Do Financial Factors Drive Aggregate Productivity? Evidence from Indian Manufacturing Establishments*

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Abstract

Numerous countries have implemented financial reforms in the past three decades, but how these reforms affect economic growth has not been established. I develop a dynamic model with heterogeneous firms and endogenous leverage to isolate the effects of financial development on aggregate productivity growth. Financial development affects aggregate productivity by shifting the allocation of resources across firms. However, productivity growth that is common to all firms but unrelated to finance also changes the allocation of resources across firms, because firms respond to productivity growth by changing leverage. I calibrate the model to plant-level data from India and find that resource re-allocation consistent with financial development explains 2%–7% of Indian labor productivity growth from 1990 to 2011. My work suggests that factors that affect productivity within firms are more important determinants of aggregate productivity than financial development.

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1 Introduction

Although dozens of countries have instituted financial reforms in the past three decades, economists still debate whether and how these reforms affect economic growth. One channel through which better-functioning financial markets might increase economic growth is a superior allocation of resources, which raises aggregate productivity. In this paper I gauge the quantitative importance of reallocation for productivity growth in a large developing economy, India.

In this paper, I separate shocks to financial development from common shocks to productivity by comparing their effects on the joint distribution of size and productivity across firms. I derive a dynamic model in which the allocation of resources across firms is endogenous, depends on the degree of financial frictions, and affects aggregate productivity. Reductions in financial frictions increase aggregate productivity by reducing the cost of capital for borrowing firms, causing these firms to grow faster. Because in equilibrium borrowing firms are also more productive, financial development strengthens the positive relationship between size and productivity, without directly affecting productivity within firms. In contrast, an increase in productivity common to all firms directly affects both within-firm productivity and the relationship between size and productivity, because more-productive firms respond to an increase in their productivity by borrowing and investing more. Tracking the size-productivity distribution over time thus allows the model to distinguish changes in financial frictions from other shocks that affect the common component of aggregate productivity.

[Figure 1 about here.]

To illustrate the quantitative implications of the model, I calibrate it to establishment-level microdata from Indian manufacturing plants, and then use it to distinguish financial development from changes in aggregate productivity in India over the 1990–2011 period. India is an ideal setting for testing the effects of financial development: it began a major series of financial reforms in 1991, after the start of my sample period, and reforms continued over the next two decades. Importantly, the labor productivity of the Indian manufacturing sector has also grown tremendously over time (Figure 1). My methodology allows me to quantify how much of this improved performance is attributable to the increased efficiency of the financial sector.

I find that financial development accounts for between 2% and 7% of aggregate labor productivity growth over the 1990 to 2011 period. Common shocks to total-factor productivity explain both the within-firm and the across-firm components of aggregate labor productivity growth for most of the sample period. Although the level of financial frictions is important in understanding how common shocks to productivity affect the allocation of resources across firms, financial frictions themselves are not diminishing in India since 1995. Thus, to understand productivity growth in India since 1995 we need to understand the factors that lead to productivity growth within firms, rather than the factors that lead to reallocation across firms.

The key ingredient in my quantification exercise is a model in which a simple decomposition of aggregate labor productivity, the Olley-Pakes decomposition, separates changes in financial frictions from changes in total-factor productivity that are common to all firms. Olley and Pakes (1996) decompose aggregate productivity into a within-firm component and an across-firm component representing the cross-sectional covariance between size and productivity. The second term of the decomposition is commonly interpreted as a measure of allocative efficiency; I derive a structural model in which the across-firm component is positive in equilibrium, and increases in response to both financial development and common shocks to productivity. I then use the model to understand which of these two forces is more important for understanding aggregate productivity growth.

I find that rising productivity across all firms explains both terms of the Olley-Pakes decomposition over time in India. The mechanism that allows a productivity shock common to all firms to explain both terms of the Olley-Pakes decomposition relies on three assumptions: (1) firm-level productivity is persistent, (2) borrowers want to smooth consumption, and (3) borrowers can default on their debt. Borrowers pay a premium for any leverage (liabilities over assets) beyond the collateral rate sufficient to cover the lender's losses should they default. For any level of leverage, more-productive firms represent a better credit risk, and thus enjoy a lower interest rate. In addition, because borrowers want to smooth consumption and productivity is persistent, at any interest rate more-productive firms want to borrow more. Taken together, in equilibrium, more-productive firms will choose higher leverage ratios, grow faster on average, and thus be larger than less-productive ones. In addition, more-productive firms respond more to the same increase in productivity than less-productive firms, because they increase their borrowing in response to reduced interest rates. I find that the common increase in productivity that best explains the within-firm component of the Olley-Pakes decomposition also explains most of the variation in the across-firm component.

I also show that standard difference-in-difference regressions can be misleading when productivity is growing over time for all firms. Following the literature on financial development, I separate firms into those that are more or less sensitive to financial development, and show that more-sensitive firms and industries exhibited greater leverage and aggregate output growth in India from 1990 to 2011. However, I simulate data from the model to show that such regressions can violate the parallel trends assumption, that in the counterfactual world without financial development, the more-sensitive firms would exhibit the same growth rate as the less-sensitive firms. The violation arises because even though both types of firm receive the same productivity shock, that shock affects borrowing firms—i.e., those more affected by financial development—more than others, by reducing their cost of borrowing. Thus, positive coefficients in difference-in-difference regressions are neither necessary nor sufficient for distinguishing financial development from productivity growth.

My paper furthers our understanding of how financial markets affect economic growth. Since the pioneering work of King and Levine (1993), a vast literature has shown that economic growth is robustly correlated with larger and more-efficient financial sectors; however, this literature struggles to impute causality. Levine (2005) notes that "If finance is to explain economic growth, we need theories that describe how financial development influences resource allocation decisions in ways that foster productivity growth." In this paper, I propose a model in which financial development can affect economic growth, through a reallocation channel. I fit the model to Indian establishment-level microdata and use it to gauge the likely magnitude of financial development on aggregate productivity relative to other factors, represented by a common shock to productivity. I find that the reallocation channel explains less than 10% of Indian labor productivity growth since 1990.

2 Literature Review

I contribute to a literature that uses cross-country regression evidence to show that economic growth is highly correlated with the size of the financial sector. Since its beginnings, this literature has struggled to show that causality runs from finance to growth. King and Levine (1993) argued that because financial development predicts subsequent economic growth over the next thirty years, causality does not run from growth to finance. Subsequent work has deepened the basic finance-growth correlation using panel-IV methods (Levine, Loayza and Beck 2000) and by showing that more financial intermediation is primarily associated with higher productivity, and not more savings or capital accumulation (Beck, Levine and Loayza 2000). La Porta, Lopez-De-Silanes and Shleifer (2002) show that in addition to the indicators used in Beck, Levine and Loayza (2000), higher government ownership of the banking sector is strongly associated across countries with lower aggregate productivity. This finding is important because since reforms began in 1991, policymakers in India have worked continually to reduce the proportion of the banking sector controlled by the government (Rajan et al. 2009, Rajan 2016).

A related literature uses industry- and firm-level data, rather than country-level aggregate variables, to draw out the mechanisms whereby financial innovation affects economic growth; I contribute to this literature by directly linking financial frictions at the firm level to aggregate productivity, using both theory and empirical evidence. In a seminal paper, Rajan and Zingales (1998) argue that better-functioning financial markets should disproportionately affect industries that are more dependent on external finance, by lowering their cost of capital; they propose a measure of such external dependence, and use it to show that financial development exerts a greater influence on the growth rates of externally dependent industries across countries. Wurgler (2000) also uses industry-level data to show that the elasticity of investment growth to value-added growth is higher in more financially-developed countries, suggesting that better-functioning financial markets improve the allocation of resources. Beck, Demirgüç-Kunt and Maksimovic (2005) use cross-country firm-level survey data to show that financial factors are an important obstacle to firm growth, especially for small firms. The results of this literature suggest that improvements in financial markets remove constraints on the growth of some industries and firms, enhancing allocative efficiency, aggregate productivity, and economic growth.

In an influential paper, Olley and Pakes (1996) derive an empirical decomposition to measure changes in allocative efficiency over time. They decompose aggregate productivity into a within-plant term and an across-plant term representing the extent to which larger plants are more productive. They then relate changes in the second term to regulatory changes that affected the telecommunications industry in the United States. Bartelsman, Haltiwanger and Scarpetta (2013) perform the same decomposition for all manufacturing industries in a number of European countries and interpret the across-plant term as an overall measure of allocative efficiency, showing that it is high for more-developed economies, such as the United States and Germany, but lower—though growing over time—for some formerly Communist countries in Eastern Europe. I derive an equilibrium model that relates the across-plant term directly to economic shocks—in particular, changes in financial frictions and common shocks to productivity. I then apply the model to India, another country in which the across-plant term has grown over time, and use it to infer the sources of that growth.

My paper also contributes to a related literature that ties productivity differences across countries to the misallocation of resources. In an important paper, Hsieh and Klenow (2009) use a parsimonious model to measure how much of the productivity difference between the United States and two large developing economies is due to a poor allocation of resources. Rather than measure the distance from the realized allocation to the first-best allocation, as they do, I use of the Olley-Pakes decomposition to measure changes in the realized allocation over time. My paper is also related to recent work by Banerjee and Moll (2010) and Midrigan and Xu (2014), who propose models with financial frictions to explain a poor allocation of resources. They find that any misallocation due to financial frictions disappears quickly over time because firms can grow their way out of binding financial constraints. My model nests the Hsieh and Klenow (2009) framework, and has the same persistence of shocks as Midrigan and Xu (2014). I find that the model matches the size-productivity distribution among Indian manufacturing plants only when enough firms are constrained in equilibrium and misallocation is high. Thus, financial shocks have the potential in my model to exert a strong influence on aggregate productivity. Nevertheless, I find that changes in the size-productivity distribution of Indian manufacturing plants point to common shocks to productivity as the dominant factor driving growth, not financial development.

Finally, my paper relates to a new wave of macroeconomic research that adds financial factors to macroeconomic models. Much of this research, including Gertler and Kiyotaki (2010), Gertler, Kiyotaki and Queralto (2012), Buera and Moll (2012), and Jermann and Quadrini (2012), assumes a maximum leverage constraint as a parsimonious way to model financial frictions. By requiring that borrowing be fully collateralized, these models assume that debt is riskless; often this assumption is justified as satisfying an incentive-compatibility constraint. My paper enriches the financial side of these models by noting that in equilibrium lenders are not interested in whether individual loans are incentive-compatible; rather, they care only about the return on their entire portfolio. Lenders, who observe borrowers' productivity and leverage choices, can earn the same expected return by charging borrowers a higher interest rate to compensate for default risk. Borrowers, rather than facing a "hard" borrowing constraint, face an interest rate schedule that depends on their productivity and choice of leverage. In equilibrium, this modeling framework allows common shocks to productivity to induce reallocation across firms, because *ceteris paribus* an increase in productivity reduces the probability of default and thus firms' borrowing costs. This additional channel only matters for firms that want to borrow, which in equilibrium are more productive than non-borrowing firms.

3 Empirical Evidence

In this section, I describe the Indian economic reforms mentioned in section 1, and use microdata from Indian manufacturing plants to decompose aggregate productivity growth into a common factor within plants and a factor representing the allocation of resources across plants. In section 3.1, I describe the Indian macroeconomic and financial reforms in more detail. I describe the data in section 3.2, and perform the decomposition of aggregate productivity in section 3.3. Sections 4 and 5 derive, calibrate, and analyze a model to interpret these empirical results. For more details on the data, see section 4 of the online appendix.

3.1 Background

India instituted a number of economic reforms in 1991 after a severe balance-of-payments crisis. Many of the reforms affected the manufacturing sector directly. For instance, the goverment essentially abolished the system of industrial licensing, a major constraint on investment and output for registered manufacturing firms (Aghion et al. 2008). In addition, a large number of industries that had been reserved solely for the government were opened up to private entry (Ahluwalia 2002). Reforms in 1991 that took immediate effect also include a massive drop in tariffs, especially for capital goods, and freeing up of foreign direct investment restrictions (Joshi and Little 1996). Both of these reforms are likely to have made expanding production easier for productive firms.

In addition to reforms targeted directly at the manufacturing sector, the government also instituted various financial reforms in 1991. Chief among these were changes to the Indian banking system, which was dominated by poorly-performing government-owned banks. Accounting rules that allowed banks to hide non-performing assets were changed, and the government recapitalized public banks with negative net worth. High reserve ratios, whose main purpose was to pre-empt banking resources to finance the government deficit, fell dramatically in 1991. The government also allowed for more private entry into the banking sector, including allowing the public-sector banks to raise capital in public equity markets, diluting the amount of government ownership (Joshi and Little 1996).

Not all Indian financial reforms occurred immediately after the 1991 crisis. The Recovery of Debts Act, which established tribunals in several major cities to aid in the process of recovering bad debts, was passed in 1993 but did not become effective immediately, because of challenges in the court system (Joshi and Little 1996). Visaria (2009) analyzes the effects of these tribunals on firm borrowing and repayment decisions. The Securitisation and Reconstruction of Financial Assets and Enforcement of Security Interest Act of 2002 (known as the Sarfaesi Act) allowed for the creation of asset-reconstruction companies to which banks could auction non-performing loans (Rajan et al. 2009). However, Vig (2013) argues that the Sarfaesi Act mainly resulted in borrowers substituting away from secured debt, toward unsecured debt (which is not covered by the Act). Finally, the process of removing the government from control of the banking sector has also proceeded slowly since 1991 (Rajan et al. 2009). Rajan (2016) shows that, although the efficiency of the banking sector in India has improved tremendously, major reforms are still needed, especially among public-sector banks.

3.2 Data

I combine data from two sources, the Annual Survey of Industries (ASI) and the National Sample Survey (NSS). The ASI is a survey of manufacturing establishments in India that are registered under the 1948 Factories Act, which covers plants with more than 10 employees, or more than 20 employees if the plant doesn't use electric power. Registered plants in the ASI usually have more than 10 employees, though they remain registered even if their employment falls below 10, and include the largest factories in India. By contrast, the NSS is a survey of unregistered plants, which are typically much smaller than those in the ASI.[1] Because the NSS is administered to manufacturing plants only roughly every five years, I only have five years of combined ASI-NSS data: for 1989–1990, 1994–1995, 1999–2000, 2005–2006, and 2010–2011. Data years refer to the Indian fiscal year, which runs from April to March; for the rest of the paper, when only one year is given, I am referring to the second year of a pair.

[Table 1 about here.]

Table 1 reports various summary statistics for the NSS and ASI data sets separately. Panel A reports the total number of manufacturing plants and employees, as well as the NSS shares of firms, employment, and output (real value-added in 1993–1994 rupees) over time. The Indian manufacturing sector employs between 27 million and 46 million people, and this number has been rising over time. Over 99% of the 12–17 million manufacturing plants in India are in the unregistered, informal sector covered by the NSS. These plants account for about 80% of manufacturing employment in India, but only between a sixth and a quarter of manufacturing output. Thus, NSS plants must be much smaller, and substantially less productive, than their ASI counterparts.

Panel B of Table 1 reports the employment distribution of plants in the NSS and ASI. NSS plants employ on average only two employees, compared to 67–135 employees on average in the ASI. Median employment in the ASI is much lower at about 20 employees, reflecting the substantial positive skewness in the plant-size distribution in the ASI. The employment measure used in this paper is the average number of employees over the course of the year, including administrative and part-time employees.

Plants in the ASI are not only larger, but also much more productive than plants in the NSS. Panel C of Table 1 reports the distribution of (log) labor productivity in the two data sets. Plants in the ASI are roughly 200 log-points more productive than plants in the NSS, though productivity is rising over time for both types of plants. I define productivity as the natural logarithm of real value-added, in 1993-1994 rupees, per employee. Value-added is total revenue, including revenue from non-production activites, less total costs, including materials inputs and other expenses but excluding the cost of labor. I deflate nominal value-added using the industry-level Wholesale Price

[1] Although I will refer to plants in the NSS as "informal" plants or representing the "informal" sector, NSS respondents are not informal in the sense of operating illegally or evading taxes. They are merely manufacturers that are not covered by the 1948 Factories Act, and as a result are not part of the sampling frame of the ASI.

Indices; see section 3 of the online appendix for details.

I focus on revenue productivity, rather than physical productivity, in this paper in order to include the NSS plants in my analysis. Because the NSS plants are such a substantial fraction of employment and output in India, I opt to include them and focus on revenue productivity for the empirical analysis of this paper. Revenue productivity can differ from physical productivity when plants in the same industry charge different prices; see Foster, Haltiwanger and Syverson (2008) for an analysis of this phenomenon in the United States. Although the exogenous shocks in the model of section 4 will be to physical productivity, I construct all measures in the model (for example, of the Olley-Pakes decomposition) using endogenous revenue productivity.

[Table 2 about here.]

Although plants in the NSS are much smaller than those in the ASI, they do have access to external sources of credit that were affected by the Indian financial reforms described in section 3.1. Table 2 reports statistics on borrowing across the two data sets. Between 6% and 10% of NSS plants have loans outstanding, though this number is decreasing over time. ASI plants are much more likely to borrow; between two-thirds and three-quarters of ASI plants report having loans. Because ASI plants are so much bigger than NSS plants, and are more likely to have a loan at all, plants in NSS account for between 3% and 7% of total debt outstanding in the combined ASI-NSS data. The tiny fraction of total borrowing that NSS plants account for is consistent with the model developed in section 4, because NSS plants are so much less productive than their ASI counterparts.

[Figure 2 about here.]

Figure 2 plots the sources of credit for NSS plants over time.[2] About 60% of NSS loans come from commercial banks, including state-owned banks and other government sources (e.g., the Khadi & Village Industries Commission). This observation is significant because many of the financial reforms covered in section 3.1 were specifically targeted at making the banking sector more efficient. The remaining 40% of NSS credit is about evenly split between money-lenders, loans from friends, family, and business partners, and other sources. The NSS does not ask about loans from mciro-finance institutions until 2011; these loans represent a small fraction of total borrowing in that year.

[2] ASI plants do not break down their sources of credit.

3.3 Aggregate Productivity

In this section, I document the strong correlation between plant size and labor productivity in India. I then quantify the effect of this covariance on aggregate productivity using a simple decomposition due to Olley and Pakes (1996).

[Figure 3 about here.]

In India, larger plants are vastly more productive than smaller plants, as shown in Figure 3. The top panel plots the distribution of real log labor productivity across plants in six employment categories, where I chose the categories to account for roughly the same share of total employment.[3] Although there is substantial dispersion in productivity across plants within any size category, the entire distribution of productivity is shifted upwards for plants with more than five employees. The bottom panel of Figure 3 reports the shares of aggregate employment accounted for by each size bin, for reference. Each bin accounts for a large share of aggregate employment, although over 60% of Indian manufacturing employees (leftmost three categories) work in low-productivity establishments with fewer than five employees.

I quantify the impact of the covariance between size and productivity using a simple decomposition of aggregate productivity derived by Olley and Pakes (1996). If aggregate productivity is defined as a weighted average of plant-level productivities,[4] it can be written as the sum of a within-firm (average) productivity term and an across-firm covariance term:

$$\begin{aligned}\text{Aggregate Productivity} &\equiv \sum_{i=1}^{N} w_i z_i \\ &= \underbrace{\frac{1}{N}\sum_{i=1}^{N} z_i}_{\text{average productivity } \mathcal{Z}} + \underbrace{\sum_{i=1}^{N} (z_i - \mathcal{Z})\,(w_i - \overline{w})}_{\text{OP covariance } \mathcal{C}},\end{aligned} \tag{1}$$

where z_i is log labor productivity at plant i, w_i is plant i's share of total industry employment, and $\overline{w}$ is the unweighted average of w_i $(1/N)$. I define aggregate productivity in this identity as a

[3] The figure pools across both industries and time. Removing industry means changes the picture imperceptibly; the entire picture is shifted upwards over time, in all six categories.

