Information Diversity and Market Efficiency Spirals

Itay Goldstein and Liyan Yang*

June 2012

Abstract

We analyze a model where the value of a traded security is affected by two different fundamentals, e.g., the quality of the firm’s technology and the demand for its products, and where there are two groups of informed traders, each one informed about a different fundamental. We analyze the interaction between the informativeness of the price about the two fundamentals and characterize when it leads to attenuation and when it leads to amplification of shocks to market efficiency. Amplification occurs because the informativeness about one fundamental reduces the uncertainty in trading on information about the other fundamental and encourages traders to trade more aggressively on such information. This effect is dominant when the informativeness of the price is relatively balanced between the two fundamentals, which implies that economies with more diverse information – i.e., where the information is more evenly distributed between the two groups – will exhibit positive externalities and have higher levels of overall market efficiency. Finally, we endogenize the incentives for information production and show that the above effect leads to strategic complementarities in information production.

*Itay Goldstein: Department of Finance, Wharton School, University of Pennsylvania, Philadelphia, PA 19104; Email: itayg@wharton.upenn.edu; Tel: 215-746-0499. Liyan Yang: Department of Finance, Joseph L. Rotman School of Management, University of Toronto, 105 St. George Street, Toronto, Ontario M5S 3E6; Email: liyan.yang@rotman.utoronto.ca; Tel: 416-978-3930. For helpful comments and discussions, we thank Efthathios Avdis, Vincent Glode, Doron Levit, Igor Makarov, Konstantin Milbradt, Tarun Ramadorai, Dimitri Vayanos, Kathy Yuan, and participants at the 2012 Adam Smith Asset Pricing Conference (Oxford, UK).
1 Introduction

Does the presence of information in financial markets attract more traders to produce information and trade on it or does it keep them away from doing so? Knowing the answer to this question is crucial for our understanding of the workings of financial markets. If information in financial markets attracts the production and transmission of more information in the trading process, then shocks to market efficiency will be amplified, creating a spiral that leads to sharp changes in informativeness and efficiency. On the other hand, if information in financial markets deters traders from producing more information or from trading aggressively on the information that they have, then shocks to market efficiency will be attenuated, implying modest changes in informativeness and efficiency.

The seminal model of Grossman and Stiglitz (1980) provides an unambiguous answer to this question. In that model, when more informed traders are present in the market, others have a lower incentive to become informed and trade on their information. This is because, as more people are informed, the price provides more information, and so each trader has a lower incentive to produce information on his own, as he can get a good amount of information from the price. As a result, shocks to market efficiency are attenuated in equilibrium: The arrival of more information to the market deters others from becoming informed, and so the overall effect on price informativeness and market efficiency is modest.

But, in reality, things often seem to go in the opposite direction. As uncertainty about economic fundamentals heightens, traders tend to stay away from the market, avoiding trades even on issues related to their area of expertise. This was evident in the recent financial crisis, as markets froze, exhibiting very low trade volumes, following the increase in economic uncertainty that led market prices to be an unreliable source of information. The economic force behind this phenomenon is that risk averse traders want to avoid the risk in trading securities, whose underlying value is too uncertain.

For example, consider traders, who are informed about traditional (consumer and commercial) loans provided by a bank, that in normal times trade in the bank’s stock based on their information about the quality of these loans. Suppose that, as in the recent crisis, a new dimension of uncertainty emerges, concerning the quality of the bank’s investment in
mortgage-backed securities. The lack of information in the market about these securities might deter the traders, who are informed about traditional loans, from trading, since there is a lot of risk involved in taking positions in stocks, whose value will be determined to a large extent by the quality of the mortgage-backed securities. Moreover, the fact that investors with information about loans leave the market might deter traders with information about mortgage-backed securities from trading in the market for the same reason: It is too risky for them to trade when the value of the stock is largely determined by something they do not understand and that is not reflected in the price. Hence, we get a self-reinforcing market efficiency spiral, by which less information on one dimension of uncertainty leads to less information on another dimension of uncertainty, and so on, ending with much lower levels of informativeness and efficiency. This can explain sudden market breakdowns as in the recent crisis.

More generally, there are many cases where firms face multiple dimensions of uncertainty, and, due to costs of information production and natural areas of expertise, different speculators specialize and trade on information along different dimensions. The value of technology firms is driven by the quality of their technology and by the demand for their products, and there are traders with technical expertise trading on information about technology, while there are traders with marketing expertise trading on information about the demand for firms’ products. The value of multinational firms depends on the developments in various countries, and there are traders who understand the developments in a specific country and trade on this information, taking into account whether there are traders in the market trading on information about the developments in other countries. The idea that investors trade on one kind of information is featured in the literature on style investing (Chan, Chen, and Lakonishok, 2002; Barberis and Shleifer, 2003; Goetzmann and Brown, 2003). It is also related to behavioral arguments about investors being limited in the number of variables they can keep track of (Hong, Stein, and Yu, 2007; Brunnermeier and Oehmke, 2009). Overall, we seek to explore the interaction between different types of investors trading on different types of information, and the extent to which this interaction leads to amplification

---

1 This idea is akin to the notion of limited attention, which has been recently studied in the finance literature (e.g., Moscarini, 2004; Peng, 2005; Peng and Xiong, 2006; and Gabaix, Laibson, Moloche, and Weinberg, 2006).
or attenuation of market efficiency and price informativeness.

The model we study in this paper is an extension of Grossman and Stiglitz (1980) that considers two dimensions of uncertainty about the value of the traded security, say that the value of the security depends on the quality of the firm’s technology and on the demand for the firm’s products. We first consider an economy where traders are endowed with different types of information: Some traders are informed about the technology of the firm and others are informed about the demand for its products. Traders are risk averse, and trade in a market with uninformed traders and with noisy supply, as in Grossman and Stiglitz (1980). When they trade, they condition on the price of the security, and hence the information in the price benefits their trading decision.

We analyze the interaction between the two groups of informed traders in this model. Suppose that the size of the group of technology-informed traders increases, what is the effect on the informativeness of the price about the demand for the firm’s products? There are two effects. First, the presence of more technology-informed traders creates an adverse selection effect that deters demand-informed traders from trading aggressively. This is because when they trade, they do so without having information that is available to an increased number of other traders in the market. This effect attenuates the impact of shocks to market efficiency: An increase in efficiency due to the increase in the number of traders who are informed on one dimension will cause traders who are informed on the other dimension to trade less aggressively, making the overall effect on market efficiency modest. Second, the presence of more technology-informed traders implies that more of their information will get into the price, reducing the overall uncertainty that demand-informed traders have to face concerning technology issues when they trade. This effect amplifies the impact of shocks to market efficiency: The presence of more technology-informed traders allows demand-informed traders to trade more aggressively on what they know without being exposed to risks they do not understand, and this increases efficiency further.² This is the basis for the market efficiency spiral that motivates our paper.

Interestingly, we find that in equilibrium the amplification effect dominates the atten-

²This is similar to a firm that specializes in one type of activity and hedges itself against other unrelated risks.
uation effect when the informativeness of the price is relatively balanced between the two dimensions of uncertainty. On the other hand, the attenuation effect dominates the amplification effect when the price is much more informative about one dimension than about the other. This has important implications for the optimal structure of information in the economy. Fixing the total mass of informed traders, an economy with a diverse information structure — i.e., where there is more balance between the two groups of informed traders — will exhibit a higher level of market efficiency than an economy with a concentrated information structure — i.e., where there are many more traders informed about one dimension than on the other. This is because the former economy benefits more from the amplification in informativeness as a result of the uncertainty reduction effect described above.

We then extend our analysis to allow traders to decide whether to become informed on one of the two dimensions of uncertainty. We again analyze the interaction between the two types of information. Suppose that there are more technology-informed traders in the market, what will be the effect on the incentives of agents to acquire information about demand? On the one hand, the traditional Grossman-Stiglitz effect, reducing the incentive to produce information about demand when there are more technology-informed traders in the market, exists in our model. This is a strategic substitute that attenuates the effect on market efficiency. However, on the other hand, the uncertainty reduction effect mentioned above creates a strategic complementarity that amplifies the effect on market efficiency: Knowing that more technology information will go into the price, traders know they will face less uncertainty when trading on demand information, and hence have a stronger incentive to produce information about demand. We identify conditions under which this effect dominates, creating another form of market efficiency spiral that leads to amplification of shocks in equilibrium.

We finally compare our economy where the asset payoff has multiple dimensions of uncertainty and traders are informed of different dimensions, to the standard unidimensional economy, analyzed by Hellwig (1980) and Verrecchia (1982), where the asset payoff is modelled as a single random variable and traders are informed of the overall asset payoff. We

---

3We do not allow traders to acquire information in both dimensions, following the arguments above about traders having a comparative advantage in processing one kind of information or about the limited capacity that they have.
find that the standard result that the benefit of acquiring information decreases with the
total size of the informed traders population in the unidimensional economy is not robust.
In general, how traders respond to other traders becoming informed does not only depend
on the overall size of informed group, but also depends on its composition. If traders are
acquiring information about fundamentals that are weakly correlated, then acquiring informa-
tion on different fundamentals tend to be a complement rather than a substitute. This
result highlights the uniqueness of our analysis.

Several papers in the literature analyze models of financial markets, where the value of
the traded security is affected by more than one fundamental, on which agents are informed.
Froot, Scharfstein, and Stein (1992) show that short term investors tend to concentrate on
collecting one type of information, because prices in the short-run will most likely reflect
information that other people have. Goldman (2005) studies whether a two-division firm is
better off when it is listed as one unit rather than when its two divisions are traded sepa-
rately. Yuan (2005) demonstrates that benchmark securities allow heterogeneously informed
investors to create trading strategies that are perfectly aligned with their signals about differ-
ent fundamentals. Kondor (2012) shows that public announcement can create disagreement
and increase trading volume when traders are informed of different fundamentals and have
heterogeneous trading horizons.

More closely related is Lee (2010), which builds on Subrahmanyam and Titman (1999).
Like us, Lee (2010) investigates the interactions between different types of informed traders
in the same market and points out the possibility that learning can be a complement when
informed traders pursue different trading strategies. In his paper, however, the complemen-
tarity comes from the fact that trades based on different types of information provide noise
for each other in a market-order based model (Kyle, 1985). By contrast, our mechanism
works through the uncertainty reduction effect (which is absent in Lee, 2010) due to the fact
that information revealed in prices reduces the uncertainty faced by agents and encourages
them to trade more aggressively and produce more information. Our analysis also goes be-
yond pointing out the learning complementarity, e.g., by showing that information diversity
can have important implications for price informativeness and the cost of capital.

Another related literature analyzes models of trading in multiple securities, e.g., Admati
(1985) and Bernhardt and Taub (2008). Most closely related in this literature is the recent paper by Cespa and Foucault (2011). They study liquidity spillovers between different securities when speculators trading in one security may observe the price of the other traded security and learn information concerning the security they trade. In contrast, we show the amplification in efficiency and informativeness in a one-security setting and address different questions like the effect of information diversity on market efficiency, the effect of the cost of producing one type of information on the production of another type of information, etc.

Finally, our model contributes to the literature studying sources of strategic complementarities in financial markets, by highlighting a different mechanism by which such complementarities arise. Other papers in this literature include: Barlevy and Veronesi (2000), Ganguli and Yang (2009), Garcia and Strobl (2010), Goldstein, Ozdenoren, and Yuan (2010), Mele and Sangiorgi (2010), and Breon-Drish (2011).

The remainder of this paper is organized as follows. Section 2 presents the model and the characterization of the equilibrium. In Section 3, we analyze the interaction between the two types of information and provide a full characterization of when our model features amplification vs. attenuation of market efficiency. Section 4 studies the role of information diversity, showing that economies with more diverse information will feature greater market efficiency. In Section 5, we endogenize the information acquisition decision and analyze when it will exhibit strategic substitute vs. strategic complementarity. Section 6 checks the robustness of our results and highlights the uniqueness of our analysis, and Section 7 concludes.

2 The Model

2.1 Setup

We study an environment similar to Grossman and Stiglitz (1980). There are two assets traded in the financial market: one riskless asset (bond) and one risky asset (stock). The bond is in unlimited supply; its payoff is 1, and its price is normalized to 1. The stock has a total supply of 1 unit and has a price of $\tilde{p}$, which is determined endogenously in the financial
market, and its payoff \( \hat{v} \) is given by:

\[
\hat{v} = \hat{v}_1 + \hat{v}_2. \tag{1}
\]

As we see in (1), the payoff of the stock is composed of two ingredients, \( \hat{v}_1 \) and \( \hat{v}_2 \), sometimes referred to as fundamentals, which are independent and identically distributed (i.i.d.) according to a normal distribution function: \( \hat{v}_i \sim N(0, 1/\rho) \) \((i = 1, 2)\), where \( \rho \) represents the common prior precision of \( \hat{v}_1 \) and \( \hat{v}_2 \). The idea is that there are two dimensions of uncertainty about the payoff from the stock, captured by the variables, \( \hat{v}_1 \) and \( \hat{v}_2 \), and, as we will discuss below in more detail, they are potentially observable to different traders.

There are three types of rational traders (of a total mass of 1) trading the bond and the stock in the financial market: (1) \( \hat{v}_1 \)-informed traders (of mass \( \lambda_1 > 0 \)), who observe the realization of the first component \( \hat{v}_1 \) of the stock payoff. (2) \( \hat{v}_2 \)-informed traders (of mass \( \lambda_2 > 0 \)), who observe the realization of the second component \( \hat{v}_2 \) of the stock payoff. (3) Uninformed traders (of mass \( \lambda_u = 1 - \lambda_1 - \lambda_2 \geq 0 \)), who do not observe any information.

While we currently assume that the masses of informed traders, \( \lambda_1 \) and \( \lambda_2 \), are exogenous, we endogenize them later in Section 5, where we analyze learning complementarities. All three types of traders condition their trades on the stock price \( \hat{p} \). Their utility from consumption \( C \) is given by the usual constant-absolute-risk-aversion (CARA) function, \( -e^{-\gamma C} \), where \( \gamma \) is the risk-aversion parameter. Finally, to prevent fully revealing prices, we assume that there are noise traders who trade a random amount \( \hat{x} \sim N(0, 1/\chi) \) (with \( \chi > 0 \)) of the stock, which is independent of the realizations of \( \hat{v}_1 \) and \( \hat{v}_2 \).

