Pledgeability, Industry Liquidity, and Financing Cycles

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Abstract

Why are downturns following prolonged episodes of high valuations of firms so severe and long? Why do firms promise high external payments when they anticipate high valuations, and underperform subsequently? In this paper, we propose a theory of financing cycles where the control rights to enforce claims in an asset price boom (rights to sell assets) differ from the control rights used in more normal times (rights over cash flows that we term “pledgeability”). Firm management’s limited incentive to enhance pledgeability in an asset price boom can have long-drawn adverse effects in a downturn, which may not be resolved by renegotiation. This can also explain why involuntary asset turnover and asset misallocation to outsiders are high in a downturn, as well as why industry productivity falls. The paper highlights an adverse consequence of high anticipated liquidity, working through leverage, on the economy’s access to finance and productivity when that liquidity fails to materialize.

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Why do downturns following prolonged episodes of high firm valuations prove to be detrimental to growth and result in more protracted recessions (see Krishnamurthy and Muir (2015) and López-Salido, Stein and Zakrajšek (2015))? One traditional rationale is based on the idea of “debt overhang” – the debt built up during the boom serves to restrict investment and borrowing during the bust. However, if everyone, including the debt holders, knows that debt is holding back investment, they have an incentive to write down the debt in return for a stake in the firm’s growth. For debt overhang to be a serious concern, the firm and debt holders must be unable to undertake value enhancing contractual bargains. Another view is that borrowers cannot be trusted to take only value enhancing investments, even in a downturn. So debt overhang is needed to constrain the borrower’s investment – overhang is a second best solution to a fundamental moral hazard problem (see Hart and Moore (1995)). The immediate question raised by such an analysis is why we want to constrain borrowers more in bad times when the constraints imposed by debt are already high. Moreover, why would the moral hazard problem be so much more serious in a downturn which follows high valuations?

In this paper, we provide an explanation of the causes and consequences of financial liability overhang (including debt overhang) and explain why it is more acute following periods of high valuations and rational optimism about the future values of firms. In doing so, we differentiate between the control rights that are due to high resale prices for assets, which enable external claims to be enforced in a boom, and control rights based on pledging of cash flows, which facilitate the enforcement of external claims at other times, including downturns. The transition between these regimes, in which different types of control rights are operational, causes the external claim build up during the boom to have long-drawn adverse effects in the downturn.

Let us be more specific. Consider an industry that requires special managerial knowledge. Within the industry, there are firms run by incumbents. There are also industry insiders (those who know the industry well enough to be able to run firms as efficiently as the incumbents). Industry outsiders (financiers who don’t really know how to run industry firms but have general managerial/financial skills) are the other agents in the model. We first examine the effects of financing firms with fully state-contingent financial contracts, and then we turn to standard debt with a constant payment in a given period.

Financiers have two sorts of control rights; first, control through the right to repossess and sell the underlying asset being financed if payments are missed and, second, control over cash flows generated by the asset. The first right only requires the frictionless enforcement of property rights in the economy, which we assume. It has especial value when there are a large number of capable
potential buyers willing to pay the full price for the firm’s assets. Greater wealth amongst industry insiders (which we term industry liquidity) increases the availability of this asset-sale-based financing. Because we analyze a single industry, high levels of this industry liquidity can be interpreted as an economy-wide boom.

The second type of control right is more endogenous, and conferred on creditors by the firm’s incumbent manager as she makes the firm’s cash flows more appropriable or pledgeable over the medium term – for example, by improving accounting standards and transparency, by setting up escrow accounts and monitoring arrangements, by including debt covenants and conditions on dividend payments, or even by standardizing managerial procedures so as to make herself more replaceable as a manager. From the incumbent manager’s perspective, enhancing cash flow pledgeability is a double-edged sword. It makes it easier for her to sell the firm when she is no longer fit to run it because new buyers can borrow against future pledgeable cash flows to finance the acquisition. However, it also enables existing creditors to collect more if she stays in control, which reduces her incentive to enhance pledgeability. Thus the choice of cash flow pledgeability is subject to additional moral hazard, over and above the intrinsic reluctance of the incumbent to repay outside financiers. This limits the external financing capacity of the firm. The advantages of high pledgeability for financial capacity have been studied by Holmström and Tirole (1998). We examine the tradeoff between the advantages and disadvantages of increased pledgeability for the incumbent, as industry liquidity also varies.

Our goal is to understand how the external obligations built up in a boom affect a firm’s pledgeability choice, and its subsequent access to financing. When markets are buoyant and industry insiders have plenty of cash, repayment is enforced by the high resale value of assets. There is no additional need to rely on pledged cash flows. Industry assets trade for fundamental value (with no underpricing), as in Shleifer and Vishny (1992). The most efficient users hold the assets because they have enough cash and borrowing capacity to pay full value. The high anticipated resale value increases the amount of financing that a firm can credibly repay and thus the potential leverage of the firm.

If competitive pressures indeed force the firm to lever up, the incumbent’s incentive to enhance pledgeability further diminishes because the incumbent benefits less from a higher sale price when debt takes away much of the sale proceeds. Liquidity operates through leverage to crowd out pledgeability. With pledgeability low, a downturn, even one that is anticipated to occur with significant probability, can then impair firm performance severely. Industry insiders, also hit by the downturn, no longer have personal wealth to buy assets, nor does the low cash flow pledgeability of the firm allow them to borrow against future cash flows to pay for purchases. Since external claims are high in these episodes, the firm may be sold to outsiders. While industry outsiders have little
ability to operate the asset themselves, this may be a virtue – they have a strong incentive to improve asset pledgeability because they do not want to own the asset long term, but instead want to sell the asset back to industry insiders at a high price. Outsiders play a critical role, therefore, not because they are flush with funds but because they are not subject to moral hazard over pledgeability. Importantly, financiers have little incentive to renegotiate down fixed debt claims in a downturn, since the reallocation of the firm to industry outsiders may be the outcome that maximizes their claims, given past pledgeability choices. Consequently, in a downturn following a boom, a larger number of the new asset owners will be less-productive industry outsiders, reducing average productivity. Eisfeldt and Rampini (2006, 2008) provide evidence consistent with this.

Eventually, as the economy recovers, outsiders sell the assets back to the more productive industry insiders, as the higher pledgeability increases the insiders’ ability to raise money against future cash flows. Recoveries following periods of an asset price boom and high leverage are thus delayed, not just because debt has to be written down – and undoubtedly frictions in writing down debt would increase the length of the delay – but also because corporations have to restore the pledgeability of their cash flows to cope with a world where liquidity is more scarce. It is the latter which may make the debt hangover more prolonged.

High anticipated liquidity, therefore, not only leads to greater financial leverage, but also the combination leads to low pledgeability being chosen. This then leads to distortions in allocation, as unproductive users of the asset take control in a downturn from more productive users. Higher anticipated liquidity in some future states can therefore induce more eventual misallocation in less liquid states, a spillover effect between states that operates through leverage and pledgeability!

The liquidity-leverage overhang on pledgeability choice resembles traditional debt-overhang (Myers (1977)), where firm decisions are distorted whenever the decision causes an increase in the value of outstanding debt. However, it differs in important ways. The outside claim in our model could be any claim whose value is bolstered by the threat of outside sale or takeover in times of strong liquidity, as well as internally-set governance improvements in more normal times. So while the fixed nature of the outside claim helps in making the point, the effect generalizes to other variable (but not fully state-contingent) outside claims like equity. Moreover, the “underinvestment” is in pledgeability or governance, and the inefficiency is observed ex post, not ex ante, as assets go into the hands of low-productivity outsiders. The effects of this underinvestment are observed primarily after booms which give way to downturns that were known to be possible but were rationally overlooked. In that sense, outside-claim overhang is an industry or economy-wide effect, whereas traditional debt overhang occurs in a single firm as it levers up excessively.

Our paper explains why asset price booms based on a combination of liquidity and leverage can be fragile (see, for example, Borio and Lowe (2002), Adrian and Shin (2010), and Rajan and
Ramcharan (2015)). It also suggests a reason why credit cycles emerge, though a dynamic extension to the model is needed to explain the properties of such cycles fully (see, for example, Kiyotaki and Moore (1997)). More broadly, it suggests theoretical underpinnings for financing cycles (Borio (2012)), where a simultaneous and sustained rise in asset prices and leverage could significantly augment, and increase the persistence of, business cycle downturns.

Our paper builds on Shleifer and Vishny (1992), where the high net worth of industry participants allows assets to sell for their fundamental value because the best user of an asset can outbid less efficient users, which leads to efficient reallocation. In their paper, reallocation to inefficient users takes place only when industry insiders are less liquid (traditional debt overhang effect) than outsiders. Eisfeldt and Rampini (2008) develop a theory where capital reallocation is more efficient in good times, with key ingredients being private information about managerial ability and cyclical effects of labor market competition for managers. Good times lead to high required cash compensation to managers because reservation managerial wages become elevated. As a result, high ability managers can accept lower wages in return for the benefits of managing more assets. They use the differential compensation to bribe low ability managers to give up their assets. In bad times, managerial compensation is lower and even if high ability managers accepted zero cash compensation, it would not be sufficient to bribe low ability managers to give up their assets. This leads to a more efficient reallocation of capital in good (high compensation and therefore high manager liquidity) times and less in bad.

In both Shleifer and Vishny (1992) and Eisfeldt and Rampini (2008), adjusting for current conditions (such as industry net worth or compensation), past actions do not affect financial capacity or the efficiency of reallocation of capital today. This is unlike our model, where history matters, allowing us to explain prolonged downturns following booms, and sketch the possibility of financing cycles. Moreover, outsiders in our model are not necessarily more liquid, but still play an important role because they do not suffer from moral hazard over pledgeability. They take over the firm temporarily so as to raise future pledgeability, even though they cannot generate cash flow. Finally, and perhaps most important, higher liquidity is not problematic in their models, unlike in ours where its effects can be transmitted through greater anticipatory leverage and lower pledgeability into worse allocations.

The rest of the paper is as follows. In Section I, we describe the basic benchmark model of pledgeability choice and the timing of decisions in a three-period model. In Section II, we analyze the implications of pledgeability choice when financial contracts are fully state contingent and

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2 See Benmelech and Bergman (2011), Coval and Stafford (2007), and Shleifer and Vishny (2011) for comprehensive reviews.
pledgeability can be chosen flexibly in response to the state. The maximum amount that can be pledged to outside investors is characterized, and the fundamental tradeoffs in the model are explained, focusing on the last two periods. We then analyze the initial period. In Section III, we examine the implications of standard debt contracts and persistent pledgeability choices that are made before uncertainty is fully revealed. In Section IV, we discuss implications and conclude in Section V.

I. The Framework

A. The Industry and States of Nature

Consider an industry with 4 dates (-1, 0, 1, 2) and 3 periods between these dates, with date t marking the end of period t. A period is a phase of the financing cycle (see Borio (2014) for example), and extends over several years. The state of the industry is realized at the beginning of every period. In the good state G, the industry prospers. In the bad state B, industry-wide distress occurs. In period 0, the industry is in state $s_0 \in \{G, B\}$, with the probability of state G being $q^G$ (see Figure 1).

Similarly in period 1, the probability of state G is $q^{BG}$. In period 2, we assume the industry returns to state G for sure – this is meant to represent the long run state of the industry (we model economic fluctuations and not apocalypse). A full description of the state in period t includes the states that were realized in previous periods, but where this is unnecessary we will skip it for convenience.

![Figure 1: States of Nature](image)

B. Agents and the Asset

There are two types of agents in the economy: High types (H) are industry insiders with high ability to produce with an asset, which we call the firm. There is some mutual specialization
established over the period between the incumbent manager and the firm that creates a value to incumency. When the state is G, only a high type manager in place at the beginning of a period $t$ can produce cash flows $C_t$ with the asset over the period. In the B state, however, even a high ability manager cannot produce cash flows. A low type (L) manager has no ability to produce cash flows regardless of the state. These could be industry outsiders such as financiers who hold the asset for the purpose of reselling, or industry insiders who have lost their ability (see below). Financiers also have funds, which they will lend to others managing the firm if they expect to break even. All agents are risk neutral. We ignore time discounting, which is just a matter of rescaling the units of cash flows.

A high ability manager retains her ability into the next period only with probability $\theta^{hi} < 1$. Think of this as the degree of stability of the industry. Intuitively, the critical capabilities for success are likely to be stable in a mature industry or in an industry with little technological innovation. However, in an industry which is young and unsettled, or in an industry with significant innovation, the critical capabilities for success can vary over time. A manager who is very appropriate in a particular period may be ineffective in the next. This is the sense in which an incumbent can lose ability and this occurs with higher probability in a young or changing industry.

The incumbent’s loss of ability in the next period becomes known to all shortly before the end of the current period. Loss of ability is not an industry wide occurrence and is independent across managers. So even if a manager loses her ability, there are a large number of other industry insider managers equally able to take her place next period. If a new high ability manager takes over at the end of the current period, she will shape the firm towards her idiosyncratic management style, so she can produce cash flows with the firm’s assets in future periods in good states.

