

Testing Factor Models on Characteristic and Covariance Pure Plays

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Abstract

We test the recent Fama-French five-factor model and Hou-Xue-Zhang four-factor model using test assets from Fama-MacBeth regressions, which are pure plays on particular characteristics or covariances. Our tests resolve the errors-in-variable bias in Fama-MacBeth regressions with estimated betas. Monte Carlo evidence shows that the tests are unbiased even with time-varying stock betas and characteristics. For both factor models, characteristic pure plays generally have positive alphas, and covariance pure plays have negative alphas. The models fail especially in explaining returns to investment and when pure plays are momentum-neutral. The rejections are economically significant.

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1. Introduction

We test the factor models recently proposed by Fama and French (2015) and Hou, Xue and Zhang (2015) that include investment and profitability factors. Like Daniel and Titman (1997), we find that there are returns associated with characteristics that are not explained by covariances with the factors. We construct test assets that are bets on a specific characteristic or covariance and are neutral with respect to all others. The test assets that are bets on characteristics generally have positive alphas and the test assets that are bets on covariances generally have negative alphas. This shows that factor exposures do not explain the returns to characteristics and assets exposed to factor risks do not necessarily have risk premia commensurate with their exposures.

The models do a particularly poor job of explaining the different returns of stocks with high and low investment rates: In both models and in all versions of the models we test, the asset that is a bet on the investment characteristic has a significant positive alpha, and the asset that is a bet on the investment factor covariance has a significant negative alpha. The Fama-French model cannot explain momentum returns (this is observed by Fama and French, 2014). The Hou-Xue-Zhang model cannot explain the value effect. In fact, the models have difficulty with multiple characteristics, especially when the test assets are constructed to be momentum-neutral.

The two standard methods of testing whether returns are due to characteristics or to covariances are (i) form portfolios from intersecting sorts on a characteristic and corresponding covariance and test whether the portfolios have alphas relative to the factor model, and (ii) run Fama-MacBeth regressions of returns on characteristics and covariances and test whether the coefficients have positive means. Both methods have drawbacks. The disadvantage of the first method is that it is difficult to simultaneously control for other characteristics and covariances. It can be important to control for other characteristics. In particular, we show that it is important to control for momentum when forming bets on size and value. However, it can be impossible to do this with sorts. For example, in a five-factor model, there are ten characteristics and covariances, so intersecting tercile sorts produces 3^{10} separate portfolios, the vast majority of which must contain zero stocks. The disadvantage of the second method is the well-known errors-in-variables (EIV) bias. The fact that

covariances are estimated with error is likely to bias the results towards the conclusion that only characteristics matter (e.g., Lin and Zhang, 2013).

We combine the two methods, which resolves the issues with both. First, we run Fama-MacBeth regressions to simultaneously control for all other characteristics and covariances. Next, rather than test the hypothesis that Fama-MacBeth slope coefficients have zero means, we regress the time series of coefficients on factors and test the null hypotheses that they have zero alphas. The key insight, due to Fama (1976), is that Fama-MacBeth regression coefficients are returns to tradeable portfolios. These portfolios are pure plays on a particular characteristic or covariance and are neutral to all others. We show that the EIV bias causes covariance pure plays to have smaller factor exposures than expected, which results in the standard EIV outcome of downward-biased estimates of factor risk premia. The EIV bias also results in factor exposures for the characteristic pure plays. The time-series regressions test whether both sets of pure plays earn returns commensurate with their ex-post factor exposures. There are no generated regressors in these time-series regressions. We simply regress portfolio returns on factors. If the factor model is correct and the pure play returns and factors are stationary, then the alphas in the time-series regressions are unbiased. We provide Monte Carlo evidence that the factor-model alpha tests are indeed unbiased, even when there are errors in variables in the Fama-MacBeth regressions and even when stock betas and characteristics are time varying.

The idea to compute alphas of cross-sectional regression coefficients is not new. Brennan, Chordia and Subrahmanyam (1998) compute alphas of the coefficients from their risk-adjusted cross-sectional regressions. They call the regression coefficients “raw estimates” and the alphas “purged estimates.” We provide Monte Carlo evidence that purged Fama-MacBeth estimates work as well as the Brennan-Chordia-Subrahmanyam purged estimates. An advantage of using the Fama-MacBeth estimates is that we can determine whether test assets that are bets on estimated factor loadings have nonzero factor-model alphas. This is not possible with the Brennan-Chordia-Subrahmanyam approach, because the estimated factor loadings are absorbed in the risk adjustment stage.

It is important to compute alphas (purged estimates) rather than means (raw estimates) of cross-sectional regression coefficients. This is true for Fama-MacBeth, for the Brennan-

Chordia-Subrahmanyam methodology, and for two other methods we analyze—the EIV correction of Shanken (1992) employed by Chordia, Goyal and Shanken (2015) and the instrumental variables method proposed by Pukthuanthong, Roll and Wang (2014) and Jegadeesh and Noh (2014).^{1,2} The reason is that beta estimation errors are likely to be correlated with characteristics when betas and characteristics are time varying. Suppose, for example, that a characteristic and true beta move over time in a positively correlated manner. Suppose that both have risen recently for a particular stock. Because a past time period is used to estimate the beta, the estimated beta is likely to be smaller than the current beta, producing a negative estimation error. Because the characteristic has risen, it is likely to be larger than the cross-sectional average. Thus, negative estimation errors are likely to be associated with large values of the characteristic. This correlation between characteristics and estimation errors results in biased coefficients even for the methods that attempt to resolve the EIV bias (and which work when estimation errors are independent of characteristics). From a portfolio point of view, the biases in the coefficients correspond to unanticipated factor exposures of the portfolios for which the coefficients are returns. Regressing the portfolio returns on the factors produces alphas that are purged of the factor exposures.

Our Monte Carlo analysis supports two conclusions. First, as just stated, it is important to compute alphas rather than means of cross-sectional regression coefficients. In our simulation with time-varying betas and characteristics, none of the tests of means for any of the methodologies has the correct size. They all reject the factor model approximately 50% of the time, even though the model is true by construction in the simulation. However, all of the tests of alphas have the correct size. Second, if we are going to compute alphas, then

¹We implement the base method of Chordia, Goyal and Shanken (2015)—their Equation (5). Their appendices describe two refinements. One assumes betas are constant and is designed to deal with correlation between beta estimation errors and characteristics (for example, book-to-market) that depend on the past returns used to estimate betas. The second allows betas to be a linear function of the previous month’s characteristics. We do not implement the refinements, but they seem unlikely to correct satisfactorily for the cross-sectional correlation between beta estimation errors and characteristics that arises from general time-series correlation between betas and characteristics.

²Pukthuanthong, Roll and Wang (2014) and Jegadeesh and Noh (2014) propose instrumental variables estimators for models in which only estimated betas (not characteristics) are included as regressors. Our discussion and Monte Carlo analysis do not apply directly to those estimators but instead apply to an extension in which characteristics are also included as regressors.

there are no advantages to using EIV corrections as opposed to simply using Fama-MacBeth regressions. The alpha tests of the Fama-MacBeth coefficients perform as well as the alpha tests of any of the EIV corrected coefficients.

The pure play test assets created by Fama-MacBeth regressions can be constructed by investors in real time because the portfolio weights are based on information known prior to the month for which returns are measured. Because the test assets are long-short portfolios, their payoffs (the Fama-MacBeth slope coefficients) are excess returns. Each test asset is an isolated bet on a single variable (characteristic or estimated beta). Because we standardize the variables before running the regressions, the long side of each portfolio has a value—for the given variable—that is one standard deviation above that of the short side. For all other variables, the long and short sides of the test asset have equal values; thus, each test asset is neutral with respect to all other variables.

The test assets have zero net weights, but they are not 100% long and 100% short; that is, they are not differences of fully invested portfolios. This leverage feature does not affect the t -tests, but it does make the economic significance of the alphas somewhat difficult to judge. The sum of absolute weights of the test assets differs by month and by asset. The typical sum is approximately 1 (50% long and 50% short). We should multiply the alpha of such an asset by 2 to compare it to an alpha from a high-minus-low portfolio formed from sorts. To address this issue more systematically, we report information ratios in Section 8 and in Table 1 below. An information ratio is an alpha divided by the residual standard error. It is the Sharpe ratio of the portfolio that is long the test asset and short the portfolio of factors that best mimics the test asset (the fitted value from the factor model regression). As is always the case with Sharpe ratios, information ratios are invariant to leverage.

Table 1 provides a preview of our results. We test the two models using the characteristics from which their factors are constructed (see Sections 4 and 5). However, we also ask whether the models can explain momentum (and value in the case of the Hou-Xue-Zhang model). Furthermore, we ask whether the models can explain bets on the characteristics from which the factors are constructed when those bets are neutral with respect to momentum (and value). We accomplish this by including momentum (and value) characteristics in the Fama-MacBeth regressions. Those regressions produce the test assets for which results

are reported in Table 1. The characteristics used in the two models differ in some respects. We use generic names in Table 1 and explain the exact characteristics later. Table 1 reports the alphas and the information ratios of the test assets. To put the information ratios in perspective, it is useful to observe that the market, SMB, HML, and UMD had monthly Sharpe ratios of 0.10, 0.10, 0.13, and 0.16, respectively, over our sample period.

Table 1: **Monthly alphas and information ratios of test assets relative to the FF and HXZ models.** This table excerpts portions of Tables 8, 10, and 13. See those tables for variable descriptions. The sample excludes stocks smaller than the NYSE 20th size percentile. The size and investment characteristics are reversed (like the factors), so that small stocks have high size characteristics, and low investment stocks have high investment characteristics. The sample period is July 1964 through December 2012 for the Fama-French (FF) model and January 1972 through December 2012 for the Hou-Xue-Zhang (HXZ) model.

Test Assets	FF				HXZ			
	Characteristic		Covariance		Characteristic		Covariance	
	Alpha	I.R.	Alpha	I.R.	Alpha	I.R.	Alpha	I.R.
Size	0.07**	0.13	-0.06	-0.07	0.12**	0.19	-0.05	-0.06
Value	0.19***	0.22	-0.22***	-0.16	0.22***	0.32	-0.17**	-0.13
Profitability	0.10***	0.18	-0.07	-0.06	0.14***	0.26	-0.34***	-0.28
Investment	0.16***	0.31	-0.17**	-0.13	0.35***	0.34	-0.33***	-0.21
Momentum	0.43***	0.34	0.02	0.01	0.15	0.13	-0.23***	-0.14

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

As Table 1 indicates, the rejections of the models from these test assets are both economically and statistically significant. Furthermore, there is a consistent pattern. The test assets that are bets on characteristics have risk premia that are not explained by the factors and hence have positive alphas. The test assets that are bets on estimated betas have factor exposures (as we will show later) but they do not have risk premia commensurate with their factor exposures; hence, they have negative alphas. The largest information ratios in Table 1 are for the investment and momentum characteristics for the Fama-French model and for the investment and value characteristics for the Hou-Xue-Zhang model. It is natural to ask whether the explanation of momentum by the Fama-French model is improved by including UMD as a factor and whether the explanation of value by the Hou-Xue-Zhang model is improved by including HML as a factor. In terms of information ratios, the answer is “no” in both cases. The alphas decrease when the additional factors are included, but the

residual standard errors decrease more, so the information ratios go up rather than down.

We should emphasize that it is not a surprise to reject an asset pricing model. Even if the returns associated with characteristics are due to risk exposures, the particular factors in these models have ad hoc constructions and are surely not the precise factors that are priced. Thus, carefully designed tests will reject them. Nevertheless, it is important to document to what extent and in what manner they are rejected, to guide the development of better models. For example, our results indicate that it would be especially fruitful to improve or replace the investment factors and to improve the explanations of momentum (in the Fama French model) and value (in the Hou-Xue-Zhang model).

As is well known, the test assets derived from Fama-MacBeth regressions are maximally diversified in a certain sense (we explain this in Section 2). Thus, they are approximately equally weighted. This raises the possibility that a rejection of the factor models could be due primarily to the returns of micro-cap stocks. To avoid this possibility, our main results, including those in Table 1, are for a sample limited to stocks with market capitalizations that exceed the NYSE 20th percentile. As a robustness check, we drop this filter and find that the results are indeed stronger when micro-caps are included. We also do robustness checks by forming test assets from annually updated characteristics and by controlling for industries. Our results survive and are often stronger in these variations, which are reported in Section 7.

