Hedging macroeconomic and financial uncertainty and volatility*

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Abstract

This paper studies the pricing of shocks to uncertainty and realized volatility using options contracts directly related to the state of the macroeconomy and of financial markets. Contracts that provide protection against shocks to macroeconomic uncertainty have historically earned statistically and economically significantly positive excess returns. If uncertainty shocks were viewed as bad by investors – in the sense of being associated with high marginal utility – portfolios that hedge them should instead earn negative premia. Portfolios exposed to the realization (as opposed to the expectation) of large shocks, on the other hand, have historically earned large and negative risk premia.

1 Introduction

Uncertainty shocks play an important role in many macroeconomic and financial models. To analyze their role in explaining economic fluctuations, the literature has proposed several different measures of aggregate uncertainty, some based on financial variables (like the VIX, obtained from S&P 500 options, used for example in Bloom (2009)), some directly built using macroeconomic series (Ludvigson, Ma, and Ng (2015), and others based on text data (Baker, Bloom, and Davis (2015)).

The goal of this paper is to understand how investors perceive the risks associated with macroeconomic uncertainty shocks. To do so, it is crucial to look at markets for assets that are directly exposed to those shocks: markets where investors can easily and directly hedge uncertainty. The returns of assets that hedge uncertainty reveal the equilibrium price for protection from those shocks and therefore how agents’ marginal utility correlates with uncertainty shocks. That has direct implications for theoretical models. For example, if high uncertainty leads to declines in economic activity (as in Bloom (2009)), theory would predict that assets that hedge uncertainty shocks would earn negative average returns. On the other hand, uncertainty might be high in periods of high

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innovation and growth (Pastor and Veronesi (2009)), which could cause uncertainty shocks to earn positive premia.

While there is a large literature that estimates the risk premia for uncertainty and volatility in the S&P 500, recent evidence shows that aggregate uncertainty has multiple dimensions (Ludvigson, Ma, and Ng (2015); Baker, Bloom, and Davis (2015)). S&P 500 uncertainty is related to conditions in the financial sector, but it is possible that the driving force in the economy is actually uncertainty about other features of the macroeconomy, such as interest rates, inflation, or the availability of inputs to production, like crude oil. This paper contributes to the literature by estimating risk premia associated with uncertainty in 19 different markets covering a range of different features of the economy, including financial conditions, inflation, and real assets. Using the range of contracts, we construct portfolios that allow investors to directly hedge different types of uncertainty shocks, and also shocks to prominent recent uncertainty indexes from Ludvigson, Ma, and Ng (2015) and Baker, Bloom, and Davis (2015).

The analysis yields three key findings. First, for markets associated with macroeconomic uncertainty – futures with nonfinancial underlyings – shocks to uncertainty carry a statistically and economically significantly positive risk premium. That fact implies that state prices (marginal utilities) covary negatively with uncertainty in those markets: uncertainty is high in good times. Second, for financial underlyings, uncertainty has a risk premium that is not significantly different from zero, indicating that it has no average correlation with marginal utility. Third, for both financial and nonfinancial underlyings, realized volatility – a measure of the magnitude of realized movements in underlying variables – carries a negative premium. These results imply that forward-looking uncertainty shocks carry a zero or positive premium, whereas surprise jumps in prices robustly earn negative premia; so, on average, investors perceive periods of high uncertainty as generally good times and periods when large shocks occur as bad times.

The first step in the analysis is to document the strong relationship between the implied volatility in the 19 options markets and measures of aggregate uncertainty. In particular, we examine correlations between the implied volatilities for each market and the different measures of uncertainty developed by Jurado, Ludvigson, and Ng (2015) and Ludvigson, Ma, and Ng (LMN; 2017), as well as the text-based economic policy uncertainty (EPU) measure of Baker, Bloom, and Davis (2015). Implied volatility for the financial underlyings – the S&P 500 and Treasury bonds in particular – is primarily associated with the LMN financial uncertainty and EPU indexes, while implied volatility for the nonfinancial underlyings is much more strongly associated with LMN uncertainty about the real economy and goods prices. The relationships are strong in the sense that the implied volatilities explain 60–80 percent of the variation in the LMN and EPU indexes. Together, these results confirm that hedging shocks to implied volatility in these markets represents a good way to hedge various types of aggregate uncertainty shocks, both macroeconomic and financial, and they show why it is important to study more than just S&P 500 options.

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We next examine how shocks to realized and implied volatility are priced across the various markets by constructing portfolios that yield pure exposure to implied volatility (uncertainty) and to realized volatility shocks. It is the returns on those portfolios that measure the premia for volatility and uncertainty discussed above. Those results refer to the returns to hedging uncertainty in specific markets – oil, metals, etc. The final piece of the analysis is to construct portfolios that specifically hedge the EPU and JLN uncertainty indexes. To do so, we look at that particular portfolio of options across all markets that best hedges each uncertainty index. Consistent with the results for the individual assets, there has been historically a price close to zero for hedging shocks to the JLN financial uncertainty and EPU indexes, while portfolios that hedge the JLN macroeconomic and goods price uncertainty indexes have earned positive returns.

The results are difficult to reconcile with the view that innovations in economic uncertainty are contractionary. If increases in uncertainty were viewed as bad in the sense of raising marginal utility, then we would find a negative premium on implied volatility (investors are willing to accept negative average returns on assets that are hedges against high marginal utility states). Instead, the results imply that investors have historically viewed periods of high uncertainty and implied volatility as being good, in the sense that they are associated with low marginal utility.

The paper is related to two main strands of literature. The first studies the relationship between uncertainty and the macroeconomy. There are numerous channels that have been proposed through which uncertainty about various aspects of the aggregate economy may have real effects.\(^2\) Importantly, these models do not generate a uniform prediction that uncertainty shocks are necessarily contractionary. While there are contractionary forces, such as wait-and-see effects and Keynesian demand channels, there are also forces through which uncertainty can be expansionary, including precautionary saving and the Hi–Hartmann–Abel effect that is extensively discussed by Bloom et al. (2017) (see also Gilchrist and Williams (2005)). Our results are therefore more consistent with the expansionary forces. There is also a related empirical literature that tries to measure whether uncertainty does in fact have contractionary effects.\(^3\) This paper builds on that work by providing measures of risk premia that indicate how investors perceive the effects of aggregate uncertainty shocks. Furthermore, while the past literature has often used S&P 500 implied volatility to measure uncertainty (e.g. Bloom (2009) and Basu and Bundick (2017)), this paper covers a much broader range of assets.

The second literature we build on estimates the pricing of volatility risk in financial markets. Again, that literature primarily studies S&P 500. There are various papers that have studied specific markets, such as individual equities (e.g. Bakshi, Kapadia, and Madan (2003)) or Treasury bonds (Mueller, Vedolin, and Yen (2017)). Prokopczuk, Simen, and Symeonidis (2017) examine the variance risk premium across many of the same markets that we study (see also Trolle and

\(^2\) These include a Keynesian demand channel (Basu and Bundick (2017)), real options effects on investment (Bloom (2009), Bloom et al. (2017)), effects on labor search (Leduc and Liu (2015)), or through financial frictions and credit spreads (Gourio (2013)).

\(^3\) Recent examples include Berger, Dew-Becker, and Giglio (2017), Jurado, Ludvigson, and Ng (2015), Ludvigson, Ma, and Ng (2015), Baker, Bloom, and Davis (2015), and Alexopoulos and Cohen (2009), among many others.
Schwartz (2010)). Our contribution involves using multiple maturities in each market to isolate the premium on implied volatility as opposed to just the realized variance risk premium – the distinction between the two is crucial because it is only implied volatility, not realized volatility, that captures the forward-looking concept of uncertainty on which the theoretical models are based. We also provide evidence on the relationship between implied volatility and the macroeconomy.

The remainder of the paper is organized as follows. Section 2 describes the data and its basic characteristics. Section 3 discusses the construction of portfolios that hedge realized volatility and uncertainty. Section 4 reports the cost of hedging volatility and uncertainty in our data. Section 5 presents robustness results. To provide more confidence on some of the results, section 6 examines the crude oil market in detail. Finally, section 7 concludes.

2 Measures of uncertainty and realized volatility

This section describes our main data sources and then examines various measures of uncertainty and realized volatility.

2.1 Data

2.1.1 Options and futures

We collected data on prices of financial and commodity futures and options from the end-of-day database from the CME Group, which reports closing settlement prices, volume, and open interest for the period 1983–2015. The CME data is important for covering a broad array of features of the economy, including stock prices, interest rates, exchange rates, and prices of metals, petroleum products, and agricultural products.

Each market includes both futures and options, with the options written on the futures. The futures may be cash- or physically settled, while the options settle into futures. As an example, a crude oil call option gives its holder the right to buy a crude oil future at the strike price. The underlying crude oil future is itself physically settled – if held to maturity, the buyer must take delivery of oil in Cushing, Oklahoma.

To be included in the analysis, contracts are required to have least 15 years of data and maturities for options extending to at least six months, which leaves 14 commodity and 5 financial underlyings. The final contracts included in the data set have 18 to 31 years of data.

A number of standard filters are applied to the data to reduce noise and eliminate outliers. Those filters are described in appendix A.1.

We calculate implied volatility for all of the options using the Black–Scholes (1973) model (technically, the Black (1976) model for the case of futures).\footnote{The majority of the options that we study have American exercise, while the Black model technically refers to European options. We examine IVs calculated assuming both exercise styles (we calculate American IVs using a binomial tree) and obtain nearly identical results. Since there are no dividends on futures contracts, early exercise is only rarely optimal for the options studied here.} Unless otherwise specified, implied
volatility is calculated at the three-month maturity.

A key distinction in the analysis is between uncertainty and realized volatility. Realized volatility measures how much some factor actually varies over some period, while uncertainty represents variation in the conditional distribution of the factor. Option implied volatility theoretically measures investors’ conditional standard deviation for futures returns going forward, so we measure realized volatility analogously as the sample standard deviation of futures returns in the current month. Specifically, in the various futures markets, realized volatility is defined in month \( t \) as

\[
RV_{i,t} = \left( \frac{365}{\#\text{days} \in t} \sum_{\text{days} \in t} f_i^2 \right)^{1/2},
\]

where \( f_i \) here is a daily return on the near-month futures return in market \( i \). Realized volatility in month \( t \) is the annualized sample standard deviation during that month.

### 2.1.2 Alternative uncertainty measures

The implied volatilities of the CME options give direct measures of investor uncertainty, similar to the VIX. We also examine two other measures of uncertainty.

The first uncertainty index is developed in a pair of papers by Jurado, Ludvigson, and Ng (2015) and Ludvigson, Ma, and Ng (LMN; 2017). The construction involves two basic steps. First, realized squared forecast errors are constructed for 280 macroeconomic and financial time series. Denoting the error for series \( i \) as \( \varepsilon_{i,t} \), the basic assumption is that there is a variance process, \( \sigma^2_{i,t} \), such that \( E[\varepsilon_{i,t}^2] = \sigma^2_{i,t} \). So \( \varepsilon_{i,t}^2 \) constitutes a noisy signal about \( \sigma^2_{i,t} \). LMN then estimate \( \sigma^2_{i,t} \) from the history of \( \varepsilon_{i,t}^2 \) using a two-sided smoother and create an uncertainty index as the first principal component of the estimated \( \sigma^2_{i,t} \). We divide the 280 series among those that pertain financial markets, real activity, and goods prices, with the latter two also being combined into an overall macroeconomy group, and take the first principal component from each group to get different subindexes.

The goal of the LMN framework is to estimate uncertainty on each date, \( \sigma^2_t \). The method can also be extended to create a realized volatility index by taking the first principal component from the cross-section of the \( \varepsilon_{i,t}^2 \). We therefore construct both uncertainty and realized volatility under the LMN framework.

The second uncertainty index is the Economic Policy Uncertainty (EPU) index of Baker, Bloom, and Davis (2015). The EPU index is constructed based on media discussion of uncertainty, the number of federal tax provisions changing in the near future, and forecaster disagreement. Unlike the LMN framework, there is no distinction in this case between volatility and uncertainty, so we treat the EPU index as measuring only uncertainty.
2.2 The time series of uncertainty

Figure 1 plots option implied volatility for three major futures: the S&P 500, crude oil, and US Treasury bonds. The implied volatilities clearly share common variation; for example, all rise around 1991, 2001, and 2008. On the other hand, they also have substantial independent variation. The period around the 1991 Gulf War was a period of extremely high implied volatility for crude oil, but much lower uncertainty for stocks and bonds. Conversely, the Financial Crisis was associated with larger relative increases in stock and bond than crude oil implied volatility. So while they move together, their overall correlations (also reported in the figure) are only in the range 0.5–0.6.

