

Financial Exaggeration and the (Mis)Allocation of Scarce Resources*

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ABSTRACT

Decision-makers often face financial exaggeration when they allocate scarce resources. This is because agents usually have considerable discretion over their financial reporting policies and also have a conflict of interest when they compete for resources. Because of exaggeration, the decision-maker acquires less accurate information, which can lead to suboptimal allocations. However, we show how the decision-maker is better off with exaggeration, despite the loss of information caused by misreporting. This is because of a complementarity that exists between the agents' effort provision and their ability to exaggerate. As we demonstrate, it is often suboptimal for a decision-maker to curb misreporting or monitor outcomes, even if it is costless to do so. Our results may explain why exaggeration is ubiquitous and provide implications for settings in which allocation decisions are prevalent: money management, analyst coverage, investor attention, and other business agreements.

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“Entrepreneurs tend to exaggerate. They exaggerate the success of their business when talking to startup investors. They exaggerate the market potential of their products to find distribution partners. They exaggerate the soundness of their strategy to recruit employees.”

—*Asheesh Advani, Founder of CircleLending and CEO of Covestor*¹

1 Introduction

Financial exaggeration is ubiquitous when agents compete for scarce resources. This is because the state of the art in financial reporting grants marked discretion to forecasters² and every agent has an incentive to put his best foot forward when trying to persuade a decision-maker to choose his opportunity.³ Indeed, financial reporting depends on a complicated mix of judgement, gut instincts, biases, incentives, and possibly hubris. Because the decision-maker’s information is supplied by agents with a strong interest in her decision, she is inherently skeptical about how closely the intrinsic value of any proposed project actually matches what is reported (Milgrom and Roberts 1986). This issue is particularly relevant in settings where the main tool for aligning incentives—a complete contract—is infeasible or illegal, and prices do not fully reveal value in the market (e.g. analyst coverage decisions, investment fund choices).

In this paper, we demonstrate how financial exaggeration arises in equilibrium, how it affects agents’ efforts to develop valuable projects, and how it affects payoffs. By reducing transparency, financial exaggeration may impede the efficient allocation of resources, suggesting that financial exaggeration should (at least weakly) reduce the decision-maker’s surplus. We show that this is not necessarily the case: while financial exaggeration reduces allocative efficiency, it also motivates agents to supply more effort to develop valuable projects. Therefore, financial exaggeration is a complement to effort provision, which may actually enhance the quality of investments and increase the decision maker’s payoff. Therefore, even if the decision maker has a technology to monitor the value of projects before allocating the scarce resource, it may be optimal for her to commit *not*

¹See <http://www.entrepreneur.com/howto/startyourbiz.html> or <http://www.entrepreneur.com/article/197224>.

²For example, the discounted cash flow (DCF) method leaves its exact structure and its inputs unspecified: the length of the forecast period, the discount factor, the terminal growth rate, and the forecasted inputs, to name a few.

³Discretion in financial analysis may also have asset pricing implications and appears to be associated with a risk premium (Andrei, Carlin, and Hasler, 2015).

to monitor.⁴ As we discuss in the paper, this may help rationalize why some investments remain relatively unregulated (e.g. hedge funds).

We analyze a model of competitive persuasion. A decision-maker owns a scarce, indivisible resource that she would like to allocate to one of two agents. Each agent develops a project of uncertain value, which requires the decision maker's resource as an essential input. At the beginning of the game, each agent chooses the quality of his production technology. A higher quality is more costly for the agent, but increases the chance that his project is highly valuable. Next, each agent chooses a financial reporting policy, which determines how the realized value of his project is communicated to the decision-maker. The policy may involve exaggeration: the report may overstate the project's true value. Following this, the value of each project is realized, and each agent generates a report for the decision-maker, in a manner consistent with his financial reporting policy. The decision-maker observes each agents production technology, financial reporting policy, and the reports, and then allocates the resource based on rational expectations. The decision-makers goal is to select the most valuable project, but each agents goal is merely to attract the resource to his project, irrespective of its eventual value for the decision-maker.

When the agents are identical, in equilibrium each agent selects the same quality, generating a valuable project with the same probability. Subsequently, the choice of whether and how much to exaggerate depends on an important tradeoff: exaggeration increases the probability that the decision-maker receives a good report, but it also makes the decision-maker treat a good report with greater skepticism (as a Bayesian). If the marginal cost of quality is low, agents select high quality technologies, so that projects are likely to be valuable. In this case, agents are more concerned with avoiding the decision-maker's skepticism than with concealing bad outcomes, and the agents select reporting policies that perfectly reveal project value. However, if the marginal cost of quality is high, the agents choose lower quality technologies. Here, concealing a bad outcome becomes a more significant concern for the agents, and the degree of exaggeration is determined by a mixed strategy.

⁴This is consistent with a long line of papers that imply that people may be better off ignoring information or by committing not to collect it (Hirshleifer, 1971; Bikhchandani, Hirschleifer, and Welch, 1992; Teoh, 1997; Burguet and Vives, 2000; Morris and Shin, 2002; Angeletos and Pavan, 2007; Amador and Weill, 2012).

If financial exaggeration takes place, the decision-maker may be persuaded to allocate the resource to a lower value project. The decision-maker could eliminate this inefficiency by requiring that the agents report truthfully (i.e. monitoring). However, this turns out to be suboptimal *ex ante* because it induces the agents to choose less productive technologies. A complementarity exists between investment quality and exaggeration. Indeed, higher investment quality reduces the decision-maker's skepticism, so that each agent can exaggerate more without undermining the credibility of his good report. With a higher marginal cost of quality, the ability to exaggerate is more valuable. Thus, the incentive to invest is stronger when exaggeration is allowed. As such, we show that it is better for the decision-maker to cope with some *ex post* inefficiency in order to increase the probability that a high value project exists. Furthermore, because the decision-maker is indifferent between monitoring and not when the marginal cost of quality is low, we show that it is weakly suboptimal for the decision-maker to require truthful reporting, even if doing so is costless.

Following this, we consider an extension in which the conflict of interest between the parties is not as severe. Here, each agent still prefers that the decision-maker invests in his project, but if the decision-maker does so, the payoff to the decision-maker and agent are identical. In this case, the equilibrium financial reporting strategies perfectly reveal project values. Compared to full monitoring, there are higher levels of quality and better expected payoffs to the decision-maker. However, the comparison with the main model when exaggeration is allowed is equivocal. If the cost of quality is sufficiently high, dealing with strongly biased agents and coping with exaggeration leads to higher investment quality and superior expected payoffs to the decision-maker.

We then consider two extensions to the model in which agents are *ex ante* asymmetric. First, we suppose that one agent's marginal cost of quality is higher than the other's. Not surprisingly, whether or not the decision-maker monitors reports, equilibrium investment in quality is asymmetric, whereby the stronger agent invests more in quality. Also, consistent with the complementarity described above, when exaggeration is allowed the agent who chooses the higher quality technology exaggerates his project's value more than his competitor.⁵ Thus, the agent with the more produc-

⁵Technically, this is because the stronger agent's distribution function in the mixed strategy first-order stochastically dominates the weaker agent.

tive technology is less likely to communicate that he has generated a low value project. However, while the complementarity between exaggeration and quality increases both agents' investments, the effect is actually stronger for the weaker agent. The relative increase in quality (compared to full monitoring) is higher for the weaker agent: compared to the case of full monitoring, allowing financial exaggeration generates a relatively larger increase in the weaker agent's investment.

Subsequently, we consider a continuum of heterogeneous agents. This extension yields several novel implications. First, when the resource is very scarce, reporting by all agents is fully-informative, even if financial exaggeration is permitted. However, when the resource is in higher supply, exaggeration becomes part of the equilibrium. Second, when exaggeration arises, the highest skilled agents provide the least transparency, whereas lower skilled agents produce less quality, but compensate with more truthful reporting.⁶ This latter finding helps us understand the interaction of skill and transparency in investment markets. Ostensibly, higher skilled people work at hedge funds. Indeed, hedge fund managers are known to use more sophisticated financial instruments and more complex securities than mutual fund managers (e.g., Fung and Hsieh, 1997; Koski and Pontiff, 1999; Bookstaber, 2003). Hedge funds are also more opaque than mutual funds (Brown and Goetzmann, 2003). Our analysis therefore provides a new explanation for the industrial organization of money management markets.⁷

Our analysis treats financial exaggeration as a form of *persuasion*: agents control the *process* by which information about the value of their projects is communicated to a (Bayesian) principal, hoping to convince her that their projects are most deserving of the scarce resource. McCloskey and Klamer (1995) argue that persuasion is the “third part of economic talk” and accounts for one quarter of US GDP. The role of persuasion in communication is studied in an emerging literature. Kamenica and Gentzkow (2011) consider the persuasion of a single receiver by a single sender, while competitive persuasion processes are analyzed by Gentzkow and Kamenica (2012) and Boleslavsky

⁶Interestingly, in equilibrium these effects exactly offset: the expected quality of projects run by highly able agents, and the expected quality of projects run by less able agents—conditional on receiving the resource—is identical.

⁷This is distinct from Gervais and Strobl (2012), who posit that money managers tradeoff between signaling their skill versus giving away secrets via transparency. In their framework, high and low skill agents form hedge funds, whereas medium skill agents form mutual funds. In our framework, there is a separation between two groups: high skill agents with no transparency and lower skill agents with better reporting.