2. Portfolio Interpretation and EIV Corrections

The interpretation of Fama-MacBeth coefficients as portfolio returns motivates the alpha tests we conduct. It has been known since Fama (1976, Chapter 9) that the coefficients are portfolio returns.³ Nevertheless, we give a brief description here. We also explain how the

³Fama describes the origin of this insight in an interview for the American Finance Association’s History of Finance project: “I’ll tell you how that came about. Black, Jensen and Scholes published a paper on testing the CAPM. And in one of the arguments I had with Fischer, I said, “You know, Fischer, with that technique where you form portfolios, all you are doing is running a regression. He said, “No it’s not that.” I said, “Yes, it is.” “No, it isn’t.” “Yes, it is.” So, I said. “We are going to write this paper to show you that is what it does.” The paper doesn’t really describe it in those terms, but if you read Chapter 9 of *The Foundations of Finance*, it describes it exactly in those terms. That’s what’s going on. So basically I wrote *The Foundations of Finance* in order to show Fischer in detail that these two techniques were exactly the same.”

BCS (Brennan, Chordia and Subrahmanyam, 1998), CGS (Chordia, Goyal and Shanken, 2015), and IV (instrumental variable) coefficients can be interpreted in the same manner, and we relate the EIV bias to the portfolio betas.

2.1. Fama-MacBeth Coefficients as Pure Play Returns

Let N denote the number of stocks in the cross-section for a given month. We describe the estimates for a particular month and do not subscript variables by a time index. Let K denote the number of factors and let L denote the number of characteristics. Let B denote the $N \times K$ matrix of factor betas b_{nk} (estimated from time-series regressions using data from prior months) and let C denote the $N \times L$ matrix of characteristics c_{nl} (which are known at the beginning of the month). Let R denote the vector of excess stock returns r_n and let ι denote an N -dimensional vector of 1's. Consider the cross-sectional regression model that estimates slope coefficients λ :

$$r_n = \alpha + \sum_{k=1}^K \lambda_k b_{nk} + \sum_{\ell=1}^L \lambda_{K+\ell} c_{n\ell} + \varepsilon_n.$$

Set

$$X = \left(\iota \quad : \quad B \quad : \quad C \right),$$

with generic element x_{ni} , and set $W = X(X'X)^{-1}$. The $1 + K + L$ dimensional vector of regression coefficients is given by

$$(X'X)^{-1}X'R = W'R. \tag{1}$$

The fact that the coefficients compose the vector $W'R$ means that each coefficient is a linear combination of excess returns, with the weights in the linear combination being a column of W . Thus, each regression coefficient is the return to the portfolio in the corresponding column of W . Because the matrix W is known at the beginning of the month, the portfolios can be constructed in real time.

Because $W'X = I$, each column of W has a unit inner product with the corresponding column of X and is orthogonal to all other columns of X . Because the first column of X consists of 1's, this implies that the first column of W sums to one (is a unit-investment portfolio) and the other columns sum to zero (are zero-investment portfolios). The first

column of W produces the intercept in the regression. The other columns of W produce the slope coefficients. Thus, each slope coefficient is the return of a zero-investment portfolio. More explicitly, we have, for $i = 2, \dots, K + L + 1$,

$$\hat{\lambda}_{i-1} = \sum_{n=1}^N w_{ni} r_n, \quad (2)$$

$$0 = \sum_{n=1}^N w_{ni}, \quad (3)$$

$$1 = \sum_{n=1}^N w_{ni} x_{ni}, \quad (4)$$

$$(\forall j \neq i) \quad 0 = \sum_{n=1}^N w_{ni} x_{nj}. \quad (5)$$

We can regard the zero-investment portfolios as active weights that could be added to any unit investment portfolio (which we call a benchmark). Equations (4) and (5) state that adding one of the zero-investment portfolios to a benchmark would increase the benchmark's weighted average value of a single characteristic or estimated beta by one unit and would leave the weighted average values of the other characteristics and estimated betas unchanged. Thus, each slope coefficient is the return to a portfolio that is a pure play on a particular characteristic or estimated beta.

The above “pure play” interpretation is based on the fact that $W'X = I$. There are many solutions of the equation $W'X = I$ (whenever $N > K + L + 1$); hence, there are many sets of pure play portfolios. Among these, the matrix $W = X(X'X)^{-1}$ is distinguished by its least-squares property. Each column $(w_{1i} \dots w_{Ni})'$ of W solves the problem: minimize $w'w$ subject to $X'w = e_i$, where e_i denotes the i -th basis vector of \mathbb{R}^{1+K+L} . Thus, the columns of $X(X'X)^{-1}$ are maximally diversified pure plays.

2.2. The EIV Bias: The Cause and a Solution via a Portfolio Interpretation

The EIV bias can be interpreted as follows. Suppose a characteristic and a beta (for example, book-to-market and the HML beta) are positively correlated in the cross-section. To be more precise, suppose the characteristic is positively correlated with the true factor beta. The Fama-MacBeth coefficient for the characteristic is the return of the maximally

diversified portfolio that has a unit value for the characteristic and is orthogonal to the vector of estimated stock betas (and to other characteristics and estimated betas). Suppose that a unit value is an above average value for the characteristic and suppose that a zero beta is a below average beta. (The argument can be adapted to work even if this is not true.) Then, to achieve maximal diversification, we should try to be long stocks that have higher than average values for the characteristic but which do not have above average estimated betas. Because the characteristic is positively correlated in the cross section with the true beta, this will induce us to be long stocks for which the true beta is larger than the estimated beta. Consequently, even though the estimated portfolio beta is zero—by Equation (5)—the true portfolio beta in this circumstance will be positive. Thus, the portfolio return (the Fama-MacBeth coefficient for the characteristic) will have a risk premium even if the factor model is true. Symmetrically, the portfolio that produces the coefficient for the estimated beta will be long stocks for which the true beta is smaller than the estimated beta. Its estimated beta equals 1—by Equation (4)—but its true beta will be smaller than 1. Thus, its mean value will be smaller than the factor risk premium. This is the EIV attenuation bias.

A solution to this bias is to test whether the characteristic and covariance portfolios earn returns commensurate with their true betas. This is easily achieved using a time-series regression of these portfolio returns (λ_k in our notation) on the factors. This time-series regression is not subject to the EIV bias, because it has factor returns rather than generated regressors on the right hand side. Also, coefficients in this regression are estimated precisely, because the left hand side is a well-diversified portfolio with much lower idiosyncratic volatility than individual stocks have.⁴ The alphas are thus our tests of the factor model, and they are robust to the EIV bias. We confirm that the alphas are unbiased in the presence of errors-in-variables in Monte Carlo simulations in Section 3.

⁴Note that the coefficients estimated in these regressions are the intercepts and factor loadings of the pure play test assets, not the intercepts and factor loadings of individual stocks. If the factor model is true and the pure play returns and factor returns are stationary, then the estimated coefficients are unbiased.

2.3. Other EIV Corrections as Portfolios

The BCS, CGS, and IV estimators that correct for the EIV bias can also be interpreted as portfolios. This interpretation provides useful intuition for our analysis of the performance of EIV bias corrections in Monte Carlo simulations in Section 3.

BCS run cross-sectional regressions of risk-adjusted returns on characteristics. Let F denote the vector of factor realizations f_k and assume that the factors are excess returns. Continuing to let B denote the matrix of estimated betas and R the vector of excess returns, the vector of risk-adjusted stock returns is $R - BF$. Set

$$Y = \begin{pmatrix} \iota & \vdots & C \end{pmatrix},$$

where, as before, ι denotes an N -vector of 1's and C is the matrix of characteristics. Now, set $W = Y(Y'Y)^{-1}$. The BCS coefficients are the elements of the vector

$$(Y'Y)^{-1}Y'(R - BF) = W'(R - BF). \quad (6)$$

As before, we can interpret each column of W as a portfolio. Because $W'Y = I$, each column $i = 2, \dots, K + 1$ of Y sums to zero. Furthermore, the vector $R - BF$ is a vector of excess returns. Hence, the BCS coefficients are returns of zero-investment portfolios, like the Fama-MacBeth coefficients. Each BCS portfolio has a unit value for a single characteristic and a zero value for all other characteristics. What we are calling BCS coefficients here and what we will refer to as BCS coefficients in the remainder of the paper are what BCS call raw coefficients. As noted earlier, BCS also compute alphas as we do below, calling the alphas purged coefficients.

The portfolio construction in the BCS methodology does not depend on the estimated betas, so cross-sectional correlations between true betas and characteristics do not produce an EIV bias in the BCS methodology. However, if there is dependence between characteristics and beta estimation errors, then the BCS coefficients may be biased (this same observation is made by BCS). Such dependence arises naturally when characteristics and betas are time varying as discussed in the introduction.

The IV method uses two distinct sets of beta estimates. Let \hat{B} denote a second matrix of estimated betas. Pukthuanthong, Roll and Wang (2014) and Jegadeesh and Noh (2014)

propose using returns from odd months (or odd weeks) to form one set of estimates and returns from even months (or even weeks) to form the other. Define

$$Z = \begin{pmatrix} \iota & : & \hat{B} & : & C \end{pmatrix}.$$

The IV coefficients are the elements of the vector $(Z'X)^{-1}Z'R$. Again, we can write this as $W'R$, where now $W' = (Z'X)^{-1}Z'$. So, again, the coefficients are portfolio returns. Also, as with the Fama-MacBeth slope coefficients, we have $W'X = I$, so the IV slope coefficients are returns of zero-investment pure play portfolios.

Chordia, Goyal and Shanken (2015) employ an EIV correction as follows. Let

$$M = \begin{pmatrix} 0_{K \times 1} & : & I_{K \times K} & : & 0_{K \times L} \end{pmatrix}.$$

For each stock i , let Σ_i denote the $K \times K$ White (1980) heteroskedasticity-consistent estimator of the covariance matrix of the vector of beta estimates for stock i . The CGS coefficients are the elements of the vector

$$\left(X'X - \sum_{i=1}^N M'\Sigma_i M \right)^{-1} X'R.$$

We can again write this as $W'R$, so the coefficients are portfolio returns. However, in this case, we do not have $W'X = I$. As noted in footnote 1, CGS suggest additional refinements designed to deal with some correlation between characteristics and beta estimation errors and with some time variation in betas. However, we only implement their base method (their Equation 5).

3. Monte Carlo

We simulate a three-factor model for 3,000 stocks for 1,600 weeks. We simulate weekly returns and factor realizations, because we use weekly data to estimate factor betas in our empirical work. Each week we draw factor realizations from a multivariate normal distribution calibrated to the market excess return, SMB, and HML. At the beginning of the 1,600 weeks, for each stock, we draw a vector of true factor betas, characteristics, and log idiosyncratic standard deviations from a multivariate normal distribution calibrated to the Fama-French (1993, 1996) three-factor model, log market capitalization, and log book-to-market. In one set of simulations, we hold the true betas, characteristics, and idiosyncratic

standard deviations constant for each stock over the 1,600 week period. In another set of simulations, we draw innovations to the vector each week for each stock. The innovations are explained further below. For each set, we run 500 simulations.

The time period over which we calibrate the model is 1970–2013. Table 2 presents the parameters of the distribution from which the factor realizations and the distribution from which the true betas, characteristics, and log idiosyncratic standard deviations are drawn.⁵

Excess stock returns are defined from the factor model, using the true betas, the factor realizations, and residuals that are independently drawn from normal distributions with zero means and standard deviations specific to each stock as explained above. We run three sets of factor model regressions for each stock each week. For our base model, we run regressions using returns and factor realizations from the prior 48 weeks. For the IV model, we run two separate regressions using the prior 96 weeks—one regression using the 48 odd weeks and the other using the 48 even weeks.