As mentioned above, \( \hat{v}_1 \) and \( \hat{v}_2 \) represent two dimensions of uncertainty about the traded firm. For example, one dimension can be the demand for the firm’s products and the other one can be the quality of the firm’s technology. Then, different traders may be exposed to different types of information, depending on their expertise and background. For example, people with technical background may know more about the technology of the firm, while consumers may have assessments about the demand for the firm’s products. Also, retail investors may be exposed to information about aspects of the firm which are different than those that institutional investors are exposed to.
For the sake of tractability, we have assumed that the two ingredients $\tilde{v}_1$ and $\tilde{v}_2$ are mutually independent and that $\tilde{v}_i$-informed traders perfectly observe $\tilde{v}_i$. In Section 6 we remove these two assumptions and show that our main results go through. We have also assumed that $\tilde{v}_1$ and $\tilde{v}_2$ are identically distributed, as this assumption gives us a natural aspect of information diversity based on the proportion of traders informed about each fundamental (which is formalized by equation (22)). If instead the two fundamentals are not identically distributed, we have not found a tractable way to implement a notion of information diversity, because it is difficult to control the impact of the total amount of information while we vary parameters to examine the impact of information diversity.

2.2 Equilibrium Definition

The equilibrium concept that we use is the rational expectations equilibrium (REE), as in Grossman and Stiglitz (1980). In equilibrium, traders trade to maximize their expected utility given their information set, where $\tilde{v}_i$-informed traders know $\tilde{v}_i$ and $\tilde{p}$ ($i = 1, 2$), and uninformed traders know only $\tilde{p}$. The price $\tilde{p}$ is determined, in turn, by the market-clearing condition, whereby the sum of demands from the three types of rational traders and the noise traders is equal to the supply of the stock 1. As in most of the literature, we consider a linear equilibrium, where the price $\tilde{p}$ linearly depends on the signals $\tilde{v}_1$ and $\tilde{v}_2$ and the noisy trading $\tilde{x}$:

$$\tilde{p} = \alpha_0 + \alpha_1 \tilde{v}_1 + \alpha_2 \tilde{v}_2 + \alpha_x \tilde{x}. \tag{2}$$

The coefficients $\alpha_0$, $\alpha_1$, $\alpha_2$, and $\alpha_x$ will be endogenously determined.

2.3 Equilibrium Characterization

We start by deriving the demands of the different traders. The $\tilde{v}_i$-informed traders have information set $\mathcal{F}_i = \{\tilde{p}, \tilde{v}_i\}$. The CARA-normal setup indicates that their demand function is:

$$D_i(\tilde{p}, \tilde{v}_i) = \frac{E(\tilde{v}_i|\mathcal{F}_i) - \tilde{p}}{\gamma \text{Var}(\tilde{v}_i | \mathcal{F}_i)} = \frac{\tilde{v}_i + E(\tilde{v}_j | \mathcal{F}_i) - \tilde{p}}{\gamma \text{Var}(\tilde{v}_j | \mathcal{F}_i)}. \tag{3}$$
where the second equality follows from equation (1). Given the price function (2), the information set \( \mathcal{F}_i \) is equivalent to the following signal in predicting \( \tilde{v}_j \):

\[
\tilde{s}_{ji} \equiv \frac{\hat{p} - \alpha_0 - \alpha_i \tilde{v}_i}{\alpha_j} = \tilde{v}_j + (\frac{\alpha_i}{\alpha_j}) \tilde{x}.
\]  

By Bayes’ rule, we have,

\[
E(\tilde{v}_j|\mathcal{F}_i) = E(\tilde{v}_j|\tilde{s}_{ji}) = \frac{(\alpha_j/\alpha_x)^2 \chi \tilde{s}_{ji}}{\rho + (\alpha_j/\alpha_x)^2 \chi},
\]

\[
Var(\tilde{v}_j|\mathcal{F}_i) = Var(\tilde{v}_j|\tilde{s}_{ji}) = [\rho + (\alpha_j/\alpha_x)^2 \chi]^{-1}.
\]

Plugging the above expressions into equation (3), we can express the demand function of \( \tilde{v}_i \)-informed traders as follows:

\[
D_i(\hat{p}, \tilde{v}_i) = \frac{\tilde{v}_i + E(\tilde{v}_j|\tilde{s}_{ji}) - \hat{p}}{\gamma Var(\tilde{v}_j|\tilde{s}_{ji})} = \frac{\tilde{v}_i + (\alpha_j/\alpha_x)^2 \chi \tilde{s}_{ji} - \hat{p}}{\gamma [\rho + (\alpha_j/\alpha_x)^2 \chi]^{-1}},
\]  

for \( i = 1, 2 \).

The uninformed traders only observe the price \( \hat{p} \) and their demand function is:

\[
D_u(\hat{p}) = \frac{E(\tilde{v} | \hat{p}) - \hat{p}}{\gamma Var(\tilde{v} | \hat{p})}.
\]

The price \( \hat{p} \) is equivalent to the following signal in predicting the total payoff \( \tilde{v} \):

\[
\tilde{s}_u \equiv \frac{\hat{p} - \alpha_0}{\alpha_x} = \frac{\alpha_1}{\alpha_x} \tilde{v}_1 + \frac{\alpha_2}{\alpha_x} \tilde{v}_2 + \tilde{x}.
\]

Applying Bayes’ rule, we have:

\[
E(\tilde{v} | \hat{p}) = \beta_{\tilde{v}, \hat{p}} \tilde{s}_u,
\]

\[
Var(\tilde{v} | \hat{p}) = \frac{(\alpha_1/\alpha_x - \alpha_2/\alpha_x)^2 + 2\rho \chi^{-1}}{(\alpha_1/\alpha_x)^2 \rho + (\alpha_2/\alpha_x)^2 \rho + \rho^2 \chi^{-1}},
\]  

for \( \rho = 1, 2 \).
where
\[ \beta_{\tilde{v}, \tilde{p}} = \frac{(\alpha_1 / \alpha_x) + (\alpha_2 / \alpha_x)}{(\alpha_1 / \alpha_x)^2 + (\alpha_2 / \alpha_x)^2 + \rho x^{-1}}. \] (13)

The equilibrium price is determined by the market-clearing condition for the risky asset:
\[ \lambda_1 D_1(\tilde{p}, \tilde{v}_1) + \lambda_2 D_2(\tilde{p}, \tilde{v}_2) + \lambda_u D_u(\tilde{p}) + \tilde{x} = 1. \] (14)

Plugging the expressions for \( D_i(\tilde{p}, \tilde{v}_i) \) and \( D_u(\tilde{p}) \) (equations (8) and (9)) into the above market-clearing condition, we can solve for the price \( \tilde{p} \) as a function of the variables \( \tilde{v}_1, \tilde{v}_2 \) and \( \tilde{x} \). Then, comparing coefficients with those in the conjectured price function (equation (2)), we get the following proposition that characterizes the linear rational-expectations equilibrium. The proof of all propositions are in the appendix.

**Proposition 1** For any \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \), there exists a unique linear rational-expectations equilibrium, in which
\[ \tilde{p} = \alpha_0 + \alpha_1 \tilde{v}_1 + \alpha_2 \tilde{v}_2 + \alpha_x \tilde{x}. \]

The coefficients \( \alpha_0 < 0, \alpha_1 > 0, \alpha_2 > 0, \) and \( \alpha_x > 0 \) are given as a function of the exogenous parameters of the model in the proof in the appendix.

3 Trading Incentives and the Interaction between the Two Types of Information

3.1 Trading Incentives and Price Informativeness

Going back to Grossman and Stiglitz (1980), price informativeness is often defined as the ratio between the sensitivity of the price to fundamental shocks and the sensitivity of the price to noise. In our model, there are two types of fundamentals, \( \tilde{v}_1 \) and \( \tilde{v}_2 \), and we would like to examine the informativeness of the price about each one of them and the interaction between the two. Hence, we define \( I_1 \equiv \frac{\alpha_1}{\alpha_x} \) and \( I_2 \equiv \frac{\alpha_2}{\alpha_x} \) as the price informativeness about signals \( \tilde{v}_1 \) and \( \tilde{v}_2 \), respectively.\(^4\)

\(^4\)Our definition of price informativeness \( I_i \) about \( \tilde{v}_i \) is in the same spirit as Grossman and Stiglitz (1980) who interpret price informativeness as the extent to which variations of the price come from fundamental
We are also interested in a measure of overall informativeness, i.e., how much the price reflects the total stock payoff \( \tilde{v} \), which also measures the market efficiency. As in Brunnermeier (2005), we define this as \( I_v \equiv \frac{1}{\text{Var}(\tilde{v} | \tilde{p})} \), i.e., the reciprocal of the variance of \( \tilde{v} \) conditional on the price. This corresponds to how much residual uncertainty is faced by the uninformed traders after conditioning on the price. By equation (12) and the definitions \( I_i = \frac{\alpha_i}{\alpha_x} \) \((i = 1, 2)\), we can see how the overall price informativeness measure \( I_v \) depends on the other two informativeness measures \( I_1 \) and \( I_2 \):

\[
I_v = \frac{1}{\text{Var}(\tilde{v} | \tilde{p})} = \frac{I_1^2 \rho + I_2^2 \rho + \rho^2 \chi^{-1}}{(I_1 - I_2)^2 + 2 \rho \chi^{-1}}.
\]

Note that \( I_1 \) and \( I_2 \) positively affect the overall price informativeness \( I_v \). That is, \( \frac{\partial I_v}{\partial I_1} > 0 \) and \( \frac{\partial I_v}{\partial I_2} > 0 \). This result is intuitive, as one would expect that the price system will reveal more information about the total asset payoff \( \tilde{v}_1 + \tilde{v}_2 \) when the variations of the price come more from either component of the asset payoff than from the noisy asset trading.\(^5\)

In equilibrium, price informativeness is determined by the trading incentives of the informed traders, that is, by the extent to which a change in their signal affects the amount that they trade, which is represented by \( \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i} \) for the \( \tilde{v}_i \)-informed traders. Intuitively, this is because prices reflect information precisely as a result of informed traders trading on their private signals. To see this formally, consider the following experiment. A unit increase in \( \tilde{v}_i \) will cause each of these traders to buy \( \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i} \) more stocks, and so as a group \( \tilde{v}_i \)-informed traders will buy \( \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i} \) more stocks. If it happens that the noise traders supply the same number of extra shares, then the price will not change. That is, changing \( \tilde{v}_i \) by one unit has the same price impact as changing \( \tilde{x} \) by \( \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i} \) units, and as a result, the price informativeness about \( \tilde{v}_i \) is given by:

\[
I_i = \frac{\alpha_i}{\alpha_x} = \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i}, \quad \text{for } i = 1, 2.
\]

\( \tilde{v}_i \) rather than from the noise \( \tilde{x} \). Strictly speaking, a full notion of informativeness might be the precision of \( \tilde{v}_i \) conditional on the price \( \tilde{p} \), \( \frac{1}{\text{Var}(\tilde{v}_i | \tilde{p})} = \rho \left( 1 + \frac{\rho^2}{\chi^{-1}} \right) \). But our measures \( I_1 \) and \( I_2 \) are analytically more tractable and they give us building blocks we can work with toward those variables of final interest, for example the price informativeness about the overall payoff \( \tilde{v} = \tilde{v}_1 + \tilde{v}_2 \) and the cost of capital.

\(^5\)Note, however, that the result is not trivial due to the interactions between \( I_1 \) and \( I_2 \) in determining \( I_v \). Hence, establishing that \( \frac{\partial I_v}{\partial I_1} > 0 \) and \( \frac{\partial I_v}{\partial I_2} > 0 \) requires us to rely on the relation between \( I_1 \) and \( I_2 \) specified in equation (18). For details, see equations (57) and (58) in the appendix.
3.2 The Interaction between the Two Types of Information

Using equation (3) for the demands of the two types of informed speculators, we can see how these demands are affected by changes in the signals, and how this in turn determines price informativeness:

\[
I_i = \lambda_i \frac{\partial D_i(\hat{p}, \hat{v}_i)}{\partial \hat{v}_i} = \lambda_i \left( \frac{1}{\gamma \text{Var}(\hat{v}_j | F_i)} + \frac{1}{\gamma \partial \hat{v}_i \text{Var}(\hat{v}_j | F_i)} \right) = \lambda_i \left( \gamma^{-1}(\rho + \chi I_j^2) - \gamma^{-1} \chi I_i I_j \right),
\]

where the last equality follows from equations (4)-(7).

An increase in the signal \( \hat{v}_i \) has two effects on the trading of a \( \hat{v}_i \)-informed trader, as can be seen in equation (17). First, as we can see in the first term in the parentheses, he is going to demand more of the asset due to the direct effect that \( \hat{v}_i \) has on the payoff of the asset. Note that this effect gets smaller when there is a higher residual variance about \( \hat{v}_j \), since a higher variance implies that the trader takes more risk when holding a position in the asset, and this makes him trade less aggressively. Second, as we can see in the second term in the parentheses, he is going to demand less of the asset due to the indirect effect via the expectation that he has about \( \hat{v}_j \): Holding the price constant, an increase in \( \hat{v}_i \) implies a lower expectation about \( \hat{v}_j \), and so reduces the trader’s demand of the asset.

Equation (18) also demonstrates the interaction between the informativeness of the price about \( \hat{v}_i \) and the informativeness of the price about \( \hat{v}_j \). Consider equation (18) for the \( \hat{v}_i \)-informed traders. An increase in the informativeness of the price about \( \hat{v}_j \), \( I_j \), has two effects on their trading incentives. First, it reduces the residual uncertainty they face when they trade, and hence makes them trade more aggressively on their information about \( \hat{v}_i \). We label this channel the “uncertainty reduction effect”. It is represented in the first term in the parentheses. Second, it makes them use the price more to infer information about \( \hat{v}_j \), and so trade less aggressively on their information about \( \hat{v}_i \). (An increase in \( \hat{v}_i \), holding the price constant, provides now stronger indication that \( \hat{v}_j \) is low, and this makes the traders rely less on their private information when trading.) We label this channel the “adverse selection effect”. It is represented in the second term in the parentheses.

