Corporate Risk Management: Integrating Liquidity, Hedging, and Operating Policies*

Andrea Gamba† Alexander J. Triantis‡

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ABSTRACT

We present a dynamic structural model of integrated risk management that incorporates several motivations for managing risk. Risk management is enabled through a coordination of operating flexibility, liquidity management, and derivatives hedging policies. We analyze the value created by such integrated risk management strategies, and disintegrate this value in several ways to separate out the marginal impacts of specific frictions and of different risk management solutions. We highlight the importance of distress costs as well as a convexity due to personal taxes on equity income. We show that liquidity serves a critical role in risk management, providing a rationalization for seemingly high levels of cash reserves. The value attributable to derivatives usage does not appear to be significant in the presence of other risk management mechanisms, though we identify circumstances where this value is larger, thus helping to resolve conflicting empirical evidence on this issue. We examine why a significant portion of the value loss due to frictions in the presence of uncertainty still remains even under the integrated risk management strategy we employ. Finally, we evaluate the net impact of risk management policies in the presence of financial agency problems that distort these policies.

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†Warwick Business School, University of Warwick, Scarman Road, Coventry, CV4 7AL, UK. Tel: +44 (0) 024 7652 4542. email: andrea.gamba@wbs.ac.uk
‡Robert H. Smith School of Business, University of Maryland, College Park, MD, USA. Tel: +1 301 405 2246. email: atriantis@rhsmith.umd.edu
Introduction

Risk management has become a critical dimension of corporate financial policy, particularly in light of recent global financial and economic crises. It is widely appreciated that companies can enhance their values if they are able to mitigate some of the costly impacts of risk. The corporate risk management literature focuses on the potential benefits of hedging with financial derivatives, addressing questions of why companies do (or should) use derivatives and under what circumstances value is created through such use. A separate strand of the corporate finance literature analyzes the rationale and benefit of cash holdings, including understanding the precautionary motive for liquidity in the presence of uncertainty. Yet another branch of corporate finance explores the value of operating flexibility, such as real options to shut down facilities or switch modes of production.

In practice, all three of these mechanisms are commonly used to manage risk, and it is thus important to understand how they can be coordinated in an integrated fashion, and what the relative contribution of each is in the presence of the others. In this paper, we provide some new insights on corporate risk management from a normative perspective through a dynamic model that incorporates liquidity, hedging and operating policies. The key contributions of this paper are briefly outlined below.

First, we examine the simultaneous impact of several common motivations for risk management, and separate out the relative impact of each in the presence of the others. While these rationales for managing risk have been examined elsewhere in the risk management literature, each has been typically studied in isolation. We specifically focus on the reduction of expected tax payments (Graham and Smith (1999), Graham and Rogers
(2002)), the avoidance of external financing costs (Froot, Scharfstein, and Stein (1993)), and the mitigation of financial distress and default costs (Smith and Stulz (1985)).

Our results indicate that avoiding distress costs is the strongest motivation to create value through risk management. The recent survey evidence from Lins, Servaes, and Tufano (2010) indicates that managers view the most important role of cash as a buffer against future cash flow shortfalls, supporting our finding. The empirical evidence on firms using derivatives to avoid financial distress is somewhat more mixed, as shown most comprehensively in Table 5 of Aretz and Bartram (2010), which captures the results of more than twenty-five studies that have examined this issue using around twenty different distress proxies. As Guay and Kothari (2003) conclude from their empirical study, financial hedging may be used only to fine-tune a risk management strategy that is otherwise implemented using operational and other strategies, and this is particularly the case for distress cost avoidance as we argue below.

Second, we examine the impact of personal taxes in the presence of uncertainty as a motivation for risk management. Even if there are no direct costs of issuing equity, there is an indirect cost to tapping equity markets since shareholders receive payouts net of personal tax, while they provide capital at gross value. This asymmetry, a type of personal tax convexity, can impact value significantly if income variability is high and outside equity must be frequently raised. This additional motivation for risk management does not appear to have surfaced in the existing literature.

Third, by allowing for operating flexibility, corporate liquidity, and derivatives hedging, our structural model provides insights on the marginal benefits of each risk man-

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1To bound the complexity of our framework, we do not incorporate all potential rationales for risk management, such as managerial performance measurement (DeMarzo and Duffie (1991) and Breeden and Viswanathan (1996)), or particular concave operating value functions (MacKay and Moeller (2007)). While we do examine financial agency problems later in the paper, we do not incorporate managerial agency problems, as explored in Han (1996), Tufano (1996) and Geczy, Minton, and Schrand (2007).
agement mechanism when others are present, and when there are costs associated with each type of activity (an endogenously determined tax penalty to holding cash, transactions costs associated with a dynamic hedging program, and adjustment costs associated with managing operating flexibility). We show that while there is some substitutability between liquidity, hedging, and operating flexibility as risk management mechanisms, these approaches are by no means redundant and must be coordinated to maximize value creation.

The presence of alternative forms of risk management in our model also helps to explain the conflicting evidence surrounding whether (and how much) value is created through the use of derivatives by corporations. Based on a set of calibrated simulations, we estimate that the marginal value of hedging with derivatives, taking into account the presence of alternative risk management mechanisms, is likely to be no more than 2% on average. This is consistent with the lack of evidence of any value creation attributable to hedging in several recent studies (e.g., Bartram, Brown, and Conrad (2009), Guay and Kothari (2003), and Jin and Jorion (2006)). However, we find that for specific firms that may face a key exposure that can be more effectively hedged, the value creation from hedging with derivatives could be significantly higher, as found by Haushalter (2000) for the oil and gas industry, Allayannis and Weston (2001) for exporters, and Carter, Rogers, and Simkins (2006) for airlines.

Fourth, our results underscore that liquidity is an important, and in many circumstances the most effective, risk management mechanism. The value created through liquidity not only comes as a result of avoiding high costs of financial distress when a firm’s risk cannot be completely hedged using derivatives, but also from helping to cover extraordinary costs (such as costly restructuring of operations) that, while tied to operating profitability, may not be easily hedged through standard derivatives contracts.
These results support the importance of liquidity cushions during the recent financial crisis, where the costs of external financing spiked upward due to systemic financial distress, and some firms simultaneously faced significant restructuring costs.

We thus provide theoretical support for the recent empirical study of Bates, Kahle, and Stulz (2009), which documents high levels of cash held by firms, particularly during recent years. They conclude that the high cash levels reflect the precautionary savings motive, rather than a managerial agency motive. Our model is designed to highlight this precautionary savings motive, particularly due to negative cash flow surprises. While the precautionary motive described by Keynes (1936) is frequently viewed more generally to include the ability to fund profitable investment opportunities in the face of adverse cash flow shocks, we use a more narrow definition focused on avoiding distress costs. This is consistent with managerial practice documented by Lins, Servaes, and Tufano (2010), who find that firms treat cash holdings as realized liquidity held as a buffer against future cash shortfalls, while lines of credit are considered to be options on liquidity available to fund future investment opportunities (Sufi (2009) documents that lines of credit average around 15% of the book value of assets).

Combining liquidity, hedging and operating flexibility in an integrated risk management strategy significantly increases firm value, certainly well beyond that from simply using derivatives. Yet, we show that even this integrated strategy falls short of the potential value creation from perfect risk management, i.e. avoiding the effects of all frictions in our model. This result underscores that firms that limit their scope of risk management activities may be undervalued relative to their potential. While there exists limited evidence on the value creation from comprehensive Enterprise Risk Management (ERM) programs, in a recent empirical analysis of U.S. insurers, Hoyt and Lieenberg
(2010) estimate an average value premium of 16.5% for insurers that have implemented such programs, clearly well beyond the value created from derivatives hedging programs.

Finally, we provide a partial analysis of agency problems in our setting. While agency problems are frequently cited as the rationale for managing risk (e.g., Bessembinder (1991), the strategies employed to mitigate risk may themselves be subject to agency problems. For instance, cash may be hoarded by managers who subsequently inefficiently invest it (Jensen (1986), Harford (1999), and Harford, Mansi, Maxwell (2004)), the flexibility inherent in some real assets may be inappropriately exploited (Mello, Parsons, and Triantis (1995)), and derivatives may be used for speculation rather than hedging (Allayannis, Brown, and Klapper (2003), Adam and Fernando (2006), Brown, Crabb, and Haushalter (2006), Faulkender (2005), Geczy, Minton, and Schrand (2007), Guay (1999), and Hentschel and Kothari (2001)). We determine the net effect of second best risk management strategies under equity value maximization, and highlight that limits on derivatives positions are critical to avoiding considerable value destruction, particularly if the firm doesn’t have sufficient liquidity and the hedging instruments are not highly correlated with the firm’s overall risk exposure.

The notion of integrated risk management is certainly not new. Shapiro and Titman (1986) point out that there may be more than one type of mechanism to manage risk. Meulbroek (2001) and Meulbroek (2002) discuss the importance of integrating various risks in an organization and coordinating alternative ways of managing the resulting net exposure. However, there has been limited work to date to develop a robust theoretical framework to analyze this issue. With few exceptions that we discuss below, studies that have considered multiple mechanisms to manage risk have typically focused on just two of the three we examine, and have typically done so in an empirical context, often by simply including an extra regression variable. We mention some of these efforts below.
The interaction between operating flexibility and hedging has been highlighted in several empirical studies, such as Allayannis, Ihrig, and Weston (2001), Bartram (2008), Bartram, Brown, and Fehle (2009), Hankins (2009), Lin, Phillips, and Smith (2008), Morellec and Smith (1988), Pantzalis, Simkins, and Laux (2001), and Petersen and Thiagarajan (2000), all of whom document a negative relation between these forms of risk management. Mello, Parsons, and Triantis (1995) construct a model to illustrate how operating flexibility and hedging interact as substitute risk management mechanisms, but also point out that there can be an element of complementarity between them. Operating flexibility changes the risk profile of profits, and in so doing also increases firm value. This presents an opportunity to increase the level of debt, and additional hedging may be required to support the higher debt and fully capitalize on the interest tax shields.

Liquidity is frequently thrown in as a regression variable in empirical studies on hedging, and the evidence suggests that users of derivatives exhibit lower short-term liquidity than those without derivatives (e.g., Bartram, Brown, and Fehle (2009), Allayannis, Brown, and Klapper (2003); Geczy, Minton, and Schrand (1997); Tufano (1996)). More recently, Allayannis and Schill (2010) examine the relationship between liquidity and hedging, as well as payout and leverage policies, and find a positive association between “conservative” policies and firm value. Since the use of derivatives is typically treated as a binary variable (or even indirectly proxied when disclosure data on derivatives is not available), there has been no direct effort to carefully analyze the interaction between liquidity and hedging as we do in our model. While Mello and Parsons (2000) provide a dynamic model that combines liquidity and hedging, cash is not a control variable as it is in our model. Rather, it accumulates or decreases based on firm profitability over time. Their key contributions are to emphasize that risk management policies that
decrease the volatility of firm value or cash flow will likely not maximize firm value, and that the marking-to-market feature of futures can result in an amplification of risk through hedging, and as a result potential value loss from risk management if hedges are not properly designed.

The interaction between operating flexibility and liquidity has received more limited attention. While empirical analyses of liquidity typically include some characteristics of a firm’s assets, such as tangibility, which may be related to flexibility, the interaction of operating flexibility with a firm’s cash management policy has not been carefully explored. The emphasis in the liquidity literature has been more directed towards understanding the role of liquidity in alleviating investment distortions (e.g., Acharya, Almeida, and Campello (2007) and Almeida, Campello, and Weisbach (2004)). Two recent studies by Bartram, Brown, and Conrad (2009) and Duchin (2010) examine the effect of corporate diversification, a type of operating flexibility, on cash holdings, and find them to be negatively related.

A recent article by Bolton, Chen, and Wang (2011) provides a dynamic model that integrates liquidity, hedging, and real asset flexibility, and thus comes closest to the model we present. They examine the relationship between investment policy and marginal q in the presence of financial constraints, and relate financing and payout to the firm’s cash/capital ratio. They also highlight the interrelated roles that derivatives hedging and liquidity management play in corporate risk management, which is one of the contributions of our paper, as noted above. Our models, however, differ in several ways, making our results complementary. In our model, operating losses can be mitigated by a restructuring of operations which is costly, while in their model partial liquidation of assets occurs in such states, producing additional cash flow. Our model captures the joint impact of corporate and personal taxes on the indirect costs of carrying cash and
issuing equity, while they impose exogenous costs of carrying cash (which is constant) and issuing equity (proportional to asset size). In terms of the impact of hedging on cash holdings, we follow the convention for over-the-counter derivatives trading that contracts are marked to market when they are closed out, and are subject to default and costly to renegotiate, consistent with Fehle and Tsyplakov (2005). In contrast, Bolton, Chen, and Wang (2011) assume market index futures are used which are subject to cash margin requirements and are thus presumably riskless. They illustrate their results assuming this contract has a very high correlation with the firm’s overall risk, while we examine a wide range of correlations which allows us to draw broader conclusions about the interplay between hedging and liquidity in practice.