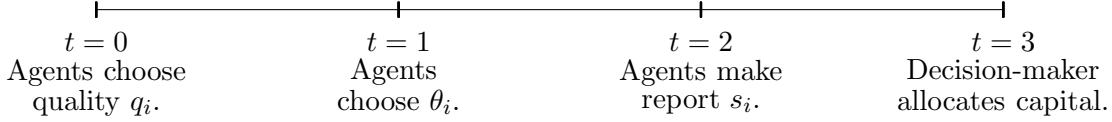


Figure 1: At $t = 0$, each agent chooses the quality of their production technology, q_i . At $t = 1$, each agent chooses the amount they exaggerate their financial performance. Then, at $t = 2$, each agent realizes their type τ_i and makes a report to the decision-maker $s_i \in \{G, B\}$. Finally, at $t = 3$, the decision-maker allocates her capital to one of the projects.

and Cotton (2014, 2015).⁸ Of particular relevance is Boleslavsky and Cotton (2015) who develop a model of school competition in which schools invest in education quality and strategically design grading policies, influencing the beliefs of employers and admissions officers in order to secure better alumni placements. Because of the level of generality with which Boleslavsky and Cotton (2015) model grading, their analysis is unsuitable for the study of financial exaggeration. Indeed, their analysis focuses exclusively on the *information content* of school grading policies, and it therefore cannot determine anything about the actual (nominal) grading strategies that schools employ. However, financial exaggeration, whereby low value projects are reported as high, is inherently a *nominal* phenomenon.⁹

2 Model

Consider that a decision-maker has a scarce resource that she wishes to allocate to one of two agents. Each agent invests in a production technology that yields a positive NPV project. As such, absent any further information, the decision-maker prefers to make an investment rather than sit out. However, her goal is to invest in the better opportunity, whereas each agent strictly wishes to attract the resource for his own use.

Figure 1 illustrates the timing of the game. At $t = 0$, each agent $i \in \{a, b\}$ simultaneously chooses the quality of his production technology, which we denote as $q_i \in [0, 1]$. Given this, agent

⁸The literature on “persuasion of a Bayesian” has two other strands: a single sender’s persuasion of groups of receivers who subsequently interact (Alonso and Camara 2015, Taneva 2015) and the persuasion of a sender with private information (Alonso and Camara 2014, Kolotin et al. 2014).

⁹These authors also do not consider a large allocation problem with a continuum of agents, as we do in Section 4.

i 's eventual project will have a high value ($\tau_i = H$) with probability q_i and low value ($\tau_i = L$) with probability $1 - q_i$. Examples of investment in q_i might be the intensity of an agent's R&D efforts or the skill of his management team. Investment in q_i is associated with a convex cost

$$C(q_i) = \frac{q_i^2}{\rho},$$

is public information, but is not contractible.¹⁰

At $t = 1$, each agent simultaneously chooses a reporting policy, which determines the transparency with which financial projections are communicated to the decision-maker.¹¹ Each agent can claim that the value of his project is high, $s_i = H$, or low $s_i = L$. Each agent always reports $s_i = H$ when he has a valuable project, ($s_i = H$ whenever $\tau_i = H$), but he may misreport when $\tau_i = L$. To capture this, suppose that agent i chooses the probability $\theta_i \in [0, 1]$ with which a low value project is reported as high value:

$$\Pr(s_i = H | \tau_i = H) = 1 \text{ and } \Pr(s_i = H | \tau_i = L) = \theta_i.$$

If $\theta_i = 0$, then agent i always reports truthfully. The greater the value of θ_i , the more the agent exaggerates his project's quality, diluting the informativeness of his report.

The agents' choice of θ_i is publicly observable. As such, it has three natural interpretations. First, in a setting where auditors are employed to monitor firms, θ_i proxies for the auditor's competency or reputation. If the agent reports $s_i = H$ and the project survives an audit by a monitor that is known to be more careful, the decision-maker can be more confident about that opportunity. The second interpretation is that θ might parameterize the effort that agents employ to stress test their financial projections to identify bad projects. For example, if the agent performs thorough sensitivity analysis and can confidently report that the project is high-type, then $\theta = 0$ (i.e., there is no possibility that the project could turn out to be a low-type). If the agent employs little sensitivity analysis, then $\theta = 1$ and projects could turn out to be low-type, even if they are reported as high-types. Thus, $\Pr(s_i = L | \tau_i = L) = 1 - \theta$, could be interpreted as the intensity or scrutiny directed by the agent toward uncovering a low quality project. Third, θ might be the degree to

¹⁰For now, the agents are ex ante identical. In Section 4, we consider what happens when $\rho_a > \rho_b$.

¹¹In Section 3.4, we reconsider the timing of the game and allow the agents to observe outcomes before choosing a reporting strategy. There we show that the our results are, if anything, magnified.

which financial information is obfuscated. In an alternative model in which a decision-maker has a cost of sorting out financial projections, $\theta = 0$ makes it easy to contrast high- and low-type projects, whereas $\theta = 1$ makes this impossible and an intermediate value of θ obfuscates project quality in a way that induces suboptimal decision-making.

At $t = 2$ each agent privately observes the value of his project $\tau_i \in \{H, L\}$ and makes a report $s_i \in \{H, L\}$ consistent with his financial reporting strategy. Finally, at $t = 3$, the decision-maker allocates the resource and then learns the outcome.

Each agent's project is worthless without the scarce resource. The agent who persuades the decision-maker to choose his project receives a payoff normalized to one. The other agent receives zero. If the decision-maker invests in a high value project, she receives a payoff normalized to one. Otherwise, her payoff is zero. Given this, the decision-maker's expected payoff with each agent is equal to the posterior probability that $\tau_i = H$. It is sequentially rational for her to invest in the project that she believes is more likely to be $\tau_i = H$. If she holds the same beliefs about each project, then we assume that she randomizes fairly between them. Once the investment is made, the true type of the project is revealed and the payoffs are realized.

This set-up assumes a contract-free environment where decision-makers rely on transparency. This is common in many settings in financial markets because agreements are either infeasible or illegal.¹² For example, in the context of our model, the decision-maker may be a money manager with a fiduciary duty to her client and the two agents are mutual funds who would like to attract capital. The mutual funds can window-dress the value of their investments, but it may be illegal for them to pay the money manager a side payment in order to attract capital. The payoff to a fund is positive if it attracts capital and the payoff is positive for the money manager if her client enjoys a high return. Another application is the market for analyst coverage. Equity analysts choose which corporations to follow and enjoy positive payoffs when they follow more successful firms. Likewise, corporations compete for this coverage because they receive benefits from attention in financial markets. Contracts between analysts and firms in this setting are not only infeasible, but also illegal. Therefore, analysts rely on financial reporting and transparency.

¹²Notwithstanding, here we begin with the most severe conflict of interest where the incentives of the decision-maker and agents are distinct. In Section 3.3, we consider preferences that are more closely aligned.

3 Equilibrium Characterization

3.1 Monitoring Benchmark

Suppose that financial reporting must be truthful (i.e., $\theta_i = \theta_j = 0$) so that the decision-maker is perfectly informed. When each agent selects his production technology, he anticipates that $\Pr(\tau_i = H) = q_i$ and $\Pr(\tau_i = L) = 1 - q_i$. For fixed investment levels (q_a, q_b) , the expected payoffs are

$$u_a(q_a, q_b) = q_a(1 - q_b) + \frac{1}{2}(q_a q_b + (1 - q_a)(1 - q_b)) = \frac{1}{2}(1 + q_a - q_b) - \frac{q_a^2}{\rho}$$

$$u_d(q_a, q_b) = 1 - (1 - q_a)(1 - q_b).$$

By inspection, the marginal benefit of improving quality for either agent is independent of the other agent's choice and each agent's investment level satisfies the first order condition: $1/2 = 2q_i/\rho$. Therefore, at $t = 0$, each agent has a dominant strategy to choose $q_i = \rho/4$ and each agent is equally likely to receive the resource. The associated expected payoffs are

$$u_i = \frac{1}{2} - \frac{\rho}{16} \quad \text{and} \quad u_d = 1 - (1 - \frac{\rho}{4})^2 = \frac{\rho}{2} - \frac{\rho^2}{16}. \quad (1)$$

In what follows, we will compare these quantities to when there is less transparency.

3.2 Equilibria with (Possibly) Less Transparency

We solve for the Perfect Bayesian Equilibria of the game and proceed by backward induction. We first characterize the equilibrium choices of θ_i for each agent given the quality of investments (q_i, q_j) made at $t = 0$. Following that, we analyze the symmetric Nash equilibrium in quality (q^*, q^*) .

If agent i chooses investment level q_i at $t = 0$ and exaggeration θ_i at $t = 1$, then the probability that a project is reported as high-type ($s_i = H$) is

$$r_i = q_i + \theta_i(1 - q_i).$$

The decision-maker's updated belief that a project with $s_i = H$ is in fact $\tau_i = H$ is

$$g_i = \frac{q_i}{q_i + \theta_i(1 - q_i)} = \frac{q_i}{r_i}.$$

An agent's choice of θ_i involves a simple tradeoff: increasing θ_i increases the probability that the decision-maker gets a good report, but causes the decision-maker to view good signals with greater skepticism.