[4] Using a weighted average of plant-level log labor productivities can be justified as a first-order approximation to the natural logarithm of total output divided by total employment, around any point where all plants have the same productivity.

weighted average of plant-level productivities, which, because the weights $\{w_i\}$ sum to 1, can be decomposed into an unweighted average $\mathcal{Z}$ across plants plus a term $\mathcal{C}$ representing the covariance between the weights (plant size) and productivity. The level of $\mathcal{C}$ indicates how much aggregate productivity would drop if, holding all plant productivities fixed, employment were re-allocated uniformly across plants. In this sense, $\mathcal{C}$ is a measure of allocative efficiency.

The two panels of Figure 4 show that the Olley-Pakes covariance $\mathcal{C}$ accounts for a substantial portion of the aggregate productivity growth in India from 1990 to 2011. Figure 4 plots values of the two terms on the right-hand side of equation (1) over time in India according to the combined ASI-NSS data as solid black lines. The top panel plots the average log labor productivity, while the bottom panel plots the Olley-Pakes covariance term. Both terms are computed within each of 100 different manufacturing industries, and then averaged across industries using each industry's share of aggregate employment in each year.

[Figure 4 about here.]

The rise in $\mathcal{C}$ over time is mainly due to changes within industries, and not to changes in the industry composition. The two dotted lines in Figure 4 plot the values of $\mathcal{Z}$ and $\mathcal{C}$, respectively, using constant aggregation weights across industries, computed either at the beginning of the sample (bottom dotted line), or at the end (top dotted line). Some of the change in average $\mathcal{C}$ is due to a shift over time toward industries with a higher value of $\mathcal{C}$, because the 2011-weights line is higher than the other two, but all three lines feature a dramatic increase in $\mathcal{C}$ over the sample period.

Changes in $\mathcal{C}$ within a country over time suggest changes in allocative efficiency, which can be driven by financial development. Bartelsman, Haltiwanger and Scarpetta (2013) show that $\mathcal{C}$ is increasing over time in all the economies they study, and it is increasing more rapidly in the developing economies of Eastern Europe. In the next section, I derive a structural model of firm investment and growth in which financial frictions have a direct impact on allocative efficiency. I calibrate the model to match moments from the Indian data and to infer interpretable fundamental shocks from the observed values of $\mathcal{Z}$ and $\mathcal{C}$.

4 Model

In this section, I derive a model in which entrepreneurs borrow to smooth consumption in the face of persistent idiosyncratic productivity shocks, but cannot commit to repaying their debt. I then put many such entrepreneurs together in industry equilibrium and compute the endogenous size-productivity distribution, enabling the model to replicate the two terms in the Olley-Pakes decomposition of section 3.3, as endogenous responses to exogenous growth in TFP and financial development. In the model, reductions in financial frictions do increase the Olley-Pakes covariance, but so do common shocks to productivity. This section derives the model and characterizes its equilibrium, while in section 5 I calibrate the model to match some features of the ASI-NSS data and show that, quantitatively, common shocks to productivity are more important drivers of aggregate productivity growth than financial development.

The heart of the model derived in this section is a borrowing decision by agents in the face of a financial friction: agents may borrow as much as they wish, but they cannot commit to paying their debt back, and lenders price debt according to the probability of default. Because defaulting entails a loss of output, an increase in productivity (all else equal) increases the cost of default to borrowers, lowering its probability, resulting in higher bond prices. In equilibrium, an increase in productivity thus mimics a reduction in the financial friction itself.

To be consistent with previous literature, and to closely match the empirical measure of productivity I use in section 3, I layer the financial friction on top of an extended version of the static model of Hsieh and Klenow (2009).[5] My model features monopolistically competitive firms combining labor and capital in a Cobb-Douglas production function, allowing the model to endogenously generate differences in physical versus revenue productivity, and total-factor versus labor productivity. In both the data and the model, I observe only revenue labor productivity, although the exogenous shocks I apply to the model are to physical total-factor productivity.

Layering the financial friction on top of a model with decreasing returns to scale in revenue works against the conclusions of the paper, because in the first-best allocation of both Hsieh and Klenow (2009) and the model in this paper, the Olley-Pakes covariance between the average revenue product of labor and firm employment is zero. Under this interpretation, a positive and increasing

[5] In section 3 of the online appendix I derive a simple static version of the model that follows even more closely the framework in Hsieh and Klenow (2009), to build intuition for the differences between that model and mine.

Olley-Pakes covariance, such as that seen in India or in all the countries analyzed by Bartelsman, Haltiwanger and Scarpetta (2013), reflects increasing misallocation over time, and not (as argued here) potential financial development. Instead, I show using the model derived in this section that financial frictions naturally imply a positive Olley-Pakes covariance that grows over time in response to positive shocks to financial development and productivity.

The rest of this section is organized as follows: in section 4.1 I describe the agents in the model, their preferences, their constraints, and their potential actions. In section 4.2 I define and characterize equilibrium in this model.

4.1 Environment

Time is countable and there is a single good that can be consumed or used for investment. There is a continuum of agents, each of whom has log utility and discounts the future at a rate $\beta \leq 1$. Agents do not live forever, but die each period with iid probability π. Each agent i has idiosyncratic productivity $z_{i,t}$, which evolves according to the AR(1) process

$$z_{i,t+1} = \rho z_{i,t} + \sigma \varepsilon_{i,t+1}, \tag{2}$$

where $\varepsilon_{i,t+1}$ is a standard normal random variable that is independent across agents and time. Physical production combines labor l and capital k according to the constant returns to scale production function

$$y_{i,t} = A e^{z_{i,t}} k_{i,t}^{\alpha} l_{i,t}^{1-\alpha}. \tag{3}$$

Agents can freely convert the consumption good into units of capital, and vice versa, at a relative price of 1; there are no capital adjustment costs. Capital does not depreciate and must be non-negative. Producers are monopolistically competitive and face individual demand curves of the form

$$p_{i,t} = y_{i,t}^{-\frac{1}{\gamma}}, \tag{4}$$

so that equations (3) and (4) together imply a decreasing returns to scale revenue function, as in Hsieh and Klenow (2009). This is isomorphic to a perfect competition model in which $p_{i,t} = 1$ but agents face a decreasing returns to scale production function, i.e. equation (3) raised to the $\frac{\gamma-1}{\gamma}$ power.

Agents solve a two-stage optimization problem: the first stage is an investment stage in which agents choose how much to consume, borrow, and invest for the future, before learning their own idiosyncratic shock ε. The second stage is a production stage, after idiosyncratic uncertainty has been resolved, in which agents choose how to allocate resources between labor and capital, given resources saved from the previous period.

In the production stage, given productivity z' and total resources κ, agents choose capital and labor to maximize revenue and undepreciated capital, subject to their budget:

$$\begin{aligned} y^*\left(\widetilde{A}, \kappa, w\right) = \max_{k,l} \quad & \underbrace{\left[\widetilde{A} k^{\alpha} l^{1-\alpha}\right]^{\frac{\gamma-1}{\gamma}}}_{py} + k \\ \text{s.t.} \quad & \\ & k + wl \leq \kappa + w\underline{L} \\ & l \geq \underline{L}, \end{aligned} \tag{5}$$

where w is the wage, and $\widetilde{A} \equiv Ae^{z'}$ depends on the current level of idiosyncratic productivity z' as well as any aggregate shock to productivity A. y^* denotes the solution to equation (5); it represents the total resources available to the agent in the next period, apart from any debt repayment.

The second-stage problem (5) has two constraints: a working-capital constraint and an overhead-labor constraint. The first constraint means that agents must invest in physical capital k, and pay the wage bill wl, before output is produced. Agents may borrow against future earnings in order to raise working capital, but to make their problem economically interesting I assume that borrowing must occur in the first stage, before idiosyncratic risk is realized.[6]

The second constraint in problem (5) is that agents must use at least $\underline{L}$ units of labor in

[6] A simpler model, in which agents invest capital k before uncertainty is realized but hire labor in frictionless markets afterwards, using part of the proceeds from production to pay the wage bill, would lead to an equilibrium where agents equalize labor productivity ex post regardless of their size or TFP. In fact in India labor productivity varies widely across plants, as I show in section 3.

production. Agents are endowed with $\underline{L}$ units of their own labor, but they must use all of this labor on their own project; thus $w\underline{L}$ appears on the right-hand side of the budget constraint and agents only pay wages for labor they hire beyond $\underline{L}$. In equilibrium, the solution to problem (5) is a labor-capital ratio m that is increasing in productivty. In the limit as productivity goes to $-\infty$, absent the overhead labor constraint, agents choose (effectively) zero labor., and make no contribution to aggregate labor productivity. In the data (see Figure 3), a massive number of very-small plants have very low labor productivity. Allowing plants in the model to "exit" by setting their labor-capital ratio close to zero precludes matching this feature of the data, and I get around this problem by assuming that each agent must invest her endowment of labor in her own project.

In the first stage, before the idiosyncratic shock is realized, agents choose consumption c, savings κ, and borrowing b to maximize the present expected discounted value of their stream of utility, knowing that their future wealth depends on the outcome of solving problem (5). Financial markets are incomplete, and the only asset that agents may trade are one-period zero-coupon bonds. To match the dispersion in productivity in the combined ASI-NSS data, I assume that agents cannot lend to each other; lenders in this economy are agents outside this production sector. Let q denote the price of one unit of face value of the agent's debt, and the agent's chosen face value b. $b > 0$ denotes that the agent is borrowing funds.

Agents are free to borrow as much as they wish, but they cannot commit to repaying their debt in the future. The timing of the default decision is as follows: at the beginning of each period, the agent learns her own productivity value $z_{i,t}$. At that moment, she decides whether she will pay back her borrowing (if $b > 0$) or default. If she defaults, she retains a fraction $1 - \theta_{i,t}$ of her own savings κ, may not produce using the solution to problem (5), and sets $b = 0$. The lender recovers a total amount $\theta_{i,t}\kappa$, that must be distributed among a total face value b, so that per bond, the lender recovers

$$\chi \equiv \min\left\{1, \theta_{i,t}\frac{\kappa}{b}\right\}. \tag{6}$$

The min ensures that lenders do not recover more than they were owed; if agents default with enough assets such that lenders would recover more than they were owed, the extra resources are lost. This kind of deadweight loss will not occur in equilibrium. Agents that default do not exit the economy, but start a new firm next period. They do not suffer any exclusion from financial markets; the only penalty for defaulting is the loss of all potential output this period and a fraction θ of their capital. Notice also that the value of θ can vary across agents i and over time t.

Lenders are perfectly competitive and require an exogenous expected return of r on their portfolios. Lenders understand that borrowers may default and that if they do, the lenders only recover $\chi \leq 1$ per bond, and they incorporate this default risk into the zero-coupon bond price q. For their part, borrowers understand that the bond price they pay per unit of face value depends on the current state and their choices for investment and borrowing.

In equilibrium, the realized return to each lender's portfolio of bonds is riskless at an exogenous level r, because lenders can lend to a sufficiently diverse cross-section of borrowers such that a law of large numbers applies and their total return across all bonds is riskless. A key tractability assumption behind this result is that $A_{i,t}$ and $\theta_{i,t}$ change after default decisions are made and production has occurred, but before new borrowing, investment, and consumption decisions are made. This assumption means that when agents borrow, the only unknowns are the realizations of the idiosyncratic shocks $\varepsilon_{i,t+1}$, which are diversifiable by lenders. The bond price q charged to any individual borrower can perfectly offset that borrower's default risk, so that the lender's expected return is constant across loans and this expected return is equal to the realized return on their entire portfolio.

Each period, agents choose consumption c, investment for next period κ', and borrowinf b'. The budget constraint for an individual agent is then

$$c - q\left(b', \kappa', z; A_{i,t}, \theta_{i,t}, w_t\right) b' + \kappa' = \begin{cases} y^* - b & \text{if old debt was repaid} \\ \left(1 - \theta_{i,t}\right) \kappa & \text{if defaulted on debt} \end{cases}$$

where the bond price q depends on the agent's choices for investment and borrowing, their productivity z, aggregate productivity $A_{i,t}$, the aggregate recovery rate $\theta_{i,t}$, and the wage w_t.

[Figure 5 about here.]

Figure 5 illustrates the timing assumptions of the model. For technical reasons, I assume that agents learn early in the period whether they will exit, immediately after their productivity z is realized. This timing assumption, which is also used by Gilchrist, Sim and Zakrajšek (2014) and Khan, Senga and Thomas (2014), ensures that agents cannot exit the model exogenously with outstanding debts. Otherwise, the exogenous death risk would enter the bond-price equation, where it would complicate the model solution without adding anything substantive.

In the investment stage, before the idiosyncratic shock is realized, agents choose consumption, savings, and leverage to maximize the present discounted value of their stream of utility, leading to the following recursive representation:

$$
\begin{aligned}
V_{i,t}\left(x,z\right) = \max_{c,\kappa,b\geq 0} \quad & \log(c) + \beta\left[\left(1-\pi\right)E\left\{V_{i,t+1}\left(x',z'\right)\right\} + \pi E\left\{\log\left(x'\right)\right\}\right] \\
\text{s.t.} & \\
& c+\kappa \leq x + qb \\
& q \equiv q\left(b,\kappa,z;A_{i,t},\theta_{i,t},w_t\right) \\
& z' = \rho z + \sigma\varepsilon \\
& x' = \max\left\{y^*\left(A_{i,t}e^{z'},\kappa,w_t\right) - b, \left(1-\theta_{i,t}\right)\kappa\right\},
\end{aligned}
\tag{7}
$$

where $q\left(b,\kappa,z;A,\theta,w\right)$ is the bond price lenders charge to ensure that their expected return is r. Agents know this bond price schedule as a function of their choice variables and internalize it. The value function $V\left(x,z\right)$ is indexed by $\left(i,t\right)$ because the values of A and θ may vary across agents or over time; these changes are deterministic and foreseen by agents.[7]

4.2 Equilibrium

The key endogenous object that allows the model to replicate the aggregate productivity decomposition of section 3.3 is the cross-sectional distribution of size (really net wealth x) and productivity. This distribution depends on the equilibrium policy functions that arise from solving problems (5) and (7). In this section I define the equilibrium and characterize it.

To ensure a stationary equilibrium, agents enter the model exogenously to "replace" those that exit due to the iid probability π of dying. Assume a continuum of agents and normalize its measure

[7] In equation (7) the second term in the continuation value arises because I have plugged in the optimal choice of κ and b should the agent receive the exit shock in the next period. Agents that receive the exit shock know that they will exit at the end of the period, so they set $\kappa = 0$; lenders know they will exit and set $q = 0$, so agents choose $b = 0$. Thus their terminal consumption is x', and their value in this case is $\log\left(x'\right)$. Recall that the value function is evaluated after default and production decisions are made; see Figure 5.

to 1. Because a measure π of agents will exit the model each period, a measure π of agents must enter the model each period to maintain stationarity. Let these new agents have net wealth x drawn from the lognormal distribution $\log x \sim \mathcal{N}\left(0, \sigma_{x_0}^2\right)$, and their independent idiosyncratic productivity z is drawn as $z \sim \mathcal{N}\left(0, \sigma_{z_0}^2\right)$.

I characterize the cross-sectional joint distribution of net wealth x and productivity z for agents of type i at time t as a cumulative distribution function $F_{i,t}(\log x^*, z^*)$. This function is the probability that a randomly-drawn agent of type i at time t has net wealth $x < x^*$ and idiosyncratic productivity $z < z^*$. Agent types are permanent, so I compute the CDFs for each type of agent separately and later aggregate across them according to the measure of each type.

I parameterize F in terms of $\log x$ instead of x because in equilibrium this parameterization will behave better computationally. For fixed values of $A_{i,t}$, $\theta_{i,t}$, w_t, and the function $F_{i,t}$, the function $F_{i,t+1}$ is given by

$$\begin{aligned} F_{i,t+1}\left(\log x^*, z^*\right) &= \pi F^e\left(\log x^*, z^*\right) \\ &\quad + (1-\pi)\int \Pi_{i,t}\left\{\begin{pmatrix} x^* \\ z^* \end{pmatrix}, \begin{pmatrix} x \\ z \end{pmatrix}; A_{i,t+1}, \theta_{i,t}, w_t\right\} dF_{i,t}(\log x, z), \end{aligned} \tag{8}$$

where $F^e(\cdot,\cdot)$ is the CDF of $(\log x, z)$ for the agents that enter exogenously each period, described above, and the function $\Pi_{i,t}$ is the conditional transition CDF for agents of type i at time t, which depends on the decision rules of individual agents, the law of motion of the idiosyncratic productivity z, and the paths of the exogenous aggregate variables $A_{i,t}$ and $\theta_{i,t}$ (see Appendix B.3 for details). I use equation (8) to compute the evolution over time of the size-productivity distribution, and also to compute the steady-state distribution where $F_{i,t+1} = F_{i,t}$.

To match the large change in aggregate productivity over the 22 years in my sample, I add an exogenous aggregate labor-supply curve to the model. In particular, I assume that the log wage is affine in aggregate labor:

$$\log w_t = \eta \log L_t + D, \tag{9}$$

where η and D are parameters and L_t is aggregate labor at time t. Individual agents take the wage rate w as given when making decisions, although their collective hiring decisions will affect w through equation (9). Without this aggregate labor-supply curve, the total-factor productivity shocks necessary to match the rise in average labor productivity would lead to too much growth in the formal sector, and far too much growth in the Olley-Pakes covariance.

I define equilibrium in this model as a collection of distribution functions $F_{i,t}(x,z)$, policy functions $\kappa_{i,t}(x,z)$, $b_{i,t}(x,z)$, $l\left(\widetilde{A},\kappa,w\right)$, and $k\left(\widetilde{A},\kappa,w\right)$, a bond-price function $q(b,\kappa,z;A,\theta)$, and a sequence of wages w_t that satisfies the following conditions, given the sequence of exogenous aggregate states $A_{i,t}$ and $\theta_{i,t}$:

1. The functions $l\left(\widetilde{A},\kappa,w\right)$ and $k\left(\widetilde{A},\kappa,w\right)$ solve equation (5), given productivity $\widetilde{A}\equiv Ae^{z'}$, cash on hand κ, and the current wage w. Denote the solution to equation (5) as $y^*\left(\widetilde{A},\kappa,w\right)$.

2. The function $q(b,\kappa,z;A,\theta,w)$ ensures that the expected return to lenders over the idiosyncratic shock ε is r, given the recovery-rate equation (6) and that borrowers default whenever $y^*\left(Ae^{z'},\kappa,w\right)-b<(1-\theta)\kappa$.

3. The functions $\kappa_{i,t}(x,z)$ and $b_{i,t}(x,z)$ solve equation (7), given $y^*\left(\widetilde{A},\kappa,w\right)$, the current values of x and z, the function $q(b,\kappa,z;A,\theta,w)$, and the entire time-series paths of $A_{i,t}$, $\theta_{i,t}$, and w_t.

4. The distribution functions $F_{i,t}(\log x^*,z^*)$ satisfy equation (8), given the time-series evolution of $A_{i,t}$, $\theta_{i,t}$, and w_t, where the transition functions $\Pi_{i,t}$ are consistent with the policy functions $\kappa_{i,t}(x,z)$ and $b_{i,t}(x,z)$.

5. The sequence of wages w_t lie on the aggregate labor-supply curve (9), give aggregate labor L_t in each period.

The solution to the model consist of five pieces: first, I solve the static problem (5). I use this solution to derive the equilibrium default policy, which determines the bond-price function $q(\cdot)$. Given both of these elements, I solve problem (7) numerically using policy function iteration. This solution then determines the transition CDF in equation (8), allowing me to characterize the endogenous distribution of net wealth x and productivity z and compute moments of interest. These four pieces all depend on a fifth piece, the time-series path for wages w_t, which I have to go back and check lies on the aggregate labor-supply curve (9).