The next section provides a simple example to illustrate some of our key results. Section II presents a more formal general model. Section III provides our results on the effects of different drivers of risk management, the relative impact of liquidity and hedging on the value attributable to risk management, the contribution of operating flexibility to corporate risk management, and the effects of financial agency problems on risk management. Section IV concludes the paper.

I. A Simple Example

To illustrate the essence of our results in the simplest way possible, we construct a one-period numerical example with some of the key features of our general model. An unlevered firm can produce one unit of output subject to a cost of $10, and with revenue $X$ of either $14$ or $8$, with equal probability. If the resulting cash flow is negative, the firm faces a distress cost equal to $20\%$ of the deficit. We assume a zero discount rate. The value of the firm is thus equal to 

\[ 0.5(14-10) + 0.5(8-10) - 0.2(8-10) = 2 - 1 - 0.2 = 0.8, \]

\]
where the expected distress cost of $0.2 represents a loss of 20% of the firm value in the absence of such frictions (i.e., the first-best value of the firm).

If the firm has access to a hedging instrument, it can increase firm value by eliminating or mitigating the distress costs. Assume first that there is a swap contract on an underlying variable $Y$ which is perfectly correlated with the product price, or most simply, $Y = X$. The swap price is $\mathbb{E}(Y) = $11, and the firm will take a short position so that for one unit of the swap contract it would receive $3 when $Y = 8$ and would pay $3$ when $Y = 14$. The firm can enter into a fraction of a unit of the short swap position, as measured by $h \in [0, 1]$. As long as $h \in [2/3, 1]$, the profit on the swap will offset (or more than offset) the loss of $2$ from production in the low state, and thus there will be no distress cost and firm value equals $1$.

Since in practice a firm’s risk will never be fully hedged, we now explore the possibility that the correlation between $X$ and $Y$, $\rho_{XY}$, is less than one. We examine the cases where $\rho_{XY}$ can equal $0, .2$, or $.6$ (as well as the perfectly correlated case). These correspond to different state probabilities for the four states of $(X,Y)$, as shown in Table I.\(^2\)

The payoffs in each state are also shown in Table I. In the first two states, the firm will never have a negative cash flow for $h \in [0,1]$. In the third state where the output price is low and the swap’s underlying value is high, the hedge works particularly poorly and thus a large distress cost is incurred, and increases with the size of the hedge. In the fourth state, the short swap position can produce a cash flow that more than offsets

\(^2\)In practice, rather than there being a swap on an underlying instrument which is less than perfectly correlated with the overall risk of the firm, it is likely that there are derivative contracts which are near perfectly correlated with certain risks that the firm faces, but a lack of derivative instruments to hedge other corporate risks. One can transform the framework we present into such a setting. For instance, the firm’s revenue may be decomposed into $X = .5Z + .5Y$, where $Z$ and $Y$ are independent, and $Y$ can be perfectly hedged with the swap while $Z$ is not hedgeable. In this case, the values for $Z$ would be $14, 20, 2, 8$ in states $1, 2, 3, 4$ respectively so as to get the correspondence between $X$ and $Y$ shown in Table I.
the operating loss as long as $h > 2/3$. A hedge ratio higher than $2/3$ will exacerbate the losses in the third state without any further benefit in the fourth state, so in this bivariate binomial example, the firm will choose an optimal $h^* = 2/3$, unless $\rho_{XY} = 1$, in which case any hedge ratio between $2/3$ and 1 maximizes firm value. The optimal hedge ratios are shown in Table II, together with the resulting expected loss. When $\rho_{XY} = 0$, there is no benefit to hedging (a flat value surface for $h \in [0, 2/3]$ and then decreasing for higher $h$), and thus there is a loss of $20\%$ of firm value due to the expected distress cost. This loss is reduced by $2\%$ for each $.1$ increase in $\rho_{XY}$, and goes to zero in the case of the perfectly correlated swap.

Table II: Optimal hedge policy and impact on expected distress costs. The table shows the optimal hedge ($h^*$) and resulting expected distress cost ($E[\text{loss}]$) assuming either firm value maximization or cash flow variance minimization, under different correlation values.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$h^*$</th>
<th>$E[\text{loss}]$</th>
<th>$\rho$</th>
<th>$h^*$</th>
<th>$E[\text{loss}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$[0, 2/3]$</td>
<td>.20</td>
<td>0</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td>.2</td>
<td>$2/3$</td>
<td>.16</td>
<td>.2</td>
<td>.188</td>
<td></td>
</tr>
<tr>
<td>.6</td>
<td>$2/3$</td>
<td>.08</td>
<td>.6</td>
<td>.092</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$[2/3, 1]$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Table II also shows what the hedge ratio would be if the objective were to minimize
the variance of the payoff. In this case, \( h^* = \rho_{XY} \), which is lower than the optimal hedge
under firm value maximization. The minimum variance hedge ratio is thus suboptimal,
and the resulting value loss is correspondingly higher.

Now, consider the case where the firm has operating flexibility that allows it to shut
down production in the low output price state before incurring any production costs,
thus mitigating any operating loss. However, there are switching costs of $1 associated
with exercising this shut down option, so there is still a loss of $1 in the low price states,
which would trigger an expected distress cost of $.10. The firm can reduce the expected
distress costs by hedging with the swap. Specifically, the firm should select \( h^* = 1/3 \). If
\( \rho_{XY} = 1 \), there would be no distress cost. For lower \( \rho_{XY} = 1 \) values, there would still be
expected distress costs due to positive probability of state 3 occurring, where \( X \) is low
and \( Y \) is simultaneously high. However, given that the potential loss is $1, rather than
$2, in the presence of operating flexibility, the expected distress costs will be half of the
expected value loss shown in Table II. Note that the presence of operating flexibility
not only mitigates distress costs, but also significantly increases firm value. Since the
firm can decrease the operating loss when \( X = 8 \) from $2 to $1, the firm value increases
by .5. Other forms of risk management can reduce the impact of frictions, but can not
raise the overall value of the firm in the same way.

Finally, consider the possibility of the firm holding cash as a precautionary measure
to mitigate distress costs triggered in the low output price states. However, there is a
cost to holding cash, potentially due to the structure of the tax code, and we assume this
cost is 2% of the amount of cash held. If the firm holds $2.04 of cash, then it can stave
off any distress costs (the cash net of the cost of holding cash will offset the operating
loss of $2 in the low output price states). The value loss in this case is simply the cost
of holding cash, which is $.04, thus reducing the loss due to the combination of frictions and uncertainty from 20% of firm value down to only 4%. As can be seen by the linear relationship between expected loss and correlation in Table II, only if $\rho_{XY} > .8$ in our example would the use of hedging dominate holding cash as a risk management tool. Thus, in most practical cases where significant parts of firm risk can not be hedged, cash will emerge as a critical element of the risk management policy.

Of course, it is possible to combine multiple risk management mechanisms. If the firm is able to shut down production, and incur only the switching cost of $1 in the low price states, if the firm also held $1.02 of cash, then distress costs would be avoided, and the net value loss is simply the $.02 cost of holding cash, lower than the $.04 in the absence of operating flexibility. In the absence of operating flexibility, the firm could combine hedging together with holding cash. The benefit of hedging is that the firm could potentially offset some of the operating loss at no cost, and thus perhaps hold less cash and avoid the associated carrying cost. However, in our example, unless $\rho_{XY} \geq .8$, the benefit from hedging to offset the operating loss in state 4 is lower than the net benefit of adding more cash to guarantee there will be no operating loss in both states 3 and 4.\textsuperscript{3} More importantly, note that even when $\rho_{XY} = 1$, the marginal benefit of hedging when cash is held is relatively small. Holding cash reduces the value loss from 20% down to 4%, and while hedging completely eliminates any value loss, this incremental benefit is only worth 4%.

We now present a much more general model that incorporates corporate and personal taxes, debt and bankruptcy costs, and equity issuance costs. Furthermore, it is dynamic with the ability to adjust cash, derivatives positions and operations over time, subject to some adjustment costs, thus leading to a richer set of insights.

\textsuperscript{3}Of course, more general state spaces could yield hybrid solutions where hedging and cash are jointly used, and we shall see such strategies emerge from our general model.
II. The Model

We model the operating, financing, and hedging decisions of a firm with a production process that can be suspended and reactivated over time in response to the fluctuations of a state variable affecting the cash flow. We use a discrete-time infinite-horizon framework.

A. Production technology

The firm has operating flexibility in that it can decide to start production if it is idle, or alternatively, it can temporarily cease operations. We denote $m$ to be the status of the firm, where $m \in \{0, 1\}$, with 1 if operations are open/active, and 0 in the closed/idle status.

The firm’s cash flow is determined by a stochastic factor, $\theta_1$, which is priced in the financial market. Under the risk-neutral probability measure, the stochastic process of the log of this price variable, $x_1 = \log \theta_1$, is described by:

$$x_1(t) - x_1(t - 1) = (1 - \kappa_1)(\bar{x}_1 - x_1(t - 1)) + \sigma_1 \epsilon_1(t),$$

where $0 \leq \kappa_1 \leq 1$ is the persistence parameter, $\sigma_1 > 0$ is the conditional standard deviation, $\bar{x}_1 = \log \bar{\theta}_1$ is the long-term mean, and $\epsilon_1$ are i.i.d. standard normal variates.

For example, one can consider the classic setting of Brennan and Schwartz (1985) in which a mine can be opened and closed over time in reaction to changes in the commodity price.
The cash flow of the firm, $R(\theta_1, m)$, is equal to the fixed production rate, $q > 0$, times the difference between the price $\theta_1$ and the average production cost per unit, $A$, if the firm is active (and zero if the firm is idle).

$$R(\theta_1, m) = \begin{cases} \frac{q(\theta_1 - A)}{m} & \text{if } m = 1 \\ 0 & \text{if } m = 0. \end{cases}$$

Opening and closing decisions entail costs. A change in the operating policy is represented by a transition from $m$ to $m'$. Hence, the cost of changing the operating status is the function

$$K(m, m') = \begin{cases} K^c & \text{if } m = 1 \text{ and } m' = 0 \\ K^o & \text{if } m = 0 \text{ and } m' = 1 \\ 0 & \text{otherwise.} \end{cases}$$

For brevity, we denote the net cash flow from the firm’s operations as

$$g(\theta_1, m, m') = R(\theta_1, m) - K(m, m').$$

### B. Hedging

The firm can take a long position in a perpetual and putable swap contract issued by a bank (for robustness purposes, we later consider other derivative structures with non-linear payoff structures and/or shorter lives, as discussed in Section III). The underlying asset of the swap is denoted $\theta_2$, and $x_2 = \log \theta_2$ follows the process

$$x_2(t) - x_2(t-1) = (1 - \kappa_2)(\bar{\theta}_2 - x_2(t-1)) + \sigma_2 \varepsilon_2(t), \quad (2)$$
where \(0 \leq \kappa_2 \leq 1\) and \(\sigma_2 > 0\), and \(\pi_2 = \log \bar{\theta}_2\). Without loss of generality, we restrict the analysis to the case where the two state variables \(\theta_1\) and \(\theta_2\) have positive correlation, \(\mathbb{E}[\varepsilon_1(t)\varepsilon_2(t)] = \rho \geq 0\) and \(\mathbb{E}[\varepsilon_1(t)\varepsilon_2(t')] = 0\) for any \(t \neq t'\). When \(0 < \rho < 1\), the swap offers an imperfect hedge of the risk of the firm. We will also examine the case where \(\rho = 1\), where the firm may seemingly be able to eliminate all the firm’s risk. For brevity, we denote \(\theta = (\theta_1, \theta_2)\) as the vector of the exogenous state variables.

