Optimal Corporate Governance in the Presence of an Activist Investor*

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July, 2010

*We thank Anat Admati, Sugato Bhattacharyya, Philip Bond, Alex Edmans, Paolo Fulghieri, Milt Harris, TJ Liu and participants at seminars at Carnegie Mellon, Chicago Booth, CSSSC Kolkata, Michigan, Texas at Austin, Toronto, UBC and Wharton and the Caesarea Center, China International, and Paris Spring Corporate Finance conferences for helpful comments.
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Abstract

We provide a model of governance in which a board arbitrates between an activist investor and a manager. Reputational concerns make the manager reluctant to implement a change in firm strategy, creating an agency conflict. The optimal level of internal governance as supplied by the board depends on both the severity of the agency conflict and the strength of external governance. In some cases, the board commits to an interventionist policy to induce participation from the activist. In other cases, board intervention exacerbates managerial misbehavior, so it is optimal to be passive. The overall relationship between internal and external governance is non-monotone. As external governance becomes stronger, internal and external governance are first substitutes, and then complements.
Shareholder activism to force policy changes at publicly-traded firms represents an increasingly important dimension of the market for corporate control. In most cases, activist investors hold relatively small stakes in a firm and so cannot exert direct control. Rather, they rely on the firm’s internal governance mechanisms to implement changes. Brav, et al. (2008) study activist hedge funds, and find that they regularly cooperate with the board and with management, achieve some success two-thirds of the time, and can have a significant impact on firm value.\textsuperscript{1} For example, in 2006, hedge fund manager Nelson Peltz launched a campaign to force Heinz to divest brands acquired earlier in the tenure of the CEO. Peltz succeeded after almost a two-year battle, despite owning only 5.4% of the firm’s equity.

An activist investor does not enter into a vacuum: The firm already has a strategy in place. The current manager, who likely chose the strategy in place, is concerned about his own reputation. As a result, he may be overly resistant to reversing his own prior decisions.\textsuperscript{2} Evidence of such reluctance can be seen in the high rate of divestitures following a change in management and the subsequent improvement in firm value.\textsuperscript{3} By continuing a range of inefficient projects for too long, a reputation-conscious manager can cause significant loss of value at a firm. This creates a role for an internal governance mechanism that can use the information of an activist investor to override a manager’s decisions. We provide a theoretical framework in which the board of directors is the natural supplier of such internal governance.

The board in our set-up creates value for shareholders by arbitrating between a manager and an activist investor who disagree about the firm’s strategic direction. The optimal policy of the board must take into account both the incentives of the activist and the response of the manager. We show that, if the manager’s concern for his reputation is low or moderate, the board should commit to being completely passive, even if intervention can improve ex post value. In the latter case, aggressive internal governance makes the manager more stubborn, unwinding the intended effect. Conversely, if both the activist’s information and the agency conflict with the manager are strong, the board optimally defers to the activist. When the agency conflict is strong and the activist’s incentives are weak, the board commits to aggressive intervention to induce the activist to enter. Here, external governance (supplied by the activist) is a substitute for internal governance (supplied by the board). With strong agency conflict and moderate activist incentives, optimal use of the activist’s information by the board results in external and internal governance being complements.

\textsuperscript{1}Gillan and Starks (2007) document several additional sources of shareholder activism, as well as its increased incidence over the years.\textsuperscript{2} For example, a manager concerned about his reputation will not divest often enough (Boot, 1992), may fail to ignore sunk costs (Kanodia, Bushman and Dickhaut, 1989), and will be inflexible about altering investment plans over time (Prendergast and Stole, 1996).\textsuperscript{3} Weisbach (1995) examines divestitures following a CEO turnover, and Bhagat, Shleifer and Vishny (1992) following a hostile takeover.
In our model, a manager chooses between two mutually-exclusive projects with uncertain payoffs. A project is interpreted as a decision about the broad strategic direction of the firm. The manager obtains a signal about the relative payoffs of the projects. The precision of his information is determined by his ability, which can be high or low. A board of directors can veto the manager’s decision. The board’s objective is to maximize firm value. The manager, on the other hand, cares both about the value of the firm and about his reputation (i.e., investors’ beliefs about his ability).

At the beginning of the game, the board may invest in a screening technology that (later in the game) produces a noisy signal of the manager’s ability. Then, the manager receives a signal about project payoffs and chooses one of the two projects. At the next stage, an activist investor (henceforth “activist” or “outsider”) chooses whether to generate additional information about the projects. Shareholder value is maximized by continuing with the initial project when the signals of the manager and the activist agree. However, if their signals disagree, the value-maximizing project depends on the manager’s ability: It is optimal to continue with a project chosen by a high-ability manager but switch to the other project if the manager’s ability is low. At this stage, the manager has the option of switching projects. The board then receives its signal about the manager’s ability, and decides whether to intervene and overrule the his decision.

We describe the model in detail in Section 2. In Section 3, we analyze the continuation game that results if the outsider chooses to acquire information. Once the outsider reveals his signal, the manager can choose to switch projects (“concede”) or stay with the original project (“fight”). If the signals of the manager and outsider agree, there is no conflict. The manager stays with the original project, and the board naturally allows this decision to stand. The more interesting case is the one in which the signals disagree.

We focus on equilibria in which the high-type manager fights with probability one. Thus, if the manager concedes, he must be a low type, and the board allows his reversal to stand. However, because the low-type manager is conscious about his reputation, he may choose to fight even when he has chosen the wrong project. If he fights, the board may either remain passive or intervene in project choice. In the latter case, it can further choose to overrule the manager only if it obtains a low signal about his type (which we term “informed” governance) or in all cases (“sledgehammer” governance).

If the low-ability manager does not care too much about his reputation, he concedes. The

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4While the CEO in our model is concerned about a reputation for being skilled, Boot, Greenbaum and Thakor (1993) and Fisher and Heinkel (2008) analyze models in which an agent attempts to acquire a reputation for honesty, which he then sometimes exploits.

5Boot (1992) considers a model in which an outside raider can engage in a hostile takeover of a firm with a stubborn manager and improve value via a divestiture. We build on his work by introducing a role for internal governance (implemented by the board) when outsiders and managers more broadly disagree about the value-maximizing strategy.
result is a separating equilibrium that implements the value-maximizing project. However, if the agency conflict is severe (i.e., he cares a lot about his reputation), he fights, resulting in a pooling equilibrium. With a moderate agency conflict, a hybrid equilibrium obtains in which he mixes between fighting and conceding. Both the pooling and hybrid equilibria are inefficient, with the less valuable project being pursued at least some of the time.

One might expect that a more active board could mitigate this inefficiency. However, in the hybrid equilibrium, improved internal governance increases the stubbornness of the low-ability manager. The intuition for this key result is that, since the board only overrules the manager when it knows he has low ability, fighting and not being overruled sends investors a positive signal about the manager’s ability. The strength of this certification effect increases with the precision of the board’s signal. As a result, when the board invests more in its signal, the low-ability manager has a stronger incentive to fight.

The pooling and hybrid equilibria with informed governance only exist when external governance is weak; i.e., the outsider’s signal is imprecise. With strong external governance, there is a pooling equilibrium with sledgehammer governance, with the board essentially free riding off the outsider’s information and always vetoing the manager. This equilibrium is also inefficient, since even the choice made by a high-ability manager is overruled.

We consider the optimal decision of the outsider in Section 4. The outsider incurs a fixed cost if she acquires information about the firm, and captures some of the resultant improvement in cash flow. She enters (i.e., acquires information about the firm) if the precision of her signal is sufficiently high, where the exact threshold depends both on the potential for agency conflict and on the equilibrium in the continuation game.

In Section 5, we consider the board’s investment in the screening technology at the start of the game. The board’s choice depends on both the potential for agency conflict reputation and the strength of external governance. When the agency conflict is mild, both types of manager will choose the value-maximizing project, so the board does not need to screen. Even with a moderate agency conflict, the board optimally remains passive to avoid exacerbating the manager’s reputational concerns. In this case, the hybrid equilibrium obtains, and the low-ability manager fights with positive probability.

When the agency conflict is severe, even the low-ability manager fights with probability one. The activist’s payoff increases in the likelihood that the board will overturn the low-ability manager. With informed governance, the latter probability in turn depends on the precision of the board’s signal. Thus, by over-investing in screening (relative to the value-maximizing level when the outsider’s presence is taken for granted), the board can induce the outsider to be active. As the quality of the activist’s signal improves, the minimum level of internal governance necessary to induce her participation falls and the board reduces its screening intensity. In this sense, external and internal governance are substitutes.

Once the outsider’s signal is sufficiently precise, the board chooses a screening level that
optimally trades off the cash flow benefit from intervention with the direct cost of its signal. The optimal level increases as the precision of the outsider’s signal increases, since it is now more valuable to overturn the low-ability manager. Thus, over this parameter range, external and internal governance are complements. Finally, when the outsider’s signal is very precise, the board simply free-rides on the outsider’s information, resulting in sledgehammer governance. The overall relationship between external and internal governance is thus complex and non-monotone.

We comment on some of the features of our model and on how alternative assumptions would affect our results in Section 6. Section 7 explores some of the empirical implications, and Section 8 concludes.

Our paper makes several contributions to the corporate governance literature. First, we identify an important new role for the board of directors, which serves as the ultimate arbitrator when different stakeholders disagree over the strategic direction of the firm. Second, we demonstrate the importance of active internal governance both in encouraging the entry of activist investors and in making optimal use of their information. Third, we identify circumstances under which the board should optimally be passive because active intervention leads to greater resistance from a manager. More generally, our model generates the rich set of interactions we observe among activist investors, managers and boards.

In contrast to the existing literature, we explicitly model the process through which an activist shareholder can influence the decisions made at a firm. Other work on governance by a blockholder assumes the blockholder can directly either affect a firm’s cash flows (e.g., Admati, Pfleiderer and Zechner, 1994) or intervene to stop managerial misbehavior (e.g., Noe, Robello and Sonti, 2008). In practice, most blockholders do not possess controlling stakes, but rather must rely on persuasion and the firm’s internal governance mechanisms to effect change. As we show, the activist’s ability to influence management’s decisions depends not only on the severity of the agency conflict within the firm, but also on the firm’s internal governance policies. Moreover, the severity of the agency conflict affects the firm’s internal governance policies, which in turn determine how efficiently an activist’s information is used.

In our model, the board is sometimes optimally passive ex ante because intervention exacerbates managerial misbehavior. Related arguments are made by Burkart, Gromb and Panunzi (1997) and Almazan and Suarez (2003), who show that the threat of loss of control (and perhaps dismissal) weakens a manager’s incentive to invest in firm-specific human capital. Along similar lines, Adams and Ferreira (2007) show that a manager will be more willing to supply information to the board of directors if the board can commit not to use this information to veto the manager’s decisions. A different perspective is provided by Hermalin and Weisbach (1998), who show that firing a CEO can reduce the joint surplus of shareholders

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6In the models of Admati and Pfleiderer (2009), Edmans (2009) and Edmans and Manso (2009), the threat that a large shareholder will sell shares and reduce the stock price exercises discipline over a manager.
and a successful incumbent. In our model, in addition to the effects on the manager, the board must consider the effects of its actions on suppliers of external governance. As a result, the board is sometimes over-active rather than passive.

The board in our model represents an internal source of governance while the activist investor represents an external source of governance. Since both forms of governance discipline managers, one may expect them to be substitutes (see, for example, Fama, 1980, Fama and Jensen, 1983, and Williamson, 1983). Acharya, Myers and Rajan (2008), on the other hand, suggest that external governance (by the board) complements internal governance (by subordinates within the firm). Immordino and Pagano (2009) also consider the interaction of internal governance (i.e., actions by a board) with external governance (in their case, the actions of an outside auditor), and find the two can be complementary under some conditions. Our model predicts that the relationship between internal and external governance depends both on the potential for agency conflict and the strength of external governance, and is non-monotone.\footnote{Empirical research on the relationship between internal and external governance has yielded mixed results. For example, Mayers, Shivdasani and Smith (1997) find that mutual insurance companies, which are hard to take over, have more outside directors than stock insurance companies, suggesting that internal governance is a substitute for external governance. Ferreira, Ferreira and Raposo (2010) find that board independence is negatively related to the informativeness of a firm’s stock price, which they argue determines the effectiveness of external governance by the market. Brickley and James (1987), on the other hand, find that banks in states that prohibit takeovers tend to have fewer outside directors, suggesting that internal governance complements external governance.}

Our work analyzes the allocation of decision-making authority within a firm, and may therefore be interpreted in the spirit of Aghion and Tirole (1997). The board retains ultimate authority in our model. Equilibria in which it is passive result in transfer of control to the manager, whereas the sledgehammer governance equilibrium implies transfer of control to the outsider. In the other equilibria, the board retains real authority. Bebchuk (2005) concludes that a greater concentration of power with shareholders (or their representatives on the board) would improve firm value. Our work is more in the spirit of Harris and Raviv (2009), who show that activist shareholders should not always have control over corporate decisions. As in their framework, an activist shareholder in our model is only partially informed.

Our model highlights that the potential for agency conflict is critical in determining the optimal level of governance. Chhaochharia and Grinstein (2007) examine the effects of the 2002 Sarbanes-Oxley Act on the performance of US firms, and find that less compliant firms (which had the greater agency conflict before the Act) experience an increase in value. John, Litov and Yeung (2008) conduct a cross-country study, and find evidence to support the hypothesis that in countries with poor investor protection (so greater potential for agency conflicts), managers invest sub-optimally.

As a final point, Gompers, Ishii and Metrick (2003) and Bebchuk, Cohen, and Ferrell (2004) have constructed widely-used empirical indices of corporate governance that lump to-
gether both internal and external governance measures. However, our results suggest that the interplay of internal and external governance can be quite complex, and simple aggregation may not be a reliable way to measure the expected effectiveness of governance. In the data, if firms are in equilibrium, changing governance cannot affect the value of a firm. Conversely, if firms are being sub-optimal, more intense governance is not necessarily beneficial. For example, in our model, there is sometimes a negative relationship between firm value and internal governance: the board can improve firm value by committing to be less active.

2 Model

A publicly-traded firm faces a choice between two mutually exclusive projects. Each project yields a cash flow of either 0 or 1 at time 4. There are two possible future states. In state $x_A$, project $A$ yields a cash flow of 1 and project $B$ earns 0. In state $x_B$, project $A$ earns 0 and project $B$ earns 1. The ex ante probability of state $x_A$ is $\frac{1}{2}$. The firm is operated by a manager who has a type $\theta \in \{\theta_H, \theta_L\}$, with $\theta_H > \theta_L \geq \frac{1}{2}$. The unconditional probability that the manager has type $\theta_H$ is given by $q \in (0, 1)$. The manager observes his own type, but other parties in the model do not.