Rearranging terms in (18), we get two response functions that jointly determine \( I_1 \) and
\[ I_i = h_i(I_j; \lambda_i, \rho, \gamma, \chi) \equiv \frac{\lambda_i(\rho + \chi I_j^2)}{\gamma + \lambda_i \chi I_j}, \]  \text{for } i, j = 1, 2 \text{ and } j \neq i. \tag{19} \]

As we can see, due to the two effects mentioned above (“uncertainty reduction effect” and “adverse selection effect”), the two response functions may have either a positive or a negative slope. Analyzing (19), we can see that \( h_i(I_j; \cdot) \) is decreasing in \( I_j \) when \( I_j \) is low, and then increasing when \( I_j \) is high. That is, the adverse selection effect dominates for low levels of informativeness and the uncertainty reduction effect dominates for high levels of informativeness. As we show in the next subsection, whether we get amplification or attenuation of shocks to market efficiency depends on the slopes of the two response functions in equilibrium.

### 3.3 Market Efficiency Multipliers

Let \( Q \) be one of the five exogenous parameters \((\lambda_1, \lambda_2, \rho, \gamma, \chi)\) that determine the price informativeness measures \((I_1 \text{ and } I_2)\) in the system defined in (19).\(^6\) The following proposition characterizes the effect of \( Q \) on \( I_i \), separating between the direct effect and the indirect effect due to the feedback loop between the two types of information. It identifies when the direct effect of \( Q \) on \( I_i \) is amplified and when it is attenuated due to the interdependence between the two price informativeness measures.

**Proposition 2**  
(i) The effect of an exogenous parameter \( Q \) on the informativeness \( I_i \) of the price about \( \tilde{v}_i \) is given by:

\[
\frac{dI_i}{dQ} = \mathcal{M}\left( \frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j} \frac{\partial h_j}{\partial Q} \right), \tag{20} \]

where the term \( \left( \frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j} \frac{\partial h_j}{\partial Q} \right) \) captures the “direct effect” of changing \( Q \) on \( I_i \) and the

\(^6\)Note that we can rewrite (19) as \( I_i \equiv \frac{\lambda_i(\rho + \chi I_j^2)}{\gamma + \lambda_i \chi I_j} \). So, the three parameters \((\gamma, \rho, \chi)\) affect (19) only through \((\rho/\chi, \gamma/\chi)\), meaning that \((\gamma, \rho, \chi)\) only have two degrees of freedom in determining the system (19). Actually, we can normalize any one of \((\gamma, \rho, \chi)\) to 1 without affecting the equilibrium price informativeness measures.
The direct effect of a shock in a parameter $Q$ on price informativeness $I_i$ can be thought of as the initial impact before the feedback effect between the two informativeness measures is considered. It is given by \( \left( \frac{\partial h_1}{\partial Q} + \frac{\partial h_2}{\partial Q} \right) \). The feedback loop then creates a multiplier that either amplifies or attenuates the direct effect. It is given by \( M = \left( 1 - \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1} \right)^{-1} \), which, as the proof of the proposition shows, is strictly positive. Whether the direct effect is attenuated or amplified in equilibrium then depends on whether $M$ is smaller than or bigger than 1, which depends on the signs of the cross derivatives \( \frac{\partial h_1}{\partial I_2} \) and \( \frac{\partial h_2}{\partial I_1} \). The proof of the proposition shows that it is impossible to have both cross derivatives negative in equilibrium (i.e., \( \frac{\partial h_1}{\partial I_2} < 0 \) and \( \frac{\partial h_2}{\partial I_1} < 0 \) at the point where the two response functions in (19) intersect). Hence, either one of them is negative and one is positive, in which case $M < 1$ and the direct effect of a shock on price informativeness is attenuated in equilibrium, or both of them are positive, in which case $M > 1$ and the direct effect of a shock on price informativeness is amplified in equilibrium.

The result shows that when $I_1$ and $I_2$ are very far from each other (specifically, when \( \frac{I_1}{I_2} < \frac{1}{2} \) or when \( \frac{I_1}{I_2} > 2 \)) – which will be true when $\lambda_1$ and $\lambda_2$ are very far from each other – the uncertainty reduction effect dominates for one response function, while the adverse selection effect dominates for the other. As a result, the interaction between the two price informativeness measures tends to attenuate the initial shock to the economy; that is, $0 < M < 1$. In contrast, when $I_1$ and $I_2$ are close to each other (specifically, when \( \frac{I_1}{I_2} < \frac{1}{2} \)) – which will be true when $\lambda_1$ and $\lambda_2$ are close to each other – the uncertainty reduction effect dominates for both response functions. As a result, the two price informativeness measures reinforce each other and any exogenous shock affecting one price informativeness measure
will lead to a “market efficiency spiral”, whereby a shock directly improving one type of price informativeness will propagate to the other type, which will in turn feed back on the original one, amplifying the effect of the initial shock and improving both types of price informativeness, as well as the total market efficiency. In this case, we have $M > 1$.\footnote{We note that the competition between the adverse selection effect and the uncertainty reduction effect is also responsible for the uniqueness of the financial market equilibrium described by Proposition 1. Specifically, the uncertainty reduction effect increases the possibility of multiple equilibria through its self-reinforcing feature, while the adverse selection effect decreases it. In our economy, both effects are present so that the strength of self-fulfilling expectations is weak and there are no multiple equilibria. In a recent study by Cespa and Foucault (2011) who consider a two-security setting, they indeed find the possibility of multiple linear equilibria, because in their economy, informed traders only specialize in one market and there is no adverse selection effect to counterbalance the uncertainty reduction effect, thereby causing the strength of self-fulfilling expectations to be particularly strong.}

Figure 1 uses a numerical example to illustrate this spiral reaction to an increase in $\lambda_1$. The figure depicts the two $h$ functions in a case where they intersect at the upward sloping parts (i.e., the uncertainty reduction effect is the dominant effect for both of them). In the figure, we set the parameters at $\rho = 50$, $\chi = 50$ and $\gamma = 3$. Suppose that initially $\lambda_1 = \lambda_2 = 0.05$, so that the equilibrium is at Point $E$ and $I_1 = I_2 = 0.83$. Now suppose that $\lambda_1$ increases to 0.1. Then, function $h_1$ moves upward to $h_1'$, and the equilibrium moves to point $E'$ ($I_1 = 1.28$ and $I_2 = 1.07$). We can see how the effect of the increase in the amount of informed trading on $\tilde{v}_1$ is amplified in equilibrium along the route $E \rightarrow A \rightarrow B \rightarrow C \ldots \rightarrow E'$. That is, when there are more informed traders who know $\tilde{v}_1$, the price is automatically more informative about $\tilde{v}_1$. Then, traders who know $\tilde{v}_2$ trade more aggressively on their information because they face less uncertainty (and the uncertainty reduction effect dominates the adverse selection effect), and so the price is more informative about $\tilde{v}_2$ as well. Now traders trade more aggressively on $\tilde{v}_1$ as well, which increases the price informativeness on $\tilde{v}_1$ further. The amplification chain continues on and on till it converges to much higher levels of informativeness on both $\tilde{v}_1$ and $\tilde{v}_2$. 

[FIGURE 1 ABOUT HERE]
4 The Impact of Information Diversity

In the previous section, we showed that the interaction between the two types of information can either amplify or attenuate shocks to market efficiency in our model, and that whether we obtain amplification or attenuation depends on the difference between the two types of informativeness. A natural implication of this result is that the diversity of information in the model should have important implications for overall market efficiency and, as a result, for the cost of capital of the traded firm. In this section, we explore this implication formally.

4.1 Price Informativeness

In our model, there are two types of informed agents. The mass of traders informed about \( \tilde{v}_1 \) is \( \lambda_1 \) and the mass of traders informed about \( \tilde{v}_2 \) is \( \lambda_2 \). We would like to analyze the effect of the difference between \( \lambda_1 \) and \( \lambda_2 \) on the informativeness of the price in our model, while keeping the total size of the informed-traders population fixed. For this purpose, we set \( \lambda_1 + \lambda_2 = \Lambda \) (where \( \Lambda \) is a constant), and define the following measure of information diversity:

\[
\Delta \equiv 1 - \frac{|\lambda_1 - \lambda_2|}{\Lambda} \in [0, 1].
\]

A higher \( \Delta \) means that the two groups of informed traders are closer in size, and so the total amount of information is more equally distributed between the two types of informed traders; hence, there is more diversity of information in the economy. (By this logic, a situation with less diversity is one where most people know the same thing and so \( \Delta \) is low.)

Following Proposition 2, we know that when \( \Delta \) is close to 0, either \( \lambda_1 \) or \( \lambda_2 \) is close to 0, and so \( I_1 \) and \( I_2 \) are far from each other, implying that the market efficiency multiplier \( M \) is smaller than 1. On the other hand, when \( \Delta \) is close to 1, we know that \( \lambda_1 \) is close to \( \lambda_2 \), and so \( I_1 \) and \( I_2 \) are close to each other, and by Proposition 2, the market efficiency multiplier \( M \) is greater than 1.

This link between \( \Delta \) and \( M \) has important implications for market efficiency. Specifically, the following proposition shows that as information diversity increases, the overall price informativeness \( I_v \) goes up.
Proposition 3 Information diversity increases the total price informativeness regarding $\tilde{v}$, that is, $\frac{dI_v}{d\Lambda} > 0$.

To understand this result, compare the following two economies with the same total mass of informed traders $\lambda_1 + \lambda_2 = \Lambda$, but with levels of diversity at the two ends of the spectrum: (1) Economy I, where $\lambda_1 = \Lambda - \varepsilon \approx \Lambda$, $\lambda_2 = \varepsilon \approx 0$, and $\Delta \approx 0$ (a “concentrated” economy); and (2) Economy II, where $\lambda_1 = \lambda_2 = \frac{\Lambda}{2}$ and $\Delta = 1$ (a “diverse” economy). These two economies can be obtained by injecting a total mass $(\Lambda - 2\varepsilon)$ of informed traders to an initial economy where there is almost no information (i.e., $\lambda_1 = \lambda_2 = \varepsilon \approx 0$) along two different paths. To obtain Economy I, we only add traders who are informed about $\tilde{v}_1$, while keeping the mass of agents informed about $\tilde{v}_2$ close to 0. In contrast, to obtain Economy II, we simultaneously add traders informed about $\tilde{v}_1$ and traders informed about $\tilde{v}_2$.

Adding informed traders along both paths improves market efficiency, since $\frac{dI_v}{d\lambda_i} > 0$ for $i = 1, 2$. However, the impact of the new information is different on the two different paths leading to the two economies because of the market-efficiency-multiplier effect identified by Proposition 2. Along the path to obtain Economy I, the multiplier $M$ is smaller than 1, and thus the impact of the new added information is attenuated, while along the path to obtain Economy II, the multiplier $M$ is greater than 1, and the impact of the new added information is amplified. As a result, the total impact of the added mass $(\Lambda - 2\varepsilon)$ of informed traders on market efficiency $I_v$ is larger in Economy II than in Economy I.

Overall, market efficiency is higher in our model when there is more diversity of information, or when there is more balance between the amount of information available on different dimensions. This is because the effect of adding more informed agents on market efficiency is greater when the two levels of informativeness are relatively close to each other, as then the uncertainty reduction effect dominates, and we get a self-reinforcing spiral, by which more information encourages more aggressive trading, increasing informativeness further.

Figure 2 demonstrates graphically the effect of information diversity in our model for the parameter values $\rho = \chi = 50$, $\gamma = 3$ and $\Lambda = 0.1$. In Panel (a), we see that the market efficiency multiplier $M$ is below 1 for low levels of diversity and above 1 for high levels of diversity. Panel (b) confirms the result of Proposition 3 that the overall price informativeness
Increases in information diversity (as a result of the multiplier effect in Panel (a)).

**FIGURE 2 ABOUT HERE**

### 4.2 Cost of Capital

One important implication of price informativeness is its effect on the cost of capital. We define the cost of capital as follows (see, e.g., Easley and O’Hara, 2004):

\[
CC = E (\hat{v} - \hat{p}).
\]  

(23)

That is, the cost of capital is the expected difference between the cash flow generated by the security and its price, which is due to the risk taken by the traders who hold the security. By Proposition 1, we have

\[
CC = -\alpha_0 = \frac{\gamma}{A_{\rho_0}},
\]  

(24)

where \( A_{\rho_0} \) is the average trading aggressiveness across all traders, and is given by equation (46) in part A of the appendix as follows:

\[
A_{\rho_0} = \frac{\lambda_1}{\text{Var} (\hat{v}|F_1)} + \frac{\lambda_2}{\text{Var} (\hat{v}|F_2)} + \frac{\lambda_u}{\text{Var} (\hat{v}|\hat{p})}
= \lambda_1 (\rho + I^2_2 \chi) + \lambda_2 (\rho + I^2_1 \chi) + (1 - \Lambda) I_v.
\]  

(25)

(26)

The right hand side of equation (25) captures the average trading aggressiveness of the different kinds of traders, which is a function of the residual risk they have to bear. Combining (25) with (24), we can see that the cost of capital increases in the risk that traders are exposed to per unit of the security: \( \text{Var} (\hat{v}|F_1) \) for \( \hat{v}_1 \)-informed traders, \( \text{Var} (\hat{v}|F_2) \) for \( \hat{v}_2 \)-informed traders, and \( \text{Var} (\hat{v}|\hat{p}) \) for uninformed traders.

The following proposition shows that information diversity lowers the cost of capital:

**Proposition 4** Information diversity lowers the cost of capital, that is, \( \frac{dCC}{d\Delta} < 0 \).

There are two effects leading to the result in Proposition 4. First, greater information diversity causes uninformed traders to bear less risk and trade more aggressively (as captured
by the term \((1 - \Lambda) I_v\) in equation (26)), because, as we showed in the previous subsection, information diversity increases the total price informativeness \(I_v\) regarding \(\hat{v}\). This raises prices and lowers the cost of capital. Second, the two groups of informed traders, in aggregate, also bear less risk and trade more aggressively (as captured by the term \(\lambda_1 (\rho + I_2^2 \chi) + \lambda_2 (\rho + I_2^2 \chi)\) in equation (26)). To understand this, suppose, without loss of generality, that there are more \(\hat{v}_1\)-informed traders than \(\hat{v}_2\)-informed traders, so that in general, the price system reflects more information regarding \(\hat{v}_1\) than \(\hat{v}_2\) (i.e., \(I_1 > I_2\)), and thus \(\hat{v}_2\)-informed traders are exposed to less risk when they trade than \(\hat{v}_1\)-informed traders (i.e., \((\rho + I_1^2 \chi) > (\rho + I_2^2 \chi)\)). Increasing information diversity is equivalent to replacing some \(\hat{v}_1\)-informed traders with \(\hat{v}_2\)-informed traders, and as a result, the average risk that informed traders are exposed to decreases. This leads to an increase in price and a reduction in the cost of capital. Panel (c) of Figure 2 demonstrates the effect of information diversity on the cost of capital.

