Informed trading, indexing, and welfare*

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Abstract

We study the implications of informed trading for the welfare of market participants within the canonical rational expectations model (Grossman and Stiglitz (1980), Hellwig (1980)). We find that as the informed population grows, or their signals become more precise, uninformed investors' welfare falls. We extend the model to allow for multiple assets, in order to study the consequences of indexing, i.e., committing to invest in risky assets only via the market portfolio. We show that indexing imposes a negative externality on agents. More indexing makes informed trading in the market more profitable, which decreases welfare by distorting risk-sharing.

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1 Introduction

The standard investment recommendation that academic financial economists offer to retail investors it to purchase a low-fee index fund. This recommendation is often justified by an argument along the following lines. The "average investor theorem" suggests that departures from holding the market portfolio constitute a zero-sum game. Since retail investors are unlikely to be winners in this zero-sum game, they would be well-advised to restrict themselves to trading only the market portfolio, i.e., to index.[1] In this paper we ask: what are the equilibrium consequences of this advice, both for prices and welfare?

We first study the effect of information production within the canonical model of informed trade with a single asset, namely a version of Grossman and Stiglitz (1980) and Hellwig (1980) in which "liquidity trades" are explicitly motivated as stemming from stochastic exposure to economic shocks that are correlated with the cash flows of traded assets. A subset of agents observe informative signals about cash flows prior to trading. As the quality of these signals increases, equilibrium prices contain more information about cash flows, i.e., price efficiency increases. This has two opposing effects on risk sharing, and hence welfare. On one hand, when prices reveal more information about cash flows, this reduces agents' ability to share cash flow risk, as in Hirshleifer (1971). On the other hand, if prices reveal more information about cash flows, they must reveal *less* information about other determinants of prices—broadly, "discount rates," which in this class of models are driven by aggregate endowment shocks. This facilitates risk sharing with respect to differences in individual endowments. We show the former effect dominates in the canonical model: increasing the

[1] For example, Cochrane (2013) writes: "The average investor theorem is an important benchmark: The average investor must hold the value-weighted market portfolio. Alpha, relative to the market portfolio, is by definition a zero-sum game. For every investor who over-weights a security or invests in a fund that earns positive alpha, some other investor must underweight the same security and earn the same negative alpha. Collectively, we cannot even rebalance. And each of us can protect ourselves from being the negative-alpha mark with a simple strategy: hold the market portfolio, buy or sell only the portfolio in its entirety, and refuse to trade away from its weights, no matter what price is offered. If every uninformed trader followed this strategy, informed traders could never profit at our expense." French (2008) makes a similar argument.

precision of signals makes uninformed agents worse off, and under many conditions, makes informed agents worse off also. This result is both of independent interest, and central to our results on indexing.

In order to evaluate the recommendation to index, we extend our model to a case with multiple (two) distinct assets.[2] Investors in our model have heterogeneous abilities to acquire information about future asset payoffs. We consider what happens when a subset of agents with below-average information acquisition ability adopt an indexing strategy.

Our main result is that as the incidence of indexing increases, the utility of all investors falls. In other words, indexing exerts a negative pecuniary externality. The intuition for this result is as follows. It is useful to first note that portfolio positions in a two-asset economy can be decomposed into a holding of a market index and a "spread" asset that is long one asset and short another. Indexing represents a decision not to trade in the spread asset. Consequently, indexing reduces the amount of liquidity trading in the spread asset. Ceteris paribus, this makes the price of the spread asset more informative about future cash flows, which in turn leads informed investors to acquire less information about the spread asset. Informed investors then substitute towards acquiring more information about the market asset. By the welfare result described above, this increase in information production worsens risk-sharing, and decreases the welfare of uninformed investors. Moreover, a simple revealed preference argument further shows that informed agents are also worse off.

Even when trading the market asset, uninformed agents suffer a disadvantage relative to informed investors, since, conditional on the price, informed investors buy exactly when future cash flows are likely to be high, while uninformed investors do just the opposite. This disadvantage only grows when informed investors acquire more information, reducing the welfare of uninformed investors.[3]

[2]Note that in Admati (1985) and other previous analyses of multiple-asset economies, liquidity trades are entirely exogenous.

[3]In addition, note that indexing directly reduces the welfare of an indexer. This is an immediate implication of the canonical model, since indexing is simply a constraint on portfolios. In more detail: a low asset

In addition to showing that, in equilibrium, indexing imposes a negative pecuniary externality, our analysis produces a number of empirical implications. First, indexing reduces the market risk premium. Second, indexing leads to higher levels of "price efficiency," as measured by the ability of prices to predict future cash flows. Third, indexing leads to a shift in trading strategies of informed investors, who shift from trades based on cross-sectional "mispricing" to ones that focus instead on aggregate factors. These implications are broadly consistent with the recent empirical literature on hedge funds and ETFs (Fung et al., 2008; Sun, Wang, and Zheng, 2012; Ramadorai, 2013; Pedersen, 2015; Israeli, Lee, and Sridharan, 2015). Cremers et al. (2016) show how active and passive funds interact with each other, with indexing increasing (both index funds and ETFs) been associated with more active trading, also in line with our main results.

Our model essentially extends Admati (1985) by explicitely modelling endowments shocks, as in Diamond and Verrecchia (1981). While uninformed agents are unable to observe any signal about cash flows, we allow the informed agents to choose the precisions of signals, as in Verrecchia (1982), on both the aggregate market, or on the relative value of the two stocks, via the "spread" asset. We note that our indexing agents can still time the market, but they cannot tilt their portfolios towards one particular asset, or, using the lingo from Admati et al. (1986), they can do timing, but not selectivity. In contrast to Van Nieuwerburgh and Veldkamp (2009), who focus on information acquisition in multi-asset markets using "capacity" constraints defined over the whole variance-covariance matrix of signals' errors, we explicitely model the signals that informed agents can receive, and allow the informed agents to choose the precision of each signal's errors. Our results are in line with

price may reflect either that informed investors have negative expectations about the asset's future cash flows, or that the aggregate exposure to economic shocks correlated with these cash flows is high, generating a high discount rate for these cash flows. Consequently, if an investor observes a low price for an asset, and has no exposure to economic shocks, he should take a long position in the asset, since its conditional expected return is high. That is, an investor can profit from buying "value" stocks, which an indexing strategy would preclude. Although this point is often overlooked, it is nonetheless a standard implication of the canonical model (see, e.g., Biais, Bossaerts, and Spatt, 2010).

the empirical evidence in Gerakos, Linnainmaa, and Morse (2017), which argues that active managers are using more timing, versus selectivity, in their trades, coinciding with the rise of index investments and ETFs.

While with very different focus, Marín and Rahi (1999), Ganguli and Yang (2009) and Manzano and Vives (2011) are closely related. Like these papers, we also use endowment shocks as the modelling device that prevents prices from fully revealing the informed agents' signals. The main reason for such a choice is two fold: (1) we want a setting where trading motives are explicitely modelled, (2) we can do welfare analysis without having to judge what to do with, say, Kyle (1985)'s noise traders. An important implication of this assumption is that even when uninformed agents do not receive signals on asset payoffs, they do have private signals on aggregate endowment shocks, provided by their individual endowment realization. Relative to the afore-mentioned papers, we model an economy with multiple risk assets, and we explicitly characterize welfare, our main contribution to the literature.

Our paper is also closely related to the recent literature on the rise of indexing investing. Stambaugh (2014) documents a large drop in individual equity ownership over the last three decades, coupled with a similar drop in the share of institutional money that is actively managed. Our paper shares with his model the fact that more indexing means "less noise" in financial markets, although the mechanisms are quite different: we explicitely model trading stemming from endowment shocks, whereas Stambaugh (2014)'s noise is closer to the behaviorally biased agents of Black (1986).

TBW: Discussion of related papers on ETFs, indexing, and benchmarking.

The rest of the paper is structured as follows. In Section 2 we introduce the one asset model, and we solve for equilibrium asset prices, and we study the impact on welfare of more informed trading. In Section 3 we extend our model to two assets, and we present our main results on the effects of indexing on agents' welfare. Section 4 concludes.

2 Informed trading and welfare

We start by studying a model with a single risky asset, a mix of informed and uninformed traders, and noise (Black, 1986) stemming from endowment shocks. The model follows closely that in Ganguli and Yang (2009) and Manzano and Vives (2011).

2.1 The model

There is a continuum of agents, indexed by the unit interval, $i \in [0, 1]$. Each agent i has preferences with constant absolute risk aversion (CARA) over terminal W_i, and a coefficient of absolute risk aversion of γ.

There is one risky asset available for trading, with payoff X, where we assume that this asset follows a Gaussian distribution, $X \sim \mathcal{N}(\mu_x, \sigma_x^2)$. We will use τ_x to denote the precision of random variable x, i.e. $\tau_x = 1/\sigma_x^2$. The equilibrium price of the asset is P.

The risky asset is in positive net supply, distributed equally among agents, with each agent having an initial endowment $\bar{s}$. We denote by θ_i agent i's position in the risky asset after trade.

In addition, agents also have other sources of income (e.g., labor income) that are correlated with the cash flow of the risky asset. For simplicity, we assume the correlation is perfect, and write agent i's income from sources other than the risky asset as $e_i X$, where e_i is (privately) known to agent i at the trading date. Differences in "exposures" e_i across agents motivate trade in the risky asset.

Hence the terminal wealth of agent i is determined by trading profits, along with the combination of asset endowment $\bar{s}$ and other income $e_i X$:

$$W_i = \bar{s}P + e_i X + \theta_i (X - P) = (\bar{s} + e_i) P + (\theta_i + e_i)(X - P). \tag{1}$$

Although each agent knows his own income exposure e_i, he does not know that of other agents. Specifically, we assume $e_i = Z + u_i$, where $Z \sim \mathcal{N}(0, \sigma_z^2)$ and $u_i \sim \mathcal{N}(0, \sigma_u^2)$. Note that total income thus has an aggregate component ZX, where both Z and X are unknown to individual agents, although their own income exposures e_i provide (private) signals about Z.

There are two types of traders in our model. The first set are informed agents, who observe private signals prior to trading in financial markets. We think of institutional investors as playing this role, although we step away from a formal model of the mutual fund/hedge fund industry (see Garcia and Vanden, 2009). The rest of agents are uninformed, mimicking the retail sector. We assume there is a mass λ_I of informed agents, and, similarly, we let λ_U denote the number of uninformed agents, where $\lambda_I + \lambda_U = 1$.

Informed agents receive information about the payoffs of the risk asset prior to trading. In particular, agent i observes the signal

$$Y_i \;=\; X + \epsilon_i,$$

where $\epsilon_i \sim \mathcal{N}(0, \tau_\epsilon^{-1})$. We denote by $\mathcal{F}_i$ the information set agent i has at the time of trading, which consists of private signals Y_i and the endowment shock Z_i if informed (only Z_i if uninformed), as well as the price of the risky asset.

The equilibrium definition is standard, and follows competitive rational expectations models (Grossman and Stiglitz (1980), Hellwig (1980)).

