \mathbb{Q} \) and some small \( \epsilon_1, \epsilon_2 \in \mathbb{R}^+ \setminus \mathbb{Q} \), \( P(D, -d, \theta, \gamma) = \hat{P}(D, -d, \theta, \gamma) - \epsilon_1(d + 1) \) and \( P(C, f, d, \theta, \gamma) = \hat{P}(C, f, d, \theta, \gamma) - \epsilon_2(d + 1) \) for all \( f, d, \theta, \gamma \).

\[\text{A clique is a network in which all nodes are adjacent to each other.}\]
always default. Thus from the viewpoint of any single firm $n_i$, if it gets a good shock, and $f \leq R(d, \gamma_i)$ many firms get bad shocks, in the cooperating equilibrium it continues and incurs losses due to $f$ bad counterparties since all other good firms continue as well. If $n_i$ gets a good shock, but $f > R(d, \gamma_i)$, then it defaults and gets the fixed outside option for the good firms. If it gets a bad shock, it gets the fixed outside option for the bad firms. Thus, its payoff is identical to $V(d, \gamma_i)$. 

Finally, define the set of **safe $\gamma$-counterparty degrees** $S(\gamma) := \{d \in \mathbb{N}_0 : R(d, \gamma) \geq d - 1\}$. This is the set of degrees such that having a $\gamma$-type counterparty of such degree does not carry any second-order counterparty risk. Consider a good firm $n_i$, and consider a counterparty $n_j$ of $n_i$ with degree $d_j$. If $n_i$ finds it optimal to default directly, or indirectly due to losses from firms other than $n_j$, then $n_i$ already gets a fixed outside option and does not worry about $n_j$’s action. Otherwise, even if all other $d_j - 1$ counterparties of $n_j$ default, resilience of $n_j$, $R(d_j, \gamma_j)$, is still sufficiently large for $n_j$ to continue if $n_i$ continues. Thus, conditional on $n_i$ being a good firm, having a partner $n_j$ with a safe $\gamma_j$-counterparty degree does not bring more counterparty risk to $n_i$ than what $n_j$ already brings individually as first-order counterparty risk. That is, a counterparty with a safe counterparty degree has the highest resilience a firm could possibly need in its counterparties.

**Proposition 3.** Consider two counterparties $n_i$, $n_j$ that both achieve their ideal expected payoffs conditional on their degrees. Then, either

- they both have unsafe counterparty degrees, their set of their counterparties are identical except each other, and they have the same resilience, or
- they both have safe counterparty degrees.

*Proof. For any $n_x, n_y \in N$, let $d_{xy} = |N_x \cap N_y|$. Take an arbitrary $E$, and a firm $n_x$ with degree $d_x$. Take any counterparty $n_y \in N_x$. Suppose that $min \{R(d_x), d_{xy}\} + (d_y - d_{xy} - 1) > R(d_y)$. Then in the event that $min \{R(d_x), d_{xy}\}$ many firms in $N_x \cap N_y$ and all the $d_y - d_{xy} - 1$ many firms in $(N_y \setminus \{n_x\}) \setminus N_x$ get bad shocks, and all else get good shocks, $n_y$ would default, and that would cause $n_x$ to incur a non-zero loss on top of the direct costs from bad partners. That is, there is second-order counterparty risk for $n_x$ through $n_y$. Due to the existence of such a positive probability event, conditional on the event that both $n_x$ and $n_y$ are good, and less than $R(d_x)$ many counterparties of $n_x$ are bad, the distribution of the number of defaulting counterparties of $n_x$ in $(N, E)$ first order stochastically dominates the same distribution in the case when $n_x$ were at the center of a star with $d_x$ leaves, i.e. no second-order counterparty risk case. Hence, $n_x$’s expected payoff is strictly less than $V(d_x, \gamma_x)$. That is, if $n_x$ achieves $V(d_x, \gamma_x)$, for all counterparties $n_y$ of $n_x$, $min \{R(d_x), d_{xy}\} + (d_y - d_{xy} - 1) \leq R(d_y)$ is satisfied.
Since both $n_i$ and $n_j$ achieve their ideal payoffs conditional their degrees, the inequality is satisfied for both. \[
\min \{R(d_i), d_{ij}\} + d_j - d_{ij} - 1 \leq R(d_j) \] and \[
\min \{R(d_j), d_{ij}\} + d_i - d_{ij} - 1 \leq R(d_i).
\]

If one of them, say $n_i$ has a safe counterparty degree, $R(d_i, \gamma_i) \geq d_i - 1 \geq d_{ij}$, so that \[
\min \{R(d_i, \gamma_i), d_{ij}\} = d_{ij}.
\] Then the inequality becomes $d_j - 1 \leq R(d_j)$. Thus, the other $n_j$ must also have a safe counterparty degree.

Consider the case in which both have unsafe counterparty degrees. $d_j \notin S(\gamma_j)$ so $R(d_j, \gamma_j) < d_j - 1$. Then \[
\min \{R(d_i, \gamma_i), d_{ij}\} < d_{ij}.
\] That implies \[
\min \{R(d_i, \gamma_i), d_{ij}\} = R(d_i, \gamma_i)
\] so that \[
R(d_i, \gamma_i) + d_j - d_{ij} - 1 \leq R(d_j, \gamma_j).
\] Similarly if $d_j \notin S(\gamma_j)$, \[
R(d_j, \gamma_j) + d_i - d_{ij} - 1 \leq R(d_i, \gamma_i).
\] Add both up to get $d_i + d_j \leq 2(d_{ij} + 1)$. That implies that $d_i = d_j = d_{ij} + 1$, which in turn implies that $N_i \setminus \{n_j\} = N_j \setminus \{n_i\}$. Put that back into the inequalities to get $R(d_i, \gamma_i) = R(d_j, \gamma_j)$. \hfill \Box

The only way two counterparties with unsafe counterparty degrees get their ideal payoff conditional on their degrees is that none of them increases the second-order counterparty risk of the other. This is only possible if they have exactly the same counterparties and resilience. Another remark is that a firm with a safe counterparty degree cannot achieve its ideal payoff conditional on its degree if it has any counterparty with an unsafe counterparty degree.

**Corollary 1.** Take any component.\textsuperscript{36} All firms in the component achieve their ideal payoffs given their degrees if and only if either

- they all have unsafe counterparty degrees, the component is a clique (hence all have same degree), and they all have the same resilience, or
- they all have safe counterparty degrees.

Note that this corollary does not state anything about what the ideal degrees are. This is solely conditional on given degrees. In the next subsection, I pin down the networks firms form using Propositions 1, 2, and 3. Then I investigate measures of systemic risk.

### 3.2 Strongly stable networks

**Homogenous firms:** Here I consider the case when all firms are of type $\gamma$. Therefore, suppress the dependence on $\gamma$ in the notation for simplicity until further notice. By Propositions 1 and 2, a disjoint clique is an ideal configuration for all firms in it. The idea is

\textsuperscript{36}Two nodes are connected if one can be reached from the other in a sequence of adjacent nodes. A subnetwork is connected if any two nodes in it are connected. A component is a maximally connected subnetwork: it is connected and if any other node is added to the subnetwork it is not connected anymore.
that a disjoint clique eliminates any second-order counterparty risk for members, because all counterparties of a firm’s counterparties are its counterparties, and there are no second-order counterparties. The reason is partly that, any ‘second-order counterparty risk’ in a clique is already accounted for in the first-order counterparty risk since all counterparties of a firm’s counterparties are already its counterparties in the clique. Indeed, as pointed out, a firm with degree $d$ can achieve $V(d)$ only if it can eliminate second-order counterparty risk completely. Propositions 1, 2, and 3 lead the way to the main theorems of the section without government.

**Theorem 1.** (Pareto strongly stable networks) Let $d^* = d^*(k) + 1$. The set of Pareto strongly stable networks is as follows.

- If $d^* \not\in S$, and $k$ is divisible by $d^* + 1$: $\frac{k}{d^*+1}$ many disjoint cliques of order $d^* + 1$.
- If $d^* \in S$ and $kd^*$ is an even number: any $d^*$-regular network.
- Otherwise, Pareto strongly stable networks do not exist.

Proof. If there is any firm who is not achieving $V(d^*)$ payoff, the ideal payoff among $k$ firms, then this firm, and $d^*$ other firms could deviate to forming a disjoint clique of order $d^* + 1$ and all get $V(d^*)$. This would be a Pareto improvement. Hence, in any Pareto strongly stable network, all firms must achieve $V(d^*)$. The only way this is possible is as follows. First, all firms must have degree $d^*$. Also, by Propositions 1, 2, 3, if $d^*$ is an unsafe counterparty degree, network must be in disjoint cliques, which is only possible when $k$ is divisible by $d^* + 1$. If $d^*$ is a safe counterparty degree, network must be any $d^*$-regular structure, which is possible only when $kd^*$ is even. In these configurations, all firms get their ideal payoffs among $k$ firms, so there are no Pareto profitable deviations.

Pareto strongly stable networks may not exist due to integer problems. However, strongly stable networks always exist. Stating the set of strongly stable networks requires some additional notation. Construct a sequence iteratively as follows. Set $n_0 = k$. For $t \geq 1$, as long as $d^*(k_t) \not\in S$, set $k_t = k_{t-1} - d^*(k_{t-1}) - 1 \geq 0$. Let $k_\kappa$ be the last element of the sequence: $d^*(k_\kappa) \in S$. That is, find the ideal degree among the remaining number of firms, and separate that many plus one firms aside. Iterate, and stop when ideal degree is a safe counterparty degree.

**Theorem 2.** (Strongly stable networks)

- (Existence) The following is a strongly stable network: There are $\kappa$ disjoint cliques with

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37 Order of a subnetwork is the number of nodes in it.
38 A network is $d$-regular if all nodes have degree $d$.
39 Non-existence is due to cycles of deviations that arise solely due to integer problems.
orders $d^*(k_{t-1}) + 1$, for $t = 1, 2, ..., \kappa$, and another disjoint residual subnetwork which is almost-$d^*(k_\kappa)$-regular\(^{40}\) among the $k_\kappa$ remaining nodes.

- **(Almost uniqueness)** In any strongly stable network, there are $\kappa$ disjoint cliques with orders $d^*(k_{t-1}) + 1$ nodes, for $t = 1, 2, ..., \kappa$. The remaining $k_\kappa$ nodes constitute an approximately-$d^*(k_\kappa)$-regular\(^{41}\) network.\(^{42}\)

Proof. (Existence) As I stated before, by Propositions 1 and 2, being part of a clique with order $d^*(k_0) + 1$ gives the highest payoff any configuration can achieve for a firm among a network of $k_0$ firms. Therefore, nodes in the clique with order $d^*(k_0) + 1$ have no incentive to deviate to any other network. The argument can be applied iteratively for the $\kappa$ cliques. As for the remaining almost-$d^*(k_\kappa)$-regular part, all nodes have degree $d^*(k_\kappa) \in S$ (except possibly one which is not connected to anyone). That is, all these remaining nodes (except the singleton) have safe counterparty degrees. Then there is no second-order counterparty risk and two good counterparts are sufficient for each other to rationalize continuing. Hence for any firm (except the singleton) has $V(d^*(k_\kappa))$ expected payoff, which is the highest any can achieve among $k_\kappa$ people. If there is a singleton left-over firm with degree 0, it cannot convince anyone to deviate either, because everyone else is already getting their maximum possible payoff among people they could convince to deviate.

(Almost uniqueness) Take any strongly stable network. Let $d^* = d^*(k_0)$. First consider $d^* \notin S$. If all nodes have strictly less than $V(d^*)$ expected payoff, $d^* + 1$ of them can deviate to a $(d^* + 1)$-clique and improve. Hence, there is at least one firm who gets $V(d^*)$ payoff, say $n_{i_0}$. Then $d_{i_0} = d^* \notin S$.

For any counterparty of $n_{i_0}$ which gets $V(d^*)$, say $n_j$, it must be that $d_j = d^* \notin S$. By Proposition 3, $N_{i_0} \setminus \{n_j\} = N_j \setminus \{n_{i_0}\}$. Let $N_0 = N_{i_0} \cup \{n_i\}$. Thus all firms in $N_0$ which get $V(d^*)$ are connected to all other firms in $N_0$, and none else.

Consider firms in $N_0$ that get less than $V(d^*)$, say $N_1$. Suppose that $N_1 \neq \emptyset$. Consider the deviation by $N_1$ in which they keep all existing edges with $N_0$, they connect all of the missing edges in $N_1$, and they cut all edges they have with $N_0^C$. After this deviation, $N_0$ becomes a

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\(^{40}\)A network is almost-$d$-regular if all nodes, except at most one of them, have degree $d$ and the possible residual node has degree 0. An almost-$d$-regular network always exists among $d + 1$ or more nodes.

\(^{41}\)A network is approximately-$d$-regular if all nodes, except at most $d$ of them, have degree $d$.

\(^{42}\)Concerning the remaining $k_\kappa$ nodes, more can be said on the structure of the subnetwork using Erdos-Gallai Theorem. If the degree sequence of the remaining $k_\kappa$ firms is given by $x_1,...,x_\kappa$, then the sequence $d^*(k_\kappa) - x_\kappa,...,d^*(k_\kappa) - x_1$ cannot be a graphic sequence.

A graphic sequence is sequence of integers such that there is a simple graph whose node degrees are given by the sequence. Erdos-Gallai Theorem provides a necessary and sufficient condition for a sequence being graphic.
is an even number. Suppose that for each safe class $\iota$, call this class a $d^\iota$ and the resulting resiliences are the same, say that the equivalence classes by $k$ relation. Consider configuration with the same degree sequence.

Therefore, $N_1 = \emptyset$, so that $N_0$ is already a $(d^\iota + 1)$-clique.

All in all, in any strongly stable network of $k_0$ nodes, if $d^\iota(k_0) \not\in S$, there exists a disjoint clique of order $d^\iota(k_0) + 1$. Now the same arguments can be repeated for firms in the remaining $k_1 = k_0 - d^\iota(k_0) - 1$ nodes. Then among those, there must be a clique with $d^\iota(k_1) + 1$ nodes, then $d^\iota(k_2) + 1$ nodes.... as long as $d^\iota(k_t) \not\in S$.

When $d^\iota(k_\kappa) \in S$ first time in the sequence, for the remaining $k_\kappa$ people, among them there cannot be $d^\iota(k_\kappa) + 1$ or more people that have degree other than $d^\iota(k_\kappa)$ because then $d^\iota(k_\kappa) + 1$ many would deviate and form a clique, and get $V(d^\iota(k_\kappa))$.