5 Endogenous Information Acquisition

So far, we assumed that the proportions of agents who are informed about the two fundamentals \(\hat{v}_1\) and \(\hat{v}_2\) – \(\lambda_1\) and \(\lambda_2\), respectively – were exogenous. We now endogenize these parameters and examine how they are determined in light of the incentives to become informed in our model. The new result that we get relative to the literature is that sometimes there will be a dominant strategic complementarity effect, whereby the increase in the mass of agents acquiring information on one fundamental will lead more agents to acquire information about the other fundamental. This is because of the uncertainty reduction effect identified earlier.

5.1 Information Acquisition in Equilibrium

We assume that there are two groups of traders of sizes \(\lambda_1, \lambda_2 > 0\), \(\lambda_1 + \lambda_2 = 1\). Traders in \(\lambda_1\) can acquire the signal \(\hat{v}_1\) at cost \(c_1 > 0\), and traders in \(\lambda_2\) can acquire the signal \(\hat{v}_2\) at cost \(c_2 > 0\). Traders who choose to acquire \(\hat{v}_1\) (\(\hat{v}_2\)) become part of the \(\lambda_1\) (\(\lambda_2\)) group in the trading model described in previous sections, while those who choose not to acquire
information become part of the \( \lambda_u \) group.

Note that, for simplicity, we assume that any trader has an opportunity to become informed only about one of the two signals. This can be viewed as an extreme case of asymmetric expertise in information acquisition. That is, there are two groups of traders, and each group can only gather a particular type of information at a finite cost. This seems particularly reasonable when speculators acquire and trade on inside information. But, even without inside information, it is reasonable to think of different traders having potential access to different pieces of news based on their different expertise. Moreover, our setting can be justified based on limited ability to process information, e.g., Brunnermeier and Oehmke (2009) argue that investors may be subject to computational limitations or limited in the number of variables that they can keep track of. This idea is akin to the notion of limited attention, which has been recently studied in the finance literature (e.g., Moscarini, 2004; Peng, 2005; Peng and Xiong, 2006; and Gabaix, Laibson, Moloche, and Weinberg, 2006).

Using arguments similar to those in Grossman and Stiglitz (1980), we can show that for given fractions \((\lambda_1, \lambda_2)\) of informed traders (and hence given values of \(I_1\) and \(I_2\), which are determined by the system in (19)), the expected net benefit from purchasing information \(\tilde{v}_i\) to a potential purchaser is:

\[
B_i(I_1, I_2) = \log \left( \frac{\text{Var}(\tilde{v}|\tilde{p})}{\text{Var}(\tilde{v}|F_i)} \right) - 2\gamma c_i = \log (\rho + I_{ij}^2\chi) - \log (I_v) - 2\gamma c_i, \text{ for } i, j = 1, 2 \text{ and } j \neq i.
\]

(27)

Intuitively, the net benefit from acquiring information is increasing in the quality of information available to the trader after purchasing the signal \(\tilde{v}_i\) (given by \(\rho + I_{ij}^2\chi\)), decreasing in the quality of information available to the trader by only observing the price (given by \(I_v\)), and decreasing in the cost of information production.

We focus on interior solutions, where \(0 < \lambda_1^* < \bar{\lambda}_1\) and \(0 < \lambda_2^* < \bar{\lambda}_2\). Hence, conditions \(B_1(I_1^*, I_2^*) = 0\) and \(B_2(I_1^*, I_2^*) = 0\) characterize the equilibrium price informativeness measures \(I_1^*\) and \(I_2^*\), such that speculators are indifferent between acquiring information and not.
acquiring information. These equilibrium price informativeness measures, in turn, characterize the equilibrium fractions \( (\lambda_1^*, \lambda_2^*) \) of informed traders through the system (19). While we cannot prove analytically that the equilibrium in the information-acquisition stage is unique, in simulation analysis, the equilibrium always appears to be unique.

5.2 Strategic Complementarities vs. Strategic Substitutes

We now analyze the strategic interactions among traders producing information. In particular, we show that learning the two independent pieces of information \( \tilde{v}_i \) and \( \tilde{v}_j \) can be complementary, in the sense that an increase in the mass of agents acquiring information on one fundamental will increase the incentive of agents to acquire information about the other fundamental.

Specifically, by equation (27), an increase in the population \( \lambda_j \) of \( \tilde{v}_j \)-informed traders has two offsetting effects on the benefit \( B_i \) of acquiring signal \( \tilde{v}_i \), which are reflected by the two terms, \( \log (\rho + I_j^2 \chi) \) and \( \log (I_v) \). First, an increase in \( \lambda_j \) will improve the informativeness \( I_j \) of the price about signal \( \tilde{v}_j \) (as \( \frac{dI_j}{d\lambda_j} > 0 \) by Proposition 2). This increased \( I_j \) directly reduces the remaining uncertainty of a \( \tilde{v}_j \)-informed trader, which allows him to trade more aggressively and increases his welfare due to the uncertainty reduction effect identified in earlier sections. This increases the benefit of becoming \( \tilde{v}_j \)-informed, which is reflected by the term \( \log (\rho + I_j^2 \chi) \) in equation (27). Second, an increase in \( \lambda_j \) also causes the price to be more informative about the total cash flow \( \tilde{v} \) (as \( \frac{dI_v}{d\lambda_j} > 0 \) by Proposition 2 and equation (15)). This reduces the incentive of uninformed traders to become informed about \( \tilde{v}_i \), which is part of \( \tilde{v} \), as they can now gain more information about \( \tilde{v} \) from the price. This effect is the standard Grossman-Stiglitz substitution effect, whereby having more informed traders, reduces the incentive to become informed. This negative effect is reflected by the term \( \log (I_v) \) in equation (27).

When the positive uncertainty reduction effect dominates the negative Grossman-Stiglitz effect, an increase in \( \lambda_j \), the size of \( \tilde{v}_j \)-informed traders, can increase \( B_i \), the benefit of acquiring signal \( \tilde{v}_i \), thereby causing acquiring the two types of information to be complementary. We can show that this is true when \( I_j > I_i \); that is, \( \frac{\partial B_i}{\partial \lambda_j} > 0 \) if and only if \( I_j > I_i \). Intuitively, when \( I_j > I_i \), the price \( p \) is mainly a public signal about the component \( \tilde{v}_j \). As a
result, acquiring $\tilde{v}_i$ will be particularly profitable when used in combination with $\tilde{p}$, a public signal about $\tilde{v}_j$, in the trading stage (i.e., a strong uncertainty reduction effect); and at the same time, the knowledge of $\tilde{v}_i$ will not be passed on to the price very much (i.e., a weak Grossman-Stiglitz effect), since the price is mainly informative about $\tilde{v}_j$.

Interestingly, learning the same information is still a strategic substitute, i.e., $\frac{\partial B_i}{\partial \lambda_i} < 0$: An increase in the population $\lambda_i$ of $\tilde{v}_i$-informed traders will always decrease the benefit $B_i$ of acquiring signal $\tilde{v}_i$. This is because the uncertainty reduction effect discussed above operates through the price informativeness $I_j$ about the other component $\tilde{v}_j$, while increasing $\lambda_i$ mainly increases $I_i$. Thus, the complementarity effect in acquiring different information is not present in the traditional unidimensional Grossman-Stiglitz framework, and can only be uncovered by considering the two-dimension framework in our paper. We summarize the above discussion in the following proposition.

**Proposition 5** Acquiring information on the same fundamental is a strategic substitute: As more traders become informed of $\tilde{v}_i$, the benefit $B_i$ of acquiring $\tilde{v}_i$ decreases; that is, $\frac{\partial B_i}{\partial \lambda_i} < 0$. Acquiring information on different fundamentals can be a strategic substitute or a complement: As more traders become informed of $\tilde{v}_i$, the benefit of acquiring $\tilde{v}_j$ can decrease or increase, and $\frac{\partial B_i}{\partial \lambda_j} > 0$ if and only if $I_j > I_i$.

### 5.3 The Impact of Information Acquisition Cost

In this subsection, we will conduct comparative-statics analysis, examining the impact of changing the cost $c_i$ of acquiring information $\tilde{v}_i$ on the equilibrium fractions $(\lambda_1^*, \lambda_2^*)$ of informed traders, on market efficiency $I^*$, and on the cost of capital $CC^*$ in the overall equilibrium. The comparative-statics analysis is based on the equilibrium conditions $B_1 (I_1^*, I_2^*) = 0$ and $B_2 (I_1^*, I_2^*) = 0$ in the information-acquisition stage and on the system in (19) charac-

---

Formally, we can argue that the uncertainty reduction effect generated by an increase in the size of traders informed of the same information either works in the same direction as the Grossman-Stiglitz effect or is small in magnitude. Specifically, the uncertainty reduction effect is captured by $\log (\rho + I_j^2 \chi)$ in equation (27) and its strength is related to $\frac{dI_j}{\delta I_j}$. By Proposition 2, we have $\frac{dI_j}{\delta I_j} = \frac{\partial b_i}{\partial \lambda_i} \frac{d\lambda_i}{d\lambda_i}$, which can be positive or negative, depending on the sign of $\frac{\partial b_i}{\partial \lambda_i}$. If $\frac{\partial b_i}{\partial \lambda_i} < 0$, then $\frac{dI_j}{\delta I_j} < 0$ and the uncertainty reduction effect works in the same direction as the Grossman-Stiglitz effect. If $\frac{\partial b_i}{\partial \lambda_i} > 0$, we have $\frac{dI_j}{\delta I_j} < 1$ by equation (52), and hence the effect of $\lambda_i$ on $I_j$ is smaller that its effect on $I_i$; therefore, the uncertainty reduction effect is limited.
characterizing price informativeness measures in the trading stage. The cost of information \( c_i \) represents a measure of the easiness of acquiring information on one fundamental \( \tilde{v}_i \): A proliferation of sources of information about the firm (say, abundant disclosure, large analyst/media coverage, and advanced communication technologies) leads to easier access to information and corresponds to a low value of \( c_i \) (e.g., Fishman and Hagerty, 1989; Kim and Verrecchia, 1994). Our results depend on the strategic interactions between traders producing information on the two different dimensions, as we discussed above. We now turn to describe these effects and the comparative-statics results that they generate.

As we will show in Proposition 6, a decrease in the cost \( c_i \) of acquiring signal \( \tilde{v}_i \) increases the equilibrium size \( \lambda_i^* \) of the population of \( \tilde{v}_i \)-informed traders. This is intuitive since a lower \( c_i \) corresponds to a higher net benefit of knowing \( \tilde{v}_i \). More interestingly, the complementarity effect emphasized in Proposition 5 implies that a decrease in \( c_i \), the cost of acquiring \( \tilde{v}_i \), can increase the equilibrium size \( \lambda_j^* \) of the population of \( \tilde{v}_j \)-informed traders, too. As a result, \( \lambda_i^* \) and \( \lambda_j^* \) move in the same direction in response to the change in \( c_i \), which is another form of a positive market efficiency spiral. Proposition 6 shows that \( \frac{d\lambda_i^*}{dc_i} < 0 \) if and only if \( c_i < c_j \). Specifically, when \( c_i < c_j \), we have \( I_i^* > I_j^* \), which, according to Proposition 5, implies \( \frac{d\lambda_i^*}{d\lambda_i} > 0 \), so that the increased \( \lambda_i^* \) (due to the decrease in \( c_i \)) increases the incentive of traders to acquire \( \tilde{v}_j \), leading to a higher equilibrium size \( \lambda_j^* \) of the population of \( \tilde{v}_j \)-informed traders.

The proposition also shows that a decrease in the cost \( c_i \) of acquiring signal \( \tilde{v}_i \) always leads the price to reveal more about the fundamental of the underlying asset, and as a result, the total price informativeness measure \( I_v^* \) increases. Finally, the equilibrium cost of capital \( CC^* \) is given by equations (24) and (26) as follows:

\[
CC^* = \frac{\gamma}{A_{p0}^*},
\]

where

\[
A_{p0}^* = \lambda_1^* (\rho + I_2^2 \chi) + \lambda_2^* (\rho + I_1^2 \chi) + (1 - \lambda_1^* - \lambda_2^*) I_v^*
\]

\[
= [\lambda_1^* (\rho + I_2^2 \chi - I_v^*) + \lambda_2^* (\rho + I_1^2 \chi - I_v^*)] + I_v^*. \tag{28}
\]
Clearly, the increased $I_1^*$ caused by the decrease in $c_i$ tends to decrease $CC^*$. We can also show that the bracket term in equation (28), which captures the willingness to bear risk by all informed traders, increases. Thus, a decrease in $c_i$ will decrease the equilibrium cost of capital $CC^*$. This suggests that disclosure, reducing the cost of information acquisition by speculators improves market efficiency and reduces the cost of capital.

The proposition is stated as follows:

**Proposition 6** A decrease in the cost $c_i$ of acquiring information $\hat{v}_i$

(i) increases the equilibrium size $\lambda_1^*$ of $\hat{v}_1$-informed traders (i.e., $\frac{d\lambda_1^*}{dc_i} < 0$);

(ii) increases the equilibrium size $\lambda_j^*$ of $\hat{v}_j$-informed traders if and only if $c_i < c_j$ (i.e., $\frac{d\lambda_j^*}{dc_i} < 0$ if and only if $c_i < c_j$);

(iii) increases the overall price informativeness (i.e., $\frac{dI^*}{dc_i} > 0$); and

(iv) decreases the equilibrium cost of capital $CC^*$ (i.e., $\frac{dCC^*}{dc_i} > 0$).

Figure 3 plots $\lambda_1^*$ (Panel (a)), $\lambda_2^*$ (Panel (b)), and $I_1^*$ and $I_2^*$ (Panel (c)) as functions of $\frac{1}{c_1}$ for the same numerical example as in Figure 1 with $c_2 = 0.1$ and $c_1 \in [0.05, 0.13]$. We can see that, for low values of $c_1$, we have $I_1^* > I_2^*$, and so in equilibrium learning the two types of information is complementary to each other, as $\lambda_2^*$ and $\lambda_1^*$ both become larger when $c_1$ decreases. In contrast, for high values of $c_1$, the Grossman-Stiglitz substitution effect dominates and information choices are strategic substitutes.

