On the Equivalence of Private and Public Money*

Markus K. Brunnermeier† Dirk Niepelt‡

January 1, 2019

Abstract

We propose a generic model of money and liquidity. We provide sufficient conditions under which a swap of private (inside) against public (outside) money leaves the equilibrium allocation and price system unchanged. We apply the results to Central Bank Digital Currency, the "Chicago Plan," and the Indian de-monetization experiment.

Keywords: Money creation, Monetary system, Inside money, Outside money, Equivalence, CBDC, Chicago Plan, Sovereign money

*For comments and discussions, we thank Joseph Abadi, Hans Gersbach, Martin Gonzalez-Eiras, Sebastian Merkel, Ricardo Reis, Christian Wolf and conference and seminar participants at the Graduate Institute Geneva, the JME-SNB-SCG conference on money creation and competition, Princeton University, and the Riksbank. Brunnermeier acknowledges research support from MFM.

†Princeton University; CEPR; CESifo; NBER. Princeton, NJ, 08544, USA. `markus@princeton.edu`, `http://scholar.princeton.edu/markus`.

‡Study Center Gerzensee; University of Bern; CEPR; CESifo. P.O. Box 21, CH-3115 Gerzensee, Switzerland. `dirk.niepelt@szgerzensee.ch`, `http://alum.mit.edu/www/niepelt`.

1 Introduction

Modern economies rely on public—central-bank issued—and private means of payment. The latter are gaining ground although the recent financial crisis is partially attributed to excessive private money creation. FinTechs reshape the payment system and compete with traditional payment service providers to supply digital monies; in China, technology firms are already the front runners. On the user side, consumers in Scandinavia increasingly avoid cash and bank on payment apps and mobile phones instead, as do consumers and firms in developing countries. Governments, in the meantime, discourage cash transactions with the intention to fight tax evasion and money laundering.

As a result of these developments, debates about the "right" monetary architecture resurface and classic questions about the nature of money return to the fore. A key point of contention concerns the optimal balance between public and private money. Proponents of a strong government role fear that private money creation breeds instability and shifts seignorage rents from taxpayers to shareholders. In the "Chicago Plan" of the 1930s and the recently rejected Swiss constitutional referendum on "Vollgeld" (sovereign money), they propose to severely restrict or even ban money creation by anyone except the central bank. Less drastic proposals aim at electronic Central Bank Digital Currency (CBDC) for use by non-banks. Monetary authorities in countries such as Canada, Singapore, and Sweden currently evaluate the introduction of such "Reserves for All;" the Banco Central del Uruguay has successfully tested the model; and other central banks consider it.

Skeptics, on the other hand, warn of severe macroeconomic risks due to the replacement of private by public means of payment. In their view, a reduction of bank issued inside money will hamper credit extension by banks to firms and households, with negative implications for growth; and the introduction of CBDC will provide a safe haven asset for depositors to run into at the slightest hint of a crisis, rendering bank runs more likely and thereby threatening financial stability.

To assess these arguments, we develop a generic model of money and liquidity. Within this framework, we establish sufficient conditions under which it is irrelevant whether the public or the private sector issues means of payment. Our equivalence theorem is not meant to suggest that public and private money are perfect substitutes in the real world. It rather aims at transparently identifying the fundamental conditions for equivalence and thus, the manifold possible reasons for *non*-equivalence. In this sense, our research strategy is inspired by Modigliani and Miller (1958), Barro (1974), and the many other equivalence results in the economics literature.

Our approach is based on a comparison of choice sets. We establish that under specific conditions, swaps of public against private money leave the relevant choice sets of all agents unchanged (and also satisfy the government's budget constraint). Hence, the optimal choices and the equilibrium allocations "before" and "after" the swap are the same. Since we focus on choice sets our equivalence conditions are sufficient rather than necessary. Moreover, since our framework is generic the equivalence result applies beyond a specific economic environment.

For the choice sets of the users of money to remain unchanged a swap of public against private means of payment must not alter the wealth of agents and the liquidity

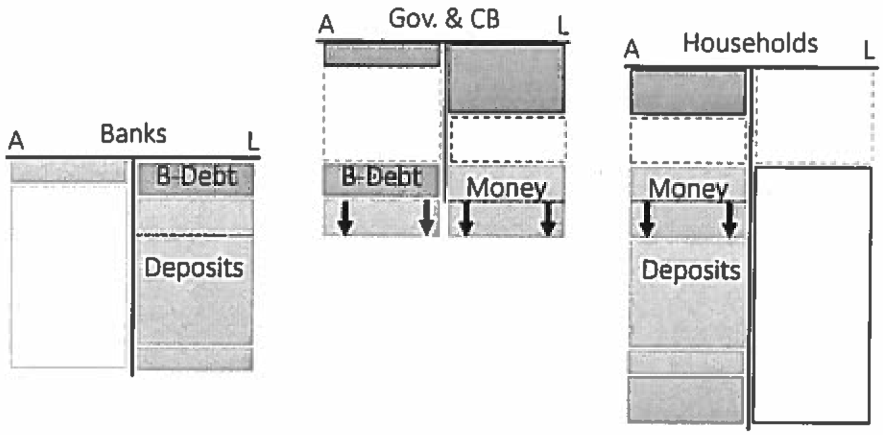

Figure 1: Pass-through funding of banks.

of their portfolios. Liquidity-neutrality can be satisfied even if monies differ in price, payoff characteristics (e.g., due to bank runs), or "liquidity payoffs" (usefulness to relax monetary frictions). While the details of the frictions matter, the key condition for liquidity-neutrality is that some of them are relaxed by a weighted sum of the two monies.

Wealth-neutrality requires, in addition, that a swap occurs in the form of open-market operations, possibly augmented by contingent transfers to accommodate the effects of the swap on subsequent portfolio payoffs. If existing assets replicate the payoff of the swap then no transfers are needed. If the swap does not affect the wealth distribution in general equilibrium then neither transfers nor open-market operations are needed. This is the case, for example, when the direct wealth effects of the swap are offset by indirect effects from changes in tax burdens, due to Ricardian equivalence, i.e. the fact that households "own" the central bank (Barro, 1974).

Equivalence follows whether banks are competitive or not, and without the central bank assuming assets from commercial banks. In fact, the central bank can completely insulate the banking system from a swap, by intermediating between non-banks and banks: If households and firms reduce deposits in exchange for central bank money, the central bank passes its new sources of funding through to banks, at the same terms as non-banks used to supply deposits. Figure 1 illustrates pass-through funding in the special case in which deposits and central bank money have the same value, for instance because they are equally liquid and have the same return. The open-market operation therefore does not involve a third security.

From the perspective of banks the swap therefore does not alter the market environment. Moreover, it leaves national saving and its sectoral components unchanged, such that there is no crowding out. And it does not affect the allocation of control rights—who screens, implements, manages, and monitors investment projects—, thereby avoiding risks of capital misallocation. Equivalence does require, however, that public and private liquidity creation generates the same social costs that is, that deposits and reserves, say, enter the resource constraint symmetrically. This is a weaker condition than the usual assumption that the social cost of liquidity creation equals zero (Friedman, 1969).

The social cost of liquidity generation may substantially differ from the private value of owning a liquid security. We show that a security's market price can be decomposed into three parts: The value of the fundamental payoffs; the value of liquidity payoffs; and a bubble component. The asset pricing formula has to be adjusted as the usual stochastic discount factor has to be multiplied by what we call a "liquidity kernel."

The bubble component may exceed zero for two reasons: Because markets are incomplete and consumption is volatile such that the expected value of the stochastic discount factor remains high as the horizon approaches infinity; or because monetary frictions magnify the stochastic discount factor, to the same effect. Intuitively, the bubble is valued either because it constitutes a safe store of value or because it relaxes a monetary friction, for instance by serving as a means of payment that helps overcome the double-coincidence of wants problem. We show, for example, that binding monetary frictions in a deterministic environment lower the (real) required rate of return on money, potentially below the economy's growth rate, g, even if the required rate on an illiquid security, r, exceeds g. That is, liquidity renders a "money bubble" possible even if $r > g$.

Both liquidity payoffs and bubble components imply that entities that issue liquid securities reap seignorage rents. A swap of public against private money reallocates these rents and thus, redistributes wealth unless the effects are compensated. The compensation may have to be initiated, by means of transfers, or it might take place automatically, through general equilibrium mechanisms. For example, a swap of deposits against central-bank issued money may be wealth neutral at the household level if bank shareholders pay all the taxes. Our wealth- and liquidity-neutrality conditions account for this.

We apply our results to two proposals for monetary reform: CBDC, and the more drastic "Chicago Plan." We find that the introduction of CBDC need not change macroeconomic outcomes, independently of whether deposits are subject to bank runs or not. If bank runs are a feature of the current system then the equivalent monetary regime with CBDC has state contingent transfers from the private sector to the central bank; if it does not, for instance due to a generous deposit insurance scheme, then no such transfers are needed.

Contrary to the prevailing view that CBDC would make bank runs more likely, our analysis concludes that it might well make them less likely. With pass-through funding, the central bank becomes a large depositor that internalizes run externalities, unlike small depositors. This makes the financial system less fragile.

Regarding the Chicago Plan, we also find that the conditions for equivalence are met provided that banks receive appropriate compensation for lost seignorage rents, or that the ownership structure of banks is aligned with the distributions of tax burdens. An important motivation for the "Vollgeld" proposal to outlaw banks from creating liquid assets was that banks should be forced to relinquish these rents. This would transfer seignorage from bank shareholders to taxpayers, undermining wealth-neutrality.

We also apply our results to the recent Indian de-monetization experiment. We find that it could not have been neutral because cash-based transactions at black-market prices could not have been replaced by deposit-based transactions.

Related Literature Fisher (1935) offers one of the first discussions of the Chicago plan. Gurley and Shaw (1960) introduce the distinction between inside money issued by banks and outside money supplied by the government. Tobin (1963; 1969; 1985) provides seminal contributions on the fractional reserve banking system.

Wallace (1981) derives an equivalence result in a deterministic overlapping generation (OLG) economy; he shows that it is irrelevant whether households hold physical capital directly or indirectly, via their money holdings at a central bank invested in capital. Bryant (1983) summarizes important equivalence results. Chamley and Polemarchakis (1984) establish that open-market operations are neutral when money does not serve as a medium of exchange. Sargent (1987, 5.4) presents results on equivalent fiscal-monetary policies in the opposite case.

Andolfatto (2018) studies the macroeconomic consequences of banks' money creation in an OLG framework. Benes and Kumhof (2012) set up a New Keynesian DSGE model and argue that banks' money creation ex nihilo destabilizes the economy. Faure and Gersbach (2018) develop a model of banking and contrast the welfare under a fractional reserve banking architecture and an architecture with 100% reserve banking. Niepelt (2018) discusses CBDC and offers an informal equivalence result according to which a substitution of outside for inside money does not affect macroeconomic outcomes.