The swap price for a unit of product, \(s\), is a given constant. Thus, if a firm enters into a swap agreement for a notional physical amount \(h \geq 0\), at each subsequent date \(t\) it pays \(\theta_2(t)\) and receives \(s\) for each unit of notional capital, i.e., the net payoff from the swap to the firm is \(h(s - \theta_2(t))\). The par value of the derivative contract for a unit notional amount, \(h = 1\), excluding counterparty risk and the put provision, at time \(t\) with \(\theta_2 = \theta_2(t)\), is\(^5\)

\[
SP(\theta_2) = \sum_{i=1}^{\infty} \frac{s - F_t(\theta_2, t + i)}{(1 + r)^i} < \infty, \tag{3}
\]

where \(F_t(\theta_2, t + i) = \mathbb{E}_t[\theta_2(t+i)]\) is the forward price at time \(t\) for delivery of the asset at date \(t + i\), \(\mathbb{E}_t[\cdot]\) is the expectation under the risk-neutral probability measure, conditional on the information \(\theta = \theta(t)\), and \(r\) is the risk-free rate. It can be shown that (see Appendix A)

\[
F_t(\theta_2, t + i) = \theta_2^{\kappa_2^i} \theta_2^{(1-\kappa_2^i)} \exp \left( \frac{\sigma_2^2}{2} \frac{1 - \kappa_2^i}{1 - \kappa_2^2} \right), \tag{4}
\]

with \(F_t(\theta_2, t) = \theta_2\).

The firm can default on the swap obligation, and it also may choose to change the notional amount from \(h \geq 0\) to a higher or lower level \(h' \geq 0\). If it does wish to alter its position in the swap, it redeems the current contract at the *par value*, and enters into a new agreement at the current *fair value* denoted \(SF\). Hence, the net payoff from the

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\(^5\)In our results, we numerically compute the value of the swap by approximating the infinite summation in (3) with a finite summation with \(T = 500\) terms.
transaction is $h \cdot SP - h' \cdot SF$. We assume that each transaction (closing the old contract and opening the new one) also entails a negotiation cost, $nc$, proportional to the value of the notional amount, so that the direct cost of adjusting the hedge is: $nc \cdot (h' + h)$.

At the inception of the swap agreement, the fair value of the contract, $SF$, reflects both default risk and the option to close the swap position in the future, and thus it can be different from the par value, $SP$. This implies that there is an indirect cost associated with adjusting a swap position at a later date.\footnote{This indirect cost is $h' \cdot (SF - SP)$, and thus the payoff from adjusting the hedge, net of both indirect and direct costs, can thus be rewritten as: $(h - h') \cdot SP - h' \cdot (SF - SP) - nc \cdot (h' + h)$.} We assume for simplicity that the bank selling the swap is not subject to default risk, and thus the only credit charge in the price of the contract is related to the default risk of the firm. Hence, the fair value of the swap contract at time $t$ with $\theta_2 = \theta_2(t)$ is

$$SF(\theta_2) = \mathbb{E}_t \left[ \sum_{i=1}^{T_d \wedge T_p} \frac{s - \theta_2(t + i)}{(1 + r)^i} \right] + \mathbb{E}_t \left[ \chi_{\{T_d \geq T_p\}} \frac{SP(\theta_2(t + T_p))}{(1 + r)^{T_p}} + \chi_{\{T_d < T_p\}} \frac{RS(\theta(t + T_d))}{(1 + r)^{T_d}} \right], \quad (5)$$

where $T_d$ is the default date for the firm, $T_p$ is the date when the swap position is closed, and $RS(\cdot)$ is the bank’s recovery value on the swap if the firm defaults.\footnote{$RS(\cdot)$ depends on the value of the firm, as will be shown below.} In equation (5), $a \wedge b = \inf\{a, b\}$, and $\chi_{\{A\}}$ is the indicator function of the event $A$. In particular, $\{T_d \geq T_p\}$ is the set of paths such that default happens after the position in the swap is closed, and $\{T_d < T_p\}$ is the set where the opposite happens.

Interpreting equation (5), the first line is the present value of the net payoff to the firm before either the firm exercises the option to close the swap agreement or default happens; the second part is the payoff to the bank if the firm terminates the swap.
agreement due to rebalancing of its hedge position, or if it defaults. The bank is paid
the par value in the former case, and a reduced recovery value in the latter.

$T_d$ and $T_p$ are stopping times with respect to the process $\{\theta(t)\}$. Through them,
the fair value of the swap contract depends on the corporate policy decided by equity
holders. This policy will be determined endogenously as shown later in the context of
the valuation problem.

Since the swap price, $s$, is fixed, $SF$ (and $SP$) can be either positive or negative.
Thus, as opposed to the typical swap contract where the swap price is set up so that
$SF = 0$ at inception, here there may be an upfront payment: a cash inflow for the firm
if $SF < 0$ or an outflow if $SF > 0$. Because of the credit charge and the possibility of
renegotiating the hedging contract, we can anticipate that in general $SF \neq SP$.

C. Financial policy

The firm may retain a liquidity balance, $b$, in the form of cash (or cash-equivalent assets)
earning a rate of return of $r$ per period. This cash balance can increase by retaining
after tax operating earnings, by issuing equity, or by entering into a swap contract with
negative value, or conversely by closing out a swap with positive value. If external equity
is raised, we assume that a proportional flotation cost, $\lambda$, is incurred. The cash balance
will decrease if the firm uses its cash to cover operating losses, pays opening and closing
costs, enters into a positive value swap or closes out a negative value swap, or provides
a payout to equityholders.

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8We choose to fix the swap price at a constant level, rather than setting the price to yield a zero swap
value, in order to simplify our model. However, as a robustness check, we have rerun our numerical
results using different swap price levels, and this does not affect our results.
We assume that the firm has issued a non-redeemable defaultable consol bond with face value $d$. The firm pays a coupon rate $r$ on the debt at the end of every period. The debt level is assumed to be constant over time for simplicity. As Gamba and Triantis (2008) show, when there are direct or indirect costs associated with debt rebalancing, firms will choose to actively manage their financial flexibility by adjusting the cash balance rather than the debt level, which is the structure we model here.

D. Personal and corporate taxes

Corporate pre-tax earnings depend on the state at a particular time and the set of decisions made at that time. We use $w$ as shorthand for the earnings $w(\theta, m, b, h, m', b', h')$:

$$w = g(\theta_1, m, m') - r(d-b) + h(s-\theta_2) + (hSP(\theta_1) - h'SF(\theta, m', b', h') - nc(h' + h)) \chi_{\{h' \neq h\}}$$

where $\chi_{\{h' \neq h\}}$ is the indicator function of event $h' \neq h$.

We assume a convex corporate tax function

$$\tau_c(w) = \tau_c^+ \max\{w, 0\} + \tau_c^- \min\{w, 0\},$$

where $\tau_c^+ \geq \tau_c^-$ and $\tau_c^- > 0$ to model a limited loss offset provision.

Personal taxes are levied at rates $\tau_e \geq 0$ on payments to equityholders, and $\tau_d \geq 0$ on payments to bondholders. With respect to taxes on payouts to equityholders, we do not distinguish between dividends and equity repurchases (nor do we separately consider the capital gains of selling shareholders).
E. Financial distress and default

If the after-corporate-tax cash flow from operations, net of the payoff from the swap contract and from a change in the hedging policy, plus current cash balance is lower than the interest payment on net debt,

\[ rd > g(\theta_1, m, m') + (1 + r)b + h(s - \theta_2) \]

\[ + (hSP(\theta_1) - h'SF(\theta, m', b', h') - nc(h' + h)) \chi_{h' \neq h} - \tau_c(w), \]

or \( w - \tau_c(w) + b < 0 \), the firm is in a liquidity crisis. We model financial distress costs as a proportion, \( dc \), of the cash shortfall from the coupon payment. In addition to raising cash from equityholders to cover the shortfall (which has an effective cost of \((1+dc)(1+\lambda)\) per dollar of external equity in this case of distress), the firm may decide to raise further capital to create a positive liquidity balance going forward. If it does so, the additional amount raised from equityholders is subject only to the standard proportional cost of \( \lambda \).

If \( w - \tau_c(w) + b < 0 \), and the equityholders choose not to make up this shortfall, the firm is in default. We assume that in this case the swap contract has priority over the debt contract.\(^9\) This means that the bankruptcy proceeds, net of proportional verification costs, \( \gamma \), are first used to pay the bank (swap counterparty) if the swap values is negative, and debtholders receive the remainder, if positive. If default occurs and the swap value is positive, the bank must pay to settle the swap contract, and this liquidation value becomes part of the firm value that is accessible to debtholders.\(^10\)

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\(^9\)The U.S. Bankruptcy Code generally allows swap counterparties to exercise contractual rights in connection with their agreement without violating the automatic stay that arises in connection with a Chapter 11 bankruptcy petition (Sections 560 and 362(b)(17)). Financial swaps can thus be settled and paid ahead of debt obligations giving them effective priority over debt.

\(^10\)In some cases, there may be “walk-away” clauses in the swap agreement that prevent this, but lenders often prohibit borrowers from entering into agreements with such provisions (see Gooch and Klein (2002)).
F. Valuation of Securities

In our analysis, we will consider two possible goals of the firm’s policy: either firm value (i.e., first best) maximization, or equity value (i.e., second best) maximization. Although we will predominantly focus on the first best solution, for expositional convenience we begin with the case of equity value maximization.

The cash flow to equity holders at $t$ is

$$ e_t = c(\theta, m, b, h, m', b', h') = \max\{cfe, 0\}(1 - \tau_e) + \min\{cfe, 0\}(1 + \lambda) $$

where

$$ cfe = \max\{w - \tau_c(w) + b, 0\} + \min\{w - \tau_c(w) + b, 0\}(1 + dc) - b' $$

can be positive, in which case the payout to equityholders is taxed at the personal tax rate $\tau_e$, or negative, in which case equity is raised, subject to the flotation cost $\lambda$. The cash flow depends on the current state, $(\theta, m, b, h)$, as well as the decisions regarding next period’s operating state, $m'$, cash balance level, $b'$, and swap position, $h'$.

The value of equity at time $t$, with $\theta = \theta(t)$, under second best maximization is

$$ E(\theta, m, b, h) = \max_{\{(m_i, b_i, h_i), i=1,...,T_d\}} \mathbb{E}_{\theta, m, b, h} \left[ \sum_{i=0}^{T_d} \beta^i e_i \right] $$

(6)

where $\beta = (1 + r_z(1 - \tau_e))^{-1}$ is the discount factor for valuing equity flows, with $r_z = r(1 - \tau_d)/(1 - \tau_e)$ denoting the certainty equivalent rate of return on equity (see Sick (1990)).
and $E_{\theta,m,b,h} [\cdot]$ is the expectation conditional on $(\theta, m, b, h)$. The Bellman equation for the solution to (6) is

$$E(\theta, m, b, h) = \max \left\{ \max_{(m', b', h')} \left\{ c(\theta, m, b, h, m', b', h') + \beta E_{\theta,m',b',h'} [E(\theta', m', b', h')] \right\}, 0 \right\}$$

(7)

where $(m', b', h')$ denotes the firm’s decision made at the beginning of the period, soon after the current state, $\theta$, is observed, and $\theta'$ is the unknown state at the end of the period. When the firm is not solvent, the value of equity is maximized by exercising the limited liability option, i.e., $E(\theta, m, b, h) = 0$, as per the outer maximization in equation (7).

We denote the firm’s policy as $\varphi(\theta, m, b, h)$. The optimal choice is $(m^*, b^*, h^*) = \varphi(\theta, m, b, h)$ if the firm is solvent. In the case of default, the firm is turned into an unlevered concern, i.e., all the cash balance is paid out and a new cash balance is set at $b = d$, the swap contract is liquidated ($h = 0$), and the operating policy is left unchanged (i.e., in default, the optimal policy is $(m, d, 0)$). We denote $\delta(\theta, m, b, h)$ as the default indicator, that is, the indicator of the event $E(\theta, m, b, h) = 0$.