Definition 1. *A rational expectations equilibrium is a set of trading strategies $\{\theta_i\}_{i\in[0,1]}$ and a price function $P(X, Z)$ such that*

1. *Markets clear:*

$$\int_0^1 \theta_i di = \bar{s}. \tag{2}$$

2. Taking prices as given, agent i's trading strategy is optimal:

$$\theta_i \in \arg\max_{\tilde{\theta}_i} \mathbb{E}\left[u(W_i)|\mathcal{F}_i\right]. \tag{3}$$

There are a few features worth remarking upon. First is the fact that the model's "noise," which prevents prices from being fully revealing (Black, 1986) comes from endowment shocks to agents' private portfolios (see Diamond and Verrecchia (1981), Ganguli and Yang (2009) and Manzano and Vives (2011)). This means that all agents have some private signal, their individual endowment shock e_i that they receive, which is correlated with the aggregate shock Z. More importantly for our purposes, explicitely modelling the source of noise allows us to look at welfare issues, since we can evaluate the expected utility of all players. The alternative of using "noise traders," as in Kyle (1985), introduces the decision of whether to include them in welfare calculations, and/or how much weight to give them (Leland, 1992).

We emphasize that this is a canonical setting, in which risk-sharing benefits drive the existence of a financial market.

2.2 Equilibrium

We follow the literature and focus on linear equilibria (existence is formally established in Proposition 1 below). In such an equilibrium, the optimal trading strategy by agents in our model has the standard mean-variance form,

$$\theta_i + e_i = \frac{1}{\gamma} \frac{\mathbb{E}[X - P|\mathcal{F}_i]}{\mathrm{var}(X - P|\mathcal{F}_i)}, \tag{4}$$

where we note the expectations in (4) depend on the privately observed signals, and also on the price and agent i's individual exposure e_i. Since the privately observed exposures e_i contain information about the aggregate shock Z, and this in turn moves prices, so agents optimally use the information in their Bayesian updating.

In Proposition 2 we characterize equilibrium welfare in this economy. The key to our derivation of a simple expression for welfare is to link the risk premium $\mathbb{E}[X - P]$ to the average amount of risk that needs to be shared, which is related to the per-capita net supply of the asset $\bar{s}$. In the special case in which no trader receives information about X, i.e., $\tau_\epsilon = 0$, the expected payoff and variance are the same for all traders, i.e., $\mathbb{E}[X|\mathcal{F}_i] = \mu_x$ and $\mathrm{var}(X|\mathcal{F}_i) = \tau_x^{-1}$. In this case, market clearing (2) immediately implies

$$\mathbb{E}[X - P] = \gamma\bar{s}\mathrm{var}(X).$$

Our first result generalizes this identity to the case in which different traders have different private information.

Lemma 1. *In any linear equilibrium, prices satisfy*

$$\mathbb{E}[X - P] = \gamma\bar{s}\mathrm{cov}(X - P, X) \tag{5}$$

The proof of Lemma 1 is short, and because of its importance we give it here. First, observe that whenever the information set $\mathcal{F}_i$ consists of a set of normally distributed random variables,

$$\frac{\partial}{\partial X}\mathbb{E}\left[X|\mathcal{F}_i\right] = 1 - \frac{\mathrm{var}\left[X|\mathcal{F}_i\right]}{\mathrm{var}\left[X\right]}. \tag{6}$$

(See Lemma A-1 for a formal proof.) Next, note that, similar to the special case with no informative signals, optimal portfolio choice (4) and market clearing (2) imply both

$$\frac{\mathbb{E}\left[X - P\right]}{\gamma}\int_i \frac{di}{\mathrm{var}\left[X|\mathcal{F}_i\right]} = \bar{s} \tag{7}$$

$$\int_i \frac{\frac{\partial}{\partial X}\mathbb{E}[X - P|\mathcal{F}_i]}{var\left[X|\mathcal{F}_i\right]}di = 0. \tag{8}$$

Substituting (6) into (8) gives

$$\int_i \frac{1 - \frac{\mathrm{var}[X|\mathcal{F}_i]}{\mathrm{var}[X]} - \frac{cov[P,X]}{\mathrm{var}[X]}}{\mathrm{var}\left[X|\mathcal{F}_i\right]} di = 0,$$

which in turn implies that the equilibrium price P must satisfy

$$\int_i \frac{di}{\mathrm{var}\left[X|\mathcal{F}_i\right]} = \int_i \frac{di}{\mathrm{var}\left[X\right]\left(1 - \frac{\mathrm{cov}[P,X]}{\mathrm{var}[X]}\right)} = \frac{1}{\mathrm{cov}\left[X - P, X\right]}$$

A final substitution into (7) then delivers the result.

The remaining equilibrium properties are standard, and in particular, are established in Ganguli and Yang (2009) and Manzano and Vives (2011). As in their work, it is worth noticing that there are two equilibria. However, Manzano and Vives (2011) show that just one of these is stable. We summarize these results in:

Proposition 1. *Assume that $4\gamma^2(\tau_z^{-1} + \tau_u^{-1}) < \tau_x$ and $\gamma^2 > 4\lambda_I \tau_\epsilon \tau_u$. Then there is a unique stable linear equilibrium of the form*

$$P = \mu_P + bX - dZ. \tag{9}$$

In particular, the price coefficients satisfy

$$\rho \equiv \frac{b}{d} = \frac{\gamma}{2\tau_u} - \sqrt{\left(\frac{\gamma}{2\tau_u}\right)^2 - \lambda_I \frac{\tau_\epsilon}{\tau_u}} \tag{10}$$

$$b = \frac{\rho^2\left(\tau_z + \tau_u\right) + \lambda_I \tau_\epsilon}{\tau_x + \rho^2\left(\tau_z + \tau_u\right) + \lambda_I \tau_\epsilon} \tag{11}$$

Note that the first of the parametric assumptions simply assures that expected utility is well-defined in the autarchic outcome of the economy with no trade. While our model does require the parametric assumption $\gamma^2 > 4\lambda_I \tau_\epsilon \tau_u$ for existence, Manzano and Vives (2011), in a closely related model, show that non-existence is not generally an issue.[4]

As usual in this class of models, the key equilibrium parameters are the relative price coefficients $\rho = b/d$. We note that ρ measures the price efficiency of the risky asset, since

$$\mathrm{var}(X|P)^{-1} = \tau_x + \rho^2 \tau_z \tag{12}$$

$$\mathrm{var}(X|P, e_i)^{-1} = \tau_x + \rho^2 (\tau_z + \tau_u) \tag{13}$$

$$\mathrm{var}(X|P, e_i, Y_i)^{-1} = \tau_x + \rho^2 (\tau_z + \tau_u) + \tau_\epsilon \tag{14}$$

These expressions measure the ability of an outside observer, an uninformed trader, and an informed trader, respectively, to forecast the cash flow X.

From (10), one can see that, as usual, price efficiency ρ is increasing in the size of the informed population λ_I, as well as in the quality of their signals τ_ϵ.

We remark on the drivers of the equity premium $\mathbb{E}[X - P]$. Substituting (11) into (5) gives

$$\mathbb{E}[X - P] = \frac{\gamma \bar{s}}{\tau_x + \rho^2 (\tau_z + \tau_u) + \lambda_I \tau_\epsilon}. \tag{15}$$

The risk premia in our model is driven by the amount of aggregate risk, as measured by $\bar{s}$ and τ_x, as well as the risk tolerance in the economy, γ. Informed trading affects the risk premia by changing the amount of information revealed by prices. The term in the denominator of (15) is the conditional precision of the risky asset, as in (13), plus a term that increases with the size of the informed population, as well as with the quality of their signals. Rather intuitively, the more information revelation, the less risk agents face when investing in the market portfolio, which in turn lowers the equilibrium risk premia.

[4] The main extension in Manzano and Vives (2011) relative to Ganguli and Yang (2009) is that they allow for the error terms in the trader's signals to be correlated. Non-zero correlation eliminates the existence issues in our model. Since our focus is on welfare, we choose to study the slightly more tractable model with conditionally independent estimation errors.

2.3 Welfare

Our model is driven solely by the interaction of risk-sharing opportunities and the information obtained by active traders, part of which is impounded in prices. As in Hirshleifer (1971), the impounding of information into the price distorts risk sharing. In our setting, this is true both of information about X, and information about Z. While the effect of information about X is standard, it is worth elaborating on the effect of information about Z. Risk-sharing consists of traders with high-e_i realizations selling to traders with low-e_i realizations. When prices impound information about Z, this leads to prices that vary with Z, imposing additional risk on traders, and harming risk-sharing.

As is standard in financial economics, our measure of price efficiency ρ captures how much information the price contains about the cash flow X. However, one can also consider how much information the price contains about the aggregate endowment shock Z (essentially, the discount rate), which is measured by ρ^{-1}. So the more information that the price contains about X, the less it contains about Z.

This discussion implies that, in principle, price efficiency—again, defined to mean information about cash flow X—may either improve or worsen risk sharing. The negative effect of price efficiency is as in Hirshleifer (1971). The positive effect of price efficiency aligns with a basic intuition for why price efficiency is often considered valuable, namely that price fluctuations not attributable to cash flow fluctuations are undesirable.

The next Proposition provides one of our main contributions, a characterization of the value of information in this class of canonical models. In closely related settings, Vives and Medrano (2004) argue that "the expressions for the expected utility of a hedger . . . are complicated," whereas Kurlat and Veldkamp (2015) write that "there is no closed-form expression for investor welfare." The complications, common to our model as well, stem from the role of the endowment shocks as signals regarding price movements, on top of the standard risk sharing role that motivates trade.

Proposition 2. *The expected utility of an uninformed agent, conditional on the realization of his endowment shock e_i, is given by*

$$\mathbb{E}\left[u(W_i)|e_i\right] = -\Omega \exp\left(-\gamma(\bar{s}+e_i)\mu_x + \frac{\gamma^2(\bar{s}+e_i)^2}{2}\sigma_x^2 - \Lambda e_i^2\right) \tag{16}$$

where

$$\Omega = \sqrt{\frac{\text{var}(X|P,e_i)}{\text{var}(X-P|e_i)}}, \tag{17}$$

$$\Lambda = \frac{\left(\text{cov}(P,e_i)/\text{var}(e_i) + \gamma\text{cov}(X-P,X)\right)^2}{2\text{var}(X-P|e_i)}. \tag{18}$$

The welfare of uninformed traders is decreasing in both the size of the informed population λ_I and the precision of their signals τ_ϵ. The welfare of informed agents is also decreasing in τ_ϵ for τ_ϵ sufficiently high.

The utility expression in (16) measures the welfare of uninformed agents in our model. The determinant term, measured by Ω, has a familiar form (Verrecchia (1982)), in so far as it only depends on the conditional variance of payoffs, $\text{var}(X|P,e_i)$, and how much prices fluctuate around fundamentals, $\text{var}(X-P|e_i)$.

The term in the exponent has an unconditional piece, with the mean and variance of the risky assets' and agents' endowments, and the term Λ, which is driven by the covariance of prices and endowments, as well as between profits and fundamentals. In our model, prices and the endowments are negatively correlated. Price efficiency lowers the correlation, $|\text{cov}(P,e)|$, improving risk sharing. While information generates this positive effect, it also affects the second term, as prices and fundamentals covary more strongly the more informative prices are. The Proposition formalizes the fact that this second negative effect of information dominates.

The proof shows that uninformed investors' welfare is decreasing in the price efficiency parameter ρ. From the stock market equilibrium results in Proposition 1, it follows that both the size of the informed population λ_I and the precision of their signal τ_ϵ will have a negative effect on the uninformed population.

