Rare Disasters, Financial Development, and Sovereign Debt

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Abstract

We study the implications of the interaction between rare disasters and financial development for sovereign debt markets. In our model, countries vary in their financial development, by which we mean the extent to which shocks can be hedged in international capital markets. The model predicts that low levels of financial development generate a key feature of sovereign debt in emerging economies known as "debt intolerance": high credit spreads associated with lower debt-to-output ratios than those of developed countries.

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1 Introduction

One intriguing fact about sovereign debt markets is that emerging economies pay high credit spreads on their sovereign debt, despite generally having much lower debt-output ratios than developed countries. Reinhart, Rogoff and Savastano (2003) call this phenomenon "debt intolerance."

This debt intolerance is at odds with the predictions of the classic Eaton and Gersovitz (1981) model of sovereign debt.[1] A key cost of defaulting in this model is the loss of access to capital markets. Since real output growth is generally more volatile in emerging markets than in developed countries, the loss of market access is more costly for emerging markets. So, other things equal, emerging markets should be less likely to default, pay lower credit spreads on their sovereign debt, and have higher debt capacity.

The idea that high output volatility in emerging markets makes their default cost high and their probability of default low contradicts another finding stressed by Reinhart et al (2003): emerging markets tend to be serial defaulters.

In this paper, we propose a model of sovereign debt where countries vary in their level of financial development. By financial development, we mean the extent to which countries can hedge shocks to their economies in international capital markets.[2] We show that low levels of financial development generate debt intolerance.

We write our sovereign-debt model in continuous time.[3] A significant technical advantage of this approach is that the model can be solved in closed form up to an ordinary differential equation (ODE) with intuitive boundary conditions.

The representative agent has the continuous-time version of Epstein and Zin (1989) preferences proposed by Duffie and Epstein (1992a). These preferences allow the model to generate empirically plausible average debt-to-output ratios without assuming the very high discount rates (generally in excess of 20 percent per year) used in the literature. Our cali-

[1] See Aguiar and Amador (2014) and Aguiar, Chatterjee, Cole, and Stangebye (2016) for recent surveys of the sovereign debt literature.

[2] Another aspect of financial development might reflect the country's access to commitment mechanisms such as posting collateral or depositing money in escrow accounts that can be seized by creditors. We do not consider these mechanisms because sovereign debt is in practice generally unsecured. Mendoza, Quadrini, and Rios-Rull (2009) also emphasize the importance of financial development, which they interpret as a country's ability to enforce domestic financial contracts to hedge idiosyncratic risks.

[3] Other sovereign debt models in continuous time include Nuño and Thomas (2015), Tourre (2017), Bornstein (2017), and DeMarzo, He, and Tourre (2018). See Brunnermeier and Sannikov (2016) for a review of the growing continuous-time macro-finance literature.

bration combines a conventional value of the discount factor (5.2 percent per year) with a low elasticity of intertemporal substitution (EIS) and moderate risk aversion. We interpret the low EIS as reflecting expenditure commitments that are difficult to change. The Duffie-Epstein-Zin preferences are key to making this calibration work. With standard expected utility, a low EIS implies a high risk aversion that would create an incentive to avoid the debt region, generating a low average debt-to-output ratio.

Output follows the jump-diffusion process considered by Barro and Jin (2011), in which the size distribution of jumps is governed by a power law. The presence of rare disasters is key to generating default in our model.[4]

As in Aguiar and Gopinath (2006) and Arellano (2008), we assume that upon default the country suffers a decline in output and loses access to international capital markets. It then regains access to these markets with constant probability. Outside of the default state, the country can invest in a risk-free international bond, hedge diffusion shocks, partially hedge rare-disaster risk, and issue non-contingent debt that can be defaulted upon.

Our model includes two frictions that make markets incomplete: limited commitment and limited spanning.[5] To isolate the impact of limited commitment, suppose there is full spanning so that all shocks can be hedged. As in Kehoe and Levine (1993) and Kocherlakota (1996), the country's debt capacity is reduced to a level such that in equilibrium the country weakly prefers repaying its outstanding debt over defaulting on it.[6] Since hedging is more cost effective than defaulting in terms of managing the country's risk, the country never defaults and the credit spread on sovereign debt is zero.

The second form of market incompleteness is limited spanning. In practice, manifestations of limited spanning include a country's inability to issue debt with long maturities or debt denominated in local currency. To simplify the analysis, we model limited spanning as the country's limited access to financial securities that can be used to hedge rare disasters. In the tradition of Eaton and Gersovitz (1981), we assume that these limits are

[4]Rare disasters have proved useful in modeling many other asset pricing phenomena. Examples include the equity premium (Rietz (1988), Barro (2006), Barro and Jin (2011), and Gabaix (2012)), the predictability of excess returns (Wachter (2013)), the corporate bond spread (Bhamra and Strebulaev (2011)), and the returns to the carry trade (Burnside, Eichenbaum and Rebelo (2011) and Farhi and Gabaix (2015)).

[5]Bai and Zhang (2010) combine these two forms of market incompleteness to explain the Feldstein-Horioka puzzle. They consider a limited-enforcement model in the spirit of Kehoe and Levine (1993), so in their model default does not occur in equilibrium.

[6]For other early important contributions on the implications of limited commitment, see Alvarez and Jermann (2000, 2001), Kehoe and Perri (2002), Albuquerque and Hopenhayn (2004), and Cooley, Marimon, and Quadrini (2004).

exogenous. This exogeneity assumption is consistent with the key finding of the literature on the original-sin hypothesis: the degree of market incompleteness is more closely related to the size of the economy than to the soundness of fiscal and monetary policy or other fundamentals (Hausmann and Panizza (2003) and Bordo, Meissner, and Redish (2004)).

One key result is that the more limited is the spanning of assets at a country's disposal, the more severe is its debt intolerance. When spanning is limited, it is not optimal of fully hedge risks that can be hedged.[7] The country uses the available hedging instruments to increase its debt capacity by ensuring that default is not triggered by shocks that can be hedged. So, countries with more limited spanning hedge less and endure more volatility in consumption. These countries are also more likely to default, so lenders charge them a higher credit spread to cover the expected default losses. Reducing the span of assets available to a country reduces debt capacity, increases credit spreads, and limits the ability to smooth consumption. In other words, limited spanning produces debt intolerance.

Our model suggests that Shiller's (1993) proposed creation of a market for perpetual claims on countries' Gross Domestic Produce (GDP) could significantly improve welfare in emerging markets. By increasing a country's ability to hedge its risks, GDP-linked bonds would lower credit spreads, increase debt capacity and reduce consumption volatility.

The paper is organized as follows. Section 2 presents the model and Section 3 discusses the solution method. Sections 4 and 5 summarize the solution for the first-best and the limited-commitment case, respectively. Section 6 calibrates our model and explores its quantitative properties. Section 7 performs sensitivity analysis with respect to key parameters of the model. Section 8 discusses an expected-utility version of our calibration. Section 9 concludes.

2 Model Setup

We consider a continuous-time model where the country's infinitely-lived representative agent receives a perpetual stochastic stream of output. As we show in Section 3, default occurs in equilibrium. Upon default, the country endures distress costs that take the form of a fall in output and temporary exclusion from capital markets. We call the regime in which the country does not have access to financial markets autarky and the regime in which it has access to financial markets the normal regime.

[7] This result is related to work by Caballero and Krishnamurthy (2003). These authors use a different class of models that builds on the work of Holmstrom and Tirole (1998) to show that limited financial development exacerbates underinsurance in emerging markets.

Below, we introduce the law of motion that governs the output process.

2.1 Output Process

Output Process in the Normal Regime. We model output in this regime, Y_t, as a jump-diffusion process. This process is consistent with the evidence presented in Aguiar and Gopinath (2007) which suggests that permanent shocks are the primary source of fluctuations in emerging markets. Both diffusion and jump shocks are important in generating our model's main predictions.

The law of motion for output, Y_t, is given by:

$$\frac{dY_t}{Y_{t-}} = \mu dt + \sigma d\mathcal{B}_t - (1 - Z)d\mathcal{J}_t \,, \quad Y_0 > 0 \,, \tag{1}$$

where μ is the drift parameter, σ is the diffusion-volatility parameter, $\mathcal{B}$ is a standard Brownian motion process, and $\mathcal{J}$ is a pure jump process. Let $\tau^{\mathcal{J}}$ denote the jump arrival time. If a jump occurs at t, i.e., when $t = \tau^{\mathcal{J}}$, $d\mathcal{J}_t = 1$ and output falls from Y_{t-} to $Y_t = ZY_{t-}$, where $Y_{t-} \equiv \lim_{s\uparrow t} Y_s$ denotes the left limit of Y. We call $Z \in [0, 1]$, the fraction of output retained by the country after the jump, the recovery fraction. If a jump does not occur at t, i.e., when $t \neq \tau^{\mathcal{J}}$, $d\mathcal{J}_t = 0$ and $Y_t = Y_{t-} \equiv \lim_{s\uparrow t} Y_s$ as the Brownian motion is continuous.

We assume that Z follows a well-behaved cumulative distribution function, $G(Z)$ and that jumps are governed by a Poisson process with a constant arrival rate, λ. There is no limit to the number of jumps that can occur over a fixed time interval and the occurrence of a jump does not affect the likelihood of future jumps.

Since the expected percentage output loss upon the arrival of a jump is $(1 - \mathbb{E}(Z))$, the expected growth rate of output in levels is given by:

$$g = \mu - \lambda(1 - \mathbb{E}(Z)) \ . \tag{2}$$

Here, the term $\lambda(1 - \mathbb{E}(Z))$ represents the reduction in the expected growth rate associated with jumps.

We can write the dynamics for logarithmic output, $\ln Y_t$, in discrete time as follows:

$$\ln Y_{t+\Delta} - \ln Y_t = \left(\mu - \frac{\sigma^2}{2}\right) \Delta + \sigma\sqrt{\Delta}\, \epsilon_{t+\Delta} - (1 - Z)\nu_{t+\Delta} \,, \tag{3}$$

where the time-t conditional distribution of $\epsilon_{t+\Delta}$ is a standard normal and $\nu_{t+\Delta} = 1$ with probability $\lambda\Delta$ and zero with probability $(1 - \lambda\Delta)$. Equation (3) implies that the expected change of $\ln Y$ over a time increment Δ is $(\mu - \sigma^2/2)\,\Delta - \lambda(1 - \mathbb{E}(Z))\Delta$. The term $\sigma^2/2$ is the Jensen-inequality correction associated with the diffusion shock.

Output Process under Autarky. Let $\tau^{\mathcal{D}}$ denote the endogenous time of default. Upon default, the country enters autarky. There are two costs of defaulting. The first cost is the loss of access to financial markets, so consumption equals output in autarky.

The second cost is an output loss that proxies for the disruptions of economic activity associated with default. As in Aguiar and Gopinath (2006), we assume that upon default output drops permanently from $Y_{\tau^{\mathcal{D}}-} \equiv \lim_{s\uparrow\tau^{\mathcal{D}}-} Y_s$, the output in the normal regime just prior to default, to $\alpha Y_{\tau^{\mathcal{D}}-}$, where $(1-\alpha)$ is the default cost.