The solution to the static second-stage problem in equation (5) is given in the following proposition:

Proposition 1. *The solution* (k^*, l^*) *to problem (5) is given by*

$$k^* = \frac{\kappa + w\underline{L}}{1 + wm^*}$$
$$l^* = m^* k^*,$$

where m^* *is either the value of* m *that solves*

$$m^{1-\zeta} = A^* \Big(1 + wm\Big)^{\frac{1}{\gamma}} \left(\frac{1-\alpha}{w} - \alpha m\right), \tag{10}$$

with

$$A^* \equiv \frac{\gamma - 1}{\gamma} \left(Ae^{z'}\right)^{\frac{\gamma-1}{\gamma}} (\kappa + w\underline{L})^{-\frac{1}{\gamma}} \tag{11}$$

and $\zeta \equiv (1-\alpha)\frac{\gamma-1}{\gamma}$, *or*

$$m = \frac{\underline{L}}{\kappa},$$

whichever is larger.

So long as $\frac{1-\alpha}{\alpha} < \gamma$ *, there is a unique solution* $m^*\left(z', \kappa\right)$ *to equation (10) that is weakly increasing in* z' *and decreasing in* κ.

Proof. See Appendix A. □

Apart from the overhead labor constraint, the optimal labor-capital ratio m^* that solves (5) is increasing in productivity z', and decreasing in total resources κ, because the agent retains capital k after production occurs. Intuition for this result is clearer for the negative case: as revenue productivity goes to zero (either because $z' \to -\infty$ or because $\kappa \to \infty$), the agent derives little benefit from production and consequently hires the minimum amount of labor, setting $k = \kappa$ and $l = \underline{L}$. Increasing revenue productivity raises the benefit to production relative to undepreciated capital and induces some resources to be spent on labor, raising the labor-capital ratio.

Given the solution to equation (5), agents will default whenever their resources from production and repayment, $y^* - b$, are less than the resources they derive from defaulting, $(1-\theta)\,\kappa$. The following proposition characterizes their optimal default behavior:

Proposition 2. *Suppose $\theta < 1$, and $\alpha < \min\left(\frac{1}{2}, \frac{1}{\gamma-1}\right)$. Fix A, z, b, and κ. The default threshold equation*

$$y^*\left(Ae^{\rho z+\sigma\underline{\varepsilon}}, \kappa, w\right) - b = (1-\theta)\,\kappa \tag{12}$$

either has a unique solution $\underline{\varepsilon}$, or has no solution. Agents default if and only if $\varepsilon < \underline{\varepsilon}$ when equation (12) has a unique solution, or they never default. The $\underline{\varepsilon}$ that solve equation (12) are decreasing in z and increasing in κ. If $b \leq \theta\kappa$, the probability of default is zero ($\underline{\varepsilon} = -\infty$).

Proof. See Appendix A. □

The proof of Proposition 2 does more than establish the result, which is straightforward from the fact that y^* is increasing in z': it gives a formula for $\underline{\varepsilon}$ in closed form, which allows the bond price, given by

$$q = \frac{1}{1+r}\left[1 - \left(1 - \min\left\{1, \theta\frac{b}{\kappa}\right\}\right)\Phi\left\{\underline{\varepsilon}\left(b, \kappa, z; A, \theta, w\right)\right\}\right], \tag{13}$$

to also be derived in closed form, irrespective of the κ or b policies or the value function of the agent. Equation (13) allows for tractable numerical solutions to problem (7) and equation (8), which I describe in the next section.

5 Results

In this section, I calibrate the model derived in section 4 and use it to back out the fundamental shocks that drove Indian manufacturing productivity growth from 1990 to 2011. I find that common shocks to productivity explain the lion's share of aggregate productivity growth, including much of the rise in the covariance between size and productivity after 1995. According to the model, financial development occurs only in 1990–1995, although its contribution to growth continues after

1995. Over the entire sample period, financial development accounts for at most 7% of aggregate productivity growth.

The results in this section are based on a numerical approximation to the model's solution conditional on a set of parameters. In section 5.1 I calibrate the parameters of the model to match some key moments o the ASI-NSS data. In section 5.2 I use the calibrated model to back out a time series of (A, θ) to match the observed values of $(\mathcal{Z}, \mathcal{C})$. In section 2 of the the online appendix I use a simpler version of the model in section 4 to perform the same exercise, with very similar results.

5.1 Calibration

To match the size and productivity differences across ASI and NSS plants in the data, in this section I assume two types of agents who differ in their average physical productivity, given by the parameter A. NSS or informal firms have a much lower level of A than ASI or formal firms, so that in equilibrium the agents with lower physical productivity will be smaller and, because their overhead-labor constraint is more likely to bind, they will also have lower revenue labor productivity. Both types of agents may borrow, although those with very low values of A are much less likely to do so in equilibrium. Common shocks to productivity increase the A values of formal and informal firms by the same percentage.

[Table 3 about here.]

Table 3 reports the calibrated parameters used in this section. I group parameters into two categories: Panel A lists the parameters that are standard and that I do not use to calibrate the model. These include the riskless interest rate r, the rate of time preference β, the demand elasticity parameter γ, the capital share α, the persistence of the productivity shock ρ, and the share of low-TFP plants μ. The model requires persistent productivity shocks because without them, agents would not respond to the current value of their productivity. I do not calibrate ρ because I do not observe plants over time in the combined ASI-NSS data. I set the measure of low-productivity plants to a constant 0.99, even though the share of NSS plants is changing over time (see Panel A of Table 1). Results of this section are similar for extended versions of the model where I vary the entry share each period (but hold the exit probability fixed) to match the changing NSS share over time.

Panel B of Table 3 reports the parameters of the model I use to calibrate to the data. All the moments listed in the table are determined by all the parameters jointly, but I group data moments with the parameters that most affect them. I set the exogenous exit rate π to 1/12 to match the average age of plants in the data of 12 years. The model-implied average age of 11.95 years is less than 12 because I assume that defaulting firms have their age reset to 1; default rates in equilibrium are relatively low, so this correction has little effect on average firm age. I set the standard deviation of the exogenous idiosyncratic productivity shock σ to 0.45 to match the high standard deviation of labor productivity in the data; the value of 1.07 is the standard deviation of labor productivity in the data after removing industry means.

I set the intercept in the labor-supply curve D, the base rate of productivity A, and the productivity difference across agent types $\Delta_i \log A$ to match the NSS shares of aggregate labor and revenue, and the labor productivity spread between ASI and NSS plants. The decreasing returns to scale in revenue imply that the cross-sectional labor-productivity distribution will be a compressed version of the TFP distribution. Thus to ensure a log labor productivity difference of about 2 between formal and informal firms, the model requires an average log TFP difference of 3.7. The current calibration matches the share of economic activity accounted for by low-TFP firms relatively well, but does so with too great a spread between their labor productivity and the labor productivity of high-TFP firms. Matching this moment more tightly is a priority for future work. I set th elasticity of the wage with respect to aggregate labor, η, to match the change in the informal share of labor over time.

5.2 Aggregate Productivity

In this section I use the calibrated model to analyze how the Olley-Pakes decomposition in equation (1) is affected by common shocks to productivity and financial development. I perform two exercises: first, I demonstrate that when comparing stochastic steady-states across economies, a higher level of financial development θ or higher aggregate total-factor productivity A increases the Olley-Pakes covariance. Second, I find a sequence of shocks to A and θ that replicate the evolution of both terms of the Olley-Pakes decomposition plotted in Figure 4. I then use model-implied counterfactuals to quantify the contribution of productivity shocks and financial development to aggregate productivity growth over time; I find that from 1990 to 2011 financial development accounted for between 2% and 7% of total aggregate productivity growth.

Improvements in financial development (through the parameter θ) and increases in total-factor productivity increase both the Olley-Pakes covariance term and average labor productivity. Two features of the model work against this conclusion. The first is that, because firms have an optimal scale of production, in principle they have the ability to accumulate enough capital to "grow out of" their financial constraints. In this case, increasing θ would have little to no real effects in the aggregate. The second is that, in the first-best allocation, labor productivity depends only the wage w (and other parameters). Thus increasing aggregate total-factor productivity should only affect total output and the size of firms, not labor productivity. In equilibrium, enough firms are constrained that these effects are smaller than the real effects of changes in θ and A on the allocation of resources.

[Figure 6 about here.]

Figure 6 illustrates the effects of changing financial development and average productivity by plotting four comparative statics of the model, for $\theta \in \{0.4, 0.8\}$ and $A \in \{0.34, 0.425\}$. Each bar represents an aggregate value at the ergodic steady-state size-productivity distribution for the indicated parameter values. The top panel reports the Olley-Pakes covariance, and the bottom panel plots the average log labor productivity, in each of these four economies. Formulas and other calculation details can be found in Appendix C. The top panel of Figure 6 shows that the Olley-Pakes covariance increases with both financial development θ and with aggregate TFP A. The bottom panel shows that, although the wage is fixed, average labor productivity responds to productivity shocks in equilibrium.

The results reported in Figure 6 compare distinct economies with different values of aggregate productivity and financial development, once the firm-size distribution has converged to a stochastic steady-state: individual firms still experience idiosyncratic shocks, but aggregate values are constant. In the rest of this section I explore the dynamic evolution of the model from such a stochastic steady-state, choosing values of A_t and θ_t in order to match the Olley-Pakes decomposition plotted in Figure 4.

To solve the model, I start at the final period where the exogenous variables are no longer moving. I solve for the steady-state value and policy functions at this date according to equation (7), and then step backwards one period at a time, solving for value and policy functions in equation (7) using next period's value function as the continuation value. This procedure gives me the sequence of policy functions at each date. I then solve for the $t = 0$ endogenous distribution of (x, z) in equation (8) by solving the steady-state problem for the $t = 1$ aggregate variables, and iterate this distribution forward using equation (8) and the policy functions at each date. I repeat this process until the implied aggregate labor demand and wage lie on the labor supply curve given in equation (9).

[Figure 7 about here.]

The top panel of Figure 7 reports the exogenous aggregate values of A_t and θ_t that replicate, in the model, the observed values of $\mathcal{Z}$ and $\mathcal{C}$ in the Olley-Pakes decomposition from Figure 4. Common productivity is rising throughout the sample, although it rises slowly in 2000–2006. Financial development occurs only in 1990–1995; θ is falling in every other time period in the model. θ falls, despite the increase in the Olley-Pakes covariance $\mathcal{C}$ throughout the sample period, for two reasons: first, the effects of the initial rise in θ take several years to work their way through the size-productivity distribution. Second, the model implies that increases in A also raise $\mathcal{C}$, and the model requires θ to fall to counteract this effect and match the values of $\mathcal{C}$ observed in the data.

The bottom panel of Figure 7 converts the values plotted in the top panel of Figure 7 into contributions to aggregate productivity growth. Each bar represents the contribution to aggregate productivity growth from each shock over the indicated time span. I compute these contributions as counterfactuals: for each aggregate variable, I solve the dynamic model assuming that variable is constant at its 1990 value, and allowing the other variable to move as in the top panel of Figure 7. I then compute aggregate productivity growth in this scenario, and report the difference between that value and the observed growth in aggregate productivity as the contribution from that shock, in annualized percentage points. Because the effect of shocks to θ depend on the level of A, and vice versa, the sum of the two contributions may not equal realized productivity growth. I plot the difference in Figure 7 as the "joint" contribution.

Figure 7 shows that the contributions to productivity growth from financial development are small or even negative. The largest contribution is in 1990–1995, when the large increase in θ accounted for between 9% and 18% of productivity growth (depending on how one allocates the joint term). Looking across the entire 1990-2011 sample, changes in θ contributed between 2% and 7% of aggregate productivity growth.

[Figure 8 about here.]

To illustrate how the model uses the Olley-Pakes decomposition to distinguish between changes in A and changes in θ, Figure 8 plots the implied series of the Olley-Pakes decomposition in each counterfactual exercise. The left panels plot the implied path of average productivity $\mathcal{Z}$ (top panel) and the Olley-Pakes covariance $\mathcal{C}$ (bottom panel) for the case where θ_t is constant at 0.05, but A_t evolves as in the top panel of Figure 7. The right panels repeat the exercise when θ_t evolves as in the top panel of Figure 7, but A_t is constant at its 1990 value. In both counterfactuals, the values of w_t adjust according to the implied aggregate labor demand and equation (9).

The four panels of Figure 8 show that changes in financial development over time only affect the Olley-Pakes covariance, while changes in total-factor productivity affect both terms in the Olley-Pakes decomposition. This is what leads to identification in the model: because changes in θ_t do not move average productivity (very much), I can choose the time series of A_t to match the rise in average productivity, and then set θ_t to match the residual changes needed to fit the Olley-Pakes covariance.[8]

[Figure 9 about here.]

The small contribution to productivity growth from financial development in the bottom panel of Figure 7 is not driven by the fall in θ after 1995. Figure 9 reports results from another counterfacual exercise where θ grows from 0.05 to 0.7 from 1990 to 1995, as before, but is then constant at 0.7 for the rest of the sample. This path for θ_t will not replicate the observed path of Olley-Pakes covariance term in the data, but is simpler to interpret. The results are similar to those reported in Figure 7: the largest contribution to productivity growth from financial development is in 1990-1995, between 9% and 11% of total growth, and over the entire sample financial development

[8]This description is meant as a heuristic device only: because the aggregate time-series are deterministic, each value in every time period affects the agents' decisions in all other time periods, and the level of A_t determines the endogenous effects of changing θ_t (and vice versa).

accounts for between 5% and 7% of productivity growth. The contribution from θ remains positive after 1995, even though θ is constant, because the large increase in θ takes time to work its way through the endogenous size-productivity distribution. Also, the financial development contribution in 1990-1995 is somewhat smaller in Figure 9 than in Figure 7, because in the latter scenario agents know that future values of θ will be lower, and thus they respond more to the current shock.

6 Difference-in-Difference Regressions

The overwhelming majority of empirical studies linking financial development to economic growth, and attributing causality as running from the former to the latter, rely on regression analyses. In this section I explore alternative interpretations of a common regression specification used in the financial growth literature, difference-in-difference regression. Specifically, I use my calibrated model to show that, although this method does correctly identify financial development when it occurs, it can also indicate financial development when none has occurred, but there has been productivity growth common to all firms. In technical terms, the problem is a failure of the parallel trends assumption: firms that are more affected by financial development are also more affected by common shocks to productivity.

The rest of this section is organized as follows: in section 6.1 I run some difference-in-difference regressions that, under conventional interpretations, suggest that financial development led to growth in India from 1990 to 2011. However, I show in section 6.2 that common shocks to productivity can generate the same observed regression results even in the absence of financial development. Combined with the evidence in section 5, I conclude that common shocks to productivity are a more likely driver of these regression estimates than financial development.

6.1 Empirical Evidence

In a perfect difference-in-difference scenario, only one set of agents receives the treatement, and this set of agents is chosen randomly. To use a difference-in-difference design to understand the effects of an aggregate shock, however, treatment must be defined in terms of sensitivity. In principle, financial development is a treatment that applies to all firms in an economy; the identification argument in this case is instead that some firms are more or less sensitive to the same shock.

For example, firms in industries that (for whatever reason) tend to borrow more, or have more collateralizable assets, are presumably more affected by a change in financial development than other firms.

I assign firms to "treated" and "control" groups using industry-wide characteristics that are commonly used in the literature, including dependence on external finance (Rajan and Zingales 1998) and share of fixed assets in total assets (Vig 2013). The treatment industries I use all have higher leverage, are more likely to borrow, and have a higher share of firms in the formal sector than the control industries. All these characteristics suggest that the treatment firms are more affected by financial development.

I define the second difference as time: I set the "pre" period to the observations in 1990, 1995, and 2000, and the "post" period to 2006 and 2011. These choices correspond roughly to the passage of the Sarfaesi Act in 2002; therefore these regressions can be interpreted as a long-run estimate of the effect of the Sarfaesi Act on leverage and aggregate output.

Table 4 reports the industries I selected for this analysis. The Pharmaceuticals industry is ranked by Rajan and Zingales (1998) as the most externally-dependent manufacturing industry. It also has a low share of employment accounted for by informal plants, and a high percentage of plants that report positive leverage in the pre-period (1990–2000). In contrast, the Tobacco industry is ranked by Rajan and Zingales (1998) as the least externally-dependent industry. It has a low NSS share of employment, and a low percentage of borrowing plants in the pre period. Ex ante, it is natural to assume that Pharmaceutical plants will be more sensitive to financial development than Tobacco plants.

I also included the Motor Vehicles and Motorcycles & Parts industries in the "affected" category. Motor Vehicles have a high external-dependence measure according to Rajan and Zingales (1998), and unlike the industries that have a slightly higher ranking (such as Ships or Office & Computing), they are responsible for a non-trivial share of Indian manufacturing employment, especially after includng Motorcycles & Parts along with them. These two industries have a high percentage of borrowing plants and low NSS shares of employment, as well.

[Table 4 about here.]

In terms of less affected industries, I added Other Chemical Products because it is likely to be similar to Pharmaceuticals in many respects, though it ranks lower in the Rajan and Zingales (1998) external-dependence measure and has a lower percentage of firms borrowing in the pre period. It also has a relatively low share of NSS employment, though it is higher than the more affected industries. I included Carpentry & Joinery goods because it has low percentage of borrowing plants, a high share of employment in NSS plants, and accounts for roughly the same share of aggregate employment as all the more affected industries combined (unlike Tobacco, which accounts for much more employment).

For another pair of affected versus not affected industries, I chose Cement & Plaster Products and Carpentry & Joinery Goods. Both industries are in a similar sector of the economy (construction materials), and it is reasonable to assume that Cement plants are more affected by financial development because more Cement plants borrow, and the Cement industry has a smaller share of employment in the informal sector. Cement plants in the ASI also have a slightly higher average fixed-asset share than Carpentry & Joinery plants.

Finally, I included Sugar as a less affected industry because among ASI plants, the Sugar industry has a low average fixed-asset share. Vig (2013) uses the share of fixed assets in total assets as a measure of asset tangibility, and argues that firms with low asset tangibility are less affected by the Sarfaesi Act than firms with higher asset tangibility. Indeed, Vig uses the fixed-asset share to assign firms to "treatment" and "control" groups in his difference-in-difference regressions. Although I can only compute the fixed-asset share for plants in the ASI, I assume that NSS plants in the same industry are likely to have a similar-enough fixed-asset share that I can classify an industry as high or low tangibility based on the average ASI values. According to this scheme, Sugar plants on average tend to have lower asset tangibility than the three more affected industries in Table 4.

Panel B of Table 4 reports differences in mean leverage and log labor productivity for the selected industries. In general, the more affected industries have higher pre-period leverage, and more leverage growth, between 1990–2000 and 2006–2011. In addition, with the exception of Tobacco plants, the less affected industries have positive leverage growth; this growth is smaller on average than for the more affected industries. This difference-in-difference result is consistent with financial development occurring over the 1990–2011 period; this development seems to have exerted a greater influence on industries we expect to be more affected. The final column of

Panel A of Table 4 shows that, with the exception of the Sugar industry, the more affected industries also had greater aggregate output growth over this period than the less affected industries. This industry-level measure is used by Rajan and Zingales (1998) in cross-country regressions to gauge the extent of financial development.

Tables 5 and 6 present the evidence from the first three columns of Panel B of Table 4 in regression form. I run the following regression;

$$\begin{aligned}\text{lev}_{i,t} = \alpha_s + \beta_0 \mathbb{1}\{i \in \text{treat}\} + \beta_1 \mathbb{1}\{t > 2000\} + \beta_3 \mathbb{1}\{t > 2000\} \times \mathbb{1}\{i \in \text{treat}\} \\ + \beta_4 \text{log labor prod}_{i,t} + \varepsilon_{i,t}, \end{aligned} \tag{14}$$

where i indexes plant, t indexes time, α_s is a state fixed-effect, $\mathbb{1}\{i \in \text{treat}\}$ is a dummy equal to 1 if a firm is in a treated industry, and $\mathbb{1}\{t > 2000\}$ is a dummy equal to 1 for years after 2000. The estimated coefficient β_3 represents the extent to which treated industries increased their leverage by more than non-treated industries. The different columns of Tables 5 and 6 define treatment and control industries in different ways.

[Table 5 about here.]