The value of corporate debt is

$$D(\theta, m', b', h') = \beta E_{\theta,m',b',h'} [cfd(\theta', m', b', h', \varphi)] ,$$

(8)
where we express $\beta = (1 + r(1 - \tau_d))^{-1}$ to explicitly reflect the after personal tax rate of return on debt (but is equivalent to $\beta = (1 + r_e(1 - \tau_e))^{-1}$ as shown above), and the end of period after personal tax cash flow to debt holders is

$$
cfd(\theta', m', b', h', \varphi) = (1 - \delta(\theta', m', b', h')) (rd(1 - \tau_d) + D(\theta', m'', b'', h'')) + \delta(\theta', m', b', h') \min \{rd(1 - \tau_d) + d, \\
\max \{0, b' + (1 - \gamma)E(\theta', m', d, 0) + h ((s - \theta_2^') + SP(\theta_2'))\}\}.
$$

The first line is the payoff to bondholders when the firm is solvent. As above, the value of debt, $D$, is defined in relation to the policy $\varphi(\theta', m', b', h') = (m'', b'', h'')$. For this reason, $\varphi$ is an argument of $cfd$. The second and third lines are the residual payoff to bond holders, considering their lower priority with respect to the swap. Note that if $\theta_2'$ is such that $(s - \theta_2') + SP(\theta_2') > 0$ (typically when $\theta_2$ is low), then the bond holders receive more than the unlevered going concern value of the firm net of bankruptcy costs (although less than $d$). Otherwise, they receive less than $b' + (1 - \gamma)E(\theta', m', d, 0)$ due to their lower priority at default. Note also that in the definition of the cash flow to debt holders, we incorporate $\varphi$ to reflect that the end of period value depends on the future decisions $(m'', b'', h'') = \varphi(\theta', m', b', h')$.

Under firm value maximization, we solve the equation

$$
V(\theta, m, b, h) = \max_{(m', b', h')} \{e(\theta, m, b, h, m', b', h') + \beta \mathbb{E}_{\theta, m', b', h'} [E(\theta', m', b', h')] \\
+ rd(1 - \tau_d) + D(\theta, m', b', h')\} \quad (9)
$$
where the equity value that results from the policy \( (m', b', h') \),

\[
E(\theta, m, b, h) = e(\theta, m, b, h, m', b', h') + \beta E(\theta, m', b', h') [ E(\theta, m', b', h') ],
\]
is positive. In equation (9), \( D(\cdot) \) satisfies equation (8), although for a first best policy function \( \varphi \). If at the optimal solution the firm is solvent \( (E(\theta, m, b, h) > 0) \), then the optimal policy is \( (m^*, b^*, h^*) \). Otherwise the firm is in default and hence \( (m, d, 0) \) is the optimal solution.

The fair value of the swap (from the firm’s viewpoint) incorporates the credit charge and reflects the optimal hedging policy of the firm, as well as the optimal liquidity and operating policies (whether first or second best), which in turn depend on the current state \( (\theta, m, b, h) \). Hence, given a decision \( (m', b', h') \) at the beginning of the period (assuming the firm is solvent) and given the default policy, the end of period cash flow to the firm from the swap once \( \theta' \) is observed, per unit of notional amount, is

\[
cfs(\theta', m', b', h', \varphi) = (1 - \delta(\theta', m', b', h')) \left[ (s - \theta'_2) + \text{SP}(\theta'_2) \chi_{h'' \neq h'} \right. \\
+ \text{SF}(\theta', m'', b'', h')(1 - \chi_{h'' \neq h'}) \\
+ \delta(\theta', m', b', h') \max \left\{ (s - \theta'_2) + \text{SP}(\theta'_2), -(b' + (1-\gamma)E(\theta', m', d, 0))/h' \right\}.
\]

The first two lines of this equation represent the case where the firm is solvent. The first term in this expression is the standard cash flow from the swap; the second term is the par value of the swap which is paid (or received) if the position in the swap will be changed \( (h'' \neq h') \); the third term is the new swap value in case the notional amount is unchanged, based on the firm’s operating and cash policies, \( m'' \) and \( b'' \), respectively. \( \text{SF}(\theta', m'', b'', h') \) is technically a continuation value rather than a cash flow, but this will allow us to properly capture the beginning of period value of the swap, as shown below.
The third line of the equation is the payoff in the case where the firm defaults. If \( \theta_2' \) is such that \((s - \theta_2') + SP(\theta_2') > 0 \) (typically when \( \theta_2' < s \)), then it is a cash inflow for the firm. Otherwise, the firm pays the minimum between \(- (s - \theta_2') - SP(\theta_2')\) and the after bankruptcy costs value of the unlevered asset, \((b' + (1 - \gamma)E(\theta', m', d, 0)) / h'\), per unit of notional amount. As a consequence of default, the swap contract ceases and \( h'' = 0 \).

As in the case of debt, the end of period value in the cash flow to the swap depends on the future decisions \((m'', b'', h'') = \varphi(\theta', m', b', h')\).

From the above, the fair value of the swap (per unit of notional capital) at the current state \((\theta, m, b, h)\), based on the current decision \((m', b', h')\) if the firm is solvent, is

\[
SF(\theta, m', b', h') = \beta_0 \mathbb{E}_{\theta, m', b', h'} [cfs(\theta', m', b', h', \varphi)],
\]

where \( \beta_0 = (1 + r)^{-1} \) is the appropriate discount factor for the swap.

Summarizing, under firm value maximization, the solution of the valuation problem is found by solving the system of simultaneous equations (9), (8) and (10), whereas under equity value maximization, the solution requires solving the system of equations (7), (8) and (10). The numerical technique we use is value function iteration on a discretized version of the continuous state problem. Appendix B describes the approach we use to find a discrete–state approximation of the Markov process \( \{\theta(t)\} \).

### III. Results

In this section, we explore the impact of the underlying drivers of risk management, and analyze the marginal and joint risk management contributions of liquidity, hedging and operating flexibility. Most of the analysis is conducted assuming firm value maximiza-
tion. We do so in order to focus on the normative question of how to minimize the impact of frictions, and in so doing, how much value can be created in a first-best environment. In practice, the risk management decisions of managers will clearly reflect a number of factors including the structure of managerial incentives as well as contractual terms and regulatory controls designed to prevent managerial and financial agency problems. We do provide a partial analysis of agency problems later in this section to illustrate some of the agency issues that may arise in the application of risk management strategies, and to show that effective controls can lead to risk management solutions that are close to first best.

A. Parameter values

While it might be appealing to attempt a structural estimation of our model, we are constrained in doing so given the lack of good empirical data on the magnitude of hedging and the characterization of operating flexibility, and we recognize that despite the complexity of our model, its limitations could lead to unreliable parameter estimates. Instead, we chose the approach of presenting a calibrated model in order to obtain results whose magnitudes may not be precise, but indicate important relationships that help to close the gap in explaining existing empirical evidence, or suggest empirical implications that bear further examination. We also perform numerous robustness checks throughout our analysis in order to build confidence in these results.

The base case parameters for our analysis are shown in Table III. The rationales for each of the parameter choices are described in detail below, but in each case the selection either corresponds directly to typical values found in the literature or allowed
us to obtain simulated moments for key variables that were consistent with those in recent empirical studies. The simulation procedure we use is detailed in Appendix C.

The volatility and persistence of the log of the product price (and revenue, since the production rate \( q \) is set equal to one) are set to \( \sigma_1 = 15\% \) and \( \kappa_1 = .80 \) (on an annual basis), which are consistent with the range of values used in recent articles with similar processes (e.g., Hennessy and Whited (2005) and Zhang (2005)). The volatility of operating profitability is also affected by the long-term mean of \( \theta_1 \), which is set to 1 for simplicity (i.e., \( \bar{\theta}_1 = 0 \)), the fixed production cost \( A = .97 \), and the closing and opening costs \( (K^c = .3 \) and \( K^o = .1) \). These values lead to a simulated volatility of the rate of return on equity of 0.24 (where the return is calculated inclusive of both change in equity value and the payout to equityholders). This simulated volatility is close to the average stock return volatility of .29 reported in Wei and Zhang (2006) for the period 1976-1995, particularly considering that the empirical return volatility reflects a stochastic discount rate, which we do not capture in our model.\(^{11}\) Our simulated volatility for the return on the firm’s assets is approximately 18\%, which is reasonably close to an asset volatility of around 21\% for firms with investment grade bonds, as calculated by Acharya, Davydenko, and Strebulaev (2007) using the weighted average returns on the debt and equity of these firms.\(^ {12}\)

The underlying asset for the swap is assumed to have the same mean, volatility, and persistence as the product price in order to lead to simpler interpretation of our results (particularly in the special case of perfect correlation). The swap price is fixed at \( s = 1 \) for simplicity. However, we ran two robustness checks regarding this swap price specification. First, we used alternative fixed swap prices in the range of \((0.92, 1.18)\),

\(^{11}\)The market-adjusted return volatilities reported in Wei and Zhang (2006) are very similar to those in Campbell, Lettau, Malkiel, and Xu (2001) for the longer period 1962-1997.

\(^{12}\)Acharya, Davydenko, and Strebulaev (2007) also find that equity volatilities increase monotonically from 23\% to 35\% for firms whose bonds range from AAA to BBB.
which corresponds to the range of feasible swap prices given the mean reverting process for $\theta_2$. Second, we ran several cases where the swap price is reset every time the position is renegotiated, such that the fair market value of a newly initiated swap is equal to zero. These two sets of tests confirmed that our results are not sensitive to the specification of the swap price.\textsuperscript{13}

In our model, $\theta_1$ represents the entire risk exposure of our firm, and the correlation $\rho$ measures the ability of the firm to hedge this risk exposure using the derivatives contract. We believe that for most non-financial corporations, this correlation is likely to be quite low based on the following empirical evidence. Tufano (1996) finds that few gold mining companies hedge more than half of their three–year forward production, and the mean “delta–percentage” is closer to 25%, indicating that even firms with key exposures for which there are corresponding derivatives markets can not or do not fully hedge these risks. Carter, Rogers, and Simkins (2006) find that airline companies hedge on average 15% of the next year’s jet fuel exposure (though some hedge almost half), but they also report that jet fuel represents on average only about 14% of total operating expenses (though it may represent a higher percent of the total risk exposure). Bartram, Brown, and Minton (2010) find that financial hedging reduces foreign exchange exposure by about 45% for a global sample of manufacturing firms, but these firms would also face many other risks that are not as readily hedgeable. In fact, Campello, Lin, Ma, and Zou (2010) find that the total notional value of interest rate and foreign currency derivatives contracts are on average 14% of the total assets of the firm, conditional on the firms hedging (of which only half of them do in their sample), which is similar to Graham and Rogers (2002) estimate of 13.2%. More conservatively, Guay and Kothari (2003)

\textsuperscript{13}For instance, as will be reported in the next section, the value of risk management in our base case (having a cash balance and a swap with $\rho = .1$) is equal to 14.1% at the long run mean value of $\theta_1 = 1$. Using an alternative model where the swap price is reset at renegotiation so that the swap value is zero, the value of risk management at $\theta_1 = 1$ is equal to 15.3%, which is not significantly different.
estimate that even under very generous assumptions, the median firm in their sample holds derivatives that can only hedge between 3% to 6% of their aggregate FX and interest rate risk.

In general then, since firms face a myriad of risks, including operational, legal, regulatory, reputational, strategic and others which can’t be readily hedged with derivatives, we believe it is reasonable to use $\rho = .1$ as the base case. We also present a case with $\rho = .5$ for the sake of robustness, and a case with perfect correlation in order to analyze an extreme case where the firm’s risk exposure could be perfectly hedged using the swap, thus providing an upper bound for the value created by such hedging. The perfect correlation case will also permit us to highlight some limitations of hedging with derivatives.

The personal tax on equity income ($\tau_e$) is assumed to be 12%, equal to the estimate provided by Graham (2000). The personal tax on bond income ($\tau_d$) is set at 25%, consistent with Hennessy and Whited (2005). The corporate tax rate on income ($\tau_c^-$) is set to the current marginal tax rate of 35%, and, due to limited carrybacks and carryforwards, the effective tax on losses ($\tau_c^-$) is assumed to be somewhat lower at 30%. This convexity is roughly consistent with the corporate tax function applied in Hennessy and Whited (2005). Overall, the effective tax benefit (or penalty) from interest payments on debt (or return on cash) is between 18% and 13% (depending on whether there are profits or losses).\textsuperscript{14}

The issuance cost ($\lambda$) is assumed to be 5% of the level of equity raised, a number that is consistent with empirical studies that directly estimate these costs (e.g., Altinkilic and

\textsuperscript{14}As a robustness check, we also report results in the next subsection based on $\tau_c^- = 20\%$ rather than 30\%, which yields much higher corporate tax convexity.
Hansen (2000)) as well as those that indirectly estimate them through structural models (Hennessy and Whited (2005) and Hennessy and Whited (2007)).