We compound returns over each 4-week period to form monthly returns. We run cross-sectional Fama-MacBeth regressions each month. After discarding the first 96 weeks, there are 376 monthly regressions. We compound the factor realizations to form monthly factor realizations, and we run time-series regressions of cross-sectional coefficients on the factors. Each time-series regression consists of 376 observations. We conduct t -tests of the alphas of the time-series regressions. We also conduct t -tests of the time-series means of the cross-sectional coefficients. In addition to the Fama-MacBeth regressions, we also compute the BCS, CGS, and IV coefficients. We also regress those coefficients on the factors and conduct t -tests of the alphas from the regressions and of the time-series means of the coefficients. We present the results for the beta and characteristic that are calibrated to HML and log book-to-market. We focus on these because the factor risk premium is larger for HML than for SMB in our calibration. The results are qualitatively the same for the beta and

⁵We are drawing true betas, but the data to which the distribution is calibrated includes only estimated betas. If true betas and beta estimation errors are uncorrelated in the cross-section, then the cross-sectional variance of estimated betas is the sum of the cross-sectional variance of true betas and the cross-sectional variance of estimation errors. In recognition of this, we scale down the cross-sectional variances of estimated betas by a factor of 0.7 to calibrate the variances of true betas. We find that, with this scaling factor, the cross-sectional standard deviations of estimated betas in our simulations roughly match those of estimated betas in the data. Our qualitative results are invariant to the precise calibration of the model.

characteristic calibrated to SMB and log size.

3.1. Constant Betas and Characteristics

We first report results for the simulations in which stock betas, characteristics, and idiosyncratic volatilities are constant over time in each simulation. Table 3 reports results for the time-series means of coefficients for the characteristic calibrated to log book-to-market and for the estimated beta calibrated to HML. Because we are simulating under the factor model, if there were no EIV bias, then the coefficient for the characteristic should be zero and the coefficient for the estimated beta should equal the HML risk premium.

As expected, there are too many false positives for the Fama-MacBeth characteristic coefficient (Table 3, Panel A). In the absence of bias, 2.5% of the simulations should have significant positive means, but instead 45% do. The EIV corrections work reasonably well for the characteristic coefficient, except that a slight bias seems to remain for the CGS method. The false positives for the Fama-MacBeth test can be traced to the pure play portfolio described in the previous section having a true beta that is higher than its estimated beta. Each of its estimated factor betas is zero, due to Equation (5). However, its true HML beta is positive, as shown in Table 3. (This discrepancy between estimated and true portfolio betas is discussed in Section 2.2.) As a result of the discrepancy, the true risk premium of the coefficient is positive, resulting in false positive t -tests. The true HML betas of the CGS and IV coefficients are approximately equal to 0 and the true risk premia are approximately equal to 0, so the mean coefficients are close to 0, as desired. Table 3 shows that there is no bias in the BCS coefficients for either the HML beta estimate or the risk premium estimate. Therefore, the risk adjustment of the returns is accurate and the mean coefficient is 0, as desired.

Panel B of Table 3 reports the same data for the beta coefficient calibrated to HML. In the absence of bias, the mean beta coefficient would equal the HML risk premium, which is approximately 0.36% per month in our simulation. Table 3 shows that the IV method is unbiased, the CGS method is slightly biased, and the Fama-MacBeth method is significantly biased, as expected. The difference between the mean coefficient and the HML risk premium for the Fama-MacBeth method is primarily due to the difference between the true and estimated HML betas of the pure play portfolios. The estimated HML beta of

the Fama-MacBeth portfolio is 1 due to Equation (4) and the other estimated betas are 0 due to Equation 5. Therefore, the estimated risk premium equals the HML risk premium. However, the mean coefficient, which is approximately equal to the true risk premium, is smaller because the true HML beta is smaller than the estimated HML beta. This difference between the true and estimated risk premia is the EIV attenuation bias.

Panel C of Table 3 reports the alphas of the coefficients. For all methods and both coefficients, the mean alphas are approximately zero and all of the t -tests, including the test for the Fama-MacBeth coefficients, have approximately the correct size (2.5% positive and significant and 2.5% negative and significant).

Figure 1 provides additional information about the distributions of the t statistics for the characteristic coefficients across simulations. The distribution for the t statistics for the means is shifted to the right for the Fama-MacBeth method due to the EIV bias. Consistent with Table 3, Figure 1 shows that the t statistics for the means of the EIV-corrected coefficients are roughly centered around 0, indicating that the EIV corrections work well. The exception is the distribution for the CGS method, which is shifted slightly to the right.

The simulation confirms that all of the methods proposed to alleviate the EIV bias—BCS, CGS, and IV—work reasonably well when stock betas, characteristics, and idiosyncratic volatilities are constant over time. The usual t -tests based on the time-series means of the coefficients have the correct size. This is not true in the simulation of time-varying betas and characteristics described in the next subsection.

Figure 1 also shows that the distributions of the t statistics for the alphas are all centered roughly around zero. Thus, the alpha tests are unbiased. This is true even for the Fama-MacBeth method. As we will see in the next subsection, the alpha tests are also unbiased when betas and characteristics are time-varying, even though the EIV corrections do not work for the means.

3.2. Time-Varying Betas and Characteristics

In this section, we demonstrate that time variation of betas and characteristics renders the EIV corrections for the mean coefficients inadequate. However, t -tests of alphas still have the correct size, and the alpha tests for the simplest method—the Fama-MacBeth

coefficients—work as well as any.

We construct time-varying betas, characteristics, and idiosyncratic volatilities using the multivariate normal distribution described above. The parameters of that distribution are shown in Panel B of Table 2. Let μ denote the mean vector and let Σ denote the covariance matrix. Let x_{it} denote the vector of true betas, characteristics, and log idiosyncratic volatilities for stock i in week t . We draw x_{i1} from the normal (μ, Σ) distribution independently for each stock i . Then, for each stock i and week $t > 1$, we draw x_{it} as

$$x_{it} = \mu + \delta(x_{t-1} - \mu) + \varepsilon_{it}$$

for a fixed scalar $\delta < 1$, where ε_{it} is drawn from the normal $(0, (1 - \delta^2)\Sigma)$ distribution. This implies that the stationary distribution of x_{it} is the normal (μ, Σ) distribution for each i . We take $\delta = 0.97$, which implies that the half-life of shocks is roughly 6 months (24 weeks).

Panel A of Table 4 shows that there are too many false positives for all of the t -tests of means. This is not surprising for the Fama-MacBeth coefficient, because of the known EIV bias. What is noteworthy in Table 4 is that the BCS, CGS, and IV methods do not work in the presence of time-varying betas and characteristics. As discussed in the introduction, a likely cause of failure is correlation between characteristics and beta estimation errors. Negative (positive) beta estimation errors are likely to be associated with large (small) characteristic values, when betas and characteristics move together and past returns are used to estimate betas. In contrast to the simulation with constant characteristics and betas, the true HML betas and true risk premia in this simulation are higher than the estimated HML betas and estimated risk premia for the BCS, CGS, and IV methods. This produces positive mean coefficients and false positive t -tests.

Figure 2 presents the distributions of the t statistics for the characteristic coefficients in this simulation. For the means of the coefficients, roughly half—rather than the desired 2.5%—of the mass of each distribution lies to the right of the positive critical value for the t -test. However, for the alphas of the coefficients, the distributions of the t statistics are centered on 0 with approximately 2.5% of the mass of each being below the negative critical value and 2.5% being above the positive critical value. This is confirmed in Panel B of Table 4, which presents the exact proportions of the simulations for which the alpha t

statistics lie outside the critical values. Thus, the alpha tests are unbiased even with time-varying betas and characteristics.

4. The Five Factor Fama-French Model

Fama and French (2015)—hereafter, FF—propose a five factor model with the factors being the market excess return, size (Small Minus Big), value (High Minus Low), operating profit (Robust Minus Weak), and asset growth (Conservative Minus Aggressive). To test the model, we first estimate factor betas of individual stocks by running time series regressions over the prior 52 weeks. Then, we run Fama-MacBeth regressions of excess returns on the five estimated betas and four characteristics. The third and final step is to compute the alphas of the Fama-MacBeth coefficients in time series regressions on the factors.

We use the same characteristics that FF use to define the factors. The characteristics are⁶

- NME = *negative* log market equity,
- BM = log book-to-market ratio,
- OP = operating profit,
- NAG = *negative* log of one plus asset growth.

We use the negative of log market equity and the negative of the log of one plus asset growth so that each of the characteristics is ordered the same as the corresponding factor (SMB is low minus high size, and CMA is low minus high asset growth). We winsorize each variable at the 1% level.

Before running the Fama-MacBeth regressions, we standardize the estimated betas and characteristics, dividing each by its cross-sectional standard deviation each month. The motivation for standardizing characteristics can be seen from Figure 3, which plots the time series of cross-sectional standard deviations of the nonstandardized characteristics. Each of the cross-sectional standard deviations has a positive time trend. Lewellen (2015) makes this same observation and also shows that Fama-MacBeth coefficients tend to converge

⁶See Appendix A for a description of the sample and for precise variable definitions.

towards 0 over time, in part because cross-sectional dispersions of characteristics grow over time. Standardization may help to make the coefficients stationary, which is important for the time-series regressions we run of the coefficients on the factors. (Standardization has the side benefit of facilitating the interpretation of the Fama-MacBeth coefficients: each is the return to a one-standard-deviation increase in a variable.) There are no apparent time trends in the dispersions of estimated betas, but, for consistency, we standardize them by dividing each by its cross-sectional standard deviation each month.

Figure 4 plots the time series of coefficients for the four characteristics from Fama-MacBeth regressions on the four standardized characteristics and five standardized estimated betas. There are no apparent time trends in the coefficients, so the standardization appears to have been successful. Though we do not report them, there are also no apparent time trends in the regression coefficients for the standardized estimated betas. Table 5 reports the average cross-sectional correlations of the winsorized and standardized characteristics and estimated betas.

Panel A of Table 6 reports the mean Fama-MacBeth coefficients for the four characteristics and the four corresponding estimated betas. The coefficients are biased, so we cannot rely heavily on them. However, it still seems noteworthy that none of the beta coefficients is significant. This could be due merely to attenuation bias, but we can show that it is not by examining the alphas of the coefficients.

Panel B of Table 6 reports the alphas of the Fama-MacBeth coefficients for the four characteristics and the four corresponding estimated betas. These are from standard time-series regressions of the coefficients on the factors, treating the coefficients as excess returns of test assets as described in Section 2. The operating profit and asset growth characteristic coefficients have significant positive alphas. Each loads significantly on the corresponding factor (OP on RMW and NAG on CMA);⁷ however, their mean returns are higher than their factor exposures predict. Conversely, the HML beta and CMA beta coefficients have

⁷We explain in the introduction that the magnitudes of the alphas are somewhat difficult to interpret because of a leverage issue: The test assets are long-short portfolios, but they are not 100% long and 100% short. Instead, they are in general closer to being 50% long and 50% short. This same leverage issue affects the factor loadings in the regressions of the test assets on factors, so we should not a priori expect the loading of a characteristic or beta test asset on the corresponding factor to be approximately 1. See Section 8 for further discussion.

significant negative alphas. Again, each loads significantly on the corresponding factor, but the mean returns are lower in this case than is predicted by their factor exposures. Thus, factor exposures do not fully explain returns. We discuss the economic significance of these rejections in Section 8.

The R^2 's of the regressions reported in Table 6 are reasonably high (between 29% and 71%). Thus, factor exposures explain substantial parts of the returns. The characteristic portfolios have factor exposures due to the EIV bias. As explained in Section 2, their estimated factor exposures, based on the estimated stock betas, are zero. However, as shown by our Monte Carlo analysis, their actual factor exposures are nonzero due to the EIV bias. The Monte Carlo analysis also shows that the factor regressions provide the proper adjustment for the factor exposures, rendering the alpha tests unbiased. Thus, the significant alphas for the operating profit and asset growth characteristic portfolios in Table 6 are the result of the factor model failing, not bias in the tests. It is noteworthy that the portfolios for the estimated beta coefficients also have significant factor exposures. This shows that, despite measurement error and potential time variation in betas, the estimated stock betas contain significant information about future realized betas.