Note that, in addition to the two risk-sharing effects discussed above, changes in the number and knowledge of the informed traders (λ_I and τ_ϵ) also affect welfare in third way, namely the division of the gains from risk sharing between informed and uninformed populations. In particular, in equilibrium informed traders have an information advantage over uninformed traders, generating trading profits. (Uninformed traders accept trading losses because they still benefit from risk sharing.)

The effect of changes in λ_I and τ_ϵ on the information advantage of informed traders is a priori non-obvious, since it depends on the amount of information contained in equilibrium prices. Hence an increase in the knowledge of informed traders (τ_ϵ) increases their information advantage only if the direct increase in signal quality outweighs the increase in the information content of prices. The proof of Proposition 2 establishes that an informed trader's information advantage is decreasing in signal quality τ_ϵ when τ_ϵ is large, thereby establishing that in this case even the welfare of informed traders falls if they receive higher quality signals. In contrast, for uninformed traders the net effect of changes in the division of risk-sharing surplus with the risk-sharing effects discussed above is always such that uninformed welfare decreases in price efficiency.[5]

To gain further insight into informed traders's profits, let Θ_I denote the average trade of the informed population, and Θ_U that of the uninformed population. Expanding (4), an

[5]For simplicity, we have modeled uninformed traders as receiving no signal at all about cash flow X. A perturbed version of our model in which they instead receive a signal that is less precise than that of informed traders is straightforward to solve, and has very similar equilibrium properties. In this perturbed model, one can then consider the comparative static of increasing the signal precision of "uninformed" traders. The result is similar to our discussion of raising the precision of informed trader signals. On the one hand, the increase in signal precision increases equilibrium price efficiency, which worsens risk sharing. On the other hand, the information disadvantage of uninformed traders is reduced, which improves their share of the gains from risk sharing.

individual informed trader's position is given by

$$\theta_i + e_i = \frac{1}{\gamma}\left(\tau_x\mu_x + (\tau_z + \tau_u)\,\rho^2\frac{P-\mu_P}{b} + \tau_u\rho e_i + \tau_\epsilon Y_i - P\left(\tau_x\mu_x + (\tau_z+\tau_u)\,\rho^2 + \tau_\epsilon\right)\right),$$

with a parallel expression for an individual uninformed trader's position. Averaging across traders then implies

$$\Theta_I = \Theta_U + \frac{\tau_\epsilon(X-P)}{\gamma}, \tag{19}$$

i.e., on average, informed traders buy more than uninformed traders precisely when the asset is undervalued ($P < X$).

To obtain a more empirically interpretable version of (19), note that market clearing (2) implies that

$$\frac{1}{\gamma}\mathbb{E}\left[X-P|P\right]\int_i \frac{1}{\mathrm{var}\left[X|\mathcal{F}_i\right]} = \bar{s} + \mathbb{E}\left[Z|P\right].$$

Since $E\left[Z|P\right]$ is decreasing in P, it follows that:

Corollary 1. *In equilibrium,* $\mathbb{E}[X-P|P]$ *is decreasing in* P*.*

Consequently, an outside observer (econometrician) would observe that, on average, assets are reallocated from informed to uninformed investors as the price P rises, and that subsequent returns are then low. This is consistent with the empirical evidence in Ben-Rephael, Kandel, and Wohl (2012).[6]

3 Indexing and welfare

We extend the model in the previous section to allows for multiple assets, which allow us to study the role of indexing, The main result from the previous section is that informed trading hurts uninformed investors (despite increasing price efficiency and improving at least

[6]There is a sizeable empirical literature studying the relation between flows between different types of investors and subsequent returns. Results depend significantly on how flows are measured.

some dimensions of risk sharing). Could a strategy such as indexing, where agents shy away from picking individual assets and commit to only trade the market, help the uninformed avoid the welfare losses of Section 2?

3.1 The model

As before, there is a continuum of CARA agents, $i \in [0,1]$, with risk-aversion parameter γ

There are now two risky assets available for trading, with payoffs X_1 and X_2, where we assume that both assets follow a Gaussian distribution, $X_j \sim \mathcal{N}(\mu_j, \sigma_j^2)$, and that they are uncorrelated. The assets are symmetric: $\mu_1 = \mu_2 = \mu_x$, and $\sigma_1 = \sigma_2 = \sigma_x$. The price of assets $j = 1,2$ is denoted P_j.

The two assets are again in positive net supply, initially distributed equally among agents, so that each agent i starts with an endowment $\bar{s}$ of each of assets $1,2$. Let θ_{ij} denote agent i's position in asset j after trade.

Similar to before, agents also have other sources of income, now correlated with the cash flows of both assets X_1 and X_2. Specifically, agent i has income $e_{i1}X_1 + e_{i2}X_2$, where e_{i1} and e_{i2} are privately known at the time of trading. For example, cash flows X_1 and X_2 may correspond to two sectors of the economy, or two aggregate sources of risk (e.g., retail versus technology, or value versus growth stocks), and agents differ in the extent to which their non-trading income is exposed to these two sources of risk. Similar to before, $e_{ij} = Z_j + u_{ij}$ for $j = 1,2$, where $Z_j \sim \mathcal{N}(0, \sigma_z^2)$ and $u_{ij} \sim \mathcal{N}(0, \sigma_u^2)$. Hence agent i's wealth after trading (and excluding information acquisition costs, described below) is

$$\sum_{j=1,2} \left(\bar{s}P_j + e_{ij}X_j + \theta_{ij}\left(X_j - P_j\right)\right)$$

There are three types of traders in our model. The first set are informed agents, who observe private signals prior to trading in financial markets. The rest of agents are uninformed, mimicking the retail sector. There are two types of uninformed traders: "active traders," who are unconstrained, and "indexers," who only invest in the market portfolio. Given our symmetry assumptions, this means that each indexer always takes the same position in each of the two traded assets, e.g., $\theta_{i1} = \theta_{i2}$, so that asset 1's weight in an indexer's portfolio of risky assets coincides with asset 1's weight in the market, i.e.,

$$\frac{\theta_{i1} P_1}{\theta_{i1} P_1 + \theta_{i2} P_2} = \frac{\bar{s} P_1}{\bar{s} P_1 + \bar{s} P_2}.$$

Importantly, even though indexers do not engage in stock selection, they still face a "timing" decision: they must choose how much of the market index to buy or sell. As before, we denote the number of informed investors by λ_I. We write λ_U for the number of active uninformed agents, and η for the number of indexers. At this point we do not provide a micro-foundation for the number of indexers (though see discussion further below). We note that $\lambda_I + \lambda_U + \eta = 1$.

In contrast to Section 2, we explicitly model informed traders' acquisition of information. In practice, financial analysts and traders spend a great deal of time assessing the *relative* value of different assets. A largely separate activity is attempting to forecast macroeconomic developments and aggregate market events. To capture this distinction, each informed trader i can acquire information about the relative value of the two assets, $X_s \equiv \frac{1}{\sqrt{2}} (X_1 - X_2)$, which we will refer to as the "spread portfolio;" and about market value $X_m \equiv \frac{1}{\sqrt{2}} (X_1 + X_2)$. In particular, agent i observes two signals

$$\begin{aligned} Y_{im} &= \frac{1}{\sqrt{2}} (X_1 + X_2) + \epsilon_{im}, \\ Y_{is} &= \frac{1}{\sqrt{2}} (X_1 - X_2) + \epsilon_{is}, \end{aligned}$$

where $\epsilon_{ij} \sim \mathcal{N}(0, \tau_{ij}^{-1})$ for $j = m, s$. Agents must pay a cost $\kappa(\tau_{im}, \tau_{is})$ to observe these signals. Let κ_j denote the first derivative of κ with respect to τ_{ij}, for $j = m, s$, and similarly for higher order derivatives. The cost function κ has the following properties:

Assumption 1. *The information cost function κ satisfies:*

1. $\kappa_m \geq 0$, $\kappa_s \geq 0$, $\kappa_{mm} > 0$, $\kappa_{ss} > 0$, $\kappa_{ms} > 0$, *and* $\kappa_{mm}\kappa_{ss} - \kappa_{ms}^2 \geq 0$ *where the inequalities are strict expect for at* $(\tau_{im}, \tau_{is}) = (0, 0)$.

2. *If* $\tilde{\tau}_{im} \geq \tau_{im}$ *and* $\tilde{\tau}_{is} \leq \tau_{is}$, *then* $\kappa_m(\tilde{\tau}_{im}, \tilde{\tau}_{is}) \leq \kappa_m(\tau_{im}, \tau_{is})$ *implies* $\kappa_s(\tilde{\tau}_{im}, \tilde{\tau}_{is}) \leq \kappa_s(\tau_{im}, \tau_{is})$ *and* $\kappa_s(\tilde{\tau}_{im}, \tilde{\tau}_{is}) \geq \kappa_s(\tau_{im}, \tau_{is})$ *implies* $\kappa_m(\tilde{\tau}_{im}, \tilde{\tau}_{is}) \geq \kappa_m(\tau_{im}, \tau_{is})$, *along with the symmetric property.*

The first set of assumptions are quite mild. The second assumption makes the marginal cost of information about the market portfolio more responsive to changes in the amount of information collected about the market portoflio (τ_{ϵ_m}) than the marginal cost of information about the spread asset.[7]

3.2 Market and spread assets

Although we have defined the primitive economy in terms of assets 1 and 2, analytically it is very convenient to effectively change basis and study the economy in terms of a market asset that pays X_m and a spread asset that pays X_s. Importantly, note that that the cash flows of these assets are uncorrelated, i.e.,

$$\mathrm{cov}(X_m, X_s) = 0. \tag{20}$$

The most important benefit of this change of basis is as follows. Because, by assumption, indexers always trade assets 1 and 2 together, indexers' trades introduce a common factor

[7] A simple example of a cost function satisfying these properties is $\kappa(\tau_{im}, \tau_{is}) = \frac{1}{2}\tau_{im}^2 + \frac{1}{2}\tau_{is}^2 + \delta\tau_{im}\tau_{is}$ for any $\delta \leq 1$.

into the prices of assets 1 and 2. In contrast, the indexing assumption can alternatively be stated as: indexers trade only the market asset, and do not trade the spread asset. Combined with (20), this means that the market and spread assets effectively trade independently, and can be analyzed separately.

Note that $\text{var}(X_m) = \text{var}(X_s) = \sigma_x^2$, $\mathbb{E}[X_m] = \sqrt{2}\mu_x$ and $\mathbb{E}[X_s] = 0$. Each agent has an endowment $\bar{s}_m \equiv \sqrt{2}\bar{s}$ of the market asset, and no endowment of the spread asset (i.e., $\bar{s}_s \equiv 0$).

Finally, define $Z_m \equiv \frac{1}{\sqrt{2}}(Z_1 + Z_2)$, $Z_s \equiv \frac{1}{\sqrt{2}}(Z_1 - Z_2)$, $e_{im} \equiv \frac{1}{\sqrt{2}}(e_{i1} + e_{i2})$, and $e_{is} \equiv \frac{1}{\sqrt{2}}(e_{i1} - e_{2i})$. Note that $\text{cov}(Z_m, Z_s) = \text{cov}(e_{im}, e_{is}) = 0$; $\text{var}(Z_m) = \text{var}(Z_s) = \sigma_z^2$; $\text{var}(e_{im}) = \text{var}(e_{is}) = \sigma_z^2 + \sigma_u^2$; and $\mathbb{E}[Z_m] = \mathbb{E}[Z_s] = \mathbb{E}[e_{im}] = \mathbb{E}[e_{is}] = 0$.