[FIGURE 3 ABOUT HERE]

### 6 An Extension: Correlated Payoff Ingredients and Noisy Signals

In this section, we analyze an extended economy where the two ingredients of the asset payoff are correlated and where informed traders receive noisy signals about the two ingredients. The analysis serves two purposes. First, it demonstrates the robustness of our main results – that information diversity improves overall price informativeness and reduces the cost of capital and that there are strategic complementarities at the stage of learning – by showing
that they hold in this extended economy. Second, it highlights the importance of multiple dimensions of fundamentals in driving our results, as the extended economy nests both the traditional unidimensional model analyzed by Hellwig (1980) and Verrecchia (1982) and the new two-dimensional economy analyzed by this paper, thereby allowing a direct comparison between them to sharpen the distinctions.

6.1 Setup

The asset payoff is still given by the sum of two ingredients: \( \tilde{v} = \tilde{v}_1 + \tilde{v}_2 \). To accommodate the correlation between \( \tilde{v}_1 \) and \( \tilde{v}_2 \), we assume that they are generated by the following loading structure:

\[
\begin{align*}
\tilde{v}_1 &= \sqrt{\phi} \tilde{\theta} + \sqrt{1-\phi} \tilde{n}_1, \\
\tilde{v}_2 &= \sqrt{\phi} \tilde{\theta} + \sqrt{1-\phi} \tilde{n}_2,
\end{align*}
\]

where \( \tilde{\theta} \sim N(0,1/\rho) \) is the common factor affecting both ingredients and where \( \tilde{n}_i \sim N(0,1/\rho) \) is an idiosyncratic factor. We assume that \( (\tilde{\theta}, \tilde{n}_1, \tilde{n}_2) \) are mutually independent. Thus, parameter \( \phi \in [0,1] \) is the correlation coefficient between \( \tilde{v}_1 \) and \( \tilde{v}_2 \).

As in the our baseline model analyzed in previous sections, we assume that a continuum \([0,1]\) of traders can acquire signals regarding \( \tilde{v}_1 \) and \( \tilde{v}_2 \) at costs \( c_1 > 0 \) and \( c_2 > 0 \). Still, each trader can only acquire one signal, and let \( \lambda_i \) denote the size of traders who decide to become informed of \( \tilde{v}_i \). But now we do not assume that those informed traders observe \( \tilde{v}_i \) perfectly. Instead, an informed trader \( j \) receives a noisy signal about \( \tilde{v}_i \):

\[
\tilde{s}_i^j = \tilde{v}_i + \tilde{\varepsilon}_i^j, \text{ with } \tilde{\varepsilon}_i^j \sim N(0,1/n), \ n > 0,
\]

for \( j \in [0,\lambda_i] \) and \( i = 1,2 \). Let \( \tilde{x} \sim N(0,\chi^{-1}) \) be the noise traders’ demand for the risky asset. We assume that \( (\tilde{\theta}, \tilde{n}_1, \tilde{n}_2, \{\tilde{\varepsilon}_i^j\}_{i,j}, \tilde{x}) \) are mutually independent. Our baseline model corresponds to the special case of \( \phi = 0 \) and \( n = \infty \) (i.e., the noise-to-signal ratio \( \frac{\tilde{\varepsilon}}{n} = 0 \)), and the traditional unidimensional model analyzed by Hellwig (1980) corresponds to the special case of \( \phi = 1 \) and \( n \in (0,\infty) \). Thus, this structure is general enough to serve our two
purposes mentioned above – demonstrating the robustness of our results and highlighting
the uniqueness of our analysis.

The price $\tilde{p}$ still linearly depends on the signals $\tilde{v}_1$ and $\tilde{v}_2$ and the noisy trading $\tilde{x}$, given
by equation (2). We still use $I_i = \frac{\alpha_i}{\alpha_x}$ to measure the price informativeness regarding $\tilde{v}_i$, and
use $I_v = \frac{1}{Var(\tilde{v}|\tilde{p})}$ to measure the overall price informativeness regarding $\tilde{v} = \tilde{v}_1 + \tilde{v}_2$.

### 6.2 Equilibrium Characterization

The CARA-normal feature implies that the demand of investor $\tilde{j}$ informed of $\tilde{v}_i$ is:

$$D_i(\tilde{p}, \tilde{s}_i^j) = \frac{E(\tilde{v}|\tilde{p}, \tilde{s}_i^j) - \tilde{p}}{\gamma Var(\tilde{v}|\tilde{p}, \tilde{s}_i^j)}.$$  

(32)

Applying Bayes’ rule yields:

$$E(\tilde{v}|\tilde{p}, \tilde{s}_i^j) = \beta_i^{(1)} \frac{\tilde{p} - \alpha_0}{\alpha_x} + \beta_i^{(2)} \tilde{s}_i^j,$$

(33)

$$Var(\tilde{v}|\tilde{p}, \tilde{s}_i^j) = \frac{(1 - \phi) (\rho/\chi) + 2 (\rho/\chi) (\rho/n) + (1 - \phi) (\rho/n) (I_1 - I_2)^2}{(1 - \phi^2) I_j^2 + (\rho/\chi) (1 + (\rho/n)) + (\rho/n) (I_1^2 + I_2^2 + 2\phi I_1 I_2)} (1 + \phi) \rho^{-1},$$

(34)

with

$$\beta_i^{(1)} = \frac{(1 + \phi) [(\rho/n) I_i + ((\rho/n) + 1 - \phi) I_j]}{(1 - \phi^2) I_j^2 + (\rho/\chi) (1 + (\rho/n)) + (\rho/n) (I_1^2 + I_2^2 + 2\phi I_1 I_2)}.$$  

(35)

$$\beta_i^{(2)} = \frac{(1 + \phi) [(\rho/\chi) + (1 - \phi) I_j^2 - (1 - \phi) I_1 I_2]}{(1 - \phi^2) I_j^2 + (\rho/\chi) (1 + (\rho/n)) + (\rho/n) (I_1^2 + I_2^2 + 2\phi I_1 I_2)}.$$  

(36)

The system determining the price informativeness measures $I_1$ and $I_2$ in the financial
market is:

$$I_i = \int_0^{\lambda_i} \partial E(\tilde{v}|\tilde{p}, \tilde{s}_i^j) d\tilde{j} = \frac{\lambda_i \beta_i^{(2)}}{\gamma Var(\tilde{v}|\tilde{p}, \tilde{s}_i^j)}, \text{ for } i = 1, 2.$$  

(37)

Equations (34), (36) and (37) jointly determine the equilibrium $I_1$ and $I_2$. Bayes’ rule also
implies that the overall price informativeness is:

$$I_v = \frac{1}{Var(\tilde{v}|\tilde{p})} = \frac{(\rho/\chi) + I_1^2 + I_2^2 + 2\phi I_1 I_2}{2(\rho/\chi) + (1 - \phi) (I_1 - I_2)^2} (1 + \phi) \rho^{-1}.$$  

(38)

26
The cost of capital is:

\[ CC = E(\hat{\nu} - \hat{p}) = \frac{\gamma}{\lambda_1 \text{Var}^{-1}(\hat{v}|\hat{p}, \hat{s}_1) + \lambda_2 \text{Var}^{-1}(\hat{v}|\hat{p}, \hat{s}_2) + (1 - \lambda_1 - \lambda_2) I_v} \]  

(39)

Going back to the information acquisition stage, the learning benefit (certainty equivalent multiplied by \(2\gamma\)) of observing signal \(\hat{s}_i\) is:

\[ B_i(I_1, I_2) = \log \left( \frac{\text{Var}(\hat{v}|\hat{p})}{\text{Var}(\hat{v}|\hat{p}, \hat{s}_i)} \right) - 2\gamma c_i \]  

(40)

The equilibrium fractions \(\lambda_1^*\) and \(\lambda_2^*\) of informed traders are obtained in two steps: first, set \(B_i(I_1^*, I_2^*) = 0\) (for \(i = 1, 2\)) to solve \(I_1^*\) and \(I_2^*\) for the overall equilibrium; and second, use the system defining \(I_1^*\) and \(I_2^*\) at the trading stage, that is, equations (34), (36) and (37), to back out \(\lambda_1^*\) and \(\lambda_2^*\).

### 6.3 Robustness of Our Results

We now examine our two main results: (i) at the trading stage, information diversity increases overall price informativeness and decreases the cost of capital; and (ii) at the learning stage, there is strategic complementarity in information acquisition.

The information diversity \(\Delta\) is still defined by equation (22):

\[ \Delta \equiv 1 - \frac{|\lambda_1 - \lambda_2|}{\Lambda} \in [0, 1] \]

where \(\Lambda\) is a fixed total size of the informed traders population. The complexity of the equations defining the equilibrium \(I_1\) and \(I_2\) precludes simple analytical analysis. Instead we use numerical analysis to examine the implications of information diversity and present the result in Figure 4. We have checked a wide range of parameters and found that the results hold for all parameter configurations.

[FIGURE 4 ABOUT HERE]

In Figure 4, we choose parameter values similar to those in Figure 2. That is, we set
\[ \rho = \chi = 50, \gamma = 3 \text{ and } \Lambda = 0.1. \] In Panel (a), we set \( \phi = 0.5 \) and \( n = \infty \) (i.e., the noise-to-signal ratio \( \frac{\theta}{\phi} \) is 0) to isolate the impact of the correlation \( \phi \) between the two ingredients \( \tilde{v}_1 \) and \( \tilde{v}_2 \). In Panel (b), we set \( \phi = 0 \) and \( n = 250 \) (i.e., the noise-to-signal ratio \( \frac{\theta}{n} \) is 0.2) to isolate the impact of the noise in the signal \( \tilde{s}_1^j \). We find that, consistent with Propositions 3 and 4 in Section 4, increasing \( \Delta \) increases \( I_e \) and decreases \( CC \) in both panels.

We use Figures 5 and 6 to investigate the implications of \( \phi \) and \( n \) for learning complementarities, respectively. In both figures, we conduct an exercise similar to Figure 3 to examine Proposition 6 - we fix \( c_2 \) and decrease \( c_1 \), and check whether there is a region for which \( \lambda_1^* \) and \( \lambda_2^* \) move in the same direction in equilibrium.\(^{10}\) Since \( \lambda_1^* \) always decreases with \( c_1 \), we only plot \( \lambda_2^* \) against \( (c_2/c_1) \), and a U-shaped \( \lambda_2^* \) indicates the existence of learning complementarities.

(Figures 5 and 6 about here)

Specifically, in both figures, we set \( \rho = \chi = 50 \) and \( \gamma = 3 \). Then, in each panel, we set \( c_2 \) at a value \( \bar{c} \) such that when \( c_1 = c_2 = \bar{c} \), the equilibrium fractions of informed traders are \( \lambda_1^* = \lambda_2^* = 0.05 \). We see that, consistent with Panel (b) of Figure 3, for all economies in Figures 5 and 6, when \( c_1 \) is sufficiently low, so that the equilibrium \( I_e^1 \) is sufficiently large, \( \lambda_2^* \) increases with \( (c_2/c_1) \) and hence moves in the same direction as \( \lambda_1^* \). Figure 5 also suggests the turning point of \( (c_2/c_1) \) increases with \( \phi \). This is because when \( \phi \) is high, the Grossman-Stiglitz effect is strong: The high correlation between \( \tilde{v}_1 \) and \( \tilde{v}_2 \) implies that much information contained in signal \( \tilde{s}_2^j \) can be learned from observing prices, which aggregate many new acquired \( \tilde{s}_1^j \) signals due to the drop in \( c_1 \).

### 6.4 Two-Dimensional Economy versus Hellwig (1980)

In this last subsection, we will compare our economy with the unidimensional economy studied by Hellwig (1980) and Verrecchia (1982) to highlight the role of multiple dimensions of fundamentals in delivering our results. As we mentioned before, when we choose \( \phi = 1 \) and

\(^{10}\)Note that Proposition 6 is an equilibrium result of complementarities in information acquisition described in Proposition 5. So, Figures 5 and 6 are a stronger test of learning complementarities. We will examine Proposition 5 more closely in the next subsection when we compare our economy with the standard unidimensional Hellwig (1980) economy.
\( n \in (0, \infty) \), the two ingredients of asset payoff reduce to one common factor and informed traders acquire a noisy signal about this common factor representing the overall asset payoff. In this case, using the equations characterizing the equilibrium, we can analytically compute the price informativeness measures, the cost of capital, and the benefit of acquiring signals, which all depend on the total size \( \Lambda \) of the informed traders population. In particular, unlike Propositions 3-5 in previous sections, now information diversity \( \Delta \) does not influence the overall price informativeness \( I_v \) and the cost of capital \( \frac{\partial B}{\partial \lambda} = \frac{\partial B}{\partial \lambda^j} = 0 \), and acquiring information is always a substitute \( \frac{\partial B_i}{\partial \lambda_i} < 0 \), which suggests that our results are driven by the fact that informed traders are informed of non-perfectly correlated components of asset payoff (i.e., \( \phi \neq 1 \)).\(^{11}\) We summarize these results in the following proposition.

**Proposition 7 [Hellwig (1980)]** Suppose \( \phi = 1 \) and \( n \in (0, \infty) \). Then, (i) \( I_1 = \frac{\lambda n}{2\gamma} \), \( I_2 = \frac{\lambda n}{2\gamma} \), \( I_v = \frac{\phi}{4} + \left( \frac{\lambda n}{4\gamma} \right)^2 \chi \), \( CC = \frac{4\gamma}{\rho + \Lambda n + \left( \frac{\lambda n}{4\gamma} \right)^2 \chi} \), and hence \( \frac{\partial B_i}{\partial \Delta} = \frac{\partial B}{\partial \Delta} = 0; \) (ii) \( B_i(\lambda_1, \lambda_2) = \log \left[ \frac{n}{\rho + \left( \frac{\lambda n}{4\gamma} \right)^2 \chi} + 1 \right] - 2\gamma c_i \), and hence \( \frac{\partial B_i}{\partial \lambda_i} = \frac{\partial B}{\partial \lambda_i} < 0 \), for \( i = 1, 2 \) and \( i \neq j \).