C. Financial Contracts

Any manager can raise money from financiers against the asset by writing one period financial contracts. Although our ultimate goal is to understand the effects of debt contracts, we begin by analyzing an economy in which contracts are allowed to be state contingent, so promised payments at the end of period $t$ are $D_t^s$.

Having acquired control of the firm, a manager would like to keep the realized cash flow for herself rather than share it with financiers. Two sorts of control rights force the manager to repay the external claims. First, the financier automatically gets paid a portion that we call “pledgeable” of the cash flows produced over the period, up to the amount of the financier’s claim. Second, just before the end of the period, the financier gets the right to auction the firm to the highest bidder if he has not been paid in full. Below we describe the two control rights in detail.
D. Control Rights over Cash Flow: Pledgeability

Let us define pledgeability as the fraction of realized cash flows that are automatically directed to an outside financier. In practice, it is determined by a variety of factors: the information possessed by the financier and hence the nature of the financier (“arm’s length” like a bond investor or “relationship” like a banker); the nature of financing (for example, dispersed or concentrated); the financial contract (covenant lite or covenant heavy); the quality of the accounting systems in place; the transparency of the organizational structure and the system of contracting (e.g., the absence of pyramids, the rules governing related party transactions, etc.); and the checks and balances that are imposed on the manager by the organization (the quality and independence of the board, the replaceability of the CEO, the independence of the auditor and the audit committee, etc.).

The incumbent chooses pledgeability this period, but it is embedded only by next period, and will then persist for the entire period. So pledgeability $\gamma_{t+1}$ chosen in period $t$ is the fraction of period $t+1$’s cash flows that can be automatically paid to outside financiers. $\gamma_{t+1} \in [\underline{\gamma}, \overline{\gamma}]$, where the range of feasible values is determined by the economy’s institutions supporting corporate governance (such as regulators and regulations, investigative agencies, laws and the judiciary). Also, $0 \leq \underline{\gamma} < \overline{\gamma} \leq 1$. To set $\gamma_{t+1} > \overline{\gamma}$, it costs $\epsilon \geq 0$, where $\epsilon$ is the cost of actions such as hiring a reputable accountant. Our results will be presented primarily for the case where $\epsilon \to 0$, and a positive $\epsilon$ will only alter the results quantitatively.

While a low-type incumbent cannot generate cash flows, she too can set next period’s pledgeability—she does not have industry-specific managerial capabilities but has governance capabilities.

E. Control Rights over Assets: Auction and Resale

If the financier has not been paid in full from the pledged cash flow and any additional sum the incumbent voluntarily pays, then the financier gets the right to auction the firm to the highest bidder at date $t$. One can think of such an auction as a form of bankruptcy. Therefore, the incumbent can retain control by either paying off the financier in full (possibly by borrowing once again against future pledgeable cash flows) or by paying less than the full contracted amount and outbidding other bidders in the auction.

F. Initial Conditions and Wealth

At date 0, the incumbent has initial wealth $\omega_0^{L,0} \geq 0$. Let industry insiders start out with wealth $\omega_0^{H,0} \geq 0$ which, recall, is termed industry liquidity. If the state is good in period $t$, we assume
that both the wealth of the incumbent and industry insiders go up by $\rho C_t$ (the industry boom lifts the private income of all insiders whether owning a firm or working as contractors, consultants or employees). Furthermore, the wealth of the incumbent increases by an additional $(1 - \gamma_1) C_t$, the unpledged cash flow she generates within the firm.

**G. Efficiency**

The measure of unconstrained economic efficiency we use through the rest of this paper is the extent to which the asset is in the hands of the most productive owner at that time. We do not model investment, instead assuming that the asset exists and is owned by an incumbent.³

**H. Timing**

We will start by examining incentives in period 1. The timing of events is described in Figure 2. We assume that the incumbent learns the state, then sets pledgeability $\gamma_2$, knowing the amount of payment that is due at date 1. Next, her ability in period 2 is realized. Subsequently, production takes place and the pledgeable fraction $\gamma_1$ of cash flows (set in the previous period) goes to financiers automatically. She either pays the remaining due or enters the auction. The period ends with potentially a new incumbent in place.

![Figure 2: Timing and Decisions in Period 1](image)

**II. Pledgeability Choice with State-Contingent Contracts**

We now focus on decisions in period 1. What determines pledgeability? How does the level of promised payment $D^H_t$ influence the incumbent’s incentive to set pledgeability? We will show that both the choice of pledgeability and the maximum state-contingent payments are determined by

³ Alternatively, we can put a minimum scale on the value of real inputs to be assembled into the firm at the initial date -1, and assume the firm starts at that date only if enough funding is available. As a result, inefficient underinvestment may occur if incentives to make cash flows pledgeable or to transfer the firm to more efficient producers are sufficiently weak, for bids may be reduced at date -1 below this floor.
two forms of interacting moral hazard. First, incumbents can withhold cash flows from financiers except for what they are forced to pay by pre-set pledgeability or the financier’s threat to seize and auction assets. Second, the incumbent can set future pledgeability low, potentially reducing the amount that financiers are able to collect. We assume that period 1 starts with a high type manager in place. In this setting, all outcomes will be efficient, and many of our positive implications will be clear. In Section 2.5, we extend the analysis to period 0 and show an inefficient real outcome: the asset can possibly be sold to a low type at date 0, and period 1 can start with a low type manager in place.

2.1. Date 2

Since the economy ends at date 2, and there is no uncertainty over the state in period 2, a high type industry insider who bids for control at date 1 can borrow up to $D_2 \equiv \gamma_2 C_2$ where $\gamma_2$ is preset by the incumbent in period 1. The incumbent can also borrow up to $\gamma_2 C_2$ at date 1 if she remains a high type and bids to retain control into period 2.

2.2. Date 1

Let $D_1^h$ be the promised payment to the financier at date 1 in state $s_1$, $s_1 \in \{G, B\}$. If the incumbent in period 1 is an industry insider and $s_1=G$, cash $\gamma_1 C_1$ goes directly to the financier (up to the value of her promised claim), where $\gamma_1$ is pledgeability that was set in period 0. The remaining payment due is $D_1^h = D_1^h - \text{Min}[\gamma_1 C_1, D_1^h]$. If $s_1=B$, then $D_1^h = D_1^h$.

*Industry Insider Bid*

In any date 1 auction for the firm, industry outsiders or financiers do not bid to take direct control of the firm since the firm generates no cash flow in their hands in the last period, and the firm has no residual value. Industry insiders, however, bid using their date 1 wealth and any amount that can be borrowed at date 1 by pledging period 2’s output. Their wealth increases over period 1 by $\rho C_1$ in state G, and remains unchanged in state B, i.e., $\omega_1^{H,G} = \omega_0^{H,G} + \rho C_1$ and $\omega_1^{H,B} = \omega_0^{H,B}$. Together with the amount $\gamma_2 C_2$ they can borrow, the total amount that they can pay is $\omega_1^{H,h} + \gamma_2 C_2$. Of course, they will not bid more than the total value of cash flow, $C_2$. So the maximum auction bid at date 1 is $B_1^{H,h}(\gamma_2) = \text{Min}[\omega_1^{H,h} + \gamma_2 C_2, C_2]$.

A measure which will help understand the model better is *potential underpricing*, which is the difference between the present value of future cash flows accruing to an industry insider if he buys the firm and the amount that he can bid if the incumbent has set period-2 pledgeability to be low. It equals
\( C_2 - B_{i}^{H,s_i}(\gamma) \) at date 1. By choosing different levels of period-2 pledgeability, the incumbent can vary industry insiders’ bids between \( B_{i}^{H,s_i}(\gamma) \) and \( B_{i}^{H,s_i}(\overline{\gamma}) \), thus altering the realized underpricing, which is the difference between the present value of future cash flows and the actual bid, i.e., \( C_2 - B_{i}^{H,s_i}(\gamma) \).

**Incumbent Bid**

The incumbent has to repay the financier in full or outbid others in an auction if she wants to retain control into period 2. That is, she pays \( \text{Min} \left[ \tilde{D}_{i}, B_{i}^{H,s_i}(\gamma) \right] \). The cash she has at date 1 is the initial wealth level, \( \omega_{0}^{j,s_i} \), augmented by \( \rho C_{1} \) in state G, plus the non-pledgeable portion of cash flows generated during period 1. At date 1, the incumbent has cash \( \omega_{1}^{j^{G}} = \omega_{0}^{j,s_i} + (1 - \gamma_{1} + \rho)C_{1} \) if the period 1 state is G, and \( \omega_{1}^{j^{B}} = \omega_{0}^{j,s_i} \) if the state is B. In addition, if she knows she is going to keep her ability in period 2, she can also raise funds against period 2’s output, \( \gamma_{2}C_{2} \). Therefore, the incumbent can pay as much as \( B_{i}^{j,s_i} = \text{min} \left\{ \omega_{1}^{j^{G}} + \gamma_{2}C_{2}, C_{2} \right\} \) to the financier. The incumbent will retain control if the amount she can pay is (weakly) greater than \( \text{Min} \left[ \tilde{D}_{i}, B_{i}^{H,s_i}(\gamma) \right] \). Since the continuation value of the asset, \( C_{2} \), is identical for the incumbent and industry insiders, the incumbent is always willing to hold on to the asset if she is able to, unless she has lost her ability, or she is a low type to begin with, when she will want to sell out since she cannot generate cash flow next period.

Regardless of who wins, if the incumbent in period 1 is a high type, the financier recoups \( \text{Min} \left[ \gamma_{1}C_{1}, D_{i}^{G} \right] + \text{Min} \left[ \tilde{D}_{i}, B_{i}^{H,s_i}(\gamma) \right] \) if the state is G and \( \text{Min} \left[ D_{i}^{B}, B_{i}^{H,s_i}(\gamma) \right] \) if the state is B. The financier’s threat of seizing and selling assets is therefore a powerful instrument for him to extract repayment. The value of that threat depends on the bid \( B_{i}^{H,s_i}(\gamma) \) by industry insiders, which in turn depends on the wealth of industry insiders and the future pledgeability of the asset \( \gamma_{2} \).

We now show the incumbent’s choice of pledgeability \( \gamma_{2} \) and the maximal credible payment, \( \tilde{D}_{i}^{s_i,\text{Max}} \), are jointly determined, depending on whether the incumbent can outbid industry insiders. It is easily shown that because of linearity, the incumbent never sets pledgeability at an interior level in the range. We identify three cases: (i) Pledgeability does not matter for repayment. (ii) The incumbent can never outbid industry insiders. (iii) The incumbent can always outbid industry insiders. We solve explicitly for the maximal credible payment \( \tilde{D}_{i}^{s_i,\text{Max}} \) in all these cases.
(i) Pledgeability does not matter for repayment (no potential underpricing)

When $B_{1}^{H,s}(\gamma) = C_2$, industry liquidity is sufficiently high that high-type insiders can pay the full price of the asset, even if the incumbent has chosen low pledgeability, so $\tilde{D}_{1}^{s,Max} = C_2$. In this case, there is no potential underpricing and pledgeability does not matter for repayment. As a result, the incumbent will set pledgeability to be low. External payments are committed to through the high resale price of the asset. High pledgeability is neither needed nor desired by anyone in this case.

(ii) Incumbent cannot outbid industry insiders in an auction

Let $C_2 > B_{1}^{H,s}(\gamma)$ so there is potential underpricing. When $B_{1}^{H,s}(\gamma) > B_{1}^{l,s}(\gamma)$, the industry insider can always outbid the incumbent no matter what level pledgeability is set at.\(^4\) By setting payments at or below $\tilde{D}_{1}^{h,Max} = B_{1}^{H,s}(\bar{\gamma}) - \epsilon$, the incumbent is incentivized to set next period’s pledgeability at $\bar{\gamma}$. She recoups the cost $\epsilon$ of setting pledgeability high because the promised payment is at least $\epsilon$ below the auction bid. It is easy to check that the incumbent’s payoff would never increase if she set pledgeability lower.