Write θ_{im} and θ_{is} for agent i's positions in the market and spread asset. Then agent i's terminal wealth W_i is given by an analogous expression to (1). As before, W_i is determined by trading profits combined with initial asset endowments and endowment shocks, along with information acqusition costs (for informed traders):

$$W_i = -\kappa(\tau_{im}, \tau_{is}) + \sum_{j=m,s} \left((\bar{s}_j + e_{ij}) P_j + (\theta_{ij} + e_{ij})(X_j - P_j) \right). \tag{21}$$

Finally, we write the equilibrium definition directly in terms of market and spread assets. The definition extends the equilibrium definition of the one-asset economy to multiple assets (Admati (1985)), and incorporates individually-optimal information acquisition (Verrecchia (1982)):

Definition 2. *A rational expectations equilibrium is a set of trading strategies* $\{\theta_{im}, \theta_{is}\}_{i\in[0,1]}$*, a pair of price functions* $\{P_m(X_m, X_s, Z_m, Z_s), P_s(X_m, X_s, Z_m, Z_s)\}$*, and a set of signal precisions chosen by the informed agents,* $\{\tau_{im}, \tau_{is}\}_{i\in[0,\lambda_I]}$*, such that*

1. *Markets clear: for* $k = m, s$*,*

$$\int_0^1 \theta_{ik} di = \bar{s}_k; \tag{22}$$

2. Taking prices as given, agent i's trading strategy is optimal:

$$(\theta_{im}, \theta_{is}) \in \arg \max_{(\tilde{\theta}_{im}, \tilde{\theta}_{is})} \mathbb{E}\left[u(W_i)|\mathcal{F}_i\right]. \tag{23}$$

3. Informed agents' choices of information are optimal: For $i \in [0, \lambda_I]$,

$$(\tau_{im}, \tau_{is}) \in \arg \max_{(\tilde{\tau}_{im}, \tilde{\tau}_{is})} \mathbb{E}\left[u(W_i)\right]. \tag{24}$$

Note that although the above equilibrium definition allows for the possibility that the price P_m of the market asset depends on factors related to the spread asset (X_s, Z_s), and vice versa, it will turn out that this dependence does not arise in equilibrium.

3.3 Equilibrium asset prices and information production

We first study the equilibrium of the trading stage, fixing the information acquistion decisions of informed traders. We then solve for the information acquisition decision of the informed active traders.

Because both the cash flows and exposure shocks related to the market and spread assets are uncorrelated, the two-asset economy fully "decouples" into two single-asset economies at the trading stage. (Looking ahead, the two assets will be related via the information acquisition choices.) Consequently, the next result is a straightforward extension of Lemma 5 and Proposition 1:

Proposition 3. *Assume that $4\gamma^2(\tau_z^{-1}+\tau_u^{-1}) < \tau_x$, active traders have signals with precisions $\tau_{im} = \tau_{\epsilon m}$ and $\tau_{is} = \tau_{\epsilon s}$ for all $i \in [0, \lambda_I]$; and $\gamma^2 > 4\lambda_I \tau_{\epsilon m}\tau_u$ and $\gamma^2 > 4\frac{\lambda_I}{1-\eta}\tau_{\epsilon s}\tau_u$. Then there* *is a unique stable linear equilibrium of the form*

$$P_m = \mu_m + b_m X_m - d_m Z_m,$$
$$P_s = b_s X_s - d_s Z_s.$$

In particular, the price coefficients satisfy

$$\rho_m \equiv \frac{b_m}{d_m} = \frac{\gamma}{2\tau_u} - \sqrt{\left(\frac{\gamma}{2\tau_u}\right)^2 - \lambda_I \frac{\tau_{\epsilon m}}{\tau_u}} \tag{25}$$

$$\rho_s \equiv \frac{b_s}{d_s} = \frac{\gamma}{2\tau_u} - \sqrt{\left(\frac{\gamma}{2\tau_u}\right)^2 - \frac{\lambda_I}{(1-\eta)} \frac{\tau_{\epsilon s}}{\tau_u}} \tag{26}$$

$$b_m = \frac{\rho_m^2 (\tau_z + \tau_u) + \lambda_I \tau_{\epsilon m}}{\tau_x + \rho_m^2 (\tau_z + \tau_u) + \lambda_I \tau_{\epsilon m}} \tag{27}$$

$$b_s = \frac{\rho_s^2 (\tau_z + \tau_u) + \frac{\lambda_I}{1-\eta} \tau_{\epsilon s}}{\tau_x + \rho_s^2 (\tau_z + \tau_u) + \frac{\lambda_I}{1-\eta} \tau_{\epsilon s}} \tag{28}$$

Moreover,

$$\mathbb{E}[X_m - P_m] = \gamma \bar{s}_m \mathrm{cov}(X_m - P_m, X_m) \tag{29}$$

Note that the analogue of (29) for the spread asset holds trivially, since $\mathbb{E}[X_s] = \mathbb{E}[P_s] = \bar{s}_s = 0$.

As Proposition 3 makes clear, indexing does not directly affect the price of the market asset. Instead, as we show below, its effect operates via changing incentives for information production. The absence of a direct effect stems in part from CARA preferences, which in this case imply that indexing traders do not take on more (or less) risk in the market asset when they stop trading the spread asset.

One can also see that indexing affects pricing of the spread asset in an intuitive way. The market asset is traded by a measure 1 of traders, of whom λ_I are informed traders. In constrast, the spread asset is traded by only $1 - \eta$ traders, and again, λ_I are informed traders. As reflected in (26) and (28), it is the ratio of informed traders to total traders that matters, i.e., $\frac{\lambda_I}{1-\eta}$ for the spread asset.

Holding information acquisition constant, one can see that indexing (η) increases price efficiency in the spread asset. Economically, indexing reduces trading in response to exposure shocks, which functions like "noise." In contrast, indexing does not affect informed trading. Clearly the prediction that indexing increases price efficiency depends on the way we have modelled indexing as originating from uninformed traders. We discuss this point more in Section 4 below.

Note that, having solved for the prices of the market and spread assets, it is then straightforward to recover the prices of the underlying assets 1 and 2. Specifically, since

$$X_1 = \frac{1}{\sqrt{2}}\left(X_m + X_s\right) \tag{30}$$

$$X_2 = \frac{1}{\sqrt{2}}\left(X_m - X_s\right), \tag{31}$$

it follows that

$$P_1 = \frac{1}{\sqrt{2}}\left(P_m + P_s\right) \tag{32}$$

$$P_2 = \frac{1}{\sqrt{2}}\left(P_m - P_s\right). \tag{33}$$

A key force in our analysis is the reaction of "smart money" to an increase in indexing. The next proposition characterizes the information acquisition decision of informed agents in our multi-asset model.

Proposition 4. *If information production costs are sufficiently high, then a symmetric equilibrium at the information acquisition stage exists, and it is unique in the class of symmetric equilibria. Active informed traders all choose* $\tau_{im} = \tau_{\epsilon m}$ *and* $\tau_{is} = \tau_{\epsilon s}$*, where* $(\tau_{\epsilon m}, \tau_{\epsilon s}), \rho_m, \rho_s$

solve:

$$2\gamma\kappa_m(\tau_{\epsilon m}, \tau_{\epsilon s}) = \frac{1}{\tau_x + \rho_m^2(\tau_z + \tau_u) + \tau_{\epsilon m}} \tag{34}$$

$$2\gamma\kappa_s(\tau_{\epsilon m}, \tau_{\epsilon s}) = \frac{1}{\tau_x + \rho_s^2(\tau_z + \tau_u) + \tau_{\epsilon s}} \tag{35}$$

along with (25) and (26).

Proposition 4 closes the model at the information gathering stage. Equations (34) and (35) are simply the first-order conditions for aninformed agent's decision regarding the precision of his signals. Agents' choices are driven by the trade-off between the extra precision's effect on ex-ante utility, the right-hand side terms in (34) and (35), and the marginal cost of acquiring such precision. We remark that the fact that price efficiency ρ_m and ρ_s are determined by information acqusition decisions $\tau_{\epsilon m}$ and $\tau_{\epsilon s}$ (formally, see (25) and (26)) precludes closed-form solutions.[8]

Lemma A-4 in the appendix characterizes how information acquisition responds to exogenous changes in price efficiency. First, information gathered about the market (spread) asset decreases in the price efficiency of the market (spread) asset. This is the standard substitution effect of Grossman and Stiglitz (1980): more information revelation makes becoming informed less valuable. On the other hand, if price efficiency increases in one of the assets, then because agents substitute away from collecting information about that asset, they in turn substitute towards collecting information about the other asset. This follows from Assumption 1, and in particular, the fact that the marginal cost of precision in one dimension of information is increasing in the amount of precision acquired about the other dimension of information. This is the key channel via which indexing affects the market asset.

[8] In the quadratic cost function case, $\kappa(\tau_{im}, \tau_{is}) = \frac{1}{2}\tau_{im}^2 + \frac{1}{2}\tau_{is}^2 + \delta\tau_{im}\tau_{is}$, one can verify (34) and (35) reduce to polynomials of degree four.

3.4 Indexing, asset prices and welfare

The focus of our paper is on the role of indexing on equilibrium asset prices. We start by analyzing what happens to the information acquired by agents as indexing increases.

Proposition 5. *Consider two different levels of indexing η^* and η', with associated equilibrium price coefficients (ρ_m^*, ρ_s^*) and (ρ_m', ρ_s'). If $\eta^* > \eta'$, then $\rho_m^* \geq \rho_m'$, and $\rho_s^* \geq \rho_s'$, with strict inequality if $\rho_m^* > 0$.*

Proposition 5 establishes that price efficiency increases in *both* the market and spread assets as indexing increases. Intuitively, when agents decide to index, the "noise" in the spread asset, created by the endowment shocks, is reduced. This makes prices more efficient, which, in turn, decreases the information production of informed traders in the spread asset, and so (as discussed above) increases their investment in signals about the market asset. As a consequence prices in the market asset become more efficient.

Note that price efficiency in the spread asset unambiguously increases, even though information production decreases. To see why the reduction in "noise" must be the dominant effect, suppose to the contrary that price efficiency decreased: in this case, information production would increase, which together with the reduction in noise would imply that price efficiency increases, a contradiction.

In other words, by reducing "noise" in the spread asset, indexing makes it harder for informed traders to profitably trade the spread asset. This leads each individual informed trader to devote fewer resources to relative valuation. Informed traders then reallocate some of these "saved" information acquisition resources to aggregate valuation. In turn, this means that uninformed investors, including indexers, face informed investors with better information when undertake "timing" trades, i.e., trades of the market asset.

It is also interesting to consider how much prices reveal about the underlying cash flows of the two assets, namely X_1 and X_2. Straightforward calculation yields:

Corollary 2. *An increase in indexing increases the information that prices reveal about each of cash flows X_1 and X_2. That is, for $j = 1, 2$, $\mathrm{var}(X_j | P_m, P_s)$ is decreasing in η.*

Because indexing increases both price efficiency and information production about market cash flows, Proposition 5 combines with (15), (32), and (33) to deliver

Corollary 3. *Indexing reduces the risk premium on the market asset, and on each of assets* 1 *and* 2*.*

Economically, when prices are more efficient, traders take less risk then they trade assets, and so the required risk premium falls.