There are two stages at time 0. First, the board of directors of the firm invests firm resources in a noisy screening technology that provides information about the type of the manager. The amount invested is observed by all parties. The technology specifies the regular reports that the manager is compelled to supply and also incorporates soft information about the manager’s ability the board may gather from conversations with the manager and others.

The signal produced by the internal governance mechanism takes some time to generate, and is observed only at time 2. The signal is binary, with $s_B \in \{H, L\}$. We assume that $\text{Prob}(s_B = H \mid \theta_H) = 1$ (so the high-ability manager generates signal $L$ with probability 0) and $\text{Prob}(s_B = H \mid \theta_L) = 1 - \alpha$ (so the low-ability manager generates signal $L$ with probability $\alpha$). Thus, the board signal is completely uninformative when $\alpha = 0$ and becomes fully informative as $\alpha$ approaches 1. A signal of precision $\alpha$ is obtained at a cost $c(\alpha)$. The cost function is strictly increasing and strictly convex in $\alpha$. In addition, we assume that $c(0) = 0$, $c'(0) = 0$ and $\lim_{\alpha \to 1} c'(\alpha) = \infty$. The restrictions ensure that the board will choose a level of $\alpha$ strictly less than 1.

After the board has chosen $\alpha$, the manager receives a signal $s_M \in \{A, B\}$ about the true state, and embarks on the project. The signal is informative, with $\text{Prob}(s_M = k \mid X = x_k) = \theta$. Since $\theta_H > \theta_L$, the high type has a more precise signal.

At time 1, an outsider chooses whether to generate a signal about the true state, $s_E \in \{A, B\}$, or to stay out of the game. The outsider is a financier who acquires shares and becomes an activist investor, and is external to the current power structure of the firm. If the outsider enters (i.e., generates a signal), she communicates her signal to the board and
the manager. We assume the outsider’s presence is observed by all parties.

The outsider’s signal is less precise than the signal of a high-ability manager, but more precise than the signal of a low-ability manager. In particular, \( \text{Prob}(s_E = k \mid X = x_k) = \psi \), where \( \theta_L < \psi < \theta_H \). Thus, if the manager’s and outsider’s signals disagree, the efficient outcome accords with the manager’s signal if the manager has high ability, but with the outsider’s signal if the manager has low ability.

At time 2, regardless of whether the outsider entered, the manager has the opportunity to switch projects. After the manager has made his choice, the board obtains its signal about the manager’s ability, and decides whether to uphold the manager’s decision or implement the alternative project.

At time 3, investors in the broader market form posterior beliefs about the type of the manager. Let \( \mu \) denote the posterior probability at time 3 that the manager has type \( \theta_H \). Investors observe the presence of the outsider, the actions of the manager and the actions of the board, but not the signals of either the outsider or the board.

Finally, at time 4, the cash flow from the project is realized as either 0 or 1. The project is therefore a long-term project, whose outcome is not known in the short-run. However, the manager’s labor market opportunities depend on investors’ short-run beliefs over his ability. Figure 1 displays the sequence of events in the model.

\[
\begin{array}{cccccc}
  t = 0 & t = 1 & t = 2 & t = 3 & t = 4 \\
\hline
\text{Board} & \text{Manager} & \text{Outsider} & \text{Manager} & \text{Board} & \text{Investors} & \text{Project} \\
\text{chooses } \alpha & \text{chooses to continue or switch project} & \text{generates signal} & \text{chooses signal; chooses whether to overturn manager’s project choice} & \text{generates signal; chooses whether to overturn manager’s project choice} & \text{update beliefs about manager type} & \text{cash flow realized} \\
\text{observes } s_M \in \{A, B\}; & \text{or stays out} & \text{or stays out} & \text{or stays out} & \text{or stays out} & \text{or stays out} & \text{or stays out} \\
\text{Chooses project} & \text{or stays out} & \text{or stays out} & \text{or stays out} & \text{or stays out} & \text{or stays out} & \text{or stays out} \\
\end{array}
\]

Figure 1: Sequence of events

Let \( v \) be the value of the firm at time 4; that is, \( v \) is the cash flow of the project minus the cost of the board’s signal, \( c(\alpha) \). Further, let \( \theta^\mu = \mu \theta_H + (1 - \mu) \theta_L \) be investors’ posterior

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8Because the outsider has a financial stake in the firm, her communication is credible. If she were, say, a stock analyst or an uninvolved third party, her communication would not be credible and could be ignored.

9Under SEC regulations, any party with a greater than 5% stake in a public firm must disclose its investment. This assumption is not essential but yields simpler expressions for posterior beliefs.

10For example, if the manager were to leave the firm at time 1, his compensation in the new job would depend on his perceived ability (see, e.g., Harris and Holmström, 1982).
expectation (at time 3) of manager type. The manager’s payoff is then

\[ U_M = \beta v + (1 - \beta)\theta^\mu, \]

where \( \beta \in (0, 1) \). Thus, the manager cares both about the success of the project and about his reputation, i.e., investors’ beliefs about his type.

The board represents the shareholders, who care only about the overall value of the firm (that is, expected project cash flow less any resources spent on acquiring a signal about the manager). Thus, the board’s payoff function is just \( U_B = v \). We defer a discussion of the outsider’s payoff to Section 4. All parties are risk-neutral, and so maximize their respective expected payoffs.

We interpret \( \psi \), the precision of the outsider’s signal, as a proxy for the strength of external corporate governance. The outsider’s signal represents a factor outside the direct control of the board that can nevertheless affect the manager’s behavior. Although the signal itself does not require the manager to undertake a particular action, it plays two roles in the governance process. First, it influences the manager’s choice of action, since the manager cares about the payoff on the project. Second, the board can use the outsider’s signal to veto managerial decisions.

The board also plays two roles in the governance process. First, at time 0, it chooses an optimal level of screening by deciding how much to invest in the internal governance mechanism, which in turn affects the manager’s action at time 2. Second, at time 2, it decides whether to directly intervene in the operations of the firm and implement a project contrary to the manager’s choice.

We consider a perfect Bayesian equilibrium of the game. Therefore, the board cannot commit to its overturning strategy at time 2. Instead, its action must be a best response given its own choice of \( \alpha \) at time 0, the outsider’s entry decision at time 1, and the strategy of the manager. Further, the beliefs of the board at time 2 and investors at time 3 about the type of the manager must be consistent with Bayes’ rule whenever possible.

We focus on equilibria in which, at time 0, the manager chooses the project that is favored by his signal. Thus, if \( s_M = A \), project \( A \) is chosen, and if \( s_M = B \), project \( B \) is chosen.\(^{11}\) Then, at time 2, if \( s_E = s_M \) or if the outsider chooses to stay out, the manager has no reason to switch to the other project, and will continue with the project he had chosen earlier. In this case, there is no reason for the board to intervene at time 2.

Thus, the continuation game at time 2 is relevant only if the outsider enters and \( s_E \neq s_M \) (that is, the manager and outsider receive conflicting signals). Under this scenario, the

\(^{11}\) An infinitesimal cost to switching projects (which may come from a small impact of the time 0 project choice on the firm’s cash flows or the possibility that the activist is not present at all) will rule out any equilibrium in which the manager chooses the wrong project at time 0. However, such a cost adds no qualitative insight to the model, so is omitted for brevity.
The manager must decide whether to continue with the current project, or switch to the other project. In keeping with the symmetry of the game, we consider equilibria that are symmetric in the true state and hence invariant to the actual realization of $s_M$ and $s_E$. Let $\sigma_k$, for $k \in \{L, H\}$, denote the probability the manager continues with the current project at time 2, when the manager’s type is $\theta_k$ and $s_E \neq s_M$. Such a continuation puts the manager in direct conflict with the outsider, and we refer to this choice of strategy as “Fight”. If the manager instead adopts the project favored by the outsider’s signal, we refer to his action as “Concede.” The board must then decide whether to overturn the manager’s choice of project.

Suppose the signals of the manager and outsider disagree, and the manager concedes. In keeping with our view of the board as an arbiter of disputes between the manager and the outsider, we consider equilibria in which the board allows the concession to stand. In Section 3, we exhibit equilibria in the continuation game at time 2. For these equilibria, we show that it is a best response for the board to not intervene when the manager concedes.

Next, suppose the signals of the manager and outsider disagree, and the manager fights. If the board obtains signal $L$, it knows the manager has the low type, and will overturn the manager’s decision. If it obtains signal $H$, the board has imperfect information about the manager’s type. Let $\gamma$ denote the probability it overturns the manager in this case. If $\gamma = 0$, the board overrules the manager only on obtaining signal $L$; we call this “informed” governance. If $\gamma = 1$, the manager is overruled regardless of the board’s signal, which we term “sledgehammer” governance.

Let $\xi$ denote the outsider’s optimal decision at time 0, with $\xi = 0$ implying that the outsider stays out (i.e., does not acquire information about the firm) and $\xi = 1$ that the outsider enters (i.e., generates a signal). Let $\sigma = (\sigma_H, \sigma_L)$. With a slight abuse of terminology, we describe an equilibrium only in terms of $(\alpha, \xi, \sigma, \gamma)$, with beliefs for the board at time 2 and investors at time 3 that are consistent with Bayes’ rule, wherever possible.

### 3 Optimal Strategies of Manager and Board at Time 2

We begin by considering the continuation game starting at time 2. The board has chosen $\alpha$ at $t = 0$; for now we hold this choice of $\alpha$ fixed. Since the board will never choose $\alpha = 1$, we fix $\alpha$ to be strictly less than 1. If the outsider stays out at time 1, it is optimal for the board to simply allow the manager to proceed with his chosen project (since $\theta_L \geq \frac{1}{2}$). Hence, in this section, we focus on the case where the outsider enters at time 1, and $s_E \neq s_M$.

Since we consider equilibria that are symmetric in the true state, without loss of generality assume the manager observes signal $A$. Then, the manager chooses project $A$ at $t = 0$. Let $\lambda_i = \theta_i (1 - \psi) + \psi (1 - \theta_i)$ be the probability that the signals of the manager and the outsider disagree when the manager has type $\theta_i$. Define $\delta_i$ as the probability that $x_A = 1$ if $s_M = A$ and $s_E = B$, when $\theta = \theta_i$. Note that the expected cash flow from project $A$ is $\delta_i$ and that
from project $B$ is $1 - \delta_i$. Further, $\delta_i = \frac{\theta_i(1-\psi)}{\lambda_i}$ for $i = L, H$, with $\delta_L < \frac{1}{2} < \delta_H$.

Recall that $\gamma$ denotes the probability with which the board overturns the manager when the signals of manager and outsider disagree, the manager fights, and the board receives signal $H$. Then, if a high-type manager fights, the expected cash flow from the project is $\delta_H$ with probability $1 - \gamma$ and $1 - \delta_H$ with probability $\gamma$. Suppose the board allows a concession by the manager to stand. If the high type concedes, the expected cash flow is $1 - \delta_H$. Since $\delta_H > 1 - \delta_H$, a high type finds it costly to concede if $\gamma < 1$. For the low-type manager, the expected cash flow if he fights is $\delta_L$ with probability $(1 - \alpha)(1 - \gamma)$ and $1 - \delta_L$ with probability $1 - (1 - \alpha)(1 - \gamma)$. If he concedes, the expected cash flow is $1 - \delta_L$. Since $\delta_L < 1 - \delta_L$, the low-type manager finds it costly to fight if $\gamma < 1$. Therefore, when $\gamma < 1$, the high type has a greater tendency to fight than the low type.

We show that equilibria in the continuation game can be characterized as follows. If the board overturns the manager with probability less than 1 when it obtains signal $H$ (i.e., if $\gamma < 1$), then it must be that either the high-type manager fights with probability one, or both types of manager fight with probability zero. If, instead, the board always overturns the manager when it receives the high signal, both types of manager must fight with equal probability. Recall that $\sigma_i$ is the probability that the type $\theta_i$ manager fights.

**Lemma 1.** Consider an equilibrium of the continuation game at time 2 in which, if the manager concedes, the board does not intervene.

(i) If $\gamma < 1$, either $\sigma_H = 1$ or $\sigma_H = \sigma_L = 0$.

(ii) If $\gamma = 1$, $\sigma_H = \sigma_L$.

Consider any equilibrium of the continuation game in which both types of manager concede with probability one ($\sigma_H = \sigma_L = 0$). Such an equilibrium is sustained by an off-equilibrium belief that there is a sufficiently large probability a manager who fights has the low type. Suppose that $\gamma < 1$, so that a manager who fights is not always overruled. Now, suppose the high-type manager deviates. Then, the expected cash flow of the firm is strictly greater following the deviation. Conversely, if the low-type manager were to deviate, and the board responds with $\gamma < 1$, the expected cash flow of the firm strictly falls. The high-type manager therefore has a greater incentive to deviate, so that such an equilibrium does not survive the refinement condition D1 introduced by Cho and Kreps (1987). Therefore, going forward, in considering equilibria in which $\gamma < 1$, we focus on the case $\sigma_H = 1$; that is, the high-type manager fights with probability one.

In some of the equilibria we consider, the low-type manager concedes with positive probability. When the low-type manager also fights with probability one, the equilibrium can
be sustained by the off-equilibrium belief that a concession comes from the low type. Since the low-type manager has a stronger incentive to concede, this off-equilibrium belief survives condition D1. Then, following a concession, it is optimal for the board to allow the manager to proceed with his ultimate choice of project.

We first consider the case in which the board exhibits informed governance, that is, chooses \( \gamma = 0 \). If the low type concedes, the firm implements project \( B \) and investors learn that the manager is a low type (since the high type never concedes). The low type then obtains a payoff \( \beta(1 - \delta_L) + (1 - \beta)\theta_L \). He receives exactly the same payoff if he fights and is overruled by the board. The low-type manager is therefore indifferent between these two outcomes.\(^{12}\) Thus he fights if and only if his payoff from fighting and \( not \) being overruled exceeds his payoff from conceding. In this scenario, the firm implements the wrong project. However, this allows the low-type manager to pool with the high type. Thus the low-type manager obtains a reputational benefit, since investors’ posterior expectation about his type must exceed \( \theta_L \). Therefore, he concedes only if \( \beta \) (the extent to which he cares about firm value) is high enough to outweigh the reputational benefit from fighting. Specifically, define

\[
\beta_s(\psi) = \frac{1}{1 + \frac{1 - 2\delta_L}{1 - \theta_H - \theta_L}}. \tag{1}
\]

Note that \( \beta_s \) declines in \( \psi \) (since \( \delta_L \) decreases when \( \psi \) increases), but is independent of \( \alpha \), the precision of the board’s signal.

**Proposition 1.** If (and only if) \( \beta \geq \beta_s(\psi) \), there exists a separating equilibrium in the continuation game at time 2 that induces efficient project selection. In this equilibrium, \( \sigma_H = 1, \sigma_L = 0 \) and \( \gamma = 0 \).