(iii) Incumbent always can outbid industry insiders

Now let $B_{1}^{H,s}(\gamma) \geq B_{1}^{H,s}(\gamma)$ so that the incumbent can outbid the industry insider regardless of her choice of pledgeability. She chooses $\gamma_2 = \bar{\gamma}$ iff

$$\theta^H (C_2 - Min[\tilde{D}_{1}^{h}, B_{1}^{H,s}(\bar{\gamma})]) + (1 - \theta^H)(B_{1}^{H,s}(\bar{\gamma}) - Min[\tilde{D}_{1}^{h}, B_{1}^{H,s}(\bar{\gamma})]) - \epsilon \geq \theta^H (C_2 - Min[\tilde{D}_{1}^{h}, B_{1}^{H,s}(\gamma)]) + (1 - \theta^H)(B_{1}^{H,s}(\gamma) - Min[\tilde{D}_{1}^{h}, B_{1}^{H,s}(\gamma)])$$

(1)

The left hand side is the incumbent’s continuation value if she chooses $\gamma_2 = \bar{\gamma}$, while the right hand side is the one if she chooses $\gamma_2 = \gamma$. The first term on each side of (1) is the residual amount the incumbent expects if she remains a high type in period 2. The second term on each side is the expected residual amount if she loses her ability and has to auction the firm at date 1. Note that a higher $\gamma_2$ (weakly) increases the amount the incumbent has to pay the financier when she retains

\(^4\) Strictly speaking, there is one more case because we break ties in favor of the incumbent. If $C_2 = B_{1}^{H,s}(\bar{\gamma}) = B_{1}^{l,s}(\bar{\gamma})$ and $B_{1}^{H,s}(\gamma) > B_{1}^{l,s}(\gamma)$, the incumbent retains control if she chooses high general pledgeability and continues to be a high type, because she is able to pay the full value of the asset $C_2$, and insiders will not outbid her. By contrast, if she chooses low pledgeability and debt is above $B_{1}^{l,s}(\gamma)$, she loses control because the high promised payment is enforceable and higher than what she can pay. The maximum level of debt is as in case (ii).
capability and control, therefore (weakly) decreasing the first term, while it (weakly) increases the amount the incumbent gets in the auction if she loses capability, thus (weakly) increasing the second term. The incumbent therefore trades off higher possible repayments against higher possible resale value in choosing $\gamma_2$.

Importantly, a higher outstanding promised payment reduces the incumbent’s incentive to choose higher $\gamma_2$ because more of the pledgeable cash flows are captured by financiers if the incumbent stays in control, and more of the resale value also goes to financiers if the asset is sold. This is the source of moral hazard over pledgeability. The maximum level of promised payment $\hat{D}_1^{i,\text{Max}}$ that still gives her an incentive to choose $\gamma_2 = \overline{\gamma}$ is

$$D_1^i = \theta H B_1^{i,s}(\gamma) + (1 - \theta H)B_1^{i,s}(\overline{\gamma}) - \epsilon,$$

where superscript “PayIC” indicates the payment is incentive compatible. Note that it is easier to incentivize the incumbent, and thus support higher payment, when the probability she loses skills $(1 - \theta H)$ is high, for this enhances the likelihood of sale. Lemma 2.1 summarizes the results when $s_i = s \in \{G, B\}$.

**Lemma 2.1**

(i) If $B_1^{i,s}(\gamma) = C_2$, $\hat{D}_1^{i,\text{Max}} = C_2$ and $\gamma_2 = \gamma$ . For any promised payment $\hat{D}_1^i \leq \hat{D}_1^{i,\text{Max}}$, the incumbent expects $V_1^{i,s}(\hat{D}_1^i) = C_2 - \hat{D}_1^i$.

(ii) If $C_2 > B_1^{i,s}(\gamma)$ and $B_1^{i,s}(\gamma) < B_1^{i,s}(\gamma)$, $\hat{D}_1^{i,\text{Max}} = B_1^{i,s}(\overline{\gamma}) - \epsilon$ and $\gamma_2 = \overline{\gamma}$ . For any promised payment $\hat{D}_1^i \leq \hat{D}_1^{i,\text{Max}}$, the incumbent expects $V_1^{i,s}(\hat{D}_1^i) = B_1^{i,s}(\overline{\gamma}) - \hat{D}_1^i - \epsilon$ if $B_1^{i,s}(\overline{\gamma}) < \hat{D}_1^i \leq \hat{D}_1^{i,\text{Max}}$, and expects $V_1^{i,s}(\hat{D}_1^i) = \theta H C_2 + (1 - \theta H)B_1^{i,s}(\overline{\gamma}) - \hat{D}_1^i - \epsilon$ if $\hat{D}_1^i \leq B_1^{i,s}(\gamma)$.

(iii) If $C_2 > B_1^{i,s}(\gamma)$ and $B_1^{i,s}(\gamma) \geq B_1^{i,s}(\gamma)$, $\hat{D}_1^{i,\text{Max}} = D_1^{i,\text{PayIC}}$ and $\gamma_2 = \overline{\gamma}$ . For any promised payment $\hat{D}_1^i \leq \hat{D}_1^{i,\text{Max}}$, incumbent expects $V_1^{i,s}(\hat{D}_1^i) = \theta H C_2 + (1 - \theta H)B_1^{i,s}(\overline{\gamma}) - \hat{D}_1^i - \epsilon$.

**Proof:** See Appendix.

In Case (i), there is no potential underpricing and the choice of pledgeability does not matter for payment. In Case (ii), the incumbent loses control whenever she enters an auction. The maximal promised payment is set by industry liquidity and the need to compensate the incumbent for incurring the cost of setting pledgeability high. In case (iii), however, the incumbent is able to hold onto the asset for any choice of pledgeability, provided she retains capability. Therefore, the maximal promised payment has to be significantly lower to incentivize high pledgeability choice – the
incumbent’s higher ability to retain control makes a higher outside bid price less attractive and increases the moral hazard over pledgeability.

This is why, as illustrated by the top left panel of Figure 3, the maximum credible payment \( \tilde{D}_1^{s, \text{Max}} \) decreases (weakly) with incumbent wealth \( \omega_1^{s_1} \). When \( \omega_1^{s_1} \) is low as in case (ii), the promised payments can be set as high as \( B_1^{H, s}(\overline{\tau}) - \varepsilon \). However, as \( \omega_1^{s_1} \) increases and the incumbent gains the ability to retain control, her incentives start mattering, resulting in a lower credible payment \( \tilde{D}_1^{s, \text{PayIC}} \). Of course if there is no potential underpricing to begin with, the incumbent’s wealth does not matter for repayment. Although \( \tilde{D}_1^{s, \text{Max}} \) decreases with \( \omega_1^{s_1} \), the continuation value accruing to an incumbent for a given payment \( \tilde{D}_1^{s} \) always increases with \( \omega_1^{s_1} \) (Figure 3, the top right panel).

![Figure 3: \( \tilde{D}_1^{s, \text{max}} \) (left) and \( V_1^{s, \omega}(\tilde{D}_1) \) (right) as functions of \( \omega_1^{s_1} \) (top) and \( \omega_1^{H, s} \) (bottom)](image)

Other parameters: \( \omega_1^{H, s} = 0.2, \overline{\tau} = 0.3, \overline{\gamma} = 0.1, C_2 = 1, \theta^H = 0.5, \varepsilon = 0, \tilde{D}_1^* = 0.6 \)

As discussed earlier, an increase in stability \( \theta^H \) decreases the maximum incentive compatible payment \( \tilde{D}_1^{s, \text{PayIC}} \), and hence weakly decreases \( \tilde{D}_1^{s, \text{Max}} \). Intuitively, the higher is stability,
lower the probability that a sale will be necessary. For any debt level, this increases the attractiveness for the incumbent manager to choose low pledgeability to reduce the enforceable payment.\(^5\)

Finally, an increase in *industry liquidity* \(\omega^{H,s_i}\) always raises \(\tilde{D}_t^{s,\text{Max}}\). There are two channels at work here. An increase in industry liquidity pushes up the amount industry insiders can pay, \(B_1^{H,s}(\gamma_s)\), for any level of pledgeability. It also expands the parameter ranges in which either there is no potential underpricing or the incumbent cannot retain control. Consequently, again, the maximum pledgeable payment increases. The bottom left panel of Figure 3 illustrates this by plotting \(\tilde{D}_t^{s,\text{Max}}\) against \(\omega^{H,s_i}\). It is easily seen that \(V_{1,s}^{i,s}\left(\tilde{D}_t^s\right)\) varies with \(\omega^{H,s_i}\) in a non-monotonic manner for a fixed \(\tilde{D}_t^s\).

**Corollary 2.1:** \(\tilde{D}_t^{s,\text{Max}}\) increases (weakly) with \(\omega^{H,s_i}\), decreases (weakly) in \(\theta^H\) and \(\omega^{i,s_i}\). \(V_{1,s}^{i,s}\left(\tilde{D}_t^s\right)\) increases with \(\omega^{i,s_i}\), and varies non-monotonically with \(\omega^{H,s_i}\).

Proof: Follows from discussion above.

### 2.3. Involuntary Management Turnover with State-contingent Contracts

Define an involuntary turnover as one where an incumbent who retains ability has to sell an underpriced firm. We now study how involuntary management turnover varies across different states.

As we just saw, Lemma 2.1 indicates equilibrium outcomes are determined by three state variables: industry liquidity, whether the incumbent can make the payment, and whether the incumbent can outbid industry insiders once she enters an auction. If promised payments are chosen so as to maximize payout, industry liquidity, \(\omega^{H,s_0}\), fully determines pledgeability choice: low pledgeability is chosen if and only if there is no potential underpricing.

Industry-wide liquidity is unambiguously the highest in state GG, which is meant to capture long-term booms. There is no potential underpricing in state GG if \(\omega_{0}^{H,G} \geq (1-\gamma)C_2 - \rho C_1\). Potential underpricing is guaranteed in all other states if \((1-\gamma)C_2 > \omega_{0}^{H,G}\) and \((1-\gamma)C_2 - \rho C_1 > \omega_{0}^{H,B}\). This implies the result in Proposition 2.1.

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\(^5\) There is a parallel here to Jensen (1986)’s argument that free cash flows in mature industries lead to greater waste. In his view, the paucity of investment needs in mature industries results in firms generating substantial free cash flows (and hence poorer governance because of a lower need to return to the market for funding). In our model, the lower probability of the need to sell the firm to managers with different capabilities (or equivalently, the lower need to issue financial claims to raise finance for unmodeled investment) in a mature or stable industry reduces the need to maintain better outside pledgeability.
Proposition 2.1: If \((1 - \gamma) C_2 > \omega_0^{H,G} \geq (1 - \gamma_2) C_2 - \rho C_0 > \omega_0^{H,B}\), low pledgeability \(\gamma_2 = \gamma\) is chosen if and only if the state is GG.

Clearly, by definition, involuntary management turnovers take place only when there is potential underpricing. In this case, the other two state variables also become crucial. Clearly, if the incumbent can make the contracted payment \(\tilde{D}_1\), no auction or involuntary turnover takes place. Let us now see what happens if the incumbent cannot pay and is tipped into the auction.

Even in the absence of any moral hazard over pledgeability, actual date-1 repayment is capped by \(B_1^{H,s}(\bar{\gamma})\), the maximal possible bid that an incumbent encounters in an auction. This necessarily implies that repayment is low when industry liquidity is low, such as in state BB. Second, the need to provide incentives for the incumbent to choose high pledgeability will further reduce maximum repayment. If the incumbent either cannot retain the asset or can only retain by paying the full price (case (ii) in Lemma 2.1), maximal repayment is reduced by a small amount, \(\epsilon\). If the incumbent can retain the asset (case (iii)), the credible amount that she can promise is reduced by a significant amount from \(B_1^{H,s}(\bar{\gamma})\) to \(D_1^{IC,PayIC}\). Thus maximal repayment falls with the severity of the incentive problem, which turns on whether the incumbent can outbid industry insiders if she were forced into an auction, which in turn depends on the wealth of the incumbent relative to industry insiders.

Note that in both state GB and state BB, the incumbent has strictly less wealth than industry insiders for two reasons. First, she does not produce any cash flow over the period. Second, she will has used up her previous wealth in making payments – intuitively, when the incumbent can write state-contingent contracts, it never makes sense for her to retain non-state contingent cash when future payments are positive – it is better for her to pay out everything and make future payments lower and more favorably state contingent. Thus, as long as she has to make payments at date 1, the incumbent will not leave date 0 possessing any cash. Consequently, in both state GB and BB, repayment is set at or below \(B_1^{H,s}(\bar{\gamma}) - \epsilon\), leaving her enough to incentivize high pledgeability choices, even though she always loses the firm involuntarily at date 1 in these states.

Of course, all this is contingent on the incumbent not being able to pay the contracted payment. The contracted payment may, however, be lower in state GB. This is because the incumbent is wealthier in state G (she has \((1 - \gamma_0) C_0 + \rho C_0\) than in state B (0), and since she uses this wealth to reduce what needs to be raised at date 0, she may issue lower state-contingent claims at date 0 in state G. The incumbent may thus be able to reduce the required payment in state GB, \(\tilde{D}_1^{GB}\) below what she
can raise, $B_1^{GB} \left( \overline{F} \right)$, make the payment and thus avoid an auction for control. This means involuntary turnover may be lower in state GB than in state BB.

In state BG, the incumbent may actually have more wealth than industry insiders. To see this, suppose industry insiders start with liquidity $\omega_{-1}$ on date -1. If the state is BG in period 1, the difference between incumbent and industry insiders is $(1 - \gamma_1) C_1 - \omega_{-1}$. If $(1 - \gamma_1) C_1 > \omega_{-1}$, the incumbent is able to outbid insiders in state BG, involuntary turnovers do not occur even if the auction is triggered, and a significant reduction in required payment (from $B_1^{H, BG} \left( \overline{F} \right)$ to $\tilde{D}_1^{BG, PayIC}$) is needed to incentivize the incumbent to choose high pledgeability.