The tighter condition mentioned in the footnote is also necessary. If the sequence $d^\iota(k_\kappa) - x_1, ..., d^\iota(k_\kappa) - x_\kappa$ is graphic, then an appropriate isomorphism of the graph with this particular degree sequence can be joined with the existing remainder, so that all deviators increase their degree to $d^\iota(k_\kappa)$. This way, all firms achieve their ideal payoffs among $k_\kappa$ firms, so that all deviators get strictly better off.

Heterogenous firms: Now consider heterogenous firms again. Let $\Gamma = \{\gamma^1, ..., \gamma^9\}$. For any number of firms $m \in \mathbb{N}$ and any two types $\gamma, \gamma' \in \Gamma$, if their ideal degree among $m$ firms, and the resulting resiliences are the same, say that $\gamma$ and $\gamma'$ are $m$-similar: $d^\iota(m, \gamma) = d^\iota(m, \gamma')$ and $R(d^\iota(m, \gamma), \gamma) = R(d^\iota(m, \gamma'), \gamma')$. Notice that $m$-similarity is an equivalence relation. Consider $k$ firms in $N$, and the equivalence classes induced by $k$-similarity. Index the equivalence classes by $\iota$. Let $k^\iota$ be the number of firms in equivalence class $\iota$. For an equivalence class $\iota$, denote the ideal degree and induced resilience of the class with $d^{\iota\iota} = d^\iota(k, \gamma)$ and $R^{\iota\iota} = R(d^\iota(k, \gamma), \gamma)$, where $\gamma$ is an element of the equivalence class. If for an equivalence class $\iota$, the ideal degree among $k$ firms is a safe counterparty degree, $R^{\iota\iota} \geq d^{\iota\iota} - 1$, call this class a safe class, otherwise unsafe class.

Suppose that for each safe class $\iota$, $k^\iota$ is divisible by $d^{\iota\iota} + 1$, and for each unsafe class $\iota$, $d^{\iota\iota}k^\iota$ is an even number.

Proposition 4. (Pareto strongly stable networks) The following is the set of Pareto strongly stable networks. Disjoint cliques of $k$-similar unsafe classes with their ideal order among $k$ firms, $d^{\iota\iota} + 1$, and a disjoint subnetwork of safe classes, in which each has their ideal degree among $k$ firms.\footnote{In the “safe” part of the network, firms can also become counterparties with other classes with respect to $k$-similarity since they all have safe counterparty degrees.} \footnote{Such a remainder subnetwork exists: an example is disjoint cliques of ideal order. It can be any other configuration with the same degree sequence.} \footnote{Under these divisibility conditions, strongly stable networks that are not Pareto strongly stable can be}
Proof. Similar to Theorem 1.

Note that the cliques can have different orders since members of separate equivalence classes may demand various degrees. This result illustrates that network formation theorems are not artifacts of symmetry of firms, they are rather consequences of matching and sorting.

Proposition 4 does not exhaust all possibilities for Pareto strong stability. Under divisibility conditions on the numbers of each type, different than those in the proposition, there could still exist Pareto strongly stable networks, which is not the case in Theorem 2.

As for strongly stable networks, construct a sequence in the following way, \( k_0 = k \). Pick any type \( \gamma^1 \), let \( k_1 = k_0 - d^*(k_0, \gamma^1) - 1 \). Pick any type \( \gamma^2 \) (it can be the same with \( \gamma^1 \)), let \( k_2 = k_1 - d^*(k_1, \gamma^2) \). At any step \( \kappa \), if for all types \( \gamma \in \Gamma \), \( k_\kappa \in S(\gamma) \) or the number of \( k_\kappa \)-similar types of \( \gamma \) are less than \( d^*(k_\kappa, \gamma) + 1 \), stop. Call each such sequence \( k_0, ..., k_\kappa \) a feasible sequence.

**Proposition 5. (Strongly stable networks, necessary condition)** Any strongly stable network satisfies the following. There exists a feasible sequence \( \{k_t\}_{t=0}^{\kappa} \) such that, in the network there are \( \kappa \) disjoint cliques which consist of \( k_{t-1} - k_t \) many \( k_{t-1} \)-similar nodes, for \( t = 1, 2, ..., \kappa \), and another disjoint subgraph with \( k_\kappa \) nodes.

Proof. Similar to necessity part of Theorem 2.

When there is heterogeneity, the remainder term is problematic due to integer problems that arise. If the partition induced by the equivalence classes on \( \Gamma \) with respect to \( k_\kappa \)-similarity is not the trivial partition with one element, then the sorting argument fails. Firms, whose ideal degrees among the remainder \( k_\kappa \) firms are unsafe counterparty degrees, are not able to achieve their ideal payoff among the remaining \( k_\kappa \) firms anymore. Thus sorting trick does not work any further. This may lead to non-existence of strongly stable networks. However, if there are appropriate numbers of firms from each type in \( N \), so that integer problems do not arise in the remainder, existence and uniqueness is restored already for Pareto strongly stable networks.

### 3.3 Illustrations and phase transition

Here I mainly focus on how the network topology and resulting systemic risk evolves as the number of firms increase. The function \( V \) encodes the changes in the network topology.\footnote{Different only in the remainder subnetwork by at most \( \tilde{d} \) firms where \( \tilde{d} \) is the largest ideal degree among \( k_\kappa \) firms across firms in the remainder.}
The limit behavior of $V$ dictates a particular structure for all networks above a certain size. However, the transition to large networks from small networks can be erratic. In particular, for relatively small numbers of firms, the network topology can exhibit discontinuous changes, a phase transition, when one more firm is added to the economy. I use homogenous types for illustrations, so drop $\gamma$ from notation for now.

**Large networks:** Recall that $d^*(m)$ is a weakly increasing in $m$. Let $d^* = \lim \sup_{m \to \infty} d^*(m)$. If $V(d)$ has a global maximizer, it is $d^* < \infty$. Otherwise $d^* = \infty$.

**Corollary 2. (Large networks)** If $d^* < \infty$, for $k > d^*$, Pareto strongly stable networks are $d^*$-regular, in cliques or arbitrary configurations depending on resilience $R(d^*)$. The network is sparse. If $d^* = \infty$, Pareto strongly stable networks are complete for infinitely many $k$. The network is dense.

**Example 2.** $\alpha = 0.75$; $u(d) = d$, $c(f) = 3f$.

![Figure 6: Measures of systemic risk for Example 2](image)
Example 3. $\alpha = 0.75$; $u(d) = d$, $c(f) = 5f$.

Plot of $V(d)$; Disjoint cliques of order 28 formed

Counterparty risk persists

Figure 7: Measures of systemic risk for Example 3

As the reader might have already noticed, there is a relationship between the long term behavior and the comparison between expected cost of a single edge vs. gain from a single. Consider the specification in examples: additively separable payoffs in $d$ and $f$. Suppose that $u : \mathbb{R}^+ \to \mathbb{R}^+$ is $C^2$, increasing, concave, whereas $c : \mathbb{R}^+ \to \mathbb{R}^+$ is $C^2$, strictly increasing, and convex. Let $l = \lim_{d \to \infty} \frac{u(d)}{c(d(1-\alpha))}$. Notice that if $l < 1$, then $V$ goes to 0 as $k \to \infty$, so $V$ has a global maximizer, say $d^* + 1$. Hence Pareto strongly stable networks consist of $(d^* + 1)$-cliques for $k > d^*$. If $l > 1$, $V$ is unbounded. Hence Pareto strongly stable networks are complete for infinitely many $k$.

The limit of the rate of return from having more edges and the expected cost of these edges (modulo contagion costs which is eliminated by the clique structure) determine whether the
network grows unboundedly or not. For low expected rates of return from having counterparties, in order to prevent contagion becoming almost certain, firms persist in isolating clusters, and contagion persists in the limit at a bounded rate. For higher rates of return, the one clique, complete network, keeps growing since contagion diminishes in the limit due to high rate of return.

**Small networks:** Recall that \( d^*(m) \) is weakly increasing in \( m \). Hence, the size of the cliques formed never decrease when new firms are added to the economy. Here I look into the rate at which the size increases with \( m \). That is, as more firms are added, would the cliques grow smoothly, or would there be an abrupt jump in the size? The significance of this question is as follows. When the economy is growing in the sense that the number of firms is increasing, if the network topology changes radically after a threshold number of firms leading to a jump in systemic risk, this may call for network related policy measures as a function of the size of the economy with regards to the number of firms.

**Corollary 3. (Phase transition)** If \( V(d) \) has a local maximum which is not a global maximum the network topology exhibits phase transition in the number of firms. Formally, for some \( k \), the order of cliques in the network increases by more than the number of firms added to the economy. For any \( k \) for which such a jump happens, the network actually jumps to a complete network \( K_{k+1} \).

This situation can occur for various reasons regarding the fundamentals. One possibility is that benefits are ‘more convex’ than costs, but costs are relatively large for small degrees.

**Example 4.** \( \alpha = 0.75; \ u(d) = d^{1.2}; \ c(f) = 15f \).

![The expected payoff in a clique](image-url)

*Figure 8: \( V(d) \) for Example 4*
Here in this example, the network exhibits a phase transition. For \( k \leq 5 \), \( d^*(k) = k - 1 \) so a complete network is formed. For \( 6 \leq k \leq 276 \), \( d^*(k) = 4 \) and there are as many cliques of order 5 as possible, and possibly a residual subnetwork.\(^{46}\) For \( k \geq 277 \), \( d^*(k) = k - 1 \) and a complete network is formed.

\[
\begin{align*}
\text{Figure 9: Example 4; Phase transition of strongly stable networks: } & k = 80, 160, 275, 277 \\
\end{align*}
\]

The phase transition of the network architecture causes a radical jump in systemic risk, the probability of systemwide failure.

\[
\begin{align*}
\text{Figure 10: Measures of systemic risk for Example 4} \\
\end{align*}
\]

**Corollary 4.** If \( V(d) \) does not have a local maximum which is not a global maximum, the network topology changes smoothly in the number of firms. Formally, the order of cliques in the network never increases by more than the number of firms added to the economy.

\(^{46}\)These are strongly stable networks. Pareto strongly stable networks exists only for \( k \) divisible by 5 if \( k \) is between 6 and 276.
Then, it is important for economic intuition to pin down conditions on fundamentals that determine when $V$ can have a local maximum that is not a global maximum. This situation occurs typically when costs dominate benefits for small degrees, yet benefits can catch up for large degrees. In this case, firms in a “small economy” prefer to have some counterparties in order to have any small expected payoff even if it is very unlikely. However, they refrain from having more counterparties since that makes losses more likely. The potential losses are already very large compared to benefits so that very few counterparty losses would already force the firm into default. Once some positive expected return is ensured, the highest priority is to reduce the likelihood of counterparty losses. As soon as economy is “large” enough that potential benefits finally catch up with expected costs, firms form a giant cluster.

Technically, this would happen if $P$, as a function of $d$ or $f$, has derivative\(^{47}\) which is not monotonic, such as an elliptic curve. For $P$ that is concave or convex in $d$ or $f$, benefits being “more convex” than costs results in local maxima for smaller values, but is later superseded after a threshold since benefits are “more convex”.

Another situation in which such a jump may happen is when firms have some ‘equity’ which does not depend their degree. In this case, firms fear the risk losing the equity if they incur too many counterparty losses. Hence, as long as expected benefits are not large enough, firms do not form any links. When the economy is large enough so that the cost of losing the equity is small compared to the opportunity cost of not forming a giant cluster, firms form all links possible. That is, even if costs are more ‘convex’ than benefits, due to the equity, there is more to lose, hence firms wait for connecting until it becomes sufficiently beneficial to do so.

I have illustrated via examples how systemic risk changes with the network topology. Simulations verify that clique structure consisting of cliques with order $d + 1$ indeed minimizes certain measures of systemic risk across all $d$-regular networks. Such a result would formalize the idea that payoff maximizing firms, as a byproduct, minimize some certain measures of systemic risk.

Conjecture 1. Take any $d$, and any $k$ which is divisible by $d + 1$. Consider the class of $d$-regular networks. The expected number of first-order indirect defaults (that is the good firms that default due to their bad counterparties), and the probability that all firms default, are both minimized when the network consists of cliques of order $d + 1$ (hence total number of defaults is also minimized since all defaults in a clique are of first order).

Summarizing remarks of the section:

\(^{47}\)The derivative of an appropriately selected continuously differentiable interpolation of $P$
Remark 1: Jointly achieving ideal payoffs conditional on a given level of (degree) connectivity is a matter of completely eliminating second-order counterparty risk from the component. If not all firms are at naturally safe levels of connectivity for their counterparties, then the only way to eliminate second-order counterparty risk is to form an isolated dense cluster, which is possible only if all firms have identical connectivity and resilience.

Remark 2: Each firm first evaluates the optimal level of first-order counterparty risk exposure it individually prefers. Then firms collectively implement these levels in dense isolated clusters. Cluster structure eliminates second-order counterparty risk and locally minimize contagion. Thusly, firms maximize their payoff.

Remark 3: Even in the absence of intervention, the endogenous network topology may feature a discontinuous phase transition as the economy gets larger. The transition may lead to abrupt changes in systemic risk.

Remark 4: For large numbers of firms, if the expected rate of return from having one more counterparty is positive, firms form complete network and first-order counterparty risk diminishes. However, if the expected rate of return is negative, first-order counterparty risk, hence ‘firebreaks’, persist.

4 Presence of government intervention

4.1 Preliminaries

It is natural to think that fundamentally good firms with high counterparty losses could continue operations in the market instead of defaulting. Hence a good firm being forced into indirect default due to counterparties may cause some welfare loss. Government, then, may wish to step in and save good firms to increase welfare. The following example illustrates this possibility in the context of my model.

Recall Example 1: $u(d) = d$, $c(f) = 2f$. 2 firms get bad shocks, and 6 get good shocks as depicted in the lefthand network. The network on the right illustrates the order in which firms default during contagion.
In this example, if government could convince the first wave of indirect defaults, labelled by “1”, to continue instead of defaulting, contagion would stop. Only bad firms would fail and all good firms would survive. Four good firms would be saved.

**Intervention:** Consider a government that has the ability to intervene in the market at the end of stage two. Government announces and commits to a transfer scheme, think capital injections, \( \{T_i(\tilde{a} | \tilde{\theta}, E, \tilde{\gamma})\}_{i \in N} \geq 0 \). Here \( T_i \) describes the amount of transfer to firm \( n_i \). The intervention is time-consistent in the sense that the government cannot commit to an intervention policy before stage two. However, it can commit from the end of stage two onwards. For simplicity, drop the notation \( (\tilde{\theta}, E, \tilde{\gamma}) \) since those are all set when the government intervenes.