5. The Four Factor Hou-Xue-Zhang Model

Hou, Xue and Zhang (2015)—hereafter, HXZ—propose a four-factor model in which the factors are the market excess return, a size factor (r_{ME}), a profitability factor (r_{ROE}), and an investment factor ($r_{I/A}$). We follow the same procedure to test this model that we employed for the Fama-French model. First, we estimate the time series of factor betas for individual stocks using returns over the prior 52 weeks. We use the same characteristics that HXZ use to define the factors. We winsorize and standardize the characteristics and estimated betas, and then we run Fama-MacBeth regressions on the four standardized estimated betas and three standardized characteristics. The last step is to compute the factor model alphas of the Fama-MacBeth coefficients. The characteristics are

- NME = *negative* log market equity,
- ROE = return on equity,

- NAG = *negative* log of one plus asset growth.

There is no value factor in this model, so we do not include the book-to-market characteristic. Return on equity is constructed as in HXZ using quarterly Compustat data (income before extraordinary items divided by one quarter lagged book-equity). This differs from the operating profitability variable in the Fama-French model both in terms of the definition and in updating frequency.

Panel A of Table 7 reports the mean coefficients from the Fama-MacBeth regressions. Again, none of the beta coefficients is significant (and all of the characteristic coefficients are positive and significant). As before, this is suggestive but not definitive evidence that there are returns associated with characteristics rather than with factor exposures. To obtain definitive evidence, we regress the coefficients on the factors. Panel B of Table 7 shows that, as with the FF model, each characteristic portfolio and each beta portfolio has significant exposure to the corresponding factor. Furthermore, it shows that the bets on the ROE and NAG characteristics have significant positive alphas and the bets on the corresponding covariances have significant negative alphas. Thus, as with the FF model, the returns to characteristics are not explained by the factor model, and factor exposures do not necessarily imply correspondingly high returns. Again, we discuss the economic significance of these rejections in Section 8.

6. Momentum and Value

In this section, we test the ability of the FF model to explain momentum and the ability of the HXZ model to explain momentum and value. Equally important, we test the abilities of the models to explain bets on characteristics (size, value, profitability, and investment) that are momentum-neutral. We re-run the Fama-MacBeth regressions including a momentum characteristic (the compound return for months $t - 12$ through month $t - 2$) on the right-hand side. We also include the estimated UMD beta on the right-hand side. In the case of the HXZ model, we also include log book-to-market and the estimated HML beta on the right-hand side. As with the others, these characteristics and estimated betas are winsorized and standardized to have unit cross-sectional standard deviations each month before the Fama-MacBeth regressions are run.

The importance of controlling for momentum can be seen by comparing Panel A of Table 6 to Panel A of Table 8. The mean size and book-to-market coefficients double in magnitude and become highly significant when momentum is included in the regression. This is quite natural, because bets on size and value that do not control for momentum are likely to be bets on negative momentum. This illustrates the importance of creating test assets that control for other characteristics. As remarked before, this is much easier done with the Fama-MacBeth method than with sorts. This is our primary motivation for proposing pure plays from Fama-MacBeth regressions as test assets for factor models.

The test of the FF model on the new set of test assets is reported in Panel B of Table 8. Overall, the performance of the model is worse. First, as pointed out by Fama and French (2014), the model cannot explain momentum. The momentum portfolio has a large and significant alpha. The five-factor regression for the momentum portfolio has a relatively low R^2 , and only the loading on the market is significant. Second, the momentum-neutral size and value portfolios have significant positive alphas. This is consistent with the increased magnitudes of the mean size and book-to-market coefficients shown in Panel A. Including momentum in the Fama-MacBeth regression forces both the size portfolio and the value portfolio to have zero values for the momentum characteristic—that is, to be equally long and short high momentum stocks. The FF model cannot explain these momentum-neutral size and value bets. In fact, Table 8 shows that all of the momentum-neutral characteristic portfolios have significant positive alphas. Also, the HML beta and CMA beta portfolios continue to have significant negative alphas, as in Table 6.

It is common to use the Carhart (1997) UMD factor in conjunction with the Fama-French (1993) factors, so we include UMD with the FF factors to see if the six factor model can explain these test assets. Fama and French (2014) perform the same exercise (with different test assets). Table 9 reports the results. Adding the momentum factor reduces the alpha of the momentum portfolio; however, the alpha is still large and significant.⁸

Furthermore, the alphas of all other characteristic test assets remain significant. Thus, the UMD factor adds little to the explanatory power of the five-factor FF model for this

⁸Likewise, Fama and French (2014) find that including the UMD factor “improves model performance, but leaves nontrivial unexplained momentum returns among small stocks.”

set of test assets. Section 8 shows that the information ratios of all of the characteristic test assets, including momentum, increase when UMD is included as a factor. This is due to residual standard errors falling more than alphas.

The test of the HXZ model for the new set of test assets is reported in Table 10. HXZ argue that value and momentum are redundant factors, because their model is able to price book-to-market and prior-return sorted portfolios. Our results are in agreement with theirs regarding momentum. The momentum portfolio has an insignificant alpha, primarily because of the portfolio's large loading on the profitability (ROE) factor. Novy-Marx (2014) also observes that it is the ROE factor that explains momentum in the HXZ model. He relates it to post-earnings-announcement drift.

In contrast with its success with momentum, the HXZ model is not able to explain the value effect. The momentum-neutral value portfolio has a large and significant alpha. Furthermore, both the HML and the UMD beta portfolios have significant negative alphas. The UMD beta portfolio, like the momentum portfolio, loads on the profitability factor. However, its mean return is not commensurate with the loading. The HML beta portfolio loads heavily on the investment factor and fails to have a commensurate mean return. In fact, with the exception of the momentum characteristic and the size factor beta, all of the test assets that are bets on characteristics have significant positive alphas and all of the test assets that are bets on betas have significant negative alphas.

In parallel with including UMD with the FF factors, we include HML and UMD as additional factors in the HXZ model. Table 11 reports the test of the six factor model using the same test assets as in Table 10. Including the two additional factors makes matters worse: The momentum portfolio now also has a significant positive alpha. Although the magnitude of the alpha remains the same, its standard error declines. Furthermore, Section 8 shows that, as with the FF model, including the additional factors does not reduce any of the information ratios of the characteristic test assets.

7. Annual Updating, Micro-Caps, and Industry-Neutral Test Assets

Panel A of Table 12 presents various robustness tests for the FF model. We report results for Fama-MacBeth regressions that do not include momentum. These are variations

on the results reported in Table 6 and Section 4. First, we examine the effect of updating characteristics and betas less frequently when constructing test assets. We re-run the monthly Fama-MacBeth regressions on characteristics and estimated betas that are updated only once per year, at the end of June. This annual updating is consistent with the FF factor construction methodology. Using annual updating rather than monthly updating does not affect the results substantially. The primary change is that the RMW beta test asset becomes marginally significant (and negative).

The second robustness check we do is to include micro-cap stocks. We drop the filter that stocks must have market capitalizations exceeding the NYSE 20th percentile. This strengthens the results substantially. The size and value test assets have significant positive alphas when micro-caps are included. The operating profit and investment test assets continue to have significant positive alphas and their magnitudes increase. The HML and CMA beta test assets continue to have significant negative alphas and their absolute magnitudes increase.

The final robustness check we do is to control for possible industry effects. We re-run the Fama-MacBeth regressions including dummy variables for 11 of the 12 Fama-French industries on the right-hand side. The portfolios for which the Fama-MacBeth coefficients are returns are orthogonal to all other variables. Being orthogonal to an industry dummy means that the portfolio is equally long and short in the industry—it has no net industry exposure. Because the portfolios are zero-investment portfolios, being equally long and short in 11 of the 12 industries obviously implies that they are equally long and short in the 12th industry as well. Thus, this procedure produces industry-neutral test assets. Requiring the test assets to be industry neutral has only a minor effect on the results. The primary change is that the SMB beta test asset becomes marginally significant (and negative).

Panel B of Table 12 presents the same robustness tests for the HXZ model. Annual updating is inconsistent with the construction of the HXZ profitability factor, which is based on quarterly data. However, for consistency, we update all characteristics and betas, including the HXZ profitability characteristic, annually.⁹ The results are very similar to

⁹Our construction of the factors is the same as before. In particular, we construct the HXZ profitability factor using quarterly data.

the results for monthly updating. The same characteristic test assets and beta assets are significant and have the same signs, though the alphas of the profitability and investment test assets do become smaller.

As with the FF model, including micro-cap stocks in the test assets for the HXZ model strengthens the results. In particular, the size test asset has a significant positive alpha when micro-caps are included. Requiring the test assets to be industry neutral also has the same effect for the HXZ model as for the FF model—the primary change is that the size beta test asset becomes marginally significant (and negative).

8. Information Ratios and Economic Significance

As mentioned in the introduction, it is difficult to directly compare the alphas of our test assets to risk premia of other assets because of differing leverage. Our test assets have zero net weights but are not differences of fully invested portfolios. In fact, they are generally de-levered, meaning that the sum of absolute weights is something less than 2. To get an idea of the extent of the de-leverage, we can compare the standard deviations of the test asset returns to the standard deviations of known high-minus-low portfolio returns. As an illustration, the characteristic test assets for the FF model in Table 6 have standard deviations that compare to the factor standard deviations as follows:

- Size test asset: 1.19% versus 3.13% for SMB
- Value test asset: 1.45% versus 2.92% for HML
- Profitability test asset: 0.79% versus 2.17% for RMW
- Investment test asset: 0.65% versus 2.03% for CMA

While the standard deviations of the test assets provide some indication of the lower leverage, the residual standard errors are more directly relevant. When we subtract the fitted part of a factor model regression from the excess return on the left-hand side, we are left with an excess return of the form $\alpha + \varepsilon$, where ε is the regression residual. This is the fitted excess return of the portfolio that is long the test asset and short the mimicking factor portfolio. The Sharpe ratio of this portfolio is $\alpha / \text{stdev}(\varepsilon)$, which is the information ratio of

the test asset. Table 13 reports the information ratios for the various tests of the FF model (Panel A) and the HXZ model (Panel B). The “All” columns report the maximum Sharpe ratios that can be achieved from portfolios of the test assets after hedging the factor risks (that is, Sharpe ratios of portfolios of the $\alpha + \varepsilon$ assets).

Table 13 shows that, when the test assets are not momentum-neutral, it is the investment test asset that the FF model is least capable of explaining. The information ratio of that test asset is much larger than the information ratios of the other characteristic test assets and contributes almost all of the information ratio attainable by combining the characteristic test assets. When momentum is included as a characteristic and all of the other characteristic test assets are constructed to be momentum-neutral, the momentum characteristic test asset has the highest information ratio (by a slight margin) and the information ratio of the value characteristic test asset is much larger than when it is contaminated by momentum. The information ratios of 30%+ for the value, investment, and momentum test assets annualize to more than 100%, and the 60%+ information ratios for the portfolios of characteristic test assets annualize to more than 200%. The economic significance of these findings is clear.

Panel B of Table 13 shows that it is also the investment test asset that causes the HXZ model the most difficulty, when the test assets are not constrained to be neutral with respect to momentum and value. Furthermore, the test asset that is a bet on the investment beta is also large in magnitude. Shorting that test asset produces a large positive information ratio. When momentum and value test assets are included and the others are constrained to be neutral with respect to momentum and value, the information ratios of the investment, value, and profitability test assets are all large and clearly economically significant.

The effect of including UMD as a factor in the FF model can be seen by comparing the last two rows of Panel A of Table 13. All of the information ratios of the characteristic test assets increase when UMD is included as an explanatory factor. Likewise, the effect of including UMD and HML as explanatory factors in the HXZ model can be seen by comparing the last two rows of Panel B of Table 13. None of the information ratios of the characteristic test assets fall when the additional factors are included.

9. Conclusion

We use test assets from Fama-MacBeth regressions to test the factor models recently proposed by Fama and French (2015) and Hou, Xue and Zhang (2015). The test assets are bets on individual characteristics or estimated betas that are neutral with respect to all others. We show that neutralizing can be important. Bets on size and value that are neutral with respect to momentum have larger means and alphas than similar bets that are contaminated by momentum. The advantage of creating test assets from Fama-MacBeth regressions is that neutralization is much easier with Fama-MacBeth regressions than it is with sorts. It is generally considered that the errors in variables bias is a disadvantage of Fama-MacBeth regressions, but we provide Monte Carlo evidence that the alphas of the Fama-MacBeth test assets are unbiased even when there are errors in beta estimates and even when stock betas and characteristics are time varying.