The outcome in Proposition 1 is efficient: the value-maximizing project is undertaken regardless of the type of the manager. The same outcome can also be achieved by a separating equilibrium in which the high-type manager concedes and the low-type manager fights. To obtain the efficient outcome, the board must overrule the manager always, whether he concedes or fights. Along similar lines, there are outcome-equivalent equilibria in which the high type concedes and the board overrules him that correspond to the equilibria with informed governance we consider in Propositions 2 and 3. A strategy in which only the high type concedes and is always overruled by the board (which then puts him into the right project)

\(^{12}\)Of course, the board could discipline the manager by, for example, firing him if he fights and is overruled. This would make fighting and being overruled more costly to the manager than conceding. Since the skill of the manager lies in selecting rather than implementing a project in our model, such a policy is costly to shareholders as well, if the new manager faces a learning curve. Nevertheless, our results are robust to the introduction of such a punishment, provided it is not too large.
is unrealistic. We focus therefore on the more realistic case in which the high type fights and any concession comes from the low type.

When $\beta$ is high, manager and shareholder interests are well-aligned. Therefore, the manager responds to the arrival of the outsider’s signal by choosing the project with the highest expected payoff. On the other hand, if $\beta$ is below the threshold value $\beta_s$, any continuation equilibrium will be characterized by some degree of pooling and hence of inefficiency in terms of project choice. If $\beta$ is very low, the manager focuses primarily on his reputation and places little weight on firm value. In this case, the low-type manager will pool with the high-type manager by fighting. Such pooling results in the implementation of the inefficient project, unless the board intervenes. Its decision to intervene depends on the precision of the outsider’s signal.

If the outsider’s signal is relatively precise, disagreements with the manager’s signal are more likely to occur when the manager has a low type. Given such a disagreement, the expected payoff of project $B$ increases with the precision of the outsider’s signal, while that of project $A$ falls. Both of these factors imply that the benefit to the board of overruling the manager increases with $\psi$. In fact, if $\psi$ is sufficiently high, the board is willing to overrule the manager even when it obtains signal $H$ (sledgehammer governance). Therefore, a necessary condition for a pooling equilibrium with informed governance is that $\psi$ is sufficiently low. Specifically, let

$$
\psi_f(\alpha) = \frac{q\theta_H + (1-\alpha)(1-q)\theta_L}{q + (1-\alpha)(1-q)}.
$$

(2)

It is straightforward to show that $\psi_f(\alpha)$ increases in $\alpha$. The board implements informed governance (i.e., sets $\gamma = 0$) only if $\psi \leq \psi_f(\alpha)$.

Of course, for the low-type manager to fight with probability one when he expects the board to exhibit informed governance, it must be that $\beta$ is low. Define

$$
\beta_\ell(\alpha, \psi) = \frac{1}{1 + \frac{1-2q\lambda_H}{\theta_H - \theta_L} \left[ 1 + \frac{(1-\alpha)(1-q)\lambda_L}{q\lambda_H} \right]}.
$$

(3)

Since $\alpha < 1$, it follows that $\beta_\ell(\alpha, \psi) < \beta_s(\psi)$. Further, notice that $\beta_\ell(\alpha, \psi)$ increases in $\alpha$. A pooling equilibrium with informed governance exists when $\beta \leq \beta_\ell(\alpha, \psi)$ and $\psi \leq \psi_f(\alpha)$.

**Proposition 2.** A pooling equilibrium with informed governance exists in the continuation game at time 2 if and only if $\beta \leq \beta_\ell(\alpha, \psi)$ and $\psi \leq \psi_f(\alpha)$. In such an equilibrium, both types of manager fight and the board overrules the manager only if it obtains the low signal. That is, $\sigma_H = \sigma_L = 1$ and $\gamma = 0$. 

12
When $\beta$ is in an intermediate range, the manager is somewhat, but not overly, conscious about his reputation. In this case, there exists a hybrid equilibrium with informed governance, in which the high-ability manager fights and the low-ability manager mixes between fighting and conceding. The board allows the manager’s project to continue if it receives signal $H$, and overturns the manager only if it receives signal $L$. As with the pooling equilibrium in Proposition 2, for such a hybrid equilibrium to exist, the board must find it optimal to not overrule the manager when it obtains signal $H$. Define
\[
\beta_b(\psi) = \frac{1}{1 + \frac{2(\delta_H - \delta_L)}{\theta_H - \theta_L}}.
\] (4)

Since $\delta_H > 1/2$, it follows that for each value of $\psi$, $\beta_b < \beta_s$.

Suppose that investors believe the low-type manager concedes with probability one. Then, on observing that the manager fights and is not overruled by the board, they believe he has the high type. This provides the low-type manager an incentive to fight, since the reputational component of his payoff improves. If $\beta < \beta_s$, the low-type manager does not care enough about firm value to concede with probability one. Conversely, suppose that investors believe the low-type manager fights with probability one. In this case, investors’ posterior expectation about type when the board receives signal $H$ is lower than $\theta_H$. Thus, the reputational benefit of fighting is smaller than in the previous case. Therefore, if $\beta$ is sufficiently high (but lower than $\beta_s$), the low-type manager is not willing to fight with probability one.

In the hybrid equilibrium, the low-type manager is indifferent between fighting and conceding. The probability that he fights, $\sigma_L$, depends on the parameters $\beta$, $\psi$, and $\alpha$. In particular, we show that it increases with $\alpha$, the precision of the board’s signal, and decreases with $\psi$, the precision of the outsider’s signal. We show in the proof of the Proposition that $\beta_b > \beta_k$ if $\psi > \psi_f(\alpha)$ and $\beta_b < \beta_k$ if $\psi < \psi_f(\alpha)$.

**Proposition 3.** (i) A hybrid equilibrium with informed governance exists in the continuation game at time 2 if and only if $\beta \in (\max\{\beta_l(\alpha, \psi), \beta_b(\psi)\}, \beta_s(\psi))$. In such an equilibrium, the high-type manager fights, the low-type manager mixes between fighting and conceding, and the board overrules the manager only if it obtains the low signal. That is, $\sigma_H = 1$, $\sigma_L \in (0, 1)$ and $\gamma = 0$.

(ii) In a hybrid equilibrium of the continuation game at time 2, the probability that the low-type manager fights increases with the precision of the board’s signal about manager ability and decreases with the precision of the outsider’s signal about the state. That is, $\frac{\partial \sigma_L}{\partial \alpha} > 0$ and $\frac{\partial \sigma_L}{\partial \psi} < 0$.

Part (ii) of Proposition 3 establishes a crucial insight of this paper: In the hybrid equi-
librium, while stronger external governance leads the low-ability manager to fight less often, stronger internal governance in the form of better screening by the board leads the low-ability manager to fight more often. Surprisingly, the actions of the manager and board can be thought of as strategic substitutes in this region, in terms of their impact on firm value.

A higher value of $\psi$ affects the low-type manager’s tendency to fight in two ways. First, the probability that the manager is a low type, conditional on the outsider’s signal disagreeing with his own, increases with the precision of the outsider’s signal. This reduces the reputational benefit of fighting. Second, an increase in the precision of the outsider’s signal increases the cash flow gain from choosing the value-maximizing project. Both effects lead to the low type fighting less often.

A higher value of $\alpha$ implies that the low-type manager is more likely to be overturned if he fights. If he fights and is overruled, he obtains the exact same payoff (both in terms of firm value and on the reputational component) as he does on conceding. If he fights and is allowed to proceed with his choice of project, the effect on his payoff is more complicated. The inefficient project is implemented, which is costly. On the other hand, being allowed to proceed by the board provides a noisy certification of his ability, which increases the reputational component of his payoff. The posterior probability that the manager is a high type when he fights and is not overturned increases with $\alpha$. Therefore, holding $\sigma_L$ fixed, the low-type manager’s payoff from fighting increases with $\alpha$. In turn, this results in an increase in $\sigma_L$, which reduces the reputational benefit of fighting and not being overruled so that, in equilibrium, the expected payoff from fighting and conceding are again equalized.

Next, we consider the case of an interventionist board. The board’s own signal is noisy. Therefore, if the outsider’s signal is sufficiently precise and the board believes that the low-type manager fights often enough, it may be optimal for the board to overturn the manager even when it obtains signal $H$. Of course, the board always overturns the manager on obtaining signal $L$. Thus, in such an equilibrium, the board’s action is independent of its own signal. An immediate implication is that knowing a manager was overturned has no information content for investors. Further, if a manager is always overturned by the board, both types are indifferent between fighting and conceding. Thus, in a continuation equilibrium with sledgehammer governance, the high type also may fight with probability less than one.

**Proposition 4.** An equilibrium with sledgehammer governance exists in the continuation game at time 2 if and only if $\psi \geq \psi_f(\alpha)$. In such an equilibrium, $\sigma_H = \sigma_L \in (0,1]$ and $\gamma = 1$.

Equilibria in which both types of manager mix between conceding and fighting cannot be dismissed by a refinement of beliefs, since there is no unreached information set. However,
note that for both types of manager to mix, the expected cash flow of the firm must be the same regardless of whether the manager concedes or fights. Hence, imposing a selection on this class of equilibria does not affect the expected cash flow of the firm, and so does not affect the optimal action of the board. Therefore, when considering equilibria with $\gamma = 1$, without loss of generality we focus on the case that $\sigma_H = \sigma_L = 1$; that is, both types of manager fight with probability one.

Now, suppose that $\psi > \psi_f(\alpha)$ and $\beta > \beta_b$. Then, there are multiple equilibria in the continuation game. From Proposition 4, an equilibrium with sledgehammer governance exists when $\psi > \psi_f(\alpha)$, regardless of the value of $\beta$. However, Proposition 1 shows that if $\beta \geq \beta_s$, there is also a separating equilibrium. Further, from Proposition 3, if $\beta \in (\beta_b, \beta_s)$, there is also a hybrid equilibrium with informed governance. Whenever there are multiple equilibria in the continuation game for a fixed value of $\alpha$, we select the equilibrium that maximizes the expected payoff of the board, conditional on $s_E \neq s_M$. We show that the board prefers the separating or hybrid equilibrium to the pooling equilibrium with sledgehammer governance.

**Lemma 2.** Suppose $\psi \geq \psi_f(\alpha)$. Then, if $s_E \neq s_M$ and the manager fights:

(i) If $\beta \geq \beta_s(\psi)$, the board’s expected payoff is higher under a separating equilibrium than under the equilibrium with sledgehammer governance.

(ii) $\beta \in [\beta_b(\psi), \beta_s(\psi))$, the board’s expected payoff is higher under a hybrid equilibrium than under the equilibrium with sledgehammer governance.

Therefore, if $\beta \geq \beta_s(\psi)$, we assume the separating equilibrium is played in the continuation game at time 2, regardless of the value of $\psi$. If $\psi > \psi_f(\alpha)$ and $\beta \in [\beta_b(\psi), \beta_s(\psi))$, we fix the equilibrium in the continuation game to be the hybrid equilibrium.

Observe that when $\psi = \psi_f(\alpha)$, the board is indifferent between $\gamma = 0$ and $\gamma = 1$. For this particular value of $\psi$, there exist equilibria in which the board plays a mixed intervention strategy; i.e., chooses a value of $\gamma$ strictly between 0 and 1. These equilibria all offer the same payoff to the board, so for convenience we select the equilibrium with $\gamma = 0$. In Figure 2, we illustrate the equilibria we consider at time 2 for different values of $\beta$ and $\psi$. The parameters for this figure are set to $\theta_H = 0.9, \theta_L = 0.55, q = 0.4$, and $\alpha = 0.5$.

From Figure 2, it may be observed that improved external governance (i.e., an increase in $\psi$) generally improves managerial behavior. In our model, improved external governance corresponds to better information about project cash flows, and makes it more costly for managers to choose the wrong project. As can be seen from the figure, an increase in $\psi$ generally results in a shift towards an equilibrium at time 2 in which the low type fights less often (e.g., from pooling with informed governance to a hybrid equilibrium and from a hybrid
This figure represents the equilibria we consider at time 2, for different values of $\psi$ and $\beta$. The other parameters used to generate the figure are $\theta_H = 0.9$, $\theta_L = 0.55$, $q = 0.4$, and $\alpha = 0.5$.

Figure 2: Equilibria in the Continuation Game at Time 2 when Signals of Manager and Outsider Disagree

4 Optimal Decision of Outsider at Time 1

We now step back to time 1, and consider the optimal decision of the outsider. The board has chosen $\alpha$ at time 0, and the outsider anticipates that, if she intervenes and generates a contrary signal, a continuation equilibrium $(\sigma, \gamma)$ will be played at time 2.

We assume that the outsider can acquire a fraction $\eta$ of the shares in the firm before
time 2, that is, before she acquires information about the firm. For convenience, we further assume that the market values these shares at the expected value of the firm assuming the outsider will not enter.\textsuperscript{13} If the outsider chooses to stay out, she does not acquire a stake in the firm. In this case, the board does not intervene (since $\theta_L \geq \frac{1}{2}$, it is optimal to leave the manager alone even if the board finds out he has the low type). Let $F_0 = q\theta_H + (1 - q)\theta_L$ denote the expected cash flow from the project in this case. The value of the firm is then $F_0 - c(\alpha)$.

Suppose, instead, the outsider does acquire a stake in the firm. Formally, she incurs a fixed cost $\tilde{\kappa}$ to acquire a fraction $\eta$ of the firm when the firm is valued at $F_0 - c(\alpha)$. Let $F$ denote the expected cash flow from the project after the outsider enters, where the expectation is ex ante with respect to the outsider’s signal; that is, the expectation is taken before the outsider knows her signal. The expected value of the firm after the outsider enters is then $F - c(\alpha)$.

Since the cost of the board’s signal, $c(\alpha)$, is sunk at time 0, the outsider’s decision depends only on the change in the expected cash flow from the project if she intervenes. The improvement in expected cash flow depends on $\psi$, the manager’s strategy, and on the likelihood that the manager is overturned by the board when the signals of the manager and the outsider disagree. Importantly, the expectation of cash flow in the next lemma is taken before the outsider has observed her own signal.

**Lemma 3.** Suppose that the outsider enters, and, if $s_M \neq s_E$, the continuation equilibrium $(\sigma, \gamma)$ at time 2 has $\sigma_H = 1$. Then, the expected cash flow from the project at time 1 before the outsider sees her signal is $F = q[\theta_H - \gamma(\theta_H - \psi)] + (1 - q)[\psi - \sigma_L(1 - \alpha)(1 - \gamma)(\psi - \theta_L)]$.