If θ_i and θ_j are such that $g_i > g_j$, then the decision-maker will invest in project i whenever both projects are reported to be high-types. In this case, agent i receives the resource whenever he gets a good realization, regardless of the other agent's draw, and will also receive the resource with probability $\frac{1}{2}$ when both agents report $s_i = L$. Therefore, following the investment decisions (q_i, q_j) at $t = 0$, agent i 's expected payoff as a function of the exaggeration levels (θ_i, θ_j) is:

$$u_i(\theta_i, \theta_j | q_i, q_j) = \begin{cases} r_i(1 - r_j) + \frac{1}{2}(1 - r_i)(1 - r_j) & \text{if } g_i < g_j \\ r_i + \frac{1}{2}(1 - r_i)(1 - r_j) & \text{if } g_i > g_j \\ r_i(1 - \frac{1}{2}r_j) + \frac{1}{2}(1 - r_i)(1 - r_j) & \text{if } g_i = g_j. \end{cases} \quad (2)$$

We make two observations about best responses. First, given g_j , agent i always prefers to select a θ_i for which g_i is marginally higher than g_j , rather than for which $g_i = g_j$. This means that an equilibrium involving $g_i = g_j$ is possible only at $g_i = g_j = 1$ (i.e., $\theta_i = \theta_j = 0$). Second, notice that among all financial reporting policies for which $g_i < g_j$, agent i prefers $g_i = q_i$ (i.e., $\theta_i = 1$).¹³ Taken together, these observations suggest that the equilibrium of the exaggeration stage is either a fully-revealing pure strategy equilibrium $\theta_i = \theta_j = 0$, or a mixed strategy equilibrium.¹⁴

Proposition 1. *Suppose that $q_a + q_b \geq 1$. Then, the unique Nash equilibrium is $g_a = g_b = 1$ with payoffs to agent i of*

$$u_i(q_i, q_j) = \frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho}. \quad (3)$$

If $q_a + q_b < 1$, a mixed strategy equilibrium exists in which there is financial exaggeration. In this case, the expected payoffs to the agents are

$$u_i(q_i, q_j) = \frac{q_i}{q_i + q_j} - \frac{q_i^2}{\rho} \quad (4)$$

¹³If $g_i < g_j$ then project i will be selected for certain if and only if his reported type is high and j 's is low; otherwise project i will either not be chosen, or the decision-maker will randomize. By choosing an uninformative reporting policy, agent i assures that its reported type will be high, and therefore guarantees that its project will be selected if the other agent's reported type is low. Therefore, if an asymmetric pure strategy equilibrium exists, then one agent must play an uninformative strategy. However, because of an open set issue, the best response to an uninformative policy is undefined.

¹⁴The mixed strategy equilibrium requires that agents select policies without perfectly anticipating the other agent's choices, and that adjusting one's policy is infeasible after $t = 1$.

and the higher quality technology leads to more exaggeration (i.e., θ_a first-order stochastically dominates θ_b in the mixed strategies).

According to Proposition 1, if $q_a + q_b \geq 1$, then $\theta_a = \theta_b = 0$. Otherwise, financial exaggeration is part of the equilibrium and there is lower transparency. This inequality can be appreciated as follows. Suppose that agent b always tells the truth and consider agent a 's best response. If $\tau_b = L$ and $\tau_a = H$, then there is no gain to dishonesty. Agent a gets the resource anyway. If $\tau_b = H$ and $\tau_a = L$, dishonesty is futile: the decision-maker allocates the resource to agent b because his probability of having a high-type project is one. The only time that dishonesty has a potential benefit or cost is when $\tau_a = \tau_b$. If $\tau_a = \tau_b = L$, then dishonesty has a positive payoff to agent a because the agent's probability of having a high-type project is zero (agent b tells the truth). However, if $\tau_a = \tau_b = H$, then exaggeration hurts agent a . In this case, $\theta_a > 0$ causes the decision-maker to be skeptical and allocate the resource to agent b .

The probability that $\tau_a = \tau_b = H$ is $q_a q_b$ and the probability that $\tau_a = \tau_b = L$ is $(1 - q_a)(1 - q_b)$. If $q_a + q_b \geq 1$, then $q_a \geq 1 - q_b$ and $q_b \geq 1 - q_a$. This means, in turn, that $q_a q_b \geq (1 - q_a)(1 - q_b)$. Therefore, $q_a + q_b \geq 1$ implies that it is more likely for the agents to both get a high-type outcome than for them to both get a low-type. In this case, the expected cost of exaggeration is higher than its benefit, and the agents report truthfully. Intuitively, when good outcomes are likely, both agents would rather avoid the skepticism of the decision-maker. In contrast, when $q_a + q_b < 1$, $q_a q_b < (1 - q_a)(1 - q_b)$, and the expected benefit of exaggeration is higher than its cost. In such case, bad outcomes are more likely and it is better for each agent to exaggerate and make the project look more attractive.

Lemmas A2-A3 in the appendix characterize the mixed strategy equilibria that arise with exaggeration. According to Proposition 1, the agent with a higher quality production technology exaggerates more-aggressively. This implies that there is a complementarity between investment in a production technology and financial exaggeration. The more an agent invests in his production technology, the more optimistic the decision-maker is about his project's quality. Thus, the agent can exaggerate to a larger degree, while maintaining the decision-maker's perception that the project is higher quality. Exaggeration is valuable to the agent, because it reduces the probability

with which the decision-maker observes a bad report. Thus, the agent with the more productive technology exaggerates more in equilibrium, and expects a higher equilibrium payoff.

Based on Proposition 1, the payoff to each agent then depends on their relative investment in quality:

$$u_i(q_i, q_j) = \begin{cases} \frac{q_i}{q_i + q_j} - \frac{q_i^2}{\rho} & \text{if } q_i + q_j \leq 1 \\ \frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho} & \text{if } q_i + q_j > 1 \end{cases}$$

This payoff function is continuous everywhere and is differentiable everywhere except for possibly $q_i = q_j$.¹⁵ The non-differentiability arises when the investment levels change the equilibrium from one of full disclosure to one in which exaggeration is present. Despite this non-differentiability, characterizing the symmetric equilibria of the game is straightforward.

Proposition 2. *The symmetric Nash equilibrium of the investment game is as follows:*

(i) *Suppose that $\rho > 2$. Then, $q_a = q_b = \frac{\rho}{4}$ at $t = 0$ and $\theta_a = \theta_b = 0$ at $t = 1$.*

(ii) *Suppose that $\rho \leq 2$. Then, $q_a = q_b = \frac{\sqrt{2\rho}}{4}$ at $t = 0$ and the agents engage in financial exaggeration at $t = 1$.*

According to Proposition 2, the cost of investment affects the agents' tendency to exaggerate if given the opportunity to do so. When ρ is sufficiently high, $C(q_i)$ is lower and less convex, making it easier for the agents to invest in quality. In this case, the agents do not exaggerate because they find it better to avoid the decision-maker's skepticism. Given symmetry in equilibrium, the inequality in Proposition 1 may be written as $q^* + q^* \geq 1$, or $q^* \geq \frac{1}{2}$. Substituting $q^* = \frac{\rho}{4}$ yields $\rho \geq 2$, which is true by assumption. Therefore, when the marginal cost of quality is sufficiently low, the decision-maker can rely on the agents to tell the truth.

When ρ is lower, making $C(q_i)$ higher and more convex, the agents tend to exaggerate the performance of their projects. In this case, the agents are more constrained and invest less. Revisiting the inequality in Proposition 1, $q^* < \frac{1}{2}$, which means that exaggeration is part of the equilibrium. In this case, the agents are more concerned with concealing bad outcomes than with

¹⁵While it is not the focus of our paper, it is interesting to note that the mixed strategy equilibrium of the financial reporting stage generates a payoff function of the "ratio-form" commonly used in the study of contests (Tullock 1984).

the decision-maker's skepticism. However, as we show in the next corollary, investment by the agents is higher with exaggeration than without it. In essence, the ability to exaggerate relaxes the constraint imposed by the high cost function and introduces a complementarity between investment and misreporting.

Corollary 1. *Suppose that $\rho \leq 2$. Then, in a symmetric equilibrium, there is more investment in quality when exaggeration is allowed than in the fully-informative benchmark.*

This result can be appreciated by comparing the quality choices in Proposition 2 to those in the fully-revealing benchmark. Indeed, exaggeration leads to higher quality if

$$\frac{\sqrt{2\rho}}{4} \geq \frac{\rho}{4},$$

or $\rho \leq 2$. This implies that the ability to exaggerate allows for a complementarity between investment in project quality and future financial exaggeration. If the marginal cost of investment in quality is sufficiently high, then it might be preferable for a decision-maker to allow agents to misreport. We explore this formally in the next proposition.

Proposition 3. *(Decision-maker's Payoff) Suppose that $\rho < 2$. If no exaggeration is allowed, the payoff to the decision-maker is*

$$u_p^{NE} = \frac{\rho}{2} - \frac{\rho^2}{16}. \quad (5)$$

If exaggeration is allowed, the payoff to the decision-maker is

$$u_p^{EX} = \begin{cases} \frac{19}{12} \frac{\sqrt{2\rho}}{4} & \text{if } \rho \leq \frac{8}{9} \\ \frac{1}{12(\frac{\sqrt{2\rho}}{4})^2} (1 - 9\frac{\sqrt{2\rho}}{4} + 27(\frac{\sqrt{2\rho}}{4})^2 - 8(\frac{\sqrt{2\rho}}{4})^3) & \text{if } \frac{8}{9} \leq \rho \leq 2. \end{cases} \quad (6)$$

For all values of $\rho < 2$, $u_p^{EX} > u_p^{NE}$.

According to Proposition 3, the decision-maker is strictly better off allowing the agents to exaggerate. In total, then, for any ρ , the decision-maker is weakly better off not monitoring the agents. Indeed, for $\rho > 2$, there is truth-telling anyway and $u_p = \frac{\rho}{2} - \frac{\rho^2}{16}$. As such, it is never optimal for the decision-maker to monitor the agents.

3.3 Preference Alignment

So far, we have only considered a setting in which there is a strong conflict of interest between the agents and the decision-maker. The agent who gets chosen receives a fixed payoff, irrespective of whether the outcome is high or low. Now, let us reconsider the analysis when the agent receives a payoff if the decision-maker accepts his project *and* it turns out to be a high type.

The set-up is unchanged from Section 2 except that an agent receives a benefit only when his project is funded and $\tau = H$. It is easy to show that $\theta_i = 0$ is a weakly dominant strategy for each agent. This is because the agent's payoff when $\tau = L$ is zero, no matter what type of reporting strategy is chosen. Given this, calculating the equilibrium investment levels is straightforward. The expected payoff to each agent is

$$u_i(q_i, q_j) = q_i \left(1 - \frac{q_j}{2}\right) - \frac{q_i^2}{\rho}.$$

Since this function is strictly concave in q_i , the best response for agent i is defined by the following first order condition that is linear in the choices of each agent:

$$\left(1 - \frac{q_j}{2}\right) - \frac{2q_i}{\rho} = 0.$$

Solving yields $q = \frac{2\rho}{\rho+4}$.