Figures 7 and 8 further use numerical examples to compare our economy and the Hellwig (1980) economy. In both figures, Panel (a) corresponds to the one-dimensional economy studied by Hellwig (1980): \( \phi = 1 \). Panel (b) corresponds to a two-dimensional economy where the two payoff ingredients are non-perfectly correlated: \( \phi = 0.5 \). In both economies, we assume that informed traders observe noisy signals about the asset payoff by setting \( n = 250 \) (i.e., the noise-to-signal ratio \( \frac{\phi}{4} = 0.2 \)). The other parameter values in both panels are: \( \rho = \chi = 50 \) and \( \gamma = 3 \). In Figure 7, Panel (a) illustrates Proposition 7 by showing that the overall price informativeness \( I_v \) and the cost of capital \( CC \) do not vary with information diversity \( \Delta \), while Panel (b) confirms Propositions 3 and 4 in the two-dimensional economy.

\(^{11}\) An alternative view is that the Hellwig (1980) economy is the limiting case of our two-dimensional economy and our results hold trivially. Specifically, since all traders are effectively informed of only common factor, then (i) in effect, there are no distinctions between \( \lambda_1 \) and \( \lambda_2 \), and the variable of information diversity \( \Delta \) effectively takes only one value of 0, and thus the overall price informativeness \( I_v \) and the cost of capital \( CC \) take only one value accordingly, which is consistent with Propositions 3 and 4 trivially (as there are no variations of \( \Delta \)), and (ii) for the same reason, \( \frac{\partial B}{\partial \lambda} \) effectively captures the strategic substitute in acquiring information on the same fundamental, which is consistent with Proposition 5, too. But both views highlight the fact that our results can only be developed in a two-dimensional economy like the one analyzed in this paper.
by showing that $I_v$ increases with $\Delta$ and that $CC$ decreases with $\Delta$.

[FIGURES 7 AND 8 ABOUT HERE]

In Figure 8, for both the Hellwig economy and the two-dimensional economy, we conduct two exercises to examine learning complementarities, which correspond respectively to Propositions 5 and 6. First, to examine Proposition 5 which directly describes learning complementarities through the impact of the number of informed traders on the benefit of acquiring information, we draw the simplex $\{(\lambda_1, \lambda_2) \in \mathbb{R}^2_+: \lambda_1 + \lambda_2 \leq 1\}$ – which is the whole range of $(\lambda_1, \lambda_2)$ – and use “+” to indicate the complementarity region for which the benefit $B_2$ of acquiring signal about $\tilde{v}_2$ increases with the size $\lambda_1$ of $\tilde{v}_1$-informed group (i.e., $\frac{\partial B_2}{\partial \lambda_1} > 0$). We see that there is no such a complementarity region in the Hellwig economy, while the size of the complementarity region is nontrivial in the two-dimensional economy. In addition, we find that $\frac{\partial B_2}{\partial \lambda_1}$ is more likely to be positive in the two-dimensional economy when $\lambda_1$ is relatively large relative to $\lambda_2$, which is consistent with Proposition 5 which says that $\frac{\partial B_2}{\partial \lambda_1} > 0$ if and only if $I_1 > I_2$ in the baseline model.

Second, we follow Figure 3 and draw the equilibrium $\lambda_2^*$ against $(c_2/c_1)$ to examine Proposition 6 which specifies the implications of changing the cost of information in the presence of learning complementarities. Specifically, we fix $c_2$ at a constant $\bar{c}$ (such that $\lambda_1^* = \lambda_2^* = 0.05$ when $c_1 = c_2 = \bar{c}$) and decrease $c_1$, and then plot $\lambda_2^*$ against $(c_2/c_1) = (\bar{c}/c_1)$. Then, a U-shaped $\lambda_2^*$ indicates the existence of learning complementarities, since the right upward-sloping branch is a region for which $\lambda_1^*$ and $\lambda_2^*$ move in the same direction in response to a change in $c_1$. We see that, in the two-dimensional economy, $\lambda_2^*$ indeed first decreases and then increases with $(c_2/c_1)$, which is consistent with Proposition 6. In contrast, in the Hellwig economy, $\lambda_2^*$ generally decreases with $(c_2/c_1)$: When $(c_2/c_1) < 1$, we have $c_1 > c_2 = \bar{c}$, and all informed traders acquire information about $\tilde{v}_2$ (which is the overall payoff $\tilde{v}$ too) since the cost of acquiring $\tilde{v}_2$ is cheaper than that of acquiring $\tilde{v}_1$ (which is $\tilde{v}$ too), and the size $\lambda_2^*$ is maintained at $\Lambda^* = 0.1$; when $(c_2/c_1) > 1$, then $c_2 = \bar{c} > c_1$ and thus no one will acquire $\tilde{v}_2$ and hence $\lambda_2^* = 0$; when $(c_2/c_1) = 1$, then $c_1 = c_2 = \bar{c}$, the exact size $\lambda_2^*$ is indeterminate, and it can take any value in $[0, \Lambda^*]$. 

30
Now the central message at the learning stage should be clear. The standard result that the benefit of acquiring information decreases with the total size of the informed traders population in the unidimensional Hellwig (1980) economy is not robust to perturbations in the correlation of different ingredients of the asset payoff. How traders respond to other traders becoming informed does not only depend on the total size of the informed group, but also on its composition. If traders are acquiring information about fundamentals that are weakly correlated, then acquiring information on different fundamentals tend to be a complement instead of a substitute, as the price will reveal more information that is unknown even after traders become informed and hence that is particularly useful in reducing uncertainty when combined with their own acquired information in a particular dimension.

7 Conclusion

Our paper addresses a fundamental question in the study of market efficiency and price informativeness: Does information in financial markets attract or deter the production and transmission of more information? Extending the seminal Grossman and Stiglitz (1980) model to include two dimensions of uncertainty in the value of the traded asset, we uncover a rich set of interactions, providing new insights into this question.

When the price becomes more informative about one fundamental, traders informed about another fundamental may reduce their trading aggressiveness due to an adverse selection effect, or increase it due to an uncertainty reduction effect. We show that the latter effect dominates in equilibrium if and only if the levels of informativeness about the two fundamentals are sufficiently close to each other, implying that greater diversity of information in the economy generates amplification of informativeness and hence greater equilibrium market efficiency. We also show that the uncertainty reduction effect can generate strategic complementarities in information production, leading to another form of a market efficiency spiral.

The paper highlights the importance of information diversity in determining the level of market efficiency. Future empirical research can test the predictions of the paper by linking the diversity of trading styles to measures of market efficiency and price informativeness.
Future theoretical research can extend the model in various directions, such as including
more dimensions of uncertainty and allowing traders to be informed in various dimensions,
to improve our understanding of the effect of information structure on the efficiency of
financial markets.

Appendix: Proofs

A. Proof of Proposition 1

Using the expressions for $D_i(p, \tilde{v}_i)$ and $D_u(p)$ (given by equations (8) and (9)) and the definition
of $\tilde{s}_{ij}$ (given by equations (4)), we can rewrite the market clearing condition (equation (14)) as
follows:

$$
\begin{align*}
\lambda_1 \left[ \rho + (\alpha_2/\alpha_x)^2 \chi - (\alpha_2/\alpha_x)^2 \chi \rho \right] & + \lambda_2 \left[ \rho + (\alpha_1/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)^2 \chi \rho \right] \\
& + \lambda_3 \left[ \rho + (\alpha_1/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)^2 \chi \rho \right] \tilde{v}_1 \\
& + \lambda_4 \left[ \rho + (\alpha_1/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)^2 \chi \rho \right] \tilde{v}_2 + \gamma \tilde{x}.
\end{align*}
$$

Note that the left-hand side of the above equation is only related to $\tilde{p}$, while the right-hand
side is only related to $\tilde{v}_1$, $\tilde{v}_2$, and $\tilde{x}$. Hence, based on (41) and (2), we form the following system of
two equations in terms of two unknowns, $(\alpha_1/\alpha_x)$ and $(\alpha_2/\alpha_x)$:

$$
\begin{align*}
(\alpha_1/\alpha_x) &= \lambda_1 \left[ \rho + (\alpha_2/\alpha_x)^2 \chi - (\alpha_1/\alpha_x) (\alpha_2/\alpha_x) \chi \right] \gamma^{-1}, \\
(\alpha_2/\alpha_x) &= \lambda_2 \left[ \rho + (\alpha_1/\alpha_x)^2 \chi - (\alpha_1/\alpha_x) (\alpha_2/\alpha_x) \chi \right] \gamma^{-1}.
\end{align*}
$$

By equation (43), we can express $(\alpha_2/\alpha_x)$ in terms of $(\alpha_1/\alpha_x)$ as follows (this is equation (19)
for $i = 2$ in the main text):

$$
\frac{\alpha_2}{\alpha_x} = \frac{\lambda_2 \left[ \rho + \chi (\alpha_1/\alpha_x)^2 \right]}{[\gamma + \lambda_2 \chi (\alpha_1/\alpha_x)]}.
$$

Then, plugging the above expression into equation (42), we have the following cubic polynomial in
$(\alpha_1/\alpha_x)$:

$$
\begin{align*}
& (\gamma \alpha_1 \lambda_2 \chi^2 + \gamma \lambda_2^2 \chi^2) (\alpha_1/\alpha_x)^3 + 2 (\gamma^2 \lambda_2 \chi - \lambda_1 \lambda_2^2 \rho \chi) (\alpha_1/\alpha_x)^2 \\
& + (\gamma \lambda_1 \lambda_2 \rho \chi - 2 \gamma \lambda_1 \lambda_2 \rho \chi + \gamma^2) (\alpha_1/\alpha_x) + (\lambda_1 \lambda_2^2 \rho^2 \chi - \gamma^2 \lambda_1 \rho) \\
& = 0.
\end{align*}
$$

32
The discriminant of the polynomial is

$$-\chi^2 \lambda_1 \lambda_2 (\gamma^2 + \rho \chi \lambda_2^2)^2 \left( 4 \gamma^4 + 24 \gamma^4 \chi \lambda_1 \lambda_2 \rho + 32 \chi^3 \lambda_2^3 \rho^3 + \gamma^2 \chi \lambda_1 \lambda_2 (26 \lambda_1^2 \rho^2 + (\lambda_1 \rho - 3 \lambda_2 \rho)^2 + 18 \lambda_2^2 \rho^2) \right) < 0.$$ 

Thus, equation (45) has only one real solution for \((\alpha_1/\alpha_x)\). In addition, this solution is positive, because when \(\lambda_1 > 0\) and \(\lambda_2 > 0\), the left-hand side of equation (45) is negative at \((\alpha_1/\alpha_x) = 0\) and becomes positive as \((\alpha_1/\alpha_x)\) is sufficiently large. By equation (44), we also know that the equilibrium coefficient ratio \(\frac{\alpha_0}{\alpha_x}\) is positive.

Using equations (9)-(13) to express out \(\frac{\tilde{p} - E(\bar{v}|\bar{p})}{\text{Var}(\bar{v}|\bar{p})}\) in equation (41) delivers

$$\left( A_{p0} - A_{px} \frac{1}{\alpha_x} \right) \tilde{p} = A_1 \tilde{v}_1 + A_2 \tilde{v}_2 + \gamma \hat{x} + \left( -A_{01} \frac{\alpha_0}{\alpha_1} - A_{02} \frac{\alpha_0}{\alpha_2} - A_{0x} \frac{\alpha_0}{\alpha_x} - \gamma \right)$$

where the coefficients of \(A\)'s are known positive values – which are determined by \((\alpha_1/\alpha_x)\) and \((\alpha_2/\alpha_x)\) – defined as follows:

$$A_{p0} = \frac{\lambda_1}{\text{Var}(\bar{v}|\mathcal{F}_1)} + \frac{\lambda_2}{\text{Var}(\bar{v}|\mathcal{F}_2)} + \frac{\lambda_n}{\text{Var}(\bar{v} \mid \bar{p})},$$

$$A_{px} = \lambda_1 (\alpha_2/\alpha_x) \chi + \lambda_2 (\alpha_1/\alpha_x) \chi + \frac{\lambda_n \beta_{\bar{v},\bar{p}}}{\text{Var}(\bar{v} \mid \bar{p})},$$

$$A_{01} = \lambda_2 (\alpha_1/\alpha_x)^2 \chi, A_{02} = \lambda_1 (\alpha_2/\alpha_x)^2 \chi, A_{0x} = \lambda_n \beta_{\bar{v},\bar{p}} \frac{1}{\text{Var}(\bar{v} \mid \bar{p})}.$$ 

Thus, we can solve for \(\alpha_x\):

$$\alpha_x = \frac{\gamma}{A_{p0} - A_{px} \frac{1}{\alpha_x}} \Rightarrow \alpha_x = \frac{\gamma + A_{px}}{A_{p0}} > 0.$$ 

Combining the known ratios \((\alpha_1/\alpha_x)\) and \((\alpha_2/\alpha_x)\) with the value of \(\alpha_x\) gives the values of \(\alpha_1\) and \(\alpha_2\), which are positive. Once we know \(\alpha_1, \alpha_2\) and \(\alpha_x\), then we can solve \(\alpha_0\) using

$$\alpha_0 = -\frac{A_{01} \frac{\alpha_0}{\alpha_1} - A_{02} \frac{\alpha_0}{\alpha_2} - A_{0x} \frac{\alpha_0}{\alpha_x} - \gamma}{A_{p0} - A_{px} \frac{1}{\alpha_x}} \Rightarrow \alpha_0 = -\frac{\gamma}{A_{p0} - A_{px} \frac{1}{\alpha_x}} + A_{01} \frac{1}{\alpha_1} + A_{02} \frac{1}{\alpha_2} + A_{0x} \frac{1}{\alpha_x}.$$ 

We can further use the solved expressions of \(A_{01}, A_{02}, A_{0x}\) and \(\alpha_x\) to simplify the denominator of the above expression of \(\alpha_0\) and show that

$$\alpha_0 = -\frac{\gamma}{A_{p0}} < 0.$$ 

This in turn implies that the cost of capital is

$$CC = E(\bar{v} - \bar{p}) = -\alpha_0 = \frac{\gamma}{A_{p0}}.$$ 

QED.
B. Proof of Proposition 2

Taking total differentiation of equation (19) (for \(i = 1, 2\)) with respect to \(Q\) implies:

\[
\frac{dI_i}{dQ} = \frac{\partial h_i}{\partial Q} + \frac{\partial h_1}{\partial I_2} \frac{dI_2}{dQ} \quad \text{and} \quad \frac{dI_2}{dQ} = \frac{\partial h_2}{\partial Q} + \frac{\partial h_2}{\partial I_1} \frac{dI_1}{dQ}.
\]

Solving for \(\frac{dI_1}{dQ}\) and \(\frac{dI_2}{dQ}\) delivers

\[
\frac{dI_1}{dQ} = \frac{\partial h_1}{\partial Q} + \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1} \quad \text{and} \quad \frac{dI_2}{dQ} = \frac{\partial h_2}{\partial Q} + \frac{\partial h_2}{\partial I_1} \frac{\partial h_1}{\partial I_2},
\]

which is equation (20).