Table 1 reports pairwise correlations of implied volatility across the 19 underlyings, and also gives the first introduction to the full list of 19 markets. The various markets are sorted in this table into related categories, with the result that the largest correlations are generally along the main diagonal. Shading denotes the degree of correlation, with darker cells representing greater correlation. The largest correlations in implied volatility are among similar underlyings – crude and heating oil, the agricultural products, gold and silver, and the British Pound and Swiss Franc. Correlations outside those groups are notably smaller, in many cases close to zero.

The eigenvalues of the correlation matrix quantify the degree of common variation. The largest eigenvalue explains 43 percent of the total variation. The remaining eigenvalues are much smaller, though – even the second largest is only 0.15. Eight eigenvalues are required to explain 90 percent of the total variation in the IVs, which is perhaps a reasonable estimate of the number of independent components in the data.

To understand the behavior of the implied volatilities in more detail, table 2 reports results from regressions of the 19 implied volatilities on various combinations of the EPU and JLN indexes. The left and middle panels of the table report results from the two regressions

$$\frac{IV_{i,t}}{SD(IV_{i,t})} = a_{1,i} + b_{1,i}LMNU_{t}^{Financial} + b_{2,i}LMNU_{t}^{Macro} + \varepsilon_{1,i,t},$$

(2)

$$\frac{IV_{i,t}}{SD(IV_{i,t})} = a_{2,i} + b_{3,i}LMNU_{t}^{Financial} + b_{4,i}LMNU_{t}^{Real} + b_{5,i}LMNU_{t}^{Price} + \varepsilon_{2,i,t},$$

(3)

where $IV_{i,t}$ denotes at-the-money implied volatility for underlying $i$ averaged over month $t$, $SD(IV_{i,t})$ is the sample standard deviation of $IV_{i,t}$, and the various $LMNU_{i}^{\cdot}$ are the LMN uncertainty series. The uncertainty series all have unit standard deviations by construction, and the implied volatilities are also normalized for the regressions. The regressions help understand how the individual implied volatility series relate to other measures of uncertainty. The table reports the five financial underlyings in our data at the top of each panel (S&P 500, T-bonds, and the three exchange rates), and the nonfinancial underlyings (starting with copper) at the bottom.

For the S&P 500 and US Treasury bonds, implied volatilities are strongly related to financial uncertainty, which is natural since measures of aggregate stock prices and interest rates are included in LMN’s set of financial indicators. Among the nonfinancial underlyings, the loadings almost entirely favor macro uncertainty – in 12 of 14 cases, the coefficient on macro uncertainty is larger.
than that on financial uncertainty. The coefficients are generally economically large: the average coefficient on macro uncertainty among the nonfinancial underlyings is 0.32. The coefficients are relatively larger for industrial products like energies and metals – all above 0.4 except natural gas; they are somewhat smaller for the agricultural products, averaging 0.23.

To further decompose those results, the second panel in table 2 reports results from the regression (3) that replaces the macro uncertainty time series with its real and price subcomponents. The nonfinancial underlyings are nearly evenly split, with six having larger loadings on the price component and eight having larger loadings on the real component. The energies, perhaps naturally, are more associated with price uncertainty, with coefficients near 0.5. Metals and agricultural products, on the other hand, are more associated with macro uncertainty, with coefficients near 0.4. Looking down the columns, the R²s range from 0.09 to 0.74. The bottom row of each panel reports results from a regression of the average of the 19 IVs on the JLN indexes. In that case, the coefficients on financial and macro uncertainty are similar, with values around 0.4, and the macro loading is split equally between real and price uncertainty. The R² in both cases is approximately 0.55.

Overall, table 2 shows that there is a statistically and economically strong relationship between implied volatility measured in futures markets and the JLN uncertainty measure constructed from aggregate time series. Past work has focused on S&P 500 implied and realized volatility, which the evidence here shows primarily measures financial uncertainty. The wide range of markets used here is therefore valuable for giving direct measures of investor uncertainty about broader features of the macroeconomy than simply the financial sector.

The right panel in table 2 report results of regressions of the IVs on the EPU index. In almost every case, the coefficients and R²s are smaller than for the JLN regressions. The R² for the average across the IVs is only 0.14. That suggests that the EPU index, in measuring policy uncertainty, captures somewhat different features of the economy from what is in the JLN indexes and our IVs. The S&P 500, Treasury bonds, currencies, and gold and silver uncertainty have the strongest relationships with EPU, suggesting that EPU more closely related to financial than nonfinancial uncertainty in our data.

### 2.3 Projecting the uncertainty indexes onto the 19 IVs

Figure 2 examines how well the 19 IVs can fit the LMN and EPU indexes. These regressions are then used to construct hedging portfolios for the indexes.

Figure 2 plots the time series of the three JLN uncertainty indexes – financial, real, and goods price uncertainty, in the first three rows – as well as the EPU index in the bottom row against the fitted values from their projection onto the 19 implied volatilities and a constant. The R²s are reported in the left-hand panels. The highest R², at 80 percent, is for financial uncertainty. The top-right panel plots the pairwise correlations of the implied volatilities in the individual markets with the fitted uncertainty. For financials, the correlation with S&P 500 implied volatility (which
is nearly identical to the VIX) is 95 percent. The next highest correlation is only 69 percent, for Treasury bonds. So figure 2 reinforces the result from table 2 that financial uncertainty is very nearly equivalent to S&P 500 implied volatility.

The second best fit for the LMN uncertainty projections is for price uncertainty in the third row, where the implied volatilities generate an $R^2$ of 73 percent. In this case, the highest correlations are for heating oil, crude oil, natural gas, gold, and copper. These results show the value of the alternative markets in helping provide a better fit to inflation uncertainty than the S&P 500.

Last, the second row plots fitted uncertainty for real variables. The same implied volatilities – gold, copper, crude oil, and heating oil – appear with the highest pairwise correlations as for price uncertainty. The $R^2$ is lower in this case, at 59 percent. The commodity options therefore appear to be slightly better at hedging financial and inflation uncertainty than in uncertainty about variables like GDP or industrial production. But the $R^2$ for real uncertainty is still substantial, and the implied volatilities seem to capture well the lower-frequency variation, missing some of the more high-frequency variation; overall, these investments still provide a hedge against a substantial fraction of real (GDP, IP, etc.) risk.

The bottom panels plot results for the EPU index. The overall $R^2$ is similar to what is obtained for JLN real uncertainty. Consistent with the results in table 2, the highest pairwise correlations are with financial IVs, Treasuries, gold, the S&P 500, and currencies. So the fit of the IVs to the EPU index comes mostly from the financial rather than the nonfinancial options, but note that Treasury and gold uncertainty have gotten relatively little attention in past work.

### 2.4 Realized volatility

Table 3 reports the correlation of implied and realized volatility for the 19 underlyings, along with their standard deviations. Realized volatility tends to be substantially more volatile than implied volatility, which is natural if implied volatility represents, even approximately, an expectation of future realized volatility. Implied and realized volatility are also strongly correlated with each other, which is again natural given that implied volatility represents expected future volatility. The key difference between the two is that realized volatility isolates realizations of extreme events (price jumps), whereas implied volatility measures expectations of the probability or size of future extreme events.

Table 4 reports the correlation matrix for realized volatility across the 19 markets. As in the IV correlation matrix, the correlations are relatively strong near the main diagonal, but they are all smaller in the RV case. The largest eigenvalue is only 0.32, compared to 0.43 for IV, implying there is less common and more idiosyncratic variation in realized than implied volatility.

Table A.1 in the appendix reports results from regressions analogous to (2)–(3), but replacing $IV$ with $RV$ and the $LMNU$ series with $LMNRV$, which is the LMN-type realized volatility index described above. The results are similar in the sense that S&P 500 and Treasury bond RV load more on the financial $LMNRV$ index, whereas the nonfinancials load more on the macro indexes.
The $R^2$s in this case are smaller than for IV, which is consistent with the result from the correlation matrices that there is more common variation in IV than RV.

Figure 3 replicates figure 2, but using realized instead of implied volatility. That is, it examines the ability of the RV series for the 19 futures markets to fit the three JLN RV indexes. As before, the $R^2$s are lower in this case. Interestingly, S&P 500 realized volatility appears to fit better to the JLN RV indexes than in the IV case. Nevertheless, it remains the case that for fitting real and price RV, the nonfinancial markets, including in particular the energies and copper, are particularly useful.

3 Using option portfolios to hedge uncertainty

Implied volatility and the uncertainty indexes are not directly tradable – only the options themselves are. This section shows how to construct option portfolios that hedge shocks to implied and realized volatility in each of the 19 markets. Furthermore, using the results from the previous section showing that the implied volatilities can provide a good fit to the uncertainty captured by the JLN and EPU indexes, we also construct portfolios of options across markets to hedge shocks also to those two alternative concepts of uncertainty and volatility.

3.1 Straddle portfolios

We study two-week returns on straddles with maturities between one and six months.\textsuperscript{5} A straddle is a portfolio holding a put and a call with the same maturity and strike, with the strike set to the current spot price. The final payoff of a straddle depends on the absolute value of the return on the underlying, meaning that they have symmetrical exposures to positive and negative returns.

Straddles give investors exposure both to realized and implied volatility. They are exposed to realized volatility because the final payoff of the portfolio is a function of the absolute value of the underlying futures return. But when a straddle is sold before maturity, the sale price will also depend on expected future volatility, meaning that straddles can give exposure to uncertainty shocks.

The exposures of straddles can be approximated theoretically using the Black–Scholes model, as in Coval and Shumway (2001), Bakshi and Kapadia (2003), and Cremers, Halling, and Weinbaum\textsuperscript{5} Past work on option returns and volatility risk premia has examined returns at frequencies of a day (e.g. Andries et al. (2017)), a week (Coval and Shumway (2001)), a month (Constantinides, Jackwerth, and Savov (2013); Dew-Becker et al. (2017)), and holding the options to maturity (Bakshi and Kapadia (2003)). The precision of estimates of the riskiness of the straddles is, all else equal, expected to be higher with shorter windows. On the other hand, shorter windows cause any measurement error in option prices to have larger effects. We choose two-week windows because they are within the typical range used and they are short enough to allow us to still calculate returns on relatively short-maturity options. Some of the existing literature, beginning with Bakshi and Kapadia (2003), examines delta-hedged returns. Bakshi and Kapadia (2003) study returns to maturity, but only on options with maturities shorter than 60 days. Even with delta hedging, the higher-order risk exposures of the straddles change substantially as the spot changes. Higher-frequency returns avoid that problem. Section 5 describes alternative specifications that we have examined to check the robustness of the main results.

\textsuperscript{5}Past work on option returns and volatility risk premia has examined returns at frequencies of a day (e.g. Andries et al. (2017)), a week (Coval and Shumway (2001)), a month (Constantinides, Jackwerth, and Savov (2013); Dew-Becker et al. (2017)), and holding the options to maturity (Bakshi and Kapadia (2003)). The precision of estimates of the riskiness of the straddles is, all else equal, expected to be higher with shorter windows. On the other hand, shorter windows cause any measurement error in option prices to have larger effects. We choose two-week windows because they are within the typical range used and they are short enough to allow us to still calculate returns on relatively short-maturity options. Some of the existing literature, beginning with Bakshi and Kapadia (2003), examines delta-hedged returns. Bakshi and Kapadia (2003) study returns to maturity, but only on options with maturities shorter than 60 days. Even with delta hedging, the higher-order risk exposures of the straddles change substantially as the spot changes. Higher-frequency returns avoid that problem. Section 5 describes alternative specifications that we have examined to check the robustness of the main results.
Appendix A.2 shows that the partial derivatives of the straddle return with respect to the underlying futures return, $f$, its square, and the change in volatility, can be approximated as

\[
\frac{\partial r_{n,t}}{\partial f_t} \approx 0, \quad (4)
\]
\[
\frac{\partial^2 r_{n,t}}{\partial (f_t/\sigma_{t-1})^2} \approx n^{-1}, \quad (5)
\]
\[
\frac{\partial r_{n,t}}{\partial (\Delta \sigma_t/\sigma_{t-1})} \approx 1, \quad (6)
\]

where $r_{n,t}$ is the return on date $t$ of a straddle with maturity $n$, $f_t$ is the return on the underlying future, $\sigma_t$ is the implied volatility of the underlying, and $\Delta$ is the first-difference operator.\(^6\)

The first partial derivative says that the straddles all have close to zero local exposure to the spot return, which is natural since their payoff is a symmetrical function of the underlying return. The second line says that the exposure of straddles to squared returns on the underlying – scaled by volatility – is approximately inversely proportional to time to maturity. Throughout the paper, we interpret exposures to squared returns as representing exposure to realized volatility, since realized volatility is calculated based on squared returns over some period. The third line shows that straddles are also exposed to changes in expected future volatility, through $\Delta \sigma_t/\sigma_{t-1}$, and that exposure is approximately constant across maturities.

Overall, then, all straddles have approximately equal exposure to proportional shifts in implied volatility, while the exposure to realized volatility decreases with maturity. Long-maturity straddle returns (for which the term $n^{-1}$ is sufficiently small) therefore reveal the premium associated with uncertainty shocks.