We denote the output process in autarky by $\widehat{Y}_t$. This process starts at time $\tau^{\mathcal{D}}$ with the value of $\widehat{Y}_{\tau^{\mathcal{D}}} = \alpha Y_{\tau^{\mathcal{D}}-}$ and follows the same output process as that for the normal regime:

$$\frac{d\widehat{Y}_t}{\widehat{Y}_{t-}} = \mu dt + \sigma d\mathcal{B}_t - (1-Z)d\mathcal{J}_t \ . \tag{4}$$

While in autarky, the country re-gains its access to financial markets with probability ξ per unit of time. Let $\tau^{\mathcal{E}}$ denote the stochastic exogenous exit time from autarky. The duration of autarky is $\tau^{\mathcal{D}} \le t < \tau^{\mathcal{E}}$. Upon randomly exiting from autarky at time $\tau^{\mathcal{E}}$, the country starts afresh with no debt and regains access to international markets. Output in the normal regime starts with $Y_{\tau^{\mathcal{E}}}$, which is equal to $\widehat{Y}_{\tau^{\mathcal{E}}-}$, the pre-exit output level under autarky: $Y_{\tau^{\mathcal{E}}} = \widehat{Y}_{\tau^{\mathcal{E}}-}$ and then follows the process given by equation (1).

2.2 Preferences

We assume that the lifetime utility of the representative agent, V_t, has the recursive form proposed by Kreps and Porteous (1978), Epstein and Zin (1989), and Weil (1990). We use the continuous-time version of these preferences developed by Duffie and Epstein (1992a):

$$V_t = \mathbb{E}_t\left[\int_t^\infty f(C_u, V_u)du\right], \tag{5}$$

where $f(C,V)$ is the normalized aggregator for consumption C and utility V. This aggregator is given by:

$$f(C,V) = \frac{\rho}{1-\psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\chi}}{((1-\gamma)V)^{\chi-1}} \,. \tag{6}$$

Here, ρ is the subjective discount rate and

$$\chi = \frac{1-\psi^{-1}}{1-\gamma} \,. \tag{7}$$

This recursive, non-expected utility formulation allows us to separate the coefficient of relative risk aversion (γ) from the elasticity of intertemporal substitution (ψ). This separation plays an important role in our quantitative analysis. The time-additive separable CRRA utility is a special case of recursive utility where the coefficient of relative risk aversion, γ, equals the inverse of the elasticity of intertemporal substitution (EIS), $\gamma = \psi^{-1}$, implying $\chi = 1$. In this case, $f(C, V) = U(C) - \rho V$, which is additively separable in C and V, with $U(C) = \rho C^{1-\gamma}/(1-\gamma)$.

2.3 Financial Assets and Market Structure

If the country could trade in a complete set of contingent assets, a setting which we refer to as full spanning, default would not occur in equilibrium. As discussed in the introduction, our model includes two sources of market incompleteness. The first is limited commitment: the country cannot commit to repaying its debt. The second is limited spanning: markets for certain shocks are incomplete. To capture the notion that some shocks are harder to hedge than others, we assume that large jump shocks might not be insurable.

We denote the country's financial wealth by W_t. Under normal circumstances, the country has four investment and financing opportunities: (1) it can insure its diffusion risk through hedging contracts; (2) it can buy insurance against certain jumps; (3) it can borrow in the sovereign debt market at an interest rate that is the sum of the risk-free rate, r, and an endogenous credit spread, π_t; and (4) it can save at the risk-free rate, r. Upon default on its sovereign debt, the country enters autarky and loses access to all four investment and financing opportunities. It regains this access with a constant probability.

Diffusion Risk Hedging Contracts. We assume that diffusive shocks are idiosyncratic and that markets for contracts that hedge these shocks are perfectly competitive. An investor who holds one unit of the hedging contract at time t receives no upfront payment, since there is no risk premium for bearing idiosyncratic risk, and receives a gain or loss equal to $\sigma d\mathcal{B}_t = \sigma\left(\mathcal{B}_{t+dt} - \mathcal{B}_t\right)$ at time $t + dt$. We normalize the volatility of this hedging contract to be equal to the output volatility parameter, σ. This hedging contract is analogous to a futures contract in standard no-arbitrage models, see e.g., Cox, Ingersoll, and Ross (1981). We denote the country's holdings of diffusion risk contracts at time t by Θ_t.

Jump Insurance Contracts and Premia. We assume that jump shocks are idiosyncratic and that markets for contracts that hedge these shocks are perfectly competitive. Consider an insurance contract initiated at time t that covers the following jump event: the *first* stochastic arrival of a downward jump in output with a recovery fraction in the interval $(Z, Z + dZ)$ at jump time $\tau^{\mathcal{J}} > t$ for $Z \geq Z^*$. Here, Z^* is a parameter that describes the level of financial development. The higher the value of Z^*, the less developed are financial markets and the fewer jump insurance opportunities the country has.

The buyer of a unit of this insurance contract makes continuous insurance premium payments. Once the jump event occurs at time $\tau^{\mathcal{J}}$, the buyer stops making payments and receives a one-time *unit* lump-sum payoff. The insurance premium payment is equal to $\lambda dG(Z)$, the product of the jump intensity, λ, and the probability $dG(Z)$ that the recovery fraction, Z, falls in the interval $(Z, Z+dZ)$ for $Z \geq Z^*$. Conceptually, this insurance contract is analogous to one-step-ahead Arrow securities in discrete-time models. In practice, this insurance contract is similar to a credit default swap.[8]

We denote the country's holdings of jump-risk insurance contracts at time t contingent on a recovery fraction Z by $X_t(Z)$. The country pays an insurance premium to hedge jump risk at a rate $X_t(Z)\lambda\, dG(Z)$ before the *first* jump with Z arrives at time $\tau^{\mathcal{J}}$. At this time, the country receives a lump-sum payment $X_t(Z)$ if the recovery fraction is in the interval $(Z, Z + dZ)$. Since the country can purchase insurance for all possible values of $Z \geq Z^*$, the total jump insurance premium per period is given by:

$$\Phi_t = \lambda \int_{Z^*}^{1} X_t(Z) dG(Z) \equiv \lambda\, \mathbb{E}\left[X_t(Z)\, \mathcal{I}_{Z \geq Z^*}\right] , \tag{8}$$

where the expectation, $\mathbb{E}[\,\cdot\,]$, is calculated with respect to the cumulative distribution function, $G(Z)$. The indicator function $\mathcal{I}_{Z \geq Z^*}$ equals one if $Z \geq Z^*$ and zero otherwise. This indicator function imposes the restriction that jump insurance is available only for $Z > Z^*$.

Sovereign Debt, Default, and Credit Spread. As in discrete-time settings, sovereign debt is borrower-specific, non-contingent, unsecured, and short term.[9] Sovereign debt is continuously repaid and reissued at the interest rate $r + \pi_{t-}$, where π_{t-} is the endogenous

[8] Pindyck and Wang (2013) discuss a similar insurance contract in a general equilibrium setting with economic catastrophes.

[9] Auclert and Rognlie (2016) show that sovereign debt models with short-term debt have a unique Markov perfect equilibrium. Sovereign-debt models with long-maturity debt include Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), and Chatterjee and Eyigungor (2012).

credit spread. The borrowing process continues until the country defaults and resumes once the borrower re-enters the sovereign-debt market. Sovereign debt is held and priced in competitive markets by well-diversified foreign investors. The maximal amount of debt that the country can issue is stochastic and endogenously determined in equilibrium by the creditors' break-even condition and the borrower's optimal default decisions.

The country has the option to default at any time on its sovereign debt. As emphasized by Zame (1993) and Dubey, Geanakoplos and Shubik (2005), the possibility of default provides a partial hedge against risks that cannot be insured because of limited financial spanning.[10]

Optimality. The country chooses its consumption, diffusion and jump risk hedging demands, sovereign debt issue, and default timing to maximize the utility of the representative agent, defined by equations (5)-(6), given the process specified in equation (1), and equilibrium pricing of sovereign debt and insurance contracts for diffusion and jump shocks.

3 Model Solution

We solve our model using dynamic programming. We denote by $V(W_t, Y_t)$ the representative agent's value function for the normal regime and by $\widehat{V}(Y_t)$ the value function for the autarky regime. The autarky value function depends only on contemporaneous output because financial wealth is always zero in autarky.

3.1 The Normal Regime

Financial wealth, W_t, evolves according to:

$$dW_t = [(r + \pi_{t-})W_{t-} + Y_{t-} - C_{t-} - \Phi_{t-}]dt + \sigma\Theta_{t-}d\mathcal{B}_t + X_{t-}(Z)\, d\mathcal{J}_t\,. \tag{9}$$

The first term on the right side of equation (9) is interest income/expenses, $(r + \pi_{t-})W_{t-}\, dt$ plus output, $Y_{t-}\, dt$, minus consumption, $C_{t-}\, dt$, and minus the jump-insurance premium, $\Phi_{t-}dt$. When $W_{t-} > 0$, the country has no debt outstanding and $\pi_{t-} = 0$. When $W_{t-} < 0$, the country pays interest at a rate $(r + \pi_{t-})$, where π_{t-} is the equilibrium credit spread.

The second term on the right side of equation (9), $\sigma\Theta_{t-}d\mathcal{B}_t$, is the realized gain or loss from diffusion risk hedging contracts. Since diffusion shocks are idiosyncratic with zero mean,

[10]To simplify, we consider only the possibility of complete default. See Yue (2010) and Asonuma, Niepelt, and Ranciere (2017) for models with partial default.

the country incurs no up-front payment. The third term represents the lump-sum payment, $X_{t-}(Z)$, from the jump insurance contract when a jump arrives ($d\mathcal{J}_t = 1$) and the realized Z is hedgeable, i.e., $Z \in [Z^*, 1]$.

Dynamic Programming. The value function $V(W, Y)$ in the normal regime satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:[11]

$$
\begin{aligned}
0 = \max_{C,\Theta,X} \; & f(C, V(W,Y)) + [(r+\pi)W + Y - C - \Phi]\, V_W(W,Y) \\
& + \frac{\Theta^2\sigma^2}{2} V_{WW}(W,Y) + \mu Y V_Y(W,Y) + \frac{\sigma^2 Y^2}{2} V_{YY}(W,Y) + \Theta\sigma^2 Y V_{WY}(W,Y) \\
& + \lambda \mathbb{E}\left[(V(W+X, ZY)\mathcal{I}_{Z\geq Z^*} + V(W, ZY)\mathcal{I}_{\underline{Z}\leq Z<Z^*} + V(W,ZY)\mathcal{I}_{Z<\underline{Z}}) - V(W,Y) \right] ,
\end{aligned}
\tag{10}
$$

where the expectation $\mathbb{E}[\,\cdot\,]$ is evaluated with respect to the cumulative distribution function, $G(Z)$. The HJB equation states that at the optimum, the sum of the country's normalized aggregator, $f(C, V)$, and the expected change in the value function V (the sum of all the other terms on the right side of equation (10)) must equal zero.

The second and third terms of equation (10), describe the drift and volatility effects of wealth W on the expected change of the value function $V(W, Y)$. The fourth and fifth terms reflect the drift and volatility effects of output Y on the expected change of $V(W, Y)$. The sixth term, $\Theta\sigma^2 Y V_{WY}(W, Y)$, captures the effect of the country's intertemporal diffusive shock hedging demand on the drift of $V(W, Y)$.

The last term, which appears in the third line of equation (10), represents the expected change in the value function that occurs as a result of a jump arrival and the concomitant default decision. Diffusion shocks do not cause default because it is always more efficient to hedge diffusion shocks at the actuarially fair rate. Only jump shocks can potentially trigger default. When a jump arrives at time t ($d\mathcal{J}_t = 1$), the country decides whether to default on its debt after observing the realized recovery fraction, Z. The default decision is characterized by an endogenous, stochastic threshold rule, $\underline{Z}$.