Having established that more indexing makes prices more informative, we turn next to the welfare consequences of indexing. Our main result is:

Proposition 6. *An increase in indexing η reduces the welfare of all agents.*

Proposition 6 says that an individual agent's decision to index imposes a negative externality on other agents. For uninformed agents, whether active traders or other indexing agents, this follows from a combination of previous results. From Proposition 5, indexing increases the price efficiency of both the market and spread assets. Although this increase in price efficiency has both positive and negative effects on welfare, we know from Proposition 2 that the net effect for uniformed agents is negative. In other words, indexing reduces the welfare of uninformed traders because the increase in price efficiency it engenders worsens the risk sharing function of the financial sector. And even indexers need the risk sharing function, since their exposure to underlying cash flow risk fluctuates.

Proposition 6 also shows that indexing reduces the welfare of informed traders. The key force is again that indexing increases price efficiency, for the reasons discussed above. Individual informed traders dislike the increase in price efficiency, both because it worsens risk sharing, and also because it reduces their informational advantage relative to uninformed investors. The proof of Proposition 6 uses a simple revealed-preference argument to show that even after informed traders re-optimize their information acquisition efforts, they are still worse off when price efficiency increases.

Finally, we also note that, by assumption, a decision to index makes the indexing agent worse off. This is immediate: in our model, indexing is simply a constraint on portfolio choice (i.e., only trade the market asset). More generally, one could consider a model in which indexing allows an agent to save some amount B, perhaps corresponding to lower trading costs, or lower fees on passive as opposed to active mutual funds.[9] In such a model, an increase in B (e.g., an increase in the fee differential between active and passive mutual funds) would lead to an increase in indexing. In terms of welfare, the increase in B would have two effects: the direct effect, related to the greater direct benefit of indexing; and the negative externality identified by Proposition 6. Along these lines, we think Proposition 6 should be interpreted as pointing to a negative consequence of indexing that is rarely discussed: indexing leads informed traders to shift their information acquisition efforts towards aggregrate factors, thereby worsening the terms at which other traders, including indexers, are able to make aggregate "timing" trades.

We conclude by providing an illustrative numerical example, displayed in Figures 1 - 4. We consider the following parameter values, which ensure existence of equilibria, but are otherwise arbitrary:[10] $\tau_x = 1$, $\tau_z = 3$, $\tau_u = 5$, $\bar{s} = 1$, $\mu_x = 1$, $\gamma = 0.2$, $\kappa = 0.01$, $\lambda_I = 0.1$. Figure 1 plots price efficiency for the market and spread assets (ρ_m, ρ_s) as a function of the mass of indexers (η). As established in Proposition 5, more indexing results in more informative prices. The effect is more pronounced for the spread asset, as the removal of noise generated by the endowment shocks makes informed trading more salient.

[9] Although we have modeled active uninformed traders as participating directly in financial markets, one could also imagine that this trading activity is instead carried out by active mutual funds that tilt their portfolios in response to the information contained in market prices.

[10] With the exception of Figure 4, our analytical results above establish the qualitative patterns illustrated in the figures.

Figure 2 plots the equilibrium market risk premium as a function of the mass of indexers. As established in Corollary 3, this monotonically decreases as more people index.

Figure 3 plots welfare of the three types of trader, namely informed, active uninformed, and indexers. As established in Proposition 6, the welfare of all three groups declines as indexing increases.

Finally, we investigate the effect of indexing on the correlation of informed traders' profits with the market return. The rise in index investing over the last 2-3 decades has been accompanied by an increase in the correlation of mutual fund returns with the market (Stambaugh, 2014). In Figure 4 we plot the relationship between indexing and the correlation between the trading profits of informed agents and the market. As discussed, in our model more indexing makes the informed intensify their information gathering activities in the market, which in turn makes their trading profits more correlated with market returns. Although we have been unable to establish this result analytically, all numerical simulations that we have examined exhibit this property.

4 Discussion and Conclusion

We study the implications of informed trading for the welfare of market participants within the canonical rational expectations model (Grossman and Stiglitz (1980), Hellwig (1980)). We find that as the informed population grows, or their signals become more precise, uninformed investors' welfare falls. Under some circumstances informed welfare falls also. The welfare decline reflects worse risk sharing. We extend the model to allow for multiple assets, in order to study the consequences of indexing, i.e., committing to invest in risky assets only via the market portfolio. We show that indexing imposes a negative externality on agents. More indexing makes informed trading in the market more profitable, which decreases welfare by distorting risk-sharing.

In addition to characterizing how indexing affects equilibrium welfare, our analysis produces a number of empirical implications. We have noted these in passing above, but for clarity, we collected them here.

First, indexing reduces the market risk premium. While empirically the evidence supports a correlation between the amount of indexing and the equity risk premium, it is hard to argue casuality using empirical methods. Our model explicitly links the increase in passive investment to a lower equity risk premium.

Indexing leads to higher levels of "price efficiency," as measured by the ability of prices to predict future cash flows. More indexing also leads to a shift in trading strategies of informed investors, who shift from trades based on cross-sectional "mispricing" to ones that focus instead on aggregate factors. These two implications are broadly consistent with the recent empirical literature on hedge funds and ETFs (Fung et al., 2008; Sun, Wang, and Zheng, 2012; Ramadorai, 2013; Pedersen, 2015; Israeli, Lee, and Sridharan, 2015).

Furthermore, the correlation between hedge fund returns with equity markets is significantly higher now than it was in the last two decades.[11] This mirrors the evidence from the mutual fund literature (Stambaugh, 2014), and it is consistent with the numerical results presented in Figure 4.

It is worth highlighting that we have modeled an increase in indexing as coming at the expense of uninformed trading. As noted in footnote 9, one possible interpretation is that a rise in indexing corresponds to investors shifting away from actively managed funds that lack private information, but instead actively trade on the information prices contains about future cash flows (we highlight that in the class of models we study, prices contain predictive information future cash flows); and towards passive benchmark or "tracker" funds. However, if one instead views actively managed mutual funds as possessing significant expertise in forecasting cash flows (over and above their ability to interpret prices), then a rise in indexing

[11] See "Hedge fund correlation risk alarms investors," Financial Times, June 29th, 2014. AQR estimates the correlation to be above 0.9, up from about 0.6 in the 1990s.

would instead correspond to a reduction in informed trading. In this case, many of our results would be reversed: indexing would decrease rather than increase price efficiency; would increase rather than decrease risk premia; and would increase rather than decrease welfare.

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Appendix

Results omitted from main text

Lemma A-1. *Suppose that the information set $\mathcal{F}_i$ consists of a set of normally distributed random variables. Then*

$$\frac{\partial}{\partial X} E\left[\tilde{X}|\mathcal{F}_i\right] = 1 - \frac{var\left[\tilde{X}|\mathcal{F}_i\right]}{var\left[\tilde{X}\right]}.$$

Proof of Lemma A-1. Let Σ_{22} be the variance matrix of the random variables in $\mathcal{F}_i$; and Σ_{12} for the row vector of covariances between X and the random variables in $\mathcal{F}_i$. By the standard formulae,

$$\begin{aligned} \frac{\partial}{\partial X} E\left[\tilde{X}|\mathcal{F}_i\right] &= \Sigma_{12}\Sigma_{22}^{-1}\frac{\Sigma_{12}'}{\mathrm{var}[\tilde{X}]} \\ \mathrm{var}\left[\tilde{X}|\mathcal{F}_i\right] &= \mathrm{var}[\tilde{X}] - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}'. \end{aligned}$$

Combining these two equations yields the result. ■

Lemma A-2. *Let $X \in \mathbb{R}^n$ be a normally distributed random vector with mean μ and variance-covariance matrix Σ. Let $b \in \mathbb{R}^n$ be a given vector, and $A \in \mathbb{R}^{n\times n}$ a symmetric matrix. If $I - 2\Sigma A$ is positive definite, then $\mathbb{E}\left[\exp(b^\top X + X^\top AX)\right]$ is well defined, and given by:*

$$\mathbb{E}\left[\exp\left((b^\top X + X^\top AX\right)\right] = |I - 2\Sigma A|^{-1/2}\exp\left(b^\top\mu + \mu^\top A\mu + \frac{1}{2}(b+2A\mu)^\top(I-2\Sigma A)^{-1}\Sigma(b+2A\mu)\right) \tag{A-1}$$

Proof of Lemma A-2. Standard result. ■

Lemma A-3. *The ratio of expected informed trader utility to expected uninformed trader utility is*

$$\sqrt{\frac{\mathrm{var}(X|P,e_i,Y_i)}{\mathrm{var}(X|P,e_i)}} = \sqrt{\frac{\tau_x + \rho^2(\tau_z+\tau_u)}{\tau_x + \rho^2(\tau_z+\tau_u)+\tau_\epsilon}}.$$

Proof of Lemma A-3. The final wealth of agent i, given optimal trading (4), is

$$W_i = (\bar{s} + e_i)\,P + \frac{\mathbb{E}[X-P|\mathcal{F}_i]\,(X-P)}{\gamma\mathrm{var}(X|\mathcal{F}_i)}.$$

Hence the expected utility of a trader i, given his information set at the time of trading, is

$$\mathbb{E}\left[-\exp\left(-\gamma W_i\right)|\mathcal{F}_i\right] = -\exp\left(-\gamma\left(\left(\bar{s}+e_i\right)P + \frac{1}{2}\frac{\mathbb{E}[X-P|\mathcal{F}_i]^2}{\gamma \text{var}(X|\mathcal{F}_i)}\right)\right). \tag{A-2}$$

We compute

$$\mathbb{E}\left[\exp\left(-\frac{1}{2}\frac{\mathbb{E}[X-P|\mathcal{F}_i]^2}{\text{var}(X|\mathcal{F}_i)}\right)|P, e_i\right]. \tag{A-3}$$

To do so, we use the standard linear-quadratic formula (see Lemma A-2), letting $\xi_i = \mathbb{E}\left[X-P|\mathcal{F}_i\right]$ and $A = -1/\left(2\text{var}(X|\mathcal{F}_i)\right)$, and so the expectation (A-3) equals:

$$\mathbb{E}\left[\exp\left(\xi_i^2 A\right)|P, e_i\right] = \frac{1}{\sqrt{1-2A\text{var}(\xi_i|P,e_i)}}\exp\left(\left(A + \frac{2A^2\text{var}(\xi_i|P,e_i)}{1-2A\text{var}(\xi_i|P,e_i)}\right)\mathbb{E}[\xi_i|P,e_i]^2\right). \tag{A-4}$$

Using the law of total variance we have that

$$\text{var}(X-P|P,e_i) = \text{var}(\mathbb{E}[X-P|\mathcal{F}_i]|P,e_i) + \mathbb{E}[\text{var}(X-P|\mathcal{F}_i)|P,e_i]$$

which implies

$$\text{var}(\xi_i|P,e_i) = \text{var}(X|P,e_i) - \text{var}(X|\mathcal{F}_i)$$

and so

$$1 - 2\text{var}(\xi_i|P,e_i)A = \frac{\text{var}(X|P,e_i)}{\text{var}(X|\mathcal{F}_i)}.$$