### 3.2 Hedging RV and IV in each market

The implied sensitivities in (4)–(6) give a method for constructing portfolios that the Black–Scholes model says should give exposures only to realized volatility – squared returns, measured by $(f_{n,t}/\sigma_{t-1})^2$ – and only implied volatility, measured by $\Delta \sigma_t/\sigma_{t-1}$ (Cremers, Halling, and Weinbaum (2015)). Specifically, we construct, for each market, two portfolios,

\[
rv_{i,t} = \frac{5}{24} (r_{i,1,t} - r_{i,5,t}), \quad (7)
\]
\[
iv_{i,t} = \frac{5}{4} r_{i,5,t} - \frac{1}{4} r_{i,1,t}. \quad (8)
\]

Throughout the paper, capitalized RV and IV refer to the levels of realized and implied volatility, while lower-case $rv$ and $iv$ refer to the associated portfolio returns.

\(^6\)We ignore here the fact that options at different maturities have different underlying futures contracts. If that elision is important, it can be expected to appear as a deviation of the estimated factor loadings from the predictions of the approximations (4)–(6). Note also that the units of $n$ and $\sigma$ must match. So if $\sigma$ is expressed annual units, then a one-month straddle has $n = 1/12$ and would have a loading of 6 on $(f_{n,t}/\sigma_{t-1})^2$. 

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Given equations (4)–(6), the \( rv \) and \( iv \) portfolios will both have zero local sensitivity to \( f_t \). The \( rv \) portfolio will have a unit sensitivity to \((f_t/\sigma_t)^2\) and zero sensitivity to \( \Delta \sigma_t/\sigma_{t-1} \) in each market, while the \( iv \) portfolio will have a unit sensitivity to \( \Delta \sigma_t/\sigma_{t-1} \) and zero sensitivity to the squared returns in each market. We use the one- and five-month straddles to construct the portfolios as those are the shortest and longest maturities that we consistently observe in the data.

The purpose of constructing these portfolios is to give a simple and direct method of measuring the premia associated with realized and implied volatility that does not require any complicated estimation or data transformation. One might worry, though, that they do not obtain the desired exposures in practice. Table A.2 in the appendix reports results of regressions, for each underlying, of the returns of the two portfolios on the underlying futures return, the squared futures return, and the change in implied volatility. The table shows that while the Black–Scholes predictions do not hold perfectly, the \( rv \) portfolio is nevertheless much more strongly exposed to realized than implied volatility, and the opposite holds for the \( iv \) portfolio. The coefficients on \((f_t/\sigma_t)^2\) average 0.76 for the \( rv \) portfolio and 0.10 for the \( iv \) portfolio. Conversely, the coefficients on \( \Delta \sigma_t/\sigma_{t-1} \) average 0.03 for the \( rv \) portfolio and 0.79 for the \( iv \) portfolio. Furthermore, the \( R^2 \)s are large, averaging 72 percent across the various portfolios, implying that their returns are well described by the approximation (4). Appendix A.2 also examines the accuracy of the Black–Scholes approximation for returns in a simulated setting. Finally, section 5 reports results on the cost of hedging volatility and uncertainty that do not rely on the Black–Scholes assumptions at all, rather using exposures to IV and RV in the data without imposing any model-based assumptions. The results of that robustness exercise strongly confirm those in the baseline case based on Black–Scholes.

### 3.3 Hedging the LMN and EPU indexes

Finally, using the results in figure 2 showing that the 19 IVs span most of the variation in the LMN and EPU uncertainty indexes, we construct portfolios that optimally hedge those indexes. For each index, we obtain the weights for the hedging portfolio from the regression coefficients in sections 2.3 and 2.4. For each uncertainty index \( j \), we estimate the regression

\[
LMNU_t^j = a + \sum_i b_{ij}^t IV_{it} + \varepsilon_{j,t} \quad (9)
\]

and then construct a hedging portfolio as

\[
iv_t^{hedge,j} \equiv \sum_i b_{ij}^t iv_{it} \quad (10)
\]

the coefficients \( b_{ij}^t \) therefore tell us the weight of the \( iv \) portfolio of market \( i \) in the hedging portfolio for index \( j \). We create such portfolios for each of the four LMN uncertainty indexes and the EPU index. We also construct similarly a hedge portfolio for the LMN realized volatility series.
\[ LMNRV_t^j = a + \sum_i b_i^{RV,j} RV_{i,t} + \varepsilon_{RV,j,t} \]  
\[ rv_{t}^{hedge,j} \equiv \sum_i b_i^{j} rv_{i,t} \]

### 4 The cost of hedging

This section reports our main results on the price of hedging shocks to volatility and uncertainty. Given a hedging portfolio, the cost of hedging is the negative of the average excess return (risk premium) on the portfolios. For example, holding an \( iv \) portfolio represents holding insurance against increases in implied volatility, so if the \( iv \) portfolio earns, say, a -10 percent excess return on average, the cost of that insurance is 10 percent on average. A mean excess return cannot be interpreted without reference to the associated volatility – levering a portfolio up or down will shift the mean excess return but also the volatility – so we report all risk premia in terms of Sharpe ratios: the mean excess return divided by the standard deviation. The Sharpe ratio reveals the compensation for bearing a risk (or the cost of hedging it) per unit of risk, and is therefore more easily comparable across markets. For reference, the historical Sharpe ratio of US equities in our sample is 0.52.

The cost of hedging a risk has a simple but important economic interpretation: it measures the extent to which the risk is “bad” for the agent. To formalize that intuition, consider a factor \( X \) and an asset with returns \( R_X \) that hedges it, in the sense that \( R_X \) varies one-for-one (and is perfectly correlated) with \( X \). Then if \( M \) represents the stochastic discount factor (i.e. the Arrow–Debreu state prices divided by state probabilities), then

\[ E[R_X - R_f] = -cov(M, X) R_f, \]

where \( R_f \) is the gross risk-free rate. The equation says that the negative of the risk premium on a portfolio that hedges the risk \( X \) captures the covariance of that risk with state prices.\(^7\) So if the premium \( E[R_X - R_f] \) is negative, then the risk \( X \) itself must comove positively with state prices \( M \). That is, times when \( X \) is high are bad times, in which state prices are high (in consumption-based models, these are the times when consumption is low and the marginal utility of consumption is high).

This section asks whether portfolios that hedge uncertainty have negative risk premia; if that is the case, we then learn that investors perceive uncertainty to be high in bad times. While the section will present a variety of results, all the key results on the cost of hedging realized volatility and uncertainty appear in figure 4.

\(^7\)The last term on the right, \( R_f \), is close to 1, and is the same for all assets and all risk factors, so it plays no significant role in interpreting this equation.
4.1 Hedging uncertainty shocks

The red series in figure 4 plots sample Sharpe ratios and confidence bands for the various $rv$ and $iv$ portfolios. The top panel plots results for $iv$ and the bottom panel $rv$. The boxes are point estimates while the bars represent 95-percent confidence bands based on a block bootstrap.

Across the top panel, the $iv$ portfolios clearly tend to earn zero or even positive returns on average. For financials, the average Sharpe ratios tend to be near zero, while for the nonfinancials, all 14 sample Sharpe ratios are actually positive. To formally estimate the average Sharpe ratios, we use a random effects model, which yields an estimate of the population mean Sharpe ratio while simultaneously accounting for the fact that each of the sample Sharpe ratios is estimated with error, and that the errors are potentially correlated across contracts. The procedure is described in detail in appendix A.3. The estimated mean Sharpe ratios for just the financial and nonfinancial groups are reported in their respective sections, and the estimated population mean across both groups is in the right-hand section (“overall mean”).

For both nonfinancials and all markets overall, the estimated population mean Sharpe ratio is statistically and economically significantly positive, while for financials it is close to zero. The group-level means have the advantage of being much more precisely estimated than the Sharpe ratios for the markets individually. They show that on average, instead of there being a cost, in the form of a negative return, to hedging uncertainty shocks, uncertainty-hedging portfolios actually earn positive returns. In particular, for nonfinancials, the average Sharpe ratio is 0.48, and the lower end of the 95-percent confidence interval is 0.25. For the overall mean, the corresponding numbers are 0.39 and 0.17. These are not just statistically but economically significant – the return to portfolios hedging uncertainty shocks has earned average returns nearly as high as the overall stock market. But whereas the stock market is risky, in the sense that it rises in good times and falls in bad, the $iv$ portfolios are actually hedges, by construction giving positive returns when uncertainty rises. Even for financials, the point estimate for the average Sharpe ratio is positive, though the confidence band runs below zero.

The right-hand section of figure 4 reports the Sharpe ratios for the portfolios hedging the EPU and LMN indexes. Since those hedging portfolios are constructed combining the individual $iv$ portfolios (weighting them across the 19 markets to obtain the best hedge for the LMN and EPU indexes), it is not surprising that they are all near zero or positive. The hedging portfolios for JLN financial uncertainty and the EPU index both place relatively more weight on the financials, which have Sharpe ratios close to zero or even slightly negative, so they have overall lower Sharpe ratios. The portfolios hedging macro and price uncertainty, though, since they have larger weights on markets like crude oil, heating oil, and copper, have statistically and economically significantly positive Sharpe ratios, with point estimates both near 0.50, similar to the overall mean for the $iv$ portfolios.

The top panel of figure 4 contains all of our key results on the cost of hedging different types of uncertainty shocks. It shows that in our sample spanning almost 30 years the cost of hedging
shocks to uncertainty, whether it is uncertainty in a specific commodity or financial market or a more general macro uncertainty index, has been zero or even negative (the risk premium has been zero or positive).

If uncertainty was perceived to be bad by investors, hedging uncertainty shocks would be costly, and the point estimates in the top panel of figure 4 would be negative – the graph would be the opposite of what we actually see. But at most, some of the iv portfolios and hedging portfolios for LMN and EPU have very slightly negative Sharpe ratios. In the majority of the cases – and in particular for uncertainty about the nonfinancial macroeconomy – the Sharpe ratios are statistically and economically significantly positive. In other words, investors have been able to purchase portfolios that directly hedge them against uncertainty shocks and simultaneously earn returns as large as those on the overall stock market.

4.2 Hedging realized volatility shocks

The bottom panel of figure 4 reports analogous results for the cost of hedging realized volatility shocks. The numbers are drastically different. Whereas the iv portfolios have historically earned positive returns, the rv portfolios have almost all historically earned negative returns. For the S&P 500, this result is well known and is often referred to as the variance risk premium. The S&P 500 rv portfolio has the most negative Sharpe ratio, at -0.99 – the return to selling insurance against shocks to realized volatility is twice as large as the average return on the stock market over the same period. Treasuries also have a significantly negative return, but the other financials in our sample – all currencies – have Sharpe ratios slightly above zero. For the nonfinancials, 12 of 14 estimated Sharpe ratios are negative. So whereas the cost of hedging uncertainty shocks with the iv portfolios is consistently negative in the top panel, the cost of hedging realized volatility shocks using the rv portfolios is positive in the bottom panel (risk premia are negative).

As with the iv portfolios, we use a random effects model to calculate the population mean Sharpe ratios and report them in the three sections of the figure. In this case, all three estimates – financials, nonfinancials, and all assets – are negative. The values are again statistically and economically significant. The point estimate for the overall mean Sharpe ratio is -0.33 and the upper end of the 95-percent confidence interval is -0.08. Those values are almost the same as what we obtain for the iv portfolios, but with the opposite sign.

Finally, the right-hand section of the bottom panel of figure 4 reports the returns from the LMN rv hedging portfolios – those that hedge the realized volatility of the LMN macro series. Again, consistent with the fact that the rv portfolios themselves consistently earn negative returns, hedging the LMN indexes for realized volatility – as opposed to uncertainty – historically has a positive cost. For all three subindexes, the hedging portfolios earn extremely negative returns, with the Sharpe ratios for financial, real, and price volatility at -1.02, -0.84, and -0.82.

So in stark contrast to the results for hedging uncertainty, the bottom panel of figure 4 shows that there has historically been an extremely large cost to hedge realized volatility. That is,
contracts that, rather than loading on changes in implied volatility, load on actual realized squared returns – which the analysis above shows directly hedge extreme events in the macroeconomy – earn negative Sharpe ratios with magnitudes up to twice as large as the return on the overall stock market.

In summary, across both individual markets and also the hedging portfolios for the LMN and EPU indexes, exposure to realized volatility has consistently earned a negative premium, while exposure to implied volatility has earned a zero or positive premium. Investors have therefore historically paid money (accepted negative returns) to hedge surprise realizations of large shocks, while hedging surprises in uncertainty has had a zero or even negative cost. That is, investors purchasing portfolios that directly protected them against increases in uncertainty – measured by implied volatility (i.e. the VIX, in the case of stocks) – have historically actually earned positive returns. That result holds across a wide range of markets that provide hedges against uncertainty in both real activity and aggregate prices.