When the firm is in a situation of distress, it typically must pay a significant premium to raise capital to cover its deficit. This additional distress cost ($dc$), which is proportional to the deficit amount, is assumed to be 15%. This may reflect the need to sell assets in a fire-sale situation, or to assume costly terms in agreements for capital infusions (e.g. during Fall 2008, banks and other distressed companies issued preferred stock with large dividends and warrants). Our assumption of 15% is close to the 14% fire-sale discount estimated in Pulvino (1998), and at the mid-point of the 5-25% distress cost range assumed by Strebulaev (2007).

The cost associated with a bankruptcy restructuring ($\gamma$) is assumed to be 10% of the value of the newly unlevered firm. While there is significant variation in estimates of bankruptcy costs provided in the literature, depending on sample selection as well as method of estimation, 10% appears to be a reasonable level. It is slightly higher than the mean bankruptcy costs reported in Bris, Welch, and Zhu (2006) and the estimate produced in Hennessy and Whited (2007), but towards the bottom end of the range estimated in Andrade and Kaplan (1998) based on highly leveraged transactions. As we will report later, our results are not particularly sensitive to the level of bankruptcy costs. Overall, however, a firm typically experiences distress before entering bankruptcy, and thus the total costs associated with poor performance can be quite significant.

We assume that the cost of setting up and/or renegotiating the swap ($nc$) is 1% of the sum of the par values of the new and old swap (i.e., 1% to initiate a new position and 1% to close out an existing position). This typically results in a lower cost than

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15Hennessy and Whited (2005) and Fehle and Tsyplakov (2005) use distress costs of 40% and 200%, respectively, and thus our assumption is conservative relative to their models.
in Fehle and Tsyplakov (2005), who use a fixed cost, but may still be higher than in practice if highly liquid and standardized swaps are used. However, as we will show later, our results do not appear to be highly sensitive to this cost.

Finally, we set the debt level \( d \) equal to .4. Based on the resulting simulated leverage values, we find an average market leverage ratio of 24%. The average simulated cash balance as a percent of firm value is 19%, and the average net leverage ratio is approximately 5%. These values are consistent with recent evidence presented in Bates, Kahle, and Stulz (2009), particularly for the late 1990s. In fact, our simulated values for mean leverage and cash balance are precisely the empirical values shown for two of their sample years (1997 and 1999). We assume that the risk-free rate \( r \) is 5%, which is also the coupon rate on debt and the yield on the cash balance.

**B. Relative impact of frictions motivating risk management**

As described previously, there are several different motivations in our model for managing risk, including issuance, distress, and bankruptcy costs, and convex personal and corporate taxes. We begin by studying the relative impact of these frictions in Figure 1. The Base Case curve in Figure 1 shows the value of risk management when all these frictions are present in the model (based on the parameters in Table III). The value of risk management is calculated as the percentage increase in firm value from dynamically managing both cash balance and swap positions (with \( \rho = .1 \) between the swap’s underlying asset and the product price), relative to the firm value when neither of these risk management controls are present. A first-best (firm value maximization) setting is used for now, and we later explore the effect of potential agency problems. We also later explore the role of operating flexibility in contributing to risk management; for now,
we assume that the firm has the base level of operating flexibility (i.e., $K^o = .1$ and $K^c = .3$).

The first observation to draw from Figure 1 is that risk management becomes less valuable when firm profitability increases, as expected. Overall, the potential value from carefully managing both liquidity and the swap position can be quite significant. At the long-run mean value of $\theta_1 = 1$, the value of risk management when all frictions are present is approximately equal to 14%. For high $\theta_1$ values, the value increase is only about 5%. On the other extreme, it becomes very large (over 35%), but this is largely due to the fact that there are very small firm values for low $\theta_1$ values, and thus avoiding distress, bankruptcy and issuance costs can have a large effect on the percentage increase in firm value. This underscores that financial risk management, taking into account not only the use of derivatives but also the contribution of managing an internal cash balance, can greatly enhance corporate value. We will soon examine the relative roles of liquidity and hedging in creating value. But first, it is useful to understand the relative roles of the various frictions in driving the need for risk management, which are represented by the other curves in Figure 1.

As a conceptual check, the “no frictions, linear tax” curve is included to show what the value of risk management is when there are no issuance, distress and bankruptcy costs, and when personal and corporate taxes are perfectly linear (i.e., $\tau_c^+ = \tau_c^- = .40$, and $\lambda = -\tau_e = -.12$). As expected, there is no value associated with the ability to hold a cash balance and to use a swap (neither are used in this case despite being available), because there are no penalties associated with the firm having negative income or being in distress or bankruptcy.

The other curves in Figure 1 exhibit the same general shape for the value increase due to risk management as the base case, but each of these curves is significantly below the
base case curve, as expected given that the base case reflects all of the frictions at once. Convex corporate taxes and equity issuance cost, two key rationales for risk management frequently discussed in the literature, are the source for limited value enhancement from risk management in our context (both curves are close to the x-axis). The two more significant drivers appear to be distress costs and convex personal taxes (i.e., the fact that equity outflows are taxed while inflows receive no tax subsidy).\textsuperscript{16}

The impact of distress costs (which are shown together with equity issuance costs, since both are incurred in the event of distress) is very significant, highlighting that frictions associated with distress may be the most important drivers of the value created through risk management. Of course, the precise magnitude of this effect depends on the level of distress costs assumed, but as discussed earlier, we believe that a 15\% distress cost appears reasonable. Recent events have shown that when financial distress affects an entire industry and/or a credit freeze impacts the ability of companies across the entire economy to raise financing, then there is significant value to maintaining liquidity, despite the costs associated with doing so.\textsuperscript{17} The mean and median annual distress probability from our base case simulations are 1.6\% and 1.0\% respectively. While there does not appear to be an easy way to benchmark these figures against data, these do not seem to be particularly high distress frequencies.\textsuperscript{18} Thus, our finding on the importance of distress costs as a driver of risk management does not seem to be driven by unreasonable estimates of distress costs or frequencies.

\textsuperscript{16}If bankruptcy costs are singled out as the only friction, risk management does not add any value in the current context where firm value is being optimized. The firm will raise the necessary financing (at no cost) rather than incur bankruptcy costs. Therefore, we do not include this curve in Figure 1.

\textsuperscript{17}Campello, Giambona, Graham, and Harvey (2010) examine liquidity management during the 2008-09 financial crisis, and specifically underscore the importance of access to credit lines during this period.

\textsuperscript{18}Annual default frequencies for BBB-rated bonds are approximately .2\% a year, and one would expect that the incidence of distress would be significantly higher than default.
The role of convex personal taxes is interesting and appears not to be highlighted to date in the risk management literature. To see this effect clearly, imagine that a firm pays out a dollar to shareholders in a particular period and then goes back to the shareholders in the subsequent period to raise a dollar of equity. Even in the absence of an equity issuance cost, investors lose from this round trip payout-issuance since they are taxed on income but yet do not receive a tax subsidy when providing new equity capital to the firm. The greater the uncertainty, the more likely that this payout-issuance cycle would repeat, and thus the larger the value loss due to this personal tax convexity. While derivatives can help to decrease this value loss to the extent that it can reduce the variability in profitability, the cash balance serves a vital role in preserving value by smoothing out the outflow-inflow fluctuations. This is of course consistent with what we know from the empirical literature on payout and issuance policies in that firms don’t pay out all available cash in part because this might leave them needing to raise equity capital in a subsequent period, and as a result equity issuance is also found to be relatively rare for firms that are not in a high-growth stage. In the context of our dynamic model, we are able to attribute a specific value to the ability to smooth out the payout-issuance fluctuations in the presence of convex personal taxes (and corporate taxes that induce a tax penalty to maintaining cash within the firm).

We should note that the relative impact of the various motivations for risk management clearly depends to some extent on the parameter assumptions, and thus will be situational specific for each firm. A firm’s shareholder clientele may lead to a higher or lower effective “personal” tax rate on equity than the 12% assumed here, thus affecting the magnitude of the convex personal tax effect. Similarly, different firms may have more or less ability to carry forward or back net operating losses, affecting the convexity of corporate taxes, or may face higher or lower distress costs depending on the nature of
the business and the degree of systemic distress at a point in time.\textsuperscript{19} Nevertheless, our results provide some useful insight into the relative importance of various motivations for risk management under a set of reasonable input assumptions. Finally, note that summing up the value creation from mitigating the impact of each of the different frictions (i.e., the various single-friction value curves in Figure 1) falls short of the total value created when all frictions are considered at once (the base case curve). This points to an interesting compounding effect when multiple frictions are considered simultaneously.

These observations lead us to explore more carefully the relative value contributions of hedging and liquidity, and whether the combination of these two risk management mechanisms is relatively effective in mitigating the effects of frictions and tax convexities in the presence of uncertainty. We will then examine the contribution of operating flexibility to risk management, taking into account that the firm can also manage its liquidity and hedging positions.

C. Relative Contributions of Liquidity, Hedging and Operating Flexibility to the Value of Risk Management

Figure 2 shows the percentage increase in firm value due to risk management plotted against $\theta_1$, under three risk management scenarios: 1) the firm can dynamically manage a liquidity balance, but can’t hedge with a swap; 2) the firm can dynamically manage a swap position, but can’t keep a liquidity balance; and 3) the firm can dynamically manage both its liquidity balance and a swap position. We continue to assume for now

\textsuperscript{19}If we significantly increase the convexity of corporate taxes by setting $\tau^{-}_c = 20\%$ rather than 30\%, the curve corresponding to this higher convexity case is not surprisingly higher than the convex corporate tax curve shown in Figure 1. This higher curve actually overlaps quite closely with the personal tax curve shown, i.e., at $\theta_1 = 1$, the risk management value in the presence of the more convex corporate taxes is 1.9\%.
that $\rho = 0.1$ (i.e., the underlying asset of the swap, $\theta_2$, has correlation of 0.1 with the firm’s risk exposure, $\theta_1$), and thus the case with both liquidity balance and swap is simply the base case curve which was shown in Figure 1). As discussed earlier, the swap and/or liquidity positions are dynamically managed in such a way as to maximize firm value.

The top two curves in Figure 2 demonstrate that maintaining a cash balance, and optimally rebalancing it over time, adds significant value to the firm, due to avoiding distress and equity issuance costs. Keep in mind that there is a significant tax penalty in our model associated with holding cash, and yet liquidity still proves to be quite valuable to the firm. This finding lines up with recent empirical evidence in Bates, Kahle, and Stulz (2009) showing that cash has become an increasingly important cushion against rising risk levels in the economy.\(^{20}\) They attribute this increase in cash holdings, at least in part, to the increase in idiosyncratic risk over their sample period. As we illustrate shortly, as it becomes increasingly more difficult to hedge the risk of the firm using derivatives contracts (which is more likely for idiosyncratic risk), cash holdings play a more important role in managing risk.

The incremental value associated with the derivatives hedge (with $\rho = 0.1$) ranges from 0 to 4% of firm value when the firm also has a cash balance (and is somewhat larger if there is no cash balance), and is approximately 1.5% at the long-term mean level of $\theta_1 = 1$. This result sheds some light on the recent debate in the empirical literature regarding the value created by hedging with derivatives. Our result suggests that the value increase from hedging, averaging over a cross-section of (surviving) firms, may

\(^{20}\)Bates, Kahle, and Stulz (2009) also report that firms with entrenched manager were less likely to increase cash holdings during the period they study, indicating that managerial agency problems do not appear to be driving this increase in cash. Faulkender and Wang (2006) and Gamba and Triantis (2008) provide empirical and theoretical support, respectively, for marginal values of cash being significantly in excess of one in many scenarios.
well not be much more than 2%, and may only be significantly perceptible for firms in recessionary situations. Some empirical papers, particularly Guay and Kothari (2003) and Jin and Jorion (2006), provide evidence that is consistent with this result, finding that value increases from hedging are not statistically or economically significant.\textsuperscript{21} In contrast, Allayannis and Weston (2001) estimate that hedging with foreign exchange derivatives increases value by approximately 5% on average for their sample of non-financial multinationals (a value which Guay and Kothari (2003) view as excessive given the extent of hedging and its resulting effect on cash flows), and Carter, Rogers, and Simkins (2006) estimate value increases between 5-10% for airlines hedging their oil exposure with derivatives.