Structure of Paper The remainder of the paper is organized as follows. Section 2 lays out a general model of a monetary economy with a wide range of possible monetary and other frictions. Section 3 analyzes the effects of an asset's "liquidity" on its equilibrium price. Section 4 derives and explains the main equivalence result and Section 5 provides further discussion. In Section 6 we apply our findings to Central Bank Digital Currency, the Chicago Plan as well as the Indian de-monetization experiment. Section 7 concludes.

2 Model

We consider a dynamic, stochastic economy with unit measures of households and firms; banks; and a consolidated government sector. Time is discrete and indexed by $t \geq 0$. All variables dated t are measurable with respect to the history up to and including date t, except otherwise noted. Households, firms, and banks are indexed by h, f, and b, respectively. The government does not consume nor invest; it only issues and acquires securities and collects taxes or pays transfers. We use the terms "central bank" and "government" interchangeably; a superscript c denotes "central bank."

2.1 Households

Household h chooses commodity and portfolio sequences to maximize lifetime utility subject to constraints.

Formally, we index commodities (not contingent on date or history) by n and denote household h's consumption of commodity n at date t by $x_t^{n,h}$. When we omit time subscripts (e.g., $x^{n,h}$) then we refer to the sequence over time and histories, and when we replace the superscript n by a dot (e.g., $x_t^{\cdot,h}$) then we refer to the vector containing all n. The price of commodity n at date t is denoted q_t^n; q_t denotes the price vector at date t, and q denotes the price vectors at all dates. The numeraire at each date is the "first" good, $q_t^1 = 1$.

We index securities (not contingent on date or history but possibly on the issuer) by j and denote a security of type j held (positive position) or issued (negative) by agent i by $a_t^{j,i}$. As with commodities, we refer to sequences over time when we omit time subscripts; and to vectors when we replace the first superscript (indicating the type of security) by a dot. For example, $a_t^{\cdot,h}$ denotes all security exposures of household h at date t. The price of security j at date t is denoted p_t^j and its payoff (in units of the numeraire), z_t^j; p_t denotes the price vector at date t, and p denotes the price vectors at all dates. Note that this formulation allows for time varying, stochastic prices of securities including monies. The stochasticity allows to capture bank runs in a reduced-form way.[1]

Household h chooses $x^{\cdot,h}$ and $a^{\cdot,h}$ to maximize

$$\mathcal{U}^h(x^{\cdot,h}) \quad \text{s.t.} \quad \sum_j a_t^{j,h} p_t^j = \sum_j a_{t-1}^{j,h}(p_t^j + z_t^j) - \sum_n x_t^{n,h} q_t^n - \tau_t^h(x^{\cdot,h}, q),$$
$$\mathcal{L}_t^h(\{a_t^{j,h} p_t^j\}_j, \{a_{t-1}^{j,h}(p_t^j + z_t^j)\}_j, p_t, x^{\cdot,h}, q) \leq (=)\ 0,$$

and a no-Ponzi game condition. Function $\mathcal{U}^h$ denotes h's lifetime utility function which is smooth, concave, and strictly increasing (decreasing) in all arguments that generate utility (disutility). Vector $x_t^{\cdot,h}$ includes, for example, consumption of goods and services, labor supply, or leisure.[2] Function τ_t^h denotes a tax/transfer function.

The first constraint in the household's program represents the budget constraint. It states that the payoff from the household's portfolio, $\sum_j a_{t-1}^{j,h}(p_t^j + z_t^j)$, finances commodity purchases net of sales; tax payments net of received transfers; and securities purchases net of issuances. Asset markets can be incomplete.

The second constraint captures restrictions beyond the budget constraint. We adopt a general formulation according to which the restrictions may relate portfolio positions, their values, their returns, the household's commodity vector, and commodity prices to each other. When a household faces multiple restrictions in a period then function $\mathcal{L}_t^h$ is vector valued.

At the end of this section, we discuss different interpretations of $\mathcal{L}_t^h$ as well as the dimensionality of the constraint.

[1]Let the history include information about which households and firms are first to arrive at a specific bank when this bank is subject to a run. Histories that only differ with respect to who is early in the queue and who is late, are equally likely. Ex-ante, deposits at the bank are identical but ex post, conditional on history, the payoff on deposits at the bank differs for different households and firms; households and firms in the front of the queue receive the promised payoff and price while latecomers only receive the bankruptcy value of the bank.

[2]If we treat leisure rather than labor supply as a commodity then the budget constraint also includes a time endowment.

2.2 Firms

Firm f chooses input-output, capital investment, and portfolio sequences to maximize firm value or equivalently, the market value of the firm's dividend stream (Fisher, 1930).[3]

We denote firm f's input or output of commodity n at date t by $y_t^{n,f}$; inputs are negative entries, outputs are positive entries. To align with standard "macroeconomic" notation we exclude physical capital holdings, k_t^f, from the commodity vector $y_t^{\cdot,f}$. We let $\iota_t^{n,f}$ denote the quantity of commodity n that firm f uses to produce new physical capital, and κ_t^f the quantity of capital that firm f purchases. The price of physical capital is denoted q_t^K.

When we write f, b, or c for the type of security then we refer to the equity of firm f, bank b, or the central bank, respectively. E.g., $a_t^{f,\cdot}$ denotes firm f's shares outstanding at the end of date t, and z_t^f denotes firm f's dividends. Without loss of generality we normalize the total shares of each firm or bank or of the central bank to unity.

Letting $\mu_{t,s}$ denote the date-t stochastic discount factor for payoffs at date s, firm f chooses $y^{\cdot,f}$, $\iota^{\cdot,f}$, κ^f, k^f, $a^{\cdot,f}$, and z^f to maximize

$$\sum_t \mathbb{E}_0\left[\mu_{0,t} z_t^f\right] \quad \text{s.t.} \quad \sum_{j\neq f} a_t^{j,f} p_t^j = \sum_{j\neq f} a_{t-1}^{j,f}(p_t^j + z_t^j) + \underbrace{\sum_n \left(y_t^{n,f} - \iota_t^{n,f}\right) q_t^n - \kappa_t^f q_t^K}_{\text{operating profit}} - z_t^f,$$

$$\mathcal{F}_t^f(k_{t-1}^f, y_t^{\cdot,f}) \leq 0,$$
$$\mathcal{K}_t^f(k_{t-1}^f, \iota_t^{\cdot,f}, \kappa_t^f, k_t^f) \leq 0,$$
$$\mathcal{L}_t^f(\{a_t^{j,f} p_t^j\}_j, \{a_{t-1}^{j,f}(p_t^j + z_t^j)\}_j, p_t, y^{\cdot,f}, \iota^{\cdot,f}, \kappa^f, q) \leq (=)\ 0,$$

and a no-Ponzi game condition.

The first constraint in the firm's program represents the budget constraint. It relates dividend payouts, z_t^f, to the firm's operating profit and accumulation of net financial assets. The second and third constraints represent production constraints; function $\mathcal{F}_t^f$ represents the firm's production possibilities and function $\mathcal{K}_t^f$ represents the law of motion for physical capital, possibly accounting for depreciation and adjustment costs (Jorgensen, 1963; Tobin, 1969; Hayashi, 1982).[4] The $\mathcal{L}_t^f$-constraint parallels the $\mathcal{L}_t^h$-constraint of a household; it captures restrictions that relate to the medium-of-exchange function of securities or other financial frictions.

[3]The optimal timing of dividend payouts is indeterminate (Modigliani and Miller, 1958).

[4]We could merge the law of motion for capital and the budget constraint. Suppose for example that the law of motion takes the form

$$k_t^f = k_{t-1}^f(1-\delta) + \Phi(\iota_t^{\cdot,f}, k_{t-1}^f) + \kappa_t^f,$$

where δ denotes the depreciation rate and $\Phi(\iota_t^{\cdot,f}, k_{t-1}^f)$ denotes physical capital generated from investment $\iota_t^{\cdot,f}$, possibly subject to adjustment costs. The budget constraint can then be written as

$$\sum_{j\neq f} a_t^{j,f} p_t^j + k_t^f q_t^K = \sum_{j\neq f} a_{t-1}^{j,f}(p_t^j + z_t^j) + k_{t-1}^f(1-\delta) q_t^K + \sum_n \left(y_t^{n,f} - \iota_t^{n,f}\right) q_t^n + \Phi(\iota_t^{\cdot,f}, k_{t-1}^f) q_t^K - z_t^f.$$

To accommodate price rigidity à la Calvo (1983) we could augment the firm's program by a condition that reflects monopolistic competition in the firm's output market (Dixit and Stiglitz, 1977). This condition would relate demand for the firm's output to the price set by the firm (which would constitute an additional choice variable) as well as to choices made by other firms. The structure of the constraint would parallel the structure of the limited competition constraint that we discuss below, in the context of the bank's program. Introducing such a constraint in the firm's program would not affect our results because securities do not enter the constraint.

2.3 Banks

The program of a bank parallels the program of a firm, with two differences. First, a bank may face limited competition in the market for one of its liabilities—deposits, which enter the means-of-payment constraints of households and firms. And second, it does not produce commodities and does not face a law of motion for capital.

To accommodate the possibility that a non-competitive bank chooses the price and payoff of its deposits we include a different type of constraint, denoted by $\mathcal{C}_t^b$. The constraint reflects the equilibrium relationship between the quantity of the bank's deposits and the deposit price and rate, conditional on the choices made by the bank's competitors.

We let $j = D^b$ denote the security index of deposits issued by bank b. Bank b chooses $a^{\cdot,b}$, p^{D^b}, z^{D^b}, and z^b to maximize

$$\sum_t \mathbb{E}_0 \left[\mu_{0,t} z_t^b\right] \quad \text{s.t.} \quad \begin{aligned} &\sum_{j \neq b} a_t^{j,b} p_t^j = \sum_{j \neq b} a_{t-1}^{j,b}(p_t^j + z_t^j) - z_t^b, \\ &\mathcal{C}_t^b(a_t^{\cdot,b}, p_t^{D^b}, z_{t+1}^{D^b}, \text{state}_t^b) \leq (=)\ 0, \\ &\mathcal{L}_t^b(\{a_t^{j,b} p_t^j\}_j, p_t) \leq (=)\ 0, \end{aligned}$$

and a no-Ponzi game condition. The term "state_t^b" denotes factors that the bank takes as given. When the bank is a price taker then the function $\mathcal{C}_t^b$ is not present in the program and p^{D^b} as well as z^{D^b} do not constitute choice variables.

The restriction $\mathcal{L}_t^b$ represents regulatory constraints or restrictions implied by the functioning of interbank markets; a minimum reserve requirement constitutes a primary example. We assume that the ownership structure of the bank's liabilities is irrelevant for $\mathcal{L}_t^b$.