If a jump occurs at time t with recovery fraction $Z < \underline{Z}$, the country defaults, enters autarky, and its output falls to $\widehat{Y}_t = \alpha Y_t$, where $Y_t = ZY_{t-}$, so the value function upon entering autarky at $t = \tau^{\mathcal{D}}$ is $V(W_t, ZY_{t-}) = \widehat{V}(\widehat{Y}_t) = \widehat{V}(\alpha Z Y_{t-})$.

If a jump occurs at time t with recovery fraction $Z \geq \underline{Z}$, the country does not default.

[11] Duffie and Epstein (1992b) generalize the standard HJB equation for the expected-utility case to allow for non-expected recursive utility such as the Epstein-Weil-Zin utility used here.

Local non-satiation implies that it is not optimal to purchase jump insurances that pay off in states in which the country defaults. Therefore, $\underline{Z} \leq Z^*$.

If $Z \in [\underline{Z}\,, Z^*)$, the jump was not insurable and the country does not default, so its value function at t is $V(W_{t-}, ZY_{t-})$. If $Z \geq Z^*$, the country receives a jump-insurance payment, $X_{t-}(Z)$ and does not default, so its value function at t is $V(W_{t-} + X_{t-}(Z), ZY_{t-})$.

First-Order Conditions. As in Duffie and Epstein (1992a, 1992b), the first-order condition (FOC) for C is:

$$f_C(C, V) = V_W(W, Y)\,. \tag{11}$$

This condition equates the marginal benefit of consumption, $f_C(C, V)$, to the marginal utility of savings, $V_W(W, Y)$. While under expect utility $f_C(C, V) = U'(C)$, which does not depend on V, here $f_C(C, V)$ is non-separable in C and V.

The FOC for the diffusion-risk hedging demand is:

$$\Theta = -\frac{YV_{WY}(W, Y)}{V_{WW}(W, Y)}\,. \tag{12}$$

We verify that $V(W, Y)$ is concave in W ($V_{WW} < 0$) so the FOC (12) is sufficient to characterize optimality for Θ. Equation (12) is similar to the intertemporal hedging demand in Merton (1969) for expected utility and in Duffie and Epstein (1992b) for recursive preferences. Since the country is endowed with a long position in domestic output, we expect its hedging demand to be negative.

The optimal jump risk hedging demand, $X(Z; W, Y)$, solves the following problem:

$$\max_{X} \quad \lambda\,\mathbb{E}\left[\left(V(W + X, ZY) - XV_W(W, Y)\right)\mathcal{I}_{Z \geq Z^*}\right]\,. \tag{13}$$

This problem boils down to maximize $(V(W + X, ZY) - XV_W(W, Y))$ by choosing $X(Z; W, Y)$ for each value of Z that can be insured ($Z \geq Z^*$). The FOC for $X(Z; W, Y)$ is:

$$V_W(W + X(Z; W, Y), ZY) = V_W(W, Y)\,. \tag{14}$$

The intuition for this condition is that it is optimal to choose X to equate the pre- and post-jump marginal utility of wealth. Since output falls upon a jump arrival, without jump insurance, $V_W(W, Y) < V_W(W, ZY)$. The country chooses $X(Z; W, Y) > 0$ to equate the pre- and post-jump marginal utility of wealth.

Value Function. We conjecture and verify that the value function, $V(W, Y)$, is given by:

$$V(W,Y) = \frac{(bP(W,Y))^{1-\gamma}}{1-\gamma}\,, \tag{15}$$

where b is given by

$$b = \rho \left[\frac{r + \psi(\rho - r)}{\rho}\right]^{\frac{1}{1-\psi}}\,. \tag{16}$$

We can interpret $P(W, Y)$ as the certainty equivalent wealth, which is the time-t total wealth that makes the agent indifferent between the status quo (with financial wealth W and output process Y) and having a wealth level $P(W, Y)$ and no output:

$$V(W,Y) = V(P(W,Y),0)\,. \tag{17}$$

Next, we turn to the autarky regime.

3.2 Autarky Regime

In the autarky regime, wealth is zero and the country cannot borrow or lend, so consumption equals output and wealth is not an argument of the value function. This function, $\widehat{V}(\widehat{Y})$, satisfies the following differential equation:

$$0 = f(\widehat{Y},\widehat{V}) + \mu\widehat{Y}\widehat{V}'(\widehat{Y}) + \frac{\sigma^2\widehat{Y}^2}{2}\widehat{V}''(\widehat{Y}) + \lambda\mathbb{E}\left[\widehat{V}(Z\widehat{Y}) - \widehat{V}(\widehat{Y})\right] + \xi\left[V(0,\widehat{Y}) - \widehat{V}(\widehat{Y})\right]\,. \tag{18}$$

The first term on the right side of equation (18) is the net utility flow. The second and third terms represent the impact of the output drift and diffusion volatility on marginal utility, respectively. The fourth term describes the possibility of output jumping from Y_t to ZY_{t-} while the country is in autarky. The last term reflects the possibility of exiting from autarky, which occurs at an exogenous rate, ξ. Upon exiting from autarky at time $t = \tau^{\mathcal{E}}$ and entering the normal regime, the country's value function is $V(0, Y_{\tau^{\mathcal{E}}})$, where $Y_{\tau^{\mathcal{E}}} = \widehat{Y}_{\tau^{\mathcal{E}}-}$.

We show that the value function in the autarky regime, $\widehat{V}(\widehat{Y})$, is:

$$\widehat{V}(\widehat{Y}) = \frac{(b\,\widehat{p}\,\widehat{Y})^{1-\gamma}}{1-\gamma}, \tag{19}$$

where the coefficient b is given by equation (16) and $\widehat{p}$ is the endogenous (scaled) certainty equivalent wealth in the autarky regime.

3.3 Connecting the Normal Regime with Autarky

The value functions $V(W, Y)$ and $\widehat{V}(Y)$ are connected by recurrent transitions between the normal and autarky regimes (see the two HJB equations, (10) and (18)).

If the country defaults at time t, output drops to αY_t. Therefore, the value of W_t that makes the country indifferent between repaying its debt and defaulting, which we denote by $\underline{W_t}$, satisfies the following value-matching condition:

$$V(\underline{W_t}, Y_t) = \widehat{V}(\alpha Y_t) \,. \tag{20}$$

Condition (20) implicitly defines the default boundary $\underline{W_t}$:

$$\underline{W_t} = \underline{W}(Y_t) \,. \tag{21}$$

We refer to $-\underline{W_t}$ as the country's debt capacity, since it is the maximum level of debt that the country can issue without triggering default. Whenever the country's sovereign debt exceeds its endogenous debt capacity, i.e., when $W_t < \underline{W_t}$, the country defaults and enters autarky. Its value function in this region satisfies

$$V(W_t, Y_t) = \widehat{V}(\alpha Y_t) \,, \quad \text{when} \quad W_t < \underline{W_t} \,. \tag{22}$$

We need one more condition to pin down $\underline{W_t}$, as it is a free boundary. We present this condition after we simplify our model's solution by using the property that the value function and policy rules are homogeneous in W and Y.

3.4 Simplifying the Model Solution

It is useful to define scaled financial wealth:

$$w_t = \frac{W_t}{Y_t} \,, \tag{23}$$

which is the model's scaled state variable. Similarly, we define scaled versions of the control variables: $c_t = C_t/Y_t$, scaled diffusion hedging demand, $\theta_t = \Theta_t/Y_t$, scaled jump hedging demand, $x_t = X_t/Y_t$, and scaled jump insurance premium payment, $\phi_t = \Phi_t/Y_t$.

The jump insurance premium pricing equation, (8), can be simplified as follows:

$$\phi(w_{t-}; Z^*) = \lambda \mathbb{E}[x(w_{t-}, Z)\, \mathcal{I}_{Z \geq Z^*}] \,. \tag{24}$$

The scaled certainty-equivalent wealth, $p(w_t)$, is equal to $P(W_t, Y_t)/Y_t$. Euler's theorem implies that $P_W(W_t, Y_t) = p'(w_t)$. The value of $p'(w)$ plays a crucial role in our analysis.

As debt is issued before jump arrival, the equilibrium credit spread depends only on the pre-jump information. We express the equilibrium credit spread, π_{t-}, as a function of pre-jump scaled wealth, $\pi(w_{t-})$, which we characterize below. To calculate $\pi(w_{t-})$, it is useful to characterize the default policy in terms of a threshold rule for the recovery fraction, $\underline{Z}(w_{t-})$.

Optimal Default Threshold $\underline{Z}(w_{t-})$. When the country is in the debt region under the normal regime, $(0 < -w_{t-} \leq -\underline{w})$, the ratio between debt and debt capacity, $w_{t-}/\underline{w}$, is between zero and one. When $w_{t-}/\underline{w} = 1$, the country exhausts its debt capacity and it is indifferent between defaulting or not. When $w_{t-}/\underline{w} = 0$, the country has no debt and thus does not default. The closer this ratio is to zero, the less likely the country is to default.

Next, we show that the optimal default threshold, $\underline{Z}(w_{t-})$, as a function of the pre-jump w_{t-}, for any given level of financial development, $0 \leq Z^* \leq 1$, is given by

$$\underline{Z}(w_{t-}) = \min\{w_{t-}/\underline{w}\,, Z^*\}\ , \tag{25}$$

for a country in the debt region under the normal regime, i.e., when $0 < w_{t-}/\underline{w} \leq 1$.

Fix a value of w_{t-} such that $0 < w_{t-}/\underline{w} \leq 1$. The optimal default threshold has to satisfy:

$$\underline{Z}(w_{t-}) \leq Z^*\,. \tag{26}$$

The intuition for this property is as follows. For any value of $w_{t-}/\underline{w} \in (0, 1]$, it is optimal for the country to hedge jump shocks to avoid costly default.

Next, we describe the optimal default decision in three mutually exclusive regions. First, when $Z \geq Z^*$, the post-jump scaled wealth, $w_t^{\mathcal{J}}$, satisfies: $w_t^{\mathcal{J}} = (w_{t-} + x_{t-})/Z$. Second, when Z is sufficiently low ($Z < \underline{Z}(w_{t-})$), $w_t^{\mathcal{J}} = w/Z < \underline{w}$ and it is therefore optimal to default as the benefit of reneging on debt is greater than the cost of defaulting. Third, for any $Z \in [\underline{Z}(w_{t-}), Z^*)$, the jump shocks are not hedgeable and the country does not to default. Therefore, the post-jump scaled wealth, $w_t^{\mathcal{J}}$, satisfies: $w_t^{\mathcal{J}} = w_{t-}/Z \geq \underline{w}$.

Combining these three mutually exclusive cases, we obtain the following expression for the post-jump scaled wealth, $w_t^{\mathcal{J}}$:

$$w_t^{\mathcal{J}} = \frac{w_{t-} + x_{t-}}{Z}\mathcal{I}_{Z \geq Z^*} + \frac{w_{t-}}{Z}\,\mathcal{I}_{\underline{Z}(w_{t-}) \leq Z < Z^*} + \frac{w_{t-}}{Z} \times \mathcal{I}_{Z < \underline{Z}(w_{t-})}\,, \tag{27}$$

where $\mathcal{I}_{\mathcal{A}}$ is indicator function that is equal to one if the event $\mathcal{A}$ occurs and zero otherwise. When $Z < \underline{Z}(w_{t-})$, $w_t^{\mathcal{J}} < \underline{w}$, the country defaults, enters autarky, and therefore, $p(w_t^{\mathcal{J}}) = \alpha\widehat{p}$.