Now, we can compare this level of investment to those in the base model. First, consider the case of full monitoring. The agents always invest more when the conflict of interest is mild:

$$\frac{2\rho}{\rho+4} > \frac{\rho}{4} \iff \rho < 4.$$

This is not surprising because in both scenarios, the agents tell the truth. However, when a low-type project does not generate a payoff for the agent, there is an additional incentive to avoid bad realizations¹⁶.

We can now contrast this when exaggeration is allowed, which introduces a tradeoff for the decision-maker. On one hand, as we showed before, financial exaggeration increases the incentives to invest in quality because prior beliefs play a role in evaluation. At the same time, in the base

¹⁶The cost parameter ρ is bounded by 4, assuring that the probability q_i is maximized at one.

case, the agent may benefit even when his project is low value, which lowers the incentive to invest in quality. It turns out that for small values of ρ , investment is higher in the base case with exaggeration. To see this,

$$\frac{2\rho}{\rho+4} < \frac{\sqrt{2\rho}}{4} \iff \rho < 12 - 8\sqrt{2} \approx 0.69.$$

This increase in investment can also generate a higher payoff to the decision-maker. To make the comparison, note that exaggeration in the base case can only be better if investment is higher, which happens when $\rho < 0.69 < 8/9$. Hence, we need only compare the payoffs for low values of ρ . Comparing these payoffs yields that exaggeration with a more severe conflict of interest is preferred whenever:

$$1 - \left(1 - \frac{2\rho}{\rho+4}\right)^2 < \frac{19}{12} \frac{\sqrt{2\rho}}{4} \iff \rho < \hat{\rho} \approx 0.504.$$

Given this, if ρ is sufficiently small, the payoff to the decision-maker is higher when there is no monitoring and there exists a more severe conflict of interest. In this case, preference alignment actually decreases the decision-maker's surplus.

3.4 Uninformative Reporting

One assumption that may appear critical to the results in this paper is whether the agents can actually commit to a particular reporting technology without changing it once they observe the outcome τ_i . In what follows, we relax this assumption. Not surprisingly, this leads to a cheap talk scenario in which both agents have a weakly dominant strategy to lie when they get a low value. The decision-maker internalizes this and realizes that the reports are uninformative. However, as we show shortly, this leads to an even higher investment in quality. Indeed, the results in the paper imply that the decision-maker typically prefers less information, not more. In the limit, if the decision-maker were to ignore reports altogether, she is strictly better off.¹⁷ Given this, the results in the paper are robust to different timing and modeling assumptions.

Suppose that the reports given by the agents are completely uninformative. Then, the game between the agents is essentially a full-information, all-pay auction with a symmetric, convex cost

¹⁷We show below that uninformative reporting generates a higher payoff for the decision-maker than the equilibrium with exaggerated (but informative) reports when the agents are symmetric. This ranking may reverse when the agents are asymmetric. Analytical results that support this latter finding are available from the authors.

of bidding, and a bid cap. Each agent simultaneously chooses $q_i \in [0, 1]$. The agent with the higher value of q receives a payoff of one, but both agents lose their investments, $C_i(q_i) = \frac{q_i^2}{\rho_i}$.

Proposition 4. (*Uninformative Reporting*)

i. Suppose that $\rho \geq 2$. Then $q_i = q_j = 1$ is the unique equilibrium.

ii. If $1 < \rho < 2$, then there exists a unique mixed strategy Nash equilibrium in which

$$q_i = \begin{cases} Q \sim F(x) = \frac{x^2}{2-\rho} & \text{with probability } \frac{2}{\rho} - 1 \\ 1 & \text{with probability } 2(1 - \frac{1}{\rho}) \end{cases}$$

iii. If $\rho \leq 1$, there exists a unique mixed strategy Nash equilibrium in which

$$q_i \sim F(x) = \frac{x^2}{\rho}.$$

For all ρ , the payoff to the decision-maker is higher with uninformative reporting than with strategic exaggeration or with full monitoring.

This result arises because q is publicly observable and there is no other information to judge the situation. However, it does alleviate the concern that the main findings of this paper are simply special to the specifics of the posed model.

4 Asymmetric Agents

Now, we consider that agents may be heterogeneous. We first start by considering our base model with two agents that have different marginal costs of investing in quality. Following that, we extend the analysis to consider a continuum of types.

4.1 Two Heterogenous Agents

Let us suppose that the cost of quality for the two agents is such that

$$R_a < \rho_b < \rho_a < 2, \tag{7}$$

where R_a as defined analytically in the Appendix.¹⁸ Thus, while one agent has a higher investment cost, the weaker agent's cost is still above a certain threshold, so that the asymmetry is not too large. The following proposition characterizes the equilibrium of the game.

Proposition 5. (*Asymmetric Agents*) *Suppose that (7) holds. Then, there exists an equilibrium with exaggeration and*

$$q_a^* = \frac{\sqrt{2\rho_a(\rho_a + \rho_b)}\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b}{2(\rho_a - \rho_b)} \quad q_b^* = \sqrt{\frac{\rho_b}{\rho_a}}q_a. \quad (8)$$

Further,

$$\frac{q_b^*}{\rho_b} > \frac{q_a^*}{\rho_a}. \quad (9)$$

According to Proposition 5, $q_b^* < q_a^*$, which is not surprising given that $\rho_b^* < \rho_a^*$. Based on our characterization in Proposition 1, agent a will be more inclined than the weaker agent b to exaggerate his performance at $t = 1$. However, this has an interesting effect on both agents at $t = 0$. Higher expected exaggeration leads agent a to invest more in quality at $t = 0$, which is consistent with our previous results. However, agent b has an even higher relative incentive to increase quality due to this heightened competition when exaggeration is present.

To see this, let's compare the ratio of each agent's quality choice with exaggeration q_i^* to their choice under full monitoring $\frac{\rho_i}{4}$.

$$\frac{q_a^*}{\frac{\rho_a}{4}} < \frac{q_b^*}{\frac{\rho_b}{4}} \Rightarrow \frac{q_a^*}{\rho_a} < \frac{\sqrt{\frac{\rho_b}{\rho_a}}q_a^*}{\rho_b} \Rightarrow \frac{\rho_b}{\rho_a} < \sqrt{\frac{\rho_b}{\rho_a}}, \quad (10)$$

which is always the case. Given this, there is a spill-over effect of exaggeration. When the stronger agent will be expected to invest more in quality and exaggerate, this causes the weaker one to work harder. This result may have normative implications. Monitoring may not only be suboptimal because it lowers effort provision, but it may also remove a complementarity between asymmetric agents.

¹⁸Setting this lower bound on ρ_b implies an upper bound on the cost of quality for agent b and assures that a pure strategy equilibrium exists.

4.2 Continuum of Heterogeneous Agents

Now, consider that there is a continuum of agents who have a marginal cost of quality ρ_i that is distributed according to a continuously differentiable distribution function $F(\cdot)$ on the support $[0, 1]$ with $\mu \equiv E[\rho_i] \leq \frac{1}{2}$. Assume that each agent's cost of quality is $\frac{q_i^2}{2\rho_i}$. The decision-maker has the capacity to allocate the resource to a fraction x of the projects from the population.

4.2.1 Fully Informative Benchmark

Depending on the size of x , there are two cases to consider. For small x , the decision-maker chooses a fraction of projects with good reports (i.e., rations the scarce resource to a subset). For larger x , the decision-maker allocates the resource to all high-quality projects and a fraction of lower quality projects.

Proposition 6. (*Full Information*)

i. Suppose that $x < \mu$. Then, the decision-makers's expected payoff is x and each agent's investment in quality is $\sqrt{\frac{x}{\mu}}\rho_i$. Both are increasing in x .

ii. Suppose that $x > \mu$. The decision-maker's payoff is

$$u_d = \left(1 - \frac{1}{2\mu} \left(2\mu - 1 + \sqrt{1 - 4\mu(1-x)}\right)\right) \mu \quad (11)$$

and each agent's investment in quality is

$$q_i = \left[1 - \frac{1}{2\mu} \left(2\mu - 1 + \sqrt{1 - 4\mu(1-x)}\right)\right] \rho_i. \quad (12)$$

Both are decreasing in x .

Figure 2 plots the decision-maker payoff as a function of x for $\mu = \frac{1}{3}$ and $\mu = \frac{1}{2}$. For $x < \mu$, the agents compete vigorously for the scarce resource. As the decision-maker has more capacity, the probability of receiving the resource increases. Given this, agent investment increases and the expected payoff to the decision-maker rises. However, once the resource becomes sufficiently less scarce and lower quality projects will be picked as well, the incentives for the agents to compete

relax. As x gets large, agent investment decreases and the expected payoff to the decision-maker decreases. In what follows, we compare this outcome under full information to what arises under decreased transparency.

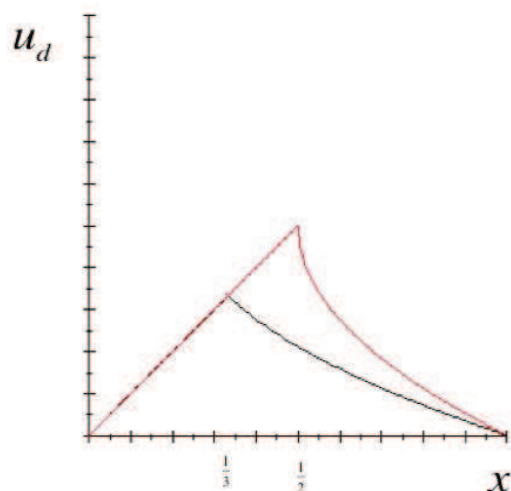


Figure 2: THE DECISION MAKER'S PAYOFF FOR $\mu = 1/2$ (RED) AND $\mu = 1/3$ (BLACK) AS A FUNCTION OF THE RESOURCE'S ABUNDANCE, x .