2.4 Central Bank

The central bank manages a balance sheet and collects taxes subject to the budget constraint

$$\sum_{j \neq c} a_t^{j,c} p_t^j = \sum_{j \neq c} a_{t-1}^{j,c}(p_t^j + z_t^j) + \int_h \tau_t^h(x^{\cdot,h}, q)dh.$$

The central bank is subject to a no-Ponzi game constraint.

Note that the model does not rely on specific assumptions about how central bank money comes into circulation. The central bank could inject money by purchasing securities from banks or other private sector agents (open-market operations) or by transferring money and paying interest on money ("helicopter drops"). In either case the central bank eventually transfers resources to the private sector unless the interest rate on central bank money exceeds the interest rate on the central bank's assets.

2.5 Equilibrium

Market clearing requires that the markets for commodities, capital goods, and securities clear at all dates:

$$\int_h x_t^{\cdot,h} dh = \int_f (y_t^{\cdot,f} - \iota_t^{\cdot,f}) df,$$
$$\int_f \kappa_t^f df = 0,$$
$$\int_i a_t^{j,i} di = 0 \text{ for all } j.$$

(We suppress endowments to simplify the notation and let i index households and firms.)

Definition 1. *A* competitive equilibrium *conditional on the policy* $(a^{\cdot,c}, \{\tau^h\}_h)$ *is an allocation,* $(\{x^{\cdot,h}\}_h, \{y^{\cdot,f}, \iota^{\cdot,f}, k^f, \kappa^f\}_f)$*; a vector of prices and payoffs,* $(p, q, \{z^j\}_j)$*; a stochastic discount factor process,* $\{\mu_{0,t}\}_t$*; and a sequence of portfolios,* $(\{a^{\cdot,h}\}_h, \{a^{\cdot,f}\}_f, \{a^{\cdot,b}\}_b)$ *such that households, firms, and banks solve their programs for given policy, prices, payoffs, and stochastic discount factors; markets clear; and the stochastic discount factor reflects the optimal household choices.*

A non-competitive equilibrium *consists of the same objects; each non-competitive bank takes* C_t^b *and* state_t^b *rather than the price and payoff of its deposits as given; and* state_t^b *is consistent with equilibrium behavior of households, firms, and other banks.*

2.6 Discussion

We conclude this section with a discussion of different interpretations of the framework as well as of the dimensionality of the $\mathcal{L}_t^h$-, $\mathcal{L}_t^f$-, and $\mathcal{L}_t^b$-constraints (in short, $\mathcal{L}_t$-constraints).

Generality of Framework Our framework captures a wide variety of models of money and liquidity. (In many of those models, the household and firms sectors are consolidated.)

First, *overlapping generation (OLG)* models à la Samuelson (1958), in which households of a certain generation, h say, derive utility, $\mathcal{U}^h$, from consumption in specific periods.[5] Money may emerge in an OLG economy in the form of a bubble if the risk-free interest rate is lower than the growth rate of the economy (Wallace, 1980). Within the

[5]See Shell (1971) on the interpretation of a cohort in an OLG economy as an infinitely lived household with preferences over consumption in only a few periods.

classic deterministic OLG setting Wallace (1981) derives an important equivalence result: It does not matter when households sell physical capital to the central bank if the latter funds the extra assets from money issuance.

Second, models in which the available assets are not sufficient to *complete the market.* Agents then demand money-like assets, as in Brunnermeier and Sannikov (2016), because these assets constitute useful hedges or a safe store of value.

Third, with the additional $\mathcal{L}$-constraint, models which emphasize the role of money as a medium of exchange that helps overcome the double-coincidence-of-wants problem in environments without centralized markets. Households or firms that face a medium-of-exchange constraint are willing to hold money despite its low rate of return because this relaxes their $\mathcal{L}$-constraints.

The *cash-in-advance constraint* imposed in Clower (1967), Grandmont and Younes (1972), Lucas (1980; 1982), Svensson (1985), and Lucas and Stokey (1987) constitutes the simplest form of an $\mathcal{L}$-constraint. In these models, the arguments of the $\mathcal{L}_t$-constraint include real balances and consumption, and the constraint requires the former to exceed the latter. Depending on the timing convention the restriction relates real balances chosen at date t to contemporaneous consumption, $x_t^{n,h}$ say, as in Lucas (1982); or to consumption in the subsequent period, $x_{t+1}^{n,h}$, as in Svensson (1985). Under the Lucas (1982) timing convention the $\mathcal{L}_t^h$-constraint is scalar valued if the household purchases a single consumption good with cash, and vector valued if it purchases several commodities with different parts of its total cash holdings. Under the Svensson (1985) timing convention the $\mathcal{L}_t^h$-constraint is vector valued in risky environments even if there is just one consumption good; this is due to the fact that the cash-in-advance constraint must hold state by state in the subsequent period.

The classic *Baumol (1952)-Tobin (1956)* model follows when the $\mathcal{L}_t$-constraint relates positive changes in an agent's stock of money holdings to some fixed resource cost that the agent must bear. Households hold low-interest bearing money in order to economize on these costs. So-called *shopping time* models follow when leisure constitutes one of the commodities and the $\mathcal{L}_t$-constraint relates "time spent shopping" to consumption purchases. Other models, for instance many New Keynesian frameworks, impose a *money-in-the-utility-function* specification (Sidrauski, 1967). Since money in the utility function is formally equivalent to a "shopping time" specification, our framework can represent these models.[6] The $\mathcal{L}_t$-constraint can also be interpreted as a reduced form element of a "New Monetarist" setup in the tradition of Kiyotaki and Wright (1993) and Lagos and Wright (2005) where the transactions of the "average" household are a function of money holdings.

Fourth, models of limited funding liquidity where agents hold money because of borrowing constraints rather than as a means of payment. In Bewley (1980) agents hold a liquid money-security for precautionary reasons since they face uninsurable income risk, due to incomplete markets, and borrowing constraints might bind after a shock. The timing in that model is similar to the timing in the Svensson (1985) cash-in-advance-constraint model. In Woodford (1990), agents demand a safe store of value, such as government

[6]See Saving (1971), Feenstra (1986), and Croushore (1993) on the equivalence.

bonds or money, because they face a tight borrowing constraint and sporadically meet profitable investment opportunities. In Holmström and Tirole (1998) entrepreneurs demand a safe store of value because funding for the continuation of a project is constrained due to a moral hazard friction; holding liquid, money-like assets relaxes the borrowing constraint in future states where large extra funds are needed. In Kiyotaki and Moore (2012) agents hold liquid money to prepare for situations in which they face investment opportunities which they cannot easily exploit due to funding illiquidity (they face a borrowing constraint) and market illiquidity (they cannot quickly sell their assets). The $\mathcal{L}_t$-constraint captures all these restrictions, possibly if it depends both on $a_t^{j,h}$ and $a_{t+1}^{j,h}$.

Finally, we note that our setup can easily accommodate models with additional, "real" frictions that are not fully reflected in the specification. This includes models with price or wage setting frictions as in the prototypical "New Keynesian" model (Clarida, Galí and Gertler, 1999; Woodford, 2003; Galí, 2008).

Multi-dimensional $\mathcal{L}_t$-constraint The $\mathcal{L}_t$-constraints may either refer to the *total* exposure to a given security or to individual *parts* of this total exposure. Consider three cases with cash-in-advance constraints. First, an economy with Lucas (1982) timing and one consumption good in each period: Only total money holdings matter in this case since the $\mathcal{L}_t$-constraint imposes a single restriction with respect to total money holdings. Second, an economy with Lucas (198[illegible]) timing and multiple consumpt[illegible] is case, it is not only total money h[illegible] that matt[illegible] use the $\mathcal{L}_t$-constraint i[illegible]s multiple restrictions, one for [illegible] mponent of [illegible] a specific good. Third, an economy with Svensson (1985) timing and one c[illegible]tion good: The $\mathcal{L}_t$-constraint again imposes multiple restrictions in this case, one for each continuation history in the su[illegible]nt peri[illegible] [illegible]theless [illegible]y total money holdings matter because each of the re[illegible]

To avoid unnecessary [illegible] introduc[illegible] different parts of holdin[illegible] same securit[illegible] an ag[illegible]feren[illegible] transactions (as in th[illegible] [illegible]ple above). Instead, we adopt the convention [illegible] a security i[illegible] fined [illegible] the "component level." That is, in the first and third example discusse[illegible]ve, w[illegible] terpret the [illegible] hold one type [illegible] while in the sec[illegible]nd exampl[illegible] we int[illegible] it to hold mu[illegible] [illegible]all with [illegible]d re[illegible]urn c[illegible]ar-acteristics), a[illegible]ent one for purchases [illegible]

3 L[illegible]uidity a[illegible]

[illegible] analyzing th[illegible] sequences [illegible] of secur[illegible] (outside) m[illegible]y private (inside) [illegible], say—then w[illegible] accou[illegible] their effects on [illegible]straints. In the [illegible]ing, we [illegible] $\mathcal{L}$-con[illegible]ints a[illegible]liquid;" similarly, we refer to char[illegible] [illegible]positions of [illegible] that [illegible]out effect on $\mathcal{L}$-constraints as illiq[illegible].

A[illegible]ow in this section, security prices, payoffs, and liquidity are tightly con-[illegible]ed. The price of a security generically is composed of three components, r[illegible]flecting the security's (i) fundamental payoffs; its (ii) liquidity payoffs; and possibly, (iii) its bubble component. An entity with the authority to create money potentially reaps seignorage rents by issuing money securities whose value primarily derives from (ii) and (iii) while it generates no or only minor payoff obligations (i).

Consider the portfolio choice problem of a household (parallel conditions apply for a firm). Let $\tilde{\mu}_t^h$ and $\tilde{\lambda}_t^h$ denote the (non-negative) Lagrange multipliers in household h's program that are attached to the date-t budget constraint and $\mathcal{L}_t^h$-constraint, respectively. The household's Euler equation for security $a_t^{j,h}$ reads

$$\tilde{\mu}_t^h p_t^j = \mathbb{E}_t\left[\tilde{\mu}_{t+1}^h(p_{t+1}^j + z_{t+1}^j)\right] \underbrace{- p_t^j \tilde{\lambda}_t^h \frac{\partial \mathcal{L}_t^h}{\partial(a_t^{j,h} p_t^j)} - \mathbb{E}_t\left[(p_{t+1}^j + z_{t+1}^j)\tilde{\lambda}_{t+1}^h \frac{\partial \mathcal{L}_{t+1}^h}{\partial a_t^{j,h}(p_{t+1}^j + z_{t+1}^j)}\right]}_{\equiv \tilde{\mu}_t^h \ell_t^h(j;p,z)}.$$

The condition states that a household that chooses to hold security j, balances costs and benefits. The cost, represented on the left-hand side, derives from the reduction of purchasing power at date t. The benefit, on the right-hand side, derives from the gain in purchasing power at date $t+1$ as well as from the relaxation (or tightening) of $\mathcal{L}_t^h$- and $\mathcal{L}_{t+1}^h$-constraints. We denote the latter component, normalized by $\tilde{\mu}_t^h$, by $\ell_t^h(j;p,z)$; it varies over time and by security, household, and possibly the prices and payoffs of all securities.