The results here are inconsistent with the view that uncertainty shocks are major drivers of economic declines. If they were – that is, if they were associated with periods of high marginal utility – the equilibrium return on assets hedging those shocks would be negative. If anything is associated with high marginal utility here, it not periods when investors are particularly uncertain about the future, but periods of high realized volatility, when large movements occur in stock, bond, and commodity markets.

4.3 Hedging average \( rv \) and \( iv \)

An alternative way to hedge aggregate uncertainty is simply to buy all the \( iv \) or \( rv \) portfolios simultaneously. Since tables 1 and 4 show that realized and implied volatility are imperfectly correlated across markets, even larger returns can be earned by holding portfolios that diversify across the various underlyings. Table 5 reports results of various implementations of such a strategy. The first row reports results for portfolios that put equal weight on every available underlying in each period, the second row uses only nonfinancial underlyings, and the third row only financial underlyings. The columns report Sharpe ratios for various combinations of the \( rv \) and \( iv \) portfolios. The first two columns report Sharpe ratios for strategies that hold only the \( rv \) or only the \( iv \) portfolios, the third column uses a strategy that is short \( rv \) and long \( iv \) portfolios in equal weights, while the final column is short \( rv \) and long \( iv \), but with weights inversely proportional to their variances (i.e. a simple risk parity strategy).

The Sharpe ratios reported in table 5 are generally larger than those in figure 4. The portfolios that are short \( rv \) and long \( iv \) are able to attain Sharpe ratios well above 1. The largest Sharpe ratios come in the portfolios that combine \( rv \) and \( iv \), which follows from the fact that they are positively correlated, so going short \( rv \) and long \( iv \) leads to internal hedging. All of that said, these Sharpe ratios remain generally plausible. Values near 1 are observed in other contexts (e.g. Broadie, Chernov, and Johannes (2009) for put option returns, Asness and Moskowitz (2013) for...
global value and momentum strategies, and Dew-Becker et al. (2017) for variance swaps).

The portfolios that take advantage of all underlyings simultaneously seem to perform best, presumably because they are the most diversified. While holding exposure to implied volatility among the financials earns a relatively small premium, it is still generally worthwhile to include financials for the sake of hedging.

Finally, it is important to note that the combined portfolios have returns that are much less skewed than those on the market-specific \(rv\) and \(iv\) portfolios. The bottom panel of table 5 reports the skewness of the various strategies from above, and, for the portfolios that include both \(rv\) and \(iv\), they range between -0.77 and 3.27. So while there may be some skewness, it does not run consistently in either direction – it is negative with equal weighting of the \(rv\) and \(iv\) portfolios, and positive for the variance weighting. That suggests that the premia from these factors can be earned without necessarily holding a portfolio that is substantially negatively skewed (as with writing puts or straddles). In fact, the risk-parity strategy that holds both financials and nonfinancials has earned a historical Sharpe ratio of 1.26 with positive skewness of 0.94.

Overall, these results show that the economic magnitudes related to hedging realized volatility and uncertainty across the 19 markets are very large, and can be obtained with portfolios that do not expose investors to particular additional risks like skewness. The results confirm that the cost of hedging realized volatility (large movements in the underlyings) in the last 30 years has been extremely high, whereas hedging uncertainty has actually yielded a large and positive risk premium.

5 Robustness

This section examines some potential concerns about the robustness of the results.

5.1 Using one-week holding period returns

Our main analysis is based two-week holding period returns for straddles, which strike a balance between having more precise estimates of risk premia and reducing the impact of measurement error in prices. We have repeated all of our analysis using one-week holding period returns, and find very similar results. We report in appendix figure A.3 the analogous of figure 4, but constructed using one-week returns. The results are qualitatively and quantitatively very similar, confirming the robustness of our analysis to the period considered.

5.2 Linear factor models

The evidence presented on the pricing of implied and realized volatility risk relies on the Black–Scholes model to give an approximation for the risk exposures of the portfolios. Appendix A.2 provides evidence that those predictions are a reasonably accurate description of the data, but our findings are not actually dependent on Black–Scholes holding with perfect accuracy. To estimate
the price of risk for realized and implied volatility purely empirically, with no appeal to exposures
from a theoretical model, we now estimate standard factor specifications and combine them across
markets using a random effects model. Rather than using the risk exposures implied by the Black-
Scholes model, we therefore estimate them freely in the data.

Typical factor models use a small number of aggregate factors. Here, though, we are interested
in the price of risk for shocks to all 19 types of uncertainty. We therefore estimate market-specific
factor models. This is similar to the common practice of pricing equities with equity-specific factors,
bonds with bond factors, currencies with currency factors, etc..

5.2.1 Specification

For each market we estimate a time-series model of the form

\[ r_{i,n,t} = a_{i,n} + \beta_{i,n} f_{i,t} + \beta_{i,n}^2 \frac{1}{2} \left( \frac{f_{i,t}}{IV_{i,t-1}} \right)^2 + \beta_{i,n} \Delta IV_{i,t} + \frac{f_{i,t}}{IV_{i,t-1}} + \varepsilon_{i,n,t}, \]  

(14)

where \( f_{i,t} \) is the futures return for underlying \( i \) and \( \Delta IV_{i,t} \) is the change in the five-month at-the-
money implied volatility for underlying \( i \). The underlying futures return controls for any exposure
of the straddles to the spot, though the Black–Scholes model predicts that effect to be small.

Much more important is the fact that straddles have a nonlinear exposure to the futures return.
\( (f_{i,t}/IV_{i,t-1})^2 \) captures that nonlinearity. Consistent with the construction and interpretation of
the \( rt \) portfolio, \( \beta_{i,n}^f \) will be interpreted as the exposure of the straddles to realized volatility, since
realized volatility is calculated based on squared returns of the underlying.\(^9\) Finally, the third
factor is the change in the at-the-money implied volatility for the specific market at the five-month
maturity.\(^10\)

We estimate a standard linear specification for the risk premia,

\[ E[r_{i,n,t}] = \gamma_i f_{i,t} + \gamma_i^2 \beta_{i,n}^f \frac{f_{i,t}}{IV_{i,t-1}} + \gamma_i^\Delta IV \beta_{i,n} \Delta IV_{i,t} + \alpha_{i,n}, \]

\[ E[f_{i,t}/IV_{i,t-1}] = \gamma_i^f Std(f_{i,t}/IV_{i,t-1}). \]

The \( \gamma \) coefficients represent the risk premia that are earned by investments that provide direct
exposure to the factors. That is, the \( \gamma \)'s are estimates of what the Sharpe ratios on the factors
would be if it were possible to invest in them directly (neither \( f_{i,t}^2 \) nor \( \Delta IV_{i,t} \) is an asset return

\(^8\)The analysis is similar to those of Jones (2006) and Constantinides, Jackwerth, and Savov (2013).
\(^9\)There are obviously numerous closely related specifications of that second term that could be substituted. We
obtain similar results when the second factor is the absolute value of the futures return instead of its square, for
example, or when it is measured as the sum of squared daily returns over the return period (recall that the straddle
returns cover two weeks, so the factor in that case is the two-week daily realized volatility). We focus on the squared
return because it can be interpreted as a second-order term in the pricing kernel and also because it allows a direct
link to the gamma of the straddles.
\(^10\)Since the IVs may be measured with error, we construct this factor by regressing available implied volatilities
on maturity for each underlying and date and then taking the fitted value from that regression at the five-month
maturity.
that one can directly purchase in our data: they are nontradable factors; the spot itself is tradable, which is why we impose the second equality. The difference between the method here and the \( rv \) and \( iv \) portfolios discussed above is that the factor model does not require assumptions about the risk exposures of the straddles, whereas the \( rv \) and \( iv \) portfolios rely on the Black–Scholes model. So the results using the factor models should be more robust, but also have more estimation error.

## 5.2.2 Results

The blue series in figure 4 plots the estimated risk premia across the various markets along with 95-percent confidence bands. The top panel plots \( \gamma_i^{\Delta IV} \), while the bottom panel plots \( \gamma_i^{f^2} \). Simple inspection shows that the results are nearly identical to those for the \( iv \) and \( rv \) portfolios. The \( \gamma_i^{\Delta IV} \) estimates are almost all positive, while the \( \gamma_i^{f^2} \) are almost all negative. Like before, we produce a random effects estimator of the mean of the risk premia in various groups. The random effects estimates of the means in the various groups are also similar, both in magnitude and statistical significance, to the main results in the red series. The main difference between the two series is that the confidence bands are wider for the factor model estimates, which is consistent with the fact that the factor model estimates impose less structure and must estimate the factor loadings of the individual straddles.

## 5.3 Liquidity

If the options used here are highly illiquid, the analysis will be substantially complicated for three reasons. First, to the extent that illiquidity represents a real cost faced by investors – e.g. a bid/ask spread – then returns calculated from settlement prices do not represent returns earned by investors. Second, illiquidity itself could carry a risk premium that the options might be exposed to. Third, bid/ask spreads represent an added layer of noise in prices. The identification of the premia for realized volatility and uncertainty depends on differences in returns on options across maturities, so what is most important for our purposes is how liquidity varies across maturities. This section shows that the liquidity of the straddles studied here is generally highly similar to that of the widely studied S&P 500 contracts traded on the CBOE, and the liquidity does not appear to substantially deteriorate across maturities.

We measure liquidity using two methods. First, since our data set does not include posted bid/ask spreads, we estimate the standard Roll (1984) effective spread using the daily returns, which is a monotone transformation of negative autocorrelation in returns.\(^{11}\) The top panel of appendix figure A.4 plots the effective bid/ask spreads for straddles at maturities of 1, 3, and 5 months for the 19 contracts that we study. The average posted bid/ask spreads for CBOE S&P 500 straddles, for which we have data since 1996, are also reported in the figure. At the one-month

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\(^{11}\)The Roll model assumes that there is an unobservable mid-quote that follows a random walk in logs and that observed prices have equal probability of being from a buy or sell order. Bid-ask bounce then induces negative autocorrelation in returns, from which the spread can be inferred (when the autocorrelation is positive, we set the spread to zero).
maturity, the effective spreads are approximately 6 percent on average, which is similar to the 6.6-percent average posted spreads for one-month CBOE S&P 500 straddles since 1996. More importantly, the spreads actually decline at longer maturities indicating that there is less observed negative autocovariance in returns for options at those maturities. For the three- and five-month options, the spreads are smaller by about half, averaging 2 to 3 percent. This is again consistent with posted spreads for CBOE S&P 500 contracts, which decline to 4.0 percent on average for 6-month options.

As a second measure of liquidity, we obtained posted bid/ask spreads for the options closest to the money on Friday, 8/4/2017 for our 19 contracts plus the CBOE S&P 500 options at maturities of 1, 4, and 7 months. Those spreads are plotted in the bottom panel of figure A.4. For the majority of the options, the spreads are less than 3 percent, consistent with the 4.1-percent bid/ask spread for one-month S&P 500 options at the CBOE. More importantly, though, across nearly all the contracts, the posted spreads again decline with maturity, consistent with the effective spreads. That said, for some of the contracts, there were no available bids or asks at the 4- and 7-month maturities on 8/4/2017. Note also, again similar to the effective spreads, for 10 of the 19 contracts, the one-month posted spreads are nearly indistinguishable from that for the S&P 500, which is typically viewed as a highly liquid market and where incorporating bid-ask spreads generally has minimal effects on return calculations (Bondarenko (2014)). For crude oil, which is studied in detail in the next section, the spreads at all three maturities are essentially identical to those for the S&P 500.

Figure A.4 yields two important results. First, it shows that the liquidity of the straddles is reasonably high, in the sense that effective and posted spreads are both relatively narrow in absolute terms for most of the contracts and that they compare favorably with spreads for the more widely studied S&P 500 options traded at the CBOE. Second, liquidity does not appear to deteriorate as the maturity of the options grows, and in fact in many cases there are improvements with increasing maturities, again consistent with CBOE data.

Finally, figure A.5 reports the average daily volume of all of the option contracts across maturities 1 to 6 months. For crude oil, which is the focus of the more in-depth study in the next section, the figure reports average daily volume in dollars; for all other contracts, it reports the the average daily volume relative to crude oil. Empirically, crude oil options have volume numbers of the same order of magnitude as the S&P 500, while there is more heterogeneity across the other markets. Looking across maturities, the general pattern is that dollar volume declines by about a factor of three in almost all the markets between the 1- and 6-month maturities – so the 6-month maturity has less volume, but far from zero.

6 Case study: crude oil

It is worthwhile to briefly delve more deeply into one market to build confidence in the robustness of the paper’s results. We choose the crude oil market for this exercise because it has one of the
longest time series available with the most maturities of any of the markets that we study, it is highly liquid (e.g. Gibson and Schwartz (1990) and Trolle and Schwartz (2010)), and it has a strong link to the macroeconomy.

