

# Overcoming Borrowing Stigma: The Design of Lending-of-Last-Resort Policies

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## Abstract

How should the government effectively provide liquidity to banks during periods of financial distress? During the most recent financial crisis, banks avoided borrowing from the Fed's Discount Window (DW) but actively participated in its Term Auction Facility (TAF), although both programs shared the same borrowing requirements. Moreover, banks bid and paid higher interest rates in TAF than the concurrent discount rate in DW. Using a model with endogenous borrowing stigma costs, we explain how the combination of DW and TAF increased banks' borrowings and willingnesses to pay for loans from the Fed. Using micro-level data on DW borrowing and TAF bidding from 2007 to 2010, we confirm our theoretical predictions about the financial conditions of banks in different facilities.

**Keywords:** discount window stigma, auction, adverse selection, lending of last resort

**JEL:** G01, D44, E58

“For various reasons, including the competitive format of the auctions, TAF has not suffered the stigma of conventional discount window lending and has proved effective for injecting liquidity into the financial system... Another possible reason that TAF has not suffered from stigma is that auctions are not settled for several days, which signals to the market that auction participants do not face an immediate shortage of funds.”

– [Bernanke \(2010\)](#) to the U.S. House of Representatives

## 1 Introduction

Financial crises are typically accompanied by liquidity shortage in the entire banking sector. How should the central bank lend to depository institutions during such episodes? The answer is not obvious. The discount window (DW) has been the primary lending facility used by the Federal Reserve, but it was severely under-utilized when the interbank market froze at the beginning of the financial crisis in late 2007. A main reason for the under-utilization is believed to be the stigma associated with DW borrowing: tapping the discount window conveys a negative signal about the borrowers’ financial conditions to their counterparties, competitors, regulators, and the public.<sup>1</sup> As suggestive evidence, banks have regularly paid more for loans from the interbank market than they could readily get from DW ([Peristiani, 1998](#); [Furfine, 2001, 2003, 2005](#)).

[Figure [1a](#) and [1b](#) about here]

In response to the credit crunch and banks’ reluctance to borrow from DW, the Fed created a temporary program, the Term Auction Facility (TAF), in December 2007. TAF held an auction every other week, providing a pre-announced amount of loans with *identical* loan maturity, collateral margins, and eligibility criteria as DW.

Surprisingly, TAF provided much more liquidity than DW: Figure [1a](#) shows that the outstanding balance in TAF far exceeded that in DW during 2007-2010.<sup>2</sup> Even more surprisingly, banks sometimes paid a higher interest rate to obtain liquidity through the auction: Figure [1b](#) shows that the stop-out rate—the rate that cleared the auction—was higher than the concurrent discount rate—the

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<sup>1</sup>Although the Fed does not disclose publicly which institutions have received loans from DW, the Board of Governors publishes weekly the total amount of DW lending by each of the twelve Federal Reserve Districts. Therefore, a surge in total DW borrowing could send the market scrambling to identify the loan recipients. Because of the interconnectedness of the interbank lending market, it is not impossible for other banks to infer which institutions went to the discount window. Market participants and social media could also infer from other activities.

<sup>2</sup>Outstanding balance in DW made up at most 33.4 percent of the total outstanding balance between 2007 and 2010. See Figure [2a](#) in the appendix for DW balance as a percent of the total balance week by week between 2007 and 2010.

rate readily available in DW—in 21 out of the 60 auctions, especially from March to September 2008, the peak of the financial crisis.<sup>3</sup>

This episode suggests the importance of the design of emergency lending programs to effectively cope with liquidity shortage. More specifically, it raises a series of questions about lending-of-last-resort policies. Why could TAF overcome the stigma and generate more borrowing than DW? Shouldn't the same stigma also prevent banks from participating in TAF? How did banks decide to borrow from DW and/or TAF? Was there any systematic difference between the banks that borrowed from the two facilities? How to further improve the program? The answers to these questions remain unclear, even to policy makers involved ([Armantier and Sporn, 2013](#); [Bernanke, 2015](#)).