The first column of Table 5 defines Pharmaceutical plants as "treated" and Tobacco plants as "control." On average, Pharmaceutical plants increased their leverage ratios by 1.16 percentage points more than Tobacco plants. Comparing Pharmaceuticals to Other Chemicals plants (column 2), the difference-in-difference estimate is slightly smaller at 74 bps. The third column of Table 5 assigns "treatment" status to Cement & Plaster Products plants, and "control" status to Carpentry & Joinery plants. These plants have much closer leverage ratios in the pre-period, though average leverage at Cement plants increases by much more. The fourth column of Table 5 compares the four more affected industries with the Sugar industry, which has a lower average fixed-asset share. Finally, the last column of Table 5 defines all four more affected industries as treatment, and all four less affected industries as control. The results are similar in all three cases: the coefficient on the interaction term is positive and large, suggesting an increase in financial development that led more affected plants to increase their leverage ratios by more than other plants.

[Table 6 about here.]

However, the final three columns in Panel B of Table 4 show that, in addition to higher average leverage and aggregate output, the more affected industries also had higher average labor productivity and higher labor productivity growth. The final three columns in Panel B of Table 4 report pre-2000 and post-2000 average log labor productivities in each industry. Labor productivity is higher on average for treated firms, and with the exception of Sugar plants, labor productivity growth was also higher among treated plants than control plants.

Table 6 examines the role of labor productivity in driving these results by including firm-level labor productivity as an explanatory variable. I also include state fixed effects. Many of the results from Table 5 are robust to including these extra controls. In particular, in all but the second column the interaction term is positive and large. In addition, the coefficient on leverage is also positive, suggesting that more productive firms borrow more on average than less-productive ones. This result is consistent with the model of section 4, in which agents borrow to move wealth from expected future productivity to the present.

An important identifying assumption when running difference-in-difference regressions is that both treatment and control groups exhibit parallel trends; that is, were the treatment group to not be treated, the change in its explanatory variable would be the same as the control group. This assumption cannot be tested directly. However, one way to argue for the parallel trends assumption is to interact the treatment variable with multiple time periods, and show that the positive estimated effects only obtain for periods after the change.

[Figure 10 about here.]

Figure 10 shows that while some of the regressions reported in Table 6 appear to violate the parallel trends assumption, others do not. Figure 10 plots coefficient estimates from running regression (14) augmented to include time dummies and interaction terms for each time period in the data. Although average Pharmaceutical leverage is higher in the last two periods than the first three (top panel of Figure 10), consistent with the results reported in Table 6, the top panel of Figure 10 is not encouraging in terms of the parallel trends assumption: Pharmaceutical leverage seems to jump the most between 1990 and 1995, and actually drops from 2006 to 2011. The first effect might be explained as a result of the major economic reforms of 1991, which included financial reforms (see section 3.1). The bottom panel of Figure 10 looks better. Average leverage in Carpentry & Joinery plants seems to be dropping slightly before 2000, but this change is very small relative to the large jump in average leverage at Cement & Plaster plants after 2001.

Overall, the results in Tables 5 and 6 and Figure 10 suggest that financial development did play a role in Indian economic growth from 1990 to 2011. However, in the next section I show that these patterns in the data can be driven by common shocks to productivity alone, without any financial development at all.

6.2 Model

In this section I use the model described in section 4 to perform idealized difference-in-difference regressions, and show that common shocks to productivity can confound inference on the role of financial development. I define financial development as a change in the collateral rate θ, which (as described above) in equilibrium reduces the cost of borrowing for firms, without affecting their exogenous total-factor productivity. In the model, I define leverage as bonds per unit of assets, $\ell \equiv \frac{b}{\kappa}$. I separate "treated" from "control" firms by assuming the former have a higher level of θ. In equilibrium, the collection of high-θ firms will have higher leverage ratios than the low-θ firms, as in the data. In addition, when both types of firms receive the same percentage shock to θ, the high-θ firms will respond more in both leverage and output growth. This effect is well-captured by a difference-in-difference regression.

The problem is that while financial development does affect some firms more than others, and in the expected ways, so do common shocks to productivity. In the model, a common shock to productivity has an additional impact on more-productive, borrowing firms, who respond by increasing leverage and output. I show that controlling for firm-level productivity does not erase this effect. In addition, even if treatment and control firms are correctly identified and randomly assigned, and financial development does occur, difference-in-difference regressions can return a negative result if the two groups happen to receive different productivity shocks.

To match the empirical results of section 6.1, in the exercises in this section I compute aggregate statistics over twenty-two time periods (years). I assume the economy is in stochastic steady-state from period 1 to period 11. In period 12, agents learn that the aggregate values of total-factor productivity (A) and/or financial development (θ) will change in period 13, to values where they will remain forever. Thus, the value and policy functions from period 12 onward are constant;

however, aggregate moments can still move because the endogenous joint distribution of (x, z), which is evolving according to equation (8), may not settle down to its stationary value immediately.

Table 7 reports model estimates for four difference-in-difference exercises. In each exercise, I assume four types of firms, two with a high level of θ (the treatment group) and two with a low level of θ (the control group). Within each type for θ are high- and low-TFP firms, where the spread in $\log A$ is reported in Table 3. These are the only dimensions along which firms vary ex ante (firms within each group vary in terms of idiosyncratic total-factor productivity z, but the distribution of z is identical across treatment and control types); in this sense, identification in the model is exact and along the same lines used by Vig (2013), who argues that high-tangibility firms are more affected by the Sarfaesi Act than low-tangibility firms. The identification used here is also similar to that in Rajan and Zingales (1998): firms with higher values of θ enjoy higher bond prices at all values of leverage and productivity, and so in equilibrium will finance a greater share of investment externally; thus the Rajan & Zingales method likely would classify high-θ firms as more "exernally dependent."

The first set of rows in Table 7 reports results from an idealized difference-in-difference scenario: both types of firms are identical in every respect other than the level of θ, which at $t = 13$ increases by 10%. Columns 4 through 6 of Table 7 show that as a result of this shock, both types increase leverage on average, but that the high types do indeed increase their leverage by about 30 bps more on average. The last column of Table 7 shows that aggregate output in the treated industry rises almost six percentage points more than the untreated industry.

The second set of rows in Table 7 add a common shock to productivity in addition to the shock to θ: both types on average become 10% more productive at $t = 13$. A difference-in-difference methdology still "works" in this case: leverage and output rise on average for both types of firms, but by more for the "treated," high-θ firms. Together, the first two groups of rows in Table 7 suggest that difference-in-difference regressions of the kind analyzed in section 6.1 will produce positive coefficients when financial develoment occurs, even if there is also productivity growth.

[Table 7 about here.]

However, the third and fourth exercises in Table 7 show that such a positive difference-in-difference result is neither necessary nor sufficient to identify financial development. In the third exercise, the two types differ in θ as before but θ is constant over time. Both types receive, as in the second exercise, a 10% productivity shock. In this case, the productivity shock lowers the cost of borrowing, but it does so by more for the high-θ firms. Thus, the high-θ types increase leverage by 35 bps more than the low-θ types. Output also grows by about 62 bps more for the high-θ types.

The third exercise in Table 7 features no financial development, but a positive difference-in-difference result; the final exercise in Table 7 has financial development, but a negative difference-in-difference result. Here I assume that θ grows by 10% for both types as before, but that the low-θ types have faster average total-factor productivity growth than the high-θ types. In this exercise, the difference in productivity growth is more important than the shock to θ, so that while both types experience output growth, and increase their leverage ratios on average, the low-θ type increases everything by more.

None of the exogenous shocks examined in Table 7 are large when compared to variation in the data; if anything, they are too small. Leverage differences in the data between types range from 5 to 9 percentages points (see Table 5), and the assumed differences in θ lead to a roughly 8 percentage point difference in Table 7. Average output growth over the entire sample period, reported in the last column of Panel A of Table 4, is between 0.4 and 2.0 in log-points, compared to at most 20% in Table 7. Likewise, labor productivity growth in the data (apart from Tobacco plants) is significantly larger than that reported in the last column of Table 7. I conjecture that larger productivity shocks would lead to even bigger movements in leverage and output, and further confound attempts to use difference-in-difference methodologies to identify financial development.

[Table 8 about here.]

As an additional robustness check, and in parallel to the regressions in Table 6, in Table 8 I run difference-in-difference regressions in the model controlling for plant-level labor productivity. Unlike the results reported in table 7 which are differences in means (integrals) that can be computed directly, controlling for labor productivity means simulating data from the model and running regressions on the simulated data. Details on these computations are in Appendix C.

The results reported in Table 8 show that, even controlling for firm-level labor productivity, aggregate productivity growth can induce a positive difference-in-difference coefficient even in the absence of financial development. As in Table 7, the first two blocks of rows show that financial development does induce a positive difference-in-difference coefficient. However, the third block also features a positive coefficient, without any financial development; moreover the coefficient is not appreciably different from the corresponding coefficient in Table 7. Finally, higher productivity growth among the low-θ firms can induce a negative difference-in-difference coefficient, even when financial development did occur. However, controlling for labor productivity reduces this effect considerably relative to the results in the last block of rows in Table 7. Finally, the coefficient on leverage in Table 8 is positive, as in the regressions results in Table 6 and in contrast to a finding of Rajan and Zingales (1995) that productivity (measuring by return on assets) and leverage are inversely correlated. I attribute the difference in the model to a different assumed rationale for borrowing (consumption smoothing rather than tax evasion), and in the data to a focus on small, privately-owned firms.

7 Conclusion

In this paper, I derive a dynamic equilibrium model that isolates aggregate financial shocks from common shocks to productivity by comparing their distinct effects on the joint distribution of firm size and productivity. Financial shocks affect the allocation of resources across firms, but hold productivity within firms constant; common shocks to productivity affect both the allocation of resources and within-firm productivity. I use the calibrated model and establishment-level microdata from India to show that despite numerous financial reforms since 1991, financial factors explain at most 7% of aggregate labor productivity growth. In addition, simple difference-in-difference regressions aimed at identifying financial development from other shocks do not provide reliable inference when firms receive common productivity shocks.

I am not suggesting that financial frictions are unimportant in determining real outcomes; indeed, the results of the paper depend on the existence of financial frictions. It is precisely because of the friction that more-productive firms are more sensitive to a common shock to productivity. Instead, I argue that *changes* in financial frictions do not appear to be an important contributor to economic growth over time, despite a rising covariance between size and productivity and positive coefficients in simple difference-in-difference regressions. I show that both of these empirical observations can be driven by common shocks to productivity in the presence of a constant financial friction, and, in the case of the Olley-Pakes decomposition, that common shocks to productivity are quantitatively more relevant than financial development.

Overall, I argue that factors other than financial frictions are important in explaining Indian economic growth. One such factor might be frictions in adopting modern management practices. Bloom et al. (2013) show that management practices are important factors driving productivity in large Indian textile firms. They argue that informational barriers, rather than financial frictions, are what prevent firms from adopting efficiency-enhancing management practices. Brooks, Donovan and Johnson (2016) illuminate a particular informational barrier in small Kenyan retail establishments, as well as how eliminating that barrier with inter-firm training and mentorhsip programs can have a large impact on productivity.

Alternatively, financial frictions that primarily affect within-firm productivity could also be driving my results. For example, it may be that adopting a higher-productivity-growth technology or entering into a more-productive sector requires large fixed costs that are difficult to finance, as in the models of Buera, Kaboski and Shin (2011) and Cole, Greenwood and Sanchez (2016). Although both papers focus primarily on differences across countries, it may be that changing financial frictions over time within a country affects average productivity through this channel. Uncovering the exact underlying factors behind the common productivity growth identified by my model is an avenue that I hope to explore in future research.

A Proofs

A.1 Proof of Proposition 1

Proof. Optimal capital k^* follows directly from the budget and the fact that revenue is strictly increasing in k, so that the budget holds with equality. Plugging this in to equation (5), the first-order condition for m becomes

$$w\frac{\kappa}{(1+wm)^2} = \frac{\gamma-1}{\gamma}\left(Ae^{z'}\right)^{\frac{\gamma-1}{\gamma}}\left(m^{1-\alpha}k^*\right)^{-\frac{1}{\gamma}}\left[(1-\alpha)\,m^{-\alpha}k^* - wm^{1-\alpha}\frac{\kappa}{(1+wm)^2}\right],$$

which can be simplified to

$$1 = \frac{\gamma-1}{\gamma}\frac{py}{mk^*}\left[\frac{1-\alpha}{w} - \alpha m\right],$$

which upon further simplification yields equation (10).

To show that the solution to equation (10) is unique, consider that the left-hand-side of equation (10) is strictly increasing in m, and equals 0 at $m = 0$. The right-hand-side of equation (10) is strictly positive so long as $m \in \left(0, \frac{1}{w}\frac{1-\alpha}{\alpha}\right)$, thus the solution m^* must also lie in this interval. The derivative of the right-hand-side of equation (10) with respect to m is given by

$$A^*\left(1+wm\right)^{\frac{1}{\gamma}}\left[\frac{w}{\gamma}\left(\frac{1-\alpha}{w} - \alpha m\right) - \alpha\right]$$

which has the same sign as the term in square braces. This term is linear and decreasing in m; at m's supremum value of $\frac{1}{w}\frac{1-\alpha}{\alpha}$, it is negative. At m's minimum value of zero, it is

$$\frac{1-\alpha}{\gamma} - \alpha,$$

which is negative if and only if $\frac{1-\alpha}{\alpha} < \gamma$. Thus, this restriction on parameters ensures that the right-hand-side of equation (10) is decreasing for $m \in \left(0, \frac{1}{w}\frac{1-\alpha}{\alpha}\right)$; because it is a continuous function of m, and is strictly greater than the left-hand side for $m = 0$ and strictly smaller than the left-hand side for $m = \frac{1}{w}\frac{1-\alpha}{\alpha}$, there must be a unique solution where the two sides are equal.

To show that m^* is increasing in z' and decreasing in κ, it suffices to show that it is increasing in A^* defined in equation (11). This follows directly from the fact that the right-hand side of equation (10) is increasing in A^*, while the left-hand side is increasing in m^* but constant with respect to A^*.

The proof up to this point has ignored the overhead labor constraint $mk \geq \underline{L}$. If $m^*k^* > \underline{L}$, then the constraint doesn't bind and is irrelevant. If instead $m^*k^* < \underline{L}$, the constraint binds and the agent sets $k = \kappa$ and $m = \frac{\underline{L}}{\kappa}$, which satisfies both the budget and the overhead labor constraint.

□

A.2 Proof of Proposition 2

Proof. Define leverage as bonds per unit of asset, $\ell \equiv \frac{b}{\kappa}$. Fix values for A, w, κ, and z. There are two cases to consider, whether the overhead-labor constraint binds or not. This depends on ε; in fact, it is straightforward to see that agents with $\varepsilon < \varepsilon^*$ choose $l = \underline{L}$, where

$$\varepsilon^* \equiv \frac{1}{\sigma}\left[\frac{\gamma}{\gamma-1}\left((1-\zeta)\log\frac{\underline{L}}{\kappa} - \log\left(\frac{1-\alpha}{w} - \alpha\frac{\underline{L}}{\kappa}\right) - \frac{1}{\gamma}\log\left(1+w\frac{\underline{L}}{\kappa}\right) - \log\frac{\gamma-1}{\gamma}\right)\right.$$
$$\left. + \frac{1}{\gamma-1}\log\left(\kappa + w\underline{L}\right) - \log A - \rho z\right], \tag{15}$$

$\zeta \equiv (1-\alpha)\,\frac{\gamma-1}{\gamma}$, and $\varepsilon^* = -\infty$ if the second term in logarithms is negative. If $\varepsilon > \varepsilon^*$ the overhead labor constraint does not bind.

Solving for the default threshold involves solving equation (12) for both the case where the overhead labor constraint binds ($\underline{\varepsilon}_1$) and where it does not ($\underline{\varepsilon}_2$). Then $\underline{\varepsilon} = \underline{\varepsilon}_2$ if $\underline{\varepsilon}_2 > \varepsilon^*$, and $\underline{\varepsilon} = \underline{\varepsilon}_1$ otherwise.

Computing $\underline{\varepsilon}_1$ is straightforward: because the labor constraint binds, $mk = \underline{L}$ and ε only appears once in equation (12). Inverting this equation for ε yields

$$\underline{\varepsilon}_1 = \begin{cases} \frac{1}{\sigma}\left[\frac{\gamma}{\gamma-1}\log\left(\ell-\theta\right) - \log A - \rho z - (1-\alpha)\log\frac{\underline{L}}{\kappa} + \frac{1}{\gamma-1}\log\kappa\right] & \text{if } \ell > \theta \\ -\infty & \text{otherwise} \end{cases}$$

Labor-constrained agents who borrow $\ell < \theta$ will never default, since regardless of the shock they always retain some output, ensuring their post-repayment value is higher than $(1-\theta)$.

Solving for $\underline{\varepsilon}_2$ requires a bit more work, since agents for whom the overhead labor constraint does not bind have ε appearing in multiple places in equation (12). Rewrite the threshold equation (12) as

$$(1+\ell-\theta)\,\frac{\kappa}{\kappa+w\underline{L}}=\left(\frac{Ae^{z'}m^{1-\alpha}}{1+wm}\right)^{\frac{\gamma-1}{\gamma}}\left(\kappa+w\underline{L}\right)^{-\frac{1}{\gamma}}+\frac{1}{1+wm},$$

which, after plugging in the optimal m from equation (10) and rearranging, yields

$$(1+wm)\,(1+\ell-\theta)\,\frac{\kappa}{\kappa+w\underline{L}}=1+\frac{\frac{\gamma}{\gamma-1}m}{\frac{1-\alpha}{w}-\alpha m}. \tag{16}$$

Equation (16) is a quadratic equation in m, $0=am^2+bm+c$, with coefficients given by

$$\begin{aligned}
a&=-\alpha w\,(1+\ell-\theta)\,\frac{\kappa}{\kappa+w\underline{L}}\\
b&=-\frac{\gamma}{\gamma-1}+\alpha+\frac{\kappa}{\kappa+w\underline{L}}\,(1-2\alpha)\,(1+\ell-\theta)\\
c&=\frac{1-\alpha}{w}\left[(1+\ell-\theta)\,\frac{\kappa}{\kappa+w\underline{L}}-1\right].
\end{aligned} \tag{17}$$

The rest of the proof consists of showing that this equation has a unique positive root $\underline{m}$, if and only if $\ell>\theta+\frac{L}{\kappa}$. Given such a root, A, κ, and z, the corresponding value for $\underline{\varepsilon}_2$ can be found using the solution to equation (10).

Suppose $\ell>\theta+\frac{w\underline{L}}{\kappa}$. Then $c>0$, and because $a<0$ it follows that $b^2-4ac>b^2$. Then regardless of the sign of b, one of the roots is positive and one is negative.

Suppose $\ell=\theta+\frac{w\underline{L}}{\kappa}$. Then $c=0$ and $b=\alpha-\frac{\gamma}{\gamma-1}<0$, so that either $m=0$ or $m=-\frac{b}{a}<0$.

Suppose $\ell<\theta+\frac{w\underline{L}}{\kappa}$. Then $c<0$ and $b^2-4ac<b^2$, so it is sufficient (using the quadratic formula and the fact that $a<0$) to show that $b<0$. Suppose it does not; then

$$\frac{\kappa}{\kappa+w\underline{L}}\,(1+\ell-\theta)\,(1-2\alpha)\geq\frac{\gamma}{\gamma-1}-\alpha,$$

so that

$$\begin{aligned}
\frac{\kappa}{\kappa+w\underline{L}}\,(1+\ell-\theta)&\geq\frac{\frac{\gamma}{\gamma-1}-\alpha}{1-2\alpha}\\
&\geq 1,
\end{aligned} \tag{18}$$

where the second line follows because the numerator is greater than 1 (since $\alpha < \frac{1}{\gamma-1}$), and $\alpha < \frac{1}{2}$ so the denominator is less than 1 but positive. But equation (18) is a contradiction, since it implies that $c \geq 0$ which was assumed false at the start. Thus $b < 0$ and there are no positive roots for this case.