There are some additional factors, however, to consider in addressing the apparent lack of consistency in the empirical findings. First, our results so far are based on $\rho = .1$, representing a situation where the extent to which derivatives are correlated with the overall risk of the firm is rather limited due to the large number of unhedgeable risks the firm faces. However, some firms may be able to hedge a more significant portion of their exposure using derivatives, particularly those in industries where certain key risk factors are present, such as a commodity output or a highly volatile input factor. In Figure 3, we plot the percentage value increase attributable to risk management as in Figure 2, but explore the effect of higher correlations between $\theta_1$ and $\theta_2$ of either 0.5 or 1. Going from the case of $\rho = .1$ to that of $\rho = .5$, the value increase attributable to risk management does become larger as expected, but rather incrementally (less than an additional .5% at $\theta_1 = 1$). However, when the swap is perfectly correlated with the

\textsuperscript{21}However, Jin and Jorion (2006) note that their sample of oil and gas firms may bias their results towards lower value creation from the use of derivatives since investors in those firms might prefer that these firms not hedge away the risk exposure.
firm’s risk, the value increase is much more substantial, indicating that the effect of correlation on the value increase due to hedging can be quite non-linear.

As discussed earlier, we believe it is unlikely that a firm’s total exposure will be highly correlated with available derivatives, even in industries where there is one key uncertainty driving profitability that can be hedged. Thus it may well be, as Jin and Jorion (2006) and others have argued, that the high levels of value increase attributed to hedging instead reflect other endogenous factors. These factors may include additional risk management efforts beyond simply hedging with derivatives, or governance practices that, while leading to the use of derivatives for hedging, increase value for other independent reasons. At a minimum, our results make clear that the extent to which a firm’s risk exposure can be hedged by derivatives contracts is a critical explanatory variable that needs to be considered in empirical studies on the value contribution of hedging programs.

The second key factor to consider in evaluating the contribution of hedging with derivatives is that we have so far only considered one type of derivative, namely a swap contract. As Adam (2002), Adam (2009), Brown and Toft (2002) and others have shown, option contracts may be more effective hedging instruments for some companies than linear derivative contracts such as forwards and swaps. Thus, by focusing only on the use of a swap contract, we may be understating the true value that could be created using alternative non-linear derivatives contracts. Note, however, that the swap position in our model can be dynamically adjusted at relatively low cost, so the hedging profile is effectively non-linear and quite flexible. In fact, the effective maturity of the swap contract in our simulations, taking rebalancing into account, is 2.24 years on average. However, to ensure that the nature of our swap is not skewing our results, we have...
explored a version of our model where one-period put options are used rather than infinite maturity (or one-period) swap contracts, and find essentially equivalent results.\footnote{When $\theta_1 = 1$, the value of risk management using one-year put options instead of the swap in the base case is 13.0\%. When a one-year swap is used, the value is 13.3\%. Both of these values are quite close to the base case value using the infinite maturity swap of 14.1\%.}

It is useful, however, to consider the value that would be created by an optimal customized hedging contract, i.e., a “perfect hedge.” This contract would offset any negative cash flows such that the firm would never be in distress or in need of issuing equity, nor subject to the effects of convex corporate taxes.\footnote{We calculate the value in this case by simply eliminating all frictions, i.e., zero issuance, distress and bankruptcy costs, and using linear functions for both personal equity tax ($\lambda = -\tau_e$) and corporate tax ($\tau_c = \tau_c^-$). Since a perfect hedge prevents a firm from incurring any such costs (or being in the lower part of the convex tax function), this no-frictions case is conceptually the same as the perfect hedge case. Implicit here is that the perfect hedge provides exactly the state-contingent payout necessary to avoid any negative cash flows, not only due to operating losses, but also those incurred when adjusting the operating policy, and that the firm in turn makes state-contingent payments whose present value is equal to the present value of the payouts received.} Figure 4 shows that the value added due to the perfect hedge contract is higher than that from dynamically rebalancing a cash balance and a swap position (with $\rho = 1$). To fully appreciate this result, first note from comparing the bottom two curves in Figure 4 that having cash is quite valuable even if the firm has access to a perfectly correlated swap contract, which can completely eliminate the variability in operating profitability. When the firm’s profitability is low, a perfectly correlated swap with swap price equal to 1 (and $h = 1$) will ensure that a profit will be locked in, but the profit is small enough that it won’t cover the closing cost if it is optimal to close production.\footnote{Note that despite locking in this profit by holding the swap, the firm may still benefit from closing down production if $\theta_1$ is low enough and operating losses are significant. The firm can then earn the profits from the swap contract (assuming the swap position is maintained) without having these profits partially offset by the operating losses it would face with an open facility.} Similarly, in states where the production is closed, the firm doesn’t have cash flow to pay for opening costs if profitability returns and it is optimal to re-open production. Maintaining a cash balance adds value by avoiding issuance costs (and the convexity in personal equity taxes)
taxes) in these two instances. The opening and closing costs in our model can be viewed more broadly as representative of restructuring costs, capital expenditures, and other extraordinary expenses faced by firms in practice that would not be offset by gains in conventional hedging positions. Liquidity thus plays an important role in an integrated risk management system by providing a mechanism to absorb some of the effects of uncertainty that are otherwise difficult to mitigate through hedging.

Returning to the “perfect hedge” in Figure 4, this contract covers deficits due not only to operating losses but also to the adjustment costs ($K^o$ and $K^c$), since it is designed to avoid all potential circumstances that would otherwise lead to equity issuance or distress.\textsuperscript{25} While a cash balance, coupled with hedging using a perfectly correlated swap, can manage much of the risk of a firm, even a substantial cash balance position will gradually be depleted if the firm continues to suffer losses over a sufficiently long period of time. Thus, even the cash and perfect hedge position will still trigger issuance and/or distress costs with some probability, though this can be minimized with a high cash balance. However, the firm may not have a sufficient string of positive outcomes to build such a large cash balance without issuing equity, and the tax penalty associated with holding a high cash balance can be very significant over time. In contrast, the perfect hedge can always cover any deficits, whether due to operating losses or adjustment costs, without any associated tax penalty.\textsuperscript{26}

\textsuperscript{25}It is not the savings from avoiding losses and operating adjustment costs that create the additional value from the perfect hedge, since the firm must pay for this coverage. It is rather the avoidance of frictions (issuance and distress costs) associated with these losses and operating adjustment costs that creates the additional value.

\textsuperscript{26}The perfect hedge also has no transaction costs associated with it, whereas the hedge with perfect correlation is adjusted over time, triggering negotiation costs. If these costs are set to zero for the “cash, hedge ($\rho = 1$)” case, the value due to risk management is equal to 22.8% at $\theta_1 = 1$, in contrast to a value of 20% shown in Figure 4 in the presence of transaction costs. Thus, the negotiation costs decrease the value of risk management by about 3% (at $\theta_1 = 1$, and when $\rho = 1$), representing part of the differential between perfect risk management versus using cash and a perfectly correlated swap.
The value increase from the perfect hedge in Figure 4 relative to the case of having cash but no hedge (in Figure 2) is quite significant, about 19% (31.7% versus 12.9%) when $\theta_1 = 1$. Thus, while the marginal benefit of having a swap contract when the firm also optimally manages its cash balance may not be that high (particularly if $\rho$ is low), as discussed earlier, the situation would be quite different if there were in fact a perfect hedge. It would clearly be difficult to design such a product in practice that would provide a blanket cover for all of a firm’s risk exposure, including occasional extraordinary costs such as those due to restructuring. In addition, we are ignoring here any transaction costs (including potential moral hazard costs) associated with such a perfect hedge. Nevertheless, our exploration of the perfect hedge provides a mechanism to bound the value attributable to risk management, and suggests that well designed enterprise risk management systems could add significant value to many firms. In a recent effort to quantify the value attributable to firm-wide ERM programs, Hoyt and Liebenberg (2010) estimate an average value premium of 16.5% for U.S. insurers that have implemented such programs, which is roughly in line with our results, particularly if transaction costs were considered.

To better understand the relative contribution of liquidity and hedging to value creation, particularly from a marginal perspective of how each contributes in the presence of the other, we examine the value surfaces shown in Figure 5. Each graph illustrates firm value for a particular current state $(\theta_1, \theta_2, m, b, h)$ against possible selections of new cash balance $(b')$ and hedge $(h')$ levels. In each graph, there is an optimal $(b', h')$ pair that maximizes firm value (i.e., corresponds to the peak of the value surface), and these values are shown above each graph. The optimal production policy is also shown above each graph as $m'$, which equals 1 if it is optimal to continue to keep production open in
the next period, and 0 if the firm should close production (the facility is currently open in all graphs).

The four panels shown in Figure 5 are organized to show the effects of both current profitability as well as existing cash balance and hedge levels, as follows. The top row assumes $\theta_1 = 1$, while the bottom row assumes a below-average profitability of $\theta_1 = 0.8$. The left column assumes the firm currently has no cash balance or swap ($b = h = 0$), while the right column assumes $b = 0.4$ and $h = .03$, which are approximately the steady-state average levels for the base case from our simulations (these statistics will be discussed in greater detail in a later subsection).\(^{27}\) In all cases, we choose the asset value underlying the swap, $\theta_2$, to be equal to 0.8, in order to have a zero fair value of the swap.\(^{28}\)

Looking across the four graphs one can see that the marginal values of liquidity and hedging are quite state specific, reflecting a variety of factors. For instance, comparing the top two graphs, note that when the firm starts with no cash, it accumulates what it makes in the current period, but doesn’t increase the cash balance further given the high cost of issuing equity, whereas in the right graph, the firm’s marginal values of cash are much higher at the lower cash balance levels since the firm already has a significant cash balance. Focusing on the two graphs in the left column, the firm in the lower row has a higher marginal value of cash given the risk of distress associated with a low $\theta_1$ value, and thus it is willing to issue equity in order to create a liquidity position that can provide protection against distress, and allow it to also potentially close production in the following period if profitability remains low. In the bottom right graph, the

\(^{27}\)Note that the values in the right column reflect the fact that the firm starts with $b = 0.4$, and thus are roughly 0.4 higher than the corresponding graphs in the left column.

\(^{28}\)While the fixed swap price, $s$, received each period is equal to one, and thus the periodic cash flow $s - \theta_2$ equals zero when $\theta_2 = 1$, the non-linearity of the swap valuation captured in Equation (5) (due to the distributional assumptions, the effect of credit risk, and the right to close the contract at any date) is such that $SF(0.8) \approx 0$. 
firm begins with a substantial cash balance and uses a large part of this to close down production, so this situation presents yet another distinct set of marginal values.

One can also see from the graphs that the marginal value of hedging appears to be less state dependent than the marginal value of cash. Since there are relatively low costs associated with rebalancing a hedge, there is less of a hysteresis effect, i.e., the existing swap position does not strongly affect the choice of future hedge position. In contrast, the choice of cash balance level is potentially affected by significant frictions such as issuance costs and convex personal taxes associated with issuing equity, as well as the tax penalty of holding cash in the firm. The lower sensitivity of the marginal value of hedging as compared to that for liquidity also reflects to some degree the relatively smaller contribution of hedging versus liquidity to the overall value of integrated risk management.

We now turn to examine operating flexibility, which is the third key component of the integrated risk management program in our framework. We focus in particular on how this risk management tool interacts with the financial risk management (FRM) mechanisms, namely liquidity and hedging. Table IV addresses this issue, showing value creation due to risk management under varying scenarios.

The first row of Table IV shows the value increase due to FRM if there is no operating flexibility (so the firm must always remain open). These values are shown for a selection of $\theta_1$ values around the long-term mean.\textsuperscript{29} The second row also shows the value increase due to FRM, but where there already is operating flexibility. The values of FRM are lower in the second row than in the first row, since operating flexibility also exists. In other words, there is some substitutability between the two forms of risk

\textsuperscript{29}Note that in looking at percentage value increases, these increases become extremely high for small $\theta_1$ values given that firm values can be very low in those cases, particularly when there is no FRM or operating flexibility.
management. This is consistent with a growing body of empirical evidence examining the interaction between financial hedging and operating flexibility across different industries, including Allayannis, Ihrig, and Weston (2001), Bartram (2008), Bartram, Brown, and Fehle (2009), Hankins (2009), Pantzalis, Simkins, and Laux (2001) and Petersen and Thiagarajan (2000). However, note that FRM still adds significant value even in the presence of operating flexibility. This is true since production is not completely flexible and the firm is still susceptible to losses, whose effects can be mitigated by using liquidity and hedging.

In the second set of rows, we see similar effects by turning the relationship around, examining the value of operating flexibility with and without FRM. We find that operating flexibility can greatly enhance firm value even if financial risk management mechanisms are in place. However, the value due to operating flexibility is considerably higher if this is the only source of risk management. Finally, in the last row we show the total potential of having an integrated risk management system comprised of both operating flexibility and financial risk management relative to the value of the firm when it is completely exposed to the impact of uncertainty without any tools to control the value loss associated with such risk. The values can clearly be quite high, though keep in mind that the percentage value increases for low $\theta_1$ values reflect the very low firm values when there is no form of risk management mechanism at work.