We show that when $\psi$ is low, the outsider stays out, regardless of the value of $\beta$. Similarly, for high values of $\psi$, the outsider always enters. However, there is also an intermediate region of $\psi$, in which the outsider enters only if $\beta$ is sufficiently high (i.e., the agency conflict is sufficiently low). Define $\psi_1 = \theta_L + \frac{\kappa}{1 - q}$, and $\psi_2(\alpha) = \theta_L + \frac{\kappa}{\alpha(1 - q)}$ if $\alpha > 0$. If $\alpha = 0$, let $\psi_2(\alpha)$ be infinite. Since $\alpha < 1$, $\psi_1$ is strictly less than $\psi_2(\alpha)$. Finally, define a function $\phi(\cdot)$ as follows:

$$
\phi(\psi) = \frac{1}{1 + \frac{\psi - \theta_L}{\theta_H - \theta_L} + \frac{(1 - q)(\psi - \theta_L) - \kappa}{q\lambda_H(\theta_H - \theta_L)}}. \tag{5}
$$

\textsuperscript{13}If investors could completely predict the presence of the outsider, the usual information acquisition problem arises: an agent will not acquire costly information if it is already incorporated into the price. Potentially, we could endow investors with a belief over $\psi$, which would then enable them to ascribe a probability to the outsider’s presence. Such an assumption complicates the analysis without changing the qualitative nature of the insights.
In the proof of Proposition 5, we show that $\phi(\cdot)$ is a strictly decreasing, and hence invertible, function of $\psi$. Provided the cost of entry, $\kappa$, is sufficiently low given other parameters, the outsider adopts the following strategy.

**Proposition 5.** Suppose $\kappa < \frac{\alpha q (\theta_H - \theta_L)}{q + (1 - \alpha)(1 - q)}$. Then, the outsider enters if $\psi > \psi_2(\alpha)$ and stays out if $\psi < \psi_1$. If $\psi \in [\psi_1, \psi_2(\alpha)]$, the outsider enters if $\beta > \phi(\psi)$ and stays out if $\beta < \phi(\psi)$.

For particular values of $\psi$ and $\beta$, the outsider is indifferent about entering (e.g., when $\psi = \psi_2$, the outsider is indifferent if $\beta \leq \phi(\psi_2)$). In the spirit of considering equilibria under which firm value is maximized, we assume the activist chooses to enter in this case.

As $\alpha$ increases, $\psi_1$ and $\phi^{-1}$ remain unchanged but $\psi_2$ decreases. In a pooling equilibrium with informed governance, an increase in $\alpha$ implies that the board weeds out the low-type manager more often. This increases the payoff to the outsider from generating her own information. Thus, as $\alpha$ increases, the outsider is more likely to enter if she anticipates a pooling equilibrium with informed governance.

Figure 3 illustrates the optimal decision of the outsider for each value of $\beta$ and $\psi$. The parameters used are the same as for Figure 2; that is, $\theta_H = 0.9, \theta_L = 0.55, q = 0.4$, and $\alpha = 0.5$. In addition, we set $\kappa = 0.04$.

## 5 Optimal Level of Screening by the Board at Time 0

We now consider the board’s optimal choice of screening intensity at time 0. The decision by the board at this stage, of course, depends on the equilibrium to be played in the continuation game when the signals of the manager and the outsider disagree. We first show that if the continuation equilibrium at time 2 is a hybrid equilibrium, small changes in $\alpha$ have no effect on the expected cash flow from the firm’s project, so that the overall effect on profit depends only on changes in the cost of the screening technology. That is, a small change in the screening intensity of the board is completely unwound by a corresponding change in the strategy of the low-ability manager. Let $\Pi$ denote the board’s expectation of firm value when it chooses $\alpha$ at time 0.

**Proposition 6.** Suppose $\psi \geq \psi_1$ and $\beta \in (\max\{\beta_L(\alpha, \psi), \beta_0(\psi)\}, \beta_4(\psi))$, so that the activist generates a signal and a hybrid equilibrium obtains in the continuation game at time 2. Then $\Pi'(\alpha) = -c'(\alpha) < 0$.

Consider a value of $\alpha$ that induces a hybrid equilibrium at time 2. All else equal, one
This figure represents the optimal decision of the outsider at time 1 and the equilibria in the continuation game at time 2, for different values of the parameters $\psi$ and $\beta$. The other parameters used to generate the figure are $\theta_H = 0.9, \theta_L = 0.55, q = 0.4, \alpha = 0.5$, and $\kappa = 0.04$.

Figure 3: Optimal Decision of Outsider for Different Values of $\beta$ and $\psi$

would expect that an improvement in screening will improve the expected cash flow from the project, since the correct project is implemented more often. However, from Proposition 3 part (ii), we know that such an increase will be met by increased intransigence on the part of the low-ability manager. As we show in the proof of Proposition 6, these two effects exactly offset each other, so that the overall profit changes only to the extent that the cost of screening changes with $\alpha$. Thus, if the board anticipates a hybrid equilibrium at time 1, it optimally chooses $\alpha = 0$ at time 0.

However, the equilibrium that obtains in the continuation game itself depends on the board’s choice of $\alpha$. For a sufficiently high value of $\alpha$ (just high enough that $\beta_L(\alpha, \psi) = \beta$), the low type fights with probability 1 and a pooling equilibrium obtains. At this point, a further increase in $\alpha$ cannot affect the strategy of the low-type manager. Therefore, the board may find it optimal to choose a high enough value of $\alpha$ to induce a pooling equilibrium.

Suppose that the activist generates a signal and a pooling equilibrium with informed governance indeed obtains in the continuation game beginning at time 2. Consider the board’s choice of $\alpha$ at time 0. The optimal value of $\alpha$ in this case must satisfy the following
first-order condition:

\[ c'(\alpha) = (1 - q)(\psi - \theta_L). \]  

(6)

Let \( \alpha_c \) denote the level of \( \alpha \) that satisfies equation (6). When it chooses the screening level \( \alpha_c \), the board makes optimal use of the outsider’s information. Since \( c(\cdot) \) is convex, it is immediate that \( \alpha_c \) increases as \( \psi \) increases.

Higher values of \( \psi \) imply a greater benefit to overturning the low-type manager. If the outsider’s signal is sufficiently strong, the board may choose instead to completely delegate the decision to the activist by overturning the manager regardless of its signal. If it anticipates an equilibrium with such sledgehammer governance at time 1, it should optimally choose \( \alpha = 0 \) at time 0. Define a threshold value \( \psi_g \) as the value of \( \psi \) that solves the implicit equation

\[ \psi = \frac{q\theta_H + (1 - q)(1 - \alpha_c)\theta_L - c(\alpha_c)}{q + (1 - q)(1 - \alpha_c)}, \]

(7)

where the equation is implicit because \( \alpha_c \) depends on \( \psi \). Then, \( \psi_g \) is the maximum value of \( \psi \) at which the board invests in learning about the manager’s type rather than simply opting for sledgehammer governance.

**Lemma 4.** Suppose the board anticipates that the outsider will enter and a pooling equilibrium will obtain at time 1. Then, if \( \psi < \psi_g \), the board chooses \( \alpha = \alpha_c \) at time 0 and implements informed governance, with \( \gamma = 0 \). If \( \psi > \psi_g \), the board chooses \( \alpha = 0 \) at time 0 and implements sledgehammer governance, with \( \gamma = 1 \).

Next, we show that by choosing \( \alpha \) appropriately, the board can induce the activist to enter. The activist’s signal increases firm value only if the board uses it to overturn the manager’s decision. In a pooling equilibrium with informed governance, the likelihood that the board uses the activist’s signal to improve decision-making increases with \( \alpha \). Thus, there is a threshold value of \( \alpha \) above which the activist is willing to acquire information about the firm. Define this threshold value as \( \alpha_e = \frac{\kappa}{(1 - q)(\psi - \theta_L)}. \) It is immediate that \( \alpha_e \) declines in \( \psi \), the precision of the outsider’s signal. As \( \psi \) increases, the outsider’s signal becomes more accurate. Thus, keeping the board’s policy fixed, the outsider’s payoff conditional on entering is higher. As a result, the board can reduce its screening intensity and still induce the outsider to incur the cost of entering.

**Lemma 5.** Suppose the outsider anticipates a pooling equilibrium with informed governance (i.e., \( \sigma_L = 1 \) and \( \gamma = 0 \)). Then, she enters if and only if \( \alpha \geq \alpha_e \).
To characterize the optimal strategy of the board, we need to define several thresholds in the parameter space. Recall that \( \psi_1 = \theta_L + \frac{1}{q} \), and let \( \psi_3 = \theta_L + \frac{\kappa}{(1-q)c^{-1}(\kappa)} \). Since \( c^{-1}(x) \) is less than 1 for any \( x \) less than \( \infty \), \( \psi_1 < \psi_3 \). Next, define the following threshold value of \( \beta \) at which the board is indifferent between choosing the cash flow maximizing investment \( \alpha_c \) and inducing a pooling equilibrium with informed governance, and choosing \( \alpha = 0 \) and inducing a hybrid equilibrium at time 2. Let

\[
\beta_c(\psi) = \frac{1}{1 + \frac{1-2\delta L}{\theta_H - \theta_L} \left[ 1 + \frac{(1-q)\lambda L}{qL} \left( 1 - \alpha_c + \frac{c(\alpha_c)}{c'(\alpha_c)} \right) \right]} \tag{8}
\]

Define \( \beta_m(\psi) = \max\{\phi(\psi), \beta_c(\psi), \beta_b(\psi)\} \). As we show in the proof of the next proposition, \( \beta_m \) equals \( \phi(\psi) \) for low values of \( \psi \) and \( \beta_b \) for high values of \( \psi \). If \( \kappa \) is not too high, there also exists an intermediate range of \( \psi \) for which \( \beta_m \) equals \( \beta_c(\psi) \). Finally, let \( \kappa_1 \) be the strictly positive solution to \( \kappa = c \left( \frac{\kappa}{qL(\theta_H - \theta_L)} \right) \), and let \( \kappa_2 = \psi_g - q\theta_H - (1-q)\theta_L \).

**Proposition 7.** Suppose \( \kappa < \min\{\kappa_1, \kappa_2\} \). Then,

(i) If \( \psi < \psi_1 \), or \( \psi \in [\psi_1, \psi_3] \) with \( \beta < \phi(\psi) \), the equilibrium is characterized by no governance. The board chooses \( \alpha = 0 \), the outsider stays out, and the board allows the manager to proceed at time 2; that is, \( \alpha = \xi = \gamma = 0 \).

(ii) If \( \psi \geq \psi_1 \) and \( \beta \geq \beta_m(\psi) \), the board continues to be completely passive, with \( \alpha = \gamma = 0 \). However, the outsider enters, so that \( \xi = 1 \). Either a separating or a hybrid equilibrium is played at time 2.

(iii) There exists a \( \psi_c \in (\psi_3, \psi_g) \) such that, if \( \psi \geq \psi_3 \) and \( \beta < \beta_m(\psi) \), then

(a) If \( \psi \leq \psi_g \), the board is informed, choosing \( \alpha = \alpha_c \), when \( \psi \in [\psi_3, \psi_c] \) and \( \alpha = \alpha_c \), when \( \psi \in (\psi_c, \psi_g] \). In both cases, the outsider enters, so that \( \xi = 1 \), and a pooling equilibrium with informed governance is played, with \( \gamma = 0 \).

(b) If \( \psi \in (\psi_g, \theta_H) \), the board chooses \( \alpha = 0 \). The outsider enters, so that \( \xi = 1 \), and a pooling equilibrium with sledgehammer governance is played, with \( \gamma = 1 \).

Figure 4 illustrates the equilibrium of the overall game for different values of \( \beta \) and \( \psi \). The parameters used are the same as for Figures 2 and 3; that is, \( \theta_H = 0.9 \), \( \theta_L = 0.55 \), \( q = 0.4 \), and \( \kappa = 0.04 \). For each value of \( \beta \) and \( \psi \), we allow \( \alpha \) to be chosen optimally by the board. We assume a cost function for the board’s signal of \( c(\alpha) = 0.1\alpha^5 \). While this cost function does not satisfy the condition \( \lim_{\alpha \to 1} c'(\alpha) = \infty \), in the example the optimal level of \( \alpha \) remains strictly below one.
This figure represents the equilibria that occur in the overall game for different values of the parameters $\psi$ and $\beta$. The other parameters used to generate the figure are $\theta_H = 0.9, \theta_L = 0.55, q = 0.4, c(\alpha) = 0.1\alpha^5$, and $\kappa = 0.04$.

Figure 4: Equilibria in the Overall Game for Different Values of $\beta$ and $\psi$, when the Outsider Acts Optimally

When $\psi$ is low, the outsider stays out, so the board cannot gain from generating a signal about the manager. Hence, there is no governance in this region. When $\beta \geq \beta_m$ and $\psi \geq \psi_1$, the outsider enters, but the board is optimally passive. It chooses to set $\alpha = 0$, and allows the manager to choose the project. If $\beta \geq \beta_s$, this achieves the first-best outcome, since the low-type manager optimally chooses the value-maximizing project. However, if $\beta \in (\beta_m, \beta_s)$, the low-type manager fights with positive probability, resulting in some inefficiency in project choice. Nevertheless, as we have shown, it is optimal for the board to be passive.

It is optimal for the board to invest in its screening technology if $\beta < \beta_m$ and $\psi \in (\psi_3, \psi_g)$. If $\psi > \psi_c$, it chooses $\alpha = \alpha_c$, which is optimal purely from a cash flow viewpoint. If $\psi < \psi_c$, the board has to over-invest in screening to induce the outsider to enter, and chooses $\alpha = \alpha_e$.

The board’s optimal policy exhibits several discontinuities when $\psi$ is large enough that the outsider enters. First, suppose $\psi \in (\psi_3, \psi_c)$ and $\beta < \beta_m$. Consider an increase in $\beta$ to $\beta_m$. At this point, the board switches from informed governance, with $\alpha \geq \alpha_c$, to being completely passive. Second, suppose $\psi > \psi_g$, and consider a similar increase in $\beta$ to $\beta_m$. The board now switches from extreme activism in the form of sledgehammer governance, in which the manager is always overturned, to complete passivity. Finally, consider the effect of an increase in $\psi$ when $\beta < \beta_m$. When $\psi$ increases to $\psi_g$, the board’s investment in screening...
drops from $\alpha_c$ to zero. Screening is substituted out in favor of extreme activism.

5.1 Internal and External Governance: Substitutes or Complements?

The relationship between internal governance and external governance is complex in our model. The strength of external governance is represented by the precision of the outsider’s signal, $\psi$. Internal governance is represented by both the screening intensity of the board $\alpha$ and the overturning probability $\gamma$.

Consider the case in which the agency conflict is severe; that is, $\beta$ is low. If the outsider’s signal is imprecise, she will stay out, so that the board is passive as well. As the precision of the outsider’s signal improves, she switches over to generating a signal, thus providing external governance. At this threshold, the board sets $\alpha = \alpha_e$, which declines in $\psi$. Near this threshold, $\alpha_e > \alpha_c$, so the board over-invests in screening relative to the level that ensures optimal use of the outsider’s information. Further, the board implements informed governance, overturning the manager only when it obtains the low signal. The overall probability that the manager is overturned is monotonic in $\alpha$, and hence also declining in $\psi$ over this region. Hence, for $\psi \in [\psi_3, \psi_c]$, internal and external governance are substitutes.