Proposition 2.2 summarizes the results.

**Proposition 2.2**: If $(1 - \gamma_1) C_1 > \omega_{-1}$ and the conditions in proposition 2.1 hold,

1) Maximal repayment is lower in all states other than GG: $\tilde{D}_1^{s, Max} = D_1^{s, PayIC}$ in state BG, and $\tilde{D}_1^{s, Max} = B_1^{H,s} \left( \overline{F} \right) - \varepsilon$ in state GB and BB.

2) Involuntary management turnover always occurs in state GB if $\tilde{D}_1^{GB} > B_1^{i,GB} \left( \overline{F} \right)$, and in state BB if $\tilde{D}_1^{BB} > B_1^{i,BB} \left( \overline{F} \right)$. Involuntary turnovers occur less frequently (i.e., for a smaller set of parameters) in state GB than in state BB. Turnover is not involuntary in state BG.

**2.4. Discussion**

We have outlined two kinds of moral hazard – moral hazard over repayment, and moral hazard over setting pledgeability. The two are related. When the economy is in a prolonged boom, industry insiders can pay full value for the firm even when pledgeability is set at a minimum level. There is no need to reduce the moral hazard over repayment by increasing pledgeability since creditors can extract full repayment through the threat of asset sales. However, when industry wide liquidity is lower, industry insiders’ bids for the firm are lower than the future cash flows it generates. This underpricing means that their bid can be raised by setting pledgeability higher. Not only does this raise what the incumbent can get if she has to sell the firm, it also increases the financier’s ability to extract repayment from her if she does not. Thus when the firm finances in the midst of more normal industry liquidity, not only is higher pledgeability necessary to reduce moral hazard over repayment, but also the incumbent faces moral hazard over pledgeability if payments are set too high.

Importantly, this last moral hazard is quite different than the effects of fixed payments in the standard risky debt overhang models. In those models, the incumbent avoids investment because the returns are largely captured by existing debt in certain states of the world. Outside claim overhang in
our model stems from the incumbent trying to reduce payments she must make in the future, by strategically reducing financiers’ enforcement capability. She has a particular incentive to do this, not in the states where everyone is liquid so she has little capability of altering enforcement (such as state GG), nor in the states where she is illiquid so she has little prospect of retaining control (such as states GB and BB), but in states of moderate overall liquidity (such as state BG). The fixed nature of the outside claim across states is not critical, only that the claim’s enforcement can be altered by the incumbent. So outside equity can also be a source of overhang. We will show later that risky debt has interesting additional effects.

The past and the future of the industry thus interact in interesting ways. When the industry experienced good outcomes in the past, and the future is also expected to be good enough that there is no potential underpricing, financing capacity is the highest. Not only is the firm likely to generate more in the future, but financiers can expect to recover what they lent through the threat of selling the fully priced asset. So they are willing to lend large amounts – both asset values and debt to value ratios are high. For intermediate levels of past industry performance, industry bidders cannot bid full value for the firm’s future cash flows out of their accumulated liquidity, so pledgeability of future cash flows becomes important to getting high outside bids and repayment. But because moral hazard over pledgeability kicks in, committed payments cannot be too high so as to not discourage high pledgeability. A fall in industry performance therefore has a “double whammy” effect on financing capacity by both increasing the underpricing of the firm’s asset by other industry bidders (because of their reduced liquidity) and also reducing the maximum possible committed payment as a fraction of that lower value (because of moral hazard over pledgeability). In other words, both asset values and loan to value ratios plummet when liquidity falls.

Finally, there is an additional twist because the incumbent’s liquidity generally fluctuates more with industry performance than other industry insiders, because she has paid out her cash up front to take control over the firm. This means that if the industry has a sequence of bad outcomes, the incumbent would be unable to retain control in the face of higher industry bids for the firm. Interestingly, this will reduce moral hazard over pledgeability since the incumbent, with no hope of retaining control, focuses on getting the maximum bid for the firm. The payments that can be committed to lenders will now be a higher fraction of firm resale value, even though intrinsic firm value itself is low. Indeed, this aspect of the model is again reminiscent of Jensen’s Free Cash Flow Hypothesis (Jensen (1986)), where lower cash with the incumbent reduces moral hazard. Importantly, therefore, while the market value of assets falls with industry liquidity, loan to value ratios may not decline monotonically.

In the model thus far, we have shown that involuntary management turnover is higher in persistent downturns, and this adds to voluntary turnover as managers lose capability. In times of high
liquidity, there are no involuntary turnovers. However, incumbents may voluntarily sell out because they are getting full value for the firm, and do not have enough liquidity to match industry insider offers. So without additional specificity about parameters, it is not possible for us to say whether overall turnovers will be higher in very liquid or very illiquid states. Moreover, we have ignored another form of voluntary turnover – when the incumbent retains ability for now, but anticipates lower ability with higher probability in the future. If incumbents can choose the timing of their leaving the business, they would certainly prefer to sell out when the asset is fully priced than when the asset is priced at a fraction of its fundamental value. We would thus see voluntary control transfers increase when asset prices are high (i.e., assets are not underpriced). This effect of management exit through retirement would further increase turnover in times of high liquidity (persistent industry up turns) relative to other times. We will explore this possibility in future work.

2.5. Date 0 and the Prolonged Recovery

Let us turn now to date 0 and the incentives that determine how $\gamma_1$ is set in period 0. There are two crucial differences between the date 1 and date 0 analysis. First, financiers may be able win a bid at date 0 with the purpose of reselling the asset at date 1, while they will not bid at date 1. Second, the continuation value of the asset from date 0 onwards to the period-0 incumbent may be different from that to an industry insider even though they have the same expected capabilities -- because they have different wealth, they have different expectations of retaining control at date 1. Contrast this to date 1, when continuation value to both capable incumbent and insider is $C_s$ because there are no control contests after date 1. Let $\tilde{C}_i^{d,0}(\tilde{D}_1^G, \tilde{D}_1^B)$ and $\tilde{C}_i^{h,0}(\tilde{D}_1^G, \tilde{D}_1^B)$ be the date-0 expected continuation value of the asset to the continuing incumbent and to industry insiders respectively, where $\tilde{D}_1^G$ and $\tilde{D}_1^B$ are due on date 1 in state $s_0G$ and $s_0B$ respectively. Both continuation values share the same expression, $q^{s_0G} (C_1 + \tilde{D}_1^G + V^{i,G}_1 (\tilde{D}_1^G)) + (1 - q^{s_0G}) (\tilde{D}_1^B + V^{i,B}_1 (\tilde{D}_1^B))$, but the incumbent and industry insiders will borrow different $\left(\tilde{D}_1^G, \tilde{D}_1^B\right)$ in general.

Let $B_0^{i,0}(\gamma_1)$, $B_0^{h,0}(\gamma_1)$, and $B_0^{L,0}$ be respectively the incumbent’s, the insider’s, and the outsider/financier’s bid at date 0. As before, $B_0^{i,0}(\gamma_1)$ is the minimum of the incumbent’s ability to pay and the asset’s continuation value to the incumbent:

$$B_0^{i,0}(\gamma_1) = \max_{\tilde{D}_1^G, \tilde{D}_1^B \leq \tilde{D}_{1,\text{Max}}} \min \left(1 - \gamma_0 + \rho \right) C_0 \left[ 1 - s_{0G} + q^{s_0G} \left( \gamma_1 C_1 + \tilde{D}_1^G \right) + (1 - q^{s_0G}) \tilde{D}_1^B, \tilde{C}_i^{d,0}(\tilde{D}_1^G, \tilde{D}_1^B) \right]$$

Similarly, the insider will bid.
\[ B^H,(s)(\gamma_1) = \max_{\tilde{D}^G_{s,Max} \leq \tilde{D}^G_{s}} \min\left[ \alpha^H,0 + q^G,0 \left( \gamma_1 C_1 + \tilde{D}^G \right) + (1 - q^G,0)\tilde{D}^B,0, \tilde{C}^H,0 \left( \tilde{D}^G, \tilde{D}^B \right) \right] \]

Financiers can bid up to \( B^H,(s) = q^G B^H,0 (\gamma_1) + (1 - q^G,0)B^H,0 (\gamma) - \varepsilon \). Note that this value is always strictly less than either \( \tilde{C}^H,0 \left( \tilde{D}^G, \tilde{D}^B \right) \) or \( \tilde{C}^H,0 \left( \tilde{D}^G, \tilde{D}^B \right) \). Intuitively, the asset is always valued more in the hands of people who are capable of producing cash flows. Therefore, financiers can only acquire the firm if neither the incumbent nor industry insiders can raise sufficient liquidity. Interestingly, the reason industry insiders may not be able to raise as much liquidity as the financier is because insiders suffer from moral hazard in setting pledgeability, while the financier does not – he only wants to increase the sale value of the firm at date 2 since he can produce nothing from running it.

Let \( B^{min,s}_0 = \max \{ B^{L,s}_0, B^{H,s}_0 (\gamma) \} \) be the minimum bid the incumbent will face in the date-0 auction, and \( B^{max,s}_0 = \max \{ B^{L,s}_0, B^{H,s}_0 (\gamma) \} \) be the maximum bid the incumbent will face. Following the cases analyzed in Section 2.2, we arrive at Lemma 2.2 below. The payoff functions are omitted for simplicity.

**Lemma 2.2**

Let \( s_0 = s_2 \)

\( i \) If \( B^{max,s}_0 = B^{min,s}_0 \), \( \tilde{D}^{L,Max}_0 = B^{min,s}_0 \), and \( \gamma_1 = \gamma \).

\( ii \) If \( B^{max,s}_0 > B^{min,s}_0 \) and \( B^{L,0}_0 (\gamma) < B^{min,s}_0 \), \( \tilde{D}^{L,Max}_0 = B^{max,s}_0 - \varepsilon \), and \( \gamma_1 = \gamma \).

\( iii \) If \( B^{max,s}_0 > B^{min,s}_0 \) and \( B^{L,0}_0 (\gamma) \geq B^{min,s}_0 \), \( \tilde{D}^{L,Max}_0 = D^{L,PayIC}_0 \), and \( \gamma_1 = \gamma \).

Proof: Along the same lines as the proof to Lemma 2.1., hence omitted.

The cases in Lemma 2.2 are thus similar to those in Lemma 2.1, with some small differences. Case (i) includes three subcases: (a) There is no potential underpricing at date 0 so that:

\( B^{min,s}_0 = q^G C_1 + C_2 \); (b) There may be realized underpricing at date 0, but \( B^{L,s}_0 \geq B^{H,s}_0 (\gamma) \) so that industry insiders are constrained in how much they can raise, and thus, they are outbid by financiers at date 0 (whose bid is unaffected by increases in pledgeability). (c) There is realized underpricing at date 0 because \( B^{L,0}_0 (\gamma) = B^{min,s}_0 = B^{H,0}_0 (\gamma) \) so that the incumbent cannot raise the insider bid at date 0 by setting \( \gamma_1 \) higher because the insider is already able to pay for all the limited rents she can appropriate, and will not raise her date-0 bid. In all three subcases, high pledgeability does not increase the bid that the incumbent faces at date 0, so low pledgeability is chosen.
Case (ii) and (iii) are identical to those in Lemma 2.1. Case (ii) includes a similar scenario to case (ii) in Lemma 2.1: the incumbent can retain control (we break ties in her favor) even though she starts with less liquidity than industry insiders, simply because both have sufficient liquidity (and can raise funds) to bid for all future rents when high pledgeability is chosen.

The interesting difference at date 0 is that a financier may acquire control without any capacity to produce cash flows over period 1 if $B_0^{H,s} > B_0^{H,s}(\gamma) \geq B_0^{H,s}(\gamma)$. Instead, he makes the firm more pledgeable over the period. The likelihood of this happening is particularly acute when industry liquidity is low (low $\omega_0^{H,s}$) and period-1 moral hazard over pledgeability (choice of $\gamma_2$) is high, so industry insiders or the incumbent cannot raise much finance (low $D_{10}$). When assets move into the control of those who cannot produce output, economic recovery is delayed until pledgeability is restored and assets returned to the control of industry insiders, regardless of the movement of the underlying economic state.

While the shift in assets to the outsider or financier is inefficient in the sense that outsiders cannot produce cash flows with the assets (and total surplus is not maximized), they can restore pledgeability of the firm. Anticipating restored pledgeability, and thus higher eventual access to finance, initial bids at date -1 may be higher. If these higher bids are beneficial, for example to permit a minimum quantum of investment to be raised, then temporary outsider control is constrained efficient.