4.2.2 Equilibria with (Possibly) Less Transparency

The decision-maker is still willing to allocate resources to a fraction x of the agents. But, since there is imperfect information, there will now exist a threshold belief G such that each agent only receives the resource if the decision-maker's posterior belief about his quality $g_i = \frac{q_i}{q_i + \theta_i(1 - q_i)}$ surpasses the threshold. In any equilibrium, the measure of projects at or above G equals x . Note that at G , all projects must also be accepted, because otherwise an epsilon deviation would generate a posterior belief marginally above G and lead to acceptance with probability one. Hence, rationing can occur in equilibrium only if $G = 1$, which reduces to the previous case.

The following proposition characterizes the equilibrium for $\rho_i \sim U[0, 1]$.

Proposition 7. *Uniform Distribution and Exaggeration*

i. Suppose that $x < \frac{1}{2}$. Then, there is fully-informative reporting, decision-makers's payoff is x , and each agent's investment in quality is $\sqrt{\frac{x}{\mu}}\rho_i$.

ii. Suppose that $x > \frac{1}{2}$. Then, there is exaggeration and the decision-maker's payoff is

$$x\sqrt{2(1-x)}, \tag{13}$$

which is strictly higher than with the fully-informative benchmark. Agents with $\rho_i < 2(1-x)$ invest $q_i = \frac{\rho_i}{\sqrt{2(1-x)}}$ whereas the remainder invest $q_i = \sqrt{2(1-x)}$.

According to Proposition 7, when resources are sufficiently scarce (i.e., $x < \frac{1}{2}$), truth-telling is part of the equilibrium. The decision-maker's payoff and the investment in quality is the same as in the fully informative benchmark. However, when the resource is less scarce, the decision-maker's payoff is higher without monitoring. As in Section 3, this arises because of a complementarity between exaggeration and quality. In Figure 3, we superimpose the decision-maker's payoff on the graph in Figure 2 with $\mu = \frac{1}{2}$. Below $x = \frac{1}{2}$, the decision-maker's payoff is the same. Above $x = \frac{1}{2}$, the decision-maker's payoff is larger with exaggeration.

Equally as interesting is the optimal behavior of the agents when resources are less scarce. High-ability agents put in sufficient effort, but give completely uninformative reports to the decision-maker. Lower ability agents do the best they can, but offer more honesty to compensate. This equilibrium outcome has implications for the industrial organization of financial markets. For example, consider skill and transparency in the investment management industry. The model would predict that high skill agents would form less transparent funds, whereas lower skilled agents would work at more transparent funds. Indeed, this is consistent with the co-existence of hedge funds and mutual funds.

5 Conclusion

For decades, financial economists have focused on ways to mitigate the agency problems that arise between shareholders and managers, investors and entrepreneurs, and lenders and borrowers. Whether due to adverse selection or moral hazard, agency conflicts are ubiquitous in corporate

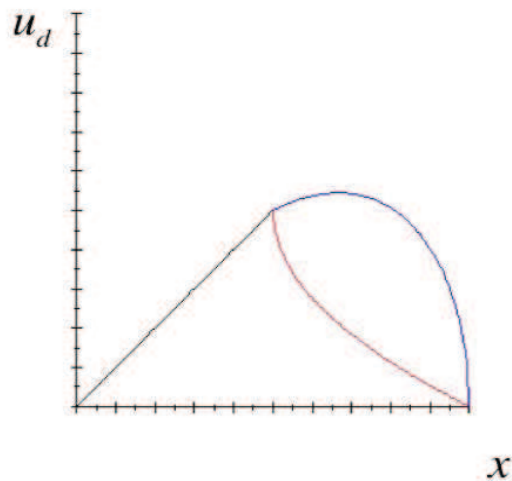


Figure 3: THE DECISION MAKER'S PAYOFF AS A FUNCTION OF THE RESOURCE'S ABUNDANCE, x UNDER FULL INFORMATION AND FINANCIAL EXAGGERATION. FOR $x < 1/2$, THESE ARE IDENTICAL. FOR $x > 1/2$ THE PAYOFF WITH FINANCIAL EXAGGERATION (BLUE) IS HIGHER THAN UNDER FULL INFORMATION (RED).

finance as an important determinant of firm size, capital structure, corporate governance, and firm value. Generally, the field has viewed perfect monitoring as a panacea, as long as it is not too costly.

In this paper, we show that this is not the complete story. If agents forecast that they will have the opportunity to hide information or act in their own self-interest, it is possible that they will invest more effort to develop strong projects when they anticipate this. In our model, agents compete for resources, which in some cases leads them to exaggerate the outcome of their projects. Knowing this, they work harder to win, which benefits the decision-maker who allocates the capital. In such cases, having less information may benefit the decision-maker.

Our findings, then, can be viewed as a more general contribution. Indeed, one can think of many settings to apply this analysis besides financial markets: marriage markets, education, litigation, and elections, to name a few. Investigating these settings is the subject of future research.

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Appendix A

Proof of Proposition 1.

The proof follows from Lemmas A1-A3. At the reporting stage, investments (q_a, q_b) with $q_a \geq q_b$ are taken as given, because these were chosen in the previous stage. Agents simultaneously choose (θ_a, θ_b) , their level of exaggeration. The posterior belief that a project with a bad report is high quality is 0, since no high quality projects receive bad reports. The posterior belief, g_i , that project i is high quality given a good report ranges from $g_i = q_i$ when $\theta_i = 1$ to $g_i = 1$ when $\theta_i = 0$. For all levels of report inflation $\theta_i \in [0, 1]$, Bayes' rule provides a one-to-one mapping between θ_i and g_i , with $g_i = q_i / (q_i + (1 - q_i)\theta_i)$, and $\partial g_i / \partial \theta_i < 0$. When characterizing the equilibrium of the reporting stage, we work with the choice of g_i directly for convenience. If an agent chooses g_i , the probability with which the posterior is equal to g_i is equal to q_i / g_i . Therefore the expected payoff of agent i is given by:

$$u_i(g_i, g_j) = \begin{cases} \frac{q_i}{g_i}(1 - \frac{q_j}{g_j}) + \frac{1}{2}(1 - \frac{q_i}{g_i})(1 - \frac{q_j}{g_j}) & \text{if } g_i < g_j \\ \frac{q_i}{g_i} + \frac{1}{2}(1 - \frac{q_i}{g_i})(1 - \frac{q_j}{g_j}) & \text{if } g_i > g_j \\ \frac{q_i}{g_i}(1 - \frac{1}{2}\frac{q_j}{g_j}) + \frac{1}{2}(1 - \frac{q_i}{g_i})(1 - \frac{q_j}{g_j}) & \text{if } g_i = g_j \end{cases}$$

Lemma A1. *If $q_b \geq 1 - q_a$ then the unique Nash equilibrium of the second stage game is $g_a = g_b = 1$.*

Proof. If agent j chooses $g_j=1$, then the best possible deviation from $g_i = 1$ is $g_i = q_i$. By choosing this deviation, agent i assures that if the other agent's project receives an L and is thus revealed to be low-type, agent i payoff is 1. Thus, $g_a = g_b = 1$ is a Nash equilibrium if and only if for each agent i

$$q_i(1 - \frac{q_j}{2}) + \frac{1}{2}(1 - q_i)(1 - q_j) \geq 1 - q_j \leftrightarrow q_i + q_j \geq 1$$

Thus, the equilibrium of the second stage is fully revealing if and only if $q_b \geq 1 - q_j$ ■

Lemma A2. *If $\frac{1}{2}(1 - q_a) < q_b < 1 - q_a$ then the mixed strategy Nash equilibrium is as follows:*

$$g_a = \begin{cases} q_a & \text{with prob } 1 - \phi_1 - \phi_2 = 1 - \frac{2q_b}{q_b + q_a} \\ G \sim F(x) = \frac{x^2 - q_a^2}{4(1 - q_b)(1 - q_a - q_b)} & \text{with prob } \phi_1 = (\frac{2q_b}{q_b + q_a})(1 - \frac{q_a + 2q_b - 1}{q_b(q_a + q_b)}) \\ 1 & \text{with prob } \phi_2 = (\frac{2q_b}{q_b + q_a})\frac{q_a + 2q_b - 1}{q_b(q_a + q_b)} \end{cases}$$

$$g_b = \begin{cases} G \sim F(x) = \frac{x^2 - q_a^2}{4(1-q_b)(1-q_a-q_b)} & \text{with prob } \lambda = 1 - \frac{q_a + 2q_b - 1}{q_b(q_a + q_b)} \\ 1 & \text{with prob } 1 - \lambda = \frac{q_a + 2q_b - 1}{q_b(q_a + q_b)} \end{cases}$$

Proof. Note, all probabilities are positive and sum to one, and the density of G is given by $f(x) = x/(2(1-q_b)(1-q_a-q_b))$. The support of G is $[q_a, 2 - q_a - 2q_b]$, and for the parameters of the proposition, the top of the support is in $[0, 1]$. To show that the proposed strategies constitute a mixed strategy Nash equilibrium, we verify that each agent is indifferent among all pure strategies inside the support and that no pure strategy outside the support delivers a better expected payoff against the mixed strategy of the other player.¹⁹

Agent b 's expected payoff from pure strategy p in the support of its mixed strategy:

$$u_b = \begin{cases} \frac{q_b}{p}(1 - \phi_1 \int_p^{2-q_a-2q_b} f(s) \frac{q_a}{s} ds - \phi_2 q_a) + \frac{1}{2}(1 - E[\frac{q_a}{g_a}])(1 - \frac{q_b}{p}) & \text{if } p \in [q_a, 2 - q_a - 2q_b] \\ q_b(1 - \frac{\phi_2 q_a}{2}) + \frac{1}{2}(1 - E[\frac{q_a}{g_a}])(1 - q_b) & \text{if } p = 1 \end{cases}$$

Substitution and simplification gives:

$$u_b = \begin{cases} \frac{q_b}{p} \frac{q_b^2 + pq_a}{(q_a + q_b)^2} + \frac{1}{2} \left(\frac{2q_b^2}{(q_a + q_b)^2} \right) (1 - \frac{q_b}{p}) & \text{if } p \in [q_a, 2 - q_a - 2q_b] \\ q_b \left(1 - \frac{q_a}{2} \left(\frac{2q_b}{q_b + q_a} \right) \frac{q_a + 2q_b - 1}{q_b(q_a + q_b)} \right) + \frac{1}{2} \left(\frac{2q_b^2}{(q_a + q_b)^2} \right) (1 - q_b) & \text{if } p = 1 \end{cases}$$

Further simplification gives $u_b = q_b/(q_a + q_b)$ in both cases. Thus all pure strategies in the support of b 's mixed strategy give the same expected payoff. Choosing any pure strategy $\hat{g} \in (2 - q_a - 2q_b, 1)$ is dominated by choosing $g = 2 - q_a - 2q_b$, because the probability of winning is the same for both pure strategies, but g is more likely to generate a good realization. It is also straightforward to verify that choosing q_b is dominated by the equilibrium mixed strategy. Thus all pure strategies in the support of b 's mixed strategy give the same expected payoff against a 's mixed strategy, and no strategy outside the support gives b a higher payoff. Thus, b 's mixed strategy is a best response to a 's mixed strategy.