In general \( T_i \) depends on the entire action profile \( \tilde{a} \). But then, the induced game in stage three may not be supermodular anymore. Hence it is necessary to restate the definition of cooperating equilibrium in a way consistent with the original definition. For now, I assume that \( T_i \) depends only on \( a_i \). This way the induced game is supermodular and cooperating equilibrium is well defined. In Section 6.4, I define the appropriate generalized cooperating equilibrium and show that the optimal policy (yet to be defined) has the property that \( T_i \) depends only on \( a_i \). I skip that for now in order not to complicate the exposition.
A transfer to \( n_i \) who would otherwise default, which makes sure that it continues in the induced cooperating equilibrium can be interpreted as bailing-out firm \( n_i \). Formally, \( T \) is said to bailout \( n_i \) if

1. \( T_i(C) > T_i(D) = 0, \)
2. in the cooperating equilibrium induced by \( T \), \( a_i = C, \)
3. for any \( T' \) with \( T'_{-i} \equiv T_{-i}, T'_i(D) = 0, T'_i(C) < T_i(C), \) in the cooperating equilibrium induced by \( T' \), \( a_i = D. \)

**Notion of welfare:** Each firm \( n_i \) generates some welfare given by \( W(a_i, d_i, \theta_i, \gamma_i) - \chi f_i \), for some constant \( \chi \geq 0 \). Total welfare is given by the sum over all firms. Welfare \( W_i \) is not necessarily the same with payoff of firm \( n_i \), which is natural in this context. I assume that good firms increase welfare by continuing:

**Assumption 3.** \( W(C, d, G, \gamma) > W(D, d, G, \gamma) \) for all \( d, \gamma. \)

Depending on the interpretation of the model appropriate notion of welfare can have more structure. In the baseline case of government intervention, bailouts are costless, the government is not restricted by a budget constraint, there is no incomplete information, bad firms reduce welfare by continuing as opposed to defaulting (\( W(D, d, B, \gamma) > W(C, d, B, \gamma) \) for all \( d, \gamma) \), and the government is not restricted by an ex-ante commitment to bailing-out only systemically important firms. Moreover counterparty losses do not enter welfare significantly, that is \( \chi = 0. \)

First, the welfare loss from a default decision of a firm \( n_i \) is already accounted for in \( W_i \), and there is no need to double-count the loss in welfare generated by each counterparty of \( n_i \). Second, I want to abstract away from the incentives of the government to subsidize firms that do not face the risk of default. This way the focus is on contagion. All of these specifications of the baseline case are relaxed one by one in Sections 5 and 6. Indeed, the insights obtained in the baseline case get stronger under these generalizations.

**Optimal policy:** An ex-post welfare maximizing transfer scheme which uses minimal transfers when indifferent is called an optimal policy. Let \( N^B \) and \( N^G \) denote the set of bad and good firms. Also, \( N_i^B = N_i \cap N^B \), \( N_i^G = N_i \cap N^G \) denote the bad and good counterparties of \( n_i \).

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48This definition of bailout does not allow for ‘overcompensating’ a firm for continuing. A more general definition is fine as well. It would only require introducing more definitions later on to distinguish between overcompensating and non-overcompensating bailouts.

49Everything goes through for sufficiently small \( \chi > 0 \) as well.
Proposition 6. The unique optimal policy $T^*$ is given by

$$T^*_i(a_i) = \begin{cases} P(D, -, d_i, G, \gamma_i) - P(C, |N^L_i|, d_i, G, \gamma_i) & \text{if } \theta_i = G, a_i = C, R(d_i) < |N^L_i| \\ 0 & \text{otherwise.} \end{cases}$$

Proof. The optimal policy must induce all good firms to continue and all bad firms to default. Then the question is how to implement this using minimal transfers. Notice that under the desired action profile, without transfers, a good firm $n_i$ has payoff $P(C, |N^B_i|, d_i, G, \gamma_i)$. In order for $n_i$ to continue, counterparty losses from bad firms, who are not getting bailed-out, must be compensated at least to an extent that makes $n_i$ indifferent between defaulting or note. That is the transfer to $n_i$ must be at least $\max \{0, P(D, -, d_i, G, \gamma_i) - P(C, |N^B_i|, d_i, G, \gamma_i)\}$. Otherwise, $n_i$ cannot rationalize continuing.

What remains is to show that this transfer profile indeed induces the desired action profile. Consider an good firm $n_i$. If it believes that all good firms continue, and $n_i$ receives $T^*_i$, then its payoff from continuing is greater than or equal to $P(D, -, d_i, G, \gamma_i)$. Hence $C$ is a best response. Therefore, if all good firms receive $T^*$, then they can all rationalize continuing by believing that all good firms continue. Hence, under $T^*$, in the cooperating equilibrium, all good firms continue and all bad firms default.

This type of bailout policy does not overcompensate firms. It makes the bailed-out firm indifferent between defaulting or not. Hence, the bailed-out firm is only as good as it defaulted. The firm’s ‘initial decision makers’ or ‘previous owners’ receive nothing extra as a result of being bailed-out. Therefore, first-order counterparty risk is unchanged in response to bailouts.

The main channel through which government intervention affects network formation is the second-order counterparty risk. Since government cannot commit to the transfer policy in stage one, firms, during network formation, know that $T^*(\cdot | \vec{\theta}, E, \vec{\gamma})$ will be implemented in stage three. As a byproduct of the optimal policy, in the presence of intervention, each firm knows that any good counterparty is, one way or the other, going to continue. This eliminates second-order counterparty risk.
Byproduct of optimal intervention: second-order counterparty risk is eliminated. ‘White circle’ does not face any risk of ‘yellow square’ indirectly defaulting due to ‘red diamonds’.

Figure 13: The main channel for intervention’s effect on the network

Therefore, firms are not concerned with the counterparties of their counterparties any further, although they are still concerned with the shocks of their immediate counterparties. Below I explore how this channel manifests itself in the networks formed.\textsuperscript{50}

4.2 Government-induced interconnectedness

Here I introduce the first effect of government intervention on the network topology. The anticipation of intervention eliminates second-order counterparty risk, and causes the cliques to dissolve. The network becomes interconnected. For the purpose of illustrating this effect it is sufficient to consider homogenous firms.

Assumption (Homogeneity). $\gamma_i = \gamma$ for all $n_i \in N$.

‘Homogeneity’ is maintained in Section 4.2. Thus I drop the $\gamma$ notation until Section 4.3.

Theorem 3. If $kd^*(k)$ is an even number, the set of Pareto stable networks is the set of $d^*(k)$-regular networks, otherwise, empty set.\textsuperscript{51}

For any $k$, any almost-$d^*(k)$-regular network is strongly stable. Any strongly stable network is approximately-$d^*(k)$-regular.

Proof. Now that firms know $T^*$ will be implemented in stage three, they know that all good firms continue and all bad firms default. Then their expected payoffs are given directly by $V$

\textsuperscript{50}In Sections 5 and 6 I consider various extensions on top of the baseline case. Typically, if there are further constraints on the government, the optimal policy may reduce first-order counterparty risk as well as eliminating second-order counterparty risk. In return, all insights of the baseline case are maintained, and further effects on the network topology emerge.

\textsuperscript{51}The clique structure is typically not Pareto strongly stable due to the residual in the presence of intervention. It is Pareto strongly stable if and only if $d^*(k) + 1$ divides $k$. 

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function independent of the topology of the network. In an almost-$d^*(k)$-regular network, all firms except possibly one is getting the maximum possible payoff. Hence, there cannot be any deviations: any almost-$d^*(k)$-regular network is strongly stable. Now take any strongly stable network. There cannot be $d^*(k) + 1$ or more people with degree other than $d^*(k)$ because then they would deviate and form a clique, and achieve $V(d^*(k))$. Hence any strongly stable network is approximately-$d^*(k)$-regular. As for Pareto strongly stable networks, even if one firm has less than $V(d^*(k))$ expected payoff, this firm and $d^*(k)$ other could deviate and form an isolated clique. It would not hurt the other deviators, but strictly benefit the first firm. Thus all firms must be achieving $V(d^*(k))$ in any Pareto strongly stable network, so that all firms must have degree $d^*(k)$. This is possible only if $d^*(k)k$ is even. It is clear that any such network is Pareto strongly stable.

After second-order counterparty risk is eliminated by government, practically, all degrees become safe counterparty degrees even if they don’t belong to $S$. Therefore, cliques dissolve into an interconnected network. However, first-order counterparty risk is not altered. Any bad firm still defaults, hence imposes same costs on its counterparties. This way firms do not overconnect or underconnect, they only make the network more interconnected.

**Example 5.** $\alpha = 0.75$, $u(d) = d$, $c(f) = 6f$, $k = 40$.

![Network topology for Example 5](image)

**Welfare and systemic risk:** Let $d^* = \limsup_{m \to \infty} d^*(m)$. If $V$ does not have a global maximizer, $d^* = \infty$. In that case, for infinitely many $k$, complete network $K_k$ is formed.
For such $k$, government intervention has no effect on the network formed. Moreover, since in a clique (a complete network is also a clique) all indirect defaults are of first order, the indirect defaults in the absence of intervention is one-to-one with bailouts in the presence of intervention. Therefore, quite trivially, government intervention increases ex-ante welfare and does not change systemic risk. If $R(k - 1) \neq k - 1$ welfare gain is strictly positive since the probability of at least one indirect default is positive.

Suppose that $d^* < \infty$. Suppose that $kd^*$ is even. If $R(d^*) \geq d^* - 1$, again, government intervention has no effect on the network formed. $d^*$-regular networks are formed in both cases. Welfare increases and systemic risk is not affected. Moreover, if $R(d^*) \neq d^*$, the welfare gain is strictly positive.

The non-trivial and most relevant case is when $d^* < \infty$ and $R(d^*) < d^* - 1$. Suppose that $k$ is divisible by $d^* + 1$. In this case, absence of intervention leads to cliques of order $d^* + 1$ whereas presence of intervention leads to $d^*$-regular networks.

**Proposition 7.** Government intervention strictly increases ex-ante welfare.

**Proof.** When there are no bailouts, every now and then some good firms default due to sufficiently many counterparty failures. This leads to a loss of welfare by $W(C, d^*, G) - W(D, d^*, G) > 0$ for each such firm. When there are bailouts, since all firms have the same degree $d^*$, the only difference in welfare is the sum of terms $W(C, d^*, G) - W(D, d^*, G) > 0$, which proves that welfare increases. Since $R(d^*) \neq d^*$, there is a positive probability of an indirect default, so that welfare increases strictly. \hfill \Box

**Note.** The clustering coefficient\textsuperscript{53} in the absence of intervention is larger than the clustering coefficient in the presence of intervention. Because, a clique structure gives the maximum clustering possible, 1.

Now I illustrate how various measures of systemic risk change with the anticipation of bailouts with another simpler example.

**Example 6.** $\alpha = 0.75$, $u(d) = d$, $c(f) = 7f$, $k = 20$.

In this example, $V(d)$ is globally maximized at $d^* = 3$. Resilience is $R(d^*) = 0$, so it takes one counterparty to force a firm into default. $R(d^*) < d^* - 1$ so firms form cliques of order 4 in the absence of intervention. In the presence of intervention, they form a 3-regular network.

\textsuperscript{52}Strongly stable networks have small remainder terms. For neat comparison I focus on Pareto strongly stable networks, which are essentially unique. That requires the divisibility assumption.

\textsuperscript{53}The number of triangles in the network divided by the total number of possible triangles. Global and average clustering coefficients are equal since the graph is regular.
Note that in the figures below, indirect ‘default’ refers to bailed-out firms for the interconnected network in presence of intervention, whereas total number of ‘defaults’ refers to direct defaults plus bailouts. The systemic risk comparison is number of indirect defaults in the benchmark case vs. bailouts in the intervention case (firms that would default indirectly if it wasn’t for bailouts). Notice that with cliques, all indirect defaults are of first order. In an interconnected network, even though first-order indirect defaults are not necessarily all indirect defaults, second-order defaults never realize since government bails-out all first-order indirect defaults. Thus, all bailouts are of first-order indirect defaults under government intervention as well. These two magnitudes are being compared: first-order (hence all) indirect defaults in absence of intervention vs. first-order (hence all) bailouts in the presence of intervention. Red dashed line labeled ‘isolated’ represents the clique structure that is formed in the benchmark case. Blue solid line labeled ‘interconnected’ represents the presence of intervention.
An immediate corollary of Conjecture 1, which simulations verify, would be the following.

**Conjecture 2.** Government intervention increases systemic risk:

1) expected number of bailouts in the presence of intervention is larger than the expected number of indirect defaults in the absence of intervention,
2) probability that all firms either default or get bailed-out in the presence of intervention is larger than the probability that all firms default in the absence of intervention.

### 4.3 Government-induced systemic importance

Now I introduce the second effect of government intervention. When there is heterogeneity in firm types $\gamma$, typically different types have different ideal degrees. Without loss of generality, there will be some firms with higher ideal degree among $k$ firms. Below is an example of how difference in $\gamma$ can alter $P$ and generate heterogeneity in demand.

**Example 7.** $\alpha = 0.75$. $\Gamma = \{\gamma^1, \gamma^2, \gamma^3, \gamma^4\}$ is given as follows. For all $\gamma \in \Gamma$, $u(d, \gamma) = d$. Moreover, $c(f, \gamma^1) = 7f$, $c(f, \gamma^2) = 6f$, $c(f, \gamma^3) = 5f$, $c(f, \gamma^4) = 4f$. 

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![Probability mass function](image1.png)

![Cumulative distribution function](image2.png)

Figure 17: Total defaults in the absence of intervention vs. Bailouts and Defaults in the presence of intervention for Example 6
If the types that demand higher degree demand too high of a degree, there would not be sufficiently many firms with \( k \)-similar types, so that these firms would not be able to convince other types to become counterparties. The reason is that the types that demand higher degrees, if not very resilient, would bring on second-order counterparty risk to the other types that prefer lower degree. However, in the presence of government intervention, second-order order counterparty risk is eliminated so that other types would also be willing to become counterparties. This way, types with high ideal degrees among \( k \) firms become central to the network, and the network typically becomes a core-periphery network. In order to illustrate this effect, it is sufficient to have two types. Arguments can be generalized to more than two types. Suppose that there is one with high demand for having counterparties, the other with low demand for counterparties. Again for simplicity of statements of results I assume away integer problems and the cases in which ideal degrees are safe counterparty degrees. Results
can be generalized to non-divisibility and unsafe counterparty degree cases analogous to the benchmark case.