This paper provides a comprehensive analysis of lending of last resort in the presence of borrowing stigma. We introduce a model in which banks have private information about their financial conditions. Weaker banks have more urgent liquidity need and enjoy higher borrowing benefits. Two lending facilities are available. An auction is held once to allocate a set amount of liquidity, and DW is always available—before, during, and after the auction. Borrowing from each facility incurs a stigma cost, which is endogenously determined by the financial conditions of participating banks.

In equilibrium, banks self select into different programs. Since DW always guarantees lending, the weakest banks borrow from it immediately, because they are desperate for liquidity and cannot afford to wait. Stronger banks, in contrast, are lured to participate in the auction because the potential of borrowing cheap renders the auction more attractive than DW. Their liquidity needs are not that imperative and they value lower expected price in the auction more than their weaker counterparts. Among the banks who participate in TAF, some may bid higher than the discount rate because they would like to avoid the discount window stigma brought by the association with the weakest banks. As a result, the clearing price in the auction may exceed the discount rate. Among the banks who have lost in TAF, relatively weaker ones might still borrow from DW.

Our model demonstrates that the introduction of TAF in addition to DW could increase liquidity provision through three channels. First, by setting a low reserve price in the auction, TAF attracted relatively weak banks to participate and take their chances of borrowing cheap. Second, participating banks can submit bids to internalize any stigma cost associated with TAF, so TAF also attracted relatively strong banks to participate. Finally, due to the selection by stronger banks into the auction, the auction stigma is endogenously lower than the discount window stigma. Hence,

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<sup>3</sup>The stop-out rate ranged from 1.5 percentage points above (September 25, 2008) to 0.83 percentage points below (December 4, 2008) the concurrent discount rate. The stop-out rate was above the concurrent discount rate for almost all auctions between March 2008 (when Bear Sterns filed for bankruptcy) and September 2008 (when Lehman Brothers filed for bankruptcy). See [Figure 2b](#) for the difference between the stop-out rate and the concurrent discount rate auction by auction.

the combination of TAF and DW expands the set of the banks who try to and may obtain liquidity, thus increasing the overall supply of short-term credit to the economy.

We use granular data on DW and TAF borrowing during the crisis to verify one of the model's main predictions that weaker banks were more likely to borrow from DW. We show that compared to TAF banks, DW banks have higher leverage, lower tier-1 capital to risk weighted asset ratio, and a lower fraction of private-labeled MBS on their balance sheets. Moreover, exploring the credit guarantee programs implemented in Canada, France, and Germany in October 2008, we show that following these policies, Canadian, German, and French banks increased their borrowing from TAF auctions and reduced their borrowing from DW, compared to their peers in the U.S. Moreover, we show that among the banks who participated in TAF, those who submitted higher bids (and thus were more likely to be winners) pledged collaterals of lower quality and were more likely to bid again in subsequent auctions (a sign of weakness). Finally, we show that prior to the borrowing dates, DW banks have persistently higher CDS spreads than TAF banks, implying that they faced higher risks of default.

Our paper improves the understanding of interventions during the financial crisis, and more specifically, contributes to the literature that studies government intervention in markets plagued by adverse selection ([Philippon and Skreta, 2012](#); [Tirole, 2012](#); [Ennis and Weinberg, 2013](#); [La'O, 2014](#); [Lowery, 2014](#); [Fuchs and Skrzypacz, 2015](#); [Gauthier et al., 2015](#); [Li et al., 2016](#); [Ennis, 2017](#); [Che et al., 2018](#)). In these studies, either there is no explicit stigma cost in government-sponsored facility, or stigmas are implicitly assumed to be uniform across all programs. Our paper endogenizes the different stigma costs associated with DW and TAF as well as banks' heterogeneous decisions on which facility to use.