Next, we examine the sign and magnitude of \(M = \left(1 - \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1}\right)^{-1}\). By equation (19), direct computation shows

\[
\frac{\partial h_i}{\partial I_j} = 1 - \frac{\gamma^2 + \chi \rho \lambda_i^2}{\left(\gamma + \chi \lambda_i I_j\right)^2},
\]

(50)

Using equations (42)-(43), we can express \(\lambda_i\) in terms of \(I_1\) and \(I_2\) as follows:

\[
\lambda_i = \frac{\gamma I_i}{\rho + I_j \chi - I_1 I_2 \chi}.
\]

(51)

Plugging the above expression into equation (50) yields:

\[
\frac{\partial h_i}{\partial I_j} = \frac{(2I_j - I_i) \chi I_i}{\rho + \chi I_j^2}.
\]

(52)

Thus, we have

\[
M^{-1} = 1 - \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1} = 1 - \frac{(2I_2 - I_1) \chi I_1 (2I_1 - I_2) \chi I_2}{\rho + \chi I_2^2} = \frac{(\rho + 2\chi I_1 I_2) \left(\rho + \chi (I_1 - I_2)^2\right)}{(\rho + \chi I_2^2) (\rho + \chi I_1^2)} > 0.
\]

That is, \(M > 0\).

Whether \(M > 1\) depends on whether \(\frac{\partial h_i}{\partial I_2}\) and \(\frac{\partial h_2}{\partial I_1}\) have the same sign. Specifically, we have three cases.

Case 1. If \(\frac{\partial h_i}{\partial I_2} = 0\) for some \(i\), then \(M = 1\). By equation (52), this will be true if and only if

\[
\frac{\partial h_i}{\partial I_j} = \frac{(2I_j - I_i) \chi I_i}{\rho + \chi I_j^2} = 0 \Rightarrow \frac{I_i}{I_j} = 2.
\]

Case 2. If \(\frac{\partial h_i}{\partial I_2} > 0\) for \(i = 1, 2\), then \(M > 1\). By equation (52), this will be true if and only if

\[
\frac{\partial h_i}{\partial I_j} = \frac{(2I_j - I_i) \chi I_i}{\rho + \chi I_j^2} > 0 \Rightarrow \frac{I_i}{I_j} < 2, \forall i \Rightarrow \frac{1}{2} < \frac{I_1}{I_2} < 2.
\]
Case 3. If $\frac{\partial h_1}{\partial \mu_t} > 0$ and $\frac{\partial h_2}{\partial \mu_t} < 0$, then $0 < M < 1$. This will be true if and only if $\frac{I_t}{\pi} > 2$, i.e., $\frac{\partial I_t}{\partial \pi} > 2$ or $\frac{\partial I_t}{\partial \pi} < 2$.

Note that it is not possible to have both $\frac{\partial h_1}{\partial \mu_t} < 0$ and $\frac{\partial h_2}{\partial \mu_t} < 0$, because these two inequalities combine to imply $2 < \frac{I_t}{\pi} < \frac{1}{2}$, which is impossible. QED.

C. Proof of Proposition 3

Without loss of generality, we assume that $\lambda_1 > \lambda_2$, and as a result, increasing diversity $\Delta$ while fixing $\Lambda$ is equivalent to decreasing $\lambda_1$ and increasing $\lambda_2$. Formally, we have

$$\lambda_1 + \lambda_2 = \Lambda \text{ and } \Delta = 1 - \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \frac{2\lambda_2}{\lambda_1 + \lambda_2}.$$ 

Then, taking total differentiation of the above system, yields:

$$d\lambda_1 + d\lambda_2 = d\Lambda = 0,$$

$$d\left(\frac{2\lambda_2}{\lambda_1 + \lambda_2}\right) = 2\left(-\frac{\lambda_2}{(\lambda_1 + \lambda_2)^2}d\lambda_1 + \frac{(\lambda_1 + \lambda_2) - \lambda_2}{(\lambda_1 + \lambda_2)^2}d\lambda_2\right) = d\Delta,$$

which implies:

$$\frac{d\lambda_1}{d\Delta} = -\frac{\Lambda}{2} \text{ and } \frac{d\lambda_2}{d\Delta} = \frac{\Lambda}{2},$$

and

$$\frac{\partial h_1}{\partial Q} = \frac{\partial h_1}{\partial \Delta} = -\frac{\partial h_1 \Lambda}{\partial \lambda_1 \Delta^2} \text{ and } \frac{\partial h_2}{\partial Q} = \frac{\partial h_2 \Lambda}{\partial \lambda_2 \Delta^2}.$$ 

in equation (20).

Now, we analyze the impact of $\Delta$ on the total price informativeness $I_v$. By equations (20) and (54), we have

$$\frac{dI_1}{d\Delta} = M \left( \frac{\partial h_1}{\partial \Delta} + \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial \Delta} \right) = M \left( \frac{\partial h_1}{\partial \lambda_1} + \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial \lambda_2} \right) \frac{\Lambda}{2}.$$ 

Then, using equations (50) and (51), we have

$$\frac{dI_1}{d\Delta} = M \frac{\Lambda}{2} \gamma \left[ -\rho + \chi I_2^2 \frac{(2I_2 - I_1) \chi I_1}{\left(\gamma + \chi \frac{\gamma I_1}{\rho + \gamma} \frac{\gamma I_1}{\rho + I_2} I_2\right)^2} + \frac{(2I_2 - I_1) \chi I_1}{\left(\gamma + \chi \frac{\gamma I_1}{\rho + \gamma} \frac{\gamma I_1}{\rho + I_2} I_2\right)^2} \frac{\rho + \chi I_2^2}{\left(\gamma + \chi \frac{\gamma I_1}{\rho + \gamma} \frac{\gamma I_1}{\rho + I_2} I_2\right)^2} \right].$$ 

Similarly, we can compute:

$$\frac{dI_2}{d\Delta} = M \frac{\Lambda}{2} \gamma \left[ -\rho + \chi I_2^2 \frac{(2I_1 - I_2) \chi I_2}{\left(\gamma + \chi \frac{\gamma I_2}{\rho + \gamma} \frac{\gamma I_2}{\rho + I_1} I_1\right)^2} - \frac{(2I_1 - I_2) \chi I_2}{\left(\gamma + \chi \frac{\gamma I_2}{\rho + \gamma} \frac{\gamma I_2}{\rho + I_1} I_1\right)^2} \frac{\rho + \chi I_1^2}{\left(\gamma + \chi \frac{\gamma I_2}{\rho + \gamma} \frac{\gamma I_2}{\rho + I_1} I_1\right)^2} \right].$$ 

35
Recall that by equation (15), we have:

\[ I_v = \frac{1}{\text{Var}(\hat{v}|\rho)} = \frac{I_1^2 \rho + I_2^3 \rho + \rho^2 \chi^{-1}}{(I_1 - I_2)^2 + 2 \rho \chi^{-1}}. \]

Direct computation shows:

\[
\begin{align*}
\frac{\partial I_v}{\partial I_1} &= \frac{2 \chi \rho (I_1 + I_2) \left( \rho + \chi I_1^2 - \chi I_1 I_2 \right)}{(2 \rho + \chi I_1^2 + \chi I_2^2 - 2 \chi I_1 I_2)^2} > 0, \\
\frac{\partial I_v}{\partial I_2} &= \frac{2 \chi \rho (I_1 + I_2) \left( \rho + \chi I_1^2 - \chi I_1 I_2 \right)}{(2 \rho + \chi I_1^2 + \chi I_2^2 - 2 \chi I_1 I_2)^2} > 0,
\end{align*}
\]

where the inequalities follow from equations (42)-(43), namely, \( (\rho + I_1^2 \chi - I_1 I_2 \chi) = \frac{\gamma I_1}{\gamma} > 0. \)

By chain rule:

\[
\frac{dI_v}{d\Delta} = \frac{\partial I_v}{\partial I_1} \frac{dI_1}{d\Delta} + \frac{\partial I_v}{\partial I_2} \frac{dI_2}{d\Delta}.
\]

Plugging equations (55)-(58) into the above equation delivers:

\[
\frac{dI_v}{d\Delta} = - \frac{\gamma \lambda \chi}{2} \frac{2 \chi^2 \rho (I_1^2 - I_2^2) \left( I_1 + I_2 \right) \rho^3 + \chi \rho^2 (I_1 + I_2)^2 + 3 \chi^2 \rho I_1 I_2 (I_1 - I_2)^2 + \chi^3 I_1 I_2 (I_1 - I_2)^4}{(2 \rho + \chi I_1^2 + \chi I_2^2 - 2 \chi I_1 I_2)^2} \frac{\gamma^2 \left( \rho + \chi I_1^2 \right) \left( \rho + \chi I_2^2 \right)}{\rho^2 (\rho + \chi I_1^2) (\rho + \chi I_2^2) (\rho + \chi I_1^2 - \chi I_1 I_2) (\rho + \chi I_2^2 - \chi I_1 I_2)} > 0,
\]

because \( I_1 > I_2 \) by \( \lambda_1 > \lambda_2 \). QED.

**D. Proof of Proposition 4**

We build on the proof of the previous proposition. By equations (25) and (24), to show \( \frac{dCC}{d\Delta} > 0 \), it suffices to show \( \frac{dA_{\rho \theta}}{d\Delta} > 0 \). By equations (26) and (53), we have

\[
\frac{dA_{\rho \theta}}{d\Delta} = \frac{d\lambda_1}{d\Delta} \left( \rho + \chi I_2^2 \right) + \lambda_1 \left( \chi 2 I_2 \frac{dI_2}{d\Delta} \right) + \frac{d\lambda_2}{d\Delta} \left( \rho + \chi I_1^2 \right) + \lambda_2 \left( \chi 2 I_1 \frac{dI_1}{d\Delta} \right) + (1 - \Lambda) \frac{dI_v}{d\Delta}
\]

\[
= \frac{\Lambda \chi}{2} \left( I_1^2 - I_2^2 \right) + (1 - \Lambda) \frac{dI_v}{d\Delta} + 2 \chi \left( \lambda_1 I_2 \frac{dI_2}{d\Delta} + \lambda_2 I_1 \frac{dI_1}{d\Delta} \right).
\]

Since \( \frac{\Lambda \chi}{2} \left( I_1^2 - I_2^2 \right) > 0 \) and \( (1 - \Lambda) \frac{dI_v}{d\Delta} > 0 \), to show \( \frac{dA_{\rho \theta}}{d\Delta} > 0 \), it suffices to show

\[
\left( \lambda_1 I_2 \frac{dI_2}{d\Delta} + \lambda_2 I_1 \frac{dI_1}{d\Delta} \right) > 0,
\]

which is true, because equations (51), (55) and (56) imply:

\[
\lambda_1 I_2 \frac{dI_2}{d\Delta} + \lambda_2 I_1 \frac{dI_1}{d\Delta} = - \frac{\Lambda \chi}{2} I_1 I_2 (I_1^2 - I_2^2) \frac{\rho^3 + \chi \rho^2 (I_1 + I_2)^2 + 3 \chi^2 \rho I_1 I_2 (I_1 - I_2)^2 + \chi^3 I_1 I_2 (I_1 - I_2)^4}{(\rho + \chi I_1^2) (\rho + \chi I_2^2) (\rho + \chi I_1^2 - \chi I_1 I_2) (\rho + \chi I_2^2 - \chi I_1 I_2)} > 0.
\]

QED.
E. Proof of Proposition 5

Let us first prove $\frac{\partial B_i}{\partial \lambda_i} < 0$. By Proposition 2, we have:

$$\frac{dI_i}{d\lambda_i} = M \frac{\gamma (\rho + \chi I^2_i)}{(\gamma + \lambda \psi I_i)^2} > 0 \quad \text{and} \quad \frac{dI_j}{d\lambda_i} = \frac{(2I_i - I_j) \chi I_j dI_i}{\rho + \chi I^2_i} d\lambda_i.$$

By equation (27) and the above expression of $\frac{dI_i}{d\lambda_i}$, we have:

$$\frac{\partial B_i}{\partial \lambda_i} = \frac{2I_J \chi}{\rho + \chi I^2_j} \frac{dI_j}{d\lambda_i} - \frac{1}{I_v} \left[ \frac{\partial I_v}{\partial I} \frac{dI_i}{d\lambda_i} + \frac{\partial I_v}{\partial I_j} \frac{dI_j}{d\lambda_i} \right]$$

$$= \left[ \frac{2I_J \chi}{\rho + \chi I^2_j} - \frac{1}{I_v} \frac{\partial I_v}{\partial I} \right] \left( \frac{2I_i - I_j}{I_v} \frac{dI_i}{d\lambda_i} \right) \frac{(2I_i - I_j) \chi I_j dI_i}{\rho + \chi I^2_i} - \frac{1}{I_v} \frac{\partial I_v}{\partial I_j} \frac{dI_j}{d\lambda_i}.$$

Then, using equations (15), (57) and (58), we can show the bracket term in the above equation is negative:

$$\frac{(2I_I - I) \chi I_I}{\rho + \chi I^2_i} \left[ \frac{\rho^2 I_1 + \rho^2 I_2 + \chi \rho I_1^3 + \chi \rho I_2^3 + \chi \rho I_1 (I_1 - I_2)^2}{(\rho + \chi I^2_1) (\rho + \chi I^2_2)} \left( \rho + \chi I^2_1 + \chi I^2_2 \right) (2(I_1^2 + \chi I^2_2) (2(\rho + \chi I^2_1 + \chi I^2_2 - 2\chi I_1 I_2)) \right] < 0,$$

where we have used the fact of $\left( \rho + \chi I^2_1 - \chi I_1 I_2 \right) = \frac{\gamma I_0}{\psi} > 0$. Thus, $\frac{\partial B_i}{\partial \lambda_i} < 0$.