A symmetric analysis applies to agent a . Simplifying the utility function gives $u_a = q_a/(q_a + q_b)$, and by a symmetric argument as above, one can show that a mixed strategy is a best response to b 's mixed strategy. ■

¹⁹We follow a similar approach in the proof to the next lemma as well. Alternative derivation of the equilibrium from the indifference conditions, which also shows uniqueness, is available for both equilibrium cases upon request.

Lemma A3. If $q_b \leq \frac{1}{2}(1 - q_a)$ then the mixed strategy Nash equilibrium is as follows:

$$g_a = \begin{cases} q_a & \text{with probability } 1 - \phi = 1 - \frac{2q_b}{q_b + q_a} \\ G \sim F(x) = \frac{x^2 - q_a^2}{4q_b(q_a + q_b)} & \text{with probability } \phi = \frac{2q_b}{q_b + q_a} \end{cases}$$

$$g_b = G \sim F(x) = \frac{x^2 - q_a^2}{4q_b(q_a + q_b)}$$

Proof. Note, all probabilities are positive and sum to one, and the density of G is given by $f(x) = x/(2q_b(q_a + q_b))$. The support of G is $[q_a, q_a + 2q_b]$, and for the parameters of the proposition, the top of the support is in $[0, 1]$. As before, we establish indifference between all pure strategies played with positive probability, and show that no pure strategy outside of the mixing distribution results in higher expected payoffs.

Consider agent b 's expected payoff from a pure strategy $p \in [q_a, q_a + 2q_b]$ in the support of its mixed strategy:

$$u_b = \frac{q_b}{p} (1 - \phi \int_p^{q_a + 2q_b} f(s) \frac{q_a}{s} ds) + \frac{1}{2} (1 - E[\frac{q_a}{g_a}])(1 - \frac{q_b}{p})$$

Substitution and simplification gives:

$$u_b = \frac{q_b}{p} (1 - \frac{2q_b}{q_a + q_b} q_a \frac{q_a + 2q_b - p}{2q_b(q_a + q_b)}) + \frac{1}{2} (1 - \frac{2q_b}{q_a + q_b} \frac{q_a}{(q_a + q_b)} - (1 - \frac{2q_a}{q_a + q_b}))(1 - \frac{q_b}{p})$$

Further simplification gives $u_b = q_b/(q_a + q_b)$. Following the same argument as in the proof to the previous lemma, one can show that b mixed strategy is a best response to a 's mixed strategy.

Consider agent a 's expected payoff from a pure strategy p in the support of its mixed strategy:

$$u_a = \frac{q_a}{p} (1 - \int_p^{q_a + 2q_b} f(s) \frac{q_b}{s} ds) + \frac{1}{2} (1 - E[\frac{q_b}{g_a}])(1 - \frac{q_a}{p}) \quad \text{if } p \in [q_a, q_a + 2q_b]$$

Substitution and simplification gives $u_a = q_b/(q_a + q_b)$. Following the same argument as before, one can show that a mixed strategy is a best response to b 's mixed strategy. ■

Proof of Proposition 2.

Proof. Consider a symmetric pure strategy Nash equilibrium $q_i = q_j = q \leq 1/2$. Because $q \leq 1/2$, $q \leq 1 - q$. Therefore, over the region $q_i \leq 1 - q$, selecting $q_i = q$ must be optimal. Hence,

a necessary condition for (q, q) to constitute a Nash equilibrium with $q \leq 1/2$ is the following stationarity condition:

$$\left. \frac{d}{dq_i} \left(\frac{q_i}{q_i + q} - \frac{q_i^2}{\rho} \right) \right]_{q_i=q} = 0 \iff 2q_i^3 + 4qq_i^2 + 2q^2q_i - q\rho \Big|_{q_i=q} = 0 \iff 8q^3 - q\rho = 0 \quad (\text{A1})$$

Note that over the region $q_i < 1 - q$ the payoff function is strictly concave in q_i , and thus this first order condition defines a maximum over this region. Because $q = 0$ cannot be an equilibrium, the only positive value of q satisfying this necessary condition that could arise in equilibrium is

$$q^* = \frac{\sqrt{2\rho}}{4}$$

To be consistent with the initial assumption that $q \leq 1/2$, it must be that $\rho \leq 2$. Thus, for $\rho \leq 2$ selecting q^* as a response to q^* dominates any other possible value of $q_i < 1 - q^*$. To show that investment level q^* constitutes the unique symmetric equilibrium, profitable deviations above $1 - q^*$ must be ruled out. Note that for $q_i > 1 - q^*$, the derivative of agent i 's payoff function is

$$\frac{d}{dq_i} \left(\frac{1}{2}(1 + q_i - q^*) - \frac{q_i^2}{\rho} \right) = \frac{1}{2} - \frac{2q_i}{\rho}$$

For $q_i > 1 - q^*$

$$\frac{1}{2} - \frac{2q_i}{\rho} < \frac{1}{2} - \frac{2(1 - q^*)}{\rho} = \frac{\rho + \sqrt{2\rho} - 4}{2\rho} \leq 0 \text{ for } \rho \leq 2$$

Hence, when $\rho \leq 2$, no deviations above $1 - q^*$ are profitable.

Next, consider a symmetric pure strategy equilibrium $q_i = q_j = q > 1/2$. Because $q > 1/2$, $q > 1 - q$. Therefore, over the region $q_i > 1 - q$, selecting $q_i = q$ must be optimal. Hence, a necessary condition for (q, q) to constitute a Nash equilibrium with $q > 1/2$ is the following stationarity condition:

$$\left. \frac{d}{dq_i} \left(\frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho} \right) \right]_{q_i=q} = 0 \iff \frac{1}{2} = \frac{2q}{\rho} \quad (\text{A2})$$

Note that over the region $q_i > 1 - q$ the payoff function is strictly concave in q_i , and thus this first order condition defines a maximum over this region. Thus, the only positive value of q satisfying this necessary condition that could arise in equilibrium is

$$q^* = \frac{\rho}{4}$$

To be consistent with the initial assumption that $q > 1/2$, it must be that $\rho > 2$. Thus, for $\rho > 2$ selecting q^* as a response to q^* dominates any other possible value of $q_i > 1 - q^*$. To show that investment level q^* constitutes the unique symmetric equilibrium, profitable deviations below $1 - q^*$ must be ruled out. Note that for $q_i < 1 - q^*$, the agent's payoff function is strictly concave, and thus has no more than one peak. The derivative of agent i 's payoff function is

$$\frac{d}{dq_i} \left(\frac{q_i}{q_i + q^*} - \frac{q_i^2}{\rho} \right) = \frac{q^*}{(q_i + q^*)^2} - \frac{2q_i}{\rho}$$

For $q_i < 1 - q^*$ the derivative is larger than at $q_i = 1 - q^*$. Hence, for $q_i < 1 - q^*$,

$$\frac{q^*}{(q_i + q^*)^2} - \frac{2q_i}{\rho} > q^* - \frac{2(1 - q^*)}{\rho} > 0 \text{ for } \rho > 2$$

Hence, when $\rho > 2$, no deviations below $1 - q^*$ are profitable. ■

Proof of Corollary 1.

Proof. In the fully-revealing benchmark, $q^* = \frac{\rho}{4}$. When exaggeration is allowed, $q^* = \frac{\sqrt{\rho}}{2\sqrt{2}}$.

$$q^* = \frac{\sqrt{\rho}}{2\sqrt{2}} > q^* = \frac{\rho}{4},$$

iff

$$\rho < 2.$$

■

Proof of Proposition 3.