Our tests reject both factor models. Test assets that are bets on characteristics generally earn positive alphas and those that are bets on covariances generally earn negative alphas. In particular, across all specifications and for both models, the test assets that are bets on investment and profitability characteristics earn significant positive alphas, while the test asset that is a bet on the investment beta has a significant negative alpha. Not a single characteristic portfolio alpha is negative and significant, and not a single covariance portfolio alpha is positive and significant in any of our specifications.

Although it is not surprising to reject factor models, the patterns and magnitudes of our results indicate areas of concern in the new models. The mispricing of the investment characteristic portfolio by both models is particularly economically large, with information ratios for this portfolio of 25% to 56% per month. The factor models also do especially poorly when characteristics and covariance portfolios are constructed to be momentum-neutral. Augmenting the Fama-French model with a momentum factor and the Hou-Xue-Zhang model with value and momentum factors does not help. Although results in Hou, Xue and Zhang (2015) and Fama and French (2015) indicate that the value factor is redundant in the sense that it has a zero alpha with respect to other factors, we show that neither model can price the momentum-neutral test asset that is a bet on value.

Our results also imply that the new factor models are imperfect benchmarks for assessing

the performance of managers or trading strategies. We show that it is possible to use only information on characteristics on which the factors are based and estimated factor betas to design trading strategies that have alphas with respect to the factors.

Appendix A. Data and Variable Definitions

Our dataset consists of all firms in CRSP/Compustat with common stock (share codes 10 or 11) outstanding. In our primary specifications for any month t , we exclude firms with market equity smaller than the NYSE 20th percentile as of month $t - 12$. All accounting variables are based on the annual Compustat file except for Return On Equity (ROE), which is based on quarterly Compustat data. In an effort to ensure that the information is publicly available, all annual Compustat data is assumed to be known 6 months after the end of the fiscal year. Following Hou, Xue and Zhang (2015), we assume that ROE is known in the month after the quarterly release date (RDQ).

Characteristics

We follow Fama and French (2014) in defining the variables underlying size, book-to-market, operating profitability, asset growth, and momentum. Size and asset growth in the HXZ model are the same as in the FF model. ROE in the HXZ model is defined differently from the operating profitability variable in the FF model; we construct ROE following the definition in Hou, Xue and Zhang (2015).

For any month t , the characteristics are defined as:

- Size (NME) is minus the log of market value at the end of month $t - 1$, where market value is absolute value of price per share times shares outstanding as reported in CRSP. When there are multiple PERMNOs in CRSP for the same GVKEY in Compustat, we retain only the security with the highest market equity and assign the combined market value across the share classes to this security.
- Book-to-market (BM) is the log book value of equity for the most recent fiscal year ending at least 6 months prior to t minus the log market value of equity at the end of month $t - 1$. Book value of equity is the book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. The book value of preferred stock is estimated as redemption, liquidation, or par value, in that order, depending on availability. Stockholders' equity is the value reported by Compustat, if it is available; otherwise, it is measured as the

book value of common equity plus the par value of preferred stock, or the book value of assets minus total liabilities, in that order.

- Operating Profitability is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses, all divided by book equity (as defined above) for the most recent fiscal year ending at least 6 months prior to t .
- Asset Growth is minus the annual changes in log total assets over the most recent fiscal year ending at least 6 months prior to t .
- Momentum is the CRSP monthly total return for the eleven months $t - 12$ through $t - 2$. The momentum characteristic is calculated only for stocks that are in CRSP for each of the 12 prior months and for which the return in month $t - 2$ is not missing; furthermore, any missing returns in months $t - 12$ through $t - 3$ must be coded -99 by CRSP ('missing return due to missing price') for the stock to be included.
- Return on equity (ROE) is income before extraordinary items divided by one-quarter-lagged book equity.

Factors and Betas

We obtain monthly FF factors from the French Data Library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. We use MKTRF, SMB, HML, RMW, and CMA from the Fama/French five Factors (2X3) file and UMD from the momentum factor file. We construct HXZ factors exactly as described in Hou, Xue and Zhang (2015). The sample period for the FF factors is July 1963 to December 2012. The sample period for the HXZ factors is Jan 1972 to December 2012. The later start is necessitated by the use of quarterly Compustat data to calculate ROE.

To estimate betas for individual stocks, we run weekly factor regressions for the prior 12 months. We only include data where the error degrees of freedom in this regression is at least 30 (for example, we require at least 36 points in the five-factor regression). We cumulate daily factor returns for both the FF and the HXZ models to obtain weekly factor returns. We use daily returns for MKTRF, SMB, HML, and UMD from the French Data Library and construct daily returns for CMA, RMW and the HXZ factors following the descriptions in the original papers.

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Table 2: **Calibration parameters.** This table reports the parameters used in simulations of three-factor models. Panel A presents parameters for factors calibrated to the FF three-factor model. Panel B presents parameters for individual stocks. Size is minus log market capitalization, and BM is the log book-to-market ratio. Betas are estimated from regressions on the FF factors over the prior 52 weeks. The calibrated distribution is based on the average cross-sectional means and the average cross-sectional covariance matrix with the exception that the average cross-sectional variance of each estimated beta is scaled by 0.7 to calibrate the variance of the corresponding true beta. Calibration parameters are estimated over the 1970–2013 period.

Panel A: Weekly factor realizations					
	Mean	Std Dev	Correlations		
			MKTRF	SMB	HML
MKTRF	0.11	2.29	1.00	0.04	-0.30
SMB	0.02	1.21	0.04	1.00	-0.12
HML	0.09	1.24	-0.30	-0.12	1.00

Panel B: True betas, characteristics, and log idiosyncratic standard deviations (IVOL)

	Mean	Std Dev	Correlations					IVOL
			Size	BM	β_{MKT}	β_{SMB}	β_{HML}	
Size	-12.97	1.38	1.00	0.22	0.003	0.47	0.04	0.48
BM	-0.62	0.78	0.22	1.00	-0.054	0.02	0.32	-0.12
β_{MKT}	1.07	0.55	0.00	-0.05	1.000	0.31	0.34	0.40
β_{SMB}	0.64	0.82	0.47	0.02	0.310	1.00	0.17	0.47
β_{HML}	0.15	0.96	0.04	0.32	0.345	0.17	1.00	-0.13
IVOL	-3.17	0.43	0.48	-0.12	0.404	0.47	-0.13	1.00

Table 3: **Simulations with constant betas, characteristics, and idiosyncratic volatilities.** We run 500 simulations of a three-factor model for 3,000 stocks and 1,600 weeks. Time-invariant betas, characteristics, and log idiosyncratic volatilities are drawn from a multivariate normal distribution with parameters in Table 2. Factors and idiosyncratic returns are also drawn from calibrated normal distributions. We first estimate betas from regressions of weekly stock returns on the three factors over the prior 48 weeks (96 for IV). We then estimate monthly cross-sectional regressions of returns on size, book-to-market, and estimated betas to the market, SMB and HML, using four methods described in the text (Fama-MacBeth, BCS, CGS, and IV). We report coefficients for variables calibrated to book-to-market and HML beta. Panels A and B present mean coefficients (in percents). Panels C and D present alphas (in percents) from regressions of the cross-sectional regression coefficients on the three factors. The first two columns report the percentage of the simulations in which the mean coefficient is significant at the 5% level (for a two-tailed test). The mean coefficients are the means across months and across simulations. The fourth and fifth columns report the estimated HML beta and the estimated risk premium. The estimated HML beta is the average (across months and across simulations) of the estimated HML beta of the portfolio for which the coefficient is the return, based on the estimated HML betas of the stocks. The estimated risk premium is the average (across months and across simulations) of the estimated risk premium given the estimated market, SMB, and HML betas and the factor risk premia. The last two columns in Panels A and B report the same statistics but using the true betas of the stocks.

	% Pos	% Neg	Mean	$\widehat{\beta}_{\text{HML}}$	$\hat{\mu}$	β_{HML}	μ
Panel A: Mean log book-to-market coefficients							
Fama-MacBeth	45.2	0.0	0.045	0.000	0.000	0.142	0.045
BCS	1.4	2.6	0.001	0.403	0.120	0.403	0.120
CGS	12.8	0.4	0.012	-0.173	-0.055	0.033	0.011
IV	3.4	2.2	0.001	-0.108	-0.034	0.000	0.000
Panel B: Mean HML beta coefficients							
Fama-MacBeth	78.2	0	0.237	1.000	0.363	0.654	0.238
CGS	78.0	0	0.332	1.417	0.513	0.920	0.334
IV	77.8	0	0.361	1.259	0.457	1.000	0.364
Panel C: Alphas of log book-to-market coefficients							
Fama-MacBeth	2.2	1.6	0.001				
BCS	1.2	2.2	0.001				
CGS	3.0	2.0	0.001				
IV	2.4	1.8	0.001				
Panel D: Alphas of HML beta coefficients							
Fama-MacBeth	2.6	2.0	-0.001				
CGS	2.4	1.8	-0.001				
IV	1.8	1.8	-0.002				

Table 4: **Simulations with time-varying betas, characteristics, and idiosyncratic volatilities.** We run 500 simulations of a three-factor model for 3,000 stocks and 1,600 weeks. Time-varying betas, characteristics, and log idiosyncratic volatilities are simulated from an autoregressive process such that its stationary distribution is multivariate normal with parameters in Table 2. Factors and idiosyncratic returns are also drawn from calibrated normal distributions. We first estimate betas from regressions of weekly stock returns on the three factors over the prior 48 weeks (96 for IV). We then estimate monthly cross-sectional regressions of returns on size, book-to-market, and estimated betas to the market, SMB and HML, using four methods described in the text (Fama-MacBeth, BCS, CGS, and IV). We report coefficients for variables calibrated to book-to-market and HML beta. Panels A and B present mean coefficients (in percents). Panels C and D present alphas (in percents) from regressions of the cross-sectional regression coefficients on the three factors. The first two columns report the percentage of the simulations in which the mean coefficient is significant at the 5% level (for a two-tailed test). The mean coefficients are the means across months and across simulations. The fourth and fifth columns report the estimated HML beta and the estimated risk premium. The estimated HML beta is the average (across months and across simulations) of the estimated HML beta of the portfolio for which the coefficient is the return, based on the estimated HML betas of the stocks. The estimated risk premium is the average (across months and across simulations) of the estimated risk premium given the estimated market, SMB, and HML betas and the factor risk premia. The last two columns in Panels A and B report the same statistics but using the true betas of the stocks.

	% Pos	% Neg	Mean	$\widehat{\beta}_{\text{HML}}$	$\hat{\mu}$	β_{HML}	μ
Panel A: Mean log book-to-market coefficients							
Fama-MacBeth	54.8	0	0.096	0.000	0.000	0.333	0.100
BCS	48.6	0	0.060	0.191	0.057	0.404	0.120
CGS	51.8	0	0.081	-0.121	-0.038	0.289	0.086
IV	53.4	0	0.096	0.010	0.002	0.338	0.100
Panel B: Mean HML beta coefficients							
Fama-MacBeth	82.4	0	0.149	1.000	0.363	0.374	0.138
CGS	81.8	0	0.238	1.619	0.588	0.599	0.221
IV	78.0	0	0.224	1.515	0.553	0.559	0.208
Panel C: Alphas of log book-to-market coefficients							
Fama-MacBeth	2.6	2.4	0				
BCS	2.6	3.0	0				
CGS	2.2	3.0	0				
IV	3.0	2.4	0				
Panel D: Alphas of HML beta coefficients							
Fama-MacBeth	2.6	1.8	0.001				
CGS	1.6	1.6	0.002				
IV	2.4	1.4	0.001				

Table 5: **Time-series averages of cross-sectional correlations.** The characteristics are negative log market equity (NME), log book-to-market (BM), operating profitability (OP), and negative asset growth (NAG). The estimated betas are from regressions of weekly individual stock returns on FF factors (MKTRF, SMB, HML, RMW, CMA) over the prior 52 weeks. Each month, all characteristics and estimated betas are winsorized and standardized to have unit cross-sectional standard deviations. Stocks with market equity smaller than the 20th NYSE percentile are excluded. The sample period is July 1964 through December 2012.