This paper, to the best of our knowledge, is the first to combine micro-level data on DW borrowing and TAF bidding and link them to information on banks' fundamentals. Existing papers largely focus on empirical estimates of either DW stigma or subsequent economic effects of TAF borrowing. [Peristiani \(1998\)](#); [Furfine \(2001, 2003, 2005\)](#) offer evidence that banks prefer the Federal Funds Market to DW, suggesting the existence of DW stigma. More recently, [Armantier et al. \(2015\)](#) show that more than half of TAF participants submitted bids above the discount rate during the 2007-2008 financial crisis. [McAndrews et al. \(2017\)](#) and [Wu \(2011\)](#) study the effect of TAF and conclude that it was effective in lowering Libor and reducing liquidity concern in the interbank lending market. [Moore \(2017\)](#) finds that TAF had a benefit on the real economy. [Cassola et al. \(2013\)](#) study the financial crisis from the bidding data in the European central bank from January to December 2007 to confirm that the banks were strategic in their bidding.

The rest of the paper is organized as follows. Section 2 describes the lending-of-last-resort facilities during the 2007-2008 financial crisis. Section 3 sets up the model. Section 4 characterizes the equilibrium of the model and discusses liquidity provision under different settings. Section 5

presents empirical evidence consistent with the predictions of the model. Section 6 concludes. The appendix contains omitted proofs, figures, and tables.

## 2 Background

The stress in the interbank lending market began to loom in the summer of 2007. In June, two of Bear Sterns' mortgage-heavy hedge funds reported large losses. On July 31, they declared bankruptcy. On August 9, BNP Paribas, France's largest bank, barred investors from withdrawing money from its investments backed by U.S. subprime mortgages, citing evaporated liquidity as the main reason. Subsequently, many other banks and financial institutions experienced dry-ups in wholesale funding (in the form of asset-based commercial paper or repurchase agreements).

With the growing scarcity of short-term funding, banks were supposed to borrow from the lender of last resort (LOLR). In the United States, the role of LOLR has been largely fulfilled by the discount window, which allows eligible institutions, mostly commercial banks, to borrow money from the Federal Reserve on a short-term basis to meet temporary shortages of liquidity caused by internal and external disruptions.<sup>4</sup> Discount window loans were extended to sound institutions with good collateral. Since its inception a century ago, the Fed has never lost a penny on a discount window loan. However, banks were reluctant to use the discount window, due to the widely held perception that a stigma was associated with borrowing from the Fed. As advised by [Bagehot \(1873\)](#), a penalty—one percentage point above the target federal funds rate—was charged on discount window loans, with the goal to encourage banks to look first to private markets for funding. However, this penalty generated a side effect on banks: Banks would look weak if it became known that they had borrowed from the Fed.

Discount window borrowing was kept confidential.<sup>5</sup> However, banks were nervous that investors, in particular money market participants, could guess when they had come to the window by observing banks' behavior or through careful analysis of the Fed's balance sheet figures, because the Fed has to disclose the level of discount window borrowing at both the aggregate and district level, another potential source of detection.<sup>6,7</sup>

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<sup>4</sup>In history, the discount window was once literally a teller window manned by a lending officer.

<sup>5</sup>The Dodd-Frank act required the disclosure of details of discount window loans after July 2010 on a two-year lag from the date on which the loan is made.

<sup>6</sup>According to [Bernanke \(2015\)](#), Ron Logue, CEO of State Street, approached the Boston Fed and checked whether the weekly district-by-district reporting of loan totals could be eliminated. The request was turned down due to legal feasibility reasons and concerns for market-wide confidence.