Finally, by the definition of the default threshold, when $Z = \underline{Z}(w_{t-})$, the country must be indifferent between defaulting or not, which means that $w_t^{\mathcal{J}} = w_{t-}/\underline{Z}(w_{t-}) = \underline{w}$. Simplifying this equation, we obtain

$$\underline{Z}(w_{t-}) = w_{t-}/\underline{w}\,, \quad \text{under the condition} \quad \underline{Z}(w_{t-}) \leq Z^*\,. \tag{28}$$

As inequality (26) always holds, we can rewrite equation (28) as equation (25).

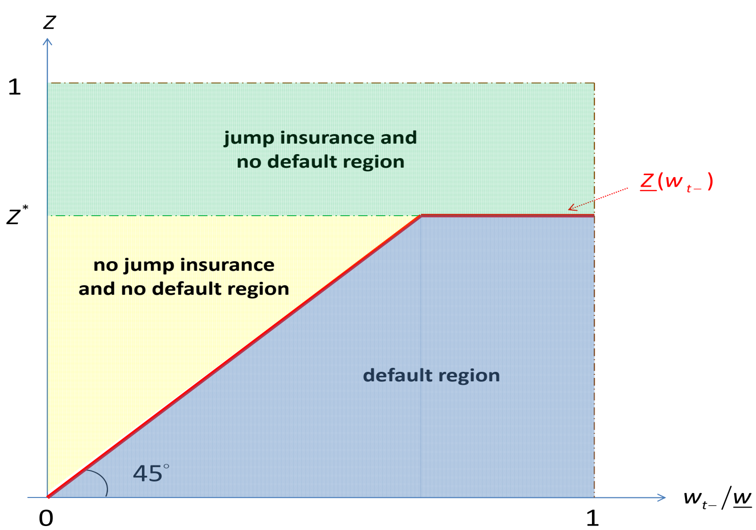

Figure 1: The optimal default threshold $\underline{Z}(w_{t-})$ and the three mutually exclusive regions: the "jump insurance and no default" (the top rectangular) region, the "no jump insurance and no default" (the upper triangular) region, and the default (the trapezoid) region. The horizontal axis is $w_{t-}/\underline{w}$. The vertical axis is the recovery fraction Z upon the jump arrival.

Figure 1 summarizes our discussions above by plotting the default threshold $\underline{Z}(w_{t-})$ given by equation (25) as a function of $w_{t-}/\underline{w}$ over the internal $(0, 1]$. This figure shows the three mutually exclusive regions discussed above: (i) the region where the country purchases jump insurances and does not default ($Z \geq Z^*$); (ii) the region where the country purchases no jump insurances does not default ($\underline{Z}(w_{t-}) \leq Z < Z^*$); and (iii) the default region where $Z < \underline{Z}(w_{t-})$.

Equilibrium Credit Spread. When the country issues debt ($W_{t-} < 0$), the competitive-market zero-profit condition for diversified sovereign debt investors implies that the credit spread, π_{t-}, satisfies:

$$-W_{t-}(1+r dt) = -W_{t-}(1+(r+\pi_{t-})dt)\left[1-\lambda\, G(\underline{Z}(w_{t-}))dt\right] + \lambda\, G(\underline{Z}(w_{t-}))dt \times 0\,. \quad (29)$$

The first term on the right side of equation (29), is the investors' expected total payment, which is given by the product of the probability of full repayment, $[1-\lambda\, G(\underline{Z}(w_{t-}))dt]$, and the cum-interest value of debt repayment, $-W_{t-}(1+(r+\pi_{t-})dt)$, if the country does not default at time t. The second term on the right side of equation (29) corresponds to the zero payment that occurs upon default. The left side of equation (29) is the expected total repayment to diversified investors.

Simplifying equation (29), we obtain the following expression for $\pi_{t-} = \pi(w_{t-})$, where[12]

$$\pi(w_{t-}) = \lambda\, G(\underline{Z}(w_{t-}))\,. \quad (30)$$

This equation ties the equilibrium credit spread to the country's default strategy. Because there is zero recovery upon default and investors are risk neutral, the credit spread is equal to the probability of default. We can generalize our model by incorporating a stochastic discount factor with jump risk premium to price sovereign debt. This generalization produces higher and more volatile credit spreads.

Dynamics for scaled wealth, w_t. Using Ito's lemma, we obtain the following law of motion for w_t in the normal regime:

$$dw_t = \mu_w(w_{t-})\, dt + \sigma_w(w_{t-})\, d\mathcal{B}_t + \left(w_t^{\mathcal{J}} - w_{t-}\right)\, d\mathcal{J}_t\,, \quad (31)$$

where $w_t^{\mathcal{J}}$ is the post-jump scaled financial wealth given in equation (27). The first term in equation (31) is the drift function, $\mu_w(w_{t-})$, given by:

$$\mu_w(w_{t-}) \;=\; \left(r+\pi(w_{t-})-\mu+\sigma^2\right) w_{t-} - \sigma^2\theta(w_{t-}) + 1 - \phi(w_{t-}) - c(w_{t-})\,, \quad (32)$$

where $\pi(w_{t-})$ is the equilibrium credit spread, which we characterize below, and $\phi(w_{t-})$ is the scaled jump insurance premium payment given by equation (24). The second term in equation (31) is the volatility function, $\sigma_w(w_{t-})$, given by:

$$\sigma_w(w_{t-}) = (\theta(w_{t-}) - w_{t-})\sigma\,. \quad (33)$$

[12] When scaled wealth is positive, there is no debt outstanding so the probability of default is zero.

We conjecture and verify that the debt capacity satisfies $\underline{W_t} = \underline{w_t} Y_t$, where $\underline{w_t}$ is a constant, $\underline{w}$. The country defaults whenever a jump reduces output to a level such that its scaled debt, $-w_t$, exceeds $-\underline{w}$. The optimal default strategy expressed in terms of the cutoff threshold rule for w_t is time invariant because debt is continuously issued, fully repaid (in the absence of default), and reissued. We next determine $-\underline{w}$.

Scaled Debt Capacity. It is useful to recall an important result: shocks that can be insured at actuarially fair prices should be hedged so that the country does not default. In order to ensure that diffusion shocks do not trigger default, the country sets the volatility of w to zero at its endogenous debt capacity:[13]

$$\sigma_w(\underline{w}) = 0\,. \tag{34}$$

Substituting this condition into equation (33), we obtain $\theta(\underline{w}) = \underline{w}$. In addition, the country sets $\mu_w(\underline{w}) \geq 0$ so that w weakly moves away from $\underline{w}$, ensuring that $w_t \geq \underline{w}$. As a result, diffusion shocks never cause default. Default occurs only in response to jumps in output that are sufficiently large. It is optimal to repay the debt in response to diffusion shocks and small jumps to preserve the option of defaulting in the future in response to larger jumps.

Endogenous Relative Risk Aversion $\widetilde{\gamma}$. In order to interpret our results, it is useful to introduce the following measure of endogenous relative risk aversion, denoted by $\widetilde{\gamma}$:

$$\widetilde{\gamma}(w) \equiv -\frac{V_{WW}}{V_W} \times P(W,Y) = \gamma p'(w) - \frac{p(w)p''(w)}{p'(w)}\,. \tag{35}$$

The first part of equation (35) defines $\widetilde{\gamma}(w)$. The second part follows from the homogeneity property.

The economic interpretation of $\widetilde{\gamma}$ is as follows. Because limited commitment results in endogenous market incompleteness, the country's endogenous risk aversion is given by the curvature of the value function $V(W,Y)$ rather than by the risk aversion parameter, γ. We use the value function to characterize the coefficient of endogenous absolute risk aversion: $-V_{WW}(W,Y)/V_W(W,Y)$.

We can build a measure of relative risk aversion by multiplying $-V_{WW}(W,Y)/V_W(W,Y)$, with "total wealth." There is no well-defined market measure of the total wealth under

[13] Bolton, Wang, and Yang (2018) derive a similar boundary condition in a corporate-finance continuous-time diffusion model where the entrepreneur has inalienable human capital.

incomplete markets. However, the certainty equivalent wealth $P(W, Y)$ is a natural measure, so we use it in our definition of $\widetilde{\gamma}$ in equation (35).[14]

Limited commitment and/or limited spanning causes the marginal certainty equivalent wealth of financial wealth to exceed one, i.e., $P_W(W, Y) = p'(w) \geq 1$. Also, in our model, $p''(w) < 0$, which implies that $\widetilde{\gamma}(w) > \gamma$ (see equation (35)). That is, limited commitment causes the representative agent to be endogenously more risk averse. In contrast, in the first-best solution that we describe below, the country fully hedges against diffusion and jump shocks and $\widetilde{\gamma}(w) = \gamma$.

4 First-Best Solution: Full Commitment and Spanning

Before discussing our results under limited commitment and limited spanning, we summarize the first-best (FB) solution that obtains when there is full commitment and full spanning. Full commitment means that the country has to honor all its contractual agreements, so the country never defaults. Full spanning means that $Z^* = 0$, which represent the highest level of financial development. We use the superscript FB to denote the variables that pertain to the FB solution.

As in Friedman (1957) and Hall (1978), we define non-financial wealth, H_t, for the case where $Z^* = 0$, as the present value of output, discounted at the constant risk-free rate, r:

$$H_t = \mathbb{E}_t \left(\int_t^\infty e^{-r(u-t)} Y_u du \right) . \tag{36}$$

Because Y is a geometric jump-diffusion process, we have $H_t = hY_t$, where h is scaled non-financial wealth given by

$$h = \frac{1}{r-g} , \tag{37}$$

and g is given by equation (2). To ensure that non-financial wealth is finite, we require that $r > g$. This convergence condition is standard in asset pricing and valuation models. To ensure that utility is finite, we require the following regularity condition:

$$\rho > (1 - \psi^{-1})\, r . \tag{38}$$

Proposition 1 *Scaled total wealth,* $p^{FB}(w) = P^{FB}(W, Y)/Y = (W + H)/Y$, *is*

$$p^{FB}(w) = w + h , \tag{39}$$

[14]See Bolton, Wang, and Yang (2018) for a similar definition in a different setting where markets are exogenously incomplete.

where h is given by equation (37) and $w_t \geq \underline{w}^{FB} = -h$. The scaled endogenous debt capacity is $-\underline{w}^{FB} = h$. The optimal consumption-output ratio, $c_t = c^{FB}(w)$, is given by:

$$c^{FB}(w) = m\, p^{FB}(w) = m(w+h)\,. \tag{40}$$

where m is given by:

$$m = r + \psi\,(\rho - r)\ . \tag{41}$$

The optimal scaled hedging demand for diffusive shocks for all values of w, $\theta^{FB}(w)$, is constant:

$$\theta^{FB}(w) = -h\,. \tag{42}$$

The optimal scaled hedging demand for jump risk, $x^{FB}(w, Z)$, is given by:

$$x^{FB}(w, Z) = (1 - Z)h\,. \tag{43}$$

The implied scaled jump insurance premium is constant: $\phi^{FB}(w) = \lambda(1 - \mathbb{E}(Z))h$. There is no default, meaning $\underline{Z}(w) = Z^* = 0$.

Equation (39) states that the scaled certainty equivalent wealth is given by the sum of w and scaled non-financial wealth h, $p^{FB}(w) = w + h$. Therefore, the country's endogenous relative risk aversion defined in equation (35), $\widetilde{\gamma}(w)$, is constant and equal to γ.