However, if $\psi$ lies between $\psi_c$ and $\psi_g$, $\alpha_e < \alpha_c$. The board no longer has to choose a higher $\alpha$ than optimally utilizes the outsider’s information in order to induce the outsider to enter. Instead, the board sets $\alpha = \alpha_c$, which is increasing in $\psi$. The intuition here is that the value to the board of overturning the low type increases as the precision of the outsider’s signal increases. Thus, it invests a greater amount in the screening technology. The board continues to implement informed governance, so the overturning probability is also increasing in $\psi$. Thus, internal and external governance are complements in this region.

Finally, if $\psi > \psi_g$ and $\beta$ is low, the board does not screen the manager, and simply acts on the outsider’s signal in deciding whether or not to overrule managerial decisions. In this sense, external governance completely substitutes for internal governance over this region of the parameter space.

Proposition 8. Suppose $\psi \in [\psi_3, \psi_g]$ and $\beta < \beta_m(\psi)$. Then, the screening intensity of the board, $\alpha$, decreases in $\psi$, the strength of external governance, when $\psi < \psi_c$ and increases in $\psi$ when $\psi > \psi_c$.

Thus, while large changes in the strength of external governance result in external governance substituting for internal governance, small changes in the strength of external governance can have complementary effects on internal governance. Overall, therefore, we find a non-monotone relationship between external and internal governance.

Our results therefore imply that corporate governance indices, such as those of Gompers,
Ishii and Metrick (2003) and Bebchuk, Cohen, and Ferrell (2004) must be interpreted with caution. If all firms have chosen an optimal level of governance, a higher index value may simply reflect the severity of the agency problem at one firm. If a firm has been sub-optimal, on the other hand, our results show that its value might be improved by less rather than more intense governance.

6 Comments on Model Features

In this section, we comment on some features of our model, and the implications of relaxing some of the assumptions.

First, the roles of the activist and board are distinct in our model: the board’s expertise lies in evaluating the manager and the activist’s in evaluating firm strategies. In practice, we expect the board to garner superior information about the manager than any outsider, given its repeated interactions with the manager. Activist investors, on the other hand, do not have the same level of access to the manager. Rather, activists specialize in evaluating strategies across different firms, in part through ongoing investments from which they can learn about the state of the world.

Suppose that, in contrast to our model, an activist investor also generates some information about the manager. Suppose further that the activist obtains such information at time 1, before deciding whether to enter. The activist will enter only if she believes the manager is likely to have a low type. This refines the board’s and outsider’s beliefs about the type of the manager if the activist enters, but does not qualitatively change the results of the paper. Alternatively, suppose the activist investor can invest in learning about the manager, but, like the board in our model, receives her information about manager type only at time 2. The activist will then withdraw from a fight if her signal suggests that the manager’s ability is high. A decision to withdraw in turn sends a noisy certification about the manager’s ability. This increases the incentives of the low type manager to fight. Thus, such information generated by the activist has broadly the same effect as the information generated by the board in our model.

Suppose now the board were also to obtain information about project payoffs. If the board obtains such information at time 0, we can simply think of the prior probabilities $\theta_H$ and $\theta_L$ as combining the information of manager and board, with the rest of the model going through. Conversely, suppose the board receives information about payoffs only at time 2, after the manager has chosen whether to fight or concede. The additional signal will simply be used to refine the posterior belief that the firm is in the right project. The presence of such information strengthens the certification effect of upholding a manager’s decision to fight. The overall effect is therefore similar to the effect of the signal about manager type.

Second, in our model the board invests in screening at time 0, with the information from
the screening process only available at time 2. In many contexts, learning about the type of an agent is a process that takes place over time. Nevertheless, there may be scenarios, say with long drawn-out activism contests, in which the board can invest in learning only after the contest begins. Suppose the board can choose $\alpha$ and $\gamma$ both at time 2 in our model. The board can no longer commit to being passive when a hybrid equilibrium is played in the continuation game. The certification effect of being upheld by the board remains important, and will tend to make the low-type manager fight more often. However, the cost of screening is no longer sunk, which makes the low type fight less often. The overall effect on $\sigma_L$ is now more ambiguous, since in addition, there may be a feedback effect from $\sigma_L$ to $\alpha$.

Third, we assume the board invests the firm’s resources rather than personal resources in learning about the manager. In any incentive compatible mechanism, the board must eventually be compensated for the cost of effort, so that the cost is eventually charged to the firm. Nevertheless, our results are robust to assuming that the board invests personal resources. Suppose the board continues to invest in learning at time 0. In this case, the board simply maximizes the value of the firm minus the personal resources it expends. As a result, the board invests less in the screening technology than in our model. However, the remaining insights of the model are unchanged.

If the board invests personal resources in screening at time 2, the effect on the manager’s behavior is unambiguous. Since the anticipated investment is no longer affecting firm value, the manager ignores its cost in deciding whether to fight or concede. Thus, a higher $\alpha$ strengthens the certification effect of having his decision upheld, and increases the low type manager’s incentive to fight.

7 Empirical Implications

At the broad level, our model captures the rich set of interactions among managers, activist investors and boards that is observed in practice. It predicts the circumstances in which there will be cooperation between managers and activist investors, fights between them, boards that are passive, boards that sometimes side with the manager and sometimes with the activist, and even heavy-handed boards that always side with activists. The comparative statics rely on the degree of agency conflict between managers and shareholders and the quality of information generated by the activist.

Specifically, we predict cooperation between activists and managers when the agency conflict between managers and shareholders is mild, and fights when the conflict is severe. Empirical proxies for the degree of the agency conflict include measures of CEO entrenchment such as very high managerial ownership and the presence of anti-takeover provisions, as well as the potential for waste as captured by measures such as free cash flow. The manager’s concern for his reputation is the specific driver of the agency conflict in our model. Reputational
concerns decline with age, so, holding managerial tenure constant, we expect to see a lower incidence of investor activism when the CEO is older.

The effect of managerial tenure is more complicated, with our model predicting an inverted-U relationship. A manager is reluctant to reverse his own prior decisions, but not those taken by previous management. Therefore, we expect to see fewer activist battles in firms that have recently changed CEOs, especially when the new CEO was hired from outside. If the manager has been in place long enough to make important strategic decisions, or has previously helped shape firm strategy as a senior insider, we expect to see more frequent investor activism campaigns. Finally, we expect that the incidence of activism will decline when a CEO has had a long tenure, since his reputation is then well-established. Thus the predicted relationship between investor activism and managerial tenure, holding age constant, has an inverted-U shape.

In our model, board intervention also depends on the agency conflict between managers and shareholders. When a manager is well-established, the agency conflict is mild, and boards will largely be passive both in their investigations of the CEO and in overruling CEO decisions. The former might be observed in fewer board meetings, lower attendance in board meetings, board members who live far from the company’s headquarters, and board members who are more likely to play an advisory role and less likely to behave in an adversarial way.

The quality of an activist’s information will depend on several factors. Outsiders are likely to have better information when the firm’s strategic options are straightforward (e.g., expand vs. contract, focus vs. diversify, grow domestically vs. expand internationally) and they have prior experience in the same industry. Here, if the agency conflict is severe, we expect frequent contests with interventionist boards. However, our model also predicts that the board will not invest in screening the manager. A board in this case is likely to have a fairly cool relationship with management, having few interactions and frequently overruling managerial decisions when confronted by activist investors.

Conversely, if the firm’s strategic options are complex or technical (e.g., which technology to adopt or how to structure the firm to be more innovative), outsiders are likely to generate imprecise information. In this case, however, boards will interact a lot with the manager. The level of board activity should first fall and then, beyond a point, increase with the precision of outsider information, leading to a U-shaped relationship. Further, the outcome of investor activism campaigns will be mixed, with the board sometimes siding with management and sometimes siding with the activist.

The above discussion focuses on predictions that emerge from equilibrium outcomes in our model. Off equilibrium, suppose that board members have a need to demonstrate their engagement with the firm. Then, if the agency conflict is moderate, a more active board should result in a larger number of activist campaigns, and fewer cases in which the manager cooperates with the activist. Similarly, if the board adds a member who is known for a high
level of involvement, the manager is likely to become more stubborn in confronting outsiders.

Finally, while we focus on the board as the source of internal governance, one can imagine settings in which the board is ineffectual and internal governance is supplied primarily through shareholder votes. The SEC has recently proposed rule changes that will make it easier for dissident investors to have their own directorial nominees included in proxy materials sent to shareholders. In principle, this should make it easier for activist investors to have managerial decisions overruled. However, such proxy contests tend to increase scrutiny of managers. Our model suggests that, as a result, managers are more likely to fight proposals by outside investors. Thus, in response to the new rules, we predict not only an increase in the incidence of proxy contests, but also a decrease in cooperation between management and activist investors.

8 Conclusion

We examine optimal internal corporate governance when a manager is concerned about his reputation and faces potential discipline from the market for corporate control, where the latter is represented by an activist shareholder. We show that the optimal policy of the board depends both on the potential for agency conflict and the strength of external governance. It is immediate that when the agency conflict is minimal, the board does not need to act. However, the board also ignores a moderate agency conflict, even though the manager sometimes chooses a project that is sub-optimal for shareholders. In this situation, by increasing its effort on screening, the board can identify an incorrect project more often. However, an increased amount of screening exacerbates a low-type manager’s reputational concerns, and as a result leads to him choosing the sub-optimal project more often. In equilibrium, this leads to the board optimally choosing to be completely passive even when the manager is moderately conscious of his reputation.

As the manager becomes more conscious of his reputation, at some point the optimal level of governance shifts discontinuously. Informed governance by the board replaces passivity at this point. The board invests a finite amount in screening, and overturns the manager if it determines he is in the incorrect project. Finally, we show that the relationship between external and internal governance is non-monotone. When external governance is weak, the board needs to over-invest in internal governance, to induce the outsider to play a role. Beyond a point, external governance then becomes a complement to internal governance. When external governance is strong, the board relies completely on the outsider and adopts an interventionist policy.

In our model, the board plays a crucial role in deciding whether control over the firm’s strategy should rest with management or the outsider. Overall, therefore, our work points to a role for the board as an arbitrator in disputes between the managers and activist shareholders.
A Appendix: Proofs

Proof of Lemma 1

Consider the beliefs held by investors. Let \( \mu_c = \text{Prob}(\theta = \theta_H \mid s_M \neq s_E, \text{ Manager concedes}) \), \( \mu_o = \text{Prob}(\theta = \theta_H \mid s_M \neq s_E, \text{ Manager fights and board overrules}) \), and \( \mu_s = \text{Prob}(\theta = \theta_H \mid s_M \neq s_E, \text{ Manager fights and board allows manager’s project to stand}) \).

The left-hand side of the last inequality represents the net gain from fighting rather than conceding, with the first term being the cash flow component of the gain and the second term the reputation component.

(i) Suppose \( \gamma < 1 \). Further, suppose that \( \sigma_L > 0 \). Then, it follows that

\[
\beta(1 - p_L)(2\delta_L - 1) + (1 - \beta)[p_L\mu_o + (1 - p_L)\mu_s - \mu_c](\theta_H - \theta_L) \geq 0.
\]

Observe that \( \delta_L < \frac{1}{2} \), so it must be that \( p_L\mu_o + (1 - p_L)\mu_s > \mu_c \).

Consider the corresponding net gain to a high-type manager from fighting rather than conceding,

\[
\beta(1 - p_H)(2\delta_H - 1) + (1 - \beta)[p_H\mu_o + (1 - p_H)\mu_s - \mu_c](\theta_H - \theta_L).
\]

Since \( \delta_H > \frac{1}{2} > \delta_L \), the cash flow component is strictly higher for the \( \theta_H \) type. Also, \( p_L = \gamma + \alpha(1 - \gamma) \geq p_H = \gamma \), with strict inequality if \( \alpha > 0 \). It therefore follows that \( \mu_s \geq \mu_o \), with strict inequality if \( \alpha > 0 \). Therefore the reputational component of the net gain from fighting is at least as large for the high-type manager as for the low-type manager. Therefore,

\[
\beta(1 - p_H)(2\delta_H - 1) + (1 - \beta)[p_H\mu_o + (1 - p_H)\mu_s - \mu_c](\theta_H - \theta_L) > 0,
\]

and it is a strict best response for the \( \theta_H \) type manager to fight. That is, if \( \sigma_L > 0 \), then \( \sigma_H = 1 \). Hence, it must be that either \( \sigma_H = 1 \), or \( \sigma_L = 0 \) and \( \sigma_H = 0 \).

Now, suppose \( \sigma_L = 0 \) and \( \sigma_H > 0 \). Then, Bayes’ rule implies that \( \mu_s = \mu_o = 1 \); that is, if the manager fights, investors believe he has type \( \theta_H \). It follows that both the cash flow and reputational components of the net gain from fighting strictly exceed zero for the high-type
manager, so it must be that $\sigma_H = 1$.

Therefore, in equilibrium, either $\sigma_H = 1$ or $\sigma_L = \sigma_H = 0$.

(ii) Suppose $\gamma = 1$. Then, $p_L = p_H = 1$. For both types of manager, the net gain from fighting rather than conceding is $(1 - \beta)(\mu_o - \mu_c)(\theta_H - \theta_L)$. If $\mu_o > \mu_c$, then it must be that $\sigma_H = \sigma_L = 1$, whereas if $\mu_o < \mu_c$, then $\sigma_H = \sigma_L = 0$. Consider the case $\mu_o = \mu_c$. Suppose $\sigma_H \neq \sigma_L$. Then, both the “fight” and “concede” information sets are reached in equilibrium. Applying Bayes’ rule, $\mu_o = \mu_c$ if and only if $\sigma_H = \sigma_L$, contradicting the conjecture that $\sigma_H \neq \sigma_L$.

**Proof of Proposition 1**

Since only the high-type manager fights, it is a best response for the board to not overturn the manager regardless of whether he fights or concedes. Bayes’ rule implies that $\mu_s = 1$ and $\mu_c = 0$. For the manager of type $\theta_i$, the net gain from fighting rather than conceding is thus

$$
\beta(2\delta_i - 1) + (1 - \beta)(\theta_H - \theta_L).
$$

As $\delta_H > 1$, the above expression is strictly positive for the high-type manager, who therefore fights with probability one. It is a best response for low-type manager to concede if the expression is weakly negative; i.e., if

$$
\beta(1 - 2\delta_L) \geq (1 - \beta)(\theta_H - \theta_L),
$$

or $\beta \geq \beta_s(\psi)$.