	NME	BM	OP	NAG	β_{MKT}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}
NME	1.00	0.22	-0.19	-0.11	-0.01	0.39	0.01	-0.02	0.00
BM	0.22	1.00	-0.34	0.26	-0.05	0.01	0.22	0.00	0.01
OP	-0.19	-0.34	1.00	-0.06	0.01	-0.04	0.01	0.15	-0.01
NAG	-0.11	0.26	-0.06	1.00	-0.07	-0.07	0.10	0.04	0.08
β_{MKT}	-0.01	-0.05	0.01	-0.07	1.00	0.16	0.14	0.11	0.08
β_{SMB}	0.39	0.01	-0.04	-0.07	0.16	1.00	0.09	0.23	0.01
β_{HML}	0.01	0.22	0.01	0.10	0.14	0.09	1.00	0.27	-0.41
β_{RMW}	-0.02	0.00	0.15	0.04	0.11	0.23	0.27	1.00	0.15
β_{CMA}	0.00	0.01	-0.01	0.08	0.08	0.01	-0.41	0.15	1.00

Table 6: **Tests of the five-factor FF model.** Monthly Fama-MacBeth regressions are run on four characteristics and five estimated betas. The characteristics are negative log market equity (NME), log book-to-market (BM), operating profitability (OP), and negative asset growth (NAG). The estimated betas are from regressions of weekly individual stock returns on FF factors (MKTRF, SMB, HML, RMW, CMA), over the prior 52 weeks. Each month, all characteristics and estimated betas are winsorized and standardized to have unit cross-sectional standard deviations. Stocks with market equity smaller than the 20th NYSE percentile are excluded. Panel A reports the time-series average of the Fama-MacBeth regression coefficients for the characteristics and corresponding betas. Panel B reports results of time-series regressions of the coefficients on the five FF factors. Newey-West standard errors are in parentheses. The sample period is July 1964 through December 2012.

	NME	BM	OP	NAG	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}
Panel A: Mean returns								
Mean return	0.07 (0.05)	0.12* (0.07)	0.18*** (0.04)	0.28*** (0.03)	-0.04 (0.05)	0.07 (0.08)	0.10 (0.07)	0.03 (0.08)
Panel B: FF five-factor regressions								
Alpha	-0.01 (0.03)	0.01 (0.07)	0.07** (0.03)	0.21*** (0.03)	-0.03 (0.04)	-0.17* (0.09)	-0.10 (0.08)	-0.18** (0.08)
MKTRF	-0.02** (0.01)	0.05*** (0.02)	0.03*** (0.01)	-0.01 (0.01)	0.03** (0.01)	-0.04** (0.02)	-0.03 (0.02)	-0.07*** (0.02)
SMB	0.33*** (0.01)	-0.01 (0.03)	0.01 (0.01)	0.00 (0.01)	0.14*** (0.02)	-0.06** (0.03)	0.07** (0.03)	-0.01 (0.03)
HML	-0.02 (0.02)	0.35*** (0.05)	0.10*** (0.02)	-0.03 (0.02)	-0.03 (0.03)	0.30*** (0.04)	0.16*** (0.05)	0.01 (0.04)
RMW	0.00 (0.02)	-0.03 (0.06)	0.20*** (0.02)	0.05* (0.03)	-0.10*** (0.03)	0.22*** (0.07)	0.52*** (0.06)	0.22*** (0.05)
CMA	0.01 (0.03)	-0.11 (0.07)	0.01 (0.03)	0.19*** (0.02)	-0.06 (0.04)	0.31*** (0.09)	-0.01 (0.08)	0.54*** (0.08)
Adj. R ²	0.71	0.34	0.39	0.29	0.36	0.53	0.48	0.44
Num. obs.	582	582	582	582	582	582	582	582

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 7: **Tests of the four-factor HXZ model.** Monthly Fama-MacBeth regressions are run on three characteristics and four estimated betas. The characteristics are negative log market equity (NME), return on equity (ROE), and negative asset growth (NAG). The estimated betas are from regressions of weekly individual stock returns on HXZ factors (MKTRF, SIZE, ROE, AG) over the prior 52 weeks. Each month, all characteristics and estimated betas are winsorized and standardized to have unit cross-sectional standard deviations. Stocks with market equity smaller than the 20th NYSE percentile are excluded. Panel A reports the time-series average of the Fama-MacBeth regression coefficients for the characteristics and corresponding betas. Panel B reports results of time-series regressions of the coefficients on the four HXZ factors. Newey-West standard errors are in parentheses. The sample period is January 1973 through December 2012.

	NME	ROE	NAG	β_{SIZE}	β_{ROE}	β_{AG}
Panel A: Mean returns						
Mean return	0.16** (0.07)	0.26*** (0.04)	0.31*** (0.04)	-0.06 (0.05)	0.04 (0.08)	0.02 (0.09)
Panel B: HXZ four-factor regressions						
Alpha	0.15 (0.10)	0.13*** (0.03)	0.21*** (0.03)	-0.05 (0.05)	-0.23*** (0.07)	-0.36*** (0.07)
MKTRF	0.00 (0.02)	0.00 (0.01)	0.00 (0.01)	0.01 (0.02)	-0.06*** (0.02)	-0.04** (0.02)
SIZE	0.27*** (0.04)	0.00 (0.01)	-0.01 (0.02)	0.17*** (0.02)	0.05* (0.03)	0.00 (0.04)
ROE	-0.14*** (0.05)	0.22*** (0.02)	-0.02 (0.01)	-0.04 (0.03)	0.52*** (0.05)	0.21*** (0.04)
AG	0.01 (0.06)	0.00 (0.02)	0.23*** (0.02)	-0.10** (0.04)	-0.10 (0.07)	0.56*** (0.05)
Adj. R ²	0.49	0.49	0.38	0.29	0.56	0.56
Num. obs.	480	480	480	480	480	480

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 8: **Tests of the five-factor FF model with momentum and with momentum-neutral test assets.** Monthly Fama-MacBeth regressions are run on five characteristics and six estimated betas. The characteristics are negative log market equity (NME), log book-to-market (BM), operating profitability (OP), negative asset growth (NAG), and momentum (MOM). The estimated betas are from regressions of weekly individual stock returns on FF factors (MKTRF, SMB, HML, RMW, CMA) and the momentum (UMD) factor, over the prior 52 weeks. Each month, all characteristics and estimated betas are winsorized and standardized to have unit cross-sectional standard deviations. Stocks with market equity smaller than the 20th NYSE percentile are excluded. Panel A reports the time-series average of the Fama-MacBeth regression coefficients for the characteristics and corresponding betas. Panel B reports results of time-series regressions of the coefficients on the five FF factors. Newey-West standard errors are in parentheses. The sample period is July 1964 through December 2012.

	NME	BM	OP	NAG	MOM	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	β_{UMD}
Panel A: Mean returns										
Mean return	0.15*** (0.05)	0.28*** (0.06)	0.20*** (0.04)	0.23*** (0.03)	0.41*** (0.06)	-0.06 (0.05)	0.01 (0.08)	0.13** (0.06)	0.02 (0.07)	0.00 (0.07)
Panel B: FF five-factor regressions										
Alpha	0.07** (0.03)	0.19*** (0.05)	0.10*** (0.03)	0.16*** (0.03)	0.43*** (0.07)	-0.06 (0.04)	-0.22*** (0.08)	-0.07 (0.07)	-0.17** (0.07)	0.02 (0.07)
MKTRF	-0.03*** (0.01)	0.04*** (0.01)	0.02*** (0.01)	0.00 (0.01)	-0.05** (0.02)	0.03** (0.01)	-0.04** (0.02)	-0.02 (0.02)	-0.06*** (0.02)	-0.04* (0.03)
SMB	0.31*** (0.01)	-0.03* (0.02)	0.01 (0.01)	0.00 (0.01)	-0.04 (0.03)	0.13*** (0.02)	-0.06** (0.03)	0.06*** (0.02)	0.00 (0.02)	0.05 (0.04)
HML	-0.02 (0.01)	0.32*** (0.03)	0.09*** (0.02)	-0.02 (0.02)	-0.05 (0.04)	-0.04 (0.03)	0.28*** (0.03)	0.13*** (0.04)	0.02 (0.03)	-0.18** (0.07)
RMW	0.00 (0.02)	-0.01 (0.05)	0.18*** (0.02)	0.05* (0.03)	0.05 (0.05)	-0.10*** (0.03)	0.20*** (0.06)	0.49*** (0.05)	0.17*** (0.04)	0.11 (0.08)
CMA	0.01 (0.03)	-0.09* (0.05)	0.00 (0.02)	0.17*** (0.02)	0.03 (0.06)	-0.05 (0.04)	0.31*** (0.08)	0.02 (0.07)	0.48*** (0.07)	0.05 (0.12)
Adj. R ²	0.74	0.45	0.39	0.28	0.05	0.35	0.55	0.50	0.45	0.09
Num. obs.	582	582	582	582	582	582	582	582	582	582

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 9: **Tests of the six-factor FF model with momentum and with momentum-neutral test assets.** Monthly Fama-MacBeth regressions are run on five characteristics and six estimated betas. The characteristics are negative log market equity (NME), log book-to-market (BM), operating profitability (OP), negative asset growth (NAG), and momentum (MOM). The estimated betas are from regressions of weekly individual stock returns on FF factors (MKTRF, SMB, HML, RMW, CMA) and the momentum (UMD) factor, over the prior 52 weeks. Each month, all characteristics and estimated betas are winsorized and standardized to have unit cross-sectional standard deviations. Stocks with market equity smaller than the 20th NYSE percentile are excluded. The table reports results of time-series regressions of the Fama-MacBeth coefficients for the characteristics and corresponding betas on the five FF factors along with UMD. Newey-West standard errors are in parentheses. The sample period is July 1964 through December 2012.

	NME	BM	OP	NAG	MOM	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	β_{UMD}
Alpha	0.09*** (0.03)	0.25*** (0.05)	0.12*** (0.03)	0.16*** (0.02)	0.25*** (0.03)	-0.07* (0.04)	-0.29*** (0.09)	-0.10 (0.08)	-0.26*** (0.08)	-0.12 (0.07)
MKTRF	-0.03*** (0.01)	0.03** (0.01)	0.02*** (0.01)	0.00 (0.01)	-0.01 (0.01)	0.03** (0.01)	-0.03 (0.02)	-0.02 (0.02)	-0.04** (0.02)	-0.02 (0.02)
SMB	0.31*** (0.01)	-0.03* (0.02)	0.01 (0.01)	0.00 (0.01)	-0.05*** (0.01)	0.13*** (0.02)	-0.06*** (0.02)	0.06*** (0.02)	-0.01 (0.02)	0.03 (0.03)
HML	-0.03*** (0.01)	0.27*** (0.02)	0.08*** (0.01)	-0.02 (0.01)	0.07*** (0.02)	-0.03 (0.02)	0.33*** (0.04)	0.16*** (0.04)	0.08** (0.03)	-0.08 (0.06)
RMW	0.00 (0.01)	0.01 (0.04)	0.19*** (0.02)	0.05* (0.03)	-0.02 (0.02)	-0.11*** (0.03)	0.17*** (0.05)	0.48*** (0.04)	0.14*** (0.03)	0.06 (0.06)
CMA	0.02 (0.02)	-0.05 (0.03)	0.01 (0.02)	0.17*** (0.02)	-0.07*** (0.03)	-0.06 (0.04)	0.27*** (0.06)	0.00 (0.05)	0.43*** (0.05)	-0.03 (0.10)
UMD	-0.02* (0.01)	-0.10*** (0.02)	-0.03*** (0.01)	0.00 (0.01)	0.25*** (0.01)	0.02* (0.01)	0.10** (0.04)	0.05* (0.03)	0.12*** (0.03)	0.20*** (0.03)
Adj. R ²	0.74	0.57	0.41	0.28	0.72	0.36	0.59	0.52	0.52	0.34
Num. obs.	582	582	582	582	582	582	582	582	582	582

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 10: **Tests of the four-factor HXZ model with value and momentum and with value-neutral and momentum-neutral test assets.** Monthly Fama-MacBeth regressions are run on five characteristics and six estimated betas. The characteristics are negative log market equity (NME), return on equity (ROE), negative asset growth (NAG), log book-to-market (BM), and momentum (MOM). The estimated betas are from regressions of weekly individual stock returns on HXZ factors (MKTRF, SIZE, ROE, AG), and the value (HML), and momentum (UMD) factors, over the past 52 weeks. Each month, all characteristics and estimated betas are winsorized and standardized to have unit cross-sectional standard deviations. Stocks with market equity smaller than the 20th NYSE percentile are excluded. Panel A reports the time-series average of the Fama-MacBeth regression coefficients for the characteristics and corresponding betas. Panel B reports results of time-series regressions of the coefficients on the four HXZ factors. Newey-West standard errors are in parentheses. The sample period is January 1973 through December 2012.