<sup>7</sup>The stigma associated with borrowing from the government is also significant in the United Kingdom. [Shin \(2009\)](#) described the storyline of the Northern Rock bank run in the United Kingdom. In the U.K, there was no government deposit insurance. Banks relied on an industry-funded program that only partially protected depositors. On September 13, 2007, BBC evening television news broadcast first broke the news that Northern Rock had sought the Bank of England's support. The next morning, the Bank of England announced that it would provide emergency liquidity

The Fed subsequently made a few changes to the discount window policies. In particular, on August 16, 2007, it halved the interest rate penalty on discount window loans.<sup>8</sup> The maturity of loans was also extended for up to thirty days with an implicit promise for further renewal. Moreover, the Fed tried to persuade some leading banks to borrow at the window, thereby suggesting that borrowing did not equal weakness. On August 17, Timothy Geithner and Donald Kohn hosted a conference call with the Clearing House Association, claiming that the Fed would consider borrowing at the discount window “a sign of strength.” Following the call, on August 22, Citi announced it was borrowing \$500 million for thirty days. JPMorgan Chase, Bank of America, and Wachovia subsequently made similar announcements that they had borrowed the same amount, increasing the total discount window borrowing amount to \$2 billion. However, the four big banks—with the borrowing stigma in mind—made it very clear in their announcements that they did not need the money. Thirty days later, the discount window borrowing fell back to \$207 million.<sup>9</sup>

To further relieve the stress in the short-term lending market, the Fed implemented the term auction facility in December 2007. The first auction held on December 17 released \$20 billion in the form of 28-day loans. The participation requirement was the same for the auction as for DW.<sup>10</sup> The Fed received over \$63 billion in bids and released the full \$20 billion to 93 different institutions. In February 2008, Dick Fuld, CEO of Lehman Brothers, urged the Board to include Wall Street investment banks in the regular TAF auctions, which would require invoking Section 13(3) to allow the Fed to have authority to lend to non-bank institutions. The final auction was held on March 8, 2010.

As shown in Figure 1, TAF was clearly more successful than DW in providing liquidity. Banks were also willing to pay a higher interest rate in TAF than the concurrent discount rate in DW.<sup>11</sup>

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support. It was only *after* that announcement, that is, after the central bank had announced its intervention to support the bank, that retail depositors started queuing outside the branch offices. Another story is in August 2007. Barclays tapped twice the emergency lending facility offered by the Bank of England. News first came out on Thursday, August 30, when the Bank of England said it had supplied almost 1.6 billion pounds as lender of last resort, without naming the borrower(s). Journalists and the market scrambled to find out. Barclays first declined to confirm that it had used the central bank’s standing borrowing facility. Later on, it cited technical breakdown in the UK clearing system as the reason for the large pile of cash. In its statement, Barclays said: “The Bank of England sterling standby facility is there to facilitate market operations in such circumstances. Had there not been a technical breakdown, this situation would not have occurred.” Its share fell 2.5 pounds immediately after the statement, which casted doubt on its 45 billion pounds bid to take over the Dutch bank ABN Amro.

<sup>8</sup>On December 11, 2007, the Fed lowered its discount rate to 4.75%.

<sup>9</sup>Records released later show that JPMorgan and Wachovia returned most of the money the next day, whereas Bank of America and Citi, already showing signs of problems, kept the money for a month.

<sup>10</sup>The rule of the auction was as follows. On Monday, banks phoned their local Fed regional banks to submit their bids specifying their interest rate (and loan amount) and to post collaterals. On Tuesday, the Fed secretly informed the winners and publicly announced the stop-out rate (as well as the number of banks receiving loans), determined by the highest losing bid (or the minimum reserve price if the auction was under-subscribed). On Thursday, the Fed released the loans to the banks. Throughout the whole auction process, banks were free to borrow from DW. The following Monday, each regional Fed published total lending from last week; banks may be inferred from these summaries or other channels.

<sup>11</sup>To this date, the specific reason for why TAF was more successful is still unclear. According to [Bernanke \(2015\)](#),

As acknowledged in [Bernanke \(2015\)](#), before implementing TAF, the policy makers were also concerned that the stigma that had kept banks away from the discount window would be attached to the auctions. The program was implemented as “give it a try and see what happens,” but turned out to be quite successful.