Figures 5 and 6 contain several plots that help illustrate the historical behavior of the crude oil market. Panel A of figure 5 plots the history of total volume for one- and five-month options (specifically, average daily dollar volume of all contracts with maturities between 15 and 45 or 135 and 165 days to maturity, respectively). The volume of contracts in both maturity bins has risen over time, peaking in 2008, with a subsequent decline. On average, there is about 5 times more volume in the one- than the five-month option, though the volume in the five-month option has been trending upward, reaching as high as 75 percent of the volume for the one-month option.

Panel B of figure 5 plots 5-year rolling sample Sharpe ratios for the iv and rv portfolios. The left-hand section plots results for crude oil, while, for reference, the right-hand panel plots results for the S&P 500. For crude oil, the rv portfolio had negative average returns in almost all five-year periods in our data, while the iv portfolio had positive returns in almost all five-year periods. The rv returns trend down over time, implying that the variance risk premium may have been growing. The iv returns are somewhat more consistent, though the returns were close to zero or even negative for short periods at the beginning and end of the sample.

The right-hand side of panel B gives further context to those results by plotting the rv and iv returns for the S&P 500 options. For the S&P, the rv portfolio has relatively more negative returns than for crude, while the iv portfolio has average returns that are generally centered on zero, rather than staying consistently positive as we observe for crude oil.

Panel C of figure 5 is similar to panel B, except instead of plotting returns on the rv and iv portfolios, it plots their constituents, the returns on the one- and five-month straddles. For crude oil, the five-month straddle has consistently positive returns, unlike the S&P 500, for which the five-month straddle tends to have negative returns. In both cases the one-month straddle has negative returns, though that effect is stronger for the S&P 500.

Overall, panels B and C have two uses. First, they show that the returns that we observe on the iv and rv portfolios are not driven by a small number of outliers; rather, they are fairly consistent over time. Second, they provide further detail on the divergences between the behavior of straddle returns for the S&P 500 compared to crude oil.

Next, to help understand how crude oil volatility relates to macroeconomic uncertainty, the top panel of figure 6 plots one-month at-the-money implied volatility for crude oil along with the LMN financial and price uncertainty series. The correlation of oil price uncertainty with the two series is immediately apparent. The various spikes upward in crude oil volatility are all traceable to spikes in either price or financial uncertainty. This figure thus underscores the utility to an investor of buying five-month crude oil straddles: they provide good protection against increases in the LMN uncertainty indexes and at the same time earn positive average returns.

Because the crude oil market is so large, it has relatively more traded maturities than the other underlyings. At any given time, the CME currently has trading in the next 12 monthly expirations
and also December expirations for a number of years into the future. Panel B of figure of figure 6 plots average returns for crude oil straddles with maturities between 1 and 11 months (not 12 because of how we interpolate to construct the monthly portfolios); panel C reports Sharpe ratios. The figure shows that the behavior at longer maturities remains similar, and returns continue to rise slightly beyond the five months examined in the main analysis (though they eventually flatten). When we calculate the \( iv \) portfolio using the 11- instead of the five-month maturity, we also obtain similar results.

Because crude oil prices are such a widely followed indicator, there are also exchange traded funds (ETFs) that track oil prices, and those ETFs have options traded on them. Appendix A.6 examines the returns on those options and shows that the results are consistent with those we obtain for the CME options, though with more noise because the ETF options were introduced in the 2000’s.

7 Conclusion

This paper studies the pricing of uncertainty and realized volatility across a broad array of options on financial and commodity futures. Uncertainty is proxied by implied volatility – which theoretically measures investors’ conditional variances for future returns – and a number of uncertainty indexes developed in the literature. Realized volatility, on the other hand, measures how large realized shocks have been. In modeling terms, if \( \varepsilon \sim N(0, \sigma^2) \), uncertainty is \( \sigma^2 \), while volatility is the realization of \( \varepsilon^2 \).

A large literature in macroeconomics and finance has focused on the effects of uncertainty on the economy. In this paper we explore empirically what they imply for investors. If uncertainty shocks have major contractionary effects so that they are associated with high marginal utility for the average investor, then assets that hedge uncertainty should earn negative average returns. Empirically, we find that such assets – constructed as portfolios of options – historically have earned positive returns. While that result was known for the S&P 500, the key contribution of this paper is to construct hedging portfolios for a range of types of macro uncertainty, including interest rates, energy prices, and uncertainty indexes. The empirical results therefore imply that uncertainty shocks are not viewed as being negative by investors, or at least not sufficiently negative that it is costly to hedge them.

What is highly costly to hedge is instead realized volatility. Portfolios that hedge extreme returns in futures markets and large innovations in macroeconomic time series earn strongly negative returns, with premia that are in many cases 1-2 times as large as the premium on the aggregate stock market over the same period. So what is high in bad times is not uncertainty, but realized volatility. Periods in which futures markets and the macroeconomy are highly volatile and display large movements appear to be periods of high marginal utility, in the sense that their associated state prices are high. This is consistent with the findings in Berger, Dew-Becker, and Giglio (2018), who provide VAR evidence that shocks to volatility predict declines in real activity in the future.
while shocks to uncertainty do not.

References


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Figure 1: Sample implied volatilities

Note: Monthly implied volatilities calculated from three-month options using the Black–Scholes model.
Figure 2: Fit to uncertainty indexes

Note: The left-hand panels plot the fitted values from the regressions of the EPU and JLN indexes on one-month implied volatility in the 19 markets. The right-hand panels plot pairwise correlations between the individual implied volatility series and the fitted values from the regressions.
Figure 3: Fit to realized volatility indexes

Note: See figure 2. This figure uses the JLN realized volatility series instead of uncertainty.
Figure 4: RV and IV portfolio Sharpe ratios and factor risk premia

**Note:** Squares are point estimates and vertical lines represent 95-percent confidence intervals. The red series plot the Sharpe ratios for the RV and IV portfolios. The blue series plots the estimated risk premia from the factor model. The confidence bands for the RV and IV Sharpe ratios are calculated through a 50-day block bootstrap, while those for the factor model use GMM standard errors with the Hansen–Hodrick (1980) method used to calculate the long-run variance. The “Fin. mean”, “Non-fin. mean”, and “Overall mean” points represent random effects estimates of group-level and overall means. The “JLN” and “EPU” points are for the portfolios that hedge those indexes.
Figure 5: Case study: crude oil (I)

(a) Volume

(b) Five-year rolling Sharpe ratios, RV and IV

(c) Five-year rolling Sharpe ratios, 1mo and 5mo straddles

Note: The top panel reports the volume in number of contracts for the 1-month and the 5-month straddles (left), and the ratio of the 5-month to the 1-month volume (right) for crude oil. The middle panel reports rolling Sharpe ratios for the RV and IV portfolios, for crude oil (left) and for the S&P 500 (right). The bottom panel reports rolling Sharpe ratios for the 1-month and 5-month straddles, for crude oil (left) and for the S&P 500 (right).
Figure 6: Case study: crude oil (II)

(a) Crude IV and Macro Uncertainty

(b) Average returns

(c) Sharpe ratios

Note: The top panel reports the Ludvigson, Ma, and Ng (2015) financial uncertainty series and the macroeconomic price uncertainty series together with the implied volatility for crude oil. The middle and bottom panels plot average returns and Sharpe ratios along with block-bootstrapped 95-percent confidence intervals.
Table 1: Pairwise correlations of implied volatility across markets

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<th>S&amp;P 500</th>
<th>Swiss Franc</th>
<th>Swiss Franc</th>
<th>Yen</th>
<th>British Pound</th>
<th>Gold</th>
<th>Silver</th>
<th>Copper</th>
<th>Crude oil</th>
<th>Heating oil</th>
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**Note:** Pairwise correlations of three-month option-implied volatility across markets. The darkness of the shading represents the degree of correlation.

Table 2: Regressions of IV onto macroeconomic uncertainty measures

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<th>IV</th>
<th>S&amp;P 500</th>
<th>T-bonds</th>
<th>GBP</th>
<th>CHF</th>
<th>JPY</th>
<th>Copper</th>
<th>Corn</th>
<th>Crude oil</th>
<th>Feeder cattle</th>
<th>Gold</th>
<th>Heating oil</th>
<th>Lean hog</th>
<th>Natural gas</th>
<th>Silver</th>
<th>Soybeans</th>
<th>Soybean meal</th>
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<td>0.00</td>
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</table>

**Note:** Regression equations are reported above the three sections. The left panel reports, in each row, the coefficients from regressions of each implied volatility onto the financial and macroeconomic uncertainty measures developed in Jurado, Ludvigson, and Ng (2015) and Ludvigson, Ma, and Ng (2017). The last row regresses the average of all 19 IVs on the LMN uncertainty measures. All variables are normalized to have unit standard deviations prior to the regressions. The middle panel repeats the exercise dividing the macro uncertainty measure in one constructed only real quantities, and one constructed using prices. The right-hand panel uses just the EPU index. Cells with coefficients that are significant at the 5-percent level are shaded.
Table 3: Correlations between RV and IV in each market

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<th>Std(IV)</th>
<th>Corr(RV,IV)</th>
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Note: The table reports, for each underlying, the standard deviation of the monthly RV and 3-month IV series, and their correlation.

Table 4: Pairwise correlations of realized volatility across markets

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<td>Soybean meal</td>
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<td>0.04</td>
<td>0.10</td>
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<td>0.10</td>
<td>0.07</td>
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</tr>
<tr>
<td>Feeder cattle</td>
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<td>0.24</td>
<td>0.13</td>
<td>0.11</td>
<td>0.17</td>
<td>0.24</td>
<td>0.28</td>
<td>0.07</td>
<td>0.09</td>
<td>0.22</td>
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<tr>
<td>Live cattle</td>
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<td>0.13</td>
<td>0.11</td>
<td>0.17</td>
<td>0.24</td>
<td>0.28</td>
<td>0.07</td>
<td>0.09</td>
<td>0.22</td>
<td>0.22</td>
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</tbody>
</table>

Note: Pairwise correlations of monthly realized volatility across markets. The darkness of the shading represents the degree of correlation.
Table 5: Portfolios of \( rv \) and \( iv \) across markets

<table>
<thead>
<tr>
<th></th>
<th>RV</th>
<th>IV</th>
<th>RV+IV</th>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Equal weight</td>
<td>Risk-parity</td>
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<tr>
<td>All underlyings</td>
<td>-0.90 ***</td>
<td>0.76 ***</td>
<td>1.34 ***</td>
<td>1.26 ***</td>
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<td>Nonfinancials</td>
<td>-0.82 ***</td>
<td>0.75 ***</td>
<td>1.16 ***</td>
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</tr>
<tr>
<td>Financials</td>
<td>-0.51 ***</td>
<td>0.12</td>
<td>0.63 ***</td>
<td>0.36 *</td>
<td></td>
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</tbody>
</table>

Panel B: Skewness

<table>
<thead>
<tr>
<th></th>
<th>RV</th>
<th>IV</th>
<th>RV+IV</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equal weight</td>
<td>Risk-parity</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>All underlyings</td>
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<td>1.73 ***</td>
<td>-0.69 ***</td>
<td>0.94 ***</td>
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<tr>
<td>Nonfinancials</td>
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<td>1.49 ***</td>
<td>-0.92 ***</td>
<td>0.72 ***</td>
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<tr>
<td>Financials</td>
<td>1.83 ***</td>
<td>3.33 ***</td>
<td>-0.77</td>
<td>3.27 ***</td>
<td></td>
</tr>
</tbody>
</table>

Note: Sharpe ratios (panel A) and the skewness (panel B) of different portfolios combining straddles across markets. For each panel, the first row reports a portfolio constructed using straddles from all available markets on each date, the second row using only nonfinancial underlyings, the third row only financial underlyings. Each column corresponds to a different portfolio. The first column is an equal-weighted RV portfolio, the second is an equal-weighted IV portfolio, the third is an equal-weighted long-short IV minus RV portfolio, and the last is the same long short portfolio but weighted by the inverse of the variance (risk-parity). The stars corresponds to tests of statistical significance obtained via block bootstrap. ***=1% confidence level, **=5% level, *=10% level.
A.1 Data filters and transformations

We focus primarily on data on futures and option prices from the CME. The observed option prices very often appear to have nontrivial measurement errors. This section describes the various filters we use and then proceeds to provide more information about the specifics of the data transformations we apply. The full details of the implementation are too numerous to describe here. The code is the definitive record and is available on request.

First, we note that the price formats for futures and strike prices for many of the commodities change over time. That is, they will move between, say, 1/8ths, 1/16ths, and pennies. We make the prices into a consistent decimal time series for each commodity by inspecting the prices directly and then coding by hand the change dates.