Hence expression (A-4) equals

$$\sqrt{\frac{\text{var}(X|\mathcal{F}_i)}{\text{var}(X|P,e_i)}}\exp\left(-\frac{1}{2}\frac{\mathbb{E}[X-P|P,e_i]^2}{\text{var}(X|P,e_i)}\right), \tag{A-5}$$

which establishes the left-hand side or the result. The right-hand side then follows from (13) and (14). ■

Proofs of results stated in main text

Proof of Proposition 1. See Ganguli and Yang (2009) and Manzano and Vives (2011). ■

Proof of Proposition 2. We start by evaluating the expected utility of an average uninformed agent,

$$\mathbb{E}\left[\exp\left(-\frac{1}{2}\frac{\mathbb{E}[X-P|P,e_i]^2}{\text{var}(X|P,e_i)} - \gamma(\bar{s}+e_i)P\right)\right] \tag{A-6}$$

or

$$\mathbb{E}\left[-\frac{1}{2}\frac{\mathbb{E}[X-P|P,e_i]^2}{\mathrm{var}(X|P,e_i)} - \gamma(\bar{s}+e_i)(P-\mathbb{E}[P|e_i]) + \gamma(\bar{s}+e_i)\mathbb{E}[X-P|e_i] - \gamma(\bar{s}+e_i)\mathbb{E}[X|e_i]\right] \tag{A-7}$$

Let $\alpha_e \equiv \mathbb{E}[X-P|e_i]$. Using (A-1), (A-6) can be written as

$$-D^{-1/2}\exp\left(\gamma(\bar{s}+e)\alpha_e - \gamma(\bar{s}+e)\mu_x - \frac{1}{2}\frac{\alpha_e^2}{\mathrm{var}(X|P,e_i)} + \frac{1}{2}\left(\frac{\alpha_e\mathrm{cov}(X-P,P|e_i)}{\mathrm{var}(P|e_i)\mathrm{var}(X|P,e_i)} + \gamma(\bar{s}+e_i)\right)^2 \frac{\mathrm{var}(P|e_i}{D} \right. \tag{A-8}$$

where

$$D = 1 + \frac{\mathrm{cov}(X-P,P|e_i)^2}{\mathrm{var}(X|P,e_i)\mathrm{var}(P|e_i)} \tag{A-9}$$

Using the law of total variance,

$$\mathrm{var}(X-P|e_i) = \mathrm{var}(X-P|P,e_i) + \mathrm{var}(\mathrm{E}[X-P|P,e_i]), \tag{A-10}$$

which using (A-9) means that

$$D = \frac{\mathrm{var}(X-P|e_i)}{\mathrm{var}(X|P,e_i)} \tag{A-11}$$

Some tedious algebra shows that (A-7) can be written as

$$-D^{-1/2}\exp\left(-\gamma(\bar{s}+e_i)\mu_x + \frac{\gamma^2(\bar{s}+e_i)^2}{2}\sigma_x^2 - \frac{1}{2}\frac{(\alpha_e - \mathrm{cov}(X-P,X)\gamma(\bar{s}+e_i))^2}{D\mathrm{var}(X|P,e_i)}\right) \tag{A-12}$$

Substituting (5) into (A-12), and noting also that

$$\alpha_e = \mathbb{E}[X-P] - \frac{\mathrm{cov}(P,e_i)}{\mathrm{var(e_i)}}e_i,$$

one arrives at (16).

Comparative statics, uninformed trader: We next argue use (16) to show that the welfare of an uninformed trader is decreasing in ρ. There are two steps to this. First, we show that (A-11) is decreasing in ρ. Expanding,

$$\frac{\mathrm{var}(X-P|e_i)}{\mathrm{var}(X|P,e_i)} = \left(\tau_x + \rho^2(\tau_z+\tau_u)\right)\left((1-b)^2\tau_x^{-1} + \left(\frac{b}{\rho}\right)^2(\tau_z+\tau_u)^{-1}\right)$$

From Proposition 1, we know

$$\tau_u \rho^2 - \gamma\rho + \lambda_I \tau_\epsilon, \tag{A-13}$$

and hence

$$1 - b = \frac{\tau_x}{\tau_x + \rho^2\tau_z + \gamma\rho}; \qquad \frac{b}{\rho} = \frac{\rho\tau_z + \gamma}{\tau_x + \rho^2\tau_z + \gamma\rho}, \tag{A-14}$$

so that

$$\begin{aligned} \frac{\mathrm{var}(X - P|e_i)}{\mathrm{var}(X|P, e_i)} &= \frac{\tau_x + \rho^2(\tau_z + \tau_u)}{(\tau_x + \rho^2\tau_z + \gamma\rho)^2}\left(\tau_x + (\rho\tau_z + \gamma)^2(\tau_z + \tau_u)^{-1}\right) \\ &= \frac{\tau_x^2 + \tau_x(\rho\tau_z + \gamma)^2(\tau_z + \tau_u)^{-1} + \tau_x\rho^2(\tau_z + \tau_u) + \rho^2(\rho\tau_z + \gamma)^2}{(\tau_x + \rho^2\tau_z + \gamma\rho)^2} \\ &= 1 + \frac{\tau_x}{\tau_z + \tau_u}\frac{(\gamma - \rho\tau_u)^2}{(\tau_x + \rho^2\tau_z + \gamma\rho)^2}, \end{aligned}$$

where the final equality follows from simple algebraic manipulations. We know $\frac{\gamma}{2} - \rho\tau_u > 0$, and so certainly $\gamma - \rho\tau_u > 0$. Hence the ratio $\frac{\mathrm{var}(X-P|e_i)}{\mathrm{var}(X-P|P,e_i)}$ is decreasing in ρ.

Second, we show that (18) is decreasing in ρ. Expanding, (18) equals

$$\frac{1}{2}\frac{\left(-\frac{b}{\rho}\frac{\tau_z^{-1}}{\tau_u^{-1}+\tau_z^{-1}} + \gamma(1-b)\tau_x^{-1}\right)^2}{(1-b)^2\tau_x^{-1} + \left(\frac{b}{\rho}\right)^2(\tau_z + \tau_u)^{-1}} = \frac{\left(\frac{b}{(1-b)}\frac{1}{\rho}\frac{\tau_u}{\tau_u+\tau_z} - \gamma\tau_x^{-1}\right)^2}{\tau_x^{-1} + \left(\frac{b}{(1-b)}\right)^2\left(\frac{1}{\rho}\right)^2\frac{1}{\tau_u+\tau_z}}. \tag{A-15}$$

From (A-14) we know $\frac{b}{1-b}\frac{1}{\rho} = \frac{\rho\tau_z+\gamma}{\tau_x}$, so that we simply have to prove that (A-15) is decreasing in $\frac{b}{1-b}\frac{1}{\rho}$. It is easy to check that (A-15) is decreasing in $\frac{b}{1-b}\frac{1}{\rho}$ if and only if

$$\frac{\rho\tau_z + \gamma}{\tau_x}\frac{\tau_u}{\tau_u + \tau_z} < \frac{\gamma}{\tau_x},$$

which indeed holds since (as noted above) $\rho \le \frac{\gamma}{\tau_u}$.

Comparative statics, informed trader: From Lemma A-3, informed trader welfare certainly falls if the ratio $\frac{\tau_x+\rho^2(\tau_z+\tau_u)}{\tau_x+\rho^2(\tau_z+\tau_u)+\tau_\epsilon}$ increases, or equivalently, if the ratio $\frac{\tau_\epsilon}{\tau_x+\rho^2(\tau_z+\tau_u)}$ decreases. From (A-13),

$$\frac{\tau_\epsilon}{\tau_x + \rho^2(\tau_z + \tau_u)} = \frac{1}{\lambda_I}\frac{\gamma\rho - \tau_u\rho^2}{\tau_x + (\tau_z + \tau_u)\,\rho^2}.$$

This ratio is decreasing in ρ if and only if the following expression is negative:

$$\begin{aligned}
& \left(\gamma - 2\tau_u\rho\right)\left(\tau_X + \left(\tau_Z + \tau_u\right)\rho^2\right) - 2\left(\gamma\rho - \tau_u\rho^2\right)\left(\tau_Z + \tau_u\right)\rho \\
= \; & \left(\gamma - 2\tau_u\rho\right)\tau_X + \gamma\left(\tau_Z + \tau_u\right)\rho^2 - 2\gamma\left(\tau_Z + \tau_u\right)\rho^2 \\
= \; & \left(\gamma - 2\tau_u\rho\right)\tau_X - \gamma\left(\tau_Z + \tau_u\right)\rho^2.
\end{aligned}$$

For ρ sufficiently close to $\frac{\gamma}{2\tau_u}$, which corresponds to τ_ϵ being sufficiently close to $\frac{\gamma^2}{4\lambda_I\tau_u}$, the above expression is indeed negative, completing the proof. ■

Proof of Proposition 3. As with the proof of Proposition 1, see Ganguli and Yang (2009) and Manzano and Vives (2011). The proof of (29) is the same as for Lemma 1. ■

Proof of Proposition 4. Given Lemma A-3, each informed trader i chooses signal precisions τ_{im}, τ_{is} to solve

$$\max_{\tilde{\tau}_{im},\tilde{\tau}_{is}} -\sqrt{\frac{\tau_x + \rho_m^2(\tau_z + \tau_u)}{\tau_x + \rho_m^2(\tau_z + \tau_u) + \tilde{\tau}_{im}}}\sqrt{\frac{\tau_x + \rho_s^2(\tau_z + \tau_u)}{\tau_x + \rho_s^2(\tau_z + \tau_u) + \tilde{\tau}_{is}}}\exp\left(\gamma\kappa\left(\tilde{\tau}_{im}, \tilde{\tau}_{is}\right)\right) \quad \text{(A-16)}$$

The first-order conditions of (A-16) are (34) and (35). We note that the Hessian is positive definite, so there is a unique pair (τ_{im}, τ_{is}) that solves (A-16).[12] Let this unique pair be denoted by $(\tau_{im}, \tau_{is}) = (g_m(\rho_m, \rho_s), g_s(\rho_m, \rho_s))$.

Lemma A-4. *The functions $g_m(\rho_m, \rho_s)$ and $g_s(\rho_m, \rho_s)$ satisfy:*

$$\frac{\partial g_m}{\partial \rho_m} < 0, \quad \frac{\partial g_m}{\partial \rho_s} > 0, \quad \frac{\partial g_s}{\partial \rho_s} < 0, \quad \frac{\partial g_s}{\partial \rho_m} > 0.$$

Proof of Lemma A-4. First, we establish $\frac{\partial g_m}{\partial \rho_m} < 0$. Suppose to the contrary that $\frac{\partial g_m}{\partial \rho_m} \geq 0$. Hence there exist ρ_s, ρ_m and $\tilde{\rho}_m > \rho_m$ such that $g_m\left(\tilde{\rho}_m, \rho_s\right) \geq g_m\left(\rho_m, \rho_s\right)$. From the first-order condition (34), it follows that $\kappa_m\left(g_m\left(\tilde{\rho}_m, \rho_s\right), g_s\left(\tilde{\rho}_m, \rho_s\right)\right) < \kappa_m\left(g_m\left(\rho_m, \rho_s\right), g_s\left(\rho_m, \rho_s\right)\right)$. Hence $g_s\left(\tilde{\rho}_m, \rho_s\right) < g_s\left(\rho_m, \rho_s\right)$. On the one hand, Assumption 1 implies $\kappa_s\left(g_m\left(\tilde{\rho}_m, \rho_s\right), g_s\left(\tilde{\rho}_m, \rho_s\right)\right) \leq \kappa_s\left(g_m\left(\rho_m, \rho_s\right), g_s\left(\rho_m, \rho_s\right)\right)$. On the other hand, (35) implies $\kappa_s\left(g_m\left(\tilde{\rho}_m, \rho_s\right), g_s\left(\tilde{\rho}_m, \rho_s\right)\right) > \kappa_s\left(g_m\left(\rho_m, \rho_s\right), g_s\left(\rho_m, \rho_s\right)\right)$. The contradiction establishes the result.