	NME	ROE	NAG	BM	MOM	β_{SIZE}	β_{ROE}	β_{AG}	β_{HML}	β_{UMD}
Mean returns										
Mean return	0.16*** (0.05)	0.33*** (0.04)	0.23*** (0.04)	0.36*** (0.06)	0.32*** (0.07)	-0.07 (0.05)	0.05 (0.08)	0.02 (0.08)	0.00 (0.09)	-0.03 (0.09)
HXZ four-factor regressions										
Alpha	0.12** (0.05)	0.22*** (0.04)	0.14*** (0.03)	0.35*** (0.08)	0.15 (0.09)	-0.05 (0.05)	-0.17** (0.08)	-0.34*** (0.09)	-0.33*** (0.12)	-0.23*** (0.08)
MKTRF	-0.01 (0.01)	-0.01 (0.01)	0.00 (0.01)	0.01 (0.02)	-0.02 (0.02)	0.02 (0.01)	-0.05** (0.02)	-0.05*** (0.02)	-0.05** (0.02)	-0.03 (0.03)
SIZE	0.30*** (0.01)	0.00 (0.02)	0.00 (0.02)	-0.08 (0.05)	0.05 (0.04)	0.13*** (0.02)	0.06* (0.03)	0.05* (0.02)	-0.04 (0.04)	0.13** (0.05)
ROE	-0.05* (0.02)	0.16*** (0.02)	0.00 (0.01)	-0.13*** (0.04)	0.25*** (0.05)	-0.05* (0.02)	0.44*** (0.07)	0.17*** (0.04)	0.06 (0.07)	0.37*** (0.07)
AG	-0.05* (0.03)	0.03 (0.02)	0.19*** (0.02)	0.21*** (0.05)	0.02 (0.05)	-0.08** (0.04)	-0.11 (0.08)	0.55*** (0.07)	0.69*** (0.10)	-0.11 (0.09)
Adj. R ²	0.71	0.29	0.31	0.17	0.23	0.25	0.44	0.50	0.47	0.25
Num. obs.	480	480	480	480	480	480	480	480	480	480

***, $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 11: **Tests of the six-factor HXZ model with value and momentum and with value-neutral and momentum-neutral test assets.** Monthly Fama-MacBeth regressions are run on five characteristics and six estimated betas. The characteristics are negative log market equity (NME), return on equity (ROE), negative asset growth (NAG), log book-to-market (BM), and momentum (MOM). The estimated betas are from regressions of weekly individual stock returns on HXZ factors (MKTRF, SIZE, ROE, AG), and the value (HML) and momentum (UMD) factors, over the past 52 weeks. Each month, all characteristics and estimated betas are winsorized and standardized to have unit cross-sectional standard deviations. Stocks with market equity smaller than the 20th NYSE percentile are excluded. The table reports results of time-series regressions of the Fama-MacBeth coefficients for the characteristics and corresponding betas on the four HXZ factors plus HML and UMD. The sample period is January 1973 through December 2012. Newey-West standard errors are in parentheses.

	NME	ROE	NAG	BM	MOM	β_{SIZE}	β_{ROE}	β_{AG}	β_{HML}	β_{UMD}
Alpha	0.12*** (0.04)	0.20*** (0.04)	0.14*** (0.03)	0.32*** (0.07)	0.15*** (0.04)	-0.05 (0.05)	-0.18** (0.08)	-0.36*** (0.08)	-0.37*** (0.10)	-0.22** (0.09)
MKTRF	-0.02* (0.01)	0.00 (0.01)	0.00 (0.01)	0.02* (0.01)	0.00 (0.01)	0.02 (0.01)	-0.04** (0.02)	-0.03** (0.02)	-0.02 (0.02)	-0.03 (0.02)
SIZE	0.31*** (0.01)	0.02 (0.01)	-0.01 (0.01)	-0.03* (0.02)	-0.03** (0.01)	0.11*** (0.02)	0.07** (0.03)	0.03 (0.02)	-0.04 (0.02)	0.07* (0.03)
ROE	-0.03** (0.02)	0.22*** (0.02)	0.00 (0.02)	0.02 (0.03)	0.02 (0.02)	-0.10*** (0.03)	0.48*** (0.05)	0.15*** (0.03)	0.11** (0.04)	0.18*** (0.07)
AG	0.01 (0.02)	-0.07** (0.03)	0.20*** (0.02)	-0.04 (0.04)	-0.07** (0.03)	-0.05 (0.04)	-0.27*** (0.10)	0.38*** (0.05)	0.28*** (0.07)	-0.03 (0.11)
HML	-0.05*** (0.01)	0.10*** (0.02)	-0.01 (0.01)	0.25*** (0.03)	0.06** (0.02)	-0.03 (0.02)	0.15** (0.06)	0.15*** (0.03)	0.39*** (0.04)	-0.09 (0.07)
UMD	-0.03* (0.01)	-0.03** (0.01)	0.00 (0.01)	-0.09*** (0.02)	0.25*** (0.01)	0.04*** (0.01)	0.00 (0.03)	0.06** (0.03)	0.05 (0.04)	0.17*** (0.03)
Adj. R ²	0.72	0.40	0.31	0.53	0.73	0.29	0.48	0.55	0.62	0.39
Num. obs.	480	480	480	480	480	480	480	480	480	480

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 12: **Robustness tests.** Four different specifications of Fama-MacBeth regressions are run on characteristics and estimated betas for each of the FF and HXZ factor models. The characteristics are negative log market equity (NME), log book-to-market (BM), operating profitability (OP), negative asset growth (NAG), momentum (MOM), and return on equity (ROE). The estimated betas are from weekly regressions of individual stock returns on FF factors (MKTRF, SMB, HML, RMW, CMA) in Panel A and on the HXZ factors (MKTRF, SIZE, ROE, AG) in Panel B. Both sets are estimated from weekly regressions over the prior 52 weeks. The table presents alphas from regressions of the Fama-MacBeth coefficients on the FF (Panel A) and HXZ (Panel B) factors. Base is the baseline specification from Table 6 for the FF model and Table 7 for the HXZ model. Annual Updating updates characteristics and estimated betas only once per year (at the end of June). All Stocks does not exclude stocks smaller than the NYSE 20th percentile. Industry Controls includes 11 dummy variables for the FF 12 industry classification. Newey-West standard errors are in parentheses. The sample period is July 1964 through December 2012 for the FF model and January 1973 through December 2012 for the HXZ model.

Panel A: FF model.								
	NME	BM	OP	NAG	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}
Base	-0.01 (0.03)	0.01 (0.07)	0.07** (0.03)	0.21*** (0.03)	-0.03 (0.04)	-0.17* (0.09)	-0.10 (0.08)	-0.18** (0.08)
Annual Updating	-0.04 (0.03)	0.03 (0.03)	0.05** (0.03)	0.15*** (0.03)	0.02 (0.04)	-0.22*** (0.07)	-0.10* (0.06)	-0.24*** (0.07)
All Stocks	0.26*** (0.06)	0.21*** (0.08)	0.11*** (0.04)	0.31*** (0.04)	-0.03 (0.06)	-0.31** (0.12)	-0.07 (0.09)	-0.24*** (0.08)
Industry Controls	0.01 (0.03)	0.07 (0.07)	0.11*** (0.03)	0.22*** (0.03)	-0.06* (0.04)	-0.13* (0.08)	-0.05 (0.06)	-0.14** (0.06)

Panel B: HXZ model.						
	NME	ROE	NAG	β_{SIZE}	β_{ROE}	β_{AG}
Base	0.06 (0.07)	0.24*** (0.04)	0.22*** (0.03)	-0.04 (0.05)	-0.04 (0.07)	-0.24*** (0.07)
Annual Updating	-0.03 (0.04)	0.10*** (0.03)	0.15*** (0.03)	0.00 (0.04)	0.00 (0.06)	-0.26*** (0.07)
All Stocks	0.30*** (0.08)	0.32*** (0.05)	0.28*** (0.04)	-0.08* (0.04)	-0.08 (0.08)	-0.22*** (0.06)
Industry Controls	0.09 (0.06)	0.26*** (0.04)	0.22*** (0.03)	-0.07* (0.04)	-0.04 (0.06)	-0.17*** (0.05)

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 13: **Monthly information ratios.** Fama-MacBeth regressions are run on different sets of characteristics and estimated betas for each of the FF and HXZ factor models. The characteristics are negative log market equity (NME), log book-to-market (BM), operating profitability (OP), negative asset growth (NAG), momentum (MOM), and return on equity (ROE). The estimated betas for the first four specifications are from weekly regressions of individual stock returns on FF factors (MKTRF, SMB, HML, RMW, CMA) in Panel A and on the HXZ factors (the MKTRF, SIZE, ROE, AG) in Panel B. The betas are estimated from weekly regressions over the prior 52 weeks. The last two specifications augment the factors with momentum (UMD) in Panel A and with momentum (UMD) and value (HML) in Panel B. The table presents information ratios from regressions of the Fama-MacBeth coefficients on the FF (Panel A) and HXZ (Panel B) factors. The information ratio is the alpha divided by the residual standard deviation from the factor regression. The "All" columns report the maximum Sharpe ratios that can be achieved from portfolios of the test assets after hedging the factor risks. Base is the baseline specification from Table 6 for the FF model and Table 7 for the HXZ model. Annual Updating updates characteristics and estimated betas only once per year (at the end of June). All Stocks does not exclude stocks smaller than the NYSE 20th percentile. Industry Controls includes 11 dummy variables for the FF 12 industry classification. Momentum and UMD Beta includes momentum and UMD beta as additional variables in the Fama-MacBeth regressions. Additional Chars and Betas includes momentum, book-to-market and betas to HML and UMD as additional variables in the Fama-MacBeth regressions. 6 Factor Model in Panel A computes information ratios of the Fama-MacBeth coefficients with respect to the five FF factors and UMD. 6 Factor Model in Panel B computes information ratios of the Fama-MacBeth coefficients with respect to the four HXZ factors, HML, and UMD. The sample period is July 1964 through December 2012 for the FF model and January 1973 through December 2012 for the HXZ model

	Characteristics											Covariances						
	Panel A: FF model																	
	NME	BM	OP	NAG	MOM	All Chars	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	β_{UMD}	All	β_{SIZE}	β_{ROE}	β_{AG}	β_{HML}	β_{UMD}	All
Base Model	-0.02	0.01	0.12	0.39		0.42	-0.04	-0.11	-0.08	-0.12		0.43						
Annual Updating	-0.06	0.05	0.09	0.28		0.31	0.02	-0.15	-0.09	-0.15		0.34						
All Stocks	0.20	0.17	0.13	0.56		0.69	-0.03	-0.17	-0.05	-0.16		0.73						
Industry Controls	0.01	0.06	0.18	0.42		0.49	-0.08	-0.09	-0.05	-0.11		0.49						
Momentum and UMD Beta	0.13	0.22	0.18	0.31	0.34	0.64	-0.07	-0.16	-0.06	-0.13	0.01	0.65						
6 Factor Model	0.16	0.33	0.21	0.32	0.37	0.63	-0.08	-0.22	-0.09	-0.21	-0.09	0.63						
	Panel B: HXZ pure plays																	
	NME	ROE	NAG	BM	MOM	All Chars	β_{SIZE}	β_{ROE}	β_{AG}	β_{HML}	β_{UMD}	All	β_{SIZE}	β_{ROE}	β_{AG}	β_{HML}	β_{UMD}	All
Base Model	0.15	0.23	0.38			0.50	-0.06	-0.19	-0.31			0.57						
Annual Updating	0.01	0.03	0.25			0.25	0.02	-0.12	-0.29			0.40						
All Stocks	0.25	0.32	0.48			0.73	-0.09	-0.22	-0.31			0.80						
Industry Controls	0.18	0.30	0.41			0.59	-0.09	-0.19	-0.27			0.64						
Additional Chars and Betas	0.19	0.32	0.26	0.34	0.13	0.62	-0.06	-0.13	-0.28	-0.21	-0.14	0.66						
6 Factor Model	0.21	0.33	0.26	0.40	0.22	0.65	-0.05	-0.15	-0.31	-0.29	-0.15	0.67						