**Proof of Proposition 2**

In the conjectured equilibrium, the “concede” information set is reached with probability zero. Assign the belief $\mu_c = 0$ at this information set to both investors and the board. That is, if the manager concedes, investors and the board both believe he has the low type with probability 1. Note that the board observes its signal (which investors do not) and this belief is consistent with both signal outcomes. Observe that $\gamma = 0$, so that $p_H = 0$ and $p_L = \alpha$. Further, $\mu_o = 0$ (investors know the manager has the low type whenever he is overruled) and $\mu_s = \frac{q\lambda_H}{q\lambda_H + (1-q)\lambda_L} > \mu_c$.

Now, for the high-type manager, the cash flow component of the net gain from fighting is $\beta(2\delta_H - 1) > 0$, and the reputational component is $(1 - \beta)(\mu_s - \mu_c)(\theta_H - \theta_L) > 0$. Therefore, he fights with probability one. For the low-type manager, it is a best response to set $\sigma_L = 1$. 

29
if and only if
\[
\beta(1 - \alpha)(2\delta_L - 1) + (1 - \beta) \left[ \frac{(1 - \alpha)q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L} \right] (\theta_H - \theta_L) \geq 0, \quad (10)
\]
or \(\beta \leq \beta_\ell(\alpha, \psi)\).

Next, consider the best response of the board. Suppose the manager fights. Since \(\gamma = 0\), when the board obtains signal \(H\), it does not overturn the manager. Therefore, investors and the board have the same beliefs over the type of the manager. That is, in this case, the board’s beliefs are also represented by \(\mu_s = \frac{q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L}\). The cost of screening, \(c(\alpha)\) is sunk at time 2 and can be ignored. If the board overturns the manager, it obtains a payoff \((1 - \mu_s)(1 - \delta_L) + \mu_s(1 - \delta_H)\). If it does not overturn, it obtains a payoff \((1 - \mu_s)\delta_L + \mu_s\delta_H\). It is then a best response for the board to set \(\gamma = 0\) if and only if
\[
(1 - \mu_s)(2\delta_L - 1) + \mu_s(2\delta_H - 1) \geq 0. \quad (11)
\]
Substituting in for the value of \(\mu_s\), the board should set \(\gamma = 0\) if and only if
\[
\frac{q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L} \geq \frac{1 - 2\delta_L}{2(\delta_H - \delta_L)},
\]
or,
\[
q\lambda_H(2\delta_H - 1) \geq (1 - q)(1 - \alpha)\lambda_L(1 - 2\delta_L). \quad (12)
\]
Now, note that for each \(i = H, L\), \(\lambda_i = \theta_i(1 - \psi) + \psi(1 - \theta_i)\) and \(\delta_i = \frac{\theta_i(1 - \psi)}{\lambda_i}\). Hence, it follows that \(\lambda_H(2\delta_H - 1) = \theta_H - \psi\), and \(\lambda_L(1 - 2\delta_L) = \psi - \theta_L\). Making these substitutions, the inequality (12) may be re-written as
\[
q(\theta_H - \psi) \geq (1 - q)(1 - \alpha)(\psi - \theta_L),
\]
or,
\[
\psi \leq \frac{q\theta_H + (1 - q)(1 - \alpha)\theta_L}{q + (1 - q)(1 - \alpha)} = \psi_f(\alpha).
\]
Hence, it is optimal for the board to set \(\gamma = 0\) if and only if \(\psi \leq \psi_f(\alpha)\).

Finally, if the manager concedes, given the board’s belief that he has the low type, it is optimal for the board to allow the concession to stand. \(\blacksquare\)

**Proof of Proposition 3**

(i) Since \(\sigma_H = 1\) and \(\sigma_L \in (0, 1)\), Bayes’ rule implies that \(\mu_c = 0\). That is, if the manager concedes, investors and the board both recognize him to have the low type.

Now, consider the high-type manager. As in the proof of Proposition 2, both the cash flow and reputational component of payoff are strictly higher when he fights. Hence, \(\sigma_H = 1\).

Next, consider the low-type manager. Since \(\gamma = 0\), \(p_L = \alpha\). Further, \(\mu_s = \frac{q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L}\).
It is a best response for him to mix between fighting and conceding if and only if
\[
\beta(1 - \alpha)(2\delta_L - 1) + (1 - \beta) \left[ \frac{(1 - \alpha)q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L\sigma_L} \right] (\theta_H - \theta_L) = 0. \tag{13}
\]
Simplifying and solving for \( \sigma_L \), we obtain
\[
\sigma_L = \frac{q\lambda_H}{(1 - \alpha)(1 - q)\lambda_L} \left[ \frac{1 - \beta}{\beta} \frac{\theta_H - \theta_L}{1 - 2\delta_L} - 1 \right]. \tag{14}
\]
Therefore, \( \sigma_L > 0 \) requires \( \beta < \beta_b(\psi) \), and \( \sigma_L < 1 \) requires \( \beta > \frac{1}{1 + \frac{2\delta_L}{\theta_L - \theta_H}} \). Substituting in \( \mu_s = \frac{q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L\sigma_L} \), the above inequality holds if and only if \( \beta \geq \frac{1}{1 + \frac{2\lambda_H}{\theta_L - \theta_H}} = \beta_b \). Hence, if \( \beta > \beta_b(\psi) \), the board maximizes its payoff by setting \( \gamma = 0 \).

Therefore, if \( \beta > \max\{\beta_b(\alpha, \psi), \beta_b(\psi)\} \) and \( \beta < \beta_b(\psi) \), a hybrid equilibrium exists in the continuation game, with \( \sigma_H = 1, \sigma_L \in (0, 1) \), and \( \gamma = 0 \).

(ii) Consider the expression for \( \sigma_L \) in equation (14). It is immediate that as \( \alpha \) increases, \( \sigma_L \) increases as well. Also, note that \( \frac{\partial}{\partial \psi} \frac{\lambda_H}{\sigma_L} = -\frac{2\psi(1 - \theta_L)(\theta_H - \theta_L)}{[\theta_L(1 - \psi) + \psi(1 - \theta_L)]^2} \), which is negative, and that \( 1 - 2\delta_L \) increases with \( \psi \). It follows that, as \( \psi \) increases, \( \sigma_L \) also increases. \[\Box\]

**Proof of Proposition 4**

Suppose there is an equilibrium in which \( \gamma = 1 \). From Lemma 1, it must be that \( \sigma_H = \sigma_L \). Further, as in the proof of Lemma 1 part (ii), the net gain to each type of manager from fighting is \( (1 - \beta)(\mu_o - \mu_e)(\theta_H - \theta_L) \). If \( \sigma_H = \sigma_L \in (0, 1) \), both information sets “concede” and “fight” are reached. It follows that \( \mu_e = \mu_o \); that is, the investors have the same beliefs over type when the manager concedes and when he fights and is overruled. Therefore, each type of manager is indifferent between fighting and conceding, so any \( \sigma_H = \sigma_L \) strictly between 0 and 1 is a best response. If \( \sigma_H = \sigma_L = 1 \), assign \( \mu_e = \mu_o = q \). It follows again that it is a best response for each type of manager to fight with probability one.

Now, when the manager fights and the board receives signal \( H \), the probability it ascribes to the manager having the high type is \( \mu_f(\alpha) = \frac{q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L} \). Modifying the proof of Proposition 2, it should set \( \gamma = 1 \) if and only if \( \mu_f(\alpha) \leq \frac{1}{1 + \frac{2\delta_L}{\theta_H - \theta_L}} \); that is, if and only if \( \psi \geq \psi_f(\alpha) \). Observe that under exactly the same condition, the board should not overturn when the manager concedes and it obtains signal \( H \). Of course, it remains a best response for
the board to overturn the manager if he fights and it obtains signal $L$, and to not overturn if he concedes and it obtains signal $L$.

**Proof of Lemma 2**

Let $z$ denote the posterior probability the manager has type $\theta_H$, conditional on $s_M \neq s_E$. Then, $z = \frac{qL}{qH + (1-q)\lambda L}$. Suppose the continuation equilibrium at time 2 is $(\sigma, \gamma)$. Then, the expected payoff of the board is the expected cash flow from the project less the cost of the screening procedure, $c(\alpha)$. That is,

$$
P = z[\gamma(1 - \delta_H) + (1 - \gamma)\delta_H] + (1 - z)[(1 - \delta_L) - \sigma_L(1 - \alpha)(1 - \gamma)(1 - 2\delta_L)] - c(\alpha).
$$

Suppose $\psi \leq \psi_f(\alpha)$ and $\beta > \beta_s$. Then, from Propositions 1 and 4, both a separating equilibrium and a pooling equilibrium with sledgehammer governance exist. In the separating equilibrium, $\sigma_L = 0$ and $\gamma = 0$. Thus, the board’s payoff is

$$
\tilde{P} = z\delta_H + (1 - z)(1 - \delta_L) - c(\alpha).
$$

In the sledgehammer equilibrium, $\sigma_L = 1$ and $\gamma = 1$. Thus, the board’s payoff is

$$
\hat{P} = z(1 - \delta_H) + (1 - z)(1 - \delta_L) - c(\alpha).
$$

Now, $\bar{P} - \hat{P} = z(2\delta_H - 1) > 0$, since $\delta_H > 1/2$.

Now suppose that $\psi > \psi_f(\alpha)$ and $\beta > \beta_h$. Then, from Propositions 3 and 4, both the hybrid and sledgehammer equilibria exist. In this equilibrium, $\gamma = 1$, and the exact expression for $\sigma_L$ is shown in equation (14). From equation (14), $\sigma_L(1 - \alpha) = \frac{qL}{(qH + (1-q)\lambda L)} \left[ \frac{1 - \beta}{\beta} \theta_H - \theta_L - (1 - 2\delta_L) \right]$. Therefore, the board’s payoff from the hybrid equilibrium is

$$
\hat{P} = z\delta_H + (1 - z)(1 - \delta_L) + z\left[ \frac{1 - \beta}{\beta} \theta_H - \theta_L - (1 - 2\delta_L) \right] - c(\alpha).
$$

Subtracting the board’s expected payoff in the equilibrium with sledgehammer governance, we have $\hat{P} - \tilde{P} = z[2(\delta_H - \delta_L) - \frac{1 - \beta}{\beta} (\theta_H - \theta_L)]$. It follows that the condition $\hat{P} \geq \tilde{P}$ is equivalent to the condition $\beta \geq \beta_h(\psi)$.

**Proof of Lemma 3**

We first prove the following claim.

**Claim**: Suppose the activist enters, and, in the continuation equilibrium, the project favored by the manager’s signal is undertaken with probability $p$ whenever $s_E \neq s_M$. Then, the expected cash flow from the project before the activist observes her signal is $p\theta_i + (1 - p)\psi$.

**Proof of Claim**:
There are two cases to consider. First, the signal of the activist investor agrees with the manager’s signal with probability $1 - \lambda_i = \theta_i \psi + (1 - \theta_i)(1 - \psi)$. If $s_M = s_E = Y \in \{A, B\}$, the true state is $x_Y$ with conditional probability $\frac{\theta_i \psi}{1 - \lambda_i}$. Hence, the expected cash flow in this case is $\frac{\theta_i \psi}{1 - \lambda_i}$.

Next, suppose $s_E \neq s_M$. This event occurs with probability $\lambda_i = \theta_i(1 - \psi) + \psi(1 - \theta_i)$. If the project favored by the manager’s signal is undertaken, the expected cash flow is $\delta_i = \frac{\theta_i(1 - \psi)}{\lambda_i}$. If the project favored by the outsider’s signal is undertaken, the expected cash flow is $1 - \delta_i = \frac{\psi(1 - \theta_i)}{\lambda_i}$.

Now, when $s_E \neq s_M$, the project favored by the manager’s signal is undertaken with probability $p$. Thus, before the activist observes her signal, the expected cash flow from the project is

$$(1 - \lambda_i) \frac{\theta_i \psi}{1 - \lambda_i} + \lambda_i \frac{p \theta_i (1 - \psi) + (1 - p) \psi (1 - \theta_i)}{\lambda_i} = p \theta_i + (1 - p) \psi.$$

Now, we return to the proof of the Lemma. Suppose the manager has type $\theta_H$ and $s_M \neq s_E$. The high-type manager fights with probability 1, and the project favored by the manager’s signal is undertaken with probability $1 - \gamma$. Hence, the expected cash flow from the project is $(1 - \gamma) \theta_H + \gamma(\psi) = \theta_H - \gamma(\theta_H - \psi)$.

Next, suppose the manager has type $\theta_L$ and $s_M \neq s_E$. With probability $1 - \sigma_L$, the manager concedes, and the project favored by the outsider’s signal is undertaken. With probability $\sigma_L$, he fights, and is overturned with probability $\alpha + (1 - \alpha) (1 - \gamma)$. Hence, with cumulative probability $1 - \sigma_L + \sigma_L \{\gamma + \alpha (1 - \gamma)\}$, the project favored by the outsider’s signal is undertaken. Note that this probability may be written as $1 - \sigma_L \{1 - \gamma - \alpha (1 - \gamma)\}$. With probability $\sigma_L \{1 - \gamma - \alpha (1 - \gamma)\}$ the project favored by the manager’s signal is undertaken. Thus, the expected cash flow from the project is $\psi - \sigma_L \{1 - \gamma - \alpha (1 - \gamma)\} (\psi - \theta_L)$.

Hence, the overall expected cash flow from the project is

$$F = q[\theta_H - \gamma(\theta_H - \psi)] + (1 - q)[\psi - \sigma_L \{1 - \gamma - \alpha (1 - \gamma)\} (\psi - \theta_L)].$$

Proof of Proposition 5

The improvement in expected cash flow following the outsider’s intervention is

$$F - F_0 = -q \gamma (\theta_H - \psi) + (1 - q) [1 - \sigma_L (1 - \alpha) (1 - \gamma)] (\psi - \theta_L).$$

Using the appropriate values of $\gamma$ in each case, it follows that the cash flow improvement if the outsider intervenes is $\Delta_S(\psi) = (1 - q)(\psi - \theta_L)$ if the continuation equilibrium is separating,
Therefore, the numerator of the payoff in any other equilibrium is lower, the outsider stays out for all values of β when ψ < ψ₁.

Next, suppose ψ ∈ [ψ₁, ψ₂]. We first show that φ(ψ) = \frac{1}{1+\frac{1-2δ_L}{θ_H-θ_L}+\frac{(1-q)(ψ-θ_L)-κ}{qλH(θ_H-θ_L)}} is strictly decreasing in ψ when ψ ≥ ψ₁. Consider the denominator of φ(ψ). Since δₗ is decreasing in ψ, it follows that \frac{1-2δ_L}{θ_H-θ_L} is increasing in ψ. Denote the third term in the denominator as Z = \frac{(1-q)(ψ-θ_L)-κ}{qλH(θ_H-θ_L)}. Then, \frac{∂Z}{∂ψ} is decreasing. Consider the numerator: λ_H(1-q) > 0, and 1 - 2θ_H < 0. Further, recall that ψ₁ = θ_L + \frac{κ}{1-q}. Hence, if ψ ≥ ψ₁, it follows that (1-q)(ψ-θ_L) - κ ≥ 0. Therefore, the numerator of \frac{∂Z}{∂ψ} is strictly positive, and hence Z is strictly increasing in ψ. Hence, the denominator of φ(ψ) is strictly increasing in ψ whenever ψ ≥ ψ₁. It follows that φ(ψ) is strictly decreasing in ψ over the same range.