Next, we prove $\frac{\partial B_i}{\partial \lambda_j} > 0$ if and only if $I_j > I_i$. Following the similar argument as above, we can show:

$$\frac{\partial B_i}{\partial \lambda_j} = \left[ \frac{2I_J \chi}{\rho + \chi I^2_j} - \frac{1}{I_v} \frac{\partial I_v}{\partial I} \frac{dI_i}{d\lambda_i} \right] \frac{dI_j}{d\lambda_j}$$

$$= \frac{2\chi (I_j - I_i) \left( \rho + \chi I^2_j - \chi I_1 I_2 \right)^2}{\left( \rho + \chi I^2_j \right) (2\rho + \chi I^2_1 + \chi I^2_2 - 2\chi I_1 I_2)} \frac{dI_j}{d\lambda_j}.$$

Since $\frac{dI_j}{d\lambda_j} > 0$, we have $\frac{\partial B_i}{\partial \lambda_j} > 0$ if and only if $I_j > I_i$. QED.

F. Proof of Proposition 6

Setting $B_i (I_1, I_2) = 0$ for $i = 1, 2$ delivers the following system:

$$\begin{cases}
\log (\rho + I^2_1) - \log (I^*_0) = 2\gamma c_1, \\
\log (\rho + I^2_1) - \log (I^*_0) = 2\gamma c_2.
\end{cases} \quad (59)$$
Suppose we decrease $c_1$. Applying the implicit function theorem to the above system delivers:

\[
\begin{align*}
-1 \frac{\partial I^*_v}{I^*_v \partial c_1} + \left( -1 \frac{\partial I^*_v}{I^*_v \partial c_1} + \frac{2 I^*_v \chi}{p + I^*_v I^*_2} \right) \frac{dI^*_v}{dc_1} &= 2 \gamma, \\
\left( -1 \frac{\partial I^*_v}{I^*_v \partial c_1} + \frac{2 I^*_v \chi}{p + I^*_v I^*_2} \right) \frac{dI^*_v}{dc_1} - \frac{1}{I^*_v \partial c_1} \frac{dI^*_v}{dc_1} &= 0.
\end{align*}
\]

Thus,

\[
\frac{dI^*_v}{dc_1} = \frac{-2 \gamma}{D} \frac{1}{I^*_v} \frac{\partial I^*_v}{I^*_v \partial c_1}
\quad \text{and} \quad
\frac{dI^*_v}{dc_1} = \frac{-2 \gamma}{D} \left( - \frac{1}{I^*_v} \frac{\partial I^*_v}{I^*_v \partial c_1} + \frac{2 I^*_v \chi}{\rho + I^*_v I^*_2} \right),
\]

where

\[
D = \frac{4 \chi^2 (I^*_1)^2 + I^*_2^2) \left( \rho + \chi I^*_2^2 - \chi I^*_1 I^*_2 \right) \left( \rho + \chi I^*_1^2 - \chi I^*_1 I^*_2 \right)}{(\rho + \chi I^*_2^2) \left( \rho + \chi I^*_1^2 + \chi I^*_2^2 \right) \left( 2 \rho + \chi I^*_1^2 + \chi I^*_2^2 - 2 \chi I^*_1 I^*_2 \right)} > 0.
\]

**Impact on $\lambda^*_1$ and $\lambda^*_2$.** By equation (51),

\[
\frac{d\lambda^*_1}{dc_1} = \gamma \frac{(\rho + I^*_2^2 \chi) \frac{dI^*_v}{dc_1} + (I^*_1 - 2 I^*_2) I^*_1 \chi \frac{dI^*_v}{dc_1}}{(\rho + I^*_2^2 \chi - I^*_1 I^*_2 \chi)^2}.
\]

Then, plugging equation (60), (15), (57) and (58) into the above expression yields:

\[
\frac{d\lambda^*_1}{dc_1} = \frac{-4 \chi^2 (\rho + \chi I^*_1^2 - \chi I^*_1 I^*_2)}{D (\rho + I^*_2^2 \chi - I^*_1 I^*_2 \chi)^2} \frac{\rho^2 I^*_1 + \rho^2 I^*_2 + \chi \rho I^*_1^2 + \chi I^*_1 I^*_2 + 2 \chi I^*_1 I^*_2^2}{\left( I^*_1 - I^*_2 \right)^2 (\rho + \chi I^*_1^2 + \chi I^*_2^2) (2 \rho + \chi I^*_1^2 + \chi I^*_2^2 - 2 \chi I^*_1 I^*_2)} < 0.
\]

Similarly, we can compute:

\[
\frac{d\lambda^*_2}{dc_1} = \gamma \frac{I^*_2 (I^*_2 - 2 I^*_1) \chi \frac{dI^*_v}{dc_1} + (\rho + I^*_1^2 \chi) \frac{dI^*_v}{dc_1}}{(\rho + I^*_2^2 \chi - I^*_1 I^*_2 \chi)^2}
\]

\[
= \frac{-4 \chi^2 (I^*_1 - I^*_2)}{D (\rho + I^*_2^2 \chi - I^*_1 I^*_2 \chi)^2} \frac{\rho + \chi I^*_1^2 - \chi I^*_1 I^*_2}{(\rho + \chi I^*_1^2 + \chi I^*_2^2) (2 \rho + \chi I^*_1^2 + \chi I^*_2^2 - 2 \chi I^*_1 I^*_2)} < 0.
\]

Thus, $\frac{d\lambda^*_1}{dc_1} < 0$ if and only if $I^*_1 > I^*_2$.

Finally, we have

\[
I^*_1 > I^*_2 \iff c_1 < c_2,
\]

because the system of determining $I^*_1$ and $I^*_2$ (equation (59)) implies that

\[
\log (\rho + I^*_2^2 \chi) - \log (\rho + I^*_1^2 \chi) = 2 \gamma (c_1 - c_2).
\]

Therefore, $\frac{d\lambda^*_2}{dc_1} < 0$ if and only if $c_1 < c_2$.

**Impact on $I^*_v$.** By the chain rule,

\[
\frac{dI^*_v}{dc_1} = \frac{\partial I^*_v}{\partial I^*_1} \frac{dI^*_1}{dc_1} + \frac{\partial I^*_v}{\partial I^*_2} \frac{dI^*_2}{dc_1}.
\]
Plugging equations (57), (58) and (60) into the above equation delivers:

\[
\frac{dI_v^*}{dc_1} = -\frac{2\gamma \partial I_v^*}{D \partial I_2} \frac{2I_1^* \chi}{\rho + I_2^2 \chi} < 0.
\]

**Impact on CC*.** Given \(\frac{dI_v^*}{dc_1} < 0\), to show \(\frac{dCC^*}{dc_1} > 0\), it suffices to show that in equation (28) the term \([\lambda_1^* (\rho + I_2^2 \chi - I_v^*) + \lambda_2^* (\rho + I_2^2 \chi - I_v^*)]\) decreases with \(c_1\). By equations (15) and (51), we have:

\[
\lambda_1^* (\rho + I_2^2 \chi - I_v^*) + \lambda_2^* (\rho + I_2^2 \chi - I_v^*) = \gamma \rho \frac{I_1^* + I_2^*}{2\rho + \chi I_1^2 + \chi I_2^2 - 2\chi I_1^* I_2^*}.
\]

Direct computation shows:

\[
\frac{d}{dc_1} \left[ \frac{\gamma \rho}{2\rho + \chi I_1^2 + \chi I_2^2 - 2\chi I_1^* I_2^*} \right] I_1^* + I_2^* \quad = \quad \gamma \rho \frac{(2\rho - \chi I_1^2 + 3\chi I_2^2 - 2\chi I_1^* I_2^*) \frac{dI_1^*}{dc_1} + (2\rho + 3\chi I_1^2 - \chi I_2^2 - 2\chi I_1^* I_2^*) \frac{dI_2^*}{dc_1}}{(2\rho + \chi I_1^2 + \chi I_2^2 - 2\chi I_1^* I_2^*)^2}.
\]

Plugging equations (57), (58) and (60) into the above equation shows:

\[
\frac{d}{dc_1} \left[ \lambda_1^* (\rho + I_2^2 \chi - I_v^*) + \lambda_2^* (\rho + I_2^2 \chi - I_v^*) \right] \quad = \quad \frac{4\gamma^2 \rho \chi (\rho I_1^3 + \chi I_2^3 + 2\chi I_1^* I_2^2 + I_1^* (\rho + \chi I_1^2 - \chi I_1^* I_2^*)) (\rho + \chi I_2^2 - \chi I_1^* I_2^*)}{D (2\rho + \chi I_1^2 + \chi I_2^2 - 2\chi I_1^* I_2^*)^2 (\rho + \chi I_1^2 + \chi I_2^2)} < 0.
\]

QED.
References


This figure plots the functions $h_1$ and $h_2$ whose intersections determine the equilibrium values of $I_1$ and $I_2$. Parameter $\lambda_1$ increases from 0.05 to 0.1. Other relevant parameter values are: $\rho = \chi = 50$ and $\gamma = 3$. 
This figure plots the impact of information diversity on market efficiency multiplier, price informativeness regarding total cash flow, and cost of capital. The parameter values are $\rho = \chi = 50$, $\gamma = 3$ and $\Lambda = 0.1$. 
Figure 3: Learning Complement vs. Substitute

This figure plots the impact of the cost $c_1$ of acquiring $\bar{p}_1$ on the endogenous fractions of informed traders and the price informativeness measures. When the two fractions $\lambda_1^*$ and $\lambda_2^*$ of informed traders move in the same direction, there exist learning complementarities; otherwise, information acquisition is a substitute. The parameter values are: $\rho = \chi = 50$, $\gamma = 3$, and $c_2 = 0.1$. The range of $c_1$ is [0.05, 0.13].
Figure 4: Impact of Diversity (Δ) in Extended Economies

This figure plots the impact of information diversity on price informativeness regarding overall cash flow and the cost of capital in the extended model described in Section 6, where the two payoff ingredients are correlated and where informed traders observe noisy signals about the two ingredients. The parameter values are $\rho = \chi = 50$, $\gamma = 3$ and $\Lambda = 0.1$. 
Figure 5: Impact of Cash Flow Correlation ($\phi$) on Learning Complementarties

This figure plots the impact of the cash flow correlation parameter $\phi$ on the response of $\lambda_2^*$ to the decrease in $c_1$ in the extended model described in Section 6, where the two payoff ingredients are correlated and where informed traders observe noisy signals about the two ingredients. For each panel, $c_2$ is fixed at a value $\bar{c}$, which is chosen to make $\lambda_1^* = \lambda_2^* = 0.05$ when $c_1 = c_2 = \bar{c}$. The other parameter values are: $\rho = \chi = 50$ and $\gamma = 3$. 
Figure 6: Impact of Noise-to-Signal Ratio ($\rho/n$) on Learning Complementarities

This figure plots the impact of the noise-to-signal ratio ($\rho/n$) of the acquired signals on the response of $\lambda_2^*$ to the decrease in $c_1$ in the extended model described in Section 6, where the two payoff ingredients are correlated and where informed traders observe noisy signals about the two ingredients. For each panel, $c_2$ is fixed at a value $\bar{c}$, which is chosen to make $\lambda_1^* = \lambda_2^* = 0.05$ when $c_1 = c_2 = \bar{c}$. The other parameter values are: $\rho = \chi = 50$ and $\gamma = 3$. 

(a) $\rho/n = 0$

(b) $\rho/n = 0.1$

(c) $\rho/n = 0.2$

(d) $\rho/n = 0.3$
Figure 7: Two-Dimensional Economy vs. Hellwig (1980): Impact of Diversity (Δ)

This figure plots the impact of information diversity on price informativeness regarding total cash flow and the cost of capital in two different economies. Panel (a) corresponds to the one-dimensional economy studied by Hellwig (1980); that is, the two payoff ingredients are perfectly correlated and informed traders observe noisy signals about the two ingredients (φ=1 and n=250). Panel (b) corresponds to a two-dimensional economy studied by Section 6; that is, the two payoff ingredients are non-perfectly correlated and where informed traders observe noisy signals about the two ingredients (φ=0.5 and n=250). The other parameter values in both panels are ρ = χ = 50, γ = 3 and Δ = 0.1.
Figure 8: Two-Dimensional Economy vs. Hellwig (1980): Learning Complementarities

Panel (a) corresponds to the one-dimensional economy studied by Hellwig (1980); that is, the two payoff ingredients are perfectly correlated and informed traders observe noisy signals about the two ingredients ($\phi = 1$ and $n = 250$). Panel (b) corresponds to a two-dimensional economy studied by Section 6; that is, the two payoff ingredients are non-perfectly correlated and where informed traders observe noisy signals about the two ingredients ($\phi = 0.5$ and $n = 250$). In both panels, on the left we use “+” to indicate the region for which the benefit $B_2$ of acquiring signal about $\tilde{v}_2$ increases with the size $\lambda_1$ of $\tilde{v}_1$-informed traders (i.e., $\frac{\partial B_2}{\partial \lambda_1} > 0$). On the right we plot the response of equilibrium $\lambda_2^*$ to the decrease in $c_1$, while $c_2$ is fixed at a constant $\bar{c}$, which is chosen to make $\lambda_1^* = \lambda_2^* = 0.05$ when $c_1 = c_2 = \bar{c}$. The other parameter values in both panels are: $\rho = \chi = 50$ and $\gamma = 3$. 

This figure plots the possibility of learning complementarities in two economies. Panel (a) corresponds to the one-dimensional economy studied by Hellwig (1980); that is, the two payoff ingredients are perfectly correlated and informed traders observe noisy signals about the two ingredients ($\phi = 1$ and $n = 250$). Panel (b) corresponds to a two-dimensional economy studied by Section 6; that is, the two payoff ingredients are non-perfectly correlated and where informed traders observe noisy signals about the two ingredients ($\phi = 0.5$ and $n = 250$). In both panels, on the left we use “+” to indicate the region for which the benefit $B_2$ of acquiring signal about $\tilde{v}_2$ increases with the size $\lambda_1$ of $\tilde{v}_1$-informed traders (i.e., $\frac{\partial B_2}{\partial \lambda_1} > 0$). On the right we plot the response of equilibrium $\lambda_2^*$ to the decrease in $c_1$, while $c_2$ is fixed at a constant $\bar{c}$, which is chosen to make $\lambda_1^* = \lambda_2^* = 0.05$ when $c_1 = c_2 = \bar{c}$. The other parameter values in both panels are: $\rho = \chi = 50$ and $\gamma = 3$. 

49