Outsider control is also reminiscent of leveraged buyout transactions (see, for example, Jensen (1997)), where firms in stable industries (where moral hazard over pledgeability is high) are taken over, and the revamped management team, which is motivated by the prospect of going public soon, focuses on finding free cash flow that has been eaten up either through inefficiency or misappropriated by staff (the proverbial company jet). The management team does not really make fundamental changes to the firm’s earning prospects in the time the firm is private, but it significantly enhances the pledgeability of future cash flows, thus enhancing bids for the firm when it goes public. Our model suggests that the leveraged buyout is a means to check moral hazard at a time of moderate to low liquidity, as opposed to outright takeovers, which are more likely when liquidity is higher.

Example:
Suppose the parameters are as follows:

$q^G = 0.8$, $q^{GG} = 0.9$, $q^{BG} = 0.1$, $\theta^H = 0.7$, $\rho = 0.1$, $\overline{\gamma} = 0.6$, $\gamma = 0.3$, $C_0 = C_1 = C_2 = 1$

$\omega_0^{H,g} = 0.1$, $\omega_0^{H,b} = 0$, $\varepsilon = 0$, $\gamma_0 = \overline{\gamma}$. 
In this example, if the incumbent in period 1 were a high type, she loses control at date 1 in state GB, but not in any other state. In states other than GB, she suffers from moral hazard problems and can raise at most \( \hat{D}_{x,PotK} \), which is significantly less than \( B_{x,I}^{H,s}(\overline{\gamma}) \). As a result, an industry insider cannot raise much at date 0 in state B against the promise of date 1 payments and \( B_0^{H,B}(\gamma_1 = \overline{\gamma}) = 0.46 \). Moreover, since her wealth \( \omega_0^{H,B} \) is zero, a financier who does not suffer from moral hazard issues in setting pledgeability can raise as much as \( \omega^{L,B} = 0.61 \), and can outbid her. Therefore, if the period-0 incumbent has to auction the firm—either because required payment is high or she loses capabilities, a financier will acquire the firm. As a result, the recovery in cash flows is delayed, even if the state improves.

2.6. When Pledgeability is Chosen

Thus far, the incumbent sets pledgeability after the state in period 1 is already realized (ex-post choice). This reflects short term attributes of pledgeability which can be changed rather quickly (such as a more reputable accountant). Now let us see what happens when the incumbent chooses pledgeability based on the probability distribution of the states, before the state for the period is known. This situation represents more durable pledgeability choices such as the specificity of the production technique or the internal organization of a firm, and implies rigidity of pledgeability across future states. The point of this section is to show that with state-contingent contracts, there is little difference between the two types of pledgeability, which is why we choose the simpler analysis with short term pledgeability to illustrate our point. Durable pledgeability choice will be more important with debt contracts.

Figure 4 below shows the timing. The incumbent makes a decision before the state is realized based on the probabilities of each state. If the cost, \( \epsilon \), is sufficiently small, there is no effect on real outcomes. High pledgeability is chosen ex-ante if the incumbent had the incentive to choose high pledgeability ex-post in at least one of the two subsequent states.

**Figure 4: Timing and Decisions with ex-ante Pledgeability Choice**
Next, we analyze what happens when $\varepsilon$ is significantly positive. If the incumbent had the incentive to choose high pledgeability in both the subsequent states when she was making the choice ex post obtaining knowledge of the state, then she would choose high pledgeability ex ante. The state-continent payments would be identical (because the payments set when choice is ex-post give an incentive to choose high pledgeability in both states). However, if liquidity is so high in one of the states (say $s_0G$) that there is no potential underpricing (or potential underpricing of less than $\varepsilon$), and hence there is no incentive to increase pledgeability merely anticipating the outcome in that state, then there must be a lower state-contingent payment in the other state (say $s_0B$) so that the incumbent has sufficient rents to cover the cost of choosing high pledgeability before the state is known\(^6\). This means the incumbent’s ability to raise funding will (weakly) fall relative to when pledgeability is chosen ex post if the cost $\varepsilon$ is significant. If the probability of the fully liquid state and the cost $\varepsilon$ are both sufficiently high, the incumbent may even choose low pledgeability ex ante, for it may not be worthwhile to lower the promised payment enough in the unlikely other state to give her the incentive to incur cost $\varepsilon$.

Broadly, however, in the baseline case of our model where $\varepsilon$ is small and contracts are state contingent, the timing of pledgeability choice is not very important, so both quickly changeable aspects and durable aspects of pledgeability can be similarly incentivized. This is not the case with debt contracts, which we now turn to.

III. Debt Contracts

Our analysis of state-contingent contracts serves as a building block to understand the effects of industry liquidity and promised payments on pledgeability choices. However, financial contracts used by most firms are less than fully state-contingent and are much closer to debt contracts, which specify a constant promised payment on a given date in all states $s$ such that $D'_s = D_s$. In this section, we study such debt contracts and focus on how they can limit pledgeability. We will continue to assume that pledgeability in a period is chosen before the state in that period is realized (for reasons we will explain at the end of section 3.1) and persists over the next period – we therefore focus on durable aspects of pledgeability consistent with the financing cycle. For simplicity, we do not add explicit frictions to make debt the optimal contract, such as costs of verifying the state.

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\(^6\) The maximal payment in state $s_0B$ is $\tilde{D}_{s_0B,\text{Max}} = \frac{q^{s_0G}}{1 - q^{s_0B}} \varepsilon$. 

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With state-contingent contracts, no one has an interest to overpromise, given the potential damage to both incumbent and financier from inducing low pledgeability and increasing underpricing. With debt contracts, however, if the incumbent must raise a large amount initially (to fund the investment or to outbid others to become the new incumbent), it is possible that this amount can only be raised by making a high fixed promised payment across states, even if it leads to low pledgeability (which will be inefficiently low in some states).

Section 3.1 formalizes the analysis of period 1 with debt contracts and pledgeability set ex ante. With state-contingent contracts, we have shown that both past and current states affect the equilibrium outcomes. With debt contracts, we will show that expectations about future states, and thus the spillover between future states, also affects pledgeability choices, asset allocation and financing capacities. In Section 3.2, we consider period 0. Similar to the case with state-contingent contracts, at date 0 the asset can be sold to a financier who has no production capabilities. With state-contingent contracts, the asset is sold inefficiently only when industry liquidity is very low and future moral hazard is at very high levels. With debt contracts, however, such inefficiency occurs even at moderate levels of industry liquidity because the low prior pledgeability chosen in the face of debt will make it hard for current incumbents or industry insiders to raise finance. This further suggests why recoveries from debt-fueled, asset-price-based expansions are slow.

3.1 Debt Contracts with Ex-ante Pledgeability Choice

Because there is a single state in period 2, the promised payment when contracts are restricted to debt contracts will be identical to that when contracts are state contingent. Next, we turn to period 1. The timing of events in the period is identical to Figure 4, with the exception that the promised payment is a constant, $D_1^s = D_1^h$. Let $V_{i,1}^{h,\gamma_1} (\tilde{D}_1, \gamma_2 = \gamma)$ and $V_{i,1}^{h,\gamma_1} (\tilde{D}_1, \gamma_2 = \gamma)$ respectively be the incumbent’s payoff when she chooses high and low pledgeability, given residual required payment $\tilde{D}_1$. Define $\Delta^h_{i,\gamma_1} (\tilde{D}_1) = V_{i,1}^{h,\gamma_1} (\tilde{D}_1, \gamma_2 = \gamma) - V_{i,1}^{h,\gamma_1} (\tilde{D}_1, \gamma_2 = \gamma)$. In the baseline model with state-contingent contracts and ex-post pledgeability choices, the maximal promised payment $\tilde{D}_1^{\gamma_1, Max}$ satisfies $\Delta^h_{i,\gamma_1} (\tilde{D}_1^{\gamma_1, Max}) = 0$ if there is potential underpricing. With a constant payment and ex-ante choice, the expected difference in payoff must be non-negative to provide incentives for high pledgeability.

Lemma 3.1 describes $\Delta^h_{i,\gamma_1} (\tilde{D}_1)$ for any level of $\tilde{D}_1$ and for low levels of $\epsilon$.

**Lemma 3.1**

(i) If $B_{i,1}^{h,\gamma} (\gamma) = C_2$, $\Delta^h_{i,\gamma} (\tilde{D}_1) \equiv -\epsilon$.
(ii) If \( C_2 > B_{H,s}^i(\gamma) \) and \( B_{i,s}^s(\gamma) < B_{i,s}^h(\gamma) \), \( \Delta^{s_i} \left( \tilde{D}_1 \right) \geq -\varepsilon \)

(iii) If \( C_2 > B_{H,s}^i(\gamma) \) and \( B_{i,s}^s(\gamma) \geq B_{i,s}^h(\gamma) \),

\[
\Delta^{s_i} \left( \tilde{D}_1 \right) \begin{cases} 
\leq 0 & \text{if } \tilde{D}_1 \geq D_1^{\text{PayIC}} \\
> 0 & \text{if } \tilde{D}_1 < D_1^{\text{PayIC}}.
\end{cases}
\]

Figure 5: \( \Delta^{s_i} \left( \tilde{D}_1 \right) \) in Different Cases

The detailed expressions for Lemma 3.1 are available in the Appendix. Figure 5 characterizes the function \( \Delta^{s_i} \left( \tilde{D}_1 \right) \) in different cases. If there is no potential underpricing in state \( s_0 s_1 \),

\( \Delta^{s_i} \left( \tilde{D}_1 \right) = -\varepsilon \) for all values of \( \tilde{D}_1 \) because raising pledgeability does not change enforceable payments, while resulting in cost \( \varepsilon \). If there is potential underpricing, however, and if the incumbent can outbid outsiders (case (iii)), \( \Delta^{s_i} \left( \tilde{D}_1 \right) = 0 \) for \( \tilde{D}_1 = D_1^{s_i,M_{Max}} \), negative for higher values of \( \tilde{D}_1 \).
and strictly positive for all lower values of $\hat{D}_1$. Higher committed payments depress the incentive to increase pledgeability, since the incumbent retains control whenever she retains her ability. Finally, if there is potential underpricing and the incumbent has no hope of retaining control once she enters an auction (Case (ii)), $\Delta^{s_2}_{s_1} \left(\hat{D}_1\right) \geq -\varepsilon$ : the incumbent sees only the upside of increasing pledgeability since the asset invariably will be sold. Even for very high promised values of $\hat{D}_1$—above the most the asset could be sold for, the only disadvantage of choosing high pledgeability is its cost, $\varepsilon$.

With one single face value $D^{s_1}_{s_1} = D^{s_1, G}_{s_1} = D^{s_1, B}_{s_1}$ and *ex-ante* pledgeability choice, there is a single incentive constraint across states. In particular, there exists a unique $D^{s_1, IC}_{s_1}$ that satisfies

$$Q^{s_1, G} \{\Delta^{s_1, G} (D^{s_1, IC}_{s_1} - \gamma; C_1) \} + (1 - Q^{s_1, G}) \{\Delta^{s_1, B} (D^{s_1, IC}_{s_1}) \} = 0$$

such that high pledgeability is chosen if and only if the face value of debt $D_1 < D^{s_1, IC}_{s_1}$. In general, $D^{s_1, IC}_{s_1}$ lies in the range $\hat{D}^{s_1, B, Max}_{s_1}$ and $\hat{D}^{s_1, G, Max}_{s_1} + \gamma_1 C_1$.

Two cases, summarized by Lemma 3.2, deserve special mention.

**Lemma 3.2**

(i) If there is no potential underpricing in state $G$, $D^{s_1, IC}_{s_1} \rightarrow \hat{D}^{s_1, B, Max}_{s_1}$ (the lowest possible value) as $\varepsilon \rightarrow 0$. High pledgeability is chosen only if promised payments are set low.

(ii) If the incumbent has no hope of retaining control in state $B$ in any auction after a payment default, $D^{s_1, IC}_{s_1} \rightarrow \gamma_1 C_1 + \hat{D}^{s_1, G, Max}_{s_1}$ (the highest possible value) as $\varepsilon \rightarrow 0$. A payment that leads to high pledgeability is always preferred.

Intuitively, in Lemma 3.2 (i), there is never any incentive to increase pledgeability coming from future state $G$, so the incentives have to be set via state $B$ alone. That is, $\Delta^{s_1, G}_{s_1} \left(\hat{D}_1\right) = -\varepsilon$ for all possible payments. The maximum payment which still provides incentives for high pledgeability in state $B$ is well below the most that can be paid in state $G$. In Lemma 3.2 (ii), in future state $B$, the incumbent will strictly prefer high pledgeability regardless of the size of the promised payment (provided she recovers cost $\varepsilon \rightarrow 0$), because she knows she has to sell. That is, $\Delta^{s_1, B}_{s_1} \left(\hat{D}_1\right) \geq -\varepsilon$ for all promised payments. With no disincentive from state $B$, there will be *ex-ante* incentives to increase pledgeability whenever there are *ex-post* incentives in state $G$ (i.e., whenever $\Delta^{s_1, G}_{s_1} \left(\hat{D}_1\right) \geq 0$).