### 3 The Model

There are  $n$  banks. Each bank is endowed with an illiquid asset that pays off a return at the end. Before the asset pays off, a liquidity shock may hit a bank. The probability a bank may be affected by the liquidity shock is privately known by the bank. Each bank can borrow from two facilities: discount window (DW), which provides liquidity before any liquidity shock hits, and term auction facility (TAF), which only provides liquidity after an early liquidity shock may have hit the bank. Borrowing banks may incur a penalty if detected of borrowing. The penalty depends on the facility one borrows from and also the average financial condition of the other banks who borrow from the same facility. [Figure 1](#) sketches the timing and sequence of events, which we will describe in detail next.

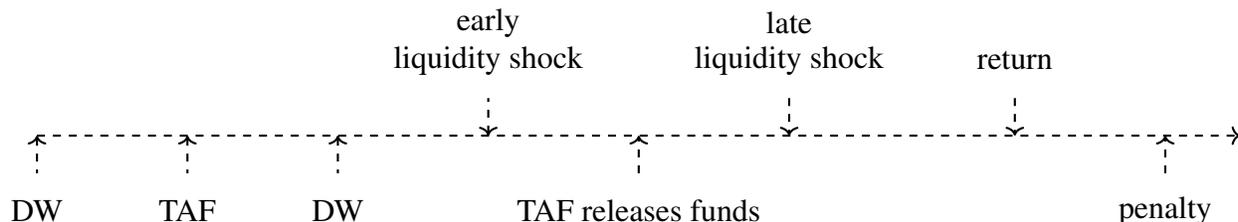


Figure 1: Timeline of the model

#### 3.1 Preferences, Technology, and Shocks

All banks are risk neutral and do not discount future cash flows. Each bank has one unit of long-term, illiquid assets that will mature at the end. The asset generates cash flows  $R$  upon maturity but nothing if liquidated early. Each bank may be hit with a liquidity shock à la [Holmström and Tirole \(1988\)](#). Let  $1 - \theta_i \in [0, 1]$  be the probability that the liquidity shock hits bank  $i$ , where  $\theta_i$  follows the independently and identically distributed cdf  $F$  with associated pdf  $f$  on the support  $[0, 1]$ . Assume  $F$  is log-concave. This assumption is not restrictive, as many standard distributions satisfy it.<sup>12</sup> It is imposed to guarantee a unique equilibrium. Throughout the paper, we assume

the auction *might possibly* reduce the stigma because interest rates would be set through a competitive auction rather than fixed. In that case, borrowers could claim they were paying a market rate, not a penalty rate.

<sup>12</sup>Distributions with a log-concave pdf, which implies a log-concave cdf, include normal, exponential, uniform over any convex set, logistic, extreme value, Laplace, chi, Dirichlet if all parameters are no less than 1, gamma if the shape

that  $\theta_i$  is private information and only known by the bank itself. We drop subscript  $i$  whenever no confusion arises. Type  $\theta$  is also referred to as a bank's financial strength.<sup>13</sup> We sometimes refer to a type- $\theta$  bank as bank  $\theta$ .

Before the liquidity shock hits, each bank has opportunities to borrow. We will describe the choices of borrowing. For now, let  $r$  be the gross interest rate of a received loan. A loan will help the bank defray the liquidity shock and bring a net benefit of  $(1 - \theta)R$  at the cost of interest rate  $r$ . The expected payoff of bank  $\theta$  from borrowing is  $(1 - \theta)R - r$  if it receives liquidity immediately, and is  $\delta(1 - \theta)R - r$  if with probability  $(1 - \delta)$  an early liquidity shock hits before it receives the liquidity.<sup>14</sup> As it becomes clear later on, the specific functional form of the borrowing benefit does not matter for any of our results. What matters is that the benefit is lower if the bank is stronger or if the interest rate is higher.

We describe the two lending facilities in the next subsection.