[12] The Hessian is given by

$$2\begin{pmatrix} \gamma\kappa_{mm}\left(\tau_{im}, \tau_{is}\right) + \frac{\tau_x + \rho_m^2(\tau_z + \tau_u)}{(\tau_x + \rho_m^2(\tau_z + \tau_u) + \tau_{im})^3} & \gamma\kappa_{ms}\left(\tau_{im}, \tau_{is}\right) \\ \gamma\kappa_{ms}\left(\tau_{im}, \tau_{is}\right) & \gamma\kappa_{ss}\left(\tau_{im}, \tau_{is}\right) + \frac{\tau_x + \rho_s^2(\tau_z + \tau_u)}{(\tau_x + \rho_s^2(\tau_z + \tau_u) + \tau_{is})^3} \end{pmatrix},$$

which given Assumption 1 is indeed positive definite.

Second, we establish $\frac{\partial g_m}{\partial \rho_s} > 0$. Suppose to the contrary that $\frac{\partial g_m}{\partial \rho_s} \leq 0$. Hence there exist ρ_m, ρ_s and $\tilde{\rho}_s > \rho_s$ such that $g_m\left(\rho_m, \tilde{\rho}_s\right) \leq g_m\left(\rho_m, \rho_s\right)$. From (34), we know $\kappa_m\left(g_m\left(\rho_m, \tilde{\rho}_s\right), g_s\left(\rho_m, \tilde{\rho}_s\right)\right) \geq \kappa_m\left(g_m\left(\rho_m, \rho_s\right), g_s\left(\rho_m, \rho_s\right)\right)$. Hence $g_s\left(\rho_m, \tilde{\rho}_s\right) \geq g_s\left(\rho_m, \rho_s\right)$, and from Assumption 1, $\kappa_s\left(g_m\left(\rho_m, \tilde{\rho}_s\right), g_s\left(\rho_m,\right.\right.$ $\kappa_s\left(g_m\left(\rho_m, \rho_s\right), g_s\left(\rho_m, \rho_s\right)\right)$. From (35), it follows that $g_s\left(\rho_m, \tilde{\rho}_s\right) < g_s\left(\rho_m, \rho_s\right)$, a contradiction, establishing the result.

The remaining two results follow by analogous arguments. ■

In order to prove uniqueness and existence, first define

$$\bar{\tau}_{\epsilon m} = \frac{\tau_u}{\lambda_I}\left(\frac{\gamma}{2\tau_u}\right)^2$$

$$\bar{\tau}_{\epsilon s} = \frac{\tau_u}{(1-\eta)\lambda_I}\left(\frac{\gamma}{2\tau_u}\right)^2$$

For any $\tau_{\epsilon m} \in [0, \bar{\tau}_{\epsilon m}]$, let $f_m(\tau_{\epsilon m})$ denote the price informativeness parameter in the market portfolio, i.e.

$$f_m(\tau_{\epsilon m}) = \frac{\gamma}{2\tau_u} - \sqrt{\left(\frac{\gamma}{2\tau_u}\right)^2 - \lambda_I \frac{\tau_{\epsilon m}}{\tau_u}}.$$

The function f_m is strictly increasing in $\tau_{\epsilon m}$, and it satisfies $f_m(0) = 0$ and $f_m(\bar{\tau}_{\epsilon m}) = \frac{\gamma}{2\tau_u}$. Similarly, let $f_s(\tau_{\epsilon m}; \eta)$ denote the price informativeness parameter in the spread asset, i.e.

$$f_s(\tau_{\epsilon s}; \eta) = \frac{\gamma}{2\tau_u} - \sqrt{\left(\frac{\gamma}{2\tau_u}\right)^2 - \frac{\lambda_I}{(1-\eta)} \frac{\tau_{\epsilon s}}{\tau_u}}.$$

The function f_s is strictly increasing in $\tau_{\epsilon s}$, and it satisfies $f_s(0; \eta) = 0$ and $f_s(\bar{\tau}_{\epsilon s}; \eta) = \frac{\gamma}{2\tau_u}$ for all $\eta \in [0, 1)$. Furthermore, the function f_s is strictly increasing in η for all $\tau_{\epsilon s} \in (0, \bar{\tau}_{\epsilon s})$.

With the above notation, an equilibrium exists if there is a fixed point to the mapping

$$(f_m(g_m(\cdot,\cdot)), f_s(g_s(\cdot,\cdot))) : \left[0, \frac{\gamma}{2\tau_u}\right]^2 \to \left[0, \frac{\gamma}{2\tau_u}\right]^2 \tag{A-17}$$

For sufficiently high information production costs, it is clear that $g_m(\rho_m, \rho_s) < \bar{\tau}_{\epsilon m}$, $g_s(\rho_m, \rho_s) < \bar{\tau}_{\epsilon s}$, for all $(\rho_m, \rho_s) \in \left[0, \frac{\gamma}{2\tau_u}\right]^2$. Existence follows from Brouwer's fixed point theorem.

In order to proof uniqueness, we proceed by contradiction. Suppose that (ρ_m^*, ρ_s^*) and (ρ_m', ρ_s') both satisfy A-17. From Lemma A-4 we have that both $\rho_m^* \neq \rho_m'$ and $\rho_s^* \neq \rho_s'$. Assume, without loss of generality, that $\rho_m' > \rho_m^*$, so that $g_m(\rho_m', \rho_s') > g_m(\rho_m^*, \rho_s^*)$. From

(34) we have that $\kappa_m(g_m(\rho'_m, \rho'_s), g_s(\rho'_m, \rho'_s)) < \kappa_m(g_m(\rho^*_m, \rho^*_s), g_s(\rho^*_m, \rho^*_s))$. This implies that $g_s(\rho'_m, \rho'_s) < g_s(\rho^*_m, \rho^*_s)$, so that $\rho'_s < \rho^*_s$. From (35) we conclude that $\kappa_s(g_m(\rho'_m, \rho'_s), g_s(\rho'_m, \rho'_s)) > \kappa_s(g_m(\rho^*_m, \rho^*_s), g_s(\rho^*_m, \rho^*_s))$. On the other hand, Assumption 1 implies $\kappa_s(g_m(\rho'_m, \rho'_s), g_s(\rho'_m, \rho'_s)) \leq \kappa_s(g_m(\rho^*_m, \rho^*_s), g_s(\rho^*_m, \rho^*_s))$, which is a contradiction. This completes the proof. ■

Proof of Proposition 5. For any $\rho_m \in \left[0, \frac{\gamma}{2\tau_u}\right]$, define $\rho_s(\rho_m; \eta)$ as the solution to $\rho_s = f_s(g_s(\rho_m, \rho_s); \eta)$. From Lemma A-4 we have that $f_s(g_s(\rho_m, \rho_s); \eta)$ is strictly decreasing in ρ_s. Furthermore, for sufficiently high information production costs, we have $f_s(g_s(\rho_m, 0); \eta) < \frac{\gamma}{2\tau_u}$. The function $\rho_s(\rho_m; \eta)$ is therefore uniquely defined. The function $\rho_s(\rho_m; \eta)$ is also increasing in η and ρ_m.

In equilibrium we have

$$\rho^*_m = f_m(g_m(\rho^*_m, \rho_s(\rho^*_m; \eta^*))).$$

From the uniqueness result of Proposition 4 we have that for all $\rho_m \in [0, \rho^*_m)$

$$f_m(g_m(\rho_m, \rho_s(\rho_m; \eta^*))) - \rho_m > 0.$$

From Lemma A-4 we have

$$f_m(g_m(\rho_m, \rho_s(\rho_m; \eta'))) - \rho_m > 0 \geq f_m(g_m(\rho_m, \rho_s(\rho_m; \eta^*))) - \rho_m$$

with strict inequality if $\rho_s(\rho_m; \eta) > 0$. It follows that $\rho'_m \geq \rho^*_m$, with strict inequality if $\rho^*_m > 0$, and that $\rho'_s \geq \rho^*_s$. ■

Proof of Corollary 2. By symmetry, is suffices to establish the result for asset 1. The information content of observing prices is the same as observing $(X_m - \rho_m^{-1} Z_m, X_s - \rho_s^{-1} Z_s)$. Defining

$$\begin{aligned}
\Sigma_{22} &= \begin{pmatrix} \operatorname{var}(X_m) + \rho_m^{-2} \operatorname{var}(Z_m) & 0 \\ 0 & \operatorname{var}(X_s) + \rho_s^{-2} \operatorname{var}(Z_s) \end{pmatrix} \\
\Sigma_{12} &= \begin{pmatrix} \frac{1}{\sqrt{2}} \operatorname{var}(X_1) & \frac{1}{\sqrt{2}} \operatorname{var}(X_1) \end{pmatrix},
\end{aligned}$$

we have

$$\begin{aligned}
\operatorname{var}(X_1 | P_m, P_s) &= \operatorname{var}(X_1) - \Sigma_{12} \Sigma_{22}^{-1} \Sigma'_{12} \\
&= \operatorname{var}(X_1) \\
&- \frac{1}{2} \operatorname{var}(X_1)^2 \left(\left(\operatorname{var}(X_m) + \rho_m^{-2} \operatorname{var}(Z_m) \right)^{-1} + \left(\operatorname{var}(X_s) + \rho_s^{-2} \operatorname{var}(Z_s) \right)^{-1} \right).
\end{aligned}$$

The result then follows from Proposition 5. ■

Proof of Proposition 6. From Proposition 3, we know

$$\tau_u \rho_s^2 - \gamma \rho_s + \frac{\lambda_I}{1-\eta} \tau_{\epsilon s}, \tag{A-18}$$

and hence

$$1 - b_s = \frac{\tau_x}{\tau_x + \rho_s^2 \tau_z + \gamma \rho_s}; \qquad \frac{b_s}{\rho_s} = \frac{\rho_s \tau_z + \gamma}{\tau_x + \rho_s^2 \tau_z + \gamma \rho_s}. \tag{A-19}$$

Hence the proof of Proposition 2 that shows that uninformed welfare is decreasing in price efficiency applies to both the market and spread assets. Consequently, Proposition 5 implies that an increase in indexing (η) reduces the welfare of all other uninformed investors, regardless of whether or not they are indexing.

Finally, we show that informed welfare also drops. To do so, let $V(\rho_m, \rho_s, \tau_{im}, \tau_{is})$ be an informed trader's expected utility from information acquisition decisions τ_{im}, τ_{is} when price efficiency in the two assets is ρ_m, ρ_s. Observe that V is decreasing in its first two arguments ρ_m, ρ_s: this follows from the just-established result that uninformed welfare is decreasing in ρ_m, ρ_s, together with the fact that the ratio of informed to uninformed welfare is given by the negative of the objective in (A-16).