An important new result is that, unlike with state-contingent contracts, $D^{s_1, IC}_{s_1}$, the level of debt which provides incentives for high pledgeability keeping in mind both future states, may not be the face value that enables the incumbent to raise the most upfront. This is most easily seen when
liquidity is plentiful, as in state G with no potential underpricing. In this case, the incumbent can pay \( B_1^{H,G}(\gamma) + \gamma C_1 = C_2 + \gamma C_1 \) in state G even if pledgeability is set low. At the same time, the anticipated high liquidity in the G state offers no incentive for the incumbent to enhance pledgeability.

This also means that the level of debt that provides incentives to set pledgeability high, \( D_1^{sI,JC} \), is weakly less than \( \tilde{D}_{\gamma}^{sI,B,Max} \), and strictly less if \( \epsilon > 0 \). If the difference between \( C_2 + \gamma C_1 \) and \( \tilde{D}_{\gamma}^{sI,B,Max} \) is large, the incumbent could raise more by setting \( D_1^{sH} = B_1^{H,G}(\gamma) + \gamma C \) and inducing low pledgeability even if there is significant probability of the low state occurring. The broader point is that the prospect of highly liquid states not only makes feasible greater promised payments, but also eliminates incentives to enhance pledgeability. To restore those incentives, keeping in mind the other states, debt may have to be set so low that funds raised are greatly reduced – something the incumbent will not do. Note that this can happen even if the probability of the low state is significant, and even if the direct cost \( \epsilon \) of enhancing pledgeability is zero.

More formally, if \( B_1^{H,G}(\gamma) + \gamma C_1 \) exceeds \( D_1^{sI,JC} \), setting promised payment at \( B_1^{H,G}(\gamma) + \gamma C_1 \) leads the incumbent to choose low pledgeability, and pay \( B_1^{H,B}(\gamma) \) in state B and \( B_1^{H,G}(\gamma) + \gamma C_1 \) in state G. This can provide a larger expected payment if

\[
q^{sG}\left[B_1^{H,G}(\gamma) + \gamma C_1 - D_1^{sI,JC}\right] + (1 - q^{sG})\left[B_1^{H,B}(\gamma) - \min\left\{B_1^{H,B}(\overline{\gamma}), D_1^{sB,PayIC}\right\}\right] > 0. \tag{2}
\]

Lemma 3.3 then describes \( D_1^{sH,Max} \), the level of promised date-1 debt that can raise the most, corresponding to the cases in Lemma 3.2 for \( \epsilon = 0 \).

**Lemma 3.3**

When \( \epsilon = 0 \),

(i) If \( B_1^{H,sG}(\gamma) \leq B_1^{sG}(\gamma) < C_2 \) and \( B_1^{H,sG}(\gamma) = C_2 \), then \( D_1^{sI,JC} = D_1^{sH,B,PayIC} \).

a) If \( q^{sG}\left(C_2 + \gamma C_1\right) + (1 - q^{sG})B_1^{H,B}(\gamma) > D_1^{sH,B,PayIC} \), then \( D_1^{sH,Max} = \gamma C_1 + C_2 \). For any promised payment \( D_1^{sH,B,PayIC} < D_1^{sH,Max} \), \( \gamma_2 = \gamma \). For any promised payment

\[
D_1^{sH} \leq D_1^{sH,B,PayIC} \text{,} \quad \gamma_2 = \overline{\gamma}.
\]

b) If \( q^{sG}\left(C_2 + \gamma C_1\right) + (1 - q^{sG})B_1^{H,B}(\gamma) \leq D_1^{sH,B,PayIC} \), then \( D_1^{sH,Max} = D_1^{sH,IC} = D_1^{sH,B,PayIC} \).

For any promised payment \( D_1^{sH} \leq D_1^{sH,Max} \), \( \gamma_2 = \overline{\gamma} \).

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(ii) If \( B^{H,s_G}_1(\gamma) < C_2 \), \( B^{i,s_G}_1(\gamma) \geq B^{H,s_G}_1(\gamma) \), and \( B^{i,s_B}_1(\gamma) < B^{H,s_B}_1(\gamma) \), then

\[
D^{\text{Max}}_{1, s} = D^{s,IC}_1 = \gamma_1 C_1 + D^{s_G, PayIC}_1.
\]

For any promised payment \( D^i_1 \leq D^{s,\text{Max}}_1 \), \( \gamma_2 = \overline{\gamma} \).

In summary, high anticipated liquidity thus crowds in debt and crowds out pledgeability, setting the stage for more severe anticipated asset misallocation than in the case with contingent contracts. Interestingly, debt will not be renegotiated, before or after the state is realized, even if renegotiation is feasible – it will not be renegotiated before because the level of debt is set to raise the maximum amount possible even if it results in low pledgeability, and will not be renegotiated after because relevant parties will not write down their claims given that pledgeability has already been set. Interestingly, both the fixed promised debt payments across states, and the act of choosing pledgeability before the state is known, have the effect of causing a spillover between anticipated states. Therefore, outcomes are somewhat similar even when pledgeability is chosen after the future state is fully known, but in that case one needs to explain why the level of debt payments cannot be renegotiated once the state is known. Specifically, when the state is known and the lender gets all surplus from renegotiation, debt becomes equivalent to fully state-contingent contracts, in part because all are risk neutral. The analysis of non-renegotiable debt when pledgeability choice is made after the state is realized is available from the authors.

### 3.2 Date 0 Choices and Prolonged Recoveries

Let us now analyze period 0 with debt contracts. With minor notational complications, the pledgeability decision in period 0 is similar to that we have just examined for period 1. We just saw, however, that debt contracts could induce low pledgeability because of cross-state spillovers. In other words, before the state in period 0 is realized, high liquidity in one anticipated state (state G) could induce both high promised debt payments at date 0 and low pledgeability (\( \gamma_1 = \gamma \)). Financiers may now outbid industry insiders at date 0 in state B simply because the latter have little ability to pledge. Such misallocation could occur even at moderate levels of liquidity in state B. This spillover in outcomes between anticipated states is a special property of debt. We explain all this in more detail.

With a bit of abuse of notation, let \( \widetilde{C}^{i,s}_1 (D^i_1) \) and \( \widetilde{C}^{H,s}_1 (D^i_1) \) be the date-0 expected continuation value of the asset to the incumbent and industry insiders if the face value of debt is \( D^i_1 \). Unlike with state-contingent contracts, default is now possible on committed payments. So let \( \tilde{D}^{s_G}_1 (D^i_1) \) and \( \tilde{D}^{s_B}_1 (D^i_1) \) be the amount that financier recovers in the two states respectively, given the face value

\[
D^i_1 . \widetilde{C}^{i,s}_1 (D^i_1) = q^{s_G} \bigg( (1 - \gamma_1) C_1 + \tilde{D}^{s_G}_1 (D^i_1) + V^{i,G}_1 (D^i_1) \bigg) + (1 - q^{s_G}) \bigg( \tilde{D}^{s_B}_1 (D^i_1) + V^{i,B}_1 (D^i_1) \bigg)
\]
\[ \tilde{C}^{H, B}(D) = q^{H, B}( (1 - \gamma_{1}) C_{1} + \tilde{D}^{H, B}(D_{1}) + H_{1}^{H, B}(D_{1})) + (1 - q^{H, B})( \tilde{D}^{H, B}(D_{1}) + H_{1}^{H, B}(D_{1})) , \]

\[ B_{0}^{L, \epsilon_{0}}(\gamma_{1}) , B_{0}^{H, \epsilon_{0}}(\gamma_{1}) , \text{and } B_{0}^{H, \epsilon_{0}}(\gamma_{1}) \text{ are then:} \]

\[ B_{0}^{L, \epsilon_{0}}(\gamma_{1}) = \max_{D_{0} \leq D_{0}^{\epsilon_{0}, \max}} \min \{ \omega_{0}^{L, \epsilon_{0}} + q^{H, G} \tilde{D}^{L, \epsilon_{0}}(D_{1}) + (1 - q^{H, G}) \tilde{D}^{H, \epsilon_{0}}(D_{1}) , \tilde{C}_{1}^{L, \epsilon_{0}}(D_{1}) \} \]

\[ B_{1}^{L, \epsilon_{0}} = q^{H, G} \left[ B_{1}^{H, \epsilon_{0}}(\overline{\gamma}) - \epsilon \right] + (1 - q^{H, G}) \left[ B_{1}^{H, \epsilon_{0}}(\overline{\gamma}) - \epsilon \right] \]

\[ B_{0}^{H, \epsilon_{0}}(\gamma_{1}) = \max_{D_{0} \leq D_{0}^{\epsilon_{0}, \max}} \min \{ \omega_{0}^{H, \epsilon_{0}} + q^{H, G} \tilde{D}^{H, \epsilon_{0}}(D_{1}) + (1 - q^{H, G}) \tilde{D}^{H, \epsilon_{0}}(D_{1}) , \tilde{C}_{1}^{H, \epsilon_{0}}(D_{1}) \} . \]

Similar to date 1, we can define \( D_{0}^{IC} \) such that \( q^{G} \Delta^{G} \left( D_{0}^{IC} - \gamma_{0} C_{0} \right) + \left( 1 - q^{G} \right) \Delta^{B} \left( D_{0}^{IC} \right) = 0 \).

Lemma 3.4 is analogous to Lemma 3.2. The definitions for \( B_{0}^{\max, \epsilon_{0}} \) and \( B_{0}^{\min, \epsilon_{0}} \) follow those in Section 2.5.

**Lemma 3.4**

i) If \( B_{0}^{\max, \epsilon_{0}} = B_{0}^{\min, \epsilon_{0}} \) so that there is no potential underpricing in state G, \( D_{0}^{IC} \rightarrow \tilde{D}_{0}^{B, \max} \) as \( \epsilon \rightarrow 0 \).

ii) If the incumbent has no hope of retaining control in state B, \( D_{0}^{IC} \rightarrow \gamma_{0} C_{0} + \tilde{D}_{0}^{G, \max} \) as \( \epsilon \rightarrow 0 \).

We now highlight one interesting case which only occurs in the dynamic context when contracts are restricted to be debt.

**Lemma 3.5**

If \( B_{0}^{H, B}(\overline{\gamma}) > B_{0}^{L, B} > B_{0}^{H, B}(\underline{\gamma}) \) and

\[ q^{G} \left[ B_{0}^{H, G}(\overline{\gamma}) + \gamma_{0} C_{0} - D_{0}^{IC} \right] + (1 - q^{G}) \left[ B_{0}^{L, B} - \min \{ B_{0}^{H, B}(\overline{\gamma}), D_{0}^{IC} \} \right] > 0 , \]

then

\[ D_{0}^{\max} = B_{0}^{H, G}(\overline{\gamma}) + \gamma_{0} C_{0} > D_{0}^{IC} = \tilde{D}_{0}^{B, \max} . \]

For any promised payment \( D_{0}^{IC} < D_{0} \leq D_{0}^{\max} \), \( \gamma_{1} = \overline{\gamma} \).

For sufficiently high probability of state G in period 0, and sufficiently high liquidity in state G, the incumbent will set date-0 debt high enough that she chooses low pledgeability for period 1. If the realized state is G, she repays everything. If the state turns out to be B, she is forced to sell the asset and the preset low pledgeability, \( \gamma_{1} = \overline{\gamma} \), restricts the amount at which she can sell, which equals
— a sale to financiers. If she had set pledgeability higher, she would have sold to an industry insider. The additional misallocation in an industry bust stems wholly from the anticipated high liquidity in the boom, which causes the incumbent to both promise high debt payments for date 0, and induces her to choose low pledgeability in period 0. When the expected boom (high \( q^G \)) instead turns out to be a bust, we have a prolonged recovery due to misallocation.

Note that the incumbent’s decision is completely rational: she knows the probability of a bust but because the debt level has to be kept very low to induce high pledgeability, she rationally ignores the consequences, even if the probability of the state is not low.\(^7\) By contrast, if pledgeability is selected after the state is realized, renegotiation would eliminate this type of misallocation since \textit{ex-post}, it is in the joint interests of the incumbent and financiers to reduce debt face value and restore incentives for high pledgeability.

**Example:**

We have shown that in the case with state-contingent contracts when \( \omega_{H,B}^0 = 0 \), the asset is sold to an industry outsider in state B at date 0, since the industry liquidity is very low. Now we increase \( \omega_{H,B}^0 \) to 0.1 and show that even at this moderate level of industry liquidity, the asset is misallocated in the same state if and only if debt contracts are used. Equally important, the asset would not be misallocated even with debt contracts \textit{if liquidity in the good state were lower}. Higher anticipated liquidity can induce more misallocation via the increase in debt!