We then remove all options with the following properties

1. Strikes greater than 5 times the spot
2. Options with open interest below the 5th percentile across all contracts in the sample
3. Price less then 5 ticks above zero
4. Maturity less than 9 days
5. Maturity greater than 7 years
6. Volume equal to zero or missing
7. Options with prices below their intrinsic value (the value if exercised immediately)

We then calculate implied volatilities using the Black–Scholes formula, treating the options as though they are European. We have also replicated the analysis using American implied volatilities and find nearly identical results (the reason is that in most cases we ultimately end up converting the IVs back into prices, meaning that any errors in the pricing formula are largely irrelevant – it is just a temporary data transformation, rather than actually representing a volatility calculation).

The data are then further filtered based on the IVs:

1. Eliminate all zero or negative IVs
2. All options with IV more than 50 percent (in proportional terms) different from the average for the same underlying, date, and maturity
3. We then filter outliers along all three dimensions, strike, date, and maturity, removing the following:
   
   (a) If the IV changes for a contract by 15 percent or more on a given day then moves by 15 percent or more in the opposite direction in a single day within the next week, and if
it moves by less than 3 percent on average over that window, for options with maturity
greater than 90 days (this eliminates temporary large changes in IVs that are reversed
that tend to be observed early in the life of the options).

(b) If the IV doubles or falls by half in either the first or last observation for a contract

(c) If, looking across maturities at a given strike on a given date, the IV changes by 20
percent or more and then reverses by that amount at the next maturity (i.e. spikes at
one maturity). This is restricted to maturities within 90 days of each other.

(d) If the last, second to last, or third to last IV is 40 percent different from the previous
maturity.

(e) If, looking across strikes at a given maturity on a given date, the IV changes by 20
percent and reverses at the next strike (for strikes within 10 percent of each other).

(f) If the change in IV at the first or last strike is greater than 20 percent, or the change at
the second or second to last option is greater than 30 percent.

At-the-money (ATM) IVs are constructed by averaging the IVs of the options with the first
strike below and above the forward price. The ATM IV is not calculated for any observation where
we do not have at least one observation (a put or a call) on either side of the forward price.

To calculate ATM straddle returns, we first construct returns for straddles with all observable
strikes. We calculate ATM straddle returns by averaging across the two closest strikes above and
below the current spot price as long as they are less than 0.5 ATM standard deviations from the
spot. Denote the returns on the four straddles in order of increasing strike as $R_1$ to $R_4$, with
associated strikes $S_1$ to $S_4$. The interpolated return is then

$$
\frac{1}{2} \left( \frac{R_2}{S_3 - S_2} + \frac{R_3}{S_3 - S_2} \right) + \frac{1}{2} \left( \frac{R_1}{S_4 - S_1} + \frac{R_1}{S_4 - S_1} \right)
$$

That is, we linearly interpolate pairwise through $R_2$ and $R_3$ and then $R_1$ and $R_4$ and average
across those two interpolations. The reason to use four straddles instead of two is to try to reduce
measurement error. The linear interpolation ensures that the portfolio has an average strike equal
to the forward price $F$. If there is only one straddle available on either side of the forward price,
we then interpolate using just a single pair of options, the nearest to the money on either side of
the forward price.

To calculate returns at standardized maturities, we again interpolate. If there are options
available with maturities on both sides of the target maturity and they both have maturities
differing from the target by less than 60 days, then we linearly interpolate. If options are not
available on both sides of the target, then we use a single option if it has a maturity within 35
days of the target. This does mean that the maturity of the option used for a portfolio at a desired
maturity can deviate from the target.
### A.2 Approximating straddle return sensitivities

This section describes the approximation of option returns used to obtain the \( rv \) and \( iv \) portfolios. \( P \) denotes the price of an at-the-money straddle. \( \sigma \) is the Black–Scholes volatility, \( n \) is the time to maturity, \( F \) is the forward price, and \( K \) is the strike. \( N \) denotes the standard Normal cumulative distribution function.

For a general straddle, we have

\[
P(F, \sigma) = e^{-rn} (K + F) (N(d_1) - N(-d_1)) \tag{A.2}
\]

where

\[
d_1(S, \sigma) = \frac{1}{\sqrt{n}} \left[ \log \frac{F}{K} + \frac{\sigma^2}{2} n \right] \tag{A.3}
\]

The first derivatives are

\[
e^{-rn} F = S \tag{A.4}
\]

\[
\frac{d}{dF} = e^{-rn} \frac{d}{dS} \tag{A.5}
\]

\[
P_F = e^{-rn} \left( N \left( \frac{\sigma \sqrt{n}}{2} \right) - N \left( -\frac{\sigma \sqrt{n}}{2} \right) \right) \approx 0 \tag{A.6}
\]

where the approximation holds for small \( \sigma \sqrt{n} \).

The second derivative of the straddle’s price is

\[
P_{FF}(F_t, \sigma_t) = 2e^{-rn} \frac{N'(\frac{\sigma \sqrt{n}}{2})}{F_t \sigma_t \sqrt{n}} \tag{A.7}
\]

The sensitivity to volatility is

\[
P_{\sigma} (F_t, \sigma_t) = 2e^{-rn} F_t N' \left( \frac{\sigma \sqrt{n}}{2} \right) \sqrt{n} \tag{A.8}
\]

The local approximation for returns that we use is

\[
\frac{\partial r_{t+1}}{\partial x_{t+1}} = \frac{\partial}{\partial x_{t+1}} \frac{P(F_{t+1}, \sigma_{t+1})}{P(F_t, \sigma_t)} \tag{A.9}
\]

and we evaluate the derivatives at the point \( F_{t+1} = F_t, \sigma_{t+1} = \sigma_t \).

We have

\[
\frac{\partial^2 r_{t+1}}{\partial F_{t+1}^2} = \frac{P_{FF}(F_t, \sigma_t)}{P(F_t, \sigma_t)} \tag{A.10}
\]

\[
= 2e^{-rn} \frac{N' \left( \frac{\sigma \sqrt{n}}{2} \right)}{F_t \sigma_t \sqrt{n}} \frac{1}{2e^{-rn} F_t \left[ N \left( \frac{\sigma \sqrt{n}}{2} \right) - N \left( -\frac{\sigma \sqrt{n}}{2} \right) \right]} \tag{A.11}
\]
We then use the approximations
\[ N\left(\frac{\sigma_t \sqrt{n}}{2}\right) - N\left(-\frac{\sigma_t \sqrt{n}}{2}\right) \approx 2N'(0) \frac{\sigma_t \sqrt{n}}{2} \] (A.12)
\[ N'\left(\frac{\sigma \sqrt{n}}{2}\right) \approx N'(0) \] (A.13)
(noting that \(N''(0) = 0\)), yielding
\[ \frac{\partial^2 r_{t+1}}{\partial F^2_{t+1}} \approx \frac{1}{F^2_t \sigma^2_t n} \] (A.14)

Since \(\frac{\partial F^2_{t+1}}{\partial F_t^2 \sigma^2_t} = \partial (f_{t+1}/\sigma_t)^2\), where \(f_t\) is the log futures return, we have
\[ \frac{\partial^2 r_{t+1}}{\partial (f_{t+1}/\sigma_t)^2} \approx \frac{1}{n} \] (A.15)

For the sensitivity to \(\sigma\), we have, following similar steps,
\[ \frac{N'(0) \sqrt{n}}{2N'(0) \frac{\sigma \sqrt{n}}{2}} = \frac{1}{\sigma_t} \] (A.16)
\[ \frac{\partial r_{t+1}}{\partial \sigma_{t+1}} = \frac{P_\sigma(F_t, \sigma_t)}{P(F_t, \sigma_t)} \approx \frac{1}{\sigma_t} \] (A.17)
\[ \frac{\partial r_{t+1}}{\partial (\Delta \sigma_{t+1}/\sigma_t)} \approx 1 \] (A.18)

### A.2.1 Accuracy

The approximation above obviously ignores higher order terms, and also has the approximation for the option price in the denominator. To study how effective the approximation is, we examine a simple simulation. We assume that options are priced according to the Black–Scholes model. We set the initial futures price to 1 and the initial volatility to 30 percent per year. We then examine instantaneous returns (i.e. through shifts in \(\sigma\) and \(S\)) on the IV and RV portfolios defined exactly as in the main text, allowing the spot return to vary between between \(+/-23.53\) percent, which corresponds to variation out to four two-week standard deviations. We allow volatility to move between 15 and 60 percent – falling by half or doubling.

The top two panels of figure A.1 plot contours of returns on the RV and IV portfolios defined in the main text, while the middle panels plot the contours predicted by the approximations for the partial derivatives, ignoring the \(\exp\left(-\sigma^2/2\right)\) terms. For the IV portfolio, except for very large instantaneous returns – 15–20 percent – the approximation lies very close to the truth. The bottom-right panel plots the error – the middle panel minus the top panel – and except for cases where
the spot has an extreme movement and the implied volatility falls – the exact opposite of typical behavior – the errors are all quantitatively small, especially compared to the overall return.

For the RV portfolio, the errors are somewhat larger. This is due to the fact that we approximate the RV portfolio using a quadratic function, but its payoff has a shape closer to a hyperbola. Again, for underlying futures returns within two standard deviations (where the two-week standard deviation here is 5.88 percent), the errors are relatively small quantitatively, especially when $\sigma$ does not move far. Towards the corners of the figure, though, the errors grow somewhat large.

These results therefore underscore the discussion in the text. The approximations used to construct the IV and RV portfolios are qualitatively accurate, and except in more extreme cases also hold reasonably well quantitatively. But they are obviously not fully robust to all events, so the factor model estimation, which does not rely on any approximations, should be used in situations where the nonlinearities are a concern.

### A.2.2 Empirical return exposures

To check empirically the accuracy of the expressions for the risk exposures of the straddles, figure A.2 plots estimated factor loadings for straddles at maturities from one to five months for each market from time series regressions of the form

$$r_{i,n,t} = a_{i,n} + \beta_{i,n}^f \frac{f_{i,t}}{IV_{i,t-1}} + \beta_{i,n}^{f^2} \left( \frac{f_{i,t}}{IV_{i,t-1}} \right)^2 + \beta_{i,n}^{\Delta IV} \frac{\Delta IV_{i,t}}{IV_{i,t-1}} + \varepsilon_{i,n,t}$$

(A.20)

The prediction of the analysis above is that $\beta_{i,n}^f = 0$, $\beta_{i,n}^{f^2} = 1/n$, and $\beta_{i,n}^{\Delta IV} = 1$.

Across the panels, the predictions hold surprisingly accurately. The loadings on the spot return are all near zero, if also generally slightly positive. The loadings on the change in implied volatility are all close to 1, with little systematic variation across maturities. And the loadings on the squared spot return tend to begin near 1 (though sometimes biased down somewhat) and then decline monotonically, consistent with the predicted $n^{-1}$ scaling.

Table A.2 reports results of similar regressions for each underlying of the returns on the RV and IV portfolios on the underlying futures return, the squared futures return, and the change in implied volatility. The table shows that while the Black–Scholes predictions do not hold perfectly, it is true that the RV portfolio is much more strongly exposed to realized than implied volatility, and the opposite holds for the IV portfolio. The coefficients on $(f_t/\sigma_{t-1})^2$ average 0.76 for the RV portfolio and 0.10 for the IV portfolio (though that average masks some variation across markets). Conversely, the coefficients on $\Delta \sigma_t/\sigma_{t-1}$ average 0.03 for the RV portfolio and 0.79 for the IV portfolio. Furthermore, the $R^2$s are large, averaging 72 percent across the various portfolios, implying that their returns are well described by the approximation (4).
A.3 Random effects models

Denote the vector of true Sharpe ratios for the straddles in market $i$ as $sr_i$. Our goal is to estimate the distribution of $sr_i$ across the various underlyings. A natural benchmark distribution for the means is the normal distribution,

$$sr_i \sim N(\mu_{sr}, \Sigma_{sr}) \quad (A.21)$$

This section estimates the parameters $\mu_{sr}$ and $\Sigma_{sr}$. $\mu_{sr}$ represents the high-level mean of Sharpe ratios across all the markets, and $\Sigma_{sr}$ describes how the market-specific means vary. The estimates of the market-specific Sharpe ratios differ noticeably across markets, but much of that is variation is likely driven by sampling error. $\Sigma_{sr}$ is an estimate of how much the true Sharpe ratios vary, as opposed to the sample estimates.

Denote the sample estimate of the Sharpe ratio in each market as $\hat{sr}_i$, and the stacked vector of sample Sharpe ratios as $\hat{sr} \equiv [\hat{sr}_1', \hat{sr}_2', ...]'$. Similarly, denote the vector of true Sharpe ratios as $sr \equiv [sr_1', sr_2', ...]'$. Under the central limit theorem,

$$\hat{sr} \Rightarrow N(sr, \Sigma_{\hat{sr}}) \quad (A.22)$$

where $\Rightarrow$ denotes convergence in distribution and the covariance matrix $\Sigma_{\hat{sr}}$ depends on the covariance between all the returns, across both maturities and underlyings, along with the lengths of the various samples.\(^1\) Appendix A.4 describes how we construct $\Sigma_{\hat{sr}}$.