Now, observe that φ(ψ₁) = \frac{1}{1-\frac{1-2δ_L}{θ_H-θ_L}} = φₙ(ψ₁). By inspection, φ(ψ) < φₙ(ψ₁) when ψ > ψ₁. Recall that βₗ(α, ψ) = \frac{1}{1+\frac{1-2δ_L}{θ_H-θ_L}+\frac{(1-α)(1-q)λ_L(1-2δ_L)}{qλH(θ_H-θ_L)}}. Now, λ_L(1-2δ_L) = ψ - θ_L, so that we can write βₗ(α, ψ) = \frac{1}{1+\frac{1-2δ_L}{θ_H-θ_L}+\frac{(1-α)(1-q)λ_L}{qλH(θ_H-θ_L)}}. Therefore, the condition φ(ψ) > βₗ(α, ψ) is equivalent to (1-q)(ψ-θ_L) - κ > (1-α)(1-q)(ψ-θ_L), or ψ < θ_L + \frac{κ}{α(1-q)} = ψ₂. Also, it follows that φ(ψ₂) = βₗ(α, ψ₂).

Finally, the condition κ < \frac{αq(1-q)(θ_H-θ_L)}{q+1-q(1-α)} is equivalent to ψ₂ < ψₚ(α). Further, it is straightforward to show that ψ < ψₚ(α) in turn implies that βₗ(α, ψ) > βₗ(α, ψ₂) (for α < ψ₂). Therefore, for ψ ∈ (ψ₁, ψ₂), φ(ψ) lies between βₗ(α, ψ) and max{βₗ(α, ψ), βₗ(α, ψ₂)}. It follows from Proposition 3 part (i) that for any ψ in this range, if β = φ(ψ), a hybrid equilibrium is played in the continuation game.

Consider the payoff improvement the outsider can expect from this hybrid equilibrium. We have ∆ₚ(ψ, α, σₐ) = (1-q)[1-σₐ(1-α)](ψ-θ_L), where σₐ is as defined in equation (14). When β = φ(ψ), \frac{1-β}{β} = \frac{1-2δ_L}{θ_H-θ_L} + \frac{(1-q)(ψ-θ_L)-κ}{qλH(θ_H-θ_L)}. Hence,

\[
\frac{1-β}{β} = \frac{1-2δ_L}{θ_H-θ_L} - 1 = \frac{(1-q)(ψ-θ_L)-κ}{qλH(1-2δ_L)}.
\]

Further, note that λ_L(1-2δ_L) = ψ - θ_L. Therefore, σₐ = \frac{(1-q)(ψ-θ_L)-κ}{(1-α)(1-q)(ψ-θ_L)}, so that [1-(1-α)σₐ] = \frac{κ}{(1-q)(ψ-θ_L)}.

Hence, ∆ₚ(ψ, α, σₐ) = (1-q)(ψ-θ_L)[1-(1-α)σₐ] = κ. That is, if ψ ∈ (ψ₁, ψ₂) and...
\[ \beta = \phi(\psi), \text{ the payoff improvement resulting from the outsider's intervention is exactly } \kappa. \]

Hence, the outsider is exactly indifferent between intervening and not.

Now, keeping \( \psi \) fixed, consider an increase in \( \beta \). From equation (14), \( \sigma_L \) declines in \( \beta \). Hence, \( \Delta_Y(\psi, \alpha, \sigma_L) \) increases as \( \beta \) increases. Therefore, for any \( \beta > \phi(\psi) \), if a hybrid equilibrium is played in the continuation game at time 2, the outsider strictly prefers to enter. If \( \sigma_L \) declines to zero, a separating equilibrium is played in the continuation game. Since \( \Delta_S(\psi) > \Delta_Y(\psi, \alpha, \sigma_L) \) for all \( \sigma_L > 0 \), the outsider again strictly prefers to enter.

Finally, consider \( \psi > \psi_2(\alpha) \). The condition \( \kappa < \frac{aq(1-q)(\theta_H - \theta_L)}{q+(1-q)(1-\alpha)} \) is equivalent to \( \psi < \psi_f(\alpha) \). Suppose first that \( \psi \in (\psi_2, \theta_f) \). Then, it is possible that, if \( \beta \) is sufficiently low, a pooling equilibrium with informed governance is played in the continuation game at time 2. The payoff improvement if the outsider intervenes is then \( \Delta_I(\psi, \alpha) = \alpha(1-q)(\psi - \theta_L) \).

If \( \psi > \psi_2 = \theta_L + \frac{\kappa}{\alpha(1-q)} \), it follows that \( \Delta_I(\psi, \alpha) > \kappa \), and the outsider strictly prefers to enter. Suppose, instead, that a hybrid equilibrium is played, the outsider strictly prefers to enter. Finally, it follows that if a separating equilibrium is played, since \( \theta > \psi_1 \), the outsider strictly prefers to enter.

Next, suppose that \( \psi > \psi_1(\alpha) \). If \( \beta \) is sufficiently low, a pooling equilibrium with sledgehammer governance is played in the continuation game at time 2. The payoff improvement if the outsider intervenes is then \( \Delta_I(\psi, \alpha) = \alpha(1-q)(\psi - \theta_L) \). Evaluating this expression at \( \psi = \psi_1 \), we have \( \Delta_I(\psi_1) = \frac{aq(1-q)(\theta_H - \theta_L)}{q+(1-q)(1-\alpha)} > \kappa \). Since \( \Delta_I(\psi) \) is strictly increasing in \( \psi \), the outsider strictly prefers to enter at all \( \psi > \psi_f(\alpha) \).

On the other hand, if \( \beta \) is high enough that a hybrid equilibrium results, since \( \beta_H(\psi) > \beta_L(\alpha, \psi) > \phi(\psi) \), the outsider again strictly prefers to enter. Finally, it follows that if a separating equilibrium is played, since \( \theta > \psi_1 \), the outsider strictly prefers to enter. \[ \blacksquare \]

**Proof of Proposition 6**

In a hybrid equilibrium \( \gamma = 0 \), so the board’s payoff may be written as

\[ \Pi(\alpha) = F - c(\alpha) = q\theta_H + (1-q)\psi - (1-q)(1-\alpha)\sigma_L(\psi - \theta_L) - c(\alpha). \] (20)

From the expression for \( \sigma_L \) in equation (14), it follows that the term \( (1-\alpha)\sigma_L \) is a constant that does not depend on \( \alpha \). It is immediate that the derivative with respect to \( \alpha \) is \( \Pi'(\alpha) = -c'(\alpha) < 0 \). \[ \blacksquare \]

**Proof of Lemma 4**

In a pooling equilibrium with informed governance, \( \sigma_H = \sigma_L = 1 \) and \( \gamma = 0 \). Substituting these into the expression for \( F \) in Lemma 3, the payoff of the board in this equilibrium may
be written as
\[ \Pi(\alpha) = \theta H + (1-q)\psi - (1-q)(1-\alpha)(\psi - \theta L) - c(\alpha). \] (21)

The first-order condition with respect to \( \alpha \) is
\[ c'(\alpha) = (1-q)(\psi - \theta L), \] (22)
and since \( c(\cdot) \) is convex, the second-order condition is satisfied. Hence, if the board anticipates a pooling equilibrium with informed governance at time 2, it should set \( \alpha = \alpha_c \). Its expected payoff is then
\[ \Pi(\alpha_c) = \theta H + (1-q)\psi - (1-q)(1-\alpha_c)(\psi - \theta L) - c(\alpha_c). \] (23)

Now, suppose the board anticipates sledgehammer governance at time 2. It should optimally set \( \alpha = 0 \). Its expected payoff is then
\[ \tilde{\Pi}(0) = \theta H + (1-q)\psi = \psi. \] (24)

Comparing the two payoffs, the board strictly prefers to set \( \alpha = \alpha_c \) and conduct informed governance when \( \theta < \psi_f(\alpha) \).

Proof of Lemma 5

In a pooling equilibrium with informed governance, \( \sigma_L = 1 \) and \( \gamma = 0 \). Thus, the cash flow improvement following the outsider’s intervention is \( (1-q)\alpha(\psi - \theta L) \). For the outsider to intervene, this expression must be weakly greater than \( \kappa \); i.e., \( \alpha \geq \frac{\kappa}{(1-q)(\psi - \theta L)} = \alpha_c \).

Proof of Proposition 7

We begin the proof with three preliminary steps.

Preliminary step 1: Board payoff expressions

For a fixed value of \( \alpha \), the payoff to the board in each of the different possible equilibria is denoted as follows: In a no-governance equilibrium, it earns a payoff \( F_N(\alpha) = \theta H + (1-q)\theta L - c(\alpha) \), in a separating equilibrium it earns \( F_S(\alpha, \psi) = \theta H + (1-q)\psi - c(\alpha) \), in a pooling equilibrium with informed governance it earns \( F_I(\alpha, \psi) = \theta H + (1-q)\theta L + (1-q)\alpha(\psi - \theta L) - c(\alpha) \), in a pooling equilibrium with sledgehammer governance it earns \( F_G(\alpha, \psi) = \psi - c(\alpha) \), and in a hybrid equilibrium it earns \( F_Y(\alpha, \psi) = \theta H + (1-q) \left[ \psi - \frac{\theta H - \theta L}{1-q} \psi L \left\{ \frac{1-\beta}{\beta} \frac{\theta H - \theta L}{1-2\theta L} - 1 \right\} \right] - c(\alpha) \). Observe that in every equilibrium in which the outsider enters, the payoff is strictly increasing in \( \psi \).
**Preliminary step 2: Threshold ψ values**

Define \( \psi_c \) as the value of \( \psi \) at which \( \alpha_c(\psi) = \alpha_c(\psi) \); i.e., as the solution to \( \psi = \theta_L + \frac{\kappa}{(1-q)\alpha_c(\psi)} \). Further, define \( \psi_d \) as the value of \( \psi \) at which \( \phi(\psi) = \beta_c(\psi) \); i.e., as the solution to \( \psi = \theta_L + \frac{\kappa + c(\alpha_c(\psi))}{(1-q)\alpha_c(\psi)} \). We show that the following ordering holds: \( \psi_1 < \psi_3 < \psi_c < \psi_d < \psi_g \).

\[ \psi_1 < \psi_3: \text{Recall that } \psi_1 = \theta_L + \frac{\kappa}{1-q} \text{ and } \psi_3 = \theta_L + \frac{\kappa}{(1-q)c^{-1}(\kappa)}. \text{ We have } c^{-1}(\kappa) < 1 \text{ since } \kappa < \infty, \text{ so } \psi_1 < \psi_3. \]

\[ \psi_3 < \psi_c: \text{ We first show that } \alpha_c(\psi_3) > \alpha_c(\psi_3). \text{ Recall that } \psi_3 = \theta_L + \frac{\kappa}{(1-q)c^{-1}(\kappa)}. \text{ Denote } c^{-1}(\kappa) = x; \text{ then } \kappa = c(x). \text{ The condition } \alpha_c(\psi_3) > \alpha_c(\psi_3) \text{ is then equivalent to } x > c^{-1}\left(\frac{c(x)}{x}\right), \text{ or } c'(x) > \frac{c(x)}{x}, \text{ which holds by convexity of } c(\cdot). \text{ Hence, } \alpha_c(\psi_3) > \alpha_c(\psi_3). \]

\[ \psi_c < \psi_d: \text{ This follows directly from the fact that } c(\alpha_c(\psi)) > 0. \]

\[ \psi_d < \psi_g: \text{ From the definition of } \psi_d, \text{ it follows that } (1-q)(\psi_d - \theta_L)\alpha_c(\psi_d) - c(\alpha_c(\psi_d)) = \kappa. \text{ Similarly, from the definition of } \psi_g, \text{ we have } (1-q)(\psi_g - \theta_L)\alpha_c(\psi_g) - c(\alpha_c(\psi_g)) = \psi_g - q\theta_H - (1-q)\theta_L = \kappa_2. \text{ Since } \kappa < \kappa_2, \text{ it is immediate that } (1-q)(\psi_d - \theta_L)\alpha_c(\psi_d) - c(\alpha_c(\psi_d)) < (1-q)(\psi_g - \theta_L)\alpha_c(\psi_g) - c(\alpha_c(\psi_g)). \text{ Now, recall that } c'(\alpha_c(\psi)) = (1-q)(\psi - \theta_L) \text{ for each } \psi. \]

Thus, the function \( (1-q)(\psi - \theta_L)\alpha_c(\psi) - c(\alpha_c(\psi)) \) is strictly increasing in \( \psi \) and \( \psi_d < \psi_g \).

**Preliminary step 3: Relationship between \( \beta_m \) and \( \psi \)**

Recall that \( \phi(\psi_d) = \beta_c(\psi_d) \). It is straightforward to show that \( \phi(\psi) < \beta_c(\psi) \) if and only if \( \psi < \psi_d \). Similarly, we can show that \( \beta_c(\psi) < \beta_b(\psi) \) if and only if \( \psi < \psi_b \), with equality when \( \psi = \psi_g \). Therefore,

\[
\beta_m = \begin{cases} 
\phi(\psi) & \text{if } \psi \leq \psi_d \\
\beta_c(\psi) & \text{if } \psi \in (\psi_d, \psi_g) \\
\beta_b(\psi) & \text{if } \psi \geq \psi_g 
\end{cases}
\]

(25)

Having established these preliminary results, we now prove each part of the proposition.

**Proof of part (i)**

Suppose first that \( \psi < \psi_1 \). Then, as shown in the proof of Proposition 5, the outsider stays out, regardless of the value of \( \alpha \) chosen by the board. It is then optimal for the board to allow the project of even the low-ability manager to stand, so it chooses \( \alpha = 0 \). Hence, there is no governance in this region.

Next, suppose \( \psi \in (\psi_1, \psi_3) \) and \( \beta < \phi(\psi) \). In this region, the board can induce the outsider to enter by choosing \( \alpha = \alpha_c \). We show that the board instead prefers the no-governance outcome.

Suppose the board chooses an \( \alpha \) such that the outsider enters. Since \( \phi(\psi) < \beta_\sigma(\psi) \) for each value of \( \psi \), the equilibrium in the continuation game cannot exhibit separation; instead,
either a hybrid or pooling equilibrium must obtain. As shown in the proof of Proposition 5, if a hybrid equilibrium results in the continuation game and $\beta < \phi(\psi)$, the outsider will not enter regardless of the value of $\alpha$.