For example, in an economy with a cash-in-advance constraint, $\partial\mathcal{L}_t^h/\partial(a_t^{j,h}p_t^j) < 0$ and $\partial\mathcal{L}_{t+1}^h/\partial(a_t^{j,h}p_{t+1}^j) = 0$ as the $\mathcal{L}_t^h$-constraint requires consumption minus real balances to be weakly smaller than zero. In an economy where the household may not sell security positions too quickly (Kiyotaki and Moore, 2012) the $\mathcal{L}_t^h$-constraint requires the negative change of the security position not to exceed a threshold value; accordingly, $\partial\mathcal{L}_t^h/\partial(a_t^{j,h}p_t^j) < 0$ and $\partial\mathcal{L}_{t+1}^h/\partial(a_t^{j,h}p_{t+1}^j) > 0$.

Define $\lambda_t^h \equiv \tilde{\lambda}_t^h/\tilde{\mu}_t^h$ and note that the stochastic discount factor (SDF) satisfies $\mu_{t,t+1}^h \equiv \tilde{\mu}_{t+1}^h/\tilde{\mu}_t^h$. We can then write the Euler equation as $p_t^j = \mathbb{E}_t\left[\mu_{t,t+1}^h(p_{t+1}^j + z_{t+1}^j)\right] + \ell_t^h(j;p,z)$ or

$$p_t^j = \mathbb{E}_t\left[\mu_{t,t+1}^h \Lambda_{t,t+1}^h (p_{t+1}^j + z_{t+1}^j)\right], \tag{1}$$

where $\Lambda_{t,t+1}^h \equiv \left(1 - \lambda_{t+1}^h \frac{\partial\mathcal{L}_{t+1}^h}{\partial a_t^{j,h}(p_{t+1}^j + z_{t+1}^j)}\right) / \left(1 + \lambda_t^h \frac{\partial\mathcal{L}_t^h}{\partial(a_t^{j,h}p_t^j)}\right)$ is the one-period "liquidity kernel." Iterating the Euler equation (1) forward yields

$$p_t^j = \lim_{T\to\infty} \mathbb{E}_t\left[\sum_{s=1}^{T} \mu_{t,t+s}^h \Lambda_{t,t+s}^h z_{t+s}^j\right] + \lim_{T\to\infty} \mathbb{E}_t\left[\mu_{t,t+T}^h \Lambda_{t,t+T}^h p_{t+T}^j\right], \tag{2}$$

where $\mu_{t,t+s}^h$ denotes the standard multi-period SDF and $\Lambda_{t,t+s}^h \equiv \prod_{\tau=t}^{t+s-1} \Lambda_{\tau,\tau+1}^h$ is the multi-period liquidity kernel.

The equilibrium security price is composed of three components. First, the fundamental value that reflects the fundamental payoffs represented in the standard pricing term, $\lim_{T\to\infty} \mathbb{E}_t\left[\sum_{s=1}^{T} \mu_{t,t+s}^h z_{t+s}^j\right]$. Second, the value of the liquidity payoffs that arise due to the fundamental payoffs, $\lim_{T\to\infty} \mathbb{E}_t \left[\sum_{s=1}^{T} \mu^h_{t,t+s}(\Lambda^h_{t,t+s} - 1)z^j_{t+s}\right]$. And third, a bubble component, which is given by the last term in Equation (2). (The first and second price component add to the first term in Equation (2).)

The bubble component may differ from zero for two reasons. First, in a setting without $\mathcal{L}$-constraint but with incomplete markets, as in Brunnermeier and Sannikov (2016), because individual consumption is highly volatile. Since the shadow value of income and hence the SDF $\mu^h_{t,t+T}$ is convex in contingent consumption, the expected value of $\mu^h_{t,t+T}$ then is sufficiently high (by Jensen's inequality) for the bubble component to be bounded away from zero as $T \to \infty$. Second, in a setting with a possibly binding $\mathcal{L}$-constraint, because the standard SDF is multiplied by a liquidity kernel which exceeds unity sufficiently strongly for the bubble component to remain strictly positive in the limit. For example, in a deterministic setting without inflation, a binding $\mathcal{L}$-constraint with $\Lambda_{t,t+1} > 1$ lowers the required rate of return on money, r^M, to $r^M = (\mu^h_{t,t+1}\Lambda_{t,t+1})^{-1} - 1$, which can fall short of the economy's growth rate, g, even if the required rate on an illiquid security, r, exceeds g because $r = (\mu^h_{t,t+1})^{-1} - 1 > g$.[7] In short, the liquidity payoff lowers the interest rate and might render "money bubbles" possible.

Decomposing the price of a security into the three parts clarifies how the creation of money entails the creation of (seignorage) rents. A central bank that is in the position to issue a bubble security reaps seignorage rents because the sale of the bubble creates value without forcing the bank to ever produce a fundamental payoff. Similarly, a private bank which holds illiquid assets with high fundamental payoffs and issues liquid inside money with low fundamental payoffs but high liquidity payoffs earns rents, too. While the bank's assets and liabilities have the same market value, the former yield a higher fundamental payoff than the latter and the discounted fundamental payoff difference between bank assets and liabilities contributes to the bank's (intangible) franchise value, which is reflected in its equity market value. Competition among banks can erode this franchise value as competitive banks have incentives to issue more money and grant more loans, driving the deposit interest rate up and the loan interest rate down.

Our discussion highlights that substituting public (outside) money for private (inside) money—or vice versa—may reallocate seignorage rents and thus, redistribute wealth (in addition to possibly having inflationary or deflationary effects). To obtain neutrality of a private-public money swap, we must design the swap so as to neutralize any wealth effects, or complement it with additional measures to the same effect. In addition, we have to make sure that the swap does not give rise to liquidity effects that is, we have to account for differential effects of inside and outside money on $\mathcal{L}$-constraints.

4 Equivalence

In this section we establish equivalence classes of monetary regimes: We derive conditions under which a regime change that involves a swap of central bank liabilities against bank liabilities is "irrelevant" in the sense that it does not affect the equilibrium allocation or

[7]Note that typically, the literature assumes some OLG setting to ensure that $r < g$. Our $\mathcal{L}$-constraint can achieve the same without assuming an OLG setting.

prices. For brevity, and without pre-committing to any specific interpretation, we refer to central bank liabilities as "cash" and to bank liabilities as "deposits."

Let Δx denote the change of a generic variable x; let $j = M$ denote the index denoting cash (outside money); and recall that $j = D^b$ denotes the index of deposits (inside money) issued by bank b. Moreover, let $\mathbb{I}$ denote the set of households and/or firms (indexed by i) whose portfolios might change as a consequence of the swap.

Definition 2. *A* one-period swap *with agents in set* $\mathbb{I}$ *at date* t *is a set of exchanges of cash against deposits that is reversed after one period, in all continuation histories. Formally,*

$$\begin{aligned} \Delta a_t^{j,i} &= -\Delta a_{t+1}^{j,i}, \ i \in \mathbb{I}, \ j = M, D^b, \\ \Delta a_s^{j,i} &= 0, \ i \in \mathbb{I}, \ j \neq M, D^b, \ \text{all } s, \\ \Delta a_s^{j,i'} &= 0, \ i' \notin \mathbb{I}, \ \text{all } j, s. \end{aligned}$$

A one-period swap is fully characterized by $\{\Delta a_t^{M,i}, \Delta a_t^{D^b,i}\}_{i \in \mathbb{I}}$.

Throughout the section we restrict attention to one-period swaps. This is without loss of generality because persistent or other, more complicated swaps can always be decomposed into elementary one-period swaps. Moreover, our notation presumes that the swap involves deposits at a single bank, b. The extension to the case with many banks is immediate; it only involves adjustment of notation to handle the larger set of financial institutions. With small (competitive) banks, any interesting swap necessarily involves a measure of financial institutions.

Our approach to demonstrating the equivalence of monetary regimes—or the "irrelevance" of certain swaps—is based on a comparison of choice sets. We establish that swaps which satisfy specific conditions, possibly accompanied by supporting measures, leave the choice sets of all private sector agents unchanged (and also satisfy the government's budget constraint). The logic of the argument is simple: If for given prices and payoffs, all agents in the private sector have the same choice sets with respect to allocations and portfolios (excluding cash and deposits) "before" and "after" the regime change then their relevant choices coincide as well and the initial equilibrium also constitutes an equilibrium in the new regime. Our equivalence conditions thus are sufficient rather than necessary, and they can be applied without detailed knowledge of all equilibrium conditions.

The swaps and supporting measures that we consider are open-market operations with transfers:

Definition 3. *An* open-market operation with compensating transfers *with agents in the set* $\mathbb{I}$ *at date* t *(conditional on a SDF and security prices and payoffs) is*

i. *at date* t*, an open-market operation with each agent* $i \in \mathbb{I}$*, consisting of a one-period swap and changes in the portfolio positions of an illiquid security,* $j = s$*,*

$$\Delta a_t^{s,i} = -\frac{p_t^M \Delta a_t^{M,i} + p_t^{D^b} \Delta a_t^{D^b,i}}{p_t^s},$$

where $a_t^{M,i} + \Delta a_t^{M,i} > 0$*,* $a_t^{D^b,i} + \Delta a_t^{D^b,i} > 0$*;*

ii. at date $t+1$, contingent, illiquid transfers to/from each agent $i \in \mathbb{I}$ such that the transfers together with the open-market operation leave financial wealth unchanged in all continuation histories,

$$T^i_{t+1} = -\left((p^M_{t+1} + z^M_{t+1})\Delta a^{M,i}_t + (p^{D^b}_{t+1} + z^{D^b}_{t+1})\Delta a^{D^b,i}_t + (p^s_{t+1} + z^s_{t+1})\Delta a^{s,i}_t\right).$$

A key element determining the choice set of a household or firm is the agent's budget set. By construction, an open-market operation with compensating transfers does not change these budget sets from date $t+1$ onward, at least if the agents do not further adjust their portfolios in response to the intervention. However, it might change agents' financial wealth at date t. This is the case if and only if the market value of an agent's contingent transfer differs from zero:

Lemma 1 (Wealth-Neutrality). *An open-market operation with compensating transfers with agents in the set $\mathbb{I}$ at date t does not change date-t financial wealth of any $i \in \mathbb{I}$ if and only if the swap does not change liquidity payoffs that is, if and only if for all $i \in \mathbb{I}$*

$$\ell^i_t(M;p,z)\Delta a^{M,i}_t + \ell^i_t(D^b;p,z)\Delta a^{D^b,i}_t = 0.$$

Proof. Fix the SDF, security prices, and payoffs. The open-market operation with compensating transfers changes financial wealth of agent i at date t by

$$p^M_t \Delta a^{M,i}_t + p^{D^b}_t \Delta a^{D^b,i}_t + p^s_t \Delta a^{s,i}_t + \mathbb{E}_t[\mu_{t,t+1}T^i_{t+1}] = \mathbb{E}_t[\mu_{t,t+1}T^i_{t+1}],$$

where we use the definition of an open-market operation as well as the fact that transfers are illiquid. By definition of an open-market operation with compensating transfers,

$$(p^M_{t+1} + z^M_{t+1})\Delta a^{M,i}_t + (p^{D^b}_{t+1} + z^{D^b}_{t+1})\Delta a^{D^b,i}_t + (p^s_{t+1} + z^s_{t+1})\Delta a^{s,i}_t + T^i_{t+1} = 0.$$

Accordingly, financial wealth changes by

$$\begin{aligned}
\mathbb{E}_t[\mu_{t,t+1}T^i_{t+1}] &= -\sum_{j=M,D^b,s} \mathbb{E}_t[\mu_{t,t+1}(p^j_{t+1} + z^j_{t+1})]\Delta a^{j,i}_t \\
&= -\sum_{j=M,D^b,s} p^j_t \Delta a^{j,i}_t + \ell^i_t(M;p,z)\Delta a^{M,i}_t + \ell^i_t(D^b;p,z)\Delta a^{D^b,i}_t \\
&= \ell^i_t(M;p,z)\Delta a^{M,i}_t + \ell^i_t(D^b;p,z)\Delta a^{D^b,i}_t,
\end{aligned}$$

where we use the fact that security s is illiquid and that we consider an open-market operation. □

Lemma 1 asserts that an open-market operation with compensating transfers does not change financial wealth positions at date t if the swap does not change the portfolio's liquidity payoffs. (By definition, the open-market operation with compensating transfers also does not change financial wealth at date $t+1$.) Lemma 1 does not impose any additional conditions. In particular, it does not restrict the payoff characteristics of security s and related, the characteristics of the transfer. Several cases of interest are worth noting.

First, in the baseline case, the open-market operation involving the swap and security s is wealth-neutral at date t and the effects of the open-market operation on financial wealth at date $t+1$ are neutralized by means of the contingent transfers.

Second, no security s is needed if the market value of the swap with each agent equals zero.

Third, no transfers may be needed if the open-market operation involves a security s whose return is linearly dependent on the returns of cash and deposits. If the fundamental payoffs of the changed portfolio positions $\{\Delta a_t^{s,i}\}_{i \in \mathbb{I}}$ exactly replicate the fundamental payoffs of the swap then no transfers are needed to maintain financial wealth in the continuation histories. A setting with risk-free cash, deposits, and security s constitutes a special case.

Finally, we can interpret security s broadly, as implicit claims vis-à-vis other agents. To see this, consider an economy with a representative household. This household "owns" the government because the household is the residual claimant (tax payer). Explicit changes in the household's wealth position vis-à-vis the government (e.g., an increase of explicit government debt) therefore are irrelevant as long as they are mirrored by changes in future taxes or implicit debt (Barro, 1974). In other words, if implicit future tax obligations were securitized then changes in the government's funding policy constituted open-market operations. The same logic applies here. In a representative household economy, an open-market operation with compensating transfers does not need to involve an explicit ownership change of a security s as well as transfers because the swap necessarily involves accompanying changes in the household's implicit claims vis-à-vis the government which exactly neutralize the financial consequences of the swap. This logic extends to settings with heterogeneous households whose exposure to implicit government debt matches their exposure to the swap.

We have discussed conditions that render the swap irrelevant as far as its consequences on the budget sets of households and firms are concerned. But for a swap not to change the choice sets of households and firms it does not suffice that the swap leaves the budget sets unchanged. In addition, the households and firms must continue to satisfy their $\mathcal{L}_t$-constraints. (In general equilibrium further conditions must be met which we discuss below.) We now turn to conditions under which this is the case:

Lemma 2 (Liquidity-Neutrality). *A one-period swap with agents in the set $\mathbb{I}$ at date t (conditional on the allocation, commodity prices, and security prices and payoffs) does not change the function value of any $\mathcal{L}_t^i$-constraint for $i \in \mathbb{I}$, nor any derivative of these functions, if the following conditions are all satisfied for all $i \in \mathbb{I}$:*

i. $\Delta a_t^{M,i}$ *and* $\Delta a_t^{D^b,i}$ *are linearly substitutable and separable in* $\mathcal{L}_t^i$ *and* $\mathcal{L}_{t+1}^i$,

$$\mathcal{L}_\tau^i(\{a_\tau^{j,i} p_\tau^j\}_j, \{a_{\tau-1}^{j,i}(p_\tau^j + z_\tau^j)\}_j, p_\tau, x^{\cdot,i}, q) = \hat{\mathcal{L}}_\tau^i(A_{\tau,\tau}^i, A_{\tau,\tau-1}^i, \text{other}_\tau^i), \ \ \tau = t, t+1,$$

where

$$\begin{aligned} A_{\tau,\tau}^i &= a_\tau^{M,i} p_\tau^M v_\tau^M + a_\tau^{D^b,i} p_\tau^{D^b} v_\tau^{D^b}, \\ A_{\tau,\tau-1}^i &= a_{\tau-1}^{M,i}(p_\tau^M + z_\tau^M) v_\tau^M + a_{\tau-1}^{D^b,i}(p_\tau^{D^b} + z_\tau^{D^b}) v_\tau^{D^b}, \\ \text{other}_\tau^i &\perp a_\tau^{M,i}, a_\tau^{D^b,i}, a_{\tau-1}^{M,i}, a_{\tau-1}^{D^b,i}, \end{aligned}$$

for some exogenous "velocity" parameters $v_t^M, v_t^{D^b}, v_{t+1}^M, v_{t+1}^{D^b} \neq 0$;

ii. $\Delta a_t^{M,i}$ and $\Delta a_t^{D^b,i}$ neutralize each other in terms of their effects on $\mathcal{L}_t^i$ and $\mathcal{L}_{t+1}^i$,

$$\begin{aligned} p_t^M v_t^M \Delta a_t^{M,i} + p_t^{D^b} v_t^{D^b} \Delta a_t^{D^b,i} &= 0, \\ (p_{t+1}^M + z_{t+1}^M) v_{t+1}^M \Delta a_t^{M,i} + (p_{t+1}^{D^b} + z_{t+1}^{D^b}) v_{t+1}^{D^b} \Delta a_t^{D^b,i} &= 0. \end{aligned}$$

Lemma 2 defines conditions under which a swap does not affect liquidity. These conditions relate both to the functional forms of $\mathcal{L}_t^i$ and $\mathcal{L}_{t+1}^i$ and the relative size of $\Delta a_t^{M,i}$ and $\Delta a_t^{D^b,i}$. The sense in which the swap is liquidity-neutral is a dual one. On the one hand, the swap does not change the $\mathcal{L}_t^i$- and $\mathcal{L}_{t+1}^i$-constraints; that is, it does not shift the agent in or our of the constraint set. On the other hand, the swap does not alter the marginal liquidity contribution of any security because it does not affect the derivatives of the $\mathcal{L}_t^i$- and $\mathcal{L}_{t+1}^i$-constraints; that is, the swap does not change the liquidity payoffs of securities (conditional on the allocation and prices and fundamental payoffs). Figuratively speaking, when plotting the $\mathcal{L}$-constraint in cash-deposit space, liquidity neutrality implies that the isoquants are linear, i.e. the marginal rate of liquidity substitution is constant.

In many applications, liquidity-neutrality imposes only mild conditions. If $(\Delta a_t^{M,i}, \Delta a_t^{D^b,i})$ only enters once (rather than multiple times) in the $\mathcal{L}_t^i$- and $\mathcal{L}_{t+1}^i$-constraints, which is the case in many of the examples discussed earlier; and if cash and deposits are at least minimally substitutable, which seems very reasonable to expect; then there always exists a $\Delta a_t^{M,i}$ for every $\Delta a_t^{D^b,i}$ such that the pair locally satisfies the conditions of Lemma 2.[8] When $(\Delta a_t^{M,i}, \Delta a_t^{D^b,i})$ enter the $\mathcal{L}_t^i$- and/or $\mathcal{L}_{t+1}^i$-constraints multiple times, however, then the requirements for liquidity-neutrality are strong and generically impossible to satisfy.

To accommodate models without $\mathcal{L}_t^i$-constraints we adopt the convention that these models satisfy the conditions of Lemma 2.

We can now state our main result. It holds for arbitrary combinations over time and histories of elementary open-market operations with compensating transfers, as defined in Definition 3, under the condition that each of these elementary interventions satisfy the conditions for liquidity-neutrality of Lemma 2. We refer to such multi-period and multi-history combinations of interventions as *liquidity-neutral open-market operations with compensating transfers.*

Theorem 1 (Equivalence). *Consider an equilibrium conditional on the policy $(a^{\cdot,c}, \{\tau^h\}_h)$ where the returns on cash and deposits lie in the asset span. A liquidity-neutral open-market operation with compensating transfers implements an equilibrium with the same allocation and price system as in the initial equilibrium.*

Proof. We prove the theorem for an elementary intervention. The result for arbitrary combinations of elementary interventions follows directly. Conjecture that the open-market operation with compensating transfers does not alter prices nor the allocation.

[8]This follows directly from the fact that for a small swap, the function $\mathcal{L}_t^i$ (or the function $\mathcal{L}_{t+1}^i$) can well be approximated to the first order.

Banks: The central bank insulates banks from the swap by replicating the deposit supply *schedule* banks face. If the swap involves a reduction of deposit holdings by households and firms then the central bank supplies deposits to banks in a way that replicates the deposit supply schedule in the initial equilibrium. If the swap involves a reduction of cash holdings by households and firms then the central bank issues deposits to households and firms. In either case, the environment for banks remains unchanged and consequently, their choice sets and choices are unaltered.

Households and firms: Lemma 2 implies that the liquidity payoff of the swap and thus, of the open-market operation with compensating transfers, equals zero for each $i \in \mathbb{I}$,

$$\ell_t^i(M;p,z)\Delta a_t^{M,i} + \ell_t^i(D^b;p,z)\Delta a_t^{D^b,i} = 0.$$

Lemma 1 then implies that financial wealth of all agents $i \in \mathbb{I}$ at date t (and at date $t+1$) remains unchanged. The open-market operation with compensating transfers therefore leaves the choices of households and firms in the initial equilibrium budget feasible and does not change the fact that households and firms meet their $\mathcal{L}_t^i$- and $\mathcal{L}_{t+1}^i$-constraints. Moreover, due to unchanged prices and fundamental payoffs as well as liquidity-neutrality, households and firms can shift purchasing power over time and across histories in exactly the same way as in the initial equilibrium, and they can do so with exactly the same consequences for their $\mathcal{L}_t^i$- or $\mathcal{L}_{t+1}^i$-constraints as in the initial equilibrium. Households and firms thus face unchanged choice sets and their choices are unaltered except for the portfolio changes due to the open-market operation.