30While there is an extensive literature on how flexibility in operation and supply chain management mitigates and exploits uncertainty, there has also been increasing recognition of the importance of strategic acquisitions as an operational risk management mechanism. Recent examples include Garfinkel and Hankins (2010) who find vertical merger activity picks up in response to an increase in risk exposure, and Duchin (2010) who finds that multi-division firms hold about half the cash that standalone firms do due to reduced risk associated with diversification.
D. Agency Problems due to Speculation

Much attention in the academic literature and in practice has been paid to the fact that derivatives can be readily used for speculative purposes within a corporation. In this section, we investigate the value impact of speculative behavior that might arise as a result of risk-shifting incentives driven by financial agency problems. We examine situations where this problem is of greatest concern, and discuss the extent to which position limits that can be effectively imposed and monitored can mitigate such problems.

In the top panel of Figure 6, we examine the percentage increase in firm value due to having a cash balance and a swap with $\rho = 0.1$, under three scenarios: 1) equity value maximization (“SB” for second-best); 2) equity value maximization, but with $h$ capped at 0.5 (“constrained”); and 3) firm value maximization (“FB” for first-best, which was also shown in Figure 2). One can see that the value created is significantly lower when decisions are made to maximize equity value rather than firm value. Given that $\theta_1$ and $\theta_2$ have low correlation, there are cases where the firm may increase the hedge position beyond the level which would serve to maximize firm value, in order to increase risk and take advantage of the positive effect of risk on shareholder value. However, if position limits can be placed on $h$ (and effectively monitored), this can help to significantly mitigate this agency problem due to speculation. The “constrained” case in the top panel illustrates that this position limit can almost restore the firm to its first-best value.

In the lower panel of Figure 6, the bottom two curves capture the firm value and equity value maximization cases when there is no cash balance. Without cash to provide some buffering effect on firm risk and at least a temporary reduction in net debt, equityholders use the swap even more aggressively. Not only does the firm not benefit
from the risk management value of hedging, it loses value as a result of the speculative behavior of shareholders. The top two curves show a contrasting case where the firm holds a cash balance, and also has access to a swap with $\rho = 1$. In this case, speculation is much less likely given that the swap clearly aligns with the risk of the firm. Indeed, we find that there is no perceptible difference between the firm values in the first-best and second-best cases (the curves lie on top of each other in the graph).

Thus, Figure 6 shows that the extent and impact of speculative use of derivatives on firm value will depend on, among other factors, the degree to which the derivatives instruments are correlated with firm risk, the ability to set and enforce position limits, and the extent to which the firm has cash, which provides a buffer against risks and ultimately represents value that would be foregone by shareholders in the event of a derivatives-induced loss.

These effects are also highlighted in Table V, where we show the mean and standard deviation (in parentheses) of the cash and hedging positions under both firm value and equity value maximization. These values are generated based on the simulation described in detail in Appendix C. Note that in the second row (corresponding to the top panel of Figure 6), the hedge level (and its standard deviation) is higher in the equity value maximization case than under firm value maximization. This increased hedge level is even more pronounced in the last row where there is no cash, consistent with the larger value loss due to speculative behavior noted above.

In contrast, when $\rho = 1$, the opportunity to speculate and drive the firm’s risk higher is limited, as noted earlier. As a result, we find that the hedge ratio statistics in the third row of Table V are virtually identical in the first-best and second-best cases, consistent with the similar values observed in the top three curves in the bottom panel of Figure 6.
Finally, note in Table V that the cash position decreases as the hedge becomes more effective (particularly increasing from $\rho = 0.5$ to $\rho = 1$), and that the cash position does not appear to be affected by financial agency problems.\textsuperscript{31} Similarly, while the probability of being open (the “Operate” column) differs somewhat from case to case, it is not greatly affected by agency conflicts, other than equityholders appearing to keep production open longer since they are less willing to incur the closing cost given that they may soon exercise their limited liability option. Overall, it is clear that the swap presents the best opportunity for equityholders to engage in risk shifting, and this incentive must be appropriately constrained by position limits that are carefully monitored.

\section*{IV. Conclusions}

Our framework incorporates several rationales for managing risk and examines the coordination of three important risk management mechanisms in a dynamic setting. Our results highlight, among other things, that hedging with derivatives may have a limited contribution to the value created through risk management, that liquidity management serves as a key risk management tool, and that distress costs are a key motivation for managing risk. Furthermore, given the costs associated with holding cash, the inability of derivatives to hedge significant drivers of firm risk and to cover extraordinary expenses such as adjustment costs, and the potential speculative use of derivatives, firms may fall quite short of achieving their risk management goal of eliminating the negative impacts of frictions on firm value.

\textsuperscript{31}Consistent with this result, Disatnik, Duchin, and Schmidt (2010) find that firms that hedge have lower cash flow levels. Similarly, Graham and Rogers (2002) determine that hedging increases a firm’s net debt capacity.
The challenges of designing an integrated risk management system extend beyond the issues that have been incorporated into our model. Most notably, we have not considered asymmetric information and managerial agency problems. Given the significant value potential associated with optimal risk management programs, further normative research into risk management design should be of high priority, coupled with empirical research informed by insights from this line of research.
A. Derivation of Equation (4)

From equation (2), omitting the subscript for simplicity, we have

\[ x(t + i) = \kappa^i x(t) + \overline{\sigma}(1 - \kappa) \sum_{j=0}^{i-1} \kappa^j + \sigma \sum_{j=0}^{i-1} \kappa^j \varepsilon(t + i - j). \]

Hence, the average of \( x(t + i) \) conditional on \( x(t) \) is

\[ \mathbb{E}_t \left[ x(t + i) \right] = \kappa^i x(t) + \overline{\sigma}(1 - \kappa) \sum_{j=0}^{i-1} \kappa^j = \kappa^i x(t) + \overline{\sigma}(1 - \kappa^i) \]

and the variance of \( x(t + i) \) conditional on \( x(t) \) is

\[ \text{Var}_t \left[ x(t + i) \right] = \sigma^2 \sum_{j=0}^{i-1} \kappa^{2j} = \sigma^2 \frac{1 - \kappa^{2i}}{1 - \kappa^2}. \]

The future price as of \( t \), given the current value \( \theta_2(t) = \theta_2 = e^{x(t)} \), for delivery at \( t + i \) is

\[ F_t(\theta_2, t + i) = \exp \left\{ \mathbb{E}_t \left[ x(t + i) \right] + \frac{1}{2} \text{Var}_t \left[ x(t + i) \right] \right\} \]

\[ = \exp \left\{ \kappa^i x(t) \right\} \cdot \exp \left\{ \overline{\sigma}(1 - \kappa^i) \right\} \cdot \exp \left\{ \frac{\sigma^2 \frac{1 - \kappa^{2i}}{2}}{1 - \kappa^2} \right\} \]

\[ = \theta_2^i \theta_2^{(1-\kappa^i)} \exp \left\{ \frac{\sigma^2 \frac{1 - \kappa^{2i}}{2}}{1 - \kappa^2} \right\}, \]

where \( \overline{\theta}_2 = e^{\overline{x}}. \)

To obtain a price of the swap consistent with a discrete-time version of Model I in Schwartz (1997), i.e., a one-factor, mean reverting process, the long term mean must be \( \overline{x} = \frac{1}{2} \sigma^2/(1 - \kappa) \) in place of \( \overline{x} \).
B. Approximation of the stationary distribution of the state variable

Given the dynamics in (1) and (2), using a vector notation we have a VAR of the form

\[ x(t) = c + K x(t-1) + \varepsilon(t) \]

where \( \varepsilon = (\varepsilon_1, \varepsilon_2) \) is a bivariate Normal variate with zero mean and covariance matrix \( \Sigma \),

\[ c = \begin{pmatrix} (1 - \kappa_1) \bar{x}_1 \\ (1 - \kappa_2) \bar{x}_2 \end{pmatrix}, \quad K = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1 \sigma_2 \rho \\ \sigma_1 \sigma_2 \rho & \sigma_2^2 \end{pmatrix}, \]

where, differently from the approach proposed in Tauchen (1986), \( \Sigma \) may be non-diagonal and singular (when \( \rho = 1 \)).

Given this alternative assumption for the error covariance matrix, we follow an efficient approach first proposed by Knotek and Terry (2011) based on numerical integration of the multivariate Normal distribution. In particular, the continuous-state process \( x(t) \) is approximated by a discrete-state process \( \hat{x}(t) \), which varies on the finite grid \( \{X_1, X_2, \ldots, X_n\} \subset \mathbb{R}^2 \). Define a partition of \( \mathbb{R}^2 \) made of \( n \) non-overlapping 2-dimensional intervals \( \{\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n\} \) such that \( X_i \in \mathcal{X}_i \) for all \( i = 1, \ldots, n \) and \( \bigcup_{i=1}^n \mathcal{X}_i = \mathbb{R}^2 \). The \( n \times n \) transition matrix \( \Pi \) is defined as

\[ \Pi_{i,j} = \text{Prob} \{ \hat{x}(t+1) \in \mathcal{X}_j \mid \hat{x}(t) = X_i \} \]

\[ = \text{Prob} \{ c + K x(t) + \varepsilon(t+1) \in \mathcal{X}_j \} \]

\[ = \text{Prob} \{ \varepsilon(t+1) \in \mathcal{X}_j' \} = \int_{\mathcal{X}_j'} f(x, 0, \Sigma) dx \]
where $X'_j = X_j - c - Kx(t)$, and $f(x, 0, \Sigma)$ is the density of the bivariate Normal with mean zero and covariance matrix $\Sigma$. The integral on the last line is computed numerically following the Monte–Carlo integration approach by Genz (1992) (extended to the singular covariance matrix case by Genz and Kwong (2000)).\footnote{We use both Genz’s and Knotek and Terry’s routines for this part of the program.} As for the choice of the grid, we follow Tauchen (1986) using a uniformly spaced scheme.

\textbf{C. Monte Carlo simulation}

Given the optimal policy function $\varphi(\cdot)$ from the solution of the valuation problem (either first or second best), we use Monte Carlo simulation to generate a sample of $\Omega$ possible future paths for our firm. In particular, we obtain the simulated dynamics of the state variable $\theta = (\theta_1, \theta_2)$ under the actual (as opposed to the risk–neutral) probability by application of the recursive formula

$$x(t) = c^* + Kx(t - 1) + \varepsilon(t)$$

where $\varepsilon(t)$ are independent samples for a bivariate Normal distribution, with covariance matrix $\Sigma$,

$$c^* = \begin{pmatrix} (1 - \kappa_1)x_1^* \\ (1 - \kappa_2)x_2^* \end{pmatrix}, \quad K = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1 \sigma_2 \rho \\ \sigma_1 \sigma_2 \rho & \sigma_2^2 \end{pmatrix},$$

and $x_i^* = \pi_i + RP_i$, for $i = 1, 2$. The risk premia are $RP_1 = 6\%$ and $RP_2 = .6\%$ (consistent with $\rho = 0.1$). Then we compute $\theta_i(t) = e^{x_i(t)}$, for $i = 1, 2$. 

\[32\]
We set as initial points for our simulation $x_1(0) = x_2(0) = 0$, and $m_0 = 0$, $b_0 = 0$, $h_0 = 0$. Based on this, we determine the optimal choice $(m_1, b_1, h_1) = \varphi(\theta_0, m_0, b_0, h_0)$. Then for the specific realization $\theta_1(\omega)$, if the firm is not in default, we apply the optimal policy function, obtaining $(m_2, b_2, h_2) = \varphi(\theta_1(\omega), m_1, b_1, h_1)$. Otherwise, if the firm is in default, $(m_2, b_2, h_2) = (m_1, d, 0)$.

The subsequent steps are a recursive application of the same principle:

$$(m_{t+1}, b_{t+1}, h_{t+1}) = \varphi(\theta_t(\omega), m_t, b_t, h_t)$$

if the firm is solvent and

$$(m_{t+1}, b_{t+1}, h_{t+1}) = (m_t, d, 0)$$

in case of default, for $t = 1, \ldots, T$ and for $\omega = 1, \ldots, \Omega$. Given the realized state $(\theta_t(\omega), m_t, b_t, h_t)$, we determine all the quantities of interest (e.g., value of equity, level of cash balance) from the simulated sample.