Equation (40) shows that $c^{FB}(w)$ is proportional to scaled total wealth, $p^{FB}(w) = w + h$. Equation (42) shows that the country fully hedges its diffusive risk by taking a short position of h units in the diffusion hedging contract so that the net exposure of its total wealth, $P^{FB}(W, Y)$, to diffusive shocks is zero. Similarly, equation (43) shows that the country fully hedges the jump risk by buying $(1 - Z)h$ units of the jump insurance contract for each possible value of Z, so that the net exposure of $P^{FB}(W, Y)$ to jump shocks is zero.

5 Limited-Commitment Solution

In this section, we discuss the solution of our model when there is limited commitment with and without limited spanning. The following proposition summarizes the main properties of the solution.

Proposition 2 *The scaled certainty equivalent wealth $p(w)$ when $w > \underline{w}$ in the normal regime and $\widehat{p}$ in the autarky regime satisfy the following two interconnected ODEs:*

$$0 = \left(\frac{m(p'(w))^{1-\psi} - \psi\rho}{\psi - 1} + \mu - \frac{\gamma\sigma^2}{2} \right) p(w) + \left[\, (r + \pi(w) - \mu) \, w + 1 - \phi(w) \, \right] p'(w)$$
$$+ \frac{\gamma^2\sigma^2 p(w)p'(w)}{2\,\widetilde{\gamma}(w)} + \frac{\lambda}{1-\gamma}\mathbb{E}\left[\left(\frac{Zp(w^{\mathcal{J}})}{p(w)} \right)^{1-\gamma} - 1 \right] p(w) \,, \tag{44}$$

$$0 = \frac{\rho\left[(b\,\widehat{p}\,)^{-(1-\psi^{-1})} - 1 \right]}{1 - \psi^{-1}} + \mu + \frac{\lambda(\mathbb{E}(Z^{1-\gamma}) - 1)}{1-\gamma} - \frac{\gamma\sigma^2}{2} + \frac{\xi}{1-\gamma}\left[\left(\frac{p(0)}{\widehat{p}} \right)^{1-\gamma} - 1 \right] , \tag{45}$$

where $w^{\mathcal{J}}$ is given by equation (27). When $w < \underline{w}$, the country defaults and hence its $p(w)$ is given by

$$p(w) = \alpha\,\widehat{p}\,. \tag{46}$$

In addition, we have the following boundary conditions:

$$p(\underline{w}) = \alpha\,\widehat{p}\,, \tag{47}$$
$$p''(\underline{w}) = -\infty\,, \tag{48}$$
$$\lim_{w\to\infty} p(w) = w + h\,, \tag{49}$$

where h is given by equation (37).

The equilibrium credit spread is $\pi(w_{t-}) = \lambda G(\underline{Z}(w_{t-})$. The scaled jump insurance premium, $\phi(w_{t-})$, is given by equation (24). The country defaults when $\tau^{\mathcal{D}} = \inf\{t : w_t < \underline{w}\}$.

In the no-default region where $w \geq \underline{w}$, the following policy rules apply. The optimal consumption-output ratio, $c(w)$, is:

$$c(w) = mp(w)(p'(w))^{-\psi}\,, \tag{50}$$

where m is given by equation (41). The scaled diffusion risk hedging demand, $\theta(w)$, is:

$$\theta(w) = w - \frac{\gamma p(w)p'(w)}{\gamma(p'(w))^2 - p(w)p''(w)} = w - \frac{\gamma p(w)}{\widetilde{\gamma}(w)}\,, \tag{51}$$

where $\widetilde{\gamma}(w)$ is the endogenous relative risk aversion given by equation (35). For $Z^* \leq Z < 1$, the optimal scaled hedging demand for jump risk, $x(w, Z)$, solves:

$$p'(w) = \left(\frac{Zp((w + x(w,Z))/Z)}{p(w)} \right)^{-\gamma} p'((w + x(w,Z))/Z)\,. \tag{52}$$

Equations (44) and (45) are the interconnected ODEs for the scaled certainty equivalent wealth: $p(w)$ in the normal regime and $\widehat{p}$ in the autarky regime.

Equation (47) follows from the value-matching condition, (20). Equation (48) follows from the zero volatility condition, (34) for w at $\underline{w}$, and $p(\underline{w}) > 0$. Equations (47) and (48) jointly characterize the left boundary, $\underline{w}$. Equation (49) states that, as $w \to \infty$, the effect of limited commitment disappears and $p(w)$ converges to $w + h$.

Equation (50) shows that consumption is a nonlinear function of w, depending on both the certainty equivalent wealth, $p(w)$, and its derivative, $p'(w)$. Equation (51) determines the hedging demand with respect to diffusive shocks. As discussed above, the country hedges to avoid default triggered by diffusive shocks and preserve the option to default in response to rare disasters. Without hedging diffusive shocks, a country that has exhausted its debt capacity ($W_t = \underline{W_t}$) would default with probability 50 percent, with default potentially triggered by very small shocks.

Substituting equation (51) into equation (33), we obtain:

$$\sigma_w(w) = (\theta(w) - w)\sigma = -\sigma \frac{\gamma p(w)}{\widetilde{\gamma}(w)} < 0\,. \tag{53}$$

In absolute value, the volatility for w is proportional to the ratio between $p(w)$ and endogenous risk aversion, $\widetilde{\gamma}(w)$. Evaluating equation (53) at $\underline{w}$ and using $\sigma_w(\underline{w}) = 0$ and $p(\underline{w}) = \alpha\,\widehat{p} > 0$, we conclude that endogenous relative risk aversion, $\widetilde{\gamma}(w)$, approaches infinity, as $w \to \underline{w}$.

Equation (52) determines the country's scaled hedging demand with respect to jump shocks, $x(w, Z)$. As discussed above, for insurable jump shocks ($Z \geq Z^*$) the country hedges its jump risk exposures to equate its pre- and post-jump marginal utility of wealth. The homogeneity property allows us to express this condition in terms of the certainty equivalent wealth, $p(w)$, and the marginal certainty equivalent value of financial wealth, $p'(w)$.

Next, we turn to the special case where there is full spanning and hence all jump risks can be hedged ($Z^* = 0$).

Full Spanning and Limited Commitment. As in Kehoe and Levine (1993), when all shocks are insurable at actuarially fair terms, the country never defaults in equilibrium. The country is better off honoring its debt and preserving its debt capacity. Doing so allows the country to borrow at the risk-free rate ($\pi = 0$). The country's temptation to default is an off-equilibrium threat that determines the country's debt capacity. This maximal amount of sustainable debt makes the country indifferent between defaulting or not.

For this full-spanning and limited-commitment case, equations (47) and (48) are the continuous-time equivalent of the limited-enforcement conditions in Kehoe and Levine (1993), Kocherlakota (1996), and Alvarez and Jermann (2000).

6 Calibration and Quantitative Results

To explore the quantitative properties of our model, we calibrate it with the eleven parameter values summarized in Table 1. We divide these parameters into two groups. The seven parameters in the first group are set to values that are standard in the literature. The four parameters in the second group are calibrated to match key features of data for Argentina.

6.1 Baseline Calibration

We first describe the parameters drawn from the literature.

Parameters from the literature. Following Aguiar and Gopinath (2006), we set the coefficient of relative risk aversion (γ) to 2, the annual risk-free rate (r) to 4 percent, and the rate at which the country exits autarky (ξ) to 0.25 per annum. This choice of ξ implies that that the country stays on average in autarky for four years, which is consistent with the estimates in Aguiar and Gopinath (2006). Following Barro (2009), we set the annual subjective discount rate (ρ) to 5.2 percent.

As in the rare-disasters literature, we assume that the cumulative distribution function of the recovery fraction, $G(Z)$, is governed by a power law:

$$G(Z) = Z^{\beta}\,. \tag{54}$$

Following Barro and Jin (2011), we call disasters jump shocks that create realized output losses greater than 10 percent ($Z < 1 - 0.1 = 0.9$). In our baseline calibration, we assume that disaster shocks cannot be hedged so we set the level of financial development, Z^*, equal to 0.9.

We choose $\beta = 6.3$ and the annual jump arrival rate, $\lambda = 0.073$, so that the annual disaster probability is $\lambda G(0.9) = 0.073 \times G(0.9) = 3.8$ percent, which is the value estimated by Barro and Jin (2011).

Table 1: Parameter Values

Parameters	Symbol	Value
risk aversion	γ	2
elasticity of intertemporal substitution	ψ	0.047
subjective discount rate	ρ	5.2%
risk-free rate	r	4%
financial development parameter	Z^*	0.9
output drift (in the absence of jumps)	μ	2.7%
output diffusion volatility	σ	4.5%
jump arrival rate	λ	0.073
power law parameter	β	6.3
default distress cost	α	97.5%
autarky exit rate	ξ	0.25
Targeted observables		
average output growth rate	g	1.7%
output growth volatility		6.66%
average debt-output ratio		15%
unconditional default probability		3%

All parameter values, whenever applicable, are continuously compounded and annualized.

Calibrated parameters from Argentinean data. We choose the parameter that controls the drift in the absence of jumps (μ), the diffusion volatility (σ), the default distress cost (α), and the intertemporal elasticity of substitution (ψ), to target the following four moments estimated using Argentinean data: an average growth rate of output of 1.7 percent per annum, a standard deviation of the growth rate of output of 6.7 percent, an average debt-to-GDP ratio of 15 percent, and unconditional default probability of 3 percent. Our empirical estimates of the average and standard deviation of the annual growth rate of real GDP for Argentina were obtained using Barro and Ursua's (2008) data for the period 1876-2009.

Our model consolidates the expenditure and borrowing decisions of the private sector and the government. For this reason, we calibrate it to match the ratio of net debt to GDP. In Argentina, as in most countries, a significant fraction of government debt is owned by the domestic private sector. We compute our target for the debt-to-output ratio by calculating the difference between Argentina's debt liabilities and debt assets using the data compiled by Lane and Milesi-Ferretti (2007) for the period from 1970 to 2011. The average net debt-to-GDP ratio during this period is 15 percent.[15] Argentina defaulted six times in roughly 200 years, so we target an annual default probability of 3 percent.[16]

We obtain the following parameter values: the EIS $\psi = 0.047$, $\mu = 2.7$ percent per annum, $\sigma = 4.5$ percent per annum, and $\alpha = 0.975$. The calibrated value of α implies that the direct costs of defaulting on sovereign debt are equal to 2.5 percent of output. This cost of default is conservative relative to the estimates reported by Hébert and Schreger (2017) for Argentina.

In this calibration, the value of the EIS ($\psi = 0.047$) is low so the representative agent has a strong preference for smooth consumption paths.[17] This preference makes the utility cost of default high. Since default is costly, debt capacity is high. At the same time, the preference for smooth consumption means that the country does not save away quickly from the debt region. These properties generate plausible debt-output ratios. We can interpret the low value of the EIS as resulting from rigidities in spending patterns and expenditure commitments that are difficult to change.

6.2 Debt Intolerance

Table 2 shows the impact of different levels of financial development ($Z^* = 0.5, 0.9, 1$) on debt intolerance. We set all other parameters to the values used in our benchmark calibration and summarized in Table 1. To better understand the intuition, we proceed in three steps.

First, we recall that in the FB case the country fully uses its debt capacity, which is the present discounted value of output, $h = 1/(r-g)$, and never defaults. For our calibration, the country borrows 4,348 percent of current output, an implication that is clearly unrealistic.

Second, we isolate the impact of limited commitment by comparing the full-spanning limited-commitment case ($Z^* = 0$) to the FB case. When $Z^* = 0$, the country never defaults in equilibrium because with full spanning it is cheaper to manage risk by hedging than by defaulting on its sovereign debt. However, debt capacity is much lower under limited commitment than in the FB case: $|\underline{w}| = 20.5$ percent versus $|\underline{w}| = 4,348$ percent.