The only other possibility in which the outsider may enter is that there is a pooling equilibrium in the continuation game. We first show that the pooling equilibrium must exhibit informed governance, and then argue that the board is better off with no governance.

**Step 1** In this parameter region, a pooling equilibrium must exhibit informed governance.

Consider the equation that defines $\kappa_1$, $\kappa = c \left( \frac{\kappa}{q(1-q)(\theta_H - \theta_L)} \right)$. The left-hand side is linear in $\kappa$, and the right-hand side is strictly convex. Hence, if $\kappa < \kappa_1$, it follows that $\kappa > c \left( \frac{\kappa}{q(1-q)(\theta_H - \theta_L)} \right)$, or $q(\theta_H - \theta_L) > \frac{\kappa}{(1-q)c^{-1}(\kappa)}$. Adding $\theta_L$ to both sides, we have $\psi_f(0) > \psi_3$. From Proposition 4, we know that a pooling equilibrium with sledgehammer governance exists only if $\psi \geq \psi_f(\alpha)$. However, $\psi_f(\alpha)$ is strictly increasing in $\alpha$. Hence, $\psi_3 < \psi_f(\alpha)$ for any $\alpha \geq 0$, and for $\psi < \psi_3$, any pooling equilibrium must exhibit informed governance.

**Step 2** The board prefers no governance.

The board must choose $\alpha \geq \alpha_e = \frac{\kappa}{(1-q)(\psi - \theta_L)}$ to induce the outsider to enter in this parameter region. Hence, the difference in payoffs between a pooling equilibrium with informed governance and the no-governance outcome is $F_I(\alpha_e, \psi) - F_N(0) = \kappa - c \left( \frac{\kappa}{(1-q)(\psi - \theta_L)} \right)$. Evaluating this last expression at $\psi = \psi_3 = \frac{\kappa}{(1-q)c^{-1}(\kappa)}$, we have $F_I(\alpha_e, \psi_3) - F_N(0) = 0$. That is, at $\psi = \psi_3$, the board is indifferent between a pooling equilibrium with $\alpha = \alpha_e(\psi)$ and a no-governance outcome with $\alpha = 0$.

By inspection, $F_I(\alpha_e(\psi), \psi) - F_N(0)$ is strictly increasing in $\psi$, so for any $\psi < \psi_3$, it follows that $F_I(\alpha_e(\psi), \psi) < F_N(0)$. That is, the board strictly prefers no governance to a pooling equilibrium with $\alpha = \alpha_e$. Since $\alpha_e > \alpha_c$, which is the optimal value of $\alpha$ conditional on the outsider entering, it follows that the board earns a higher profit in the pooling equilibrium with informed governance when it chooses $\alpha = \alpha_e$ rather than any value strictly greater than $\alpha_e$. Therefore, $F_I(\alpha, \psi_3) - F_N(0) < 0$ for any $\alpha > \alpha_e(\psi)$. Hence, the board prefers the no-governance outcome to any pooling equilibrium with informed governance in which $\alpha \geq \alpha_e(\psi)$. The board then optimally chooses $\alpha = 0$, and the outsider stays out.

**Proof of part (ii)**

Next, we consider part (ii) of the proposition. Suppose that $\psi > \psi_1$ and $\beta > \phi(\psi)$. Then, from Proposition 5, the outsider enters.
First, suppose $\beta \geq \beta_s(\psi)$. Then, if the outsider enters, a separating equilibrium is played in the continuation game at $t = 2$. Hence, the efficient outcome is obtained without board intervention and it is optimal for the board to set $\alpha = 0$.

Next, suppose that $\beta \in [\beta_m(\psi), \beta_e(\psi))$. We proceed in three steps.

Step 1 If the board chooses $\alpha = 0$ and the outsider enters, a hybrid equilibrium obtains in the continuation game.

Suppose the board chooses $\alpha = 0$. Consider $\beta_c(\psi)$ in equation (8) and $\beta_c(\alpha, \psi)$ in equation (3), substituting $\alpha = 0$ into the latter equation. Then, it follows that $\beta_c(\psi) \geq \beta_c(0, \psi)$ if and only if $\alpha_c \geq \frac{c(\alpha_c)}{c^{\prime}(\alpha_c)}$. But the last inequality follows from the convexity of $c(\cdot)$. Hence, $\beta_c(\psi) \geq \beta_c(0, \psi)$.

Now, from the definition of $\beta_m$, it follows that $\beta_m(\psi) \geq \max\{\beta_c(0, \psi), \beta_b(\psi)\}$. Hence, if $\beta \in [\beta_m(\psi), \beta_s(\psi))$, the board chooses $\alpha = 0$ and the outsider enters, from Proposition 3, a hybrid equilibrium obtains in the continuation game. Since $\psi \geq \psi_1$, it follows from the proof of Proposition 5 that the outsider enters.

Step 2 If it anticipates a hybrid equilibrium, the board optimally chooses $\alpha = 0$.

From Proposition 6, it follows that $F_Y(\alpha, \psi) = -c^\prime(\alpha) < 0$. Therefore, if the board anticipates that a hybrid equilibrium it chooses $\alpha = 0$.

Step 3 For any fixed value of $\alpha$, the board prefers a hybrid to a pooling equilibrium.

The overall payoff to the board in a hybrid equilibrium may be written as $F_Y(\alpha) = q\theta_H + (1 - q)[1 - \sigma(1 - \alpha)](\psi - \theta_L) - c(\alpha)$. Hence, the difference in payoffs between a hybrid equilibrium and a pooling equilibrium with informed governance is $F_Y(\alpha) - F_I(\alpha) = (1 - q)(1 - \sigma)(1 - \alpha)(\psi - \theta_L) > 0$. Further, from Lemma 2 part (ii), since $\beta \in [\beta_b(\psi), \beta_s(\psi))$, the board’s expected payoff is higher in a hybrid equilibrium than in a pooling equilibrium with sledgehammer governance.

Hence, when $\beta \in [\beta_m(\psi), \beta_s(\psi))$, the board chooses $\alpha = 0$, the outsider enters, and a hybrid equilibrium obtains at $t = 2$.

**Proof of part (iii) (a)**

We separately consider the cases $\psi \in (\psi_3, \psi_c]$ and $\psi \in [\psi_c, \psi_g)$, maintaining $\beta < \beta_m(\psi)$ in each case.

First, suppose $\psi \in (\psi_3, \psi_c)$. Suppose the board chooses some $\alpha \geq \alpha_e(\psi)$. It is immediate that $\psi \geq \psi_2(\alpha)$. Further, $\alpha \geq \alpha_c(\psi)$ implies that $(1 - \alpha)(1 - q)(\psi - \theta_L) \leq (1 - q)(\psi - \theta_L) - \kappa$, which further implies that $\phi(\psi) \leq \beta_c(\alpha_e(\psi), \psi)$. Next, note that $\psi_g < \psi_f(\alpha_e(\psi_g))$ since $c(\alpha_e(\psi_g)) > 0$. Hence, for any $\psi < \psi_g$, we have $\psi < \psi_f(\alpha_e(\psi))$. Also, $\alpha_e > \alpha_c$ for $\psi < \psi_c$.
Step 2 A sufficiently low $\alpha$ is strictly increasing in $\psi$, boards’s payoff is clearly maximized by choosing $\alpha = \alpha_e$. The payoff to the board in this equilibrium is then $F_I(\alpha_e(\psi), \psi)$.

Now, suppose the board chooses $\alpha < \alpha_e$. Then, one can show that $\psi < \psi_2(\alpha)$. Further, $\beta_m(\psi) = \phi(\psi)$ when $\psi \in (\psi_3, \psi_c)$, so $\beta < \beta_m(\psi)$ implies that $\beta < \phi(\psi)$. Therefore, by Proposition 5, the activist stays out and no governance results. Across these equilibria, the board’s payoff is clearly maximized by choosing $\beta = \beta_e$. The payoff to the board in this equilibrium is then $F_I(\beta_e(\psi), \psi)$.

Next, suppose $\psi > \psi_3$. Further, $F_I(\alpha_e(\psi), \psi) > F_N(0)$. Hence, the board chooses $\alpha = \alpha_e$, and the activist enters.

However, $\psi > \psi_3$ can be rewritten as $F_I(\alpha_e(\psi_3), \psi_3) > F_N(0)$. So the board does not choose $\alpha < \alpha_e(\psi)$. Therefore, the board optimally chooses $\alpha = \alpha_e(\psi)$, which results in a pooling equilibrium with informed governance in the continuation game.

Next, suppose $\psi \in (\psi_c, \psi_2)$. We proceed in three steps.

Step 1 If the board chooses $\alpha \geq \alpha_e(\psi)$, the equilibrium of the continuation game is a pooling equilibrium with informed governance.

Observe that $\psi > \psi_c$ implies that $\psi > \theta_L + \frac{\kappa}{(1-q)\alpha_c(\psi)} = \psi_2(\alpha_c(\psi))$. Since $\psi_2(\alpha) < 0$, we also have $\psi > \psi_2(\alpha_c(\psi))$ for any $\alpha > \alpha_c(\psi)$. Further, as observed earlier in considering the region $\psi \in (\psi_3, \psi_c)$, $\psi < \psi_c$ implies $\psi < \psi_f(\alpha_c(\psi))$. Since $\partial \psi_f(\alpha) > 0$, we also have $\psi < \psi_f(\alpha)$ for $\alpha > \alpha_c(\psi)$. Now, $\psi > \psi_c$ can be rewritten as $\alpha_c(\psi)(1 - q)(\psi - \theta_L) \geq \kappa$, which is equivalent to $\beta_2(\alpha_c(\psi), \psi) > \phi(\psi)$. Hence, $\beta < \beta_m(\psi)$ implies that $\beta < \beta_2(\alpha_c(\psi), \psi)$ in this range, with $\frac{\partial \beta_2}{\partial \alpha} > 0$. In sum, $\alpha \geq \alpha_c(\psi)$ results in $\psi \in (\psi_2(\alpha(\psi)), \psi_f(\alpha(\psi)))$ and $\beta < \beta_2(\alpha, \psi)$. Hence, by Proposition 2, $\alpha \geq \alpha_c(\psi)$ results in a pooling equilibrium with informed governance.

Across these equilibria, the board’s payoff is maximal at $\alpha = \alpha_c(\psi)$.

Step 2 A sufficiently low $\alpha$ may result in the outsider staying out, but the board prefers $\alpha = \alpha_c(\psi)$ and a pooling equilibrium with informed governance.

Across all continuation equilibria in which the outsider stays out, the board’s payoff is maximized by choosing $\alpha = 0$, with a payoff $F_N(0)$. Substituting in $\psi = \psi_3$, we can show that $F_I(\alpha_e(\psi_3), \psi_3) = F_N(0)$. But for $\psi > \psi_3$, $F_I(\alpha_e(\psi_3), \psi) > F_I(\alpha_e(\psi_3), \psi_3)$. Further, since $\alpha_e$ is decreasing in $\psi$, we have $F_I(\alpha_e(\psi), \psi) > F_I(\alpha_e(\psi_3), \psi)$. Finally, $F_I(\alpha_e(\psi), \psi) > F_I(\alpha_e(\psi), \psi)$, since $\alpha_e(\psi)$ maximizes $F_I(\alpha, \psi)$ over $\alpha$. Hence, $F_I(\alpha_e(\psi), \psi) > F_N(0)$, so the board prefers to set $\alpha = \alpha_e$ to the no-governance outcome.
Step 3 A low $\alpha$ may result in a pooling equilibrium with sledgehammer governance or a hybrid equilibrium, but the board prefers $\alpha = \alpha_c(\psi)$ with informed governance.

Since $\psi_f(\alpha) > 0$, a sufficiently low $\alpha$ may result in $\psi > \psi_f(\alpha)$ and a pooling equilibrium with sledgehammer governance. However, observe that $\psi < \psi_g$ implies that $F_G(0, \psi) < F_I(\alpha_c(\psi), \psi)$, so the board prefers to set $\alpha = \alpha_c$ and follow up with informed governance.

Similarly, since $\frac{\partial \beta}{\partial \alpha} > 0$, it may be that for a sufficiently low $\alpha$, $\beta > \beta_{b}(\alpha, \psi)$, resulting in a hybrid equilibrium. To induce a hybrid equilibrium (if possible), the board would optimally choose $\alpha = 0$. For $\psi < \psi_g$, we have $\beta_m(\psi) = \min\{\phi(\psi), \beta_{c}(\psi)\}$.

Finally, suppose $\beta_m(\psi) = \phi(\psi)$. For there to be a hybrid equilibrium, it must be that $\beta > \beta_{b}(\alpha, \psi)$, so $\beta_{b}(\alpha, \psi) < \phi(\psi)$. The latter inequality implies that $(1 - q)(\psi - \theta_L) - \kappa < (1 - \alpha)(1 - q)(\psi - \theta_L)$, or $\psi < \psi_2(\alpha)$. Hence, by Proposition 5, the outsider stays out, and the outcome is no governance.

In summary, when $\psi \in (\psi_c, \psi_g)$ and $\beta < \beta_m(\psi)$, the board chooses $\alpha = \alpha_c(\psi)$ and implements a pooling equilibrium with informed governance.

Proof of part (iii) (b)

Suppose that $\psi > \psi_g$ and $\beta < \beta_m(\psi)$. It can be shown that $\psi_g > \psi_f(0)$. Further, $\beta_m(\psi) = \beta_b(\psi)$ for $\psi > \psi_g$. Hence, by Proposition 4, if the board chooses $\alpha = 0$, a pooling equilibrium with sledgehammer governance results. Further, $\alpha = 0$ is optimal over all values of $\alpha$ that also yield a pooling equilibrium with sledgehammer governance.

Now, $F_G(0, \psi) = \psi$, and $F_N(0) = q\theta_H + (1 - q)\theta_L = \psi_f(0)$. Since $\psi > \psi_g > \psi_f(0)$, it follows that $F_G(0, \psi) > F_N(0)$, so the board prefers the pooling equilibrium with sledgehammer governance to any continuation equilibrium with no governance. Further, $\psi > \psi_g$ is equivalent to $F_I(\alpha_c(\psi), \psi) < F_G(0, \psi)$, so the board prefers the pooling equilibrium with sledgehammer governance to any continuation equilibrium that features pooling with informed governance. Finally, note that since $\beta < \beta_b(\psi)$, Proposition 3 part (i) implies immediately that a hybrid equilibrium cannot obtain.

Proof of Proposition 8

Recall that $\alpha_e = \kappa \frac{\kappa}{(1 - q)(\psi - \theta_L)}$. By inspection, $\alpha_e$ decreases as $\psi$ increases.

The value $\alpha_c$ is defined as the value of $\alpha$ that solves the equation $c'(\alpha) = (1 - q)(\psi - \theta_L)$. Since $c(\cdot)$ is convex, as $\psi$ increases, $c'(\alpha_c)$ must increase as well. That is, $\alpha_e$ increases.
References