Central bank: If $\int_{i\in\mathbb{I}} \Delta a_t^{M,i} di > 0$ and $\int_{i\in\mathbb{I}} \Delta a_t^{D^b,i} di < 0$ then the central bank's balance sheet expands; it issues cash to households and firms in exchange for deposits which the central bank "passes through" to banks. In the opposite case, the central bank issues deposits to households and firms and redeems cash. If the market value of the two positions differ then the central bank also gives securities s to households and firms or receives such securities. Finally, the central bank pays or receives contingent transfers at date $t+1$ whose date-t market value equals zero; these transfers may or may not be securitized at date t.

Market clearing and budgets: Since households' and firms' commodity purchases/sales as well as their production and capital accumulation decisions in the initial equilibrium remain optimal, the resource constraints continue to be satisfied. As a consequence of the open-market operation with compensating transfers all agents continue to meet their budget constraints.

Prices: Since all security positions except those for cash, deposits, and security s are unchanged; and since the markets for cash, deposits, and security s clear, all securities markets continue to clear. Since security s does not appear in the $\mathcal{L}_t^i$- or $\mathcal{L}_{t+1}^i$-constraints and as a consequence of Lemma 2, the arguments of all $\mathcal{L}_t^i$- and $\mathcal{L}_{t+1}^i$-constraints as well as the derivatives of these functions remain unchanged. Accordingly, the security prices in the initial equilibrium continue to constitute equilibrium prices. Commodity prices do not change either because markets continue to clear. We have verified the conjecture and proved the stated results. $\square$

5 Discussion of Equivalence Result

5.1 Wealth-Neutrality

Theorem 1 asserts that a liquidity-neutral open-market operation with compensating transfers leaves the wealth distribution unaltered; abstracting from liquidity considerations, all private sector agents therefore can afford to make, and make the same allocative choices. In particular, wealth-neutrality implies that the swap does not alter capital accumulation—there is no crowding in or out of physical capital.

As discussed after Lemma 1 open-market operations are interpreted broadly in that context, and depending on the degree of heterogeneity among households. Specifically, when households are homogeneous then both explicit exchanges of security s and transfers can be dispensed with because the representative household "owns" the government and lump-sum transfers between the central bank and the household sector are irrelevant, as with Ricardian equivalence (Barro, 1974). Rather than conducting open-market operations, the central bank then can institute the swap through lump-sum transfers (helicopter drops).

The same holds true when households are heterogeneous as long as the consolidated fiscal exposure of each household (i.e., its direct exposure as well as its indirect exposure through firm ownership) mirrors its consolidated exposure to the swap, along each continuation history. For example, when wealthy households pay all the taxes and the central bank swaps cash against deposits exclusively with the wealthy as counterparties, then wealth-neutrality holds. In general, what matters for wealth-neutrality is that the intervention leaves the wealth distribution unaltered in general equilibrium, not in partial equilibrium.

When households are heterogeneous wealth neutrality also requires that the measures accompanying the swap do not redistribute between the central bank and banks (unless the distributive effect on households, through their bank ownership, exactly matches their fiscal exposure). Our assumption that the central bank accommodates changes in the deposit supply by households and firms at *market rates*, guarantees that this requirement is met. The swap therefore does not change bank rents. If the central bank provided substitute funding at different rates, e.g., at rates that reflect the strong bargaining position of the central bank, then equivalence would no longer be guaranteed.

5.2 Liquidity-Neutrality

Theorem 1 asserts that the source of liquidity is irrelevant. This makes it possible for the swap to be associated with an unchanged equilibrium allocation and price system.

At the level of an individual household or firm, sources of liquidity can be substituted if effects on the $\mathcal{L}_t^i$- and $\mathcal{L}_{t+1}^i$-constraints can be avoided, for instance because the conditions of Lemma 2 are satisfied. As mentioned earlier these conditions are always met, at a minimum locally, when cash and deposits are at least minimally substitutable and when they appear only once in the $\mathcal{L}_t^i$- and $\mathcal{L}_{t+1}^i$-constraints.[9]

[9]A model which does not meet the latter requirement is the model with a cash-in-advance constraint,

At the economy-wide level, substitutability of liquidity sources follows from our assumption that liquidity is socially costless to provide (Friedman, 1969), i.e. that portfolio positions do not enter the resource constraint. But substitutability at the aggregate level follows under more general conditions: All that is needed is that the substitution of equally liquid securities does not give rise to *differential* resource costs. This seems plausible in many applications, for instance when we associate bank money with deposits and central bank money with reserves or central bank digital currency. It seems less plausible when we interpret one type of money as a cryptocurrency such as Bitcoin whose use as a medium of exchange causes substantial resource costs.

Theorem 1 also uses assumptions about the $\mathcal{L}$-constraints—or their absence—of banks and the central bank. Regarding banks, we have assumed that the ownership structure of deposits does not affect $\mathcal{L}_t^b$-constraints that is, it is irrelevant whether households, firms, or the central bank supply deposits. Regarding the central bank, we have assumed that it can extend its balance sheet without having to back monetary liabilities with gold or other commodities.

5.3 "Pass-Through" Funding and Absence of Crowding Out

One of the main arguments for a fractional reserve banking system is that such a system lowers the cost of financing real investment because it attracts cheap deposit funding. Supporters of fractional reserve banking conclude that a swap of deposits against central bank liabilities would shorten bank balance sheets and lead to less investment funding, at higher cost.

This is far from necessary, however. First, the central bank could use its newly issued liabilities to fund the investment projects previously financed by banks; we will discuss this public-investment approach shortly. Second, the central bank can serve as a pass-through entity that intermediates funds between non-banks and the banking sector, thereby effectively insulating banks from the swap. This is the pass-through approach we discussed in the proof of Theorem 1.

Consider first the public-investment approach where a reduction in non-bank deposit holdings leads banks to shed assets which the central bank assumes (the opposite direction works correspondingly). One can prove that an open-market operation with compensating transfers is wealth- and liquidity-neutral in this case. However, equivalence requires that banks are indifferent between short and long balance sheets, for example because their profits always equal zero, due to competition and free entry.[10] Moreover, and importantly, it requires that the central bank is equally capable of managing assets as banks are.

Both requirements may be violated in the real world. Banks may behave non-competitively, specifically vis-à-vis their retail depositors. And central banks may lack the skills to screen

Svensson (1985) timing, and multiple continuation histories; money balances acquired at date t appear in multiple constraints in that case, one for each continuation history. The substitutability requirement would be violated in a model where an incentive constraint depends on one security (deposits say) but not the other (cash), for instance because cash transactions cannot be monitored by a tax authority. For an application, see our discussion of the Indian de-monetization experiment in Subsection 6.3.

[10]Convex combinations of an intervention that insulates banks from the swap, as in Theorem 1, and the alternative intervention that leads to shorter or longer bank balance sheets, also work.

projects or monitor entrepreneurs and managers as efficiently as private banks. Moreover, in the face of high-powered political interests in finance, central banks may also be unable to maximize profits on their newly acquired asset portfolios. As a consequence, a substitution of bank liabilities by central bank money could give rise to a misallocation of funds as exemplified, in extreme form, in a mismanaged, centrally planned economy.

The conclusion of the supporters of fractional reserve banking still is premature; for a reduction in *private* deposit funding need not imply less *total* deposit funding. As we show in the proof of Theorem 1, banks can completely be insulated from swap induced changes in non-bank deposit holdings when the central bank engages in pass-through financing at the terms of the initial equilibrium. Equivalence then follows even if banks are non-competitive and even if central banks lack skill or face corporate governance frictions. For equivalence only requires that the ownership structure of bank liabilities be modified, not that bank assets be managed by the central bank.

Of course, pass-through financing opens the door for other potential problems. One concerns information: With non-competitive banks the central bank must replicate the deposit supply schedule when it steps in as a substitute for private depositors, and this requires the central bank to know the schedule—a mild information requirement for short-term swaps[11] but a more demanding requirement in the context of longer-term swaps.

A second potential problem concerns collateral. Equivalence could be undermined, it appears, if the central bank only lends against collateral while households and firms supply uncollateralized deposits. But this feature need not break equivalence, theoretically speaking. One could imagine an admittedly counter intuitive arrangement where after a swap of deposits for central bank money, the households and firms which previously supplied deposits to the bank, step in to submit collateral on behalf of the banks which now receive funding from the central bank. Households and firms then would effectively be in the same position as before the swap: Rather than supplying deposits to risky banks, against no collateral, they would supply deposits to the safe central bank, and collateral to banks on top of it.

Both the public-investment and the pass-through approach face potential problems when transaction costs are a major concern. At its heart, equivalence relies on the fact that unchanged wealth and liquidity positions can be supported through different gross positions, which are reflected in different balance sheet structures. When the adjustment of balance sheets or the flows associated with them are costly then equivalence may be undermined. It is not clear, however, why the costs of adjustment should be major, in particular when the central bank provides pass-through funding such that only few balance sheet positions change.

In summary, our equivalence result applies under weak assumptions about banks and bank balance sheets. It does not require the central bank to assume bank assets and it allows for arbitrary market structures. Concerns that a swap could crowd out capital thus seem unwarranted. Not only would national saving and its sectoral components remain unchanged if a swap were instituted in the form of an open-market operation with corresponding transfers; for such an intervention would be wealth-neutral and provide

[11] In the short term, deposits and deposit rates tend to be "sticky." Moreover, the central bank can observe the private sector's remaining deposit supply and draw inference from it.

no incentives for agents to change their consumption and saving choices relative to the situation before the swap (see the discussion in Subsection 5.1). But the allocation of control rights—who screens, implements, manages, and monitors investment projects—would be unchanged as well, avoiding any new risks of capital misallocation.

5.4 Multiple Equilibria and Bank Runs

Many economic models, specifically those with a focus on banks and financial markets, admit multiple equilibria. In these models, it is only through an arbitrary selection device—for instance the assumption that realizations of a "sunspot" process determine how agents coordinate their behavior—that a particular equilibrium allocation and price system is selected.

Theorem 1 can readily be applied with this type of models. Suppose, for example, that sunspots serve as the selection device. Past and present sunspot realizations then are an element of the economy's history on which outcomes are conditioned and Theorem 1 states that the history contingent equilibrium allocation and price system "before" and "after" the swap are identical. That is, for any assumed mapping from sunspots into private sector expectations, Theorem 1 implies that contingent equilibrium outcomes are unaffected by the swap.