Proof. First, we begin with the decision-maker's payoff for symmetric investments $q_a = q_b = q$. Because we consider symmetric agents, the equilibrium of the investment stage will also be symmetric. This calculation will therefore facilitate the comparison of decision-maker payoff under exaggeration to the decision-maker payoff in the no exaggeration benchmark. Given the posterior belief realizations of (g_a, g_b) generated by the equilibrium mixed strategies, define $g_m = \max[g_a, g_b]$

and $g_n = \min[g_a, g_b]$. The decision-maker's expected payoff for a particular combination of (g_m, g_n) is given by

$$u_p(g_m, g_n) = g_m \frac{q}{g_m} + (1 - \frac{q}{g_m}) g_n \frac{q}{g_n} = q(2 - \frac{q}{g_m}) \quad (\text{A3})$$

If the agent with the less-inflated disclosure policy, and higher posterior g_m , generates a good report, then the decision-maker will accept that project, giving the decision-maker a payoff of g_m . This event occurs with probability $\frac{q}{g_m}$ (the first term in expression (A3)). If the project with higher g_i generates a bad report, then the decision-maker knows for sure that the project is low quality. In this instance, if the more-inflated project generates a good report, (probability $\frac{q}{g_n}$) the decision-maker accepts it, giving the decision-maker an expected payoff of g_n . Thus, the decision-maker's ex ante expected payoff in this equilibrium is equal to

$$u_p^* = E[q(2 - \frac{q}{g_m})] = q(2 - qE[\frac{1}{g_m}]) \quad (\text{A4})$$

In order to calculate the decision-maker's ex ante expected payoff in this equilibrium, we need to determine the expected value of the inverse of the maximum order statistic from the equilibrium mixed strategies, $E[\frac{1}{g_m}]$. We consider three cases in turn:

Case I: $q \leq \frac{1}{3}$. When $q_a = q_b = q \leq \frac{1}{3}$, the equilibrium mixed strategy of each agent is to randomize over support $[q, 3q]$ using distribution function $F(x) = \frac{x^2 - q^2}{8q^2}$ with corresponding density $f(x) = \frac{x}{4q^2}$. The expectation in question, $E[\frac{1}{g_m}]$, is therefore²⁰

$$\int_q^{3q} 2(\frac{1}{x})(\frac{x^2 - q^2}{8q^2}) \frac{x}{4q^2} dx = \frac{5}{12q}$$

Thus, when $q \leq 1/3$ the decision-maker's ex ante equilibrium expected payoff is

$$q(2 - q\frac{5}{12q}) = (\frac{19}{12})q$$

Case II: $\frac{1}{3} \leq q \leq \frac{1}{2}$. Here, the equilibrium mixed strategy of each agent is as follows. With probability $\phi = \frac{3q-1}{2q^2}$ choose $g = 1$. With probability $1 - \phi$ randomize over support $[q, 2 - 3q]$

²⁰The density of the maximum order statistic g_m is $2f(x)F(x)$

using distribution function $F(x) = (x^2 - q^2)/(4(1 - q)(1 - 2q))$ with corresponding density $f(x) = x/(2(1 - q)(1 - 2q))$. Thus, $E[\frac{1}{g_m}]$ equals

$$1 - (1 - \phi)^2 + (1 - \phi)^2 \int_q^{2-3q} 2\left(\frac{1}{x}\right)\left(\frac{x^2 - q^2}{4(1 - q)(1 - 2q)}\right)\left(\frac{x}{2(1 - q)(1 - 2q)}\right)dx = \frac{1}{12q^4}(32q^3 - 27q^2 + 9q - 1).$$

Thus for $\frac{1}{3} \leq q \leq \frac{1}{2}$ the decision-maker expected payoff simplifies to

$$(1 - 9q + 27q^2 - 8q^3)/(12q^2)$$

Case III: $q \geq \frac{1}{2}$. In this case, agents use fully revealing grading strategies in the second stage and the decision-maker's ex ante equilibrium expected payoff is

$$2q - q^2$$

Summarizing, in a subgame in which $q_a = q_b = q$, the decision-maker's ex ante expected payoff when financial exaggeration is possible is equal to:

$$u_p^* = \begin{cases} \frac{19}{12}q & \text{if } q \leq \frac{1}{3} \\ (1 - 9q + 27q^2 - 8q^3)/(12q^2) & \text{if } \frac{1}{3} \leq q \leq \frac{1}{2} \\ 2q - q^2 & \text{if } q > \frac{1}{2} \end{cases} \quad (\text{A5})$$

Now, we can consider the payoff to the decision-maker for different values of $\rho < 2$. Substituting into (A5) yields (6). Taking the difference between (5) and (6) yields

$$\begin{aligned} & 1 - (1 - \frac{\rho}{4})^2 - \frac{19}{12} \frac{\sqrt{2\rho}}{4} & \text{if } \rho \leq \frac{8}{9} \\ & 1 - (1 - \frac{\rho}{4})^2 - \frac{1}{12(\frac{\sqrt{2\rho}}{4})^2} (1 - 9\frac{\sqrt{2\rho}}{4} + 27(\frac{\sqrt{2\rho}}{4})^2 - 8(\frac{\sqrt{2\rho}}{4})^3) & \text{if } \frac{8}{9} \leq \rho \leq 2 \end{aligned}$$

Plotting these payoff functions clearly shows that when investment is endogenous, the decision-maker's evaluator payoff is higher when exaggeration is allowed. ■

Proof of Proposition 4.

Proof. First consider that $\rho \geq 2$. Suppose that agent j chooses $q_j = 1$. All pure strategies $q_i \in (0, 1)$ lead to payoff zero. Because investment is costly, and given agent j 's strategy, the best deviation from $q_i = 1$ is $q_i = 0$. Thus, provided

$$\frac{1}{2} - \frac{1}{\rho} \geq 0 \leftrightarrow \rho \geq 2$$

$q_i = 1$ is a best response to $q_j = 1$, and full investment is the unique equilibrium.

Now, consider that $1 < \rho < 2$. Under the parameter range in the proposition, all values we claim are probabilities are in $[0, 1]$ and sum to one. Also, $F(x)$ is increasing on the support of Q which is $[0, \sqrt{2-\rho}]$. To show that the proposed strategies constitute a mixed strategy Nash equilibrium, we verify that each agent is indifferent among all pure strategies inside the support and that no pure strategy outside the support delivers a better expected payoff against the mixed strategy of the other player. A derivation of the equilibrium from the indifference conditions, which also shows uniqueness, is available upon request.

Consider agent i 's expected payoff from a pure strategy p in the support of its mixed strategy:

$$u = \begin{cases} 0 & \text{if } p = 0 \\ (\frac{2}{\rho} - 1)F(p) - \frac{p^2}{\rho} & \text{if } p \in [0, \sqrt{2-\rho}] \\ 1 - \frac{2(1-\frac{1}{\rho})}{2} - \frac{1}{\rho} & \text{if } p = 1 \end{cases}$$

Substituting and simplifying gives:

$$u_\beta = \begin{cases} 0 & \text{if } p = 0 \\ (\frac{2}{\rho} - 1)\frac{p^2}{2-\rho} - \frac{p^2}{\rho} = 0 & \text{if } p \in [0, \sqrt{2-\rho}] \\ 1 - \frac{2(1-\frac{1}{\rho})}{2} - \frac{1}{\rho} = 0 & \text{if } p = 1 \end{cases}$$

Thus all pure strategies in the support of agent i 's mixed strategy give the agent expected payoff zero. Choosing any pure strategy $\hat{q} \in (\sqrt{2-\rho}, 1)$ is dominated by choosing $q = \sqrt{2-\rho}$, because the probability of winning is the same for both pure strategies, but q is less costly. Thus all pure strategies in the support of i 's mixed strategy give the same expected payoff against j 's mixed strategy, and no strategy outside the support gives i a higher payoff.

Now, consider that $\rho \leq 1$. To show that the proposed strategies constitute a mixed strategy Nash equilibrium, we verify that each agent is indifferent among all pure strategies inside the support and that no pure strategy outside the support delivers a better expected payoff against the mixed strategy of the other player. A derivation of the equilibrium from the indifference conditions, which also shows uniqueness, is available upon request.

Consider agent i 's expected payoff from a pure strategy p in the support of its mixed strategy:

$$u = \begin{cases} 0 & \text{if } p = 0 \\ F(p) - \frac{p^2}{\rho} & \text{if } p \in [0, \sqrt{\rho}] \end{cases}$$

Substituting and simplifying gives that in both cases $u = 0$. Thus all pure strategies in the support of agent i mixed strategy give the agent expected payoff zero. Choosing any pure strategy $\hat{q} \in (\sqrt{\rho}, 1)$ is dominated by choosing $q = \rho$, because the probability of winning is the same for both pure strategies, but q is less costly. Thus all pure strategies in the support of i mixed strategy give the same expected payoff against j mixed strategy, and no strategy outside the support gives i a higher payoff.

Finally, we can compare payoff comparisons for all three cases. If $\rho > 2$, uninformative reporting is clearly optimal since the unique equilibrium involves $q_i = q_j = 1$. We consider the other two cases in turn and show that uninformative reporting is preferred to either strategic reporting or fully-revealing reporting. In order to make this calculation, we need to know the evaluator payoff in the absence of reporting.

Case: $1 \leq \rho \leq 2$ Observe that

$$E[Q^{(2)}] = 2 \int_0^{\sqrt{2-\rho}} \left(\frac{2x}{2-\rho}\right) \left(\frac{x^2}{2-\rho}\right) (x) dx = \frac{4}{5} \sqrt{2-\rho}$$

Where $Q^{(2)}$ represents the maximum of two draws of Q . The decision-maker's expected payoff is therefore

$$1 - \left(\frac{2}{\rho} - 1\right)^2 + \left(\frac{2}{\rho} - 1\right)^2 E[Q^{(2)}] = \frac{4}{5\rho^2} ((2-\rho)^{\frac{5}{2}} - 5(1-\rho))$$

Case: $\rho \leq 1$ Observe that the decision-maker's expected payoff is given by:

$$E[Q^{(2)}] = 2 \int_0^{\sqrt{\rho}} \left(\frac{2x}{\rho}\right) \left(\frac{x^2}{\rho}\right) (x) dx = \frac{4}{5} \sqrt{\rho}$$

A simple plot reveals that both of these payoffs dominate both fully-revealing and strategic reporting derived earlier in the paper. ■

Proof of Proposition 5.