Next, consider two indexing levels, η and $\eta' > \eta$. Write $\rho_m, \rho_s, \tau_{\epsilon m}, \tau_{\epsilon s}$ for price efficiency and equilibrium information acquisition decisions when indexing is η; and similarly, write $\rho'_m, \rho'_s, \tau'_{\epsilon m}, \tau'_{\epsilon s}$ for price efficiency and equilibrium information acquisition decisions when indexing is η'. From Proposition 5, we know $\rho'_m > \rho_m$ and $\rho'_s > \rho_s$.

Suppose that, contrary to the claimed result,

$$V(\rho'_m, \rho'_s, \tau'_{\epsilon m}, \tau'_{\epsilon s}) > V(\rho_m, \rho_s, \tau_{\epsilon m}, \tau_{\epsilon s}). \tag{A-20}$$

Individual optimality certainly implies

$$V(\rho_m, \rho_s, \tau_{\epsilon m}, \tau_{\epsilon s}) \geq V(\rho_m, \rho_s, \tau'_{\epsilon m}, \tau'_{\epsilon s}).$$

Since V is decreasing in its first two arguments,

$$V(\rho_m, \rho_s, \tau'_{\epsilon m}, \tau'_{\epsilon s}) \geq V(\rho'_m, \rho'_s, \tau'_{\epsilon m}, \tau'_{\epsilon s}).$$

This sequence of three inequalities delivers a contradiction, completing the proof. ■

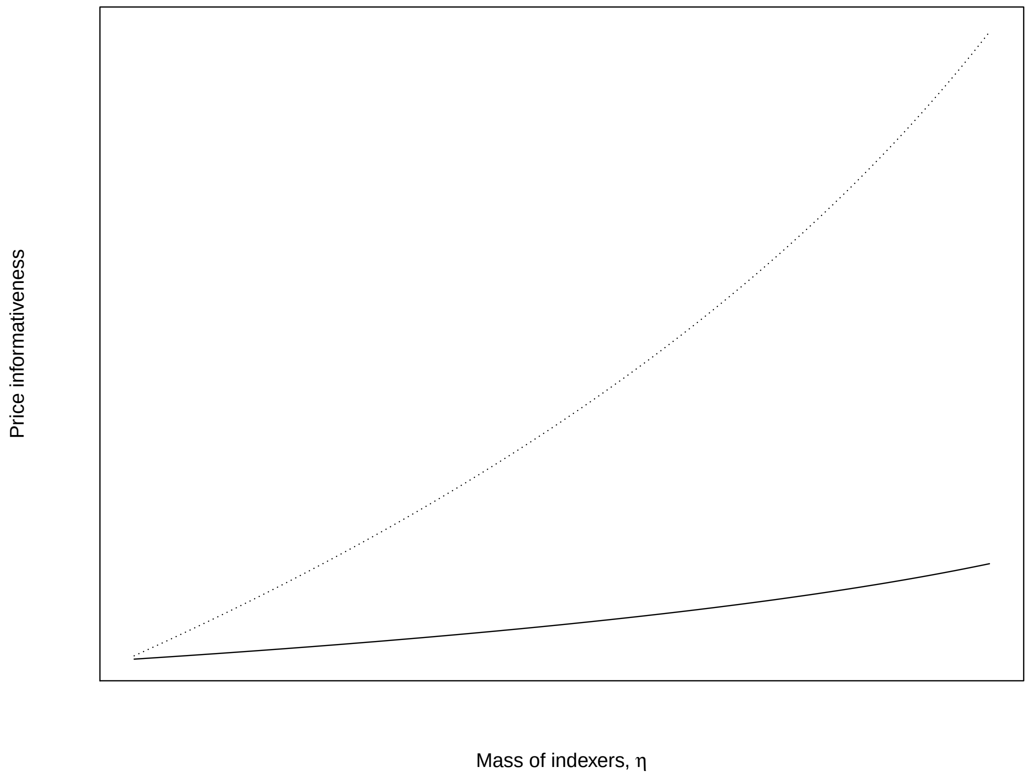

Figure 1: The graphs give the relationship between the price informativeness, $\mathrm{var}(X_j|P_m, P_s)$, for $j = m, s$, and the amount of indexers in our model, η. The solid line depicts the informativeness of the market asset, while the dotted line presents the informativeness of the spread asset.

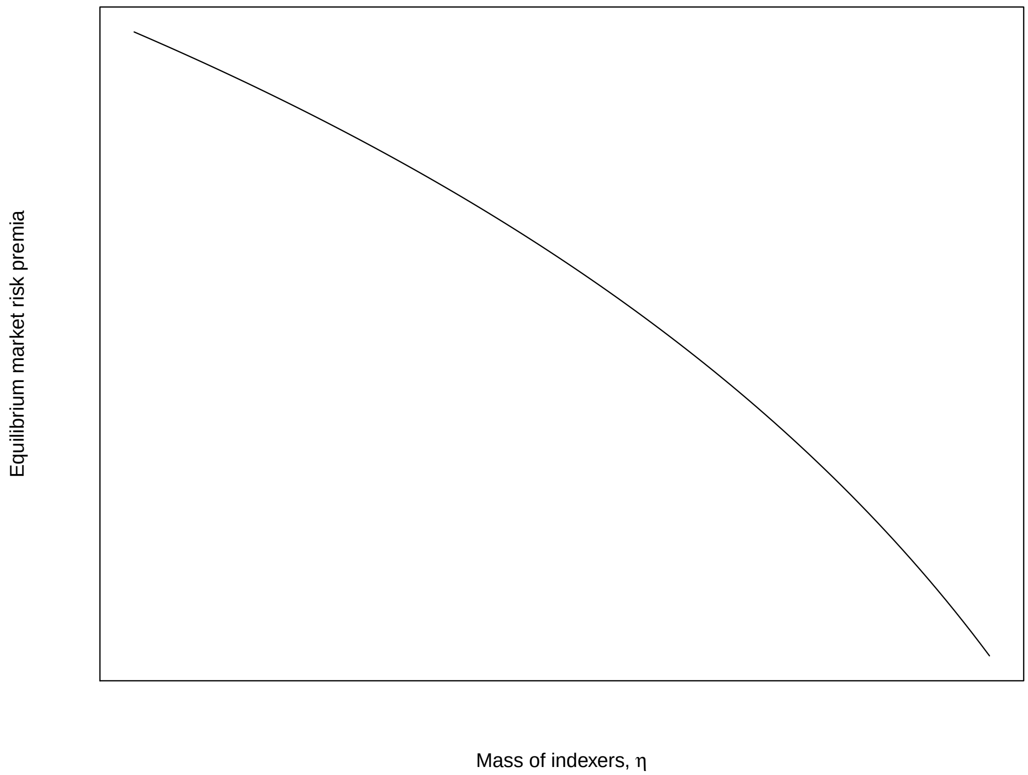

Figure 2: The graph gives the relationship between the equilibrium market risk premium in our model and the amount of indexers, η.

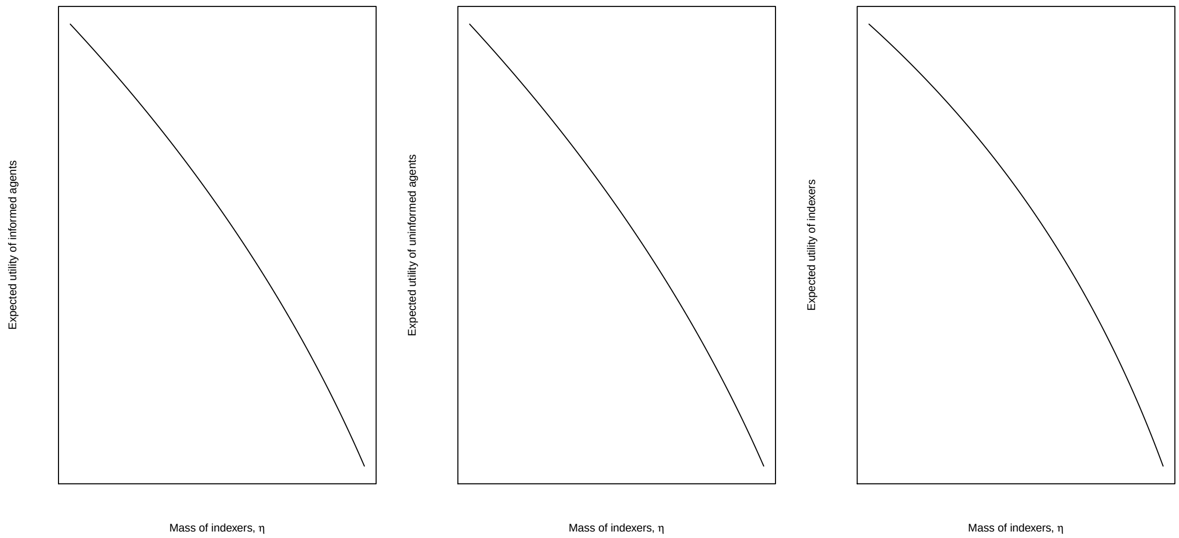

Figure 3: The graph gives the relationship between the expected utility of informed agents (left panel), uninformed active agents (middle panel), and indexers (right panel) as as function of the amount of indexers, η.

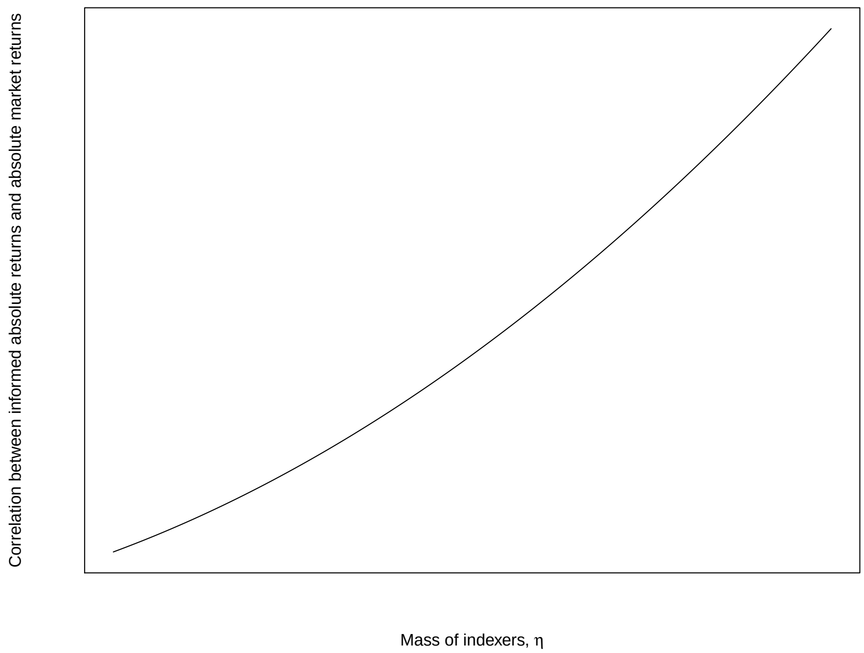

Figure 4: The graph plots the relationship between the correlation of the absolute returns of informed traders and the market absolute returns, as as function of the amount of indexers, η.