[15] Table 4 shows that the model can be easily calibrated to generate higher average debt-output ratios by increasing $(1-\alpha)$, the distress cost associated with default.

[16] Argentina defaulted in 1830, 1890, 1915, 1930, 1982, and 2001. See Sturzenegger and Zettlemeyer (2006) for a discussion.

[17] There is currently no consensus on what are empirically plausible values for the EIS (see Attanasio and Weber (2010) for a discussion). Our choice is consistent with Hall (1988) who argues that the elasticity of intertemporal substitution is close to zero. It is also consistent with the recent estimates by Best, Cloyne, Ilzetzki, and Kleven (2017) which are based on mortgage data.

Table 2: PARTIAL SPANNING AND DEBT INTOLERANCE

Z^*	Average debt-output ratio	Default probability	Debt capacity $\lvert\underline{w}\rvert$
1 (No jump hedging)	14.7%	4.0%	20.5%
0.9	**15%**	**3.1%**	**20.7%**
0.5	20.7%	0.1%	24%

All parameter values other than Z^* are summarized in Table 1.

As a result, the country's average debt-to-output ratio is only 19 percent under limited commitment despite full spanning rather than 4,348 percent in the FB case.

Third, we study the impact of financial development. Reducing spanning from full ($Z^* = 0$) to the value used in our calibration, $Z^* = 0.9$, produces a large decline of debt capacity, from 20.5 to 20.7 percent. Because of limited spanning, the country uses default to manage large jump shocks: when $Z^* = 0.9$, the probability of default is 3.1 percent.

Eliminating entirely the ability to use insurance contracts to hedge jump risk ($Z^* = 1$) results in a large rise in the probability of default relative to the benchmark case (from 3.1 to 4 percent), even though the decline in debt capacity and average debt-output ratio is small (from 20.7 to 20.5 percent and from 15 to 14.7 percent, respectively). The large rise in the probability of default occurs because when $Z^* = 1$, the only way to manage large jump risk is to default on sovereign debt. In contrast, when $Z^* = 0.9$, roughly half of the jump shocks can be hedged ($1 - G(Z^*) = 49$ percent).

Improving financial development by decreasing Z^* from 0.9 to 0.5 has a dramatic impact on the debt capacity, average debt-output ratio, and default probability: debt capacity rises from 20.7 to 24 percent of output, the average debt-output ratio increases from 15 to 20.7 percent, and the probability of default drops to close to zero, from 3.1 to 0.1 percent.

In sum, Table 2 shows that low financial development causes debt intolerance. This table also shows that improving financial development from a low level has a large positive impact on the country's ability to borrow and the credit spread of its sovereign debt.

6.3 Economic Mechanisms and Quantitative Implications

In this subsection, we use our calibration to explore the economic mechanisms in our model. Figures 2, 3, and 4 illustrate the properties of our model for different levels of financial development. Recall that in our benchmark calibration we set $Z^* = 0.9$, which means that jumps that generate output losses greater than 10 percent cannot be hedged with insurance contracts.

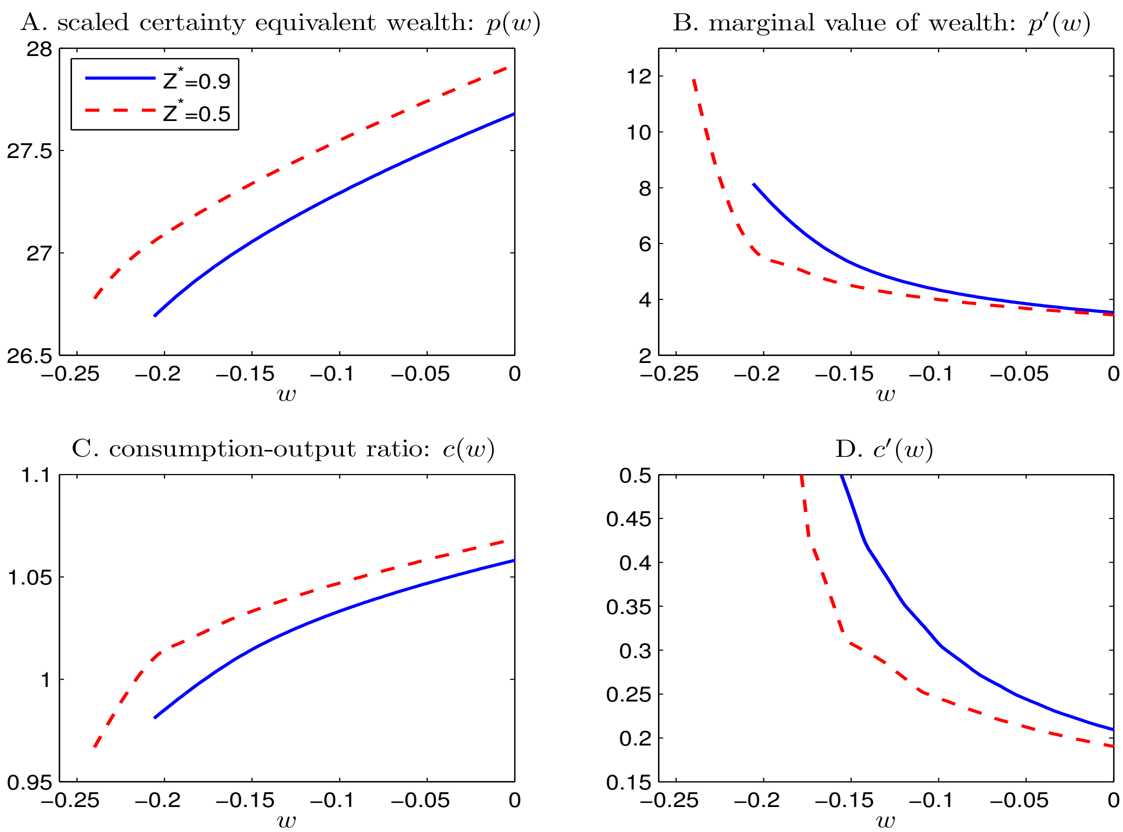

Figure 2: Scaled certainty equivalent wealth $p(w)$, marginal certainty equivalent value of wealth $p'(w)$, consumption-output ratio $c(w)$, and $c'(w)$ for two levels of financial development: $Z^* = 0.5$ and $Z^* = 0.9$. Debt capacity is equal to $-\underline{w} = 20.6$ percent and $-\underline{w} = 24.0$ percent for $Z^* = 0.9$ and 0.5, respectively.

Certainty equivalent wealth, marginal value of wealth, consumption, and the MPC. Panels A and B of Figure 2 display the scaled certainty-equivalent wealth, $p(w)$, and the marginal certainty-equivalent value of wealth, $P_W(W, Y) = p'(w)$, respectively. The function $p(w)$ is increasing and concave, which implies that $p'(w)$ is decreasing in w and $p'(w)$

is greater than one.[18] Panels C and D display the consumption-output ratio, $c(w)$, and the MPC out of wealth, $c'(w)$, respectively. The function $c(w)$ is increasing and concave, which implies that $c'(w)$ is decreasing in w. As w goes to infinity, $p(w)$ approaches $p^{FB}(w) = w+h$, $p'(w)$ approaches one, $c(w)$ approaches $c^{FB}(w) = m(w+h)$, and $c'(w)$ approaches the MPC obtained in the FB, $m = 0.041$.

Next, we discuss the impact of financial development under limited commitment. We compare our baseline case where $Z^* = 0.9$ (our proxy for the status quo in emerging markets) with an economy where $Z^* = 0.5$, which corresponds to a high level of financial development since the country can hedge jumps that generate output losses smaller than 50 percent.

The higher is financial development (lower Z^*), the higher is $p(w)$ because more risks are hedged and the representative agent faces less uncertainty. As a result, the marginal value of wealth, $p'(w)$, is lower. Consumption is higher because both a higher $p(w)$ and a lower $p'(w)$ cause $c(w)$ to be higher (see equation (50)).

To compare the two economies, consider $w = -15$ percent, which is the average debt-to-output ratio in the baseline calibration. The marginal value of wealth, $p'(-0.15)$, is equal to 5.31 in the economy with $Z^* = 0.9$, which is 18 percent higher than in the economy with $Z^* = 0.5$. Both values are much higher than one, the value of $p'(w)$ in the FB case. The MPC out of wealth, $c'(-0.15)$, is equal to 0.47 in the economy with $Z^* = 0.9$, which is 52 percent higher than in the economy with $Z^* = 0.5$. Both values are much higher than m, which is equal to 0.041, the value of the MPC in the FB case.

Jump-risk hedging demand, jump-insurance premium payment, and the credit spread. Panel A of Figure 3 plots $x(w, Z)$ as a function of Z for $w = -15$ percent, the average debt-output ratio targeted in our calibration. This panel shows that for a given Z^* and w, the hedging demand $x(w, Z)$ is decreasing in the recovery fraction Z, which means that the country insures more against bigger losses in order to smooth consumption. Panel B plots the scaled jump-insurance premium payment, $\phi(w) = \int_{Z^*}^{1} x(w, Z)dG(Z)$, which integrates the hedging demand $x(w, Z)$, displayed in Panel A, over the admissible range of $Z \geq Z^*$ for each value of w. This panel shows that the scaled jump-insurance premium payment, $\phi(w)$, increases with w.

[18] Wang, Wang, and Yang (2016) derive similar properties for the certainty equivalent wealth in a self-insurance model where labor income shocks are uninsurable and the agent can only save via a risk-free asset.

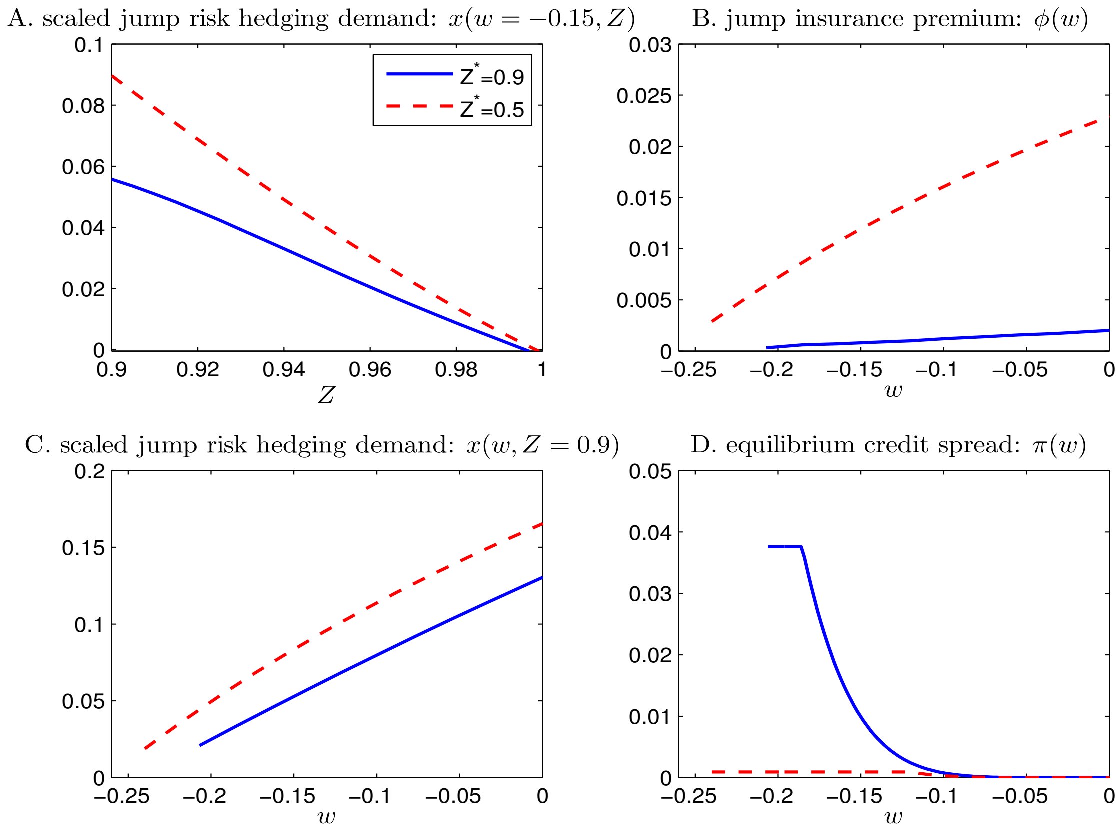

Figure 3: Scaled jump risk hedging demand $x(w, Z)$ at $w = -0.15$, jump insurance premium payment $\phi(w)$, scaled jump risk hedging demand $x(w, Z)$ at $Z = 0.9$, and the equilibrium credit spread $\pi(w)$ for two levels of financial development: $Z^* = 0.5$ and $Z^* = 0.9$. Debt capacity is equal to $-\underline{w} = 20.6$ percent and $-\underline{w} = 24.0$ percent for $Z^* = 0.9$ and 0.5, respectively.