The solution $\underline{m}$ to equation (16) is the optimal labor-capital ratio, given A, z, and κ, at which agents are indifferent between defaulting and repaying. The fact that the optimal labor-capital ratio is strictly increasing in z', means that there is a one-to-one mapping between $\underline{m}$ and $\underline{\varepsilon}$, given A, z, and κ; moreover equations (10) and (11) give this mapping in closed form (even though m^* itself as a function of A^* must be computed numerically). That $\underline{\varepsilon}$ is decreasing in z and increasing in κ then follows directly from the same facts (reversed) for m^*. $\square$

B Model Solution Approximation

In this section I describe the numerical approximations I use to solve the model described in section 4. The solution consists of three parts: solving the static production problem (5), using that solution to solve the dynamic investment problem (7), and using the implied policy functions from the dynamic problem to solve the Fredholm equation (8).

B.1 Approximating Equation (10)

I approximate the solution m^* to equation (10) as a univariate function in A^*, defined in equation (11), by inversion: that is, I guess a range of values for m^*, compute the implied A^* for a large number of m^* in that range, linearly interpolate the resulting function, and invert the linear interpolation at the appropriate value of A^*.

More specifically: define the function $A^*(m)$ implicitly from equation (10). Given a list $\vec{A}$ of values of A^* at which to evaluate m^*, I find m_0 sufficiently close to zero and m_1 sufficiently close to $\frac{1-\alpha}{\alpha}\frac{1}{w}$ such that $A^*(m_0) < \min\left\{\vec{A}\right\}$ and $A^*(m_1) > \max\left\{\vec{A}\right\}$. Because the true function $m^*(A^*)$ is monotonically increasing, the desired values $m^*\left(\vec{A}\right)$ lie in the interval (m_0, m_1).

The function $A^*(m)$ implictily defined by equation (10) is in closed form; I thus compute a large number of values of A^* on a uniform grid for m in the interval (m_0, m_1). These values of A^*, and the grid for m, define a piecewise linear function $\widetilde{A}^*(m)$, which I then invert and evaluate at the desired values $\vec{A}$ of A^* to approximate the true m^*.

The above procedure yields a univariate function $m^*(A^*)$; the bivariate function $m\left(Ae^{z'}, \kappa\right)$ can then be computed using the definition of A^* in equation (11).

B.2 Approximating the Value and Policy Functions

I compute the value and policy functions $V(x, z)$, $\kappa(x, z)$, and $\ell(x, z)$ by approximating the continuous state-space (x, z) with a set of discrete points. The grid of points is a tensor product of grids in x and z. I specify the x grid as 150 evenly-spaced points, in logs, from -6 to 11 for the high-A types and from -6 to 1 for the low-A types. I verify ex post that these bounds on $\log x$ are not binding for either type; that is, a negligible mass of firms lies at either boundary point. Because the size distribution has fat tails in equilibrium, I also check that the mass at each boundary, multiplied by the level value $\exp\{\log x\}$, is also a negligible value of the total expected value of x. I specify the z grid as 10 points using the Rouwenhorst (1995) method. I then organize the $N = 150 \times 10 = 1,500$ state-space points in a list, so that the first 10 points are the first $\log x$ point and all 10 z points, the next 20 points are the second $\log x$ point and all 10 z points, and so on (this accounting, while tedious, matters for the formulas below).

The value and policy functions in this approximation are represented by vectors of values at each gridpoint. Given a value function, at each gridpoint I search over policies $\log \kappa$ and ℓ to maximize current-period utility plus the continuation value. To do so, I specify a policy grid for $\log \kappa$ on the interval $[\log x - 5, \log x + 5]$ and a policy grid for leverage on the interval $[0, 1.3]$. I verify ex post that in equilibrium only a negligible measure of the stationary distribution of $(\log x, z)$ chooses these extreme values. These grids have $M = 150$ points each, so that there are $M^2 = 22,500$ potential actions at each state-space point.

The approximate solution to system (7) induces an $N \times N$ transition matrix Π, whose $\left(i, i'\right)$ element represents the probability of transitioning to state i' from state i. This matrix is the key component in approximating the equilibrium size-productivity distribution function $F(\log x, z)$ in equation (8), which I also approximate as a $N \times 1$ vector of values. The rest of this subsection describes the computation of Π in more detail.

First, fix the value function vector $\vec{V}$. The following procedure will then produce a new value of $\vec{V}$, and iterate to convergence. Next, fix a state-space point $i = (\log x, z)$. Everything beyond this point is then repeated N times, once for each point. Equation (7) at this point becomes

$$v_i = \max u^* + \beta \vec{P} \left[(1-\pi) \vec{V} + \pi \log \vec{x} \right], \tag{19}$$

where u^* is the $M^2 \times 1$ vector of implied period-utility values for each policy choice from point i, $\vec{x}$ is the $N \times 1$ vector of the $\log x$ values of the state (i.e., 10 copies of the first $\log x$, followed by 10 copies of the next, etc), and $\vec{P}$ is the $M^2 \times N$ matrix of transition implied by each policy choice from point i. The max is taken over the M^2 discrete policy choices, as described below. Denote the index of the optimal policy by j^*; then the ith row of Π is given by the j^*th row of $\vec{P}$. Solving equation (19) at each point i then fills out the entire Π matrix.

To compute element (j, i') of the matrix $\vec{P}$ I calculate the implied transition probabilities from point i to point i', for each i' from 1 to N. To do so, I first compute the $M^2 \times N$ transition CDF matrix $\widetilde{P}$, whose (j, i') element is the probability of moving to a $\log x$ point below that in state i' and a z point below that in state i' when the policy is j. I convert $\widetilde{P}$ to the matrix of transition probabilities $\vec{P}$ using

$$\vec{P} = \widetilde{P} T^{-1\prime}, \tag{20}$$

where $T \equiv t_{150} \otimes t_{10}$ is the matrix that converts a discrete pdf to a CDF through appropriate additions (so that T^{-1} converts a CDF vector into a pdf vector). The matrices t_n are the $n \times n$ lower-triangular matrices of 1s.

Let p_1 be the matrix of probabilities of moving from i to a $\log x$ point lower than i', and p_2, the probability of moving from i to a z point lower than i', given the current policy choice j. Then $\widetilde{P} = \min \{p_1, p_2\}$, where the minimum is element-wise.

Transition probabilties p_1 in the $\log x$ dimension depend on the agent's policy choice for $\log \kappa$ and ℓ, but transition probabilities p_2 in the exogenous z dimension do not. They are therefore simpler to compute. Let P_z be the 10×10 matrix of transition probabilities implied by the Rouwenhorst (1995) method, and let $\widetilde{i} \in \{1, 2, ...10\}$ denote the z index of the current point i. Likewise let $\widetilde{i'}$ be the z-point of the next-period point i' under consideration. Then the 10 unique values over $\widetilde{i'}$ in p_2

are given by

$$p_2\left(\widetilde{i'}\right) = \left[P_z t'_{10}\right]_{\widetilde{i},\widetilde{i'}}$$

where the subscript denotes the given element of the matrix in brackets. The above values of p_2 then need to be copied appropriately so that they correspond to the $M^2 \times N$ values of the policies and state-space. The role of the matrix t'_{10} is to convert the transition probabilities in P_z into a transition CDF; this conversion is later reversed, after being combined with the $\log x$ transition CDF described below, in equation (20).

I compute p_1 by finding the values e of ε low enough to push x' below the value x^* implied by point i', using the current policy choice j. It is possible that $e = -\infty$, for example if $x^* < (1-\theta)\,\kappa$; agents are guaranteed at least this value of x' by their default option. Solving for e at each policy choice j is complicated by the fact that the overhead-labor constraint in equation (5) may or may not bind. I handle this issue by finding the value of ε where the overhead labor constraint binds, and solving for e in each region of x^* space separately.

Fix the current policy $j = (\kappa, \ell)$. Define

$$\overline{x} \equiv y^*\left(Ae^{\rho z+\sigma\varepsilon^*}, \kappa, w\right) - \ell\kappa,$$

where ε^* is defined in equation (15). For $x^* \leq \overline{x}$, the overhead-labor constraint binds, and for $x^* > \overline{x}$, it does not. In what follows, I fix x^* (from next period's state i', i.e. column i' of $\widetilde{P}$) and consider each case in turn. Before I do so, it is important to note that I sample the x^* points not from the uniform $\log x$ grid used everywhere else in the paper, but from the midpoints of that grid, plus an additional point just past the maximum value of that grid (such that the resulting $\log x^*$ grid is uniform). The reason is that the computations below are for a CDF, i.e. probabilities that $\log x$ is less than $\log x^*$. It is important that each $\log x^*$ point be at the right end-point of each interval defined by the $\log x$ grid, as a simple application of the discretization to distributions with a known solutions (an AR(1), for example) shows. Using the same grids for the $\log x$ at which policies and other functions are evaluated, as for the CDF computationm leads to bias.

If $x^* \leq \overline{x}$, then the overhead-labor constraint binds, ε appears only once in the definition of y^*

and e is given by

$$e = \frac{1}{\sigma}\left[\frac{\gamma}{\gamma-1}\log\left[x^* - (1-\ell)\,\kappa\right] - \log A - \rho z - (1-\alpha)\log \underline{L} - \alpha\log\kappa\right]. \tag{21}$$

If instead $x^* > \overline{x}$, then ε appears in the definition of y^* in multiple places, because the labor-capital ratio m depends on it. Nevertheless, there is a unique value of ε that moves to $x' = x^*$; the proof and construction of such an ε is entirely analogous to Proposition 2, with $(1-\theta)\,\kappa$ replaced by x^*.

The details are as follows: following the algebra in Proposition 2, the value of m that sets $y^* = x^*$ is given by the quadratic equation

$$0 = am^2 + bm + c, \tag{22}$$

where

$$\begin{aligned}
a &= -\alpha w\left(\frac{x^*}{\kappa} + \ell\right)\frac{\kappa}{\kappa + w\underline{L}} \\
b &= -\frac{\gamma}{\gamma-1} + \alpha + \frac{\kappa}{\kappa + w\underline{L}}(1-2\alpha)\left(\frac{x^*}{\kappa} + \ell\right) \\
c &= \frac{1-\alpha}{w}\left[\left(\frac{x^*}{\kappa} + \ell\right)\frac{\kappa}{\kappa + w\underline{L}} - 1\right].
\end{aligned}$$

Denote the solution to equation (22) by m^*. It is straightforward to show that equation (22) has a unique solution for m^* if and only if $x^* > (1-\theta)\,\kappa$. Then e can be calculated from m^* by computing the implied value of A^* using equation (10), and converting this to ε using the definition of A^* in equation (11).

Putting all three pieces together, $e = -\infty$ if $x^* < (1-\theta)\,\kappa$. If $x^* > \overline{x}$, I compute e using the solution m^* to equation (22). Then, if $\overline{x} > (1-\theta)\,\kappa$,and x^* lies in it, I compute e from equation (21). I repeat this computation for each policy j and each future $\log x$ value in i' to completely fill the $M^2 \times N$ matrix $\vec{e}$, after which $p_1 = \Phi\left\{\vec{e}\right\}$.

The above procedure gives me the matrix $\vec{P}$ of transition probabilities from equation (20), which I compute at each state-space point i, and take the max according to equation (19). I stack the corresponding values of u^* into an $N \times 1$ vector $\vec{u}^*$, and the corresponding rows of each $\vec{P}$ into the

$N \times N$ matrix Π, and then compute a new value vector $\vec{V}$ according to

$$\vec{V} = \left[I - \beta\left(1-\pi\right)\Pi\right]^{-1}\left(\vec{u}^{*} + \beta\pi\Pi\log\vec{x}\right), \tag{23}$$

which approximately solves system (7). I then use this new value of $\vec{V}$ in the next iteration, repeat the entire procedure, and iterate until the policy functions converge (at which point the value function will also converge).

To speed up the above procedure, I ignore policy choices with negative consumption or that have ℓ beyond the Laffer peak; that is, leverage points (for each value of $\log\kappa$) in which the default probability is so high that increasing leverage actually reduces $q\ell$. Rational agents would not choose such leverage values, since they result in less resources today for a higher future liability, than other choices. Wherever possible, I vectorize calculations or compute values once and then distribute them to appropriately-sized vectors or matrices; this speeds up calculation but means that the actual computations differ somewhat from the heuristic description above. Many of these calculations are embarrassingly parallel, and I take advantage of this fact where possible.

Finally, everything described in this section is for a single value of the parameters and a single time period t—equation (23) in particular applies only for a steady-state value function. Equilibria for agents with different values of A or θ involve repeating all the steps described here for each agent type. Dynamics imply taking a future value function vector $\vec{V}$ as given, computing the resulting policy functions for period t, and generating a new value function (to be used as period $t-1$'s continutaiton value) as

$$\vec{V}_t = \vec{u}^{*} + \beta\Pi\left[\left(1-\pi\right)\vec{V}_{t+1} + \pi\log\vec{x}\right].$$

B.3 Approximating the Size-Productivity Distribution

The model implies an equilibrium distribution across log net wealth $\log x$ and productivity z. To compute aggregste quantities such as total output and capital, or the excess demand for savings, I need to integrate over this distribution. In this section I describe how I represent the endogenous distribution of (x, z) that solves equation (8).

As in Appendix B.2, I approximate the equilibrium distribution of $(\log x, z)$ as a vector of values on a grid of point for $\log x$ and z. In fact, I describe most of the work involved in the approximation in Appendix B.2, since the key ingredient is the equilibrium state-transition matrix $\Pi_{i,t}$. Given this matrix, which varies across agents of type i (which indexes their value of A and θ) and over time, the vector $f_{i,t+1}$ of the measure of agents in each state-space point in the grid that approximate equation (8) is

$$f_{i,t+1} = \pi h + (1 - \pi)\, \Pi_{i,t} f_{i,t}, \tag{24}$$

where the vector h contains the measure of agents in each state that enter exogenously each period. I describe the construction of this vector below. I solve equation (24) for steady-state values when $f_{i,t+1} = f_{i,t}$, or for a dynamic endogenous distributon whe A or θ are changing over time. Integrals over the endogenous distribution of $(\log x, z)$ are then summations over the $f_{i,t}$ vector.

I compute the vector h (which does not vary across i or t, though this extension is minor) as follows. I approximate the entry measure over $\log x$ as

$$h_1 \equiv t_{150}^{-1} \left(\Phi \left\{ \frac{\log x^*}{\sigma_{x_0}} \right\} \otimes \vec{1}_{10} \right), \tag{25}$$

where $\log x^*$ is the 150×1 vector of CDF $\log x$ points, $\Phi\{\cdot\}$ is the standard normal CDF, $\vec{1}_{10}$ is the 10×1 vector of 1s, and the matrix t_{150} is defined just below equation (20). The Kronecker product in equation (25) expands the 150×1 CDF to the 1500×1 state-space grid (10 points for each z point). The pre-multiplication by t_{150}^{-1} converts the CDF into a pdf (measure).

I compute the ergodic density of z as the unit eigenvector of the P_z transition matrix from the Rouwenhorst (1995) method. I tile this 10×1 vector 150 times to make it 1500×1; denote this vector as h_2. Then the h vector is given by

$$h = h_1 \odot h_2$$

where $\odot$ represents an element-wise product.

C Formulas

In this section I describe how I combine the policy functions computed in Appendix B.2 and the cross-sectional distribution(s) of $(\log x, z)$ computed in Appendix B.3 to approximate aggregate quantities of interest, such as aggregate aggregate output, average leverage, and the Olley-Pakes covariance. Most of the formulas in this section apply to a single type of agent, i.e. one value of A and θ. At the end of the section I describe how I aggregate across agent types.

Given a size-productivity distribution vector f^*, I approximate integrals of functions of $\log x$ and z as f^*-weighted averages of the functions evaluated at each of the 1,500 points in the $(\log x, z)$ grid. Average leverage is then given by

$$E\{\ell\} = \frac{1}{\sum_i \left(1 - \Phi\{\underline{\varepsilon}_i\}\right) f_i^*} \sum_i \ell_i \left(1 - \Phi\{\underline{\varepsilon}_i\}\right) f_i^* \tag{26}$$

where i indexes each $(\log x, z)$ point, ℓ_i is the optimal leverage choice at point i, $\underline{\varepsilon}_i$ is the implied default probability at point i implied by ℓ_i and the investment choice κ_i, and $\Phi\{\cdot\}$ is the standard normal CDF. The $1 - \Phi$ terms in equation (26) condition the expectation on not defaulting.

To compute average firm age in the model, I need to correct for firm default, the probability of which depends on the current state i. I assume that when an agent defaults, he starts a brand-new firm, and from the point of view of the econometrician this firm has just been born. This will reduce the average age of firms below $\frac{1}{\pi}$. The vector of average age in each state point $\vec{a}$ satisfies

$$\vec{a} = (1-\pi)\,\Pi' \left(I - \operatorname{diag}\left\{\vec{d}\right\}\right)\left(\vec{a} + \vec{1}\right) + \left(\pi I + \Pi' \operatorname{diag}\left\{\vec{d}\right\}\right)\vec{1} \tag{27}$$

where $\vec{d} \equiv \Phi\{\underline{\varepsilon}\}$ is the default probability in each state, $\Phi\{\cdot\}$ is the standard normal CDF, and Π is the transition matrix computed in section B.2. The first term on the right-hand side of equation (27) handles the firms who do not exit exogenously and do not default. These firms get one year older, and transition to a new state through the matrix Π'. The second term handles the firms who will be viewed as one year-olds: these include a measure π of new entrants, plus agents who have defaulted in a previous state, who transition through the matrix Π' as well. Solving equation (27) for $\vec{a}$ yields

$$\vec{a} = \left[I - (1-\pi)\,\Pi'\left(I - \text{diag}\left\{\vec{d}\right\}\right)\right]^{-1}\left(\vec{1} + \pi\Pi'\vec{d}\right)$$

so that average age is given by

$$E\{a\} = \vec{a}'f^{*}. \tag{28}$$

The expressions in equations (26) and (28) are straightforward to compute from the policy and density vectors. However, most aggregates in the model require an additional integration, because they depend not just on $\log x$ and z but also on the realization of ε, primarily through the dependence of the labor-capital ratio m. To overcome this issue, at each point in the 150×10 integration grid I use Matlab's `integral` numeric integration routine to compute the appropriate integral over ε. In addition, whether or not the $mk \geq \underline{L}$ constraint binds also depends on ε, and this needs to be taken into account in the integrals. When the overhead-labor constraint binds, I compute integrals over ε inc closed-form rather than numerically.

With this in mind, the appropriate formulas for revenue py_i and labor l_i at a given point $i = (\log x, z)$ with optimal policy (κ, ℓ) are given by

$$\begin{aligned}
py_i = &\left(e^{\frac{1}{2}\sigma^2\left(\frac{\gamma-1}{\gamma}\right)^2}\left[\Phi\left\{\overline{\varepsilon} - \sigma\frac{\gamma-1}{\gamma}\right\} - \Phi\left\{\underline{\varepsilon} - \sigma\frac{\gamma-1}{\gamma}\right\}\right]\left[\left(\frac{\underline{L}}{\kappa}\right)^{1-\alpha} \Big/ \left(1 + w\frac{\underline{L}}{\kappa}\right)\right]^{\frac{\gamma-1}{\gamma}}\right. \\
&\left. + \int_{\overline{\varepsilon}}^{\infty}\left[\frac{e^{\sigma\varepsilon}m(\varepsilon)^{1-\alpha}}{1 + wm(\varepsilon)}\right]^{\frac{\gamma-1}{\gamma}}\phi(\varepsilon)\,d\varepsilon\right)\left[Ae^{\rho z}\kappa\right]^{\frac{\gamma-1}{\gamma}} \\
l_i = &\,\underline{L}\left(\Phi\{\overline{\varepsilon}\} - \Phi\{\underline{\varepsilon}\}\right) + \kappa\int_{\overline{\varepsilon}}^{\infty}\frac{m(\varepsilon)}{1 + wm(\varepsilon)}\phi(\varepsilon)\,d\varepsilon
\end{aligned}$$

where κ is the optimal savings at this particular value of $i = (\log x, z)$, $m(\varepsilon) = m^{*}\left(Ae^{\rho z + \sigma\varepsilon}, \kappa\right)$ from Proposition 1 when the labor constraint does not bind, and $\overline{\varepsilon} \equiv \max(\underline{\varepsilon}, \varepsilon^{*})$. ε^{*} is the shock at which the labor constraint binds, given in equation (15), so the definition of $\overline{\varepsilon}$ ensures that I include, or do not include, labor-constrained agents as appropriate.