The combination of (A.21) and (A.22) represents a fully specified distribution for the data as a function of $\mu_{sr}$ and $\Sigma_{sr}$. It is then straightforward to construct point estimates and confidence intervals for $\mu_{sr}$ and $\Sigma_{sr}$ with standard methods.

To allow for the possibility that average returns differ between the financial and nonfinancial underlyings, the mean in the likelihood can be replaced by $\mu_{sr} + \mu_D I_F$, where $\mu_D$ is the difference in Sharpe ratios and $I_F$ is a 0/1 indicator for whether the associated underlying is financial. We calculate the sampling distribution for the estimated parameters through Bayesian methods, treating the parameters as though they are drawn from a uniform prior. The point estimates are therefore identical to MLE, and the confidence bands represent samples from the likelihood.\(^2\)

\(^1\)More formally, we would say that $\hat{sr}$ properly scaled by the square root of the sample size converges to a normal distribution. The expression (A.22) implicitly puts the sample size in $\Sigma_{\hat{sr}}$. The derivation of this result is a straightforward application of the continuous mapping theorem, nearly identical to the proof that a sample t-statistic is asymptotically Normally distributed.

\(^2\)We use Bayesian methods to calculate the sampling intervals because likelihood-based methods require inverting large second derivative matrices, which can be numerically unstable. The estimation in this section is performed using the Bayesian computation engine Stan, which provides functions that both maximize the likelihood and rapidly sample from the posterior distribution. Code is available on request.
A.4 Calculating the covariance of the sample mean returns

There are two features of our data that make calculating covariance matrix of sample means difficult: we have an unbalanced panel and the covariance matrix is either singular or nearly so. We deal with those issues through the following steps.

1. For each market, we estimate the two largest principal components, therefore modeling straddle returns for underlying $i$ and maturity $n$ on date $t$ as

$$r_{i,n,t} = \lambda_{1,i,n} f_{1,i,t} + \lambda_{2,i,n} f_{2,i,t} + \theta_{i,n,t}$$  \hspace{1cm} (A.23)

where the $\lambda$ are factor loadings, the $f$ are estimated factors, and $\theta$ is a residual that we take to be uncorrelated across maturities and markets (it is also in general extremely small).

2. We calculate the long-run covariance matrix of all $J \times 2$ estimated factors. The covariance matrix is calculated using the Hansen–Hodrick method to account for the fact that the returns are overlapping (we use daily observations of 2-week returns). The elements of the covariance matrix are estimated based on the available nonmissing data for the associated pair of factors. That means that the covariance matrix need not be positive semidefinite. To account for that fact, we set all negative eigenvalues of the estimated covariance matrix to zero.

Given the estimated long-run covariance matrix of the factors, denoted $\Sigma_f$, and given the (diagonal) long-run variance matrix of the residuals $\theta$, denoted $\Sigma_\theta$, the long-run covariance matrix of the returns is then

$$\Sigma_r \equiv \Lambda \Sigma_f \Lambda' + \Sigma_\theta$$  \hspace{1cm} (A.24)

where $\Lambda$ is a matrix containing the factor loadings $\lambda$.

3. Finally, it is straightforward to show that the covariance matrix of the sample mean returns is

$$\Sigma_{\bar{r}} = M \odot \Sigma_r$$  \hspace{1cm} (A.25)

where $\odot$ denotes the elementwise product and $M$ is a matrix where the element for a given return pair is equal to the ratio of the number of observations in which both returns are available to the product of the number of observations in which each return is available individually (if all returns had the same number of observations $T$, then we would obtain the usual $T^{-1}$ scaling). We then have the asymptotic approximation that

$$\hat{r} \Rightarrow N (\bar{r}, \Sigma_{\bar{r}})$$  \hspace{1cm} (A.26)

where $\hat{r}$ is a vector that stacks the $\hat{r}_i$ and $\bar{r}$ stacks the $\bar{r}_i$ and $\Rightarrow$ denotes convergence in distribution.
A.5 Calculating risk prices with unbalanced panels and correlations across markets

In estimating the factor models, we have two complications to deal with: the sample length for each underlying is different, and returns are correlated across underlyings. This section discusses how we deal with those issues.

We have the model

$$E_{T_i}[R_i] = \lambda_i \beta_i + \alpha_i$$  \hspace{1cm} (A.27)

where $E_{T_i}$ denotes the sample mean in the set of dates for which we have data for underlying $i$, $R_i$ is the vector of returns of the straddles, $\lambda_i$ is a vector of risk prices, $\beta_i$ is a vector of risk prices, and $\alpha_i$ is a vector of pricing errors. Note that these objects are all population values, rather than estimates. In order to calculate the sampling distribution for the estimated counterparts, we need to know the covariance of the pricing errors. Note that there is also a population cross-sectional regression with

$$E_{T_i}[R_i] = a_i + \beta_i E_{T_i}[f_i] + E_{T_i}[\varepsilon_i]$$  \hspace{1cm} (A.28)

where $\varepsilon_i$ is a vector of residuals and $f_i$ is a vector of pricing factors. That formula can be used to substitute out returns and obtain

$$\alpha_i = a_i + \beta_i E_{T_i}[f_i] + E_{T_i}[\varepsilon_i] - \lambda_i \beta_i$$  \hspace{1cm} (A.29)

Since $a_i$, $\lambda_i$, and $\beta_i$ are fixed in the true model, the distribution of $\alpha_i$ depends only on the distributions of the sample means $E_{T_i}[f_i]$ and $E_{T_i}[\varepsilon_i]$. Denoting the long-run (i.e. Hansen–Hodrick) covariance matrix of $f_i$ as $\Sigma_f$ and that of $\varepsilon_i$ as $\Sigma_{\varepsilon_i}$, we have

$$\text{var}(\alpha_i) = \beta_i T_i^{-1} \Sigma_f \beta_i' + T_i^{-1} \Sigma_{\varepsilon_i}$$  \hspace{1cm} (A.30)

Since the $\lambda_i$ are estimated from a regression, if we denote their estimates as $\hat{\lambda}_i$, we obtain the usual formula for the variance of $\hat{\lambda}_i - \lambda_i$

$$\text{var}\left(\hat{\lambda}_i - \lambda_i\right) = (\beta_i \beta_i')^{-1} \text{var}(\alpha_i) \beta_i (\beta_i \beta_i')^{-1}$$  \hspace{1cm} (A.31)

$$= \Sigma_f + (\beta_i \beta_i')^{-1} \Sigma_{\varepsilon_i} \beta_i (\beta_i \beta_i')^{-1}$$  \hspace{1cm} (A.32)

Beyond the variance of $\hat{\lambda}_i$, we also need to know the covariance of any pair of estimates, $\hat{\lambda}_i$ and $\hat{\lambda}_j$. Using standard OLS formulas, we have
\[
\begin{bmatrix}
\hat{\lambda}_i - \lambda_i \\
\hat{\lambda}_j - \lambda_j
\end{bmatrix} = \begin{bmatrix}
(\beta'_i \beta_i)^{-1} \beta'_i \alpha_i \\
(\beta'_j \beta_j)^{-1} \beta'_j \alpha_j
\end{bmatrix} = \begin{bmatrix}
(\beta'_i \beta_i)^{-1} \beta'_i (\beta_i E_{T_1} [f_i] + E_{T_1} [\varepsilon_{j,i}]) \\
(\beta'_j \beta_j)^{-1} \beta'_j (\beta_2 E_{T_2} [f_j] + E_{T_2} [\varepsilon_{j,i}])
\end{bmatrix}
\]  
(A.33)

(A.34)

The covariance between \( \hat{\lambda}_i \) and \( \hat{\lambda}_j \) is then

\[
\frac{T_{12}}{T_1 T_2} \left( \Sigma_{f,i,j} + (\beta'_1 \beta_1)^{-1} \beta'_1 \Sigma_{\varepsilon,i,j} \beta_2 (\beta'_2 \beta_2)^{-1} \right)
\]

where \( \Sigma_{f,i,j} \) and \( \Sigma_{\varepsilon,i,j} \) are now long-run covariance matrices (again from the Hansen–Hodrick method). Using these formulas, we then have estimates of risk prices in each market individually along with a full covariance matrix of all the estimates.

### A.6 Robustness: ETF options

This section provides an alternative check on the results for crude oil options by examining returns on straddles for options on two exchange traded funds. The first is the United States Oil Fund (USO), which invests in short-term oil futures. USO has existed since 2006, and Optionmetrics reports quotes for options beginning in May, 2007. The second fund is the Energy Select Sector SPDR fund (XLE), which tracks the energy sector of the S&P 500. XLE has existed since 1998 and Optionmetrics reports data since December, 1998.

We eliminate observations using the following filters:

1. Volume less than 10 contracts
2. Time to maturity less than 15 days
3. Bid-ask spread greater than 20 percent of bid/ask midpoint
4. Initial log moneyness – log strike divided by spot – greater than 0.75 implied volatility units in absolute value (where implied volatility is scaled by the square root of time to maturity).

We then calculate straddle returns as in the main text over two-week periods and average across the two straddles nearest to the money for each maturity, weighting them by the inverse of their absolute moneyness.

The top section of table A.6.1 reports the number of (potentially overlapping) two-week straddle return observations across maturities for USO, XLE, and the CME Group futures options used in the main analysis. Since the CME data goes back to 1983, there are far more observations for that series than the other two. More interestingly, though, the number of observations only declines by about 10 percent between the 1- and 6-month maturities, while it falls by more than 2/3 for the XLE and USO samples. The CME data therefore has superior coverage at longer horizons, which justifies its use in our main analysis.
The bottom section of table A.6.1 reports the correlations of the USO and XLE straddle returns with those for the CME on the days where they overlap. The correlations are approximately 90 percent at all maturities for USO and 50 percent for XLE. The 90-percent correlations for USO and the CME sample provide a general confirmation of the accuracy of the CME straddle returns, since we would expect the USO and CME options to be highly similar as USO literally holds futures. The lower correlation for XLE is not surprising given that it holds energy sector stocks rather than crude oil futures.

Table A.6.1.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td># obs.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USO</td>
<td>1640</td>
<td>1616</td>
<td>1721</td>
<td>1679</td>
<td>1118</td>
<td>525</td>
</tr>
<tr>
<td>XLE</td>
<td>2612</td>
<td>2545</td>
<td>2454</td>
<td>1928</td>
<td>1134</td>
<td>369</td>
</tr>
<tr>
<td>CME</td>
<td>6762</td>
<td>6645</td>
<td>6817</td>
<td>6801</td>
<td>6606</td>
<td>5998</td>
</tr>
<tr>
<td>Corr. w/</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USO</td>
<td>0.93</td>
<td>0.96</td>
<td>0.95</td>
<td>0.92</td>
<td>0.89</td>
<td>0.83</td>
</tr>
<tr>
<td>CME</td>
<td>0.43</td>
<td>0.48</td>
<td>0.50</td>
<td>0.49</td>
<td>0.50</td>
<td>0.53</td>
</tr>
<tr>
<td>XLE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the main text, the RV and IV portfolio returns are calculated using 5- and 1-month straddles. Since the number of observations drops off substantially between 4 and 5 months for both XLE and USO, though, here we examine returns on RV and IV portfolios using both 5- and 4-month straddles for the long-maturity side.

Figure A.6 plots estimated annualized Sharpe ratios along with 95-percent confidence bands for the RV and IV portfolios using 4- and 5-month straddles for the three sets of options. In all four cases, the three confidence intervals always overlap substantially. The fact that the sample for the CME options is far larger is evident in its confidence bands being much narrower than those for the other two sources. For the IV portfolios, USO has returns that are close to zero, but its confidence bands range from -1 to greater than 0.5, indicating that it is not particularly informative about the Sharpe ratio.

Table A.6.2 reports confidence bands for the difference between the IV and RV average returns constructed with the CME data and the same portfolios constructed using USO and XLE. The top panel shows that the differences for the IV portfolios are negative for USO and positive for XLE, but only the difference for USO constructed with the 4-month straddle is statistically significant. The bottom panel similarly shows mixed results for the point estimates for the differences for the RV portfolios, with none of the differences being statistically significant.

Table A.6.2. Differences between CME and USO, XLE mean returns

<table>
<thead>
<tr>
<th></th>
<th>USO minus CME, 4mo.</th>
<th>USO minus CME, 5mo.</th>
<th>XLE minus CME, 4mo.</th>
<th>XLE minus CME, 5mo.</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV return</td>
<td>-2.2</td>
<td>-2.2</td>
<td>-0.8</td>
<td>-1.4</td>
</tr>
<tr>
<td></td>
<td>[-3.9,-0.2]</td>
<td>[-4.8,0.4]</td>
<td>[-2.5,4.1]</td>
<td>[-4.1,6.3]</td>
</tr>
<tr>
<td>RV return</td>
<td>0.43</td>
<td>0.47</td>
<td>-0.27</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>[-0.6,1.4]</td>
<td>[-0.6,1.4]</td>
<td>[-1.8,1.3]</td>
<td>[-1.5,2.6]</td>
</tr>
</tbody>
</table>

Notes: the table reports percentage (two-week) returns on USO and XLE minus returns on CME RV and IV portfolios. 95-percent confidence intervals are reported in brackets.