Consider the following parameter values:

\[
\bar{\varphi} = 0.6, \quad \varphi = 0.3, \quad C_0 = 0, \quad C_1 = C_2 = 1, \quad \omega_{H,G}^0 = 0.7, \quad \omega_{H,B}^0 = 0.1, \quad \omega_{H,BG}^0 = 0.1, \quad \omega_{H,BB}^0 = 0.1, \quad \omega_{i,BG}^0 = 0.5, \quad \omega_{i,BB}^0 = 0.1, \quad \theta^H = 0.7, \quad \epsilon = 0, \quad q^G = 0.9, \quad q^{BG} = 0.9, \quad q^{BB} = 0.5, \quad \rho = 0.1,
\]

1) State-contingent contract

Suppose \( \gamma = 0.6 \), then \( B^H_{1,\gamma} (\gamma) = 0.9 > B^H_{1,BG} (\gamma) = 0.5 \) and \( B^i_{1,\gamma} (\gamma) = 0.4 = B^i_{1,BB} (\gamma) = 0.4 \).

If \( \gamma = 0.3 \) instead, \( B^H_{1,\gamma} (\gamma) = 1 > B^H_{1,BG} (\gamma) = 0.5 \) and \( B^i_{1,\gamma} (\gamma) = 0.4 = B^i_{1,BB} (\gamma) = 0.4 \). In both cases, the incumbent is able to retain control in both state BG and BB. Therefore, \( \hat{D}_{1,\gamma}^{BG,Max} = D_{1,\gamma}^{BG,PayIC} = 0.59 \) and \( \hat{D}_{1,\gamma}^{BB,Max} = D_{1,\gamma}^{BB,PayIC} = 0.49 \). At date 0, financiers (L types) bid up to \( B^L_{0,BB} = 0.75 \) while

\(^7\) The spillover between states via debt induces lower pledgeability than would be seen with state-contingent contracts. If there is significant underpricing in state G and high pledgeability increases bids sufficiently to cover its cost, high pledgeability is chosen with debt contracts even if in state B there is no value to high pledgeability, for instance because the asset is sold to industry outsiders. Nevertheless, this is not an example of \textit{“over-pledging”} induced by debt; even with state-contingent contracts, we would have the same effect of a high choice of pledgeability – the seemingly excessively high pledgeability in the B state, where it is of little use, stems from the fact that the pledgeability decision is made \textit{before} the state is known, and not from the fixed debt contract. It is in this sense that while debt induces under-pledging, it does not induce over-pledging.
industry insiders bid $B^H_B(\gamma_1 = \overline{\gamma}) = 0.94$ and $B^H_B(\gamma) = 0.79$. Finally, the incumbent bids $B^I_B(\overline{\gamma}) = 0.84$ and $B^I_B(\gamma) = 0.69$. Therefore, the incumbent always loses control to an industry insider at date 0 if the state is B.

2) Debt contract

Now consider debt contracts. Since the incumbent can retain control in both state BG and BB, $D^{B,IC}_1 = 0.58$ and the incumbent can raise more by setting $D^B = B^{H,BG}_1(\gamma) + \gamma_1 C_1 = 1.1$ which raises 0.75 in expectation. In this case, the restriction to debt contracts further constrains the amount that can be raised at date 0. It turns out therefore that $B^H_B(\gamma_1 = \overline{\gamma}) = 0.85$ and $B^H_B(\gamma) = 0.7$. However, low types are unaffected: $B^L_B = 0.75$. There is no potential underpricing in state G, GG and GB. Since $q^G = 0.9$ is very high, date-0 debt is set high and the incumbent sets $\gamma_1 = \gamma$ in period 0. If the state B is realized at the end of the period, the asset is sold to an industry outsider. Such a reallocation could have been avoided if $\gamma_1$ had been set at $\overline{\gamma}$, as in the state-contingent contract example, but that would have required issuing less debt. So debt leads to low pledgeability and greater misallocation.

3) Misallocation induced by high expected liquidity

Suppose now that date-0 liquidity were lower, so $\omega^{H,G}_0 = 0.5$. There exists potential underpricing in state G, GG and GB. Therefore, $D_0 = \gamma_0 C_0 + D^{B,PracIC}_0 = 0.745$ and $\gamma_1 = \overline{\gamma}$ would be chosen. As a result, the asset is “correctly” allocated to an industry insider if state B is realized. So lower liquidity leads to debt with a lower face value, which in turn leads to higher pledgeability and a more productive allocation of control in downturns. More generally, anticipated liquidity operating through greater leverage causes the adverse spillover between states which pushes down pledgeability and causes misallocation.

IV. Discussion and Empirical Relevance

In a boom which is likely to continue, liquidity is high and supports high debt. When borrowers finance with such high debt, however, they will choose low pledgeability, which nevertheless will be acceptable to lenders who anticipate a high probability of continued high liquidity. Liquidity, asset prices, and leverage follow each other up, while pledgeability falls. If the boom does not continue, and liquidity falls, access to finance will drop significantly. Outsiders are also more likely to take over the firm at such times and recoveries are likely to be slow. The “overhang” created by liquidity-induced leverage on pledgeability cannot be renegotiated away. Importantly, all this can occur even if the probabilities of the downturn are not insignificant. Higher anticipated liquidity is therefore not an unmitigated blessing, and can generate worse outcomes in less liquid realized states. To the extent that government or central bank policies create anticipation of liquidity, this is a concern that has to be kept in mind.
In contrast, when lower liquidity is anticipated (in normal or bad times), a choice of low pledgeability will significantly reduce the amount financiers expect to recover. Therefore, the competitive financial market will prevent “excessive” leverage (in terms of its effect on pledgeability) in times of lower liquidity because it wants incumbents to retain incentives to set high pledgeability. Importantly, all this can occur even if the probabilities of downturns are not insignificant.

Loan contracts with many covenants could be a proxy for the choice of high pledgeability. In bad to normal times we should see many covenants and relatively low levels of leverage when fresh capital structures are chosen (such as when the firm comes out of bankruptcy). In contrast, during booms we will see higher leverage and few or no loan covenants (“covenant lite”). Indeed, Christensen and Nikolaev (2014) separate loan covenants into those that are capital or balance sheet related (such as constraints on overall leverage) and those that are short-term performance or income-statement related tripwires such as return on asset ratios. Interestingly, they find that the number of balance sheet related covenants in loan agreements came down substantially in the years of high liquidity before the financial crisis in 2007-2008, only to rise during the crisis and soon after till 2010. In contrast, they find the number of performance related covenants remain fairly constant over the years. Arguably, this suggests lenders were more open to higher borrower leverage and lower asset-related constraints during the boom years of high liquidity only to tighten once conditions deteriorated.

Boom periods with covenant lite loans and high leverage could also be interpreted as an increase in the fraction of market finance (bonds or covenant lite loans) as opposed to intermediated finance (covenant-intensive bank debt).

Low pledgeability could also be caused by choice of lower accounting quality or weaker corporate governance. Compustat reports the auditor's opinion of the effectiveness of the company's internal control over financial reporting while auditing a company's financial statements, an opinion which is mandated by section 404 of the 2002 Sarbanes-Oxley Act. A material weakness is a deficiency, or a combination of deficiencies, in internal control over financial reporting, such that there is a reasonable possibility that a material misstatement of the company's annual or interim financial statements will not be prevented or detected on a timely basis. When an auditor indicates a material weakness, it signifies a previously undetected choice to degrade accounting quality which may not yet have influenced accounting reports, and can thus serve as a measure of low pledgeability. Figure 6 below indicates that the percentage of Compustat firms with material weakness of internal control—a measure of substandard accounting—started to increase in the extremely liquid period before the financial crisis, fell after the onset of the crisis, and started to increase again as central banks around the world maintained extremely liquid conditions in financial markets.
It may be useful here to see the differences between our model and the seminal work by Shleifer and Vishny (1992) (henceforth SV). They focus on liquidity varying over time. SV emphasize control rights exclusively through asset sales while we introduce control rights over cash flow through the pledgeability channel, which itself suffers from moral hazard. Our model therefore has different implications than Shleifer and Vishny (1992). As in SV, assets migrate in our model to agents who have lower ability to manage. However, the underlying rationale is different. In SV, assets get inefficiently allocated because highly ability managers have less liquidity than outsiders. Debt, which was created to resolve a free cash problem, has an overhang effect which limits the amount of liquidity obtainable by industry insiders. Therefore, if financial contracts were state-contingent (or if debt could be renegotiated), the asset would never be sold to outsiders. In our model, the asset goes to low types precisely because they do not suffer from the moral hazard over pledgeability and not because they have more liquidity. Indeed, financiers are unwilling to renegotiate debt down because they know the asset will be sold to outsiders who can pay more by making the asset more pledgeable even when burdened with high debt.

Our paper shares similar insights with a sequence of papers by Geanakoplos (for instance, Geanakoplos 2010) on leverage cycles—which are analogous to our financing cycles. Like us, Geanakoplos endogenizes the borrowing constraint, though by a different approach. In particular, he sets up a general equilibrium model in which agents have heterogeneous beliefs. Therefore,
optimists—those who assign a high probability on good states—naturally take on leverage and borrow from pessimists. Since all borrowing is collateralized, the borrowing constraint is endogenously determined by the beliefs of heterogeneous agents. The specific mechanisms are different though. In our setup, pledgeability is essentially a choice by asset holders (incumbent), whereas in his model, pledgeability is always fixed at one. The beliefs of pessimists determine their willingness to lend, for a given amount of collateral. This, together with the beliefs of optimists which determine their willingness to borrow, pins down the loan-to-value ratio in equilibrium. After a bad shock, or just increased anticipation of one, optimists lose wealth as well as their ability to borrow on leverage. Consequently, the asset migrates to more pessimistic hands and is valued less. Excessive leverage taken in booms, if followed by bad news, leads to excessive deleveraging in bad times, even before/without an actual crash in fundamentals. This constitutes the leverage cycle. The asset price is very high in the initial or overleveraged normal economy, and after deleveraging, the price is even lower than it would have been had there never been the overleveraging in the first place.

A crucial difference between the two papers is that in our model all participants could have the same beliefs about the future. If debt contracts are the best way to raise finance, high anticipated liquidity in some future states will prompt the issuance of a lot of debt today, with both the borrower and the lender rationally accepting the adverse consequences of debt spilling over into the future low-liquidity states. In hindsight, from the vantage point of the low liquidity state, it might appear that participants neglected the possibility that it would occur, or were overly optimistic. As our model suggests, they may rationally neglect to prepare (by neglecting pledgeability) for such states.

Our paper therefore also bears some resemblance to papers where a small probability of a regime change is irrationally (Gennaioli, Shleifer, and Vishny (2010)) or rationally neglected (Dang, Gorton, and Holmstrom (2009)), though our results on the effect of anticipated liquidity on leverage and pledgeability hold even if the probability of the seemingly neglected state is not small. Our point is that the overhang of debt on pledgeability cannot be reversed immediately in bad times, unlike expectations of outcomes or information acquisition, because pledgeability takes time to reset. Therefore, not only is there a collapse in access to finance, but also a restoration of access takes time.

Brunnermeier and Sannikov (2014) present a model that also explains slow recoveries. In their model, slow recoveries follow if productive agents, constrained by low wealth (industry liquidity) are forced to misallocate assets to less productive agents. In their case, increasing the wealth level of productive agents always accelerates recovery. In our model, however, we show the downside of increasing ex-post liquidity when debt leverage is allowed to respond to liquidity: productive agents can get further financially constrained, and asset misallocation can be more severe.
V. Conclusion

We have focused on two kinds of moral hazard in this paper – moral hazard over appropriation of cash flows and moral hazard over pledgeability choice. In good times, the threat of ownership change is the means of enforcing debt contracts, and plentiful liquidity makes the threat credible. The seeds of distress are sown at such times, because incumbents have no incentive to maintain cash flow pledgeability – this alternative source of commitment seems unnecessary when times largely promise to be good, and the incentive to maintain high levels is further suppressed by the high leverage that is induced by liquidity. Importantly, there may also be general equilibrium effects that we have not modeled, where institutions supporting pledgeability, such as forensic accountants, regulations, and regulators, may atrophy from disuse at such times. As bad times hit, financing capacity plunges, and outsiders who have a better ability to take on leverage may outbid insiders. Cash flow pledgeability now becomes key to debt capacity, and industry outsiders have the incentive to increase it even in the face of high debt – it is precisely their ineffectiveness in managing the asset that makes them immune from moral hazard over pledgeability. As cash flow pledgeability increases and industry cash flows recover somewhat, industry insiders can once again bid large amounts and return to controlling firms. As liquidity among industry insiders increases further, the threat of asset sales once again becomes the source of debt enforcement. The incentive to maintain cash flow pledgeability wanes once again, and the cycle resumes.

Importantly, the change in effective creditor control rights, from cash-flow-based to asset-sale-based, occurs smoothly when economic conditions continue to improve. Incumbents simply neglect to maintain pledgeability since it is not needed to raise financing. However, when boom turns to bust, past neglect of pledgeability and the distortion to incentives caused by debt overhang ensure the transition from asset-sale-based to cash-flow-based enforcement is not smooth. Economic activity can be disrupted until outside capacity to control (and thus finance) is restored. Real investment, which we do not model, could fall significantly under these circumstances, even when it is positive net present value.