Proof. Here we consider the model in which the agents' development costs are not identical, where $\rho_a > \rho_b$. If $\rho_a < 2$ and ρ_b is not too small, then a pure strategy equilibrium exists in the investment stage, which can be explicitly characterized. Consider the possibility of an equilibrium in which $q_a + q_b < 1$. Ruling out profitable deviations for agent i that are inside $[0, 1 - q_j]$ requires that the following system of first order conditions holds:

$$\frac{q_b}{(q_a + q_b^2)} - \frac{2q_a}{\rho_a} = 0 \quad \frac{q_a}{(q_a + q_b^2)} - \frac{2q_b}{\rho_b} = 0$$

The only solution of this system in which both investments are positive is

$$q_a = \frac{\sqrt{2\rho_a(\rho_a + \rho_b)\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b}}{2(\rho_a - \rho_b)} \quad q_b = \sqrt{\frac{\rho_b}{\rho_a}}q_a$$

and these are local maxima. The term inside the radical is always positive:

$$2\rho_a(\rho_a + \rho_b)\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b > 0 \iff (\rho_a + \rho_b)^2\rho_a^3\rho_b - 4\rho_a^4\rho_b^2 > 0 \iff \rho_1^3\rho_2(\rho_1 - \rho_2)^2 > 0$$

Next note that if $\rho_a < 2$ then $q_a < 1/2$ and therefore $q_a + q_b < 1$. Indeed,

$$\begin{aligned} \frac{\sqrt{2\rho_a(\rho_a + \rho_b)\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b}}{2(\rho_a - \rho_b)} &< \frac{1}{2} \iff \\ \sqrt{2\rho_a(\rho_a + \rho_b)\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b} &< (\rho_a - \rho_b) \iff \\ 2\rho_a(\rho_a + \rho_b)\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b &< (\rho_a - \rho_b)^2 \iff \\ 4(\rho_a + \rho_b)^2\rho_a^3\rho_b &< (4\rho_a^2\rho_b + (\rho_a - \rho_b)^2)^2 \iff \\ 4(\rho_a + \rho_b)^2\rho_a^3\rho_b - (4\rho_a^2\rho_b + (\rho_a - \rho_b)^2)^2 &< 0 \iff \\ -(\rho_a - \rho_b)^2((\rho_a - \rho_b)^2 + 4\rho_a^2\rho_b(2 - \rho_a)) &< 0 \iff \end{aligned}$$

To complete the characterization, we show that when q_b is sufficiently close to q_a , no deviations in the range $[1 - q_j, 1]$ are optimal for player i . In this region, each player's payoff function is

$$\frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho_i}$$

with derivative

$$\frac{1}{2} - \frac{2q_i}{\rho_i} = \frac{4(\frac{\rho_i}{4} - q_i)}{2\rho_i}$$

Hence if $q_i > \rho_i/4$ for both $i = a, b$, then each player's payoff function is decreasing in this region (when the other player plays his part of the strategy) and therefore no deviation in this interval could be beneficial. Observe first that if $q_a > \rho_a/4$ then $q_b > \rho_b/4$. Indeed,

$$q_a > \rho_a/4 \Rightarrow q_a \sqrt{\frac{\rho_b}{\rho_a}} > \sqrt{\frac{\rho_b}{\rho_a}} \rho_a/4 \Rightarrow q_b > \frac{\sqrt{\rho_a \rho_b}}{4} > \frac{\rho_b}{4}$$

Hence, $q_a > \rho_a/4$ is sufficient to rule out global deviations. For $\rho_a < 2$, this condition is satisfied whenever

$$\rho_b > R_a \equiv \frac{\rho_a^2 - 16\rho_a + 32 + (4\sqrt{2}\rho_a - 16\sqrt{2})\sqrt{2 - \rho_a}}{\rho_a}$$

$R_a < \rho_a$ for $\rho_a < 2$. Hence for $R_a < \rho_b < \rho_a < 2$, the investment levels above constitute an equilibrium.

■

Proof of Proposition 6.

Proof. Let us start with the case in which there is rationing among high-type projects. Conditional on being $\tau_i = H$, define $\phi < 1$ to be the probability of being accepted. Each agent's optimal quality choice is calculated as

$$\phi q_i - \frac{q_i^2}{2\rho} \rightarrow q_i = \phi \rho_i.$$

Given this, the overall measure of good projects developed is

$$\int_0^1 \phi \rho_i f(\rho_i) d\rho_i = \phi \mu.$$

For consistency ϕ must be the share of acceptable projects among those that are good

$$\phi = \frac{x}{\phi \mu} \rightarrow \phi = \sqrt{\frac{x}{\mu}}$$

and to be consistent with rationing, it must be that $x \leq \mu$. Hence, actual measure of good projects in this case is $\phi \mu = \sqrt{x\mu}$. Since only x are accepted, the decision-maker's payoff equals x . Each

agent's investment in quality is calculated as

$$q_i = \phi \rho_i = \sqrt{\frac{x}{\mu}} \rho_i.$$

By inspection, each agent's effort is increasing in the x .

Now, suppose that the decision-maker accepts all projects with $\tau_i = H$ and a ϕ fraction of the projects with $\tau_i = L$. Each agent's optimal quality choice is calculated as

$$q_i + \phi(1 - q_i) - \frac{q_i^2}{2\rho} \rightarrow q_i = (1 - \phi) \rho_i.$$

Given this, the overall measure of good projects developed is

$$\int_0^1 (1 - \phi) \rho_i f(\rho_i) d\rho_i = (1 - \phi) \mu.$$

and overall measure of bad projects developed is

$$1 - (1 - \phi) \mu.$$

For consistency ϕ must be the share of projects accepted among those that are bad

$$\phi = \frac{x - (1 - \phi) \mu}{1 - (1 - \phi) \mu} \rightarrow$$

$$\phi_1 = \frac{1}{2\mu} \left(2\mu - 1 + \sqrt{1 - 4\mu(1 - x)} \right)$$

$$\phi_2 = \frac{1}{2\mu} \left(2\mu - 1 - \sqrt{1 - 4\mu(1 - x)} \right)$$

Since $\mu < \frac{1}{2}$, $\phi = \phi_1$. Since all good projects are accepted, the decision-maker payoff is equal to the measure of good projects:

$$(1 - \phi) \mu = \left(1 - \frac{1}{2\mu} \left(2\mu - 1 + \sqrt{1 - 4\mu(1 - x)} \right) \right) \mu.$$

Each agent's investment in quality is calculated as

$$q_i = \left[1 - \frac{1}{2\mu} \left(2\mu - 1 + \sqrt{1 - 4\mu(1 - x)} \right) \right] \rho_i.$$

By inspection, both the decision-maker's payoff and each agent's quality choice is decreasing in x .

■

Proof of Proposition 7.

Proof. Suppose $G < 1$ and no rationing at G . An agent with ρ_i has a payoff

$$\frac{q_i}{G} - \frac{q_i^2}{2\rho}$$

from choosing $q_i < G$ and payoff $1 - \frac{G^2}{2\rho}$ from choosing $q_i = G$. The former case is equivalent to putting in less effort, but making up for it by exaggerating less. The latter case is equivalent to choosing $\theta = 1$ (i.e., an uninformative signal), but investing sufficient quality to get the resource.

Among the agents who choose $q_i < G$,

$$\begin{aligned} q_i &= \frac{\rho_i}{G} \\ u_i &= \frac{\frac{\rho_i}{G}}{G} - \frac{\left(\frac{\rho_i}{G}\right)^2}{2\rho_i} = \frac{\rho_i}{2G^2}. \end{aligned}$$

Since $q_i < G$, it follows that $\rho_i < G^2$. Note it is easy to check that for $\rho_i < G^2$, investing ρ_i/G is preferred to deviating:

$$\begin{aligned} \frac{\rho_i}{2G^2} &> 1 - \frac{G^2}{2\rho_i} \\ \frac{\rho_i}{2G^2} - \left(1 - \frac{G^2}{2\rho_i}\right) &= \frac{1}{2G^2\rho_i} (\rho_i - G^2)^2 > 0 \end{aligned}$$

If $\rho_i > G^2$, then the optimal quality is G . Hence, all types with $\rho_i < G^2$ prefer to exert effort less than G and then give a more informative signal, whereas types with $\rho_i > G^2$ exert effort G and always claim to have a good outcome. Hence, mass of agents with projects that exactly hit threshold G is equal to

$$\int_{\rho_L}^{G^2} \frac{\left(\frac{\rho_i}{G}\right)}{G} f(\rho_i) d\rho_i + (1 - F(G^2)).$$

Because no rationing is possible at G , unless G is 1, it must be that if there is no rationing in equilibrium, then :

$$\int_{\rho_L}^{G^2} \frac{\rho_i}{G^2} f(\rho_i) d\rho_i + (1 - F(G^2)) = x,$$

and if rationing, then $G = 1$, and hence,

$$\mu < \int_{\rho_L}^{\min\{\rho_H, 1\}} \rho_i f(\rho_i) d\rho_i.$$

Now, let us consider that $\rho \sim U [0, 1]$. In that case $x < \frac{1}{2}$ implies that the equilibrium is fully revealing, but $x > \frac{1}{2}$ implies that G solves

$$\int_0^{G^2} \frac{\rho_i}{G^2} d\rho_i + (1 - G^2) = x \rightarrow G = \sqrt{2(1 - x)}$$

The decision-maker's expected payoff is therefore the measure of projects that attain threshold G , i.e. x , multiplied by the expected payoff of each project, G i.e. decision-maker payoff is

$$x\sqrt{2(1 - x)}. \tag{A6}$$

Now, let us compare the decision-maker's payoff with full revelation versus exaggeration when $x > \frac{1}{2}$. Substituting $E[\rho_i] = \frac{1}{2}$ into (11) yields

$$u_d = \frac{1}{2}[1 - \sqrt{(2x - 1)}].$$

Comparing that to (A6) for $E[\rho_i] = \frac{1}{2}$, the decision-maker's payoff is higher with exaggeration if

$$F(x) = x\sqrt{2(1 - x)} - \frac{1}{2}[1 - \sqrt{(2x - 1)}] > 0. \tag{A7}$$

It is easy to show that $F(x) = 0$ for $x = \frac{1}{2}$ and $x = 1$. Further, $F'(x) > 0$ for $x \in [\frac{1}{2}, \frac{2}{3}]$ and $F''(x) < 0$ for all $x \geq \frac{1}{2}$. Therefore, (A7) holds. ■