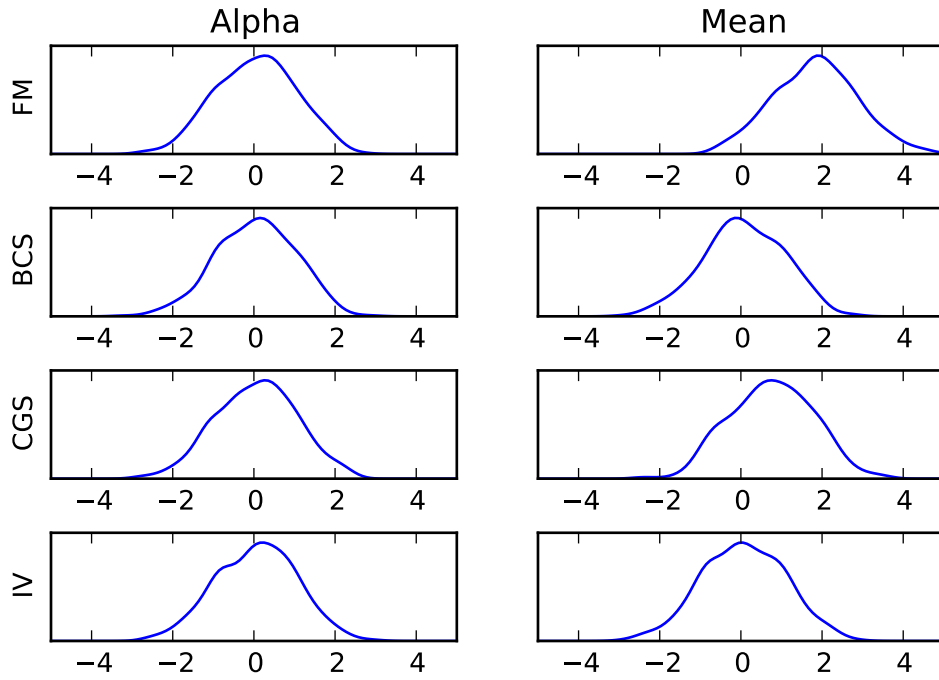


Figure 1: **Simulated distributions of t statistics for alphas and means of the book-to-market coefficients with constant betas, characteristics, and idiosyncratic volatilities.** We run 500 simulations of a three-factor model for 3,000 stocks and 1,600 weeks. Time-invariant betas, characteristics, and log idiosyncratic volatilities are drawn from multivariate normal distributions with parameters in Table 2. Factors and firm-specific returns are also drawn from calibrated normal distributions. We first run factor model regressions to compute estimated betas with data from the prior 48 weeks (96 for IV). We then estimate monthly cross-sectional regressions of returns on size, book-to-market, and estimated betas to the market, SMB and HML, using four methods described in the text (Fama-MacBeth, BCS, CGS, and IV). We report results for the variable calibrated to book-to-market. The right panel reports the distributions of t statistics of time-series means of cross-sectional coefficients. The left panel reports the distributions of t statistics of alphas from regressions of cross-sectional coefficients on the three factors.

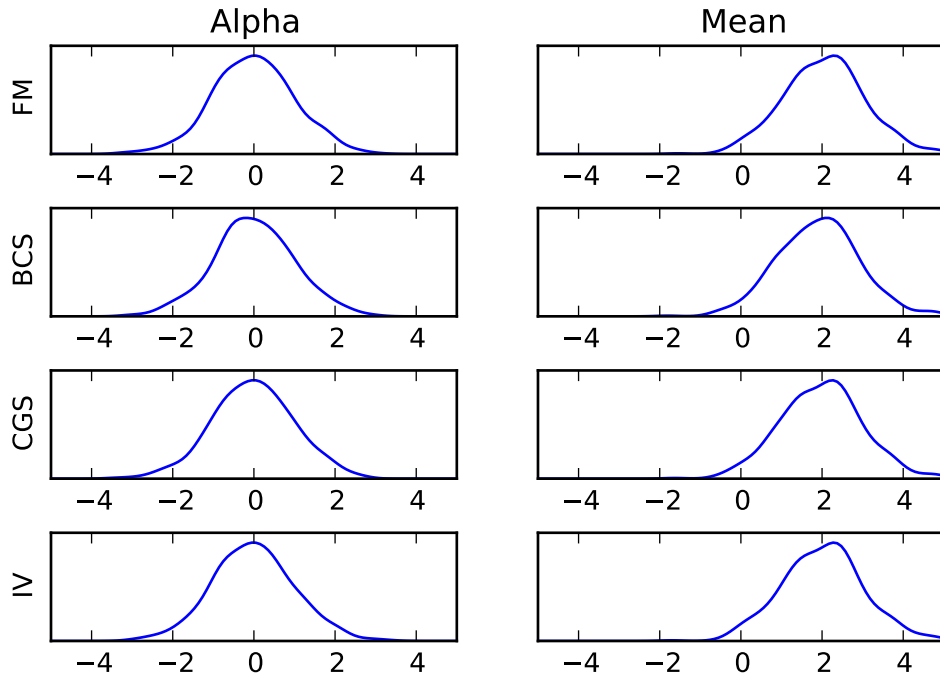


Figure 2: **Simulated distributions of t statistics for alphas and means of the book-to-market coefficients with time-varying betas, characteristics, and idiosyncratic volatilities.** We run 500 simulations of a three-factor model for 3,000 stocks and 1,600 weeks. Time-varying betas, characteristics, and idiosyncratic volatility are simulated from an autoregressive process such that its stationary distribution is multivariate normal with parameters in Table 2. Factors and firm-specific returns are also drawn from normal distributions. We first run factor model regressions to compute estimated betas with data from the prior 48 weeks (96 for IV). We then estimate monthly cross-sectional regressions of returns on size, book-to-market, and estimated betas to the market, SMB and HML, using four methods described in the text (Fama-MacBeth, BCS, CGS, and IV). We report results for the variable calibrated to book-to-market. The right panel reports the distributions of t statistics of time-series means of cross-sectional coefficients. The left panel reports the distributions of t statistics of alphas from regressions of cross-sectional coefficients on the three factors.

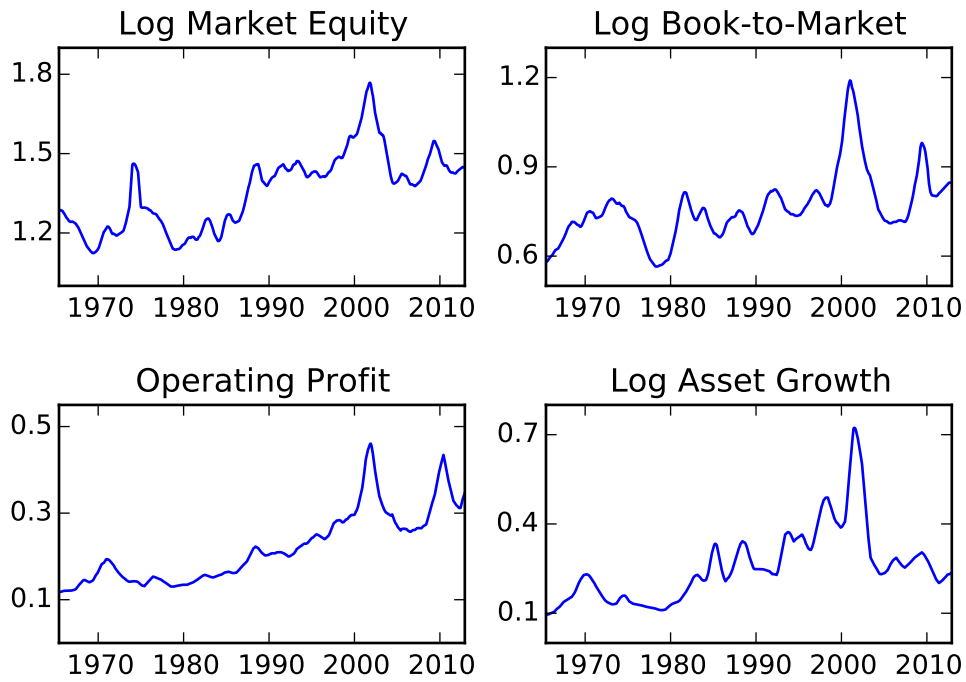


Figure 3: **Cross-sectional standard deviations of characteristics for the five-factor FF model.** This figure presents 12-month moving averages of the cross-sectional standard deviations of characteristics for the five-factor Fama and French (2015) model. Stocks with market equity smaller than the 20th NYSE percentile are excluded. The sample period is July 1964 through December 2012.

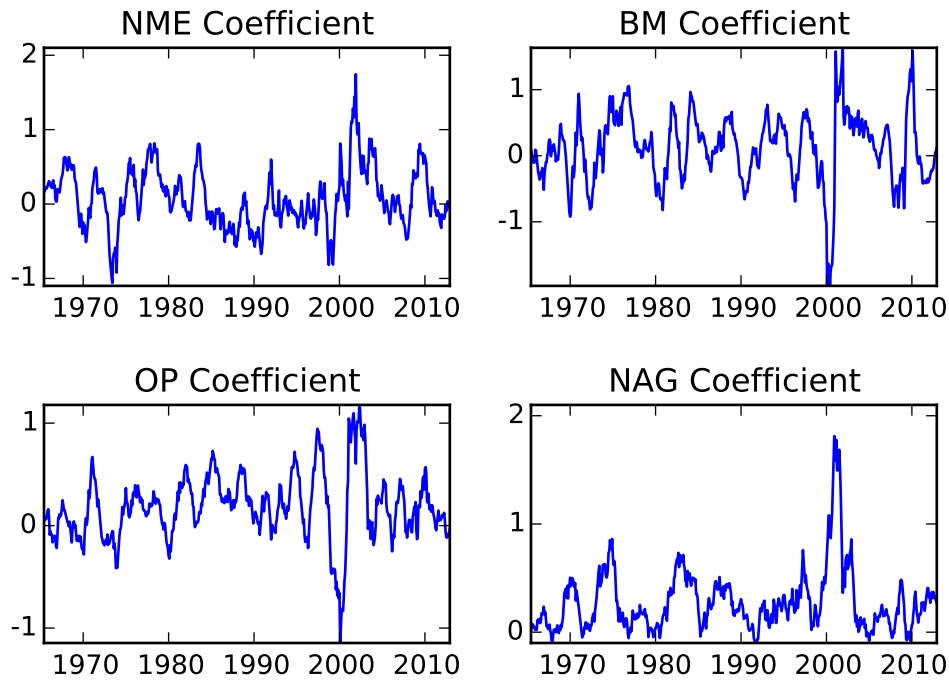


Figure 4: **12-month moving averages of the Fama-MacBeth coefficients for characteristics in the five-factor Fama and French (2015) model.** Monthly Fama-MacBeth regressions are run on four characteristics and five estimated betas. The characteristics are negative log market equity (NME), log book-to-market (BM), operating profitability (OP), and negative asset growth (NAG). The estimated betas are from regressions of weekly individual stock returns on FF factors (MKTRF, SMB, HML, RMW, CMA), over the prior 52 weeks. Each month, all characteristics and estimated betas are winsorized and standardized to have unit cross-sectional standard deviations. This figure shows 12-month moving averages of the characteristic coefficients (in %) in the Fama-MacBeth regressions. Stocks with market equity smaller than the 20th NYSE percentile are excluded. The sample period is July 1964 through December 2012.