**Assumption** (Minimal heterogeneity). Two types: $\Gamma = \{L, H\}$. There are $k_L$ many $L$ firms and $k_H$ many $H$ firms.

*Demand heterogeneity:* $V(d, L)$ is maximized at $d^*_L$ on $[0, k]$. $V(d, H)$ is increasing on $[k_H, k]$.

*Non-trivial resilience:* $R(d^*_L, L) < d^*_L - 1$, and $R(d, H) < d - 1$ for all $k - 1 \geq d \geq k_H$.

*No integer problems:* $d^*_L + 1$ divides $k_L$ and $(d_L - k_H)k_L \geq 0$ is an even integer.

‘Minimal heterogeneity’ is maintained in Section 4.3. Call $L$ the low (counterparty demanding) type and $H$ the high (counterparty demanding) type since $d^*(m, H) = m - 1$ for all $m \in [k_H, k]$.

**Theorem 4.** In the absence of intervention, the unique Pareto strongly stable network is given by disjoint cliques of order $d^*_L + 1$ of low types, and another disjoint clique of order $k_H$ of high types. In the presence of intervention, the unique Pareto strongly stable network structure is given by core-periphery: high types (core) are counterparties with all firms, and low types (periphery) form a $(d_L - k_H)$-regular subnetwork among each other aside of their links with high types.

**Proof.** Consider any Pareto strongly stable network. Similar to before, all low types must achieve their $L$-ideal payoffs. Recall the proof of Proposition 3. A low type can achieve its $L$-ideal payoff only if its all counterparties are also low type, or high type with a safe counterparty degree. But, a high type having a safe counterparty degree means its degree is less than $k_H$. That is, any high type who has a low type counterparty must have degree less than $k_H$. These high types, then, are getting less than their $H$-ideal payoff among $k_H$ firms. The other high types, those that are not adjacent to any low type firms are also getting less than or equal to their ideal degree among $k_H$ firms. Then, all high types can deviate to form an isolated clique, which Pareto improves all of them. Therefore, no low type can have any high type counterparty. Then the only Pareto strongly stable option among themselves, as in the benchmark case, is disjoint cliques since $d^*_L$ is not a safe counterparty degree. For high types, also, a clique of $k_H$ firms is the only Pareto strongly stable configuration among themselves. What remains is to show that this is indeed Pareto strongly stable. Similar to the idea outlined here, if there was a deviation involving any low type, it must be as good as its ideal payoff, which would make sure any high type that connects with must get strictly worse off due to since it must reduce its degree below $k_H - 1$. There is no Pareto deviation among high types by themselves either. Thus the network is Pareto strongly stable.
Under the anticipation of intervention, all firms are, via intervention, safe counterparties with regards to second-order counterparty risk. Then in any of the described core-periphery networks, all firms are getting their ideal payoffs among \( k \) firms. Then there is no Pareto profitable deviation. Then in any Pareto strongly stable network, all firms still must get their ideal payoffs among \( k \) firms, which implies that all high types must have degree \( k - 1 \), which then implies that all low types must have degree \( d_L - k_H \) among themselves.

**Example 8.** Continue with Example 7, with \( L = \gamma^2 \) and \( H = \gamma^4 \). That is, \( u(d, L) = d, c(d, L) = 6f, \) and \( u(d, H) = d, c(d, H) = 4f \). \( k_L = 50 \) and \( k_H = 6 \).

Under this specification, \( d_L^* = 9 \) and \( R(d_L^*, L) = 1 \). \( V(d, H) \) is increasing and \( R(d, H) = \left\lfloor \frac{d}{4} \right\rfloor \).

In the absence of intervention, network consists of 5 cliques of low types with order 10, and one clique of high types with order 6. With government, network consists of one core of 6 high types, each connected to everyone. Low types are peripheral; each have 3 links with other low types.

**Welfare and systemic risk:** There is a change in systemic risk due to the change in the architecture. Now it takes 13 out of 50 low firms to get bad shocks to force the core firms into default. In that case, however, government saves all 6 core firms, and all is under control.
This is why low types don’t mind connecting with high types at the core even when the core has very large degree. However, if 2 core firms out of 6 get bad shocks, that forces all 50 peripheral firms into default. Government now bails-out all good firms out of 50 peripheral firms. For low type firms, ex-ante it does not matter which 9 firms they have links with; it is the same first-order counterparty risk. Government takes care of second-order counterparty risk, so counterparties can very well be central high types. The following simper example illustrates such effects on measures on systemic risk.\textsuperscript{54}

**Example 9.** Continue with Example 7. \( L = \gamma^1 \) and \( H = \gamma^4 \). \( k_L = 16 \) and \( k_H = 2 \).

That is, \( u(d, L) = d \), \( c(f, L) = 7f \) and \( u(d, H) = d \), \( c(f, H) = 4f \). In this case, \( d_L^* = 3 \) and \( R(d_L^*, L) = 0 \).

\textsuperscript{54}Previous example is computationally challenging to find the exact probability distribution of the number defaults.
Core-periphery structure, makes the ‘very good’ and ‘very bad’ outcomes more likely. Very bad outcome is that core gets bad shocks, and amplifies the contagion among periphery causing most peripheral firms into indirect defaults, hence bailouts. Very good outcome is that the core gets good shocks, and mitigates the contagion among periphery by contributing to resilience of the peripheral firms.

Ex-ante welfare comparison now depends also on how the degree of high types enter welfare: \( W(a_i, d_i, \theta_i, H) \). If welfare is hurt significantly when high types operate at high degree levels, i.e. \( W(a_i, d_i, \theta_i, H) \) is decreasing ‘sharply’ in \( d_i \), ex-ante welfare could be hurt. But this would be an ad-hoc situation. If \( W(a_i, d_i, \theta_i, H) \) is not decreasing, ex-ante welfare would be increased both for inducing \( H \) firms to operate at socially better capacity and making sure good firms continue as it was the case for interconnectedness. In this sense, core-periphery may hold high benefits for welfare even though it increases volatility.

**Discussion of ‘Minimal heterogeneity’:** Note that ‘Minimal heterogeneity’ rules out the case the cases in which \( d^*_L \) is an \( L \)-safe counterparty degree (\( R(d^*_L, L) \geq d^*_L - 1 \)) and \( k - 1 \) is an \( H \)-safe counterparty degree (\( R(k - 1, H) \geq k - 2 \)). In these 4 cases, one has to further consider the sub-cases in which \( R(d^*_L, L) = d^*_L \) and/or \( R(k - 1, H) = k - 1 \) to see whether the welfare gain is strict or not. It leads to 16 cases, cumbersome notation, and long statements for results so I skip these cases. Essentially, interconnectedness and systemic importance can be de-coupled using the resilience of \( L \) and \( H \) types. Resilience of \( L \) captures the change in the interconnectedness across \( L \) firms. Resilience of \( H \) captures the change in the systemic importance of \( H \) firms. However, when resilience of these types are extremely high, there is essentially not a big change in systemic risk since absence of intervention also features little contagion due to high resilience. One exception is that, if resilience of \( H \) types is extremely
high whereas $L$ types are not extremely resilient, even in the absence of intervention, core-periphery would be formed featuring clusters of $L$ firms in the periphery. There would be volatility due to the bad shocks to $H$ firms.

A final remark is that $d^*_L \geq k_H$ is not essential for the strongly stable networks. When $d^*_L < k_H$, there is no Pareto strongly stable network due to cycles of deviations. However, under $d^*_L < k_H$ all networks in which $L$ firms are counterparties with only $H$ firms, and all $H$ firms are counterparties with each other on top of their links with $L$ firms is strongly stable. That is, it is not critical that the periphery has links among each other which requires $d^*_L \geq k_H$. Even when that is not feasible due to $d^*_L < k_H$, there are strongly stable networks that feature core-periphery structure whereas core-periphery is not strongly stable in the absence of intervention.

4.4 Individual risk behavior

For now assume ‘Homogeneity’. Since first-order counterparty risk is not altered, firms do not overconnect or underconnect. So there is a sense in which firms do not make riskier individual choices. Here I try to formalize this idea further and argue that if firms were allowed to choose the risk level of their investments, i.e. alter the distribution of shocks with their choices, government intervention would not cause any change in their choices of risk level, but they would still make the network interconnected.

Suppose that firms can choose to invest in regular projects or high risk/high return projects. Formally, firms now have payoff multipliers $\zeta$ and risk parameters $\alpha$, which are choices. Firm $n_i$ receives $\theta_i = G/B$ with probability $\alpha_i/1 - \alpha_i$, and receives a payoff $\zeta_i \times P(a_i, f_i, d_i, \theta_i)$. Normalize the outside options to 0: $P(D, -, d, \theta) = 0$ for all $d, \theta$.\footnote{Investment risk can be thought of as the risk in the cost $\tilde{c}$ of inputs/supervision in the lead example, or directly a risk in the rate of return $R$.}

A firm $n_i$ can choose $(\alpha_i, \zeta_i) \in \{(\alpha^s, \zeta^s), (\alpha^r, \zeta^r)\}$ where $\alpha^s > \alpha^r$ and $\zeta^s < \zeta^r$. $(\alpha^s, \zeta^s)$ is the regular and safer project, whereas $(\alpha^r, \zeta^r)$ is the high risk/high return project. The choice of project type is simultaneous with network formation. Alter the definition of the network to incorporate the risk choices, and the definition of feasible deviation to allow for each deviating firm to choose a different risk level as well. In the realized network, call firms that chose safer project safe firms, and the others risky firms.

Given $E$, let $d^s_i$ denote the number of safe counterparties of $n_i$ and $d^r_i$ denote the number of risky counterparties of $n_i$. Let $f(x; y, p, z, q)$ denote the probability that out of $y$ trials with a $p$-coin and $z$ trials with a $q$-coin, the total number of heads is $x$. 

\footnote{Investment risk can be thought of as the risk in the cost $\tilde{c}$ of inputs/supervision in the lead example, or directly a risk in the rate of return $R$.}
For given risk profile \( \bar{\alpha} \), a firm \( n_i \)'s expected payoff at the center of a star is

\[
V(d_i^s, d'_i, \alpha_i, \zeta_i) = \alpha_i \zeta_i \times E_{f_i} \left[ \max \left\{ P(C, f_i, d_i, H), 0 \right\} \right],
\]

where \( f_i \sim f(\cdot; d_i^s, 1 - \alpha, d'_i, 1 - \alpha^r) \).

For \( x \in \{s, r\} \), denote \( \tilde{V}(d) := E_f \left[ \max \left\{ P(C, f, d, H), 0 \right\} \right] \) for \( f \sim f(\cdot; d, 1 - \alpha^x) \). Let \( d^{x*} := \arg \max_d \tilde{V}(d) \) and \( \tilde{V}^{x*} = \tilde{V}(d^{x*}) \). The expected payoff of a firm \( n_i \) at the center of a star with \( d_i \) leaves, with all having the same risk choice \( x \in \{s, r\} \), is \( V(d_i, \alpha_i, \zeta_i) = \alpha_i \zeta_i \tilde{V}(d_i) \). Assume away divisibility issues: \( d^{x*} + 1 \) divides \( k \) for both \( x \in \{s, r\} \), and \( k \) is even. Assume that there is second-order counterparty risk at desired interconnectedness: \( R(d^{x*}) < d^{x*} - 1 \) for both \( x \in \{s, r\} \).

**Proposition 8.** In the absence / presence of intervention:

- If \( \alpha^s \zeta^s > \alpha^r \zeta^r \), all firms choose \( s \). Pareto strongly stable networks are: disjoint cliques of order \( d^{ss*} + 1 \) / \( d^{ss*} \)-regular networks.
- If \( \alpha^s \zeta^s \tilde{V}^{ss*} < \alpha^r \zeta^r \tilde{V}^{rs*} \), all firms choose \( r \). Pareto strongly stable networks are: disjoint cliques of order \( d^{rs*} + 1 \) / \( d^{rs*} \)-regular networks.
- If \( \alpha^s \zeta^s < \alpha^r \zeta^r \) but \( \alpha^s \zeta^s \tilde{V}^{ss*} > \alpha^r \zeta^r \tilde{V}^{rs*} \), there does not exist any strongly stable network in either case.

**Proof.** Define \( \tilde{V}(d_i^s, d'_i, \alpha_i, \zeta_i) = E_{f_i} \left[ \max \left\{ P(C, f_i, d_i, H), 0 \right\} \right] \) where \( f_i \sim f(\cdot; d_i^s, 1 - \alpha, d'_i, 1 - \alpha^r) \). \( F(\cdot; y, p, d - y, q) \) first order stochastically dominates \( F(\cdot; y', p, d - y', q) \) if \( y > y' \) and \( p > q \). \( \max \left\{ P(C, f, d, H), 0 \right\} \) is a decreasing function of \( f \). It decreases strictly at \( f = 0 \). So \( \tilde{V}(t, d - t, \alpha, \zeta) \) is strictly increasing in \( t \). Then, \( \tilde{V}^{rs*} = \tilde{V}(0, d^{rs*}, \alpha, \zeta) \) does not exist any strongly stable network in either case.

First part: If \( \alpha^s \zeta^s > \alpha^r \zeta^r \), in any network, any \( r \) firm would deviate unilaterally to \( s \). Thus in any (Pareto) strongly stable network, all firms choose \( s \). If all firms choose \( s \), the only candidates are, as established in benchmark/baseline cases, \( (d^{ss*} + 1) \)-cliques/\( d^{ss*} \)-regular networks. All firms get \( \alpha^s \zeta^s \tilde{V}^{ss*} \) expected payoff. These are (Pareto) strongly stable. Suppose that there is a profitable deviation. Since \( \alpha^s \zeta^s > \alpha^r \zeta^r \) and \( \tilde{V}(t, d - t, \alpha, \zeta) \) is strictly increasing in \( t \), the same network deviation with all deviators choosing \( s \) is also a profitable deviation. After this new deviation, all firms still choose \( s \). Then this must be a profitable deviation from the benchmark/baseline case, which does not exist.