Aggregates revenue and labor are then given by the appropriate summations across i, using the computed density vector f^*:

$$PY = \sum_i py_i f_i^*$$
$$L = \sum_i l_i f_i^*. \tag{29}$$

To compute the Olley-Pakes covariance, I compute log revenue labor productivity for plants $(\log x, z, \varepsilon)$, and then integrate this quantity over ε:

$$\begin{aligned}\log \mathrm{ARPL}_i = &\left(1 - \Phi\left\{\underline{\varepsilon}\right\}\right)\left(\frac{\gamma-1}{\gamma}\left(\log A + \rho z\right) - \frac{1}{\gamma}\log\left(\kappa + w\underline{L}\right)\right)\\ &+ \sigma\frac{\gamma-1}{\gamma}\left[\phi\left(\underline{\varepsilon}\right) - \phi\left(\overline{\varepsilon}\right)\right]\\ &- \left(\Phi\left\{\overline{\varepsilon}\right\} - \Phi\left\{\underline{\varepsilon}\right\}\right)\left(\left(1-\zeta\right)\log\frac{\underline{L}}{\kappa} - \frac{1}{\gamma}\log\left[1 + w\frac{\underline{L}}{\kappa}\right]\right)\\ &+ \int_{\overline{\varepsilon}}^{\infty}\left[\left(1-\zeta\right)\log m\left(\varepsilon\right) - \frac{1}{\gamma}\log\left[1 + wm\left(\varepsilon\right)\right] - \sigma\frac{\gamma-1}{\gamma}\varepsilon\right]\phi\left(\varepsilon\right)d\varepsilon,\end{aligned}$$

where the first line reflects the common expected components of revene labor productivity across agents in cell i, the second line reflects the idiosyncratic shock ε in output, and the last two lines integrate over the choice of labor-capital ratio m for agents who are constrained to set $mk = \underline{L}$, and those who are not.

Then the simple-average and weighted-average labor productivities are given by

$$\mathcal{Z} = \sum_i \log \mathrm{ARPL}_i f_i^*$$
$$\mathcal{Z}_{\mathrm{wt}} = \sum_i \log \mathrm{ARPL}_i w_i f_i^*,$$

where

$$w_i \equiv \frac{L_i}{L}$$

is cell i's share of aggregate labor. The Olley-Pakes covariance is then $\mathcal{C} = \mathcal{Z}_{\mathrm{wt}} - \mathcal{Z}$.

Finally, to compute the dispersion in labor productivity, I compute the second moment of log labor productivity $\hat{z}$ for plants $(\log x, z, \varepsilon)$ as

$$\hat{z}_i^2 = \left[\underbrace{\frac{\gamma-1}{\gamma}\left(\log A + \rho z\right) - \frac{1}{\gamma}\log\left(\kappa + w\underline{L}\right)}_{h} \underbrace{- \left(1-\zeta\right)\log m + \frac{1}{\gamma}\log\left(1+wm\right)}_{g} + \frac{\gamma-1}{\gamma}\sigma\varepsilon \right]^2. \quad (30)$$

I first integrate equation (30) over ε, at each state-space point, and then sum over f^* to get the implied second moment of log labor productivity:

$$E\left\{\hat{z}\right\} = \sum_i f_i^* \int_{\underline{\varepsilon}_i}^{\infty} \hat{z}_i^2 \phi\left(\varepsilon\right) d\varepsilon$$

Then the dispersion in log labor productivity is given by

$$\sigma_{\log \mathrm{ARPL}} = \sqrt{E\left\{\hat{z}^2\right\} - \mathcal{Z}^2}.$$

I integrate equation (30) over ε in two steps: first for $\varepsilon \in (\underline{\varepsilon}, \overline{\varepsilon})$, where the overhead-labor constraint binds. In this case, both the h and g terms in equation (30) are independent of ε, since $m = \frac{\underline{L}}{\kappa}$, so the integral is given by

$$\begin{aligned}\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \hat{z}_i^2 \phi\left(\varepsilon\right) d\varepsilon = {} & \left(h+g\right)^2 \left(\Phi\left\{\overline{\varepsilon}\right\} - \Phi\left\{\underline{\varepsilon}\right\}\right) + 2\left(h+g\right)\frac{\gamma-1}{\gamma}\sigma\left[\phi\left(\underline{\varepsilon}\right) - \phi\left(\overline{\varepsilon}\right)\right] \\ & + \left(\frac{\gamma-1}{\gamma}\sigma\right)^2 \left[\Phi\left\{\overline{\varepsilon}\right\} - \overline{\varepsilon}\phi\left(\overline{\varepsilon}\right) - \Phi\left\{\underline{\varepsilon}\right\} + \underline{\varepsilon}\phi\left(\underline{\varepsilon}\right)\right],\end{aligned}$$

which exploits standard closed-form expressions for truncated integrals over normal distributions.

The second step involves integrating equation (30) for $\varepsilon \in (\overline{\varepsilon}, \overline{\infty})$, i.e. where the overhead labor constraint does not bind. This is more complicated, because in this region m, and therefore the expression marked g in equation (30), now depends on ε:

$$\begin{aligned}\int_{\overline{\varepsilon}}^{\infty} \hat{z}_i^2 \phi\left(\varepsilon\right) d\varepsilon = {} & \left(1 - \Phi\left\{\overline{\varepsilon}\right\}\right) h^2 + \left(\frac{\gamma-1}{\gamma}\sigma\right)^2 \left[1 - \Phi\left\{\overline{\varepsilon}\right\} + \overline{\varepsilon}\phi\left(\overline{\varepsilon}\right)\right] + 2\frac{\gamma-1}{\gamma}\sigma h \phi\left(\overline{\varepsilon}\right) \\ & + \int_{\overline{\varepsilon}}^{\infty} \left(g\left(\varepsilon\right)^2 + 2\frac{\gamma-1}{\gamma}\sigma\varepsilon g\left(\varepsilon\right)\right)\phi\left(\varepsilon\right) d\varepsilon + 2h\int_{\overline{\varepsilon}}^{\infty} g\left(\varepsilon\right)\phi\left(\varepsilon\right) d\varepsilon,\end{aligned}$$

where I approximate the integrals in the second line using Matlab's `integral` function.

The above formulas are valid for a single type of agent, i.e. a single value of A and θ. Adding an additional type of agent only complicates aggregation slightly, as follows. Denote the measure of each type of agent j by μ_j. Then aggregate revenue and labor, and average leverage and age, across all agents are given by

$$PY = \sum_j \mu_j PY_j$$

$$L = \sum_j \mu_j L_j$$

$$E\{\ell\} = \sum_j \mu_j E\{\ell\}_j$$

$$E\{a\} = \sum_j \mu_j E\{a\}_j$$

where with a slight abuse of notation I have indexed aggregates on the left-hand side of equation (29) for type j with a j subscript.

The simple average of the the Olley-Pakes decomposition across types is just a weighted average of the type-specific values,

$$\mathcal{Z} = \sum_j \mu_j \mathcal{Z}_j.$$

The weighted-average term of the Olley-Pakes decomposition can be computed as a weighted averaged across types, where the weights depend on each type's share of aggregate labor:

$$\mathcal{Z}_{\text{wt}} = \sum_j \mu_j \frac{L_j}{L} \mathcal{Z}_{\text{wt},j}.$$

Finally, I aggregate the dispersion in labor productivity by aggregating over the second moment, subtracting the squared first moment, and taking a square root:

$$\sigma_{\log \text{ARPL}} = \sqrt{\sum_j \mu_j \left(\sigma^2_{\log \text{ARPL},j} + \mathcal{Z}_j^2\right) - \mathcal{Z}^2}.$$

In this final paragraph I describe the simulations used in Table 8. For each type of firm and at each time period, I draw 5 million times from the 1500 state-space points with probability proportional to the density vector f^*. For each of these draws, I compute leverage and log revenue labor productivity, and assign appropriate values to dummy variables equal to 1 for treatment firms, 1 for post-shock time periods, and the product of the two. I then regress leverage on the dummies and log labor productivity, and report the coefficients in Table 8.

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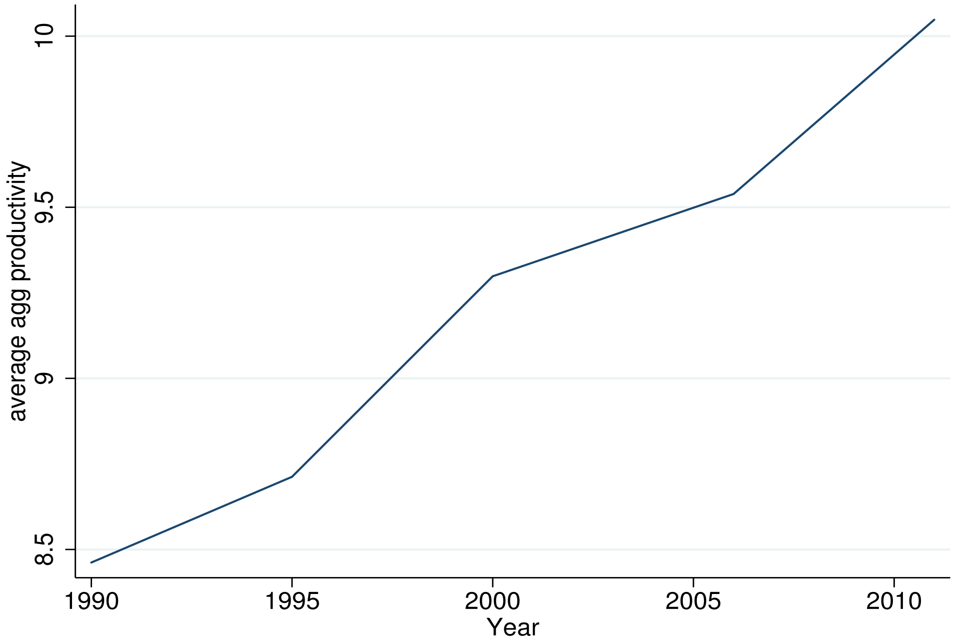


Figure 1. Aggregate Labor Productivity
The figure plots aggregate log labor productivity in India from 1989–1990 to 2010–2011. I define aggregate labor productivity as an employment-weighted average of industry-level aggregate labor productivity, where the weights are each industry's share of total employment at each date. Industry-level aggregate labor productivity is a employment-weighted average of plant-level log labor productivity, as in equation (1).

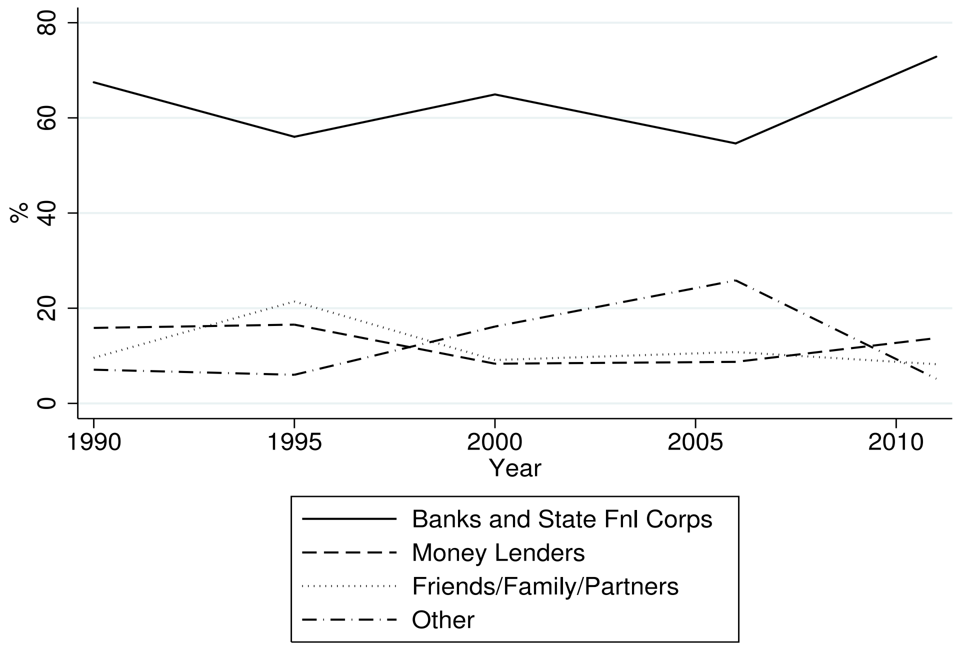
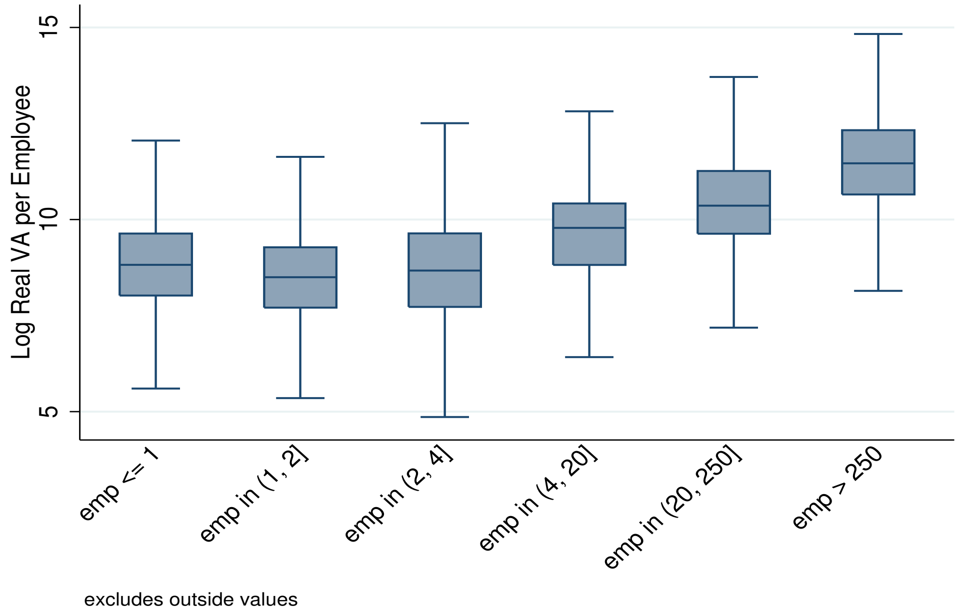
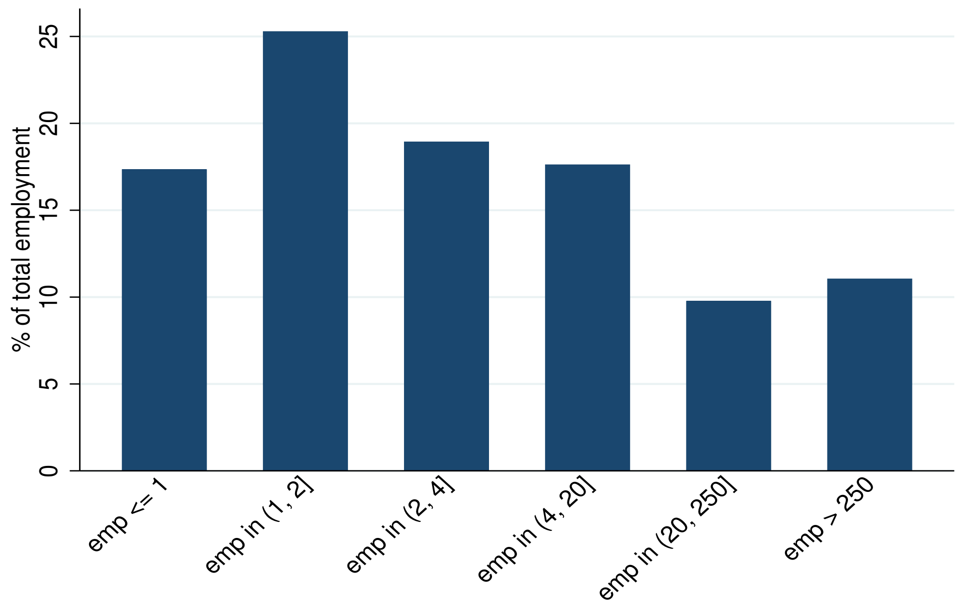


Figure 2. Sources of Credit
The figure plots the sources of credit (loans) reported by NSS plants over time. Plants in the ASI do not report their sources of credit. Each line plots the percentage of total credit reported from each source: banks and state financial corporations (including loans from government-owned commercial banks and Khadi & Village Industries Commission), money-lenders, friends, family, & business partners, and other.

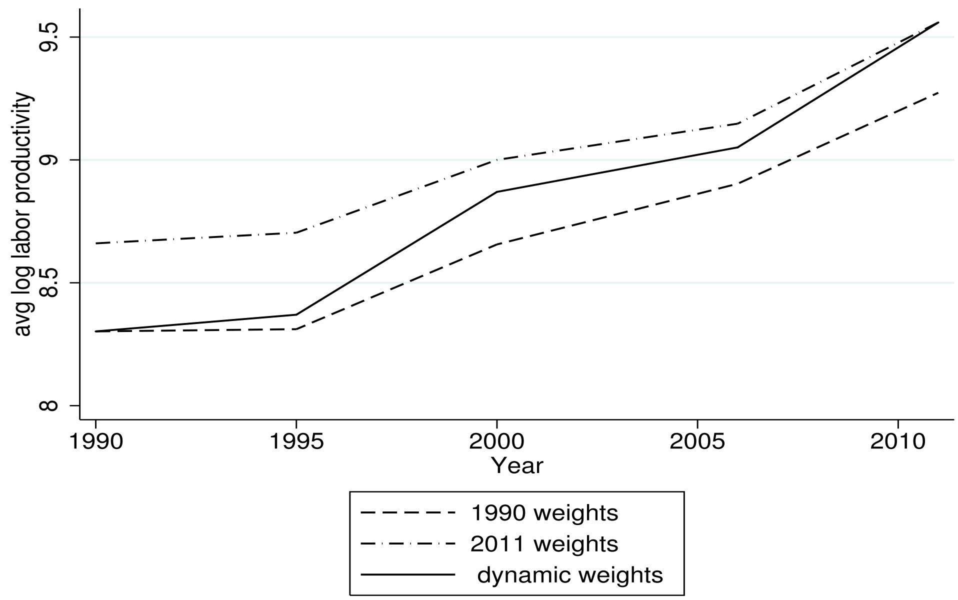


(a) Log Labor Productivity

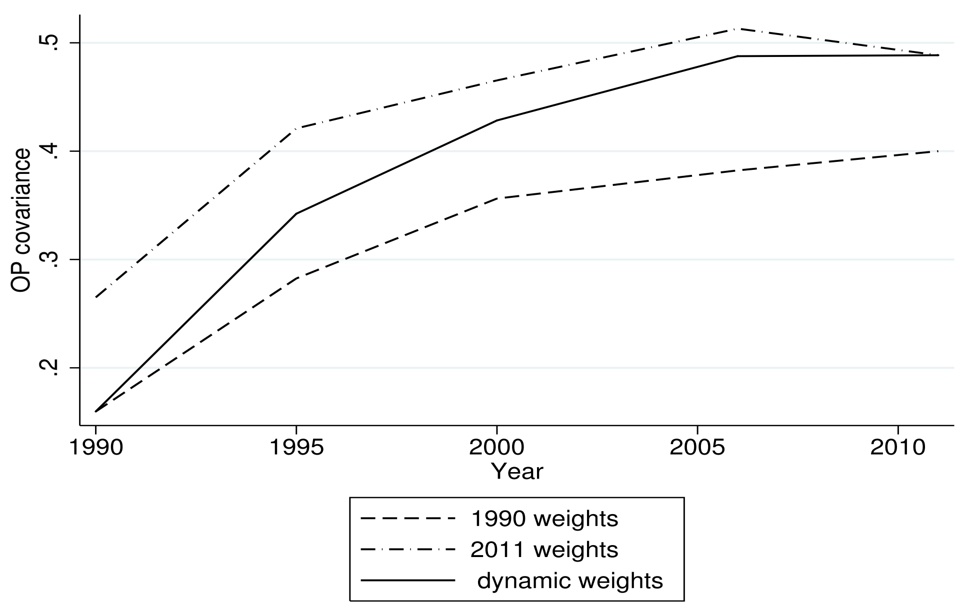


(b) Shares of Total Employment

Figure 3. Productivity and Employment Shares by Size
The top panel plots the distribution of log labor productivity, in 1993-1994 rupees of value-added per employee, across six size categories: plants with 1 employee, 2 employees, 3–4 employees, 5–20 employees, 21–250 employees, and more than 250 employees. The edges of each box represent the 25th and 75th percentiles, while the middle line inside each box is the median. The bars represent the 1st and 99th percentiles. The data are pooled across industries and dates, and I use the sample weights. The bottom panel plots the percentages of total employment accounted for by each size category.