Panel C plots the demand for jump insurance, $x(w, Z)$, against a 10 percent permanent loss in output ($Z = 0.9$). This panel shows that $x(w, Z)$ increases with w, which means that a less indebted country hedges more. That is, hedging and financial wealth are complements.

Panels A, B, and C together show that the hedging demand $x(w, Z)$ and hedging insurance premium payment $\phi(w)$ increase with financial development. Because the jump-insurance premium payment has to be paid up front, this payment is more costly in utility terms for less financially developed countries. As a result, both the hedging demand and the insurance premium payments are lower for these countries.

As the country's financial development improves (i.e., as Z^* decreases), its risk-sharing opportunities expand, causing its hedging position to increase in absolute value. This increase leads to a rise in debt capacity, $-\underline{w}$.

Panel D plots the equilibrium credit spread, $\pi(w)$, which declines with both the level of financial development and financial wealth w. The credit spread, $\pi(w_{t-})$, is constant and equal to $\lambda G(Z^*)$ in the region where $w_{t-} \leq Z^* \underline{w}$, because the default threshold of Z, $\underline{Z}(w_{t-}) = \min\{w_{t-}/\underline{w}, Z^*\} = Z^*$, is constant. That is, the value of preserving the option to default in the future is zero.

When financial development is high, the country uses jump insurance contracts to hedge most jump shocks and only uses costly default to manage rare disasters. As a result, the likelihood of default and the credit spread are low. For the case where $Z^* = 0.5$, the equilibrium credit spread is very close to zero for all values of w. In contrast, when financial development is low, the option to default is used to manage most jump shocks and hence default is likely, resulting in a high credit spread. For the case where $Z^* = 0.9$, the equilibrium credit spread is high for debt levels above 15 percent of output.

Diffusion risk hedging demand, drift, volatility, and the distribution of w. Panel A of Figure 4 shows that the scaled diffusion hedging demand, $\theta(w)$, is negative, and that its absolute value increases with w. That is, a less indebted country hedges more diffusive risk. As with the case of jump risk, hedging and financial wealth are complements. Even though the country incurs no upfront cost to hedge diffusion shocks, it is not optimal to fully hedge the diffusion risk.

Panel B plots the volatility function, $\sigma_w(w)$. Because a less indebted country has a higher $p(w)$ and a lower endogenous relative risk aversion, $\widetilde{\gamma}(w)$, the absolute value of $\sigma_w(w)$ increases with w, as one can see from equation (53). In the limit as $w \to \underline{w}$, the absolute value of σ_w reaches the minimal value, $\sigma_w(\underline{w}) = 0$. The intuition for this property, which is visible in Panel B, is that it is inefficient for the country to use default to manage continuous diffusive shocks. Since diffusion shocks do not trigger default, $\sigma_w(\underline{w}) = 0$.

Panel C shows the drift function for w, $\mu_w(w)$, which is negative for most values of w. This result follows from the observations that: (a) the country's consumption is often larger than output (see Figure 2); and (b) interest and jump insurance premium payments drain the country's financial wealth. All these forces move the country further into debt in expectation. However, as the country's debt approaches its capacity, $-\underline{w}$, the country voluntarily adjusts its consumption, insurance demand, and debt level so that $\mu_w(\underline{w}) \geq 0$. This property together with zero volatility condition for w at $\underline{w}$ discussed above are necessary

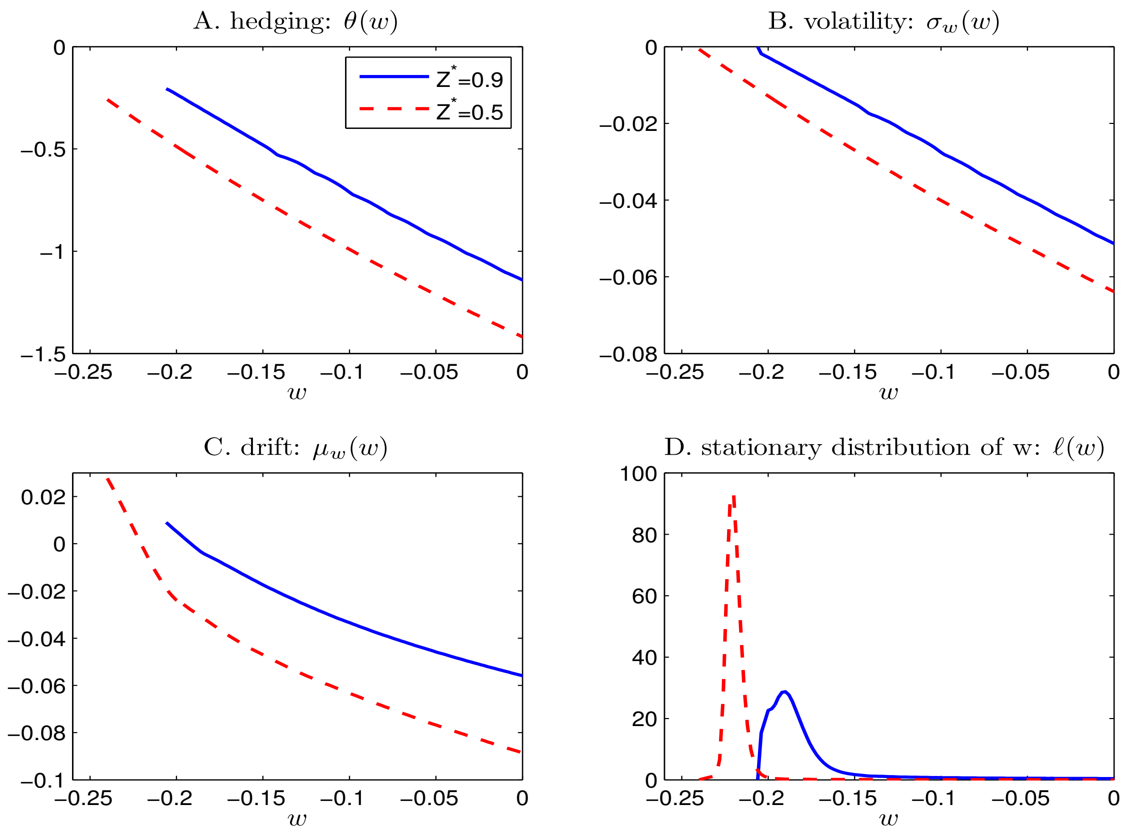

Figure 4: Scaled diffusion risk hedging demand $\theta(w)$, volatility $\sigma_w(w)$, drift $\mu_w(w)$, and the density function for the stationary distribution of w in the normal regime, $\ell(w)$, for two levels of financial development: $Z^* = 0.5$ and $Z^* = 0.9$. Debt capacity is equal to $-\underline{w} = 20.6$ percent and $-\underline{w} = 24.0$ percent for $Z^* = 0.9$ and 0.5, respectively.

to ensure that the country does not default in response to continuous diffusion shocks.

Next, we show that a more financially developed country has a larger hedging demand for diffusion shocks. Equation (53) implies that $\sigma_w(w) = -\sigma\gamma p(w)/\widetilde{\gamma}(w) < 0$ and

$$\theta(w) = w + \frac{\sigma_w(w)}{\sigma} = w - \frac{\gamma}{\widetilde{\gamma}(w)}\, p(w)\,. \tag{55}$$

The higher the level of financial development, the higher the country's certainty equivalent wealth, $p(w)$, and the lower the country's endogenous relative risk aversion, $\widetilde{\gamma}(w)$. Therefore, for a given level of w, a more financially developed country has a more negative $\sigma_w(w)$, and a more negative hedging position, $\theta(w)$.

Panel D displays the probability density function for the stationary distribution of w, $\ell(w)$, in the normal regime. This panel is consistent with the empirical observation that countries with lower levels of financial development on average have lower debt-to-output ratios. In other words, these countries are debt intolerant.

7 Sensitivity Analysis

We now discuss how a country's average debt-output ratio, average default probability, and debt capacity vary with some key parameters. We change one parameter at a time and fix all other parameters at the values reported in Table 1.

Table 3: THE EFFECT OF THE EIS ψ

ψ	debt-output ratio	default probability	debt capacity $\lvert \underline{w} \rvert$
0	15.3%	3.3%	19.8%
0.047	**15%**	**3.1%**	**20.7%**
0.25	17.9%	1.1%	34.2%
0.5	5.5%	0.7%	42.2%

All parameter values other than ψ are summarized in Table 1.

The effect of the EIS, ψ. Table 3 shows the impact of varying the EIS. Recall that risk aversion, γ, is equal to 2. Therefore, we obtain the expected-utility case when ψ is equal to 0.5. Increasing ψ from 0.047 to 0.5 substantially reduces the country's debt-output ratio from 15 percent to 5.5 percent and decreases the country's annual default probability from 3.1 percent to 0.7 percent. The intuition for this result is that a country with a higher EIS is more willing to substitute its consumption over time. As a result, this country has stronger incentives to save away from the debt region where credit spreads are relatively high.

Perhaps surprisingly, raising the EIS from 0.047 to 0.5 more than doubles the country's debt capacity, from 20.7 percent to 42.2 percent. The intuition for this result is that capital markets are more willing to lend to countries with higher intertemporal substitution, since it is less costly (in terms of utility) for these countries to cut consumption in response to adverse shocks to make their debt payments. In sum, when the EIS rises, debt capacity increases but the average debt-output ratio falls.

The effect of the distress cost, $1-\alpha$**.** Table 4 illustrates the impact of distress costs and shows that these costs play a key role in allowing the model to generate empirically plausible average debt-output ratios. Increasing the distress cost, $(1-\alpha)$, from 2.5 percent to 5 percent more than doubles the debt capacity from 20.7 percent to 49 percent, significantly raises the debt-output ratio from 15 percent to 37 percent, and decreases the annual default probability from 3.1 percent to 3.0 percent. When default is more costly, debt capacity is higher. At the same time, the country defaults less often despite borrowing more on average.