Second part: Since \( \alpha^s \zeta^s < \alpha^r \zeta^r \), in any network, any \( s \) firm would unilaterally deviate to \( r \). Thus in any (Pareto) strongly stable network, all firms choose \( r \). If all firms choose \( r \), the only candidates are, as established in benchmark/baseline cases, \( (d^{rs*} + 1) \)-cliques/\( d^{rs*} \)-regular networks. Now that \( \alpha^s \zeta^s \tilde{V}^{ss*} < \alpha^r \zeta^r \tilde{V}^{rs*} \), these networks are (Pareto) strongly stable. All
firms get $\alpha^s \zeta^s \tilde{V}r^s$ expected payoff. Suppose that there is a profitable deviation. If there are no deviators that chose $s$ in the deviation, the network deviation would be a profitable deviation in the benchmark/baseline case, which does not exist. Take any deviator that chose $s$. Suppose it has $d^s/d^r$ many $s/r$ counterparties after the deviation. Then its payoff is at most $V(d^s, d^r, \alpha^s, \zeta^s) \leq V(d^s + d^r, 0, \alpha^s, \zeta^s) \leq \alpha^s \zeta^s \tilde{V}r^{ss} < \alpha^r \zeta^r \tilde{V}r^r$. Contradiction.

Third part: Since $\alpha^s \zeta^s < \alpha^r \zeta^r$, in any network, any $s$ firm would unilaterally deviate to $r$. Thus in any (Pareto) strongly stable network, all firms choose $r$. But then $d^{ss} + 1$ many firms would get together and form an isolated clique, and all choose $s$. This would improve their payoff since $\alpha^s \zeta^s \tilde{V}r^{ss} > \alpha^r \zeta^r \tilde{V}r^r$.

The ability to choose the risk level is indeed exploited by firms. This choice also affects network architecture by altering the degrees firms have. However, anticipation of government intervention does not lead to any change in the choice of risk or degree, but just serve to interconnect the network.

It is straightforward to extend this proposition to ‘Minimal heterogeneity’ following the same proof. However, it requires cumbersome notation since there are now 9 cases instead of 3 cases. I skip formalizing this to save notation and space.

**Summarizing remarks of the section:**

**Remark 5:** In the absence of government intervention, each firm prefers that the counterparties of its counterparties are only among its own counterparties. This way, it is not exposed to any second-order counterparty risk. This force creates a market discipline guiding firms towards forming clusters, effectively serving as firebreaks. In the presence of intervention, however, second-order counterparty risk is readily eliminated as a byproduct of optimal intervention. Therefore, cliques dissolve into an interconnected network. On the other hand, firms still face the full extent of first-order counterparty risk. Hence no firm over-connects or under-connects. They just make the network more interconnected.

**Remark 6:** Time-consistent bailouts are welfare improving since they do not alter firms’ incentive to take on extra first-order counterparty risk while they save firms that suffer high counterparty losses. However, the network becomes more interconnected and susceptible to contagion since firms no longer worry about second or higher counterparty risk coming from distant parts of the network.

**Remark 7:** Anticipation of government intervention makes some small number of firms more central through the anticipation that they will be bailed-out. The network formed features core-periphery structure as a consequence of anticipation of bailouts. This endogenously correlates the counterparty risk of peripheral firms, which make up the majority of the
economy. If sufficiently many core firms get bad shocks, there would be need for an unusually high number of bailouts among the peripheral firms. If sufficiently many core firms get good shocks, core serves as a buffer. Hence good and bad events become more likely. Therefore, bailouts generate volatility through its effect on the network architecture.

*Remark 8:* Since bailouts do not overcompensate firms, government intervention in my model does not lead to moral hazard in investment decisions: all firms choose the same risk behavior individually and same number of projects. Decision makers lose everything even if their firm is bailed-out, and are only as good as defaulting. As a result there is no incentive to alter risk decisions. However, there is network hazard: second-order counterparty risk is eliminated and firms interconnect the network, leading to increased systemic risk. Also, under heterogeneity, firms form a core-periphery network without changing their risk choices in response to bailouts.

5 Robustness

In the presence of intervention high types strictly prefer the core-periphery over the clique structure. As for low type firms, they are indifferent between connecting with high types or low types in the baseline case. Also, homogenous types are indifferent between an interconnected network and the clique structure in the baseline case. Then a question of robustness arises. In this section I argue that the induced interconnectedness and core-periphery are indeed the robust outcomes. Under slight variations of the model, all firms under heterogeneity strictly prefer the core-periphery network, and all firms under homogeneity strictly prefer making the network more interconnected. The variations I consider are relaxations of the baseline case of government intervention.

Counterparty losses in welfare and subsidy via bailouts: The parameter $\chi$ can be large or small depending on the interpretation. In the baseline case, I have abstracted away from the incentives of government to subsidize firms that do not face the risk of default so that the focus has been on contagion. Besides that, the effect of a default decision by a firm $n_i$ on total welfare was already accounted for in $W_i$. In some other interpretations considered in Section 2.3, links represent transfers between counterparties which firms use for outside investments. This was another reason for $\chi$ to be small. For the sake of completeness I consider large $\chi$ as well.

Costs of bailing-out each firm: Since transfers go back to some households, bailouts possibly do not, ex-post, hurt aggregate welfare. Some scholar argue that there are indeed significant ex-post costs associated with bailouts. These costs can take various forms. It can be a
lumpsum cost of passing a bill from congress. It can be a proportional cost through the distortionary taxes imposed to fund bailouts, which alter labor choice and reduce welfare. One more possibility is the cost of managing wide scale bailouts, which would be proportional to the number of bailouts. I consider a middle case between all: there is a fixed cost to bailing out each firm $n_i$, $\lambda(d_i, \theta_i, \gamma_i) \geq 0$.

**Budget constraint:** There can be limitations on the amount of transfers rather than the number of bailouts as observed in practice. I consider a budget constraint. $b$, the budget is maximum amount of transfer government can execute.

**Incompletely informed government:** One serious problem with bailouts at time of urgently needed intervention is informational imperfections. Generally, it is costly and timely to assess which firms need bailouts and the amount of transfers they need. I leave serious treatment of information acquisition to future work. I consider two cases: government observes all shocks, or observes no shocks.

**Productive bad firms:** The shocks in the lead example are shocks to the fundamental productivity of firms via their costs. However, in other contexts, a shock could simply be a liquidity shock, like a bank-run with none or very little relation to underlying fundamentals. In that case, it might be natural to think that bailing-out bad firms could also improve welfare in and of itself.

**Assumption (Baseline government).** No subsidy to solvent firms via bailouts: $\chi = 0$.

**Costless bailouts:** $\lambda \equiv 0$.

**Unrestricted budget:** $b = \infty$.

**Bad firms reduce welfare:** $W(D, d, B, \gamma) > W(C, d, B, \gamma)$ for all $d, \gamma$.

**Government observes all shocks $\bar{\theta}$**

‘Baseline government’ was maintained in Section 4. Now I relax the components of ‘Baseline government’ one by one.

5.1 Core-periphery: subsidy aspect of bailouts

- ‘Minimal heterogeneity’ is maintained. ‘Baseline government’ is relaxed to allow for $\chi > 0$.

In this subsection, I argue that when the reduction of counterparty losses could improve welfare aside of its effect on default decisions, core-periphery result strengthens. Now, a firm with sufficiently high degree is likely to be bailed-out even if it is a bad firm, in order for the government to benefit from welfare improvement in the reduction of the losses of its counterparties. This can be seen analogous to debt guarantees. High type firms willing to
have high degree are now valuable counterparties since they get bailed-out even if they get bad shocks. In return, low type firms strictly prefer becoming counterparties with high types in order reduce first-order counterparty risk.

For simplicity, assume that \( W(D, d, B, \gamma) - W(C, d, B, \gamma) \) does not depend on \( \gamma \) or \( d \), say it is equal to a constant \( \delta \). If it were intrinsically more important that high types continue than low types, then all the following still holds. Similarly if as long as \( \{ W(D, d, B, \gamma) - W(C, d, B, \gamma) \} \) is decreasing in \( d \) everything goes through.

Government still finds it optimal that all good firms continue. Moreover, if a bad firm has degree larger than \( \delta/\chi \), by bailing this firm out, government looses \( \delta \), yet gains \( \chi d \) from counterparties of the bad firm. Thus the optimal policy involves bailing out all bad firms with degree larger than \( \delta/\chi \), and all the good firms that face default due to first-order losses from bad firms of degree less than \( \delta/\chi \). Call firms with degree larger than \( \delta/\chi \) preferred counterparties and such degrees preferred counterparty degrees. Suppose that \( \delta/\chi < k - 1 \) so this situation can indeed occur. Also assume that \( d^*_L < \delta/\chi \) so that low types are not automatically willing to have preferred counterparty degrees and face a tradeoff.

High types now are likely to become preferred counterparties. The reason is that their ideal degree among \( k \) firms is \( k - 1 \) which makes sure they get bailed-out even when they get bad shocks. This way, low types would like to be counterparties with high types. Yet, it may still be possible that low types enjoy such benefits among each other by becoming counterparties of each other at preferred counterparty degrees. Therefore, it is not immediate that low types would strictly prefer being counterparties with high types.

Assume that \( P(C, f, d, G, \gamma) \) is submodular in \( (f, d) \) and \( P(D, -, d, G, \gamma) \) is increasing in \( d \). Define \( V(d, s, \gamma) = Ef [\max_{a} \{ P(a, \max\{f - s, 0\}, d, \theta, \gamma) \}] \) where \( f \sim f(\cdot, d, 1 - \alpha) \), the binomial distribution with a \( (1 - \alpha) \)-coin. Also \( d^*(s, \gamma) := \arg\max_{d<s} V(d, s, \gamma) \).

**Proposition 9.** If \( \delta/\chi > d^*(k - 1, L) \), in any strongly stable network all high type firms are counterparts with all firms. As for the subnetwork across low types:

1) An almost-(\( d^*(k_H, L) - k_H \))-regular network among each other gives a strongly stable network.

2) In any strongly stable network, this subnetwork must be approximately-(\( d^*(k_H, L) - k_H \))-regular.

**Proof.** By the assumptions on \( P, V(d, s, \gamma) \) is supermodular in \( (d, s) \). Hence, \( d^*(s, \gamma) \) is increasing in \( s \). In particular, \( d^*(k - 1, L) > d^*(k_H, L) > d^*(0, L) = d^*_L > k_H \).
Note that the optimal policy always makes sure all good firms continue. Then the payoffs of firms are indeed given by the $V(d, s, \gamma)$ function. Consider any strongly stable network. Suppose that there is a low type firm with preferred counterparty degree $d$. Suppose that $s$ of its counterparties have preferred counterparty degrees. Since $d > \delta / \chi > d^{**}(k-1, L) > d^{**}(s, L)$. Then it would individually deviate by cutting $d - d^{**}(s, L)$; starting with non-preferred counterparties. Thus in any strongly stable network, all low types have non-preferred counterparty degrees.

For high types, since $V(d, 0, H) = V(d, H)$ is increasing in $d$ and $V(d, s, H)$ is supermodular in $(d, s)$, $V(d, s, H)$ is also increasing in $d$ for any fixed $s$. Thus, high types are happy to become counterparties with any counterparty, preferred or non-preferred. Since in any strongly stable network all low types are non-preferred the only preferred counterparties that low types can have are potentially the high types while high types are happy to become counterparties. Thus in any strongly stable network, all low types must be counterparties with all high types. Otherwise a low type would cut one link with a low type and add with the high type.

The rest follows similar ideas with proofs presented before. \hfill \Box

5.2 Core-periphery: restriction to bailing-out only systemically important firms

- ‘Minimal heterogeneity’ and ‘Baseline government’ are maintained.

In practice, there might be legal restrictions on what kinds of firms can be bailed-out. One example is to bailout only systemically important firms. Here I argue that such a commitment might actually reinforce incentives for connecting with these systemically important firms. The idea is that if government cannot bailout good but “small” firms facing failure due to bad and “large” firms, it may have to bailout these bad and “large” firms. Ex-ante, that would mean, connecting with “large” firms reduces first-order counterparty risk since they may get bailed-out even when they get bad shocks.

Suppose that there is a threshold degree $\bar{d}$ such that a firm $n_i$ with $d_i < \bar{d}$ cannot be bailed-out. Again, there are two types as described before. Also assume that $k-1 \geq \bar{d} > d^{**}(k-1, L)$ and $k_L$ is divisible by $d^*_L - k_H + 1$.

**Proposition 10.** The unique strongly stable network is given as follows. All high firms are connected to all firms, and all low firms, aside of their links with high firms, form disjoint cliques of order $d^*_L - k_H + 1$ among each other. This is also Pareto strongly stable. All low
firms receive strictly higher payoff than the case in which they formed their own cliques of order $d^*_L + 1$.

Proof. Since $\bar{d} > d^*(k-1, L)$, a low type firm is never going to have a degree higher than $\bar{d}$. Then in any configuration, it can achieve at most the payoff it gets as if it had $k_H$ many counterparties with degree larger than $\bar{d}$, and on top of that, there is no second-order counterparty risk. This payoff is achieved in this configuration. Moreover, note that, in any other configuration, either it would have less counterparties with degree larger than $\bar{d}$, or among peripheral firms, it would not be in cliques. The sorting argument of the main theorem and Proposition 3 implies that the only strong stable configuration among low types is that they form cliques of order $d^*_L - k_H + 1$ among each other, while they are all connected to all high types, who are always happy to connect with low types anyway.

5.3 Core-periphery with homogenous firms: costly bailouts

- ‘Homogeneity’ is maintained. ‘Baseline government’ is relaxed to allow for $\lambda > 0$.

In this subsection I illustrate how core-periphery structure is not an artifact of heterogeneity. The idea is as follows. When there are costs of bailing-out each firm, and the cost of bailing-out a bad firm is not significantly higher than that of bailing-out a good firm, government, in some instances, may find it optimal to bailout central bad firms instead of bailing-out its all too many good counterparties. Then if firms do not generate a topology that makes some firms more critical than others, typically, only the good firms would be bailed-out at an amount that makes them indifferent. If some firms arise to be critical, which would optimally get bailed-out even if they are bad firms, then the firms that are counterparties with the critical firms benefit from a reduction in first-order counterparty risk. All in all, some firms may want to downsize to become the periphery, in order to have a ‘safe’ core, leading to a core-periphery structure. I illustrate this possibility with an example. For the example that follows, suppose that cost and welfare does not depend on degree: $\lambda_\theta \equiv \lambda(d, \theta)$, $\Delta W_\theta \equiv W(C, d, \theta) - W(D, d, \theta)$ for all $\theta$.