(a) Average Labor Productivity $\mathcal{Z}$



(b) OP Covariance $\mathcal{C}$

Figure 4. OP Decomposition over Time
The top panel plots the average productivity $\mathcal{Z}$ term from equation (1) over the five years in the combined ASI-NSS data. I compute $\mathcal{Z}$ within each industry and then average across industries using each industry's share of total employment at each date. The upper and lower dotted lines represent averages using constant industry employment shares in 2011 and 1990, respectively. The bottom panel is identical except that it plots the OP covariance term $\mathcal{C}$ for the three sets of weights.

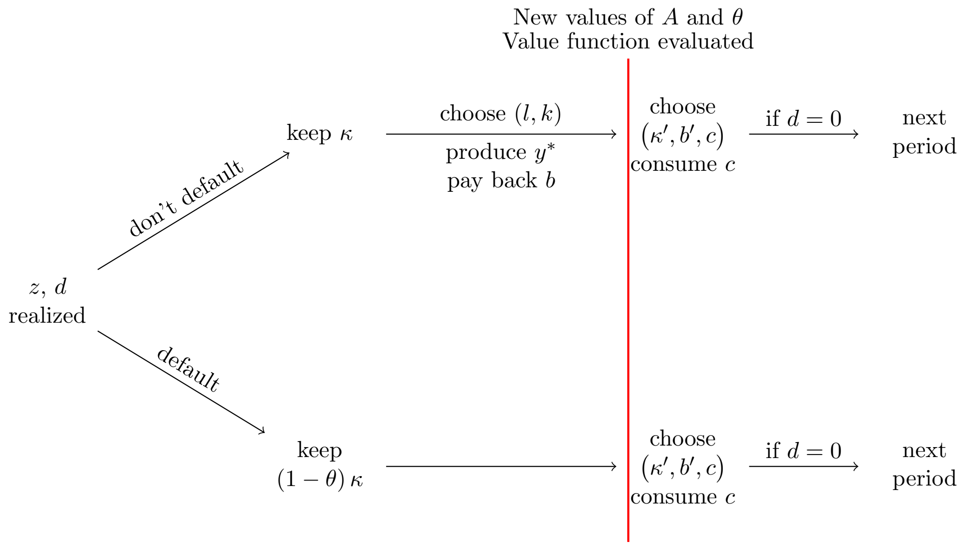
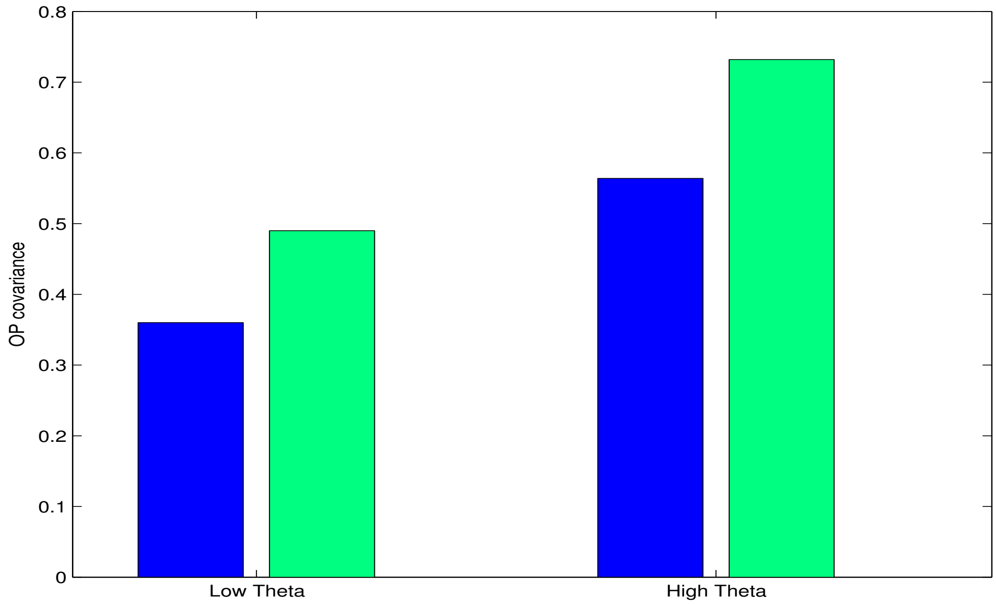
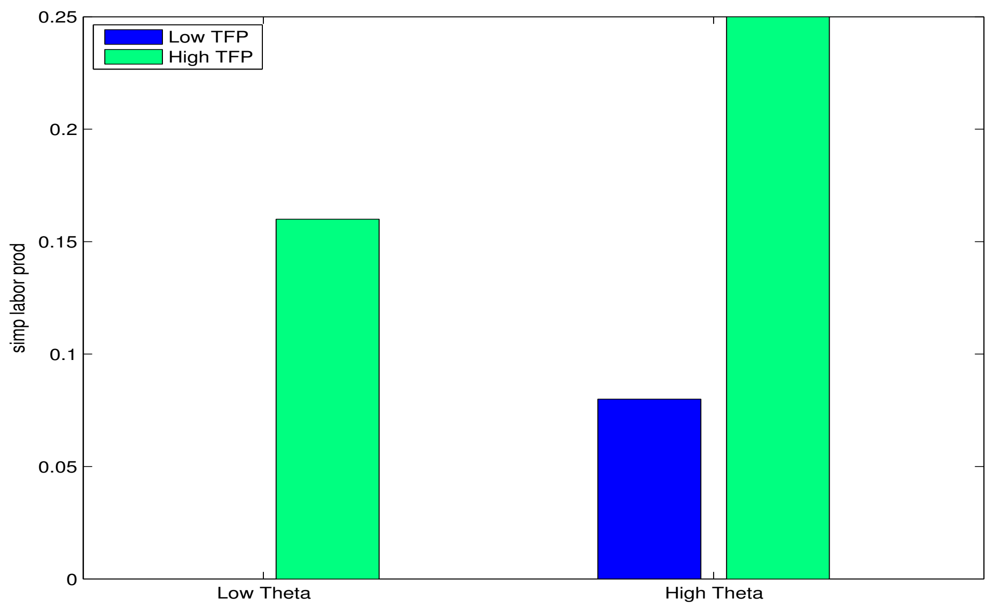


Figure 5. Timing
The figure plots the timing of shocks and decisions within the period. Agents realize their idiosyncratic productivity and exit shocks at the start of the period, and must immediately decide whether to default on their outstanding debt. Firms that default keep a fraction $1-\theta$ of their capital stock and cannot produce, but set $b = 0$. Firms that repay their debt may produce but must pay back their debt. After production occurs, but before investment and borrowing decisions are made, agents learn the values of A and θ that will obtain for future loans. The red line at this point in the timing also denotes where the value function $V_{i,t}\,(x, z)$ in equation (7) is evaluated. After choosing consumption, borrowing, and investment, agents consume and continue to the next period if they did not receive the exit shock.

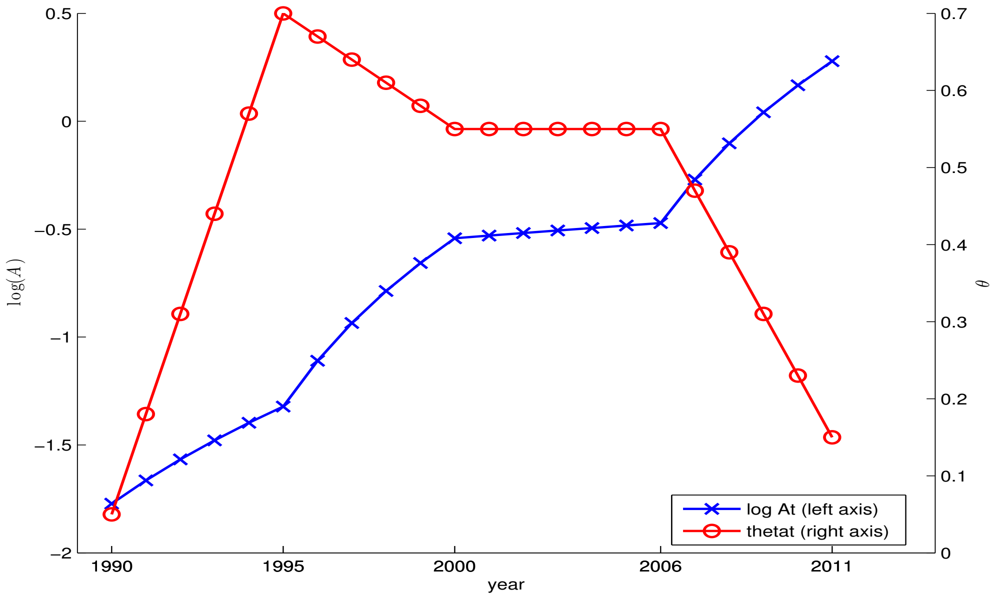


(a) OP Covariance

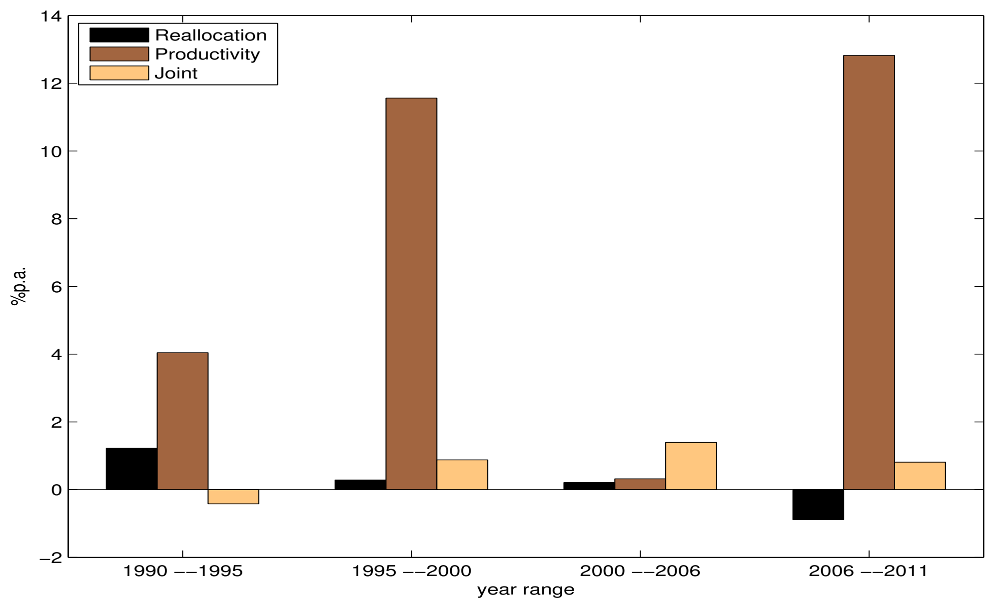


(b) Log Average Labor Productivity

Figure 6. Comparative Statics
Each figure plots a comparative static for a different model-implied value, across $\theta \in \{0.4, 0.8\}$ and $A \in \{0.34, 0.425\}$. Other parameters are listed in Table 3. The dark blue bars represent values for the low values of A, and the light green bars represent values for the high values of A. The left pair of bars represent values for the low value of θ, and the right pair of bars represent values for the high value of θ. The top panel plots the model-implied Olley-Pakes covariance, and the two panel plots average log labor productivity. The bar in the bottom two panel are normalized to the low-θ, low-A value. Formulas for these calculations are in Appendix C.

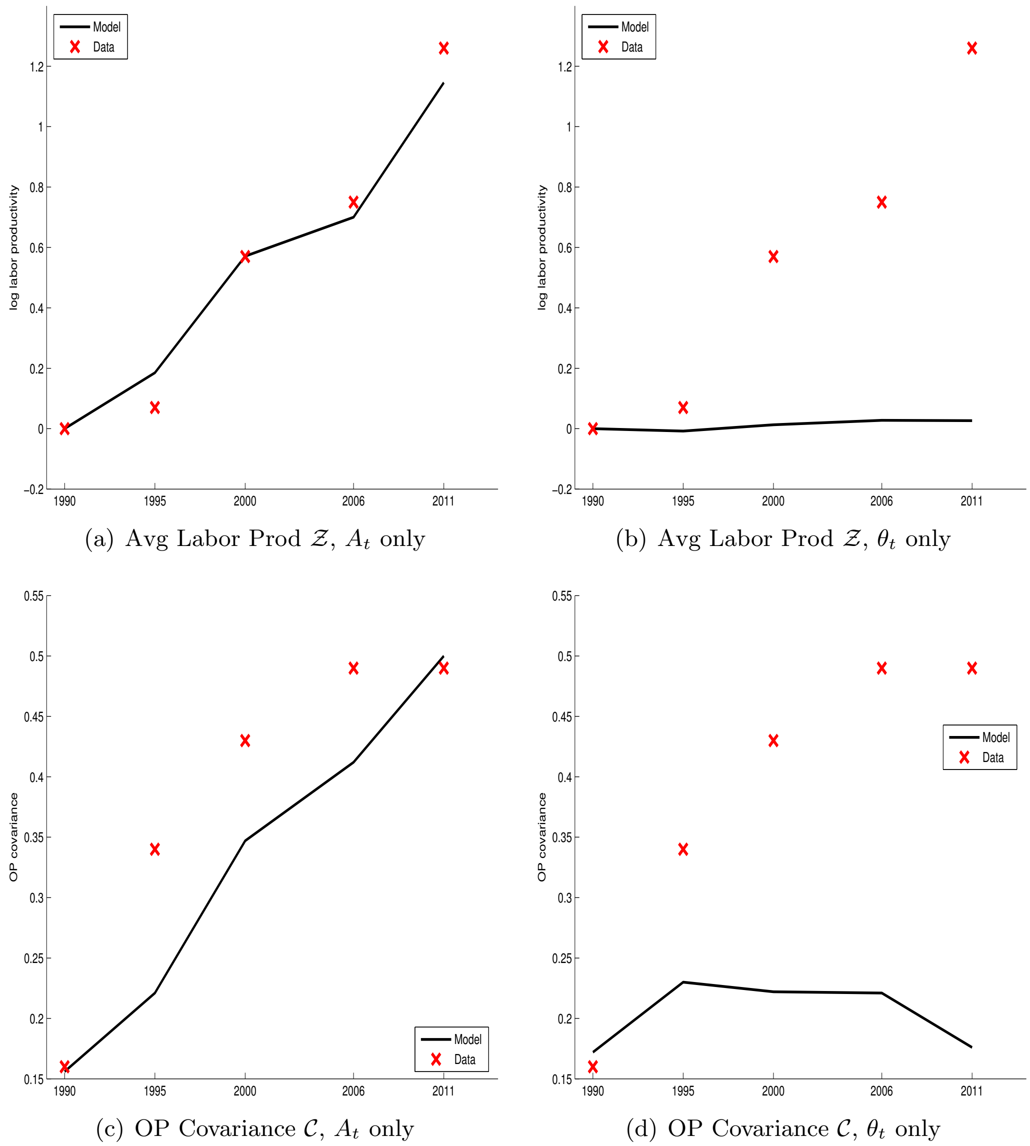


(a) Exogenous Productivity and Development



(b) Contributions to Productivity Growth

Figure 7. Contributions to Aggregate Productivity Growth
The top panel plots the exogenous aggregate time-series for A_t (left axis, in logs) and θ_t (right axis). The bottom panel plots the implied contribution to aggregate productivity in each 5-year time window, using model counterfactuals. The bars labeled "Reallocation" represent the annual productivity growth in an economy where A_t is constant at its 1990 value, but θ_t evolves as in the top panel. Likewise, the bars labeled "Productivity" report annual productivity growth in an economy with θ_t constant at its 1990 value, and A_t evolving as in the top panel. The bars labeled "Joint" make up for the difference between the sum of the two contributions and total aggregate productivity.



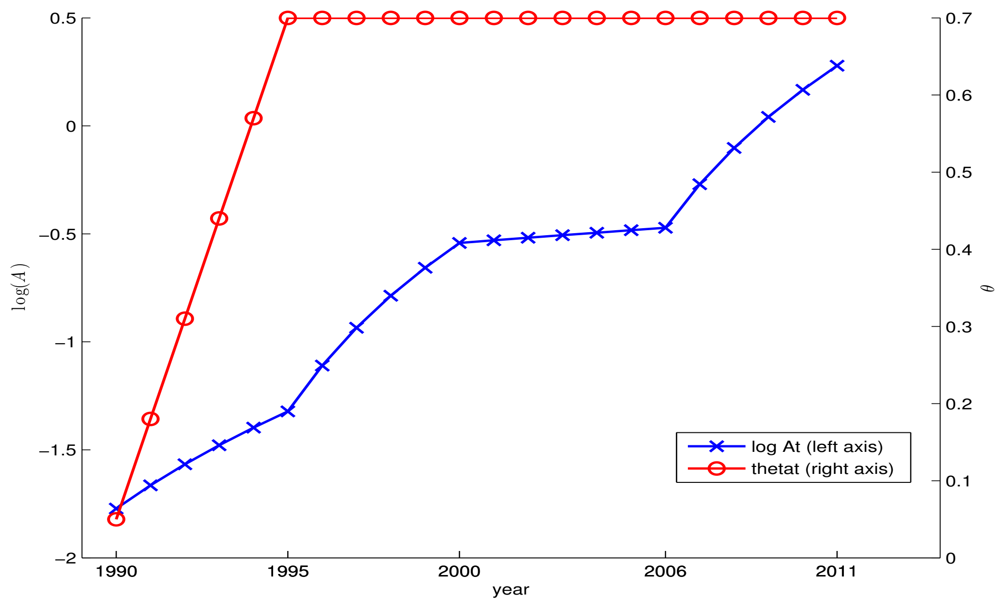
(a) Avg Labor Prod $\mathcal{Z}$, A_t only

(b) Avg Labor Prod $\mathcal{Z}$, θ_t only

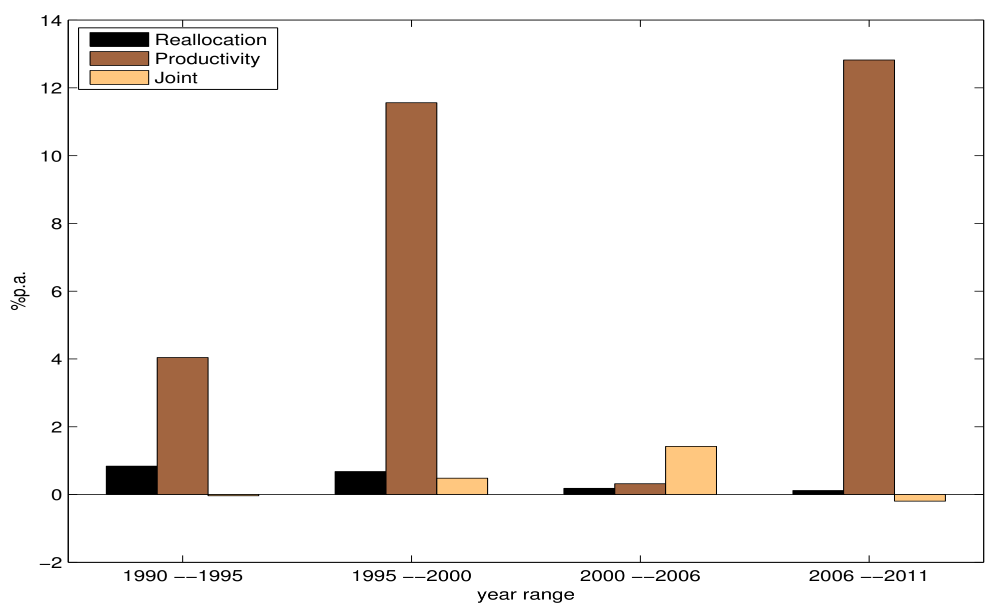
(c) OP Covariance $\mathcal{C}$, A_t only

(d) OP Covariance $\mathcal{C}$, θ_t only

Figure 8. Counterfactual Olley-Pakes Decompositions
The black lines top two panels plot the counterfactual paths of average log labor productivity ($\mathcal{Z}$ in equation 1), for the case where θ_t is constant at its 1990 value but A_t evolves as in the top panel of Figure 7 (left panel), and the case where θ_t evolves as in the top panel of Figure 7 but A_t is constant at its 1990 value (right panel). The bottom two panels repeat the exercise for the Olley-Pakes covariance ($\mathcal{C}$ in equation 1). The red X's in each figure plot the values observed in the data.