The theorem also applies if new equilibria arise as a consequence of the swap. The pre-swap equilibrium still constitutes an equilibrium in this case; the theorem is silent about the new equilibria.[12] One might think that a swap could also eliminate equilibria and thereby invalidate the theorem. For example, when run-prone deposit financing from "small" depositors is replaced with pass-through funding from a central bank then monetary policy may have the option to eliminate contingent equilibrium allocations with bank runs and to implement "better" allocations, simply because the central bank is a large player that internalizes run externalities. But this does not undermine Theorem 1 which states that there also exists a suitable open-market operation with corresponding transfers which implements the contingent allocation in the initial equilibrium.

In Section 6, where we discuss the implications of an introduction of Central Bank Digital Currency, we return to the question of bank-run risk.

5.5 Cash-Deposit-Substitutability and Gradualism

Theorem 1 relies on the assumption that a weighted sum of cash and deposits enters the $\mathcal{L}$-constraints. This assumption is typically satisfied for small cash-deposit swaps, due to a first-order approximation. Whether the assumption also is satisfied for larger swaps is unclear because the makeup of the $\mathcal{L}$-constraints is unknown. These information limitations suggest that a central bank that aims at avoiding dislocations due to a swap should follow a gradual policy of small steps.

[12] Sticking to the initial equilibrium would be the natural outcome if one applied hysteresis as an equilibrium selection criterion.

5.6 Political Economy Constraints

Theorem 1 asserts that two exogenous policies, with and without an open-market operation with corresponding transfers, implement equilibria with the same allocation and price system. The theorem does not make claims about the optimality of the two policies; nor does it establish that the policies constitute equilibrium policies in the sense of satisfying political incentive compatibility constraints. In fact, it is conceivable that one of the two policies does constitute an equilibrium policy conditional on the institutional environment, while the other does not.

Gonzalez-Eiras and Niepelt (2015) derive conditions for *politico-economic equivalence.* These conditions relate to the environments within which political decision makers operate, not to the actual policies which are endogenous and chosen sequentially. The conditions relate the state variables and sets of admissible policy instruments across the environments. Intuitively, politico-economic equivalence requires that political decision makers in the two environments effectively have the same choice sets in terms of implementable equilibrium allocations, at each date and history, even if the sets of admissible policy instruments differ across environments.

Whether an open-market operation with corresponding transfers satisfies the conditions for politico-economic equivalence depends on many aspects of the institutional and economic environment. For example, if balance sheets provide information for voters, and if improved information changes the choice sets of political decision makers, then equivalence may not hold. Similarly, equivalence may break down when explicit transfers between households/firms and the central bank differ from implicit transfers in terms of their politico-economic repercussions; or when implicit lender-of-last-resort support provided by the central bank must satisfy different political incentive constraints than the explicit support effectively enacted by a swap that is coupled with pass-through funding.[13]

6 Applications

In this section, we apply Theorem 1 to two proposals for monetary reform. The first proposal, to introduce Central Bank Digital Currency (CBDC), envisions access to electronic central bank money for the general public rather than solely for financial institutions. The second proposal, the "Chicago Plan," stipulates the elimination of private money creation. We also apply the theorem to the Indian de-monetization experiment of 2016.

6.1 Central Bank Digital Currency

The proposal to make central bank issued digital money accessible to the general public dates back at least to Tobin (1985; 1987).[14] In recent years, his arguments have been

[13]See Niepelt (2018) for a discussion.

[14]Tobin (1985) emphasizes the benefits for society of having access to electronic means of payment and at the same time relying on a robust payments system. He argues that institutional features that promote robustness, for example deposit insurance, require regulatory limits on competition and he wonders how to "strike a balance between competitive efficiency and the protection of depositors (p. 25)."

echoed by other economists.[15] Commentators have stressed that the currently prevalent monetary arrangement with restricted access to central bank money is recent; far into the twentieth century central banks commonly offered accounts not only to a select group of financial institutions but also to non-banks (BIS, 2018). Against this background, we ask whether the introduction of CBDC as a replacement for bank deposits would give rise to a different equilibrium allocation.

To apply Theorem 1, we assume that the introduction of CBDC does not alter the asset span in the economy, i.e., CBDC does not render financial markets "more complete." This assumption is satisfied, for example, if the economy is deterministic; if an infinitesimally small amount of CBDC already is in circulation in the initial equilibrium; or if the payoff characteristics of CBDC match those of a security (or a combination of securities) that are traded in the initial equilibrium.

We specify CBDC as a means of payment with the same liquidity properties as deposits. Without loss of generality we assume that at date t, CBDC and deposits trade at equal prices, $p_t^M = p_t^{D^b}$ for all banks b. The $\mathcal{L}_t^i$-constraint of a household or firm i thus takes the form

$$\mathcal{L}_t^i\left(p_t^M a_t^{M,i} + p_t^M \sum_b a_t^{D^b,i}, \ldots\right) \leq 0,$$

where arguments of $\mathcal{L}_t^i$ that we suppress are orthogonal to $a_t^{M,i}$ and $\{a_t^{D^b,i}\}_b$. This specification meets the requirements of liquidity-neutrality in Lemma 2. A swap that is part of a liquidity-neutral open-market operation with corresponding transfers therefore guarantees equivalence.

In fact, since CBDC and deposits are equally liquid, a liquidity-neutral swap does not require an additional security s; a simple open-market exchange of deposits against CBDC suffices. Whether the swap needs to be augmented by transfers between the private sector and the central bank depends on the payoff characteristics of deposits and CBDC. We consider two scenarios; for either scenario, we assume that the return on CBDC is nominally risk free.

In the first scenario, the initial equilibrium features nominally risk free deposits. From the perspective of households and firms, the swap therefore does not change portfolio payoffs and as a consequence, equivalence does not require transfers between the private sector and the central bank. Deposits might be risk-free because of a government backed deposit insurance scheme or due to central bank lender-of-last-resort provisions that is, the risk-free nature of the deposits from the perspective of non-banks might reflect contingent transfers from the government to banks. Moreover, the banks might pay some insurance premia to the public sector in exchange for these transfers. Under the equivalent arrangement, both the central bank and banks continue to making these payments to the other party. For example, if banks paid deposit insurance premia before the introduction of CBDC, then equivalence requires them to continue paying these premia even if deposits are supplied as pass-through funding from the central bank.

[15]For an overview over recent CBDC-related literature as well as an informal equivalence proposition, see Niepelt (2018).

In the second scenario, deposits bear some nominal risk in the initial equilibrium, for instance because the central bank does not stand ready to always convert bank deposits into central bank money in the event of a run. Equivalence then (additionally) requires contingent transfers between the private sector and the central bank. These transfers assure that the swap does not change the payoffs on households' and firms' portfolios (including transfers) going forward. Households and firms thus pay transfers to the central bank in histories where deposits suffer low returns, for instance because of a bank run, and they receive transfers in "normal" histories where the return on deposits exceeds the return on CBDC. Transfers are not needed, however, in the special case where the exposure of households to the government budget exactly mirrors their exposure to risky deposit returns, for instance because households are homogeneous.

A frequently made argument against the introduction of CBDC points to the danger of increased *run risk*. According to this argument, sudden deposit withdrawals in response to swings in sentiment could become more likely because withdrawals from deposit to CBDC accounts would be nearly costless.[16] More specifically, with three types of liquid securities there are possibly two types of runs (and equilibrium indeterminacies). First, a traditional bank run where non-banks withdraw deposits from banks and hold liquidity in the form of cash instead. And second, a novel form of run from deposits into CBDC. It is this second type of run that leads many observers to claim that the introduction of CBDC renders runs more likely.

However, there are important counterarguments to that line of reasoning.[17]

First, when the central bank issues CBDC and passes the funds through to the private banks, as discussed in the proof of Theorem 1, then the central bank becomes a large, possibly the largest, depositor at the private banks. A large depositor that pursues an optimal policy (not necessarily the equivalent one) internalizes the run externalities and therefor might refrain from running itself. As a consequence, the incentives for small depositors to run might also vanish. Hence, CBDC combined with pass-through funding could make run outcome less rather than more likely.

Second, with CBDC the central bank also has an informational advantage relative to conventional runs into cash because the central bank immediately notices from fund inflows when a run is in the process of starting. The central bank therefore can engage more quickly as a lender of last resort; it can more easily prevent costly fire-sales; and it can better prevent a liquidity problem to morph into a solvency crisis. Moreover, if the remaining depositors are aware of the central bank's ability to intervene earlier, and more effectively, then they may become less wary themselves which would again reduce the risk of a deposit run.[18]

[16]To assess this argument within our framework we need to consider three types of money-securities, namely cash, deposits, and CBDC. Nevertheless, Theorem 1 (which considers a swap of two money-securities) continues to apply because an exchange of three securities can be decomposed into two exchanges of two securities each.

[17]In addition to what follows, the discussion about multiplicity and banks runs from Subsection 5.1 applies.

[18]The central bank may also set an unattractive (possibly negative) interest rate on CBDC accounts to avoid that CBDC is more attractive than cash as a safe-haven asset. Of course, the central bank has to be careful that changes in the CBDC interest rate do not serve as a coordination device for households

If one takes the argument seriously that CBDC reduces run risks then critics might argue that the introduction of CBDC may even eliminate runs on insolvent zombie banks. From a welfare perspective, such runs should not be avoided. This suggests that the introduction of CBDC should be accompanied by a governance structure that prescribes how the central bank should close down insolvent financial institutions. Such an arrangement seems reasonable in light of the fact that central banks have better information about the health of a financial institution than numerous small depositors.

Related to the discussion about CBDC-induced run risk is the question whether the central bank would loose control over its balance sheet once CBDC is introduced. Indeed, a central bank that passes through funds from non-banks to banks lengthens its balance sheet, and if the size of this pass-through varies over time, so does the length of the central bank's balance sheet. There is no reason, however, to be concerned about the length of the central bank's balance sheet per se (especially if some items on the asset and liability side net out) except for the implications on credit risk exposure. This exposure can be minimized with the appropriate collateral policy.

6.2 Chicago Plan and "Vollgeld"

The proposal to end fractional reserve banking and thereby separate credit from money creation dates back to the "Chicago Plan" from the 1930s (Fisher, 1935; Fisher, 1936).[19] The Swiss "Vollgeld" (sovereign money) initiative, which was recently rejected in a popular vote, constitutes an extreme version of the Chicago Plan as it envisions a constitutional ban on inside money creation.[20] We ask whether such a complete ban on deposit taking would affect macroeconomic outcomes. We disregard a key issue of a Vollgeld regime, namely its likely lack of enforceability.

In our framework the Chicago Plan amounts to an introduction of CBDC that fully replaces deposits. Our discussion of CBDC therefore applies directly. The elimination of fractional reserve banking would be achieved by means of an arrangement where the central bank rather than households and firms supplies deposits to banks. As discussed in the proof of Theorem 1 the central bank would supply the funds to banks at the same prices and conditions as in the initial equilibrium.