**Example 10.** $k = 4$. $\alpha = 0.7$, $u(d) = \sqrt{d}$, $c(f) = (1.4) \times f$. $3\lambda_G > \lambda_B - \Delta W_B > 2\lambda_G$, $\Delta W_G - \lambda_G > 0$.

Under this specification, $V(d)$ is maximized at 3. So in the absence of intervention, complete network $K_4$ is formed. In the presence of intervention, if bailing-out bad firms was too costly, so that $\lambda_B - \Delta W_B > 3\lambda_G$, only good firms would ever be bailed-out and again $K_4$ would be formed. But now that $3\lambda_G > \lambda_B - \Delta W_B$, in some instance, it may be the optimal choice to bailout a bad firm. In particular consider a star with 3 leaves.
Note that, $R(1) = 0$. If the center of the star gets a bad shock, and all leaves get good shocks, all leaves are facing indirect default. Without intervention, all will default. Then government may bailout each good leaf and increase welfare by $3(\Delta W_G - \lambda_G) > 0$. Or bailout the center, which induces leaves to continue as well, to increase welfare by $(\Delta W_B - \lambda_B) + 3\Delta W_G > 3(\Delta W_G - \lambda_G)$. Therefore, in case of the star network, if center gets a bad shock and leaves get good shocks, government bails-out the center.

Also, $R(2) = R(3) = 1$ and $\lambda_B - \Delta W_B > 2\lambda_G$. This means that there is no other situation among 4 firms that requires bailing-out a bad firm. In all other configurations and realizations, only and all good firms are bailed-out.

Therefore, the payoff of the center is $V(3)$ indeed. But now, the leaves enjoy the possibility that a bad counterparty gets bailed-out, hence reduce first-order counterparty risk. The payoff of the leaves are $\alpha(\alpha + (1 - \alpha)\alpha^2)(u(1) - c(0))$ which is strictly larger than $V(3)$. Hence, the star network (core-periphery) is the unique strongly stable network among 4 identical firms.

### 5.4 Interconnectedness: budget restrictions

- ‘Homogeneity’ is maintained. ‘Baseline government’ is relaxed to allow for $b < \infty$.

I argue in this subsection that limitations on transfers proportional to the amount, such as a budget constraint, may have adverse effects. Firms may now strictly prefer an interconnected network. This way, they can reduce first-order counterparty risk by benefiting from bailouts of bad counterparties which are executed for other good firms instead of themselves. For this purpose, I stick to homogenous firms.

When the budget is large enough, government makes sure bad firms default in order to avoid their unproductive operations. When the budget is possibly not large enough to bailout all indirect defaults, government is not able to achieve the ‘first best’ anyways. Then depending on the welfare cost of a bad firm continuing business, and the network structure, it may become optimal to bailout bad firms in order to reduce counterparty losses and indirectly help good firms stay in business.

For example consider a star network with $d$ leaves. Suppose that the center gets a bad shock and the leaves get good shocks, but are all forced into default due to losses from the center. If the budget were to allow bailing-out all good firms on the leaves, government would do that. When budget is limited, it may very well become optimal to bailout the center only in order to achieve the second best: the welfare would be reduced by the cost of bad center staying in business, but all the leaves would continue as well.
For technical reasons, it is significantly harder to solve this problem for general parameter specifications. How many and which of bad and good firms should be bailed-out is difficult to find in closed form for an arbitrary network. Finding the network formed is even harder. In order to illuminate the effect of the necessity of bailing-out bad firms due to budget restrictions or costs, I make a simplifying but extreme assumption to focus on the case in which it is too ‘expensive’ to bailout good firms compared to bailing out bad firms, but it is still optimal to induce good firms to continue. That is, bad firms are bailed-out for the sake of good firms. For all $d, d', f$

$$|P(D, -, d, G) − P(C, f, d, G)| > b > (k − 1) \times \{P(D, -, d', B) − P(C, 0, d', B)\}.$$ 

Also, for all $d, d'$,

$$W(C, d, G) − W(D, d, G) > (d − 1) \times [W(D, d', B) − W(C, d', B)].$$

Under this assumption, government 1) cannot bailout any good firms, 2) can bailout all bad firms, 3) wants to bailout minimum number of bad firms to induce all good firms to continue. Let $d + 1 \in \mathbb{N}$ be called a **non-trivial order** if $R(d) \not\in \{0, d\}$. Notice that if $R(d) = 0$, then any bad counterparty of a good firm with degree $d$ would be bailed-out (and possibly some bad firms that are connected at a longer distance). Hence degree $d$ is completely safe with respect to counterparty losses from bad firms. If $R(d) = d$, then no good firm with degree $d$ can ever fail due to any counterparty losses anyway. If a firm has degree $d$ where $d + 1$ is trivial, then it is indifferent between any topology for the rest of the network. For nontrivial orders, the following holds.

**Theorem 5.** In any Pareto strongly stable network, there exists at most one disjoint clique with a non-trivial order.

If bailouts are anonymous\(^{56}\), in any strongly stable network, there exists at most one disjoint clique with a non-trivial order and the members of the clique have strictly worse expected payoff than any other firm (except at most one firm) in the network.

**Proof.** Fix an optimal policy $T$. Within a clique of order $d + 1$, if $R(d)$ or less firms get bad shocks, government does not intervene; all good firms continue and all bad firms default. If $f \geq R(d) + 1$ or more firms get bad shocks, then government bails-out exactly $f − R(d)$ many

\(^{56}\)Anonymity only means playing a mixed strategy when there are indifferences. It is natural to think of bailouts as not treating identities too differently and picking one name with probability 1 out of all other equally well options with other names.
bad firms. It can be any mixture over $f - R(d)$ element subsets of $f$ many bad firms. Thus
the following is the payoff of a firm in a clique with order $d + 1$:
\[
\tilde{V}(d) := \mathbb{E}_\theta \left[ \max \{ P(C, \min \{|n_j : \theta_j = B\}|, R(d), d, G), P(D, -, d, \theta_i) \} \right].
\]

Take any non-trivial $d+1$. Consider the following subgraph of $d+3$ firms: a disjoint $d$–regular
subgraph which has its complement given by a Hamilton cycle among the $d+3$ firms. Now I
show that each $d+3$ firms get strictly more than $\tilde{V}(d)$ in this configuration. Enumerate the
firms along the Hamilton cycle as $n_1, n_2, ..., n_{d+3}$. Take a firm, say $n_1$. If $n_1$ gets $L$, its payoff is
$P(D, -, d, L)$ in any case. If it gets a good shock, and $f \geq R(d) + 1$ many of its counterparties
get bad shocks, then government bails-out at least $f - R(d)$ of its counterparties, so that its
payoff is at least $P(C, R(d), d, G) > P(C, f, d, G)$. If it gets a god shock and $f \leq R(d)$ many
of its counterparties get bad shocks, then government may or not may not bailout some bad
counterparties, and in any case, $n_1$’s payoff is larger than $P(C, f, d, G) > P(C, R(d), d, G)$. Therefore,
conditional on any event, $n_1$’s payoff is at least the same, so the expected payoff is at least $\tilde{V}(d)$. Then it suffices to find a positive probability event under which $n_1$ gets
strictly higher payoff than before.

Consider the event that $n_1$ gets $G$; $n_2, n_3, ..., n_{R(d)+2}^{R(d)+3}, ..., n_{d+3}$ get $G$. Under
this shock profile, only $R(d)$ many counterparties of $n_1$ get $B$: $n_2, n_3, ..., n_{R(d)+2}^{R(d)+3}$. However, $n_{d+2}$
has exactly $R(d) + 1$ many counterparties that got $B$: $n_2, n_3, ..., n_{R(d)+2}^{R(d)+2}$. Due to optimality,
exactly one of $n_2, n_3, ..., n_{R(d)+2}^{R(d)+2}$ must be bailed-out so that $n_{d+2}$ can play $C$. Since $d$ is non-
trivial, $R(d) + 2 \geq 3$. Government is indifferent between bailing-out $n_2$ and $n_{R(d)+2}^{R(d)+2}$ so by
anonymy of the bailouts, government cannot play a pure strategy that bails-out $n_2$. Then
there is a positive probability that one of $n_2, ..., n_{R(d)+2}^{R(d)+2}$ gets bailed-out. This event gives
$n_1$ at least $P(C, R(d) - 1, d, G)$, which is strictly larger than $P(C, R(d), d, G)$. Hence, $n_1$’s
payoff is strictly improved. (Without anonymity, government can play a pure strategy. If $n_2$
is bailed-out, $n_{R(d)+3}^{R(d)+3}$ gets strictly better off. If $n_{R(d)+2}$ gets bailed-out, $n_1$ gets strictly better off. If one of $n_3, ..., n_{R(d)+1}$ gets bailed-out, both $n_1$ and $n_{R(d)+3}$ get strictly better off. Hence, this is a Pareto improvement on top of $\tilde{V}(d)$ payoffs.)

Now suppose that there are two non-trivial cliques, say with orders $d+1$ and $d'+1$. W.l.o.g.
let $\tilde{V}(d) \geq \tilde{V}(d')$. Then two firms from $(d'+1)$-clique and all firms from the $(d+1)$-
clique can form the previously mentioned configuration and get strictly better off. (Without
anonymy, get a Pareto improvement). Contradiction: there can be at most one non-trivial
clique. Furthermore, there cannot be two other firms in the network that both have less than
or equal to $\tilde{V}(d)$ payoff due to the same argument.

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5.5 Interconnectedness: costs of forming links

- ‘Homogeneity’ and ‘Baseline government’ are maintained.

Note that the payoff function $P$ encodes the costs of forming links as well. Here I illustrate how small additional costs of forming links can serve to break indifferences, and indeed make an interconnected network selected out by firms over the clique structure in the presence of bailouts. Suppose that there are very small additional ex-ante costs of links to the firms. Each link $e_{ij}$ comes with a very small extra cost $c_{ij} = c_{ji}$ to both $n_i$ and $n_j$. These additional costs $\{c_{ij}\}_{i,j}\in[N]^2$ are drawn jointly from a distribution $\mathcal{P}_\varepsilon$ with support $[0, \varepsilon]$ before the network is formed. All costs are common knowledge. Assume away integer problems: for $d^* = \arg\max_d V(d)$, $k$ is divisible by $d^* + 1$. There is second-order counterparty risk: $R(d^*) > d^* - 1$.

**Proposition 11.** There exists $\varepsilon > 0$ such that for any $\mathcal{P}_\varepsilon$:

In the absence of bailouts, any strongly stable network consists of disjoint cliques of order $d^* + 1$.\(^{57}\) Each such network is $d^*$-stable.\(^{58}\)

In the presence of bailouts, consider the following algorithmic construction. Start with, and keep taking the smallest remaining $c_{ij}$ and form the link $e_{ij}$, if that does not make any firm exceed degree $d^*$. If it does, skip and move on to the next smallest $c_{ij}$. Stop when no more links can be added. The resulting network is almost-$d^*$-regular and strongly stable.

**Proof.** Let $\varepsilon < \frac{1}{k\min_{d\neq d', d,d'<k}|V(d) - V(d')|}$. Then the lexicographic priority for firms is to get $V(d^*)$ modulo link costs. That needs having $d^*$ counterparts with no second-order counterparty risk. Following the proof of Theorem 2, if there are no profitable deviations, network must consist disjoint cliques of order $d^* + 1$.

Take any such network. Suppose that a group of firms have a profitable deviation. In this deviation, they all must be getting $V(d^*)$ modulo some reduced link costs. Then each deviator has degree $d^*$, and deviators who are counterparties have the identical set of counterparties. Take a component of deviators after the deviation. They all must have the identical set of counterparts. If these deviators have a common non-deviator counterparty, that means they were all in the same clique before the deviation, and kept all links across each other. Then if they cut some links with non-deviators within their clique, their degree is now strictly less than $d^*$. If they kept their links with non-deviators, but cut links with some other deviators, that their degree is again strictly less than $d^*$. Otherwise, they are still in the same clique.

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\(^{57}\)This is necessary, not sufficient.

\(^{58}\)t-stable: There is no profitable deviation by coalitions with t or less number of members. See Erol and Vohra (2015) for more on this solution concept.
with all links intact, meaning they actually did not deviate. Therefore, these deviators do not have a non-deviator counterparty: they all cut their links with non-deviator members from their old clique. That means these deviators are from various old cliques, and formed a genuine new clique of order \( d^* + 1 \). But that needs \( d^* + 1 \) deviators. Hence the network is \( d^* \)-stable.

In the presence of bailouts, following the algorithm, if the resulting network had two firms with degree less than \( d^* \), the algorithm would form a link between them. Consider the network formed. Suppose that there is a subset of firms that have a profitable deviation. Take the firm \( n \) among the deviators that received its \( d^* \)th link first from the algorithm among other deviators. Now this \( n \) must have degree \( d^* \) after the deviation as well, otherwise it can’t achieve \( V(d^*) \) modulo link costs. Hence, it must be replacing some links with some other links. Since this is a profitable deviation, there exists a link \( e \) that \( n \) cut which has more cost than a link \( n \) added, say \( e' \). \( n \) can’t add links with non-deviators, so all added links are with deviators. But then, the link \( e' \) would have already been assigned to \( n \) before the link \( e \) because neither \( n \) nor the counterparty incident to \( e \) had reached degree \( d^* \) when algorithm was passing through \( e' \), which is before \( e \). □

This result is does not guarantee existence of strongly stable networks in the absence of intervention, rather ensures that a \( d^* \)-stable network exists. Consider the following sub-case which guarantees existence. Consider any probability distribution \( P^* \) with support \( 2^N \). Each subset \( M \) of \( N \) is drawn with probability \( P^*(M) \). Members of \( M \) become cost-type \( \mu_1 \) and members of \( N \setminus M \) become cost-type \( \mu_2 \). \( c_{ij} = 0 \) if \( n_i \) and \( n_j \) have different cost-types. \( c_{ij} = \varepsilon \) if \( n_i \) and \( n_j \) have the same cost-types.

**Proposition 12.** There exists \( \varepsilon > 0 \) such that for any \( P^* \) the following holds. Let \( M \) be the realized subset of \( \mu_1 \) cost-types and let \( m = |M| \).

**In the absence of intervention:**

- The following is the unique strongly stable network: disjoint cliques of order \( d^* + 1 \) such that each clique includes \( \lfloor \frac{m}{k} (d^* + 1) \rfloor \) or \( \lceil \frac{m}{k} (d^* + 1) \rceil \) many \( \mu_1 \) types.