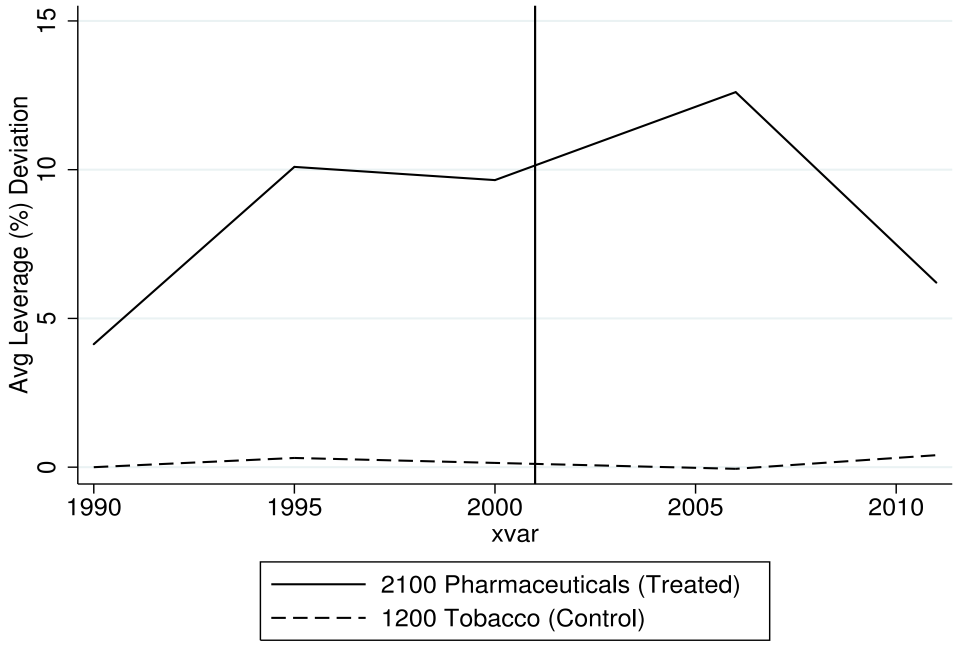


(a) Exogenous Productivity and Development



(b) Contributions to Productivity Growth

Figure 9. Contributions to Aggregate Productivity Growth, Alternate θ Path
The top panel plots the exogenous aggregate time-series for A_t (left axis, in logs) and θ_t (right axis). The bottom panel plots the implied contribution to aggregate productivity in each 5-year time window, using model counterfactuals. The bars labeled "Reallocation" represent the annual productivity growth in an economy where A_t is constant at its 1990 value, but θ_t evolves as in the top panel. Likewise, the bars labeled "Productivity" report annual productivity growth in an economy with θ_t constant at its 1990 value, and A_t evolving as in the top panel. The bars labeled "Joint" make up for the difference between the sum of the two contributions and total aggregate productivity.



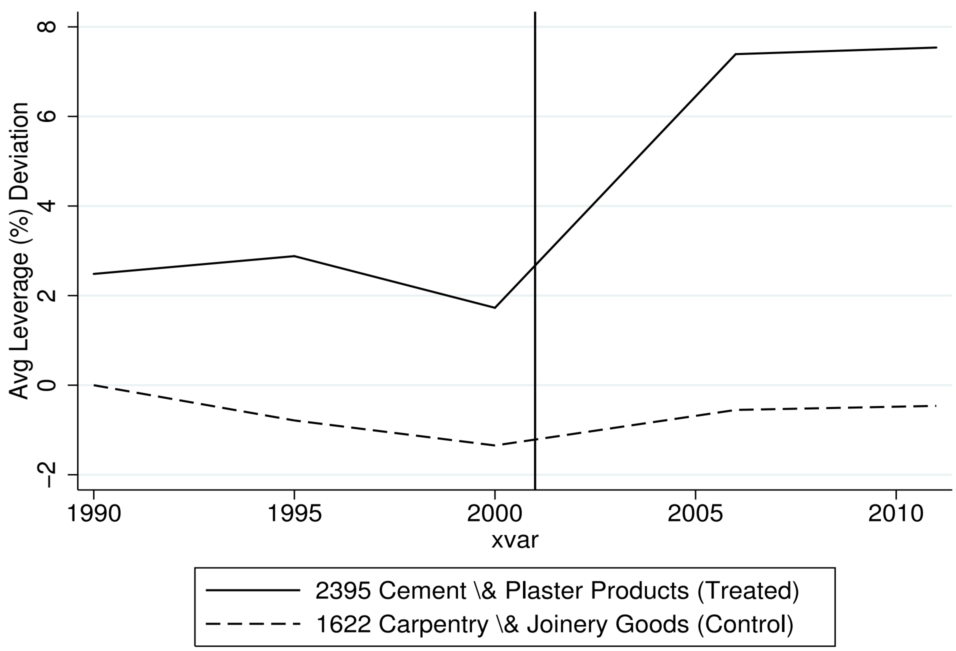


Figure 10. Interaction Effects over Time
Both figures plot regression coefficients from estimating equation (14), augmented to include time dummies and treatment interactions for each year in the data. The omitted category is control firms in 1990, so each point represents the difference in average leverage in percentage points from this value. Regressions include state fixed-effects and log labor productivity. The top panel plots coefficients from defining Pharmaceutical plants as the treatment group and Tobacco plants as the control, and the bottom panel from defining Cement & Plaster Products plants as the treatment group and Carpentry & Joinery Goods as the control.

Panel A					
	Aggregate		NSS Shares		
Year	# Firms	# Employees	% Firms	% Employment	% Output
1989–1990	12.3	26.3	99.8	86.0	31.0
1994–1995	11.9	34.5	99.4	84.7	26.0
1999–2000	14.2	36.5	99.2	80.5	24.4
2005–2006	14.1	38.7	99.2	78.5	21.8
2010–2011	17.0	45.7	99.1	74.8	18.1

Panel B: Employment					
Year	mean	s.d.	1%	50%	99%
NSS					
1989–1990	1.8	1.0	1.0	2.0	5.0
1994–1995	2.5	2.4	1.0	2.0	11.0
1999–2000	2.1	2.2	1.0	2.0	10.0
2005–2006	2.2	3.0	1.0	2.0	11.0
2010–2011	2.0	3.2	1.0	1.0	11.0
ASI					
1989–1990	135.4	661.6	1.0	23.1	1824.0
1994–1995	78.8	499.3	3.0	20.0	960.0
1999–2000	66.1	435.1	2.0	18.0	766.0
2005–2006	71.8	367.1	3.0	20.0	834.0
2010–2011	77.3	381.1	2.0	21.0	916.0

Panel C: Log Labor Productivity					
Year	mean	s.d.	1%	50%	99%
NSS					
1989–1990	8.3	1.2	5.3	8.3	11.3
1994–1995	8.2	1.2	4.9	8.3	10.7
1999–2000	8.7	1.1	5.8	8.7	11.0
2005–2006	8.8	1.2	5.6	8.8	11.1
2010–2011	9.3	1.1	6.5	9.4	11.4
ASI					
1989–1990	10.3	1.3	6.5	10.3	13.7
1994–1995	10.4	1.2	6.9	10.4	13.6
1999–2000	10.7	1.2	7.7	10.7	13.8
2005–2006	10.7	1.2	7.8	10.7	13.8
2010–2011	11.2	1.1	8.5	11.1	14.3

Table 1. NSS and ASI Summary Statistics.
The first two columns of Panel A report the total estimated number of firms and employees (in millions) in the combined ASI-NSS dataset over time. Columns 3 through 5 of report the aggregate shares of plants, employment, and output (real value added in 1993-1994 rupees) accounted for by respondents in the NSS data. Panel B reports the distribution of employment in the NSS and ASI, while Panel C reports the distribution of log labor productivity (real value-added per employee) in the two datasets. The first two columns in Panels B and C report the mean and standard deviation, respectively, while the last three columns report the 1st, 50th, and 99th percentiles.

Year	% that Borrow			NSS % of Total
	All	NSS	ASI	Borrowing
1990	10.7	10.6	66.2	2.8
1995	9.2	8.9	77.8	5.0
2000	8.1	7.6	73.0	3.2
2006	9.0	8.4	74.2	6.9
2011	6.6	6.0	73.9	2.6

Table 2. Borrowing
The first three columns report percentages of plants that borrow in the combined ASI–NSS data. The first column reports the percentage across all plants, the second column reports the percentage of NSS plants that borrow, and the third column reports the percentage of ASI plants that borrow. The last column reports the percentage of all borrowing that is accounted for by NSS plants.

Panel A: Set Parameters		
Parameter	Value	Description
r	1%	Riskless Interest Rate
β	0.99	Time Discount
γ	3	Demand Elasticity
α	0.3	Capital Share
ρ	0.95	TFP Persistence
μ	0.99	Measure of Low-A Plants

Panel B: Calibrated Parameters					
Parameter	Value	Description	Target Moment	Data	Model
π	0.083	Exit Probability	Average Age	12	11.95
D	-2.1	Labor Supply Intercept	Initial NSS % of Labor	86%	88%
η	7.1	Labor Supply Elasticity	Chg NSS % Labor	-11.2 ppt	-12 ppt
A	0.34	Average Gross TFP	NSS % of Output	31%	43%
σ	0.45	Std Dev of Prod. Shock	Labor Prod. Disp.	1.07	1.07
$\Delta_i \log A$	-3.7	log TFP Diff.	ASI-NSS Labor Prod. Diff	-2.0	-2.2

Table 3. Model Parameters
The table reports the parameter values of the dynamic model described in section 4, and analyzed in section 5. Panel A reports the fixed parameters that are standard and not used in calibration. Panel B reports the fixed parameters that are used to match the indicated data moments.

Panel A: Summary Statistics					
Industry	% of Total Employment	% that Borrow	Avg Fixed Asset %	NSS % of Employment	Log Agg Output Growth
2100 Pharmaceuticals	0.65	35.66	32.74	10.31	1.14
2910 Motor Vehicles	0.90	31.32	33.61	14.42	2.03
3091 Motorcycles & Parts	0.23	30.45	35.28	19.88	2.04
2395 Cement & Plaster Products	0.32	18.64	33.79	68.61	1.35
1200 Tobacco	9.37	2.32	29.76	88.00	0.46
2029 Other Chemical Products	0.73	8.57	32.41	50.81	0.95
1622 Carpentry & Joinery Goods	2.04	5.80	30.61	99.47	0.42
1072 Sugar	1.66	19.55	24.88	56.80	0.58

Panel B: Average Differences						
Industry	Avg Leverage			Avg Log Labor Productivity		
	Pre 2000	Post 2000	Diff	Pre 2000	Post 2000	Diff
2100 Pharmaceuticals	9.64	10.80	1.17	9.30	10.01	0.71
2910 Motor Vehicles	8.56	10.40	1.85	10.05	10.64	0.59
3091 Motorcycles & Parts	8.00	9.39	1.39	10.08	10.43	0.35
2395 Cement & Plaster Products	4.12	9.51	5.40	9.23	9.99	0.76
1200 Tobacco	0.25	0.26	0.00	8.05	7.97	-0.08
2029 Other Chemical Products	0.79	1.21	0.42	7.90	8.49	0.59
1622 Carpentry & Joinery Goods	1.19	1.53	0.34	9.19	9.59	0.41
1072 Sugar	3.82	4.03	0.21	7.69	9.44	1.75

Table 4. Industry Selection and Differences
Both panels report summary statistics for industries that are *ex ante* likely to be more or less affected by financial development than other industries. The industries in the top three rows are likely to be more affected by financial development, while the industries in the bottom four rows are likely to be less affected. The first column of Panel A reports the percentage of total employment across the ASI and NSS and all five time periods accounted for by each industry. The second column of Panel A reports the percentage of firms in 1990–2000 that have positive leverage. The third column of Panel A reports the within-industry average percentage of total assets accounted for by fixed assets; this statistic is computed among ASI plants only (NSS plants do not report fixed vs. current assets). The fourth column of Panel A reports the share of employment accounted for by NSS plants, and the last column of Panel A reports the average log productivity difference from 1990-2000 and 2006–2011. Panel B reports pre-2000 and post-2000 average differences in leverage in the first 3 columns, and differences on log aggregate output before and after 2000 in the last three columns.

	(1)	(2)	(3)	(4)	(5)
Treatment Industry	Pharmaceuticals	Pharmaceuticals	Cement	All 4 in Table 4	All 4
Control Industry	Tobacco	Other Chemical	Carpentry	Sugar	All 4 in Table 4
Treatment X Post	1.162***	0.744***	5.056***	1.363***	1.635***
	(5.340)	(3.405)	(47.71)	(10.96)	(14.83)
Treatment	9.384***	8.847***	2.926***	5.000***	8.223***
	(67.27)	(63.14)	(40.74)	(60.21)	(105.3)
Post	0.00492**	0.422***	0.339***	0.215***	-0.0570***
	(2.163)	(19.85)	(30.34)	(3.724)	(-18.83)
Constant	0.252***	0.789***	1.189***	3.820***	0.597***
	(175.1)	(59.64)	(185.9)	(135.0)	(290.1)
State Fixed Effects	NO	NO	NO	NO	NO
Observations	9,570,289	401,673	2,542,429	426,312	12,773,207
Obs unweighted	38,235	8,981	19,027	14,375	71,234
R-squared	0.024	0.086	0.023	0.027	0.022

Table 5. Leverage by Industry Regressions
The table reports results from estimating equation (14) on the combined ASI-NSS data. The dependent variable is leverage, defined as total liabilities divded by total assets times 100%. Each column includes only the industries indicated in the top two rows, and defines the "treated" industry as the industry listed in the top row. The dependent variable is leverage, defined as total liabilities divded by total assets times 100%. "Post" is a dummy variable equal to 1 for years 2006 and 2011. These regressions do not include log labor productivity or state fixed-effects as independent variables. All regressions use the ASI and NSS sample weights; the number of unweighted sample observations (survey responses) are listed below the number of weighted observations. t-statistics are reported in parentheses below each coefficient.

	(1)	(2)	(3)	(4)	(5)
Treatment Industry	Pharmaceuticals	Pharmaceuticals	Cement	All 4 in Table 4	All 4
Control Industry	Tobacco	Other Chemical	Carpentry	Sugar	All 4 in Table 4
Treatment X Post	0.476**	-0.567***	4.481***	1.971***	0.653***
	(2.171)	(-2.586)	(41.74)	(15.11)	(5.998)
Treatment	8.383***	4.435***	2.453***	2.415***	7.569***
	(58.95)	(27.74)	(33.66)	(22.12)	(94.61)
Post	0.0365***	-0.438***	0.323***	-2.898***	-0.0219***
	(14.15)	(-18.74)	(25.71)	(-40.18)	(-6.672)
Log Labor Prod	0.443***	1.678***	0.535***	1.742***	0.673***
	(110.8)	(71.22)	(46.02)	(85.12)	(199.5)
State Fixed Effects	YES	YES	YES	YES	YES
Observations	9,570,289	401,673	2,542,429	426,312	12,773,207
Obs unweighted	38,235	8,981	19,027	14,375	71,234
R-squared	0.046	0.178	0.036	0.079	0.043

Table 6. Leverage by Industry Regressions
The table reports results from estimating equation (14) on the combined ASI-NSS data. The dependent variable is leverage, defined as total liabilities divded by total assets times 100%. Each column includes only the industries indicated in the top two rows, and defines the "treated" industry as the industry listed in the top row. The dependent variable is leverage, defined as total liabilities divded by total assets times 100%. "Post" is a dummy variable equal to 1 for years 2006 and 2011. All regressions use the ASI and NSS sample weights; the number of unweighted sample observations (survey responses) are listed below the number of weighted observations. *t*-statistics are reported in parentheses below each coefficient.

Exogenous			Endogenous			
θ_0	θ_1	$\%\Delta A$	$E\{\ell\}_0$	$E\{\ell\}_1$	diff	$\%\Delta \log Y$
0.6	0.66	0	35.7	37.9	2.2	6.57
0.4	0.44	0	26.9	28.8	1.91	0.78
					0.292	5.79
0.6	0.66	10	35.7	39.4	3.73	18.9
0.4	0.44	10	26.9	29.9	3	12.3
					0.726	6.65
0.6	0.6	10	35.7	36.9	1.27	12
0.4	0.4	10	26.9	27.8	0.924	11.4
					0.35	0.624
0.6	0.66	5	35.7	38.4	2.77	12.9
0.4	0.44	20	26.9	30.8	3.91	23
					-1.13	-10.1

Table 7. Difference-in-Difference in the Model
Each panel reports model results from a difference-in-difference computation. The first three columns report the exogenous aggregate variables for each exercise. The first column reports the initial (pre) value for θ, the second column the post value of θ, and the third column reports the percentage change in total-factor productivity. The rightmost five columns report endogenous model moments from each exercise: the fourth column reports average leverage in the pre-period, the fifth column average leverage in the post-period, and the sixth column the difference between the two. The last column reports the log change in output in percent. The third row in each section reports the difference-in-difference (top minus bottom row) for leverage, output, and labor productivity. Each computation lasts for twenty-two periods (years); agents learn in period 12 that total-factor productivity and/or output will change to their post values at period 13, where they will remain. The pre-period is periods 1–12 and the post-period is period 13–22.

Exogenous			Coefficients			
θ_0	θ_1	$\%\Delta A$	Treat x Post	Treat	Post	Prod
0.6 0.4	0.66 0.44	0 0	0.245	8.79	1.72	13.2
0.6 0.4	0.66 0.44	10 10	0.661	8.78	2.01	13.4
0.6 0.4	0.6 0.4	10 10	0.363	8.77	0.155	12.8
0.6 0.4	0.66 0.44	5 20	-0.0622	8.79	2.2	13.4

Table 8. Difference in Difference in the Model, Controlling for Productivity
Each panel reports model results from a difference-in-difference regression computed on simulated data. I simulate 5 million firms of each type (high and low θ); see Appendix C for details on the simulation procedure. The first three columns report the exogenous aggregate variables for each exercise. The first column reports the initial (pre) value for θ, the second column the post value of θ, and the third column reports the percentage change in total-factor productivity. The rightmost four columns report OLS coefficient estimates for the following variables: a dummy variable equal to 1 for observations of the high-θ type in the post period, a dummy equal to 1 for observations of the high-θ type, a dummy equal to 1 for observations in the post period, and each firm's realized log revenue labor productivity. Each computation lasts for twenty-two periods (years); agents learn in period 12 that total-factor productivity and/or output will change to their post values at period 13, where they will remain. The pre-period is periods 1–12 and the post-period is period 13–22.