Our model suggests why assets that require management (such as mortgages or bank loans, or the securitized claims on such assets) may have different collateral haircuts associated with them over the cycle, unlike passively held assets such as equities. While asset values fall uniformly with the fall in liquidity, haircuts lenders apply to assets to gauge lending limits fall in proportion to both the liquidity of industry insiders (in the upturn) and the restoration of pledgeability (in the downturn), with a possible steep increase as the state of the economy switches from upturn to downturn.

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8 While we do not model investment, the point we make would become stronger still if we did. A greater share of the pie is more attractive when increasing the pie through new investment is difficult, so moral hazard over pledgeability increases still further in a downturn, over and above the effects of leverage.
Finally, the fluctuation in debt capacity may be larger if the range of possible pledgeability values is larger. To the extent that financial infrastructure such as accounting standards or collateral registries as well as contract enforcement are strong through the cycle, they may prevent large fluctuations in asset pledgeability. By allowing only moderate room to alter pledgeability, a strong institutional environment could lead to more stable credit. However, to the extent that the institutional environment is weak or responds to the cycle (regulators get complacent in good times), asset pledgeability is more endogenous, and credit may vary more over the cycle. Credit booms and busts will be more pronounced in such cases, as will asset price booms and busts.

Our model does not allow for entrenchment, but could be extended to explore its effects. Essentially, if the incumbent loses some but not all capacity to produce when she loses ability, moral hazard over pledgeability increases, and ex-ante she can borrow even less than earlier, if incentives to maintain pledgeability have to be maintained.

Finally, this paper has focused on the choice of pledgeability, assuming that both incumbent and industry insider have access to the same sources of pledgeability. Incumbent pledgeability could be different from the pledgeability industry insiders could utilize – the incumbent may be able to borrow more from relationship banks than can an industry insider who does not know the bankers. The gap between incumbent pledgeability and industry pledgeability may have independent importance over the cycle, and understanding this may be fruitful. Moreover, we can delve deeper into the sources of pledgeability and its dynamics. Institutions that were designed to raise pledgeability also change over the financing cycle. When there is a prolonged aggregate boom (with a good probability of continuing), there will be little demand for increased pledgeability. The institutions and professions which reinforce pledgeability will atrophy, and those with such specific skills will depart these professions. If we were to introduce more heterogeneity of borrowers, this would make it more difficult to increase pledgeability when other firms do not value such an increase.

We plan to explore more of these implications in future work.

References


Appendix

Proof of Lemma 2.1:

(i) If $B_{1}^{H,s}(\gamma) = C_2$, then industry insider’s bids are robust to the pledgeability choice:

$B_{1}^{H,s}(\gamma) = B_{1}^{H,s}(\gamma) = C_2$. For any level of $\tilde{D}^{*}$, the incumbent’s payoff from choosing high pledgeability is $\max\left\{ C_2 - \tilde{D}^{*}, 0 \right\} - \epsilon$ while the payoff from choosing low pledgeability is $\max\left\{ C_2 - \tilde{D}^{*}, 0 \right\}$. Hence, $\gamma_2 = \gamma$. Note here the amount $C_2$ could from either asset resale ($B_{1}^{H,s}(\gamma)$) or production in period 2. For any level of $\tilde{D}^{*} > C_2$, the incumbent always defaults and the maximal amount collected by financiers is $C_2$. Hence, $\tilde{D}^{*,\text{Max}} = C_2$ and $V^{i,s}_{1}(\tilde{D}^{*}) = C_2 - \tilde{D}^{*}$.

(ii) If $C_2 > B_{1}^{H,s}(\gamma)$ and $B_{1}^{i,s}(\gamma) < B_{1}^{H,s}(\gamma)$, we consider two subcases: $B_{1}^{i,s}(\gamma) = C_2$ and $B_{1}^{i,s}(\gamma) < C_2$.

a. If $B_{1}^{i,s}(\gamma) = C_2$, then $B_{1}^{i,s}(\gamma) = C_2$ since $B_{1}^{i,s}(\gamma) < B_{1}^{H,s}(\gamma)$. For any level of $\tilde{D}^{*}$, the incumbent’s payoff from choosing high pledgeability is $\max\left\{ C_2 - \tilde{D}^{*}, 0 \right\} - \epsilon$ while the payoff from choosing low pledgeability is $\max\left\{ C_2 - \tilde{D}^{*}, 0 \right\}$ if $\tilde{D}^{*} > B_{1}^{i,s}(\gamma)$ and $\theta^{H}\left( C_2 - \tilde{D}^{*}\right) + (1 - \theta^{H})\left( B_{1}^{H,s}(\gamma) - \tilde{D}^{*}\right)$ if $\tilde{D}^{*} \leq B_{1}^{i,s}(\gamma)$. Hence, $\gamma_2 = \gamma$ if and only if $\tilde{D}^{*} \leq C_2 - \epsilon \equiv \tilde{D}^{*,\text{Max}}$. For any promised payment $\tilde{D}^{*} \leq \tilde{D}^{*,\text{Max}}$, the incumbent expects $\max\left\{ C_2 - \tilde{D}^{*}, 0 \right\} - \epsilon$.

b. If $B_{1}^{i,s}(\gamma) < C_2$, then the incumbent is outbid for any pledgeability choice. For any level of $\tilde{D}^{*}$, the incumbent’s payoff from choosing high pledgeability is $\max\left\{ B_{1}^{H,s}(\gamma) - \min\left\{ B_{1}^{H,s}(\gamma), \tilde{D}^{*}\right\}, 0 \right\} - \epsilon$ if $\tilde{D}^{*} > B_{1}^{i,s}(\gamma)$ and $\theta^{H}\left( C_2 - \min\left\{ B_{1}^{H,s}(\gamma), \tilde{D}^{*}\right\}\right) + (1 - \theta^{H})\left( B_{1}^{H,s}(\gamma) - \tilde{D}^{*}\right)$ if $\tilde{D}^{*} \leq B_{1}^{i,s}(\gamma)$. The incumbent’s payoff from choosing low pledgeability is $\max\left\{ B_{1}^{H,s}(\gamma) - \min\left\{ B_{1}^{H,s}(\gamma), \tilde{D}^{*}\right\}, 0 \right\} - \epsilon$ if $\tilde{D}^{*} > B_{1}^{i,s}(\gamma)$ and $\theta^{H}\left( C_2 - \min\left\{ B_{1}^{H,s}(\gamma), \tilde{D}^{*}\right\}\right) + (1 - \theta^{H})\left( B_{1}^{H,s}(\gamma) - \tilde{D}^{*}\right)$ if $\tilde{D}^{*} \leq B_{1}^{i,s}(\gamma)$. Hence, $\gamma_2 = \gamma$ if and only if $\tilde{D}^{*} \leq B_{1}^{H,s}(\gamma) - \epsilon \equiv \tilde{D}^{*,\text{Max}}$.

(iii) If $C_2 > B_{1}^{H,s}(\gamma)$ and $B_{1}^{i,s}(\gamma) \geq B_{1}^{H,s}(\gamma)$, then the incumbent can outbid industry insiders for any pledgeability choice. For any level of $\tilde{D}^{*}$, the incumbent’s payoff from choosing high pledgeability is $\theta^{H}\left( C_2 - \min\left\{ B_{1}^{H,s}(\gamma), \tilde{D}^{*}\right\}\right) + (1 - \theta^{H})\left( B_{1}^{H,s}(\gamma) - \min\left\{ B_{1}^{H,s}(\gamma), \tilde{D}^{*}\right\}\right) - \epsilon$ The incumbent’s payoff from choosing low pledgeability is
Proof of Proposition 2.1:

Since \((1-\gamma)C_2 > \omega^{H, G}_0 \geq (1-\gamma)C_2 - \rho C_1 > \omega^{H, B}_0\), there is no potential rents to acquirers if and only if in state GG. Thus, according to Lemma 2.1, low pledgeability is chosen only in state GG.

Proof of Proposition 2.2:

Apply Assumption 2.1 and Lemma 2.1, it is easily verified that in state BG, the incumbent can always outbid industry insiders. In state GB and BB, however, the incumbent has strictly less wealth than industry insiders and thus can only retain control by repaying the required payment.

Proof of Lemma 3.1:

(i) If \(B^{H, s}_1(\gamma) = C_2\), \(V^{i,s}_1(\gamma_2 = \bar{\gamma}) = \max \left\{ C_2 - \tilde{D}_t^i - \varepsilon, -\varepsilon \right\}\) and \(V^{i,s}_1(\gamma_2 = \gamma) = \max \left\{ C_2 - \tilde{D}_t^i, 0 \right\}\).

Thus, \(\Delta^i = -\varepsilon\).

(ii) If \(C_2 > B^{H, s}_1(\gamma)\) and \(B^{i,s}_1(\gamma) < B^{H, s}_1(\gamma)\),

\[
V^{i,s}_1(\gamma_2 = \gamma) = \begin{cases} 
-\varepsilon & \text{if } \tilde{D}_t^i > B^{H, s}_1(\gamma) \\
B^{H, s}_1(\gamma) - \tilde{D}_t^i - \varepsilon & \text{if } B^{i,s}_1(\gamma) < \tilde{D}_t^i \leq B^{H, s}_1(\gamma), \\
\theta^H C_2 + (1-\theta^H)B^{H, s}_1(\gamma) - \tilde{D}_t^i - \varepsilon & \text{if } \tilde{D}_t^i \leq B^{i,s}_1(\gamma)
\end{cases}
\]

(iii) If \(C_2 > B^{H, s}_1(\gamma)\) and \(B^{i,s}_1(\gamma) \geq B^{H, s}_1(\gamma)\),

Thus,

\[
\Delta^{i,s}(\tilde{D}_t^i) = \begin{cases} 
\min \{-\varepsilon, B^{H, s}_1(\gamma) - \tilde{D}_t^i - \varepsilon\} & \text{if } \tilde{D}_t^i \geq B^{H, s}_1(\gamma) - \varepsilon \\
> 0 & \text{if } \tilde{D}_t^i < B^{H, s}_1(\gamma) - \varepsilon.
\end{cases}
\]
\[ V_{1,2}^s (\gamma_2 = \gamma) = \begin{cases} \theta^H \left( C_2 - B_1^{H,s} (\gamma) \right) - \varepsilon & \text{if } \tilde{D}_1^s > B_1^{H,s} (\gamma) \\ \theta^H C_2 + (1 - \theta^H) B_1^{H,s} (\gamma) - \tilde{D}_1^s - \varepsilon & \text{if } \tilde{D}_1^s \leq B_1^{H,s} (\gamma) \end{cases} \]

Thus, \[ \Delta_{s}\gamma_1 (\tilde{D}_1) \begin{cases} \leq 0 & \text{if } \tilde{D}_1 \geq D_{1,PayIC}^{s}\gamma_1 \\ > 0 & \text{if } \tilde{D}_1 < D_{1,PayIC}^{s}\gamma_1 \end{cases} \]

**Proof of Lemma 3.2:**

(i) If there is no potential underpricing in state \( G \), \( \Delta_{s}^{s,G} (\tilde{D}_1^{s,G}) = -\varepsilon \) for all \( \tilde{D}_1^{s,G} \). If there is potential underpricing in state \( B \), then \( D_{1,IC}^{s,B} \rightarrow \tilde{D}_1^{s,B,\max} \) since \( \Delta_{s}^{s,B} (\tilde{D}_1^{s,B,\max}) = 0 \).

(ii) If the incumbent has no hope of retaining control in state \( B \), \( \Delta_{s}^{s,B} (\tilde{D}_1^{s,B}) = -\varepsilon \) for \( \tilde{D}_1^{s,B} \geq \tilde{D}_1^{s,B,\max} + \varepsilon \). Since \( \Delta_{s}^{s,G} (\tilde{D}_1^{s,G,\max}) = 0 \), \( D_{1,IC}^{s,B} \rightarrow \gamma_1 C_1 + \tilde{D}_1^{s,G,\max} \).

**Proof of Lemma 3.3:**

(i) Given the parameter range, there exists no potential rents to acquirers in state \( s_0 G \). In state \( s_0 B \), the incumbent can retain control irrespective of the pledgeability choices. Apply Case (i) of Lemma 3.2, the result \( D_1^{s,IC} = D_1^{s,B,PayIC} \) follows. In case a), setting \( D_1^{s} = \gamma_1 C_1 + C_2 \) enables the incumbent to raise more in expectation despite of low pledgeability choice \( \gamma_2 = \gamma \). In case b), setting \( D_1^{s} = D_1^{s,IC} \) always raises more in expectation.

(ii) Given the parameter range, in state \( s_0 G \) the incumbent can retain control irrespective of the pledgeability choices. In state \( s_0 B \), she has no hope of retaining control. Apply Case (ii) of Lemma 3.2, the result \( \omega_0^{s,\lambda} \) follows.

**Proof of Lemma 3.4:**

Similar to the proof of Lemma 3.2.

**Proof of Lemma 3.5:**

Similar to the proof of Lemma 3.3.