**In the presence of intervention:**

- (Sufficiency) The following is strongly stable. If \( m \leq k/2 \), \( \mu_1 \) types have no edges among each other. Each \( \mu_1 \) type has \( d^* \) edges with \( \mu_2 \) types, distributed over \( \mu_2 \) types in a way that each \( \mu_2 \) type has degree \( \lfloor \frac{m}{k-m} d^* \rfloor \) or \( \lceil \frac{m}{k-m} d^* \rceil \). Then \( \mu_2 \) types have a network among each other such that each of them has \( \lfloor \frac{k-2m}{k-m} d^* \rfloor \) or \( \lceil \frac{k-2m}{k-m} d^* \rceil \) degree. If \( m \geq k/2 \), similarly for the other direction.
• (Necessity) Each strongly stable network must have the property that if $m \leq k/2$, each $\mu_1$ type has $d^*$ many $\mu_2$ counterparties and no $\mu_1$ counterparties. If $m \geq k/2$, similarly for the other direction.

When there are no bailouts, firms tend to form cliques despite the small additional costs of links. This is because second-order counterparty losses exceed the additional costs of links. However, in the presence of intervention, second-order counterparty risk is eliminated hence the network formed is dictated by the costs $c_{ij}$ modulo regularity of the network. Random realizations of costs induce an interconnected network with much higher ex-ante probability than the clique structure.

Summarizing remarks of the section:

Remark 9: If it is optimal to subsidize already solvent firms through bailing-out their defaulting counterparties, such as using debt guarantees, first-order counterparty risk gets strictly reduced by connecting with systemically important firms. In return, peripheral firms strictly prefer connecting with core firms rather than connecting with peripheral firms.

Remark 10: Committing to bailing out only systemically important firms strengthens the incentives for core-periphery structure, and reduces the incentives for interconnectedness among the periphery.

Remark 11: Government induced core-periphery is not necessarily an artifact of heterogeneity. Indeed, with homogenous firms, some firms downsize to become the periphery, in order to force the government to bailout the core even when core gets bad shocks. This way, the periphery reduces its first-order counterparty risk.

Remark 12: When there are limitations on bailouts proportional to the amount of transfer, such as a budget constraint or per-unit costs of transfers, government may be forced into selectively bailing out bad firms. In return, firms seek to benefit from the bailouts of bad firms executed for the sake of other other good firms. Firms can reduce their first-order counterparty risk by especially making the network more interconnected. This suggests increased network hazard. ‘Committing’ to less spending on bailouts may not always help in terms of systemic risk.
6 Other extensions

6.1 Productive bad firms and costly bailouts

• ‘Homogeneity’ is maintained. ‘Baseline government’ is relaxed to allow for \( \lambda > 0 \) and \( W(C, d, B) - W(D, d, B) > 0 \).

Two other aspects of bailouts that I have mentioned are the per-firm costs of bailouts and the possibility that bad firms may still have some merit in terms welfare, such as when the shocks are pure liquidity shocks. I lump these two cases into one subsection since they are technically similar.

When a firm which would otherwise default is bailed-out, the change in welfare directly from this firm is given by

\[
\zeta(d_i, \theta_i) = -\lambda(d_i, \theta_i) + W(C, d_i, \theta_i) - W(D, d_i, \theta_i).
\]

In order to avoid complicating the analysis with the dependence on degree I assume that \( \lambda \) and \( W \), thus \( \zeta \), do not depend on \( d_i \). The analysis can be extended to the general degree dependent case using the techniques in Sections 5.1 and 5.2. Henceforth, let \( \zeta_\theta \equiv \zeta(d, \theta) \), \( \lambda_\theta \equiv \lambda(d, \theta) \), \( \Delta W_\theta \equiv \zeta_\theta + \lambda_\theta \) for all \( \theta \).

In the benchmark case, since bad firms hurt welfare, \( \zeta_B < 0 \). If bad firms do not hurt welfare, and actually contribute enough relative to the cost of bailing them out, then bad firms would also be bailed-out for their own sake. Then, government then can make sure, by bailing-out all bad firms, that all firms continue, independent of the cost of bailing-out good firms. Let \( d^{**}(k) = \text{argmax}_{d<k} P(C, 0, d, G) \).

**Proposition 13.** If \( \zeta_B > 0 \), optimal policy makes sure all firms continue. Pareto strongly stable networks consist of \( d^{**}(k) \)-regular networks (modulo divisibility).

**Proof.** Take any optimal policy. If it induced an action profile in which some firms default, then this policy could be improved by bailing out all bad firms that are not bailed-out. So the optimal policy, either by bailing-out good or bad firms, makes sure all firms continue. Then a firms payoff is given by \( P(C, 0, d, G) \). The rest is similar to previous cases.

If \( \zeta_B \) is not positive, then bad firms are not bailed-out for their own sake. The other extreme is \( \zeta_B << 0 \). That is, it is never optimal to bailout any bad firms. Now, the analysis is somewhat very close to the baseline case. The only difference is that now \( \zeta_G \) now determines how welfare is affected by the costs of bailouts.

**Proposition 14.** If \( \zeta_B << 0 \) and \( \zeta_G > 0 \), the optimal policy and the network formed is identical to the baseline case.
Proof. Since $\zeta_B$ is very small, it is never optimal to bailout any bad firms. Hence any optimal policy induces all bad firms to default. Also, if there was any good firm left to default, by bailing it out, welfare would be directly improved by $\zeta_G > 0$, and possibly more through spillovers. Then the induced incentives for firms are also identical.

The treatment of bail-out costs paves the way to a more detailed welfare analysis. If $\lambda_G$ is close to zero, so that $\zeta_G$ is very close the welfare gain, welfare is improved. If, on the other hand, $\lambda_G$ is very close to the welfare gain, so that $\zeta_G$ is very close to zero, then systemic risk is reflected onto the welfare as follows. Now for each bailout, welfare gain is ex-post almost 0 from the reduction in first-order indirect defaults. Then ex-ante, each excess bailout that the interconnectedness generates on top of what would be needed in a clique structure is an ex-ante welfare loss. The welfare loss in the absence of bailouts was proportional to the total number of indirect defaults, but now in the presence of bailouts, the welfare loss is proportional to the number of bailouts. Recall that anticipation of intervention increases spillovers: the expected number of bailouts under intervention is larger than the expected number of indirect defaults under no-intervention. Therefore, welfare decreases. For each selection of the regular network formed in the presence of intervention, there is a middle threshold value of the cost of bailouts so that the welfare decreases below the threshold and increases above the threshold. The threshold depends on the extent of contagion in each particular interconnected network that might be formed in the presence of intervention. Furthermore, note that the analysis here can directly be applied to ‘Minimal heterogeneity’ as well, if degree of $H$ types do not increase welfare. Hence similar goes for core-periphery result. That is, if bailouts are sufficiently costly, but not too costly that bailouts are ex-post suboptimal, then systemic risk and volatility is reflected onto the welfare. If higher degree of $H$ firms increases welfare, the analysis relies on the comparison between bailout costs of $H$ types and welfare gains higher degree of $H$ types.

The non-trivial tradeoff between bailing-out good or bad firms is the middle case of the comparison between $\zeta_B$ and $\zeta_G$. Section 5.3 studies this case with an example. I leave the general treatment to future work.

6.2 Incompletely informed government

- ‘Homogeneity’ and ‘Baseline government’ are maintained.

Aside of the how second and first-order counterparty risk are affected by transfers, there is possibly another reason for why moral hazard might arise. If the government does not have precise information on the financial standing of firms, it may be unable to use ex-post
minimal transfers, leading to some overcompensation. That in return generates incentives to exploit the expectation of overcompensation by becoming vulnerable to counterparty risk, on purpose.

For technical tractability, assume away screening problem. Let \( P(D, -, d, B) - P(C, 0, d, B) > P(D, -, d, G) - P(C, d, d, G) > 0 \) for all \( d \). This implies that a bad firm never finds it optimal to continue and mimic a good firm in order to benefit from transfers intended for indirect defaults. Clearly, this is a very strong assumption. I leave deeper treatment of incompletely informed government to future work.

Let \( d^*(k) := \arg \max_{d<k} V(d) + \alpha \times [P(D, -, d, G) - P(C, d, d, G)] \).

**Proposition 15.** The optimal policy is to bailout all firms at an amount \( P(D, -, d, G) - P(C, d, d, G) \). Only good firms utilize the bailout and continue. Unique (Pareto) strongly stable network structure is \( (d^*(k) + 1) \)-cliques or \( d^*(k) \)-regular networks, depending on the resilience. Any almost-\( d^*(k) \)-regular network is strongly stable. Any strongly stable network is approximately-\( d^*(k) \)-regular.

**Proof.** The optimal action profile to induce is that all good firms continue and all bad firms default, if possible at all. By offering all firms \( P(D, -, d, G) - P(C, d, d, G) \), this action profile would indeed be induced due to ‘No need for screening’. Then the question is to induce this action profile using minimal transfers. Since, under any optimal policy, all bad firms default, and it is possible that a good firm \( n_i \) is surrounded by \( d_i \) many bad firms, and no amount less than \( P(D, -, d, G) - P(C, d, d, G) \) can guarantee a good firm to continue.

Given the optimal policy, the expected payoff of a firm with degree \( d \) is given by \( V(d) \) plus the expected transfer \( \alpha T_i(C) \), which makes \( d^*(k) := \arg \max_{d<k} V(d) + \alpha \times [P(D, -, d, G) - P(C, d, d, G)] \). The rest follows the proof of previous theorems.

**Lemma 1.** Suppose that \( P(D, -, d, G) - P(C, d, d, G) \) is increasing in \( d \). Then \( d^*(k) \geq d^*(k) \).

**Proof.** Increasing differences.

### 6.3 Incompletely informed firms

• ‘Baseline government’ is maintained.

Now I introduce incomplete information on the firm side into the model. Each firm observes the shocks to itself, its counterparties, and possibly some more firms. Formally, \( n_i \) observes the shocks to a subset \( I_i(E, \vec{\theta}) \) of firms which includes \( n_i \) and all counterparties of \( n_i \): \( \{n_i\} \cup N_i \subset I_i(E, \vec{\theta}) \subset N \).

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Cooperating equilibrium is defined for complete information in the model for simplicity. Vives (1990) also shows that any Bayesian game with supermodular ex-post payoff functions has a maximal pure strategy Bayesian-Nash equilibrium. Thus cooperating equilibrium is identically defined.

Most theorems hold almost identically. The idea is that, under no-bailouts, if firms form cliques, their shocks are common knowledge across clique members. This allows them to enjoy $V(d, \gamma)$ for a $(d + 1)$-clique, still. However, still under incomplete information, a firm with degree $d$ cannot get more than $V(d, \gamma)$ in any other configuration of the network. Hence similar proofs apply. On the other hand, when there are government bailouts, a firm need not know anything more than his counterparties shocks, because government makes sure that all good firms play $C$ and all bad firms play $D$. Then each firm still enjoys $V(d, \gamma)$ and still does not care about the network topology, so similar proofs apply again. Therefore the interconnection and core-periphery intuition is robust to incompletely informed firms, as long as every firm can observe its counterparties shocks.

6.4 Generalized cooperating equilibrium and transfers

• ‘Baseline government’ is maintained.

The transfer function $\{T_i\}_{i \in N}$ can depend on the action profile in general. But then, the induced game in stage three may not be supermodular so that the cooperating equilibrium may not be well defined. Here I state the appropriate definition of cooperating equilibrium for games that are not supermodular, and show that the optimal policy indeed has the property that transfers to a firm depends only on its own action.

Consider any function $\{U^*_i\}_{i \in N} : \{C, D\}^N \times \{G, B\}^N \times \mathbb{N}^2 \times \Gamma^N \rightarrow \mathbb{R}$. For fixed $(\tilde{\theta}, E, \tilde{\gamma})$, $\{U^*_i\}_{i \in N}$ induces a binary game. Let the set of Nash equilibria be denoted $\mathcal{N}(\tilde{\theta}, E, \tilde{\gamma})$. Since the game is possibly not supermodular, the set of Nash equilibria may not be a complete lattice, yet it is still a partial order.\footnote{For mixed strategies, the order is analogously defined using the probabilities on higher actions.} Take any selection, $\sigma(\tilde{\theta}, E, \tilde{\gamma}) \in \mathcal{N}(\tilde{\theta}, E, \tilde{\gamma})$ such that there is not element of $\mathcal{N}(\tilde{\theta}, E, \tilde{\gamma})$ that ranks higher in the order. Call this selection the $\sigma$-generalized cooperating equilibrium.

Suppose that the firms play $\sigma$-generalized cooperating equilibrium, for some arbitrary selection $\sigma$. Quiet trivially, for any $(\tilde{\theta}, E, \tilde{\gamma})$, the profile in which all good firms firms play $C$ and all bad firms play $D$ must be induced. Otherwise, government could use the optimal transfer scheme of the baseline case, which would make sure the game is supermodular, so that any selection $\sigma$ had to uniquely choose this profile. If, for any $(\tilde{\theta}, E, \tilde{\gamma})$, this action profile is being
induced, following the proof of Proposition 6, the transfers must be at least as much as $T^*$ in order to induce this profile. Then minimality requires that $T^*$ is still the unique optimal policy. $T^*_i$, conditional on $(\bar{\theta}, E, \bar{\gamma})$, depends only on the action of $n_i$.

**Summarizing remarks of the section:**

*Remark 13:* If the effective cost of bailouts are large, welfare gains diminish and systemic risk reflects onto welfare.

*Remark 14:* The lack of precise information on government’s side is another source of moral hazard. It leads to overcompensation and overconnection. Interconnectedness remains.

*Remark 15:* The lack of precise information on firm’s side do not have dramatic effects in my model. As long as firms can observe the financial situation of their immediate counterparties, there is no change since no firm wants to have a second-order counterparty even under complete information. As for the case of intervention, second-order counterparty is eliminated by bailouts, so firms don’t need the corresponding information.

## 7 Discussions

### 7.1 Other interpretations of the model

Here I discuss how the model can be interpreted in other ways. These are not formal micro-foundations, rather illustrations of the main forces.

**Origination and diversification of risk and investment** Each firm has one proprietary investment which has to be managed by the firm and can’t be sold, and a non-proprietary project. The firm can sell parts of the non-proprietary project to each of its counterparties, in $\varepsilon$ portions each. Each firm initially has illiquid assets of worth $A(\gamma)$ and liabilities at amount $X(\gamma)$.