Table 4: The effect of distress cost, $(1-\alpha)$

$(1-\alpha)$	debt-output ratio	default probability	debt capacity $\lvert\underline{w}\rvert$
5%	36.9%	3.0%	49%
2.5%	**15%**	**3.1%**	**20.7%**
1%	5.5%	3.2%	7.2%
0%	0.1%	3.6%	0.2%

All parameter values other than α are summarized in Table 1.

When the distress cost is zero, the only cost of default is the loss of consumption smoothing opportunities under autarky. In our calibration, the utility cost is small, so that debt capacity is essentially zero (see the last row of Table 4.)

Table 5: The effect of the probability of exiting autarky, ξ

ξ	debt-output ratio	default probability	debt capacity $\lvert\underline{w}\rvert$
0	15.8%	3.1%	21.0%
0.25	**15%**	**3.1%**	**20.7%**
0.5	14.7%	3.1%	18.2%
1	13.2%	3.2%	16.0%

All parameter values other than ξ are summarized in Table 1.

The effect of the probability of exiting autarky, ξ. Table 5 shows the impact of varying ξ. Increasing ξ reduces the expected duration of the autarky regime, $1/\xi$, lowering the cost of defaulting. Since default is less costly, the country defaults more often. In equilibrium, debt capacity falls and the country borrows less.

Decreasing the average duration of the autarky regime from four years ($\xi = 0.25$) to one year lowers the debt-output ratio from 15 percent to 13.2 percent, increases the annual default probability from 3.1 percent to 3.2 percent, and reduces the debt capacity from 20.7 percent to 16 percent. When autarky is permanent ($\xi = 0$), as in Eaton and Gersovitz (1981), debt capacity is 21 percent and the average debt-output ratio is 15.8 percent.

In sum, our quantitative results are not sensitive to the value of ξ.

The effect of risk aversion, γ. Table 6 shows that the effect of risk aversion. Increasing γ raises the cost of default since it is more costly to bear consumption volatility in the autarky regime. With default more costly, the country defaults less often and debt capacity is higher. Increasing γ from one to three increases debt capacity from 19.6 percent to 23 percent and lowers the annual default probability from 3.3 percent to 2.5 percent. The effect of γ on the average debt-output ratio is quite small.

Table 6: THE EFFECT OF RISK AVERSION γ

γ	debt-output ratio	default probability	debt capacity $\lvert\underline{w}\rvert$
1	15.6%	3.3%	19.6%
2	**15%**	**3.1%**	**20.7%**
3	15.4%	2.5%	23%

All parameter values other than γ are summarized in Table 1.

8 An Expected-utility Calibration

In this section, we restrict recursive utility to the expected-utility case generally used in the sovereign-debt literature. We explain why the calibrations traditionally used in this literature rely on a very high discount rate. Then, we show that our key result–low levels of financial development causes debt intolerance–continues to hold for these traditional calibrations.

Recall that our calibration uses a low value of the EIS. Why hasn't this type of calibration been used in the literature? One likely reason is that the literature typically works with an expected-utility specification, where a low EIS implies a high level of risk aversion ($\gamma = \psi^{-1}$). A high γ generates a large debt capacity because the utility cost of defaulting and bearing the consumption volatility associated with autarky is high. This high default cost induces the country to avoid borrowing, so the average debt-output ratio is low or even negative.

In an expected-utility setting, we need a moderate value of risk aversion to generate a realistic average debt-output ratio. We next consider an expected-utility-based calibration with $\psi = 1/\gamma = 0.5$. If we use the annual discount rate proposed by Barro and Jin (2011) and used in our calibration ($\rho = 0.052$), the model does not generate a plausible debt-output ratio for a wide range of distress costs.

To understand the intuition for this result, consider two scenarios. In the first scenario, distress costs, $(1-\alpha)$, are high, so debt capacity is also high. But a country in debt has a strong incentive to save to avoid the possibility of incurring high default costs. As a consequence, the default likelihood, credit spread, and average debt-to-output ratio are low. In the second scenario, distress costs are low, so the country is willing to borrow and the likelihood of default is high. As a consequence, credit spreads are high and debt capacity is low, resulting in a counterfactually low average debt-to-output ratio.

One approach widely used in the literature is to assume high distress costs so that debt capacity is high and also assume a very high discount rate. This configuration can generate plausible debt-output ratios because high discount rates create an incentive to borrow, even when default costs are high (see Aguiar and Gopinath (2006) and Arellano (2006)).[19] Generating the average debt-output ratio targeted in our calibration (15 percent) requires a value of ρ equal to 21 percent.[20]

We next show that association between low financial development and debt tolerance also obtains in an expected-utility setting. Table 7 uses our expected-utility calibration ($\gamma = 2$

[19]Alvarez and Jermann (2001) also find that a low risk aversion and a high discount rate are necessary to match key asset-pricing moments in a general equilibrium asset-pricing model with limited commitment.

[20]Bornstein (2017) compares discrete and continuous-time versions of Arellano's (2008) model and shows that average debt-to-output ratios are lower in the continuous-time version of the model. The reason is as follows. In discrete time, a country can respond to a negative output shock by cutting consumption to reduce its debt for next period, therefore reducing its credit spread. In continuous time, the stock of debt is continuous and an output shock leads to an instantaneous rise in the credit spread. This rise creates an incentive to save away from the debt region.

and $\rho = 21\%$) to compare economies with different levels of financial development. The same debt intolerance phenomenon that emerges in our baseline recursive-utility calibration ($\gamma = 2$, $\psi = 0.047$, and $\rho = 5.2\%$) is also present in this expected-utility setting.

In our expected-utility calibration, the probability of default is high and debt capacity is low when financial development is low: 5.7 per annum and 19 percent, respectively, for the case where $Z^* = 1$ versus 0.1 per annum and 25 percent, respectively, for the case where $Z^* = 0.5$. Countries with low financial development (e.g., $Z^* = 1$) use default on sovereign debt to manage their rare-disaster risks.

Table 7: Partial Spanning and Debt Intolerance for an Expected-utility Calibration

Z^*	Debt-output ratio	Default probability	Debt capacity $\lvert \underline{w} \rvert$
1 (No jump hedging)	13.7%	5.7%	18.8%
0.9	**14.5%**	**3.0%**	**19.0**%
0.5	21.0%	0.1%	24.8%

In this table, $\gamma = \psi^{-1} = 2$ and $\rho = 0.21$. Other parameter values excluding Z^* are summarized in Table 1.

9 Conclusion

We present a tractable model of sovereign debt that features a jump-diffusion process for output used in the rare-disasters literature, recursive preferences that separate the role of intertemporal substitution and risk aversion, and partial insurance against jump risk. We show that low levels of financial development generate the debt intolerance that we see in emerging markets: low debt levels are associated with high credit spreads.

In order to focus on the impact of financial development on sovereign debt, we abstracted from three forces that could influence sovereign-credit spreads. The first is the risk premium demanded by foreign investors to compensate their exposures to the systematic components of sovereign default risk (see, e.g. Pan and Singleton (2008), Longstaff, Pan, Pedersen, and Singleton (2011), Borri and Verdelhan (2015), and Hébert and Schreger (2017).) The second is the moral hazard problem that is associated with insurance. The third is the impact of sudden stops (Calvo (1998) and Mendoza (2010)) and debt roll-over risk. We plan to address these issues in future research.

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A Appendix: Technical Details

We conjecture that the value function in the normal regime, $V(W,Y)$, is given by equation (15) and the value function in the autarky regime, $\widehat{V}(\widehat{Y})$, is given by equation (19).

Substituting equations (15) and the first and second derivatives of $V(W,Y)$ into the HJB equation (10) and using the homogeneity property of the value function, we obtain:

$$
\begin{aligned}
0 = \max_{c,\theta,x} \; & \frac{\left(\frac{c(w)}{bp(w)}\right)^{1-\psi} - 1}{1-\psi^{-1}} \rho p(w) + \left[\, (r + \pi(w) - \mu)\, w + 1 - c(w) - \phi(w) \,\right] p'(w) \\
& + \frac{(\theta(w)\sigma)^2}{2}\left(p''(w) - \frac{\gamma (p'(w))^2}{p(w)}\right) + \frac{\sigma^2}{2}\left(w^2 p''(w) - \frac{\gamma (p(w) - wp'(w))^2}{p(w)}\right) \\
& + \theta(w)\sigma^2 \left(-wp''(w) - \frac{\gamma p'(w)(p(w) - wp'(w))}{p(w)}\right) + \frac{\lambda}{1-\gamma}\mathbb{E}\left[\left(\frac{Zp(w^{\mathcal{J}})}{p(w)}\right)^{1-\gamma} - 1\right] p(w)\,,
\end{aligned}
\tag{A.1}
$$

where $w^{\mathcal{J}}$ is given by equation (27) and $\phi(w) = \lambda \mathbb{E}[x(w;Z)\, \mathcal{I}_{Z \geq Z^*}]$.

We can simplify the first-order conditions for consumption (equation (11)) and diffusion-risk hedging demand (equation (12)) to obtain equations (50) and (51).

Simplifying the FOC for the jump risk hedging demand, given by equation (14), we obtain the following condition for the optimal scaled hedging demand for jump risk, $x(w,Z)$:

$$
p'(w) = \left(\frac{Zp((w + x(w,Z))/Z)}{p(w)}\right)^{-\gamma} p'((w + x(w,Z))/Z)\,. \tag{A.2}
$$

Substituting equations (50) and (51) into equation (A.1), we obtain ODE (44) for $p(w)$. Similarly, substituting the conjectured value functions (15) and (19) into the HJB equation (18), we obtain equation (45) for $\widehat{p}$. The value-matching condition that equates the cost of repaying debt and defaulting, given by equation (20), implies the boundary condition (47). Substituting equation (51) into (34), we obtain the boundary condition (48). Intuitively, self insurance against income shocks becomes as effective as the insurance available in the FB case as $w \to \infty$.

Next, we provides some technical details for the FB case. The conjectured certainty equivalent wealth is given by $p(w) = w + h$. Substituting this value into equations (50), (51), and (A.2), respectively, we obtain the following optimal consumption, diffusion-risk hedging demand and jump risk hedging demand rules:

$$c^{FB}(w) = m(w+h), \tag{A.3}$$

$$\theta^{FB}(w) = -h, \tag{A.4}$$

$$x^{FB}(w,Z) = (1-Z)h. \tag{A.5}$$

Substituting $p(w) = w + h$ and equation (A.5) into the ODE (44), and using the fact that $Z^* = 0$ in the FB case, we obtain:

$$0 = \left(\frac{m-\psi\rho}{\psi-1} + \mu\right)(w+h) + [(r-\mu)w + 1] + \lambda(\mathbb{E}(Z)-1)h \tag{A.6}$$

$$= \left(\frac{m-\psi\rho}{\psi-1} + r\right)w + \left(\frac{m-\psi\rho}{\psi-1} + \mu - \lambda(1-\mathbb{E}(Z))\right)h + 1. \tag{A.7}$$

As equation (A.7) must hold for all $p(w) = w+h$, we must have $\frac{m-\psi\rho}{\psi-1} + r = 0$ which implies that $m = r + \psi(\rho - r)$ as stated in equation (41). Using the fact that $m = \rho^{\psi} b^{1-\psi}$, we obtain the formula (16) for the coefficient b. Finally, substituting $m = r + \psi(\rho - r)$ into equation (A.7), we obtain the value of h:

$$h = \frac{1}{r - [\mu - \lambda(1-\mathbb{E}(Z))]} = \frac{1}{r-g}. \tag{A.8}$$