In our numerical experiments, we generate simulated samples with $\Omega = 10,000$ paths and $T = 150$ years (steps). To limit the dependence of our results on the initial conditions, we drop the first 50 steps.
References


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Tauchen, George, 1986, Finite State Markov-chain Approximations to Univariate and


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Table III: Base Case Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x}_1$</td>
<td>long-term mean of product price variable</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{x}_2$</td>
<td>long-term mean of underlying asset for swap</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>annual volatility of $x_1$</td>
<td>15%</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>annual volatility of $x_2$</td>
<td>15%</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>persistence of $x_1$</td>
<td>0.80</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>persistence of $x_2$</td>
<td>0.80</td>
</tr>
<tr>
<td>$\rho$</td>
<td>correlation between product price and swap</td>
<td>0.1</td>
</tr>
<tr>
<td>$s$</td>
<td>swap price</td>
<td>1</td>
</tr>
<tr>
<td>$A$</td>
<td>fixed production cost</td>
<td>.97</td>
</tr>
<tr>
<td>$q$</td>
<td>production rate</td>
<td>1</td>
</tr>
<tr>
<td>$K^o$</td>
<td>opening cost</td>
<td>.1</td>
</tr>
<tr>
<td>$K^c$</td>
<td>closing cost</td>
<td>.3</td>
</tr>
<tr>
<td>$r$</td>
<td>annual risk-free borrowing rate</td>
<td>5%</td>
</tr>
<tr>
<td>$d$</td>
<td>debt level</td>
<td>.4</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>personal tax rate on equity cash flows</td>
<td>12%</td>
</tr>
<tr>
<td>$\tau_d$</td>
<td>personal tax rate on bond coupons</td>
<td>25%</td>
</tr>
<tr>
<td>$\tau^+_c$</td>
<td>corporate tax rate for positive earnings</td>
<td>35%</td>
</tr>
<tr>
<td>$\tau^-_c$</td>
<td>corporate tax rate for negative earnings</td>
<td>30%</td>
</tr>
<tr>
<td>$dc$</td>
<td>proportional distress cost</td>
<td>15%</td>
</tr>
<tr>
<td>$nc$</td>
<td>swap negotiation cost</td>
<td>1%</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>variable flotation cost for equity</td>
<td>5%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>proportional bankruptcy costs</td>
<td>10%</td>
</tr>
<tr>
<td>Percentage Value Increase</td>
<td>$\theta_1 = .69$</td>
<td>$\theta_1 = .83$</td>
</tr>
<tr>
<td>---------------------------</td>
<td>---------------</td>
<td>---------------</td>
</tr>
<tr>
<td>Value of FRM, no OperFlex</td>
<td>60.7</td>
<td>28.8</td>
</tr>
<tr>
<td>Value of FRM, with OperFlex</td>
<td>25.8</td>
<td>19.6</td>
</tr>
<tr>
<td>Value of OperFlex, no FRM</td>
<td>74.6</td>
<td>33.1</td>
</tr>
<tr>
<td>Value of OperFlex, with FRM</td>
<td>36.6</td>
<td>23.6</td>
</tr>
<tr>
<td>Value of OperFlex and FRM</td>
<td>119.6</td>
<td>59.2</td>
</tr>
</tbody>
</table>

Table IV: **Contribution of Operating Flexibility to the Value of Risk Management.**
The table shows the interaction between the value of Financial Risk Management (FRM), i.e., the value gained by dynamically managing cash balance and swap positions, and the value of Operating Flexibility (OperFlex) gained from being able to close (and re-open) production subject to the base case value of $K^c = .3$ (and $K^o = .1$), relative to not having the option to close down. The first row shows the percentage value creation from having FRM if the firm does not have operating flexibility, while the second row shows the value increment of FRM if the firm does have operating flexibility. Similarly, the third and fourth rows show the percentage value addition from having operating flexibility if the firm does not have, or has, FRM, respectively. Finally, the fifth row shows the percentage value increase from having both operating flexibility and FRM relative to having neither. All values are shown for four representative $\theta_1$ levels. The values are computed based on the numerical solution of the model using 9 points for each $\theta$, 19 points for $h$, and 18 points for $b$, based on the parameter values in Table III, and current values of $h = 0$, $b = 0$ and $\theta_2 = 1$. 

<table>
<thead>
<tr>
<th></th>
<th>Cash</th>
<th>Hedge</th>
<th>Operate</th>
<th></th>
<th>Cash</th>
<th>Hedge</th>
<th>Operate</th>
</tr>
</thead>
<tbody>
<tr>
<td>CB, nH</td>
<td>0.396</td>
<td>0.000</td>
<td>0.597</td>
<td>CB, H((\rho = 0.1))</td>
<td>0.398</td>
<td>0.000</td>
<td>0.601</td>
</tr>
<tr>
<td></td>
<td>(0.287)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td>(0.287)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>CB, H((\rho = 0.1))</td>
<td>0.360</td>
<td>0.030</td>
<td>0.607</td>
<td>CB, H((\rho = 0.5))</td>
<td>0.352</td>
<td>0.048</td>
<td>0.614</td>
</tr>
<tr>
<td></td>
<td>(0.270)</td>
<td>(0.078)</td>
<td>(0.000)</td>
<td></td>
<td>(0.271)</td>
<td>(0.106)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>CB, H((\rho = 1))</td>
<td>0.358</td>
<td>0.051</td>
<td>0.606</td>
<td>CB, H((\rho = 1))</td>
<td>0.333</td>
<td>0.069</td>
<td>0.617</td>
</tr>
<tr>
<td></td>
<td>(0.265)</td>
<td>(0.086)</td>
<td>(0.000)</td>
<td></td>
<td>(0.257)</td>
<td>(0.112)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>CB, H((\rho = 1))</td>
<td>0.189</td>
<td>0.188</td>
<td>0.688</td>
<td>CB, H((\rho = 1))</td>
<td>0.189</td>
<td>0.189</td>
<td>0.692</td>
</tr>
<tr>
<td></td>
<td>(0.149)</td>
<td>(0.190)</td>
<td>(0.000)</td>
<td></td>
<td>(0.149)</td>
<td>(0.191)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>nCB, H((\rho = 0.1))</td>
<td>0.000</td>
<td>0.041</td>
<td>0.678</td>
<td>nCB, H((\rho = 0.1))</td>
<td>0.000</td>
<td>0.174</td>
<td>0.680</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.091)</td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
<td>(0.198)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

Table V: **Cash, Hedging and Operating Policies.** The table shows the mean and standard deviation (in parentheses) of the levels of cash balance \(b\), hedge \(h\) and operating activity (assumed to be 1 if operating, 0 if closed) for five different cases: for the first four rows, there is a cash balance (“CB”) and either no hedge (first row, “nH”), or a hedge (“H”) with correlation \(\rho\) of either 0.1, 0.5 or 1.0, in the second, third and fourth rows respectively; the fifth row allows for a hedge with \(\rho = 0.1\), but no cash balance (“nCB”). The first set of three columns corresponds to cases run under a first-best firm value optimization, while the second set of columns corresponds to second-best equity value maximization. The moments are based on the simulation described in Appendix C, using 10,000 runs. All other parameter values are shown in Table III.
Figure 1: Relative Impact of Various Motivations for Risk Management. The figure shows the value of risk management when different underlying frictions are turned on and off, thus allowing the relative impact of each motivation for risk management to be analyzed. The Base Case shows the value of risk management when all frictions are present (using the parameter values in Table III). The case of “No Frictions, Linear Tax” has no distress, bankruptcy or issuance costs, and linear personal and corporate taxes (i.e., \( \tau_c^+ = \tau_c^- = .40 \), and \( \lambda = -\tau_e = -12 \)), confirming that there is no value associated with risk management in the absence of frictions. The other cases are intermediate cases where only the one cost indicated in the case name is equal to its base case value, while the others are as in the “No Frictions” case (the case of "Issuance Cost and Distress Costs" is the one case where two frictions are simultaneously present, as specified in the model). The values are from the numerical solution of the model using 9 points for each \( \theta \), 19 points for \( h \), and 18 points for \( b \), based on the parameter values in Table III, except as indicated otherwise above. The plots are based on initial values of \( h = 0 \), \( b = 0 \), and \( \theta_2 = 1 \).
Figure 2: Relative Contribution of Liquidity and Hedging to the Value of Risk Management. The figure shows the value increase attributable to risk management under three different risk management scenarios: 1) dynamically managed cash balance and no swap; 2) dynamically managed hedge and no cash balance; and 3) both cash balance and swap positions are dynamically managed. The underlying asset for the swap ($\theta_2$) has a correlation ($\rho$) of 0.1 with the product price ($\theta_1$). The percentage increase in firm value from risk management is plotted against $\theta_1$. The values are from the numerical solution of the model using 9 points for each $\theta$, 19 points for $h$, and 18 points for $b$, based on the parameter values in Table III. The plots are based on initial values of $h = 0$, $b = 0$, and $\theta_2 = 1$. 
Figure 3: **Effect of Swap Correlation on Contribution of Hedging to the Value of Risk Management.** The figure shows the increase in firm value due to risk management assuming the firm holds an optimally rebalanced cash position, and either has no swap (lowest curve), or can dynamically manage a swap position, where the price of underlying asset of the swap ($\theta_2$) has correlation of 0.1, 0.5, or 1.0 with the firm’s underlying uncertainty ($\theta_1$), represented by the successively higher curves on the graph. The percentage increase in firm value is plotted against $\theta_1$. The values are from the numerical solution of the model using 9 points for each $\theta$, 19 points for $h$, and 18 points for $b$, based on the parameter values in Table III. The plots are based on initial values of $h = 0$, $b = 0$, and $\theta_2 = 1$. 
Figure 4: **Risk Management with a Perfectly Correlated Swap or with a Perfect Hedge.** The figure shows the percentage increase in firm value due to risk management under three different risk management scenarios: 1) no cash balance, but a swap with perfect correlation ($\rho = 1$); 2) cash balance, and a swap with $\rho = 1$; and 3) a “perfect hedge” (equivalent to the “no frictions” case, as explained in the text). The percentage increase in firm value is plotted against $\theta_1$, the product price. The values are from the numerical solution of the model using 9 points for each $\theta$, 19 points for $h$, and 18 points for $b$, based on the parameter values in Table III. The plots are based on initial values of $h = 0$, $b = 0$, and $\theta_2 = 1$. 


Figure 5: Marginal Values of Liquidity and Hedging. The four graphs show firm values plotted against potential new hedge ratio \((h')\) and cash level \((b')\) selections, assuming four different current states (i.e., four different \((\theta_1, \theta_2, m, b, h)\) combinations). The top row assumes an average \(\theta_1\) value (1.0), and the bottom row assumes a lower than average \(\theta_1\) value (0.8). \(\theta_2 = 0.8\) in all graphs. The left column assumes there is currently no hedge and a zero cash balance \((h = b = 0)\). The right column assumes average levels of \(h = 0.03\) and \(b = 0.4\). The optimal choice of \(h'\) and \(b'\) levels are shown above each graph, and can be seen to generate the highest value level on the surface (also seen by looking at the iso-value curves projected on the bottom plane). The optimal production decision for the firm in each panel is also shown above each graph as \(m'\), where a value of 1 signifies open production and 0 is closed. The values are from the numerical solution of the model using 9 points for each \(\theta\), 19 points for \(h\), and 18 points for \(b\), based on the parameter values in Table III.
Figure 6: **Agency Costs Associated with Speculation.** This figure illustrates the impact of potential speculation with derivatives on firm value. In the upper panel, the firm manages a liquidity balance and a hedge with $\rho = 0.1$. The three curves correspond to: first-best firm value optimization ("FB"), second-best equity value maximization ("SB"), and second-best value if a position limit of $h < 0.5$ is imposed ("constrained"). In the lower panel, there are two sets of curves. The bottom two curves represent situations where the firm does not hold a cash balance, but has a hedge with $\rho = 0.1$, and either maximizes firm value ("FB") or equity value ("SB"). For the top three curves in the lower panel, the firm manages a cash balance and has a swap with $\rho = 1$, and the cases considered are the first-best, second-best, and constrained second-best case where $h < 0.5$. All percentage firm value increases due to risk management are plotted against $\theta_1$. The values are from the numerical solution of the model using 9 points for each $\theta$, 19 points for $h$, and 18 points for $b$, based on the parameter values in Table III. The plots are based on initial values of $h = 0$, $b = 0$, and $\theta_2 = 1$. 

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