This, however, is not what the proponents of the Vollgeld proposal envisioned. According to that proposal, the central bank does not supply deposit substitutes to the banking sector but it prevents banks from issuing deposits and forces them to finance their activities through the issuance of other liabilities (e.g., central bank emergency loans). Banks would likely loose a source of profits—seignorage rents from liquidity creation—in this case and as a consequence, equivalence would be undermined, both for distributive reasons and because banks might adopt a different business model which would also affect their assets. Equivalence might hold, however, if redistribution between the central bank

to start a run.

[19]Benes and Kumhof (2012) offer a quantitative assessment of the Chicago plan; they argue that the plan would improve outcomes.

[20]See www.vollgeld-initiative.ch/english/. For the policy discussion in other countries as well as less extreme "narrow banking" proposals, see the review in Niepelt (2018).

and the banking sector does not affect household wealth, for example because the economy admits a representative household or the bank ownership structure coincides with the distribution of the tax levy; and if redistribution does not change bank incentives.

6.3 Indian De-Monetization

On November 8th, 2016 the Indian government declared more than 85% of cash in circulation illegal tender. Banknotes with a denomination of 500 rupees (about $7.50) or more had to be temporarily deposited in a bank account. The stated objectives of the measure were to reduce the size of the shadow economy, to fight corruption, and to remove counterfeited notes from circulation. Since the old bank notes were only slowly replaced with new ones the intervention increased the stock of digital money. As a consequence, cash payments became more difficult and the characteristics of certain transactions changed. For example, some transactions—most prominently real estate transactions—were conducted with prices being paid partly in cash and partly using deposits.

Our framework can easily be adapted to represent an economy with a large black market sector. Suppose that some goods and services in the commodity vector x come in two varieties, an "official" one which can be purchased in the official sector, at a price that includes goods and services tax (GST); and an "unofficial" one which can be purchased in the unofficial sector, at a price that does not include tax. Preferences exhibit a large degree of substitutability between the two varieties; the budget constraint includes both varieties, at different prices; and households' $\mathcal{L}^h$-constraints specify different cash-in-advance restrictions, say, for each variety. While the official varieties can be purchased against cash or deposits (substitutability) the unofficial varieties can only be purchased using cash (no substitutability). Subject to this complete lack of substitutability for transactions in the unofficial sector, and due to the fact that the large monetary intervention could not be delimited to the official sector, the sufficient conditions for equivalence are clearly violated.

7 Conclusions

This paper makes two contribution: It provides a general framework for the study of monetary economies; and it provides sufficient conditions for the equivalence of monetary systems with private or public money. Our equivalence result (Theorem 1) should be construed as a benchmark result that helps to organize one's thinking about complex economic relationships, in the spirit of Modigliani and Miller (1958), Barro (1974), and many other equivalence results in economics. There may be very few circumstances under which the sufficient conditions for equivalence literally apply; nevertheless, the conditions give a clear sense of the sources of non-equivalence in concrete settings.

In particular, the conditions highlight that a monetary reform should be expected to change the equilibrium allocation if it did not meet the requirements for wealth- and liquidity-neutrality. This, in turn, should be expected to be the case when the reform were associated with restrictions that prevented counteracting balance sheet adjustments, or when the reform had direct distributive implications and households were sufficiently heterogeneous.

Whether a non-neutral monetary reform would be towards the better or the worse is a question that the equivalence result cannot address. We leave answers to questions of this type for future research. They require explicit characterizations of equilibrium in concrete economies, as well as serious quantitative analyses. For policy discussions about monetary reform, our paper therefore does not propose a set of definite answers, but an analytical framework and a robust road map.

References

Andolfatto, D. (2018), Reconciling orthodox and heterodox views on money and banking, Working Paper 2018-27A, Federal Reserve Bank of St. Louis, St. Louis.

Barro, R. J. (1974), 'Are government bonds net wealth?', *Journal of Political Economy* **82**(6), 1095–1117.

Baumol, W. J. (1952), 'The transactions demand for cash', *Quarterly Journal of Economics* **67**(4), 545–556.

Benes, J. and Kumhof, M. (2012), The Chicago plan revisited, Working Paper 12/202, International Monetary Fund, Washington.

Bewley, T. F. (1980), The optimum quantity of money, *in* J. H. Kareken and N. Wallace, eds, 'Models of Monetary Economies', Federal Reserve Bank of Minneapolis, Minneapolis, pp. 169–210.

BIS (2018), Central bank digital currencies, Report, BIS: Committee on Payments and Market Infrastructures, and Markets Committee, Basel.

Brunnermeier, M. K. and Sannikov, Y. (2016), The I theory of money, Working Paper 22533, NBER, Cambridge, Massachusetts.

Bryant, J. (1983), 'Government irrelevance results: A simple exposition', *American Economic Review* **73**(4), 758–761.

Calvo, G. A. (1983), 'Staggered prices in a utility-maximizing framework', *Journal of Monetary Economics* **12**(3), 383–398.

Chamley, C. and Polemarchakis, H. (1984), 'Assets, general equilibrium and the neutrality of money', *Review of Economic Studies* **51**(1), 129–138.

Clarida, R., Galí, J. and Gertler, M. (1999), 'The science of monetary policy: A New Keynesian perspective', *Journal of Economic Literature* **37**(4), 1661–1707.

Clower, R. W. (1967), 'A reconsideration of the microfoundations of monetary theory', *Western Economic Journal* **6**(1), 1–8.

Croushore, D. (1993), 'Money in the utility function: Functional equivalence to a shoppingtime model', *Journal of Macroeconomics* **15**(1), 175–182.

Dixit, A. K. and Stiglitz, J. E. (1977), 'Monopolistic competition and optimum product diversity', *American Economic Review* **67**(3), 297–308.

Faure, S. and Gersbach, H. (2018), Money creation in different architectures, Discussion Paper 13156, CEPR.

Feenstra, R. C. (1986), 'Functional equivalence between liquidity costs and the utility of money', *Journal of Monetary Economics* **17**(2), 271–291.

Fisher, I. (1930), *The Theory of Interest, as determined by Impatience to Spend Income and Opportunity to Invest it*, Macmillan, New York.

Fisher, I. (1935), *100% Money*, Aldephi, New York.

Fisher, I. (1936), '100% money and the public debt', *Economic Forum* (April–June), 406–420.

Friedman, M. (1969), The optimum quantity of money, *in* M. Friedman, ed., 'The Optimum Quantity of Money and Other Essays', Aldine, Chicago, chapter 1, pp. 1–50.

Galí, J. (2008), *Monetary Policy, Inflation, and the Business Cycle*, Princeton University Press, Princeton.

Gonzalez-Eiras, M. and Niepelt, D. (2015), 'Politico-economic equivalence', *Review of Economic Dynamics* **18**(4), 843–862.

Grandmont, J.-M. and Younes, Y. (1972), 'On the role of money and the existence of a monetary equilibrium', *Review of Economic Studies* **39**(3), 355–372.

Gurley, J. G. and Shaw, E. S. (1960), *Money in a Theory of Finance*, Brookings Institution, Washington.

Hayashi, F. (1982), 'Tobin's marginal q and average q: A neoclassical interpretation', *Econometrica* **50**(1), 213–224.

Holmström, B. and Tirole, J. (1998), 'Private and public supply of liquidity', *Journal of Political Economy* **106**(1), 1–40.

Jorgensen, D. W. (1963), 'Capital theory and investment behavior', *American Economic Review, Papers and Proceedings* **53**(2), 247–259.

Kiyotaki, N. and Moore, J. (2012), Liquidity, business cycles, and monetary policy, Working Paper 17934, NBER, Cambridge, Massachusetts.

Kiyotaki, N. and Wright, R. (1993), 'A search-theoretic approach to monetary economics', *American Economic Review* **83**(1), 63–77.

Lagos, R. and Wright, R. (2005), 'A unified framework for monetary theory and policy analysis', *Journal of Political Economy* **113**(3), 463–484.

Lucas, R. E. (1980), Equilibrium in a pure currency economy, *in* J. H. Kareken and N. Wallace, eds, 'Models of Monetary Economies', Federal Reserve Bank of Minneapolis, Minneapolis, pp. 131–145.

Lucas, R. E. (1982), 'Interest rates and currency prices in a two-country world', *Journal of Monetary Economics* **10**(3), 335–359.

Lucas, R. E. and Stokey, N. L. (1987), 'Money and interest rates in a cash-in-advance economy', *Econometrica* **55**(3), 491–513.

Modigliani, F. and Miller, M. H. (1958), 'The cost of capital, corporation finance and the theory of investment', *American Economic Review* **48**(3), 261–297.

Niepelt, D. (2018), Reserves for all? Central Bank Digital Currency, deposits, and their (non)-equivalence, Discussion Paper 13065, CEPR.

Samuelson, P. A. (1958), 'An exact consumption-loan model of interest with or without the social contrivance of money', *Journal of Political Economy* **66**(6), 467–482.

Sargent, T. J. (1987), *Dynamic Macroeconomic Theory*, Harvard University Press, Cambridge, Massachusetts.

Saving, T. R. (1971), 'Transactions costs and the demand for money', *American Economic Review* **61**(3), 407–420.

Shell, K. (1971), 'Notes on the economics of infinity', *Journal of Political Economy* **79**(5), 1002–1011.

Sidrauski, M. (1967), 'Rational choice and patterns of growth in a monetary economy', *American Economic Review* **57**(2), 534–544.

Svensson, L. E. O. (1985), 'Money and asset prices in a cash-in-advance economy', *Journal of Political Economy* **93**(5), 919–944.

Tobin, J. (1956), 'The interest elasticity of the transactions demand for cash', *Review of Economics and Statistics* **38**(3), 241–247.

Tobin, J. (1963), Commercial banks as creators of "money", Discussion Paper 159, Cowles Foundation, New Haven.

Tobin, J. (1969), 'A general equilibrium approach to monetary theory', *Journal of Money, Credit, and Banking* **1**(1), 15–29.

Tobin, J. (1985), 'Financial innovation and deregulation in perspective', *Bank of Japan Monetary and Economic Studies* **3**(2), 19–29.

Tobin, J. (1987), The case for preserving regulatory distinctions, *in* 'Restructuring the Financial System, Proceedings of the Economic Policy Symposium, Jackson Hole', Federal Reserve Bank of Kansas City, Kansas City, chapter 9, pp. 167–183.

Wallace, N. (1980), The overlapping generations model of fiat money, *in* J. H. Kareken and N. Wallace, eds, 'Models of Monetary Economies', Federal Reserve Bank of Minneapolis, Minneapolis, pp. 49–82.

Wallace, N. (1981), 'A Modigliani-Miller theorem for open-market operations', *American Economic Review* **71**(3), 267–274.

Woodford, M. (1990), 'Public debt as private liquidity', *American Economic Review* **80**(2), 382–388.

Woodford, M. (2003), *Interest and Prices*, Princeton University Press, Princeton.