A.10
The fact that the USO and CME straddle returns are highly correlated does not necessarily mean that the CME data is accurate for the mean return on the straddles. To check whether the difference in the means observed in the USO and XLE data would affect our main results, we ask how the Sharpe ratios of the RV and IV portfolios in the CME data would change if we shifted their means by the average differences reported in table A.6.2. The bars labeled “CME, USO adj.” and “CME, XLE adj.” show how the confidence bands would change if we shifted them by exactly the point estimates from table A.6.2. Note that this is not the same as shifting the Sharpe ratio for the CME data to match that for the XLE or USO data. The reason is that the difference in table A.6.2 is calculated only for the returns on matching dates, whereas the Sharpe ratio calculated in figure A.6 is calculated using the full sample for the CME data. So the two adjusted bands take the full-sample band and then shift it by the mean difference calculated on the dates that overlap between the CME data and XLE or USO.

Figure A.6 shows that the economic conclusions drawn for the crude oil straddles are not changed if the mean returns are shifted by the differences observed in table A.6.1. The RV portfolio returns remain statistically significantly negative in all four cases, the changes in the point estimates are well inside the original confidence intervals. The top panel shows that the IV returns using 5-month straddles are similarly unaffected. For the 4-month straddles, the only difference is that with the USO options, the estimated Sharpe ratio falls by about half and is no longer statistically significantly greater than zero. So, again, out of eight cases – IV and RV with 4- and 5-month straddles – in only one is there a nontrivial change in the conclusions, and even there the Sharpe ratio on the IV portfolio does not become negative, it is simply less positive.

Overall, the period in which the USO and XLE options are traded is too short to use them for our main analysis. This section shows that the USO straddle returns are highly correlated with the CME returns. The mean returns on the XLE and CME straddles are highly similar, while they differ somewhat more for CME and USO. However, shifting the means used for the CME options in the main analysis by the observed difference between the CME and USO options does not substantially change any of the conclusions.
Figure A.1: RV and IV portfolio approximation errors

Note: The initial spot price is 1 and the initial volatility, $\sigma$, is 0.3. The top panels calculate the return on the RV and IV portfolios given an instantaneous shift in the spot price and volatility to the values reported on the axes under the assumption that the Black-Scholes formula holds. The middle panels plot returns under the approximations used in the text, including ignoring the exponential terms. The bottom panels are equal to the middle minus the top panels. All returns and errors are reported as decimals.
Figure A.2: Factor loadings

Note: Loadings of two-week straddle returns on the three risk factors.
Figure A.3: RV and IV portfolio Sharpe ratios and factor risk premia, one-week holding period

Note: Same as figure 4, but using one-week holding periods.
Figure A.4: Bid-ask spreads

Note: The top panel plots for each market the effective half-spread computed from observed option returns, calculated as in Roll (1984). The spread reported for the CBOE S&P 500 options is based on the historical mean available from Optionmetrics covering the period 1996–2015. The bottom panel reports posted bid-ask spreads for at-the-money straddles obtained from Bloomberg on of August 4, 2017 (the CBOE S&P 500 spreads on that date are also obtained from Optionmetrics).
Figure A.5: Volume across markets and maturities

Note: Average daily volume of options in different markets. The panel corresponding to crude oil reports values in dollars. All other panels show values relative to the volume in the crude oil market, matched by maturity.
Note: Sharpe ratios on RV and IV portfolios using straddles for CME crude oil futures and the XLE and USO exchange traded funds. “4-month” and “5-month” refers to the longer of the two maturities used to construct each portfolio (the short maturity is always one month). The squares are point estimates based on the full sample available for each series. The lins are 95-percent confidence bands constructed with a 50-day block bootstrap. "CME, USO adj." and "CME, XLE adj." are identical to the "CME" numbers but with the mean return in the denominator of the Sharpe ratio shifted by the point estimate for the mean difference from table A2.
Table A.1: Regressions of RV onto macroeconomic uncertainty measures

\[
RV_{i,t} = a_{i,t} + b_{i,t}LMNRV_{t}^{\text{Financial}} + b_{i,t}LMNRV_{t}^{\text{Macro}} + \varepsilon_{i,t}
\]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>0.61</td>
<td>0.19</td>
<td>0.51</td>
<td>0.63</td>
<td>0.10</td>
<td>0.08</td>
<td>0.50</td>
</tr>
<tr>
<td>T-bonds</td>
<td>0.34</td>
<td>0.32</td>
<td>0.31</td>
<td>0.39</td>
<td>0.11</td>
<td>0.16</td>
<td>0.27</td>
</tr>
<tr>
<td>GBP</td>
<td>0.04</td>
<td>0.37</td>
<td>0.16</td>
<td>0.09</td>
<td>0.19</td>
<td>0.16</td>
<td>0.13</td>
</tr>
<tr>
<td>CHF</td>
<td>0.04</td>
<td>0.14</td>
<td>0.03</td>
<td>0.06</td>
<td>0.07</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>JPY</td>
<td>0.15</td>
<td>0.13</td>
<td>0.06</td>
<td>0.18</td>
<td>0.05</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>Copper</td>
<td>0.13</td>
<td>0.40</td>
<td>0.25</td>
<td>0.18</td>
<td>0.08</td>
<td>0.34</td>
<td>0.25</td>
</tr>
<tr>
<td>Corn</td>
<td>0.10</td>
<td>0.28</td>
<td>0.11</td>
<td>0.13</td>
<td>0.09</td>
<td>0.20</td>
<td>0.11</td>
</tr>
<tr>
<td>Crude oil</td>
<td>0.07</td>
<td>0.25</td>
<td>0.09</td>
<td>0.07</td>
<td>0.15</td>
<td>0.17</td>
<td>0.10</td>
</tr>
<tr>
<td>Feeder cattle</td>
<td>0.04</td>
<td>0.14</td>
<td>0.03</td>
<td>0.07</td>
<td>0.05</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>Gold</td>
<td>0.18</td>
<td>0.33</td>
<td>0.21</td>
<td>0.21</td>
<td>0.16</td>
<td>0.19</td>
<td>0.20</td>
</tr>
<tr>
<td>Heating oil</td>
<td>0.15</td>
<td>0.24</td>
<td>0.11</td>
<td>0.15</td>
<td>0.15</td>
<td>0.16</td>
<td>0.13</td>
</tr>
<tr>
<td>Lean hog</td>
<td>0.20</td>
<td>-0.03</td>
<td>0.04</td>
<td>0.20</td>
<td>-0.11</td>
<td>0.08</td>
<td>0.04</td>
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<tr>
<td>Live cattle</td>
<td>0.07</td>
<td>0.15</td>
<td>0.04</td>
<td>0.10</td>
<td>0.01</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>Natural gas</td>
<td>0.02</td>
<td>0.15</td>
<td>0.03</td>
<td>0.00</td>
<td>0.16</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td>Silver</td>
<td>0.08</td>
<td>0.28</td>
<td>0.11</td>
<td>0.13</td>
<td>0.10</td>
<td>0.14</td>
<td>0.08</td>
</tr>
<tr>
<td>Soybeans</td>
<td>0.08</td>
<td>0.23</td>
<td>0.07</td>
<td>0.10</td>
<td>0.04</td>
<td>0.19</td>
<td>0.07</td>
</tr>
<tr>
<td>Soybean meal</td>
<td>0.07</td>
<td>0.20</td>
<td>0.06</td>
<td>0.09</td>
<td>0.02</td>
<td>0.16</td>
<td>0.05</td>
</tr>
<tr>
<td>Soybean oil</td>
<td>0.08</td>
<td>0.35</td>
<td>0.16</td>
<td>0.12</td>
<td>0.10</td>
<td>0.23</td>
<td>0.13</td>
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<tr>
<td>Wheat</td>
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<td>0.09</td>
<td>0.03</td>
<td>0.17</td>
<td>0.06</td>
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<tr>
<td>Average of the RVs</td>
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<td>0.42</td>
<td>0.35</td>
<td>0.30</td>
<td>0.15</td>
<td>0.27</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Note: See table 2. Implied is replaced with realized volatility on the left-hand side, and the LMN uncertainty indexes are replaced with the LMN realized volatility indexes. There is no EPU analog to realized volatility.

Table A.2: Risk exposures of rv and iv portfolios

<table>
<thead>
<tr>
<th>RV portfolio</th>
<th>Spot</th>
<th>Spot^2</th>
<th>ΔIV</th>
<th>R^2</th>
<th>IV portfolio</th>
<th>Spot</th>
<th>Spot^2</th>
<th>ΔIV</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>-0.06</td>
<td>1.31</td>
<td>0.05</td>
<td>0.71</td>
<td>S&amp;P 500</td>
<td>0.04</td>
<td>0.24</td>
<td>0.90</td>
<td>0.71</td>
</tr>
<tr>
<td>T-bonds</td>
<td>-0.01</td>
<td>0.80</td>
<td>-0.02</td>
<td>0.81</td>
<td>T-bonds</td>
<td>-0.11</td>
<td>1.17</td>
<td>0.80</td>
<td>0.62</td>
</tr>
<tr>
<td>GBP</td>
<td>-0.02</td>
<td>0.73</td>
<td>0.06</td>
<td>0.84</td>
<td>GBP</td>
<td>0.07</td>
<td>0.39</td>
<td>0.70</td>
<td>0.78</td>
</tr>
<tr>
<td>CHF</td>
<td>-0.02</td>
<td>0.68</td>
<td>0.07</td>
<td>0.83</td>
<td>CHF</td>
<td>0.07</td>
<td>0.48</td>
<td>0.70</td>
<td>0.85</td>
</tr>
<tr>
<td>JPY</td>
<td>-0.02</td>
<td>0.70</td>
<td>0.06</td>
<td>0.81</td>
<td>JPY</td>
<td>0.02</td>
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<td>0.74</td>
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<tr>
<td>Copper</td>
<td>-0.01</td>
<td>0.75</td>
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<td>0.62</td>
<td>Copper</td>
<td>0.16</td>
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<tr>
<td>Corn</td>
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<td>0.04</td>
<td>0.73</td>
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<td>0.14</td>
<td>0.16</td>
<td>0.73</td>
<td>0.80</td>
</tr>
<tr>
<td>Crude oil</td>
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<td>0.97</td>
<td>0.01</td>
<td>0.75</td>
<td>Crude oil</td>
<td>0.10</td>
<td>0.08</td>
<td>0.93</td>
<td>0.70</td>
</tr>
<tr>
<td>Feeder cattle</td>
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<td>0.00</td>
<td>0.74</td>
<td>Feeder cattle</td>
<td>0.15</td>
<td>-0.25</td>
<td>0.78</td>
<td>0.64</td>
</tr>
<tr>
<td>Gold</td>
<td>-0.01</td>
<td>0.66</td>
<td>0.05</td>
<td>0.71</td>
<td>Gold</td>
<td>0.16</td>
<td>-0.30</td>
<td>0.80</td>
<td>0.58</td>
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<tr>
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<td>-0.01</td>
<td>0.84</td>
<td>0.01</td>
<td>0.78</td>
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<td>0.17</td>
<td>-0.52</td>
<td>0.78</td>
<td>0.52</td>
</tr>
<tr>
<td>Lean hog</td>
<td>0.00</td>
<td>0.81</td>
<td>0.00</td>
<td>0.76</td>
<td>Lean hog</td>
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</tr>
<tr>
<td>Live cattle</td>
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<td>0.74</td>
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<td>0.81</td>
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<td>0.16</td>
<td>0.79</td>
<td>0.70</td>
</tr>
<tr>
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<td>-0.01</td>
<td>0.59</td>
<td>0.06</td>
<td>0.77</td>
<td>Silver</td>
<td>0.18</td>
<td>0.27</td>
<td>0.79</td>
<td>0.68</td>
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<tr>
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<td>0.63</td>
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<tr>
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<td>0.00</td>
<td>0.76</td>
<td>Soybean oil</td>
<td>0.02</td>
<td>-0.31</td>
<td>0.85</td>
<td>0.68</td>
</tr>
<tr>
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<td>0.61</td>
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<td>Wheat</td>
<td>0.03</td>
<td>-0.55</td>
<td>0.79</td>
<td>0.62</td>
</tr>
<tr>
<td>Average</td>
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<td>0.76</td>
<td>0.03</td>
<td>0.76</td>
<td>Average</td>
<td>0.10</td>
<td>0.10</td>
<td>0.79</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Note: The table reports regression coefficients of the rv portfolios (left panel) and iv portfolios (right panel) for each market onto three market-specific factors: the spot return, the squared spot return, and the change in IV.