All returns from all investments realize after stage three, in “stage four”. Liabilities also must be paid in stage four. Each non-proprietary project returns $r \sim F$, where $F$ has support $(\underline{r}, \bar{r})$. Each proprietary project returns $R(G, \gamma) = \bar{R}(\gamma)$ with probability $\alpha$ and $R(B, \gamma) = 0$ with probability $1 - \alpha$. The information on the proprietary projects is revealed in stage two.

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60 Think of this project as having high cost of monitoring and operating. If sold, originating firm does not exert the effort to monitor the project, so that the project fails. In return, no firm wants to buy shares of this asset.

61 These have smaller costs of monitoring so other firms are willing to buy this asset.

62 This quantities can be thought of as the scale of the project.
If a firm defaults in stage three, it liquidates its illiquid assets and its proprietary project at ratio \( \phi_1 < 1 \), so it gets \( \phi_1 (A + R(\theta, \gamma)) \). The liquidation value of its shares of non-proprietary projects are 0. The non-proprietary project that it originated fails and returns 0 to other firms that holds shares of it. Therefore,

\[
P(D, d, f, \theta, \gamma) = \phi_1 (A(\gamma) + R(\theta, \gamma)) - X(\gamma).
\]

Firms that continue in stage three move onto stage four. After the returns from non-proprietary projects are realized, if a firm’s liquid assets can pay for its liabilities, it is solvent. Thus, for \( \psi = (1 - d\varepsilon) r_i + \varepsilon \sum_{j \in N_i, a_j = C} r_j \), if \( \psi + R(\theta, \gamma) - X(\gamma) \geq 0 \), firms payoff is \( A(\gamma) + \psi + R(\theta, \gamma) - X(\gamma) \). Otherwise, it goes bankrupt, and has to liquidate its illiquid assets at rate \( \phi_2 < \phi_1 \). Accordingly, its realized payoff is given by \( \phi_2 A(\gamma) + \psi + R(\theta, \gamma) - X(\gamma) \). All in all, its payoff is \( A(\gamma) + \psi + R(\theta, \gamma) - X(\gamma) - (1 - \phi_2) \times A(\gamma) \times 1_{\psi < X(\gamma) - R(\theta, \gamma)} \). Note that in stage three, \( \psi \) is a random variable which depends on \( d \) and \( f \). Denote the distribution of the random variable \( \tilde{\psi}(d, f) \). Then the expected payoff in stage three is

\[
P(C, d, f, \theta, \gamma) = A(\gamma) + R(\theta, \gamma) - X(\gamma) + \mathbb{E}_{\tilde{\psi}} [\psi - (1 - \phi_2) \times A(\gamma) \times 1_{\psi < X(\gamma) - R(\theta, \gamma)}]
\]

where \( \tilde{\psi} \sim \psi(d, f) \). Notice that by buying assets originated by other firms, firms diversify against the risk of losing the full value of their illiquid assets \( A \).

Notice that \( P(C, d, f, \theta, \gamma) \) is strictly decreasing in \( f \) and \( P(D, d, f, \theta, \gamma) \) is constant in \( f \). Hence Assumption 1 is satisfied. As for Assumption 2, assume that \( \bar{r} < X(\gamma) - A(\gamma) \) for all \( \gamma \). That is, a firm with a bad shock is certain to go bankrupt if it continues. Hence, by continuing, it would get at most \( \phi_2 A(\gamma) - X(\gamma) + \bar{r} \). Assume that \( \bar{r} < (\phi_1 - \phi_2) A(\gamma) \) for all \( \gamma \). Thus, a firm with a bad shock strictly prefers to default. Therefore Assumption 2 is also satisfied.

**Diversification against liquidity risk** This interpretation follows the lines of Holmstrom and Tirole (1997) and Allen and Gale (2000). Firms are financial institutions. Each firm has one risky project. There is another risk that a project may require additional investments before its maturity. The liquidity shock is negatively correlated: at most one bank receives this shock. Links are credit lines across firms in order to insure each other against the liquidity risk.

There is a cost \( x \) of forming each link with other firms, such as search costs or management costs. Each credit line has a limit 1 unit of credit. After credit lines are formed in stage one, information about the productivity of projects arrive in stage two. Each project returns \( R(G, \gamma) = \bar{R}(\gamma) \) with probability \( \alpha \) and \( R(B, \gamma) \approx 0 \) with probability \( 1 - \alpha \), which is due
the end of stage four. In stage three, each firm chooses to continue managing its project or default. The cost of effort for continuing to manage the project is $y$. If effort is not exerted, project fails and returns 0. In stage four, at most one project is chosen to randomly requires an additional $\rho \sim F$ unit of investment, where $F$ has support $(0, \infty)$. The probability of being selected is $\varepsilon < 1/k$. Each firm that continued has access to 1 unit of deposits, which can be lent out via the credit lines. If a project that belongs to a firm that has chosen to continue requires additional investment, then the firm can use its credit lines. If it can’t find $\rho$ units of additional funds, it does not borrow any and defaults. Project fails. Otherwise, it provides the additional funds and project returns additional $\rho$ on top of $R(\theta, \gamma)$ which the firm uses the pay back its lenders. Thus, in stage four, a firm that continued which received the liquidity shock has payoff given by

$$P(C, d, f, \theta, \gamma) = R(\theta, \gamma) - dx - y - \varepsilon(1 - F(d - f + 1))R(\theta, \gamma).$$

A firm that defaults in stage three gets $P(D, d, f, \theta, \gamma) = -dx$.

$P(C, d, f, \theta, \gamma)$ is strictly decreasing in $f$ and $P(D, d, f, \theta, \gamma)$ is constant in $f$. Moreover, $P(D, d, -, B, \gamma) = -dx > -dx - y > P(C, d, f, \theta, \gamma)$. Therefore both Assumptions 1 and 2 are satisfied.

**Intermediation and allocation of funds** This interpretation follows the lines of Moore (2011). Firms under this interpretation are banks. Each bank has one proprietary project. Projects return small dividends over time, and finally a lumpsum return at the end of the project. Each project occasionally receives an opportunity to scale up the investment. There is a flow of depositors to each bank, who ask for dividends to keep their deposits in the bank. Banks can channel these deposits to other banks that have an opportunity to scale up their investments. The network serves the purpose intermediating and allocating funds.

Formally, there are $\bar{t}$ mini-periods of length $1/\bar{t}$ between stage one and two. Consider one bank and its proprietary project. The projects starts off at scale $s_0$. A project at scale $s_t$ in mini-period $t$ pays $\varepsilon s_t$ dividends in that mini-period. At stage two, there are no more opportunities left to rescale the investment and the scale is fixed at $s_t$. The final gross rate of return $R(\theta)$ is determined via the shock $\theta$. It takes value $R(G) = \bar{R}$ with probability $\alpha$ and $R(B) = 0$ with probability $1 - \alpha$. Thus, in stage three, the project returns $R(\theta)s_t$.

Depositors ask for $\varepsilon$ unit of dividends for 1 unit of dividends every mini-period. Therefore, the end of stage four. In stage three, each firm chooses to continue managing its project or default. The cost of effort for continuing to manage the project is $y$. If effort is not exerted, project fails and returns 0. In stage four, at most one project is chosen to randomly requires an additional $\rho \sim F$ unit of investment, where $F$ has support $(0, \infty)$. The probability of being selected is $\varepsilon < 1/k$. Each firm that continued has access to 1 unit of deposits, which can be lent out via the credit lines. If a project that belongs to a firm that has chosen to continue requires additional investment, then the firm can use its credit lines. If it can’t find $\rho$ units of additional funds, it does not borrow any and defaults. Project fails. Otherwise, it provides the additional funds and project returns additional $\rho$ on top of $R(\theta, \gamma)$ which the firm uses the pay back its lenders. Thus, in stage four, a firm that continued which received the liquidity shock has payoff given by

$$P(C, d, f, \theta, \gamma) = R(\theta, \gamma) - dx - y - \varepsilon(1 - F(d - f + 1))R(\theta, \gamma).$$

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$P(C, d, f, \theta, \gamma)$ is strictly decreasing in $f$ and $P(D, d, f, \theta, \gamma)$ is constant in $f$. Moreover, $P(D, d, -, B, \gamma) = -dx > -dx - y > P(C, d, f, \theta, \gamma)$. Therefore both Assumptions 1 and 2 are satisfied.

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Depositors ask for $\varepsilon$ unit of dividends for 1 unit of dividends every mini-period. Therefore,
all mini-period deposits are channeled to depositors. Moreover, bank promises \( r^H \) gross rate of return per 1 unit of deposits at stage three due the final gross returns from projects.

The dynamics of mini-periods are as follows. At each mini-period, exactly one bank randomly receives an opportunity to scale up its investment, whereas each bank receives one new depositor with \( x = Xk/t \) units of deposits. If the bank can promise \( \varepsilon x \) dividends each mini-period and \( r^H x \) in stage three, depositors deposit their money. Otherwise, they leave.

Banks can issue interbank bonds to borrow from other banks. The incompleteness in the market is that a bank can borrow only from banks that it has links with. Interbank bonds promise \( \varepsilon X \) units of dividends per mini-period and \( r^I x \) units of return in stage three per 1 unit borrowed.

In stage two, banks can default or not. Deposits are fully insured by the government, hence it is optimal for depositors to keep their assets in the bank in order to enjoy dividends, if the bank can promise dividends each mini-period.

All in all, in each mini-period, all dividends flow back to depositors. Through the interbank bonds, banks build exposures to each other. A bank that continues business in stage two receives a return \( R(\theta)s_0 \), pays back \( r^I \) rate of return to its lender banks, receives \( r^I \) rate of return from its borrower banks that has not defaulted, pays back \( r^H \) rate of return to its depositors.

Take \( t \to \infty \). Each bank has the rescaling opportunity \( 1/k \) portion of the mini-periods, at each of which it borrowed \( Xk/t \) from each of its counterparties. Hence, in total, the interbank bonds it has issued binds it to pay \( r^I X \) to each of its counterparties. Each bank has borrowed from depositors each time any of its counterparties or itself had the opportunity to rescale, hence, it owes the depositors \( r^I X(d+1) \). As a result, it scaled its project up by \( X(d+1) \); \( X \) directly from its own depositors, and \( Xd \) with funds from its counterparties.

A bank that defaults walks away from all obligations, pays nothing and gets 0 payoff. Therefore, a bank that continues business receives a payoff

\[
P(C, d, f, \theta) = R(\theta) \times (s_0 + X(d+1)) + (d - f)r^I X - dr^I X - r^H(s_0 + X(d+1))
\]

\[
= (R(\theta) - r^H)(s_0 + X(d+1)) - fr^I X.
\]

Notice that \( P(C, d, f, \theta) \) is strictly decreasing in \( f \) and \( P(D, d, f, \theta) = 0 \) is constant in \( f \). Moreover, \( P(C, d, f, B, \gamma) = -r^H(s_0 + X(d+1)) - fr^I X < 0 = P(D, d, f, \theta, \gamma) \). Therefore, both Assumptions 1 and 2 are satisfied.

**Joint projects and syndication of loans** This interpretation follows the lines of Erol and
Vohra (2014). Firms are financial institutions take undertake joint and safe projects. Projects are initiated and funded by deposits in stage one, with an interest rate \( r > 1 \) promised due the realization of returns from projects. Each project requires \( x \) units of investment from each from each counterparty. Projects also have to be managed by costly effort which has to be exerted at the end of stage two. Cost of effort for each firm is \( y(\theta, \gamma) < X \) per project undertaken. Each project returns \( X \) if the project succeeds. Projects succeed if and only if both counterparties exert effort. Project requires expertise of each counterparty, hence project fails if any counterparty fails.

If a firm continues business, its payoff is

\[
P(C, d, f, \theta, \gamma) = (d - f)(X - y(\theta, \gamma)) - dxr.
\]

If a firm defaults, all of its projects fail, and walks away with an outside option \( P(D, d, f, \theta, \gamma) = 0 \).

Notice that \( P(C, d, f, \theta, \gamma) \) is strictly decreasing in \( f \) and \( P(D, d, f, \theta, \gamma) \) is constant in \( f \). Assume that \( y(B, \gamma) > X - rx \) so that \( P(C, d, f, B, \gamma) \leq d(X - y(\theta, \gamma) - xr) < 0 = P(D, d, -, \theta, \gamma) \). That is, Assumptions 1 and 2 are satisfied.

### 7.2 Future work

Aside of government intervention, ex-ante regulation is another important avenue in the context of network formation.

On top of the bailout question, there is also the *bail-in* question. Transfer could be negative, meaning that government can force some firms into chipping in for bailouts of others. This would stress the insurance aspect of intervention as well. In this case, firms need to take into account their expected losses due to transfers to troubled institutions.

In the simple form of incompletely informed government, I assumed that government has no information. However, government can indeed acquire information. This might have time costs or monetary costs. *Costly information acquisition* on government’s side, and how that information would be shared with firms as a *signaling* tool can be another aspect of government’s main considerations.

The main emphasis in this paper is the network formation. A deeper treatment of the mechanism design part is an important avenue for future research.
8 Conclusion

This paper provides a framework to study contagion with endogenous networks, and the adverse effects of government intervention with the market. It can be seen as a first attempt at mechanism design with endogenous network externalities, and can be applied to various other settings. The model is tractable and has wide room for generalizations and extensions.

In the benchmark case of no intervention, second-order counterparty risk generates a market discipline and leads to the formation of isolated and dense small clusters. Intervention, in form of bailouts, however, eliminates second-order counterparty risk as a byproduct. Hence the presence of bailouts dissolves the clusters into an interconnected network. Firms do not overconnect or underconnect, they just interconnect. Moreover, firms do not make riskier individual investment choices. Interconnectedness makes the system more susceptible to various measures of systemic risk. Moreover, when second-order counterparty risk vanishes, some firms become systemically important. This way, the network becomes a core-periphery network. Core serves as both a buffer against and an amplifier of contagion, making very good and very good outcomes more likely, generating volatility. It is notable that core-periphery structure can emerge as a consequence of intervention even for homogenous firms. When there are restrictions on government with regards to how bailouts can be executed, the incentives for forming interconnected networks and forming core-periphery networks strengthen. These results suggest that the presence of bailouts might indeed be contributing to the observed interconnectedness of the financial system and the core-periphery structure of financial networks.

Several questions are immediate. Can prevention in form of capital and liquidity requirements particular to systemic-importance undo the network moral hazard (designer at both stages)? If government can use ‘bail-in’s forcing good firms to chip in to the bailouts, what kinds of effects would emerge on the network (negative transfers)? In case of incomplete information, what is the best way to acquire information for government (costly information acquisition)? I leave these to future research.

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