## 3.2 Lending Facilities

Any bank is able to borrow from either the discount window or the term auction facility.<sup>15</sup>

### 3.2.1 Discount Window

The discount window is a facility that offers loans at a fixed (gross) interest rate  $r_D$ , which is commonly referred to as the discount rate and is exogenously set by the Federal Reserve. Since a bank can always borrow from the discount window with certainty, the net borrowing benefit is  $(1 - \theta)R - r_D$ .

### 3.2.2 Term Auction Facility

The term auction facility allocates pre-announced  $m$  units of liquidity through an auction. In the auction, banks who decide to participate simultaneously submit their sealed bids, which are restricted to be higher than the pre-announced minimum bid  $r_A$ . After receiving all the bids, the

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parameter is no less than 1, beta if both shape parameters are no less than, Weibull if the shape parameter is no less than 1, and chi-square if the number of degrees of freedom is no less than 2. Distributions with a log-concave cdf but non-log-concave pdf include log-normal, Pareto, Weibull when the shape parameter is smaller than 1, and gamma when the shape parameter is smaller than 1. Student's t, Cauchy, and F distributions are non-log-concave for all parameters (Bagnoli and Bergstrom, 2005).

<sup>13</sup>In reality, one can proxy a bank's strength  $\theta$  by either its reserve of liquid assets or the level of its demandable liabilities that can evaporate in a flash.

<sup>14</sup>According to Bernanke (2015), one main reason to implement the term auction facility was it takes time to conduct an auction and determine the winning bids so that borrowers would receive funds with a delay, making clear that they were not desperate for cash.

<sup>15</sup>The interbank market essentially froze during the 2007-2008 financial crisis. We will describe in the appendix an extension in which the interbank market is well-functioned and show that no results change.

auctioneer ranks them from the highest to the lowest. The auction takes a uniform-price format: all winners pay for the same interest rate, which is referred to as the stop-out rate  $s$ , while losers do not pay anything. If there are fewer bids than the units of liquidity provided, each bidder receives a loan and pays  $r_A$ . If there are more bidders than the total liquidity, each of the  $m$  highest bidders receives one unit of liquidity by paying the highest *losing* bid. Formally, suppose there are  $l$  bidders in total. If  $l \leq m$ , bidding banks each receive a loan by paying  $s = r_A$ . If  $l > m$ , the  $m$  highest bidding banks each receive one unit of liquidity by paying the  $m + 1^{\text{st}}$  highest bid. The remaining  $l - m$  banks do not pay anything and, of course, do not receive any liquidity either.

We have modeled TAF auction as an extended second-price auction: All winning parties pay the highest losing bid. In reality, TAF is closer to an extended first-price auction: All winning banks pay the lowest winning bid. The two auctions generate the same revenue for the auction and the same expected payoffs for the bidders, by the Revenue Equivalence Theorem (Myerson, 1981), and consequently the same borrowing decisions. We present the analysis with the extended second-price auction, because it is notationally simpler, as it is a weakly dominant strategy for each bank to simply bid the maximum interest rate it is willing to pay (Vickrey, 1961).<sup>16</sup>

In reality, winners receive their TAF funds three days after the auction. We assume that there is a probability,  $1 - \delta$ , that an early liquidity shock hits each bank before it receives the funds. Hence, the expected net borrowing benefit of the winner who pays stop-out rate  $s$  is  $\delta(1 - \theta)R - s$ . Losers, upon learning the result of the auction, may borrow from the discount window if needed.

### 3.3 Stigma

Banks may incur a borrowing-dependent penalty. We have argued that a key reason that banks were reluctant to borrow from the lender of last resort is stigma cost. Detected borrowing may signal financial weakness to counter-parties, investors, and regulators. Although  $\theta$  is private information, the public can still make inference based on whether the bank has borrowed or which facility the bank has used if it has borrowed. We assume that upon detection, the public can perfectly tell whether the borrowing has been achieved through the discount window or the auction.

We capture the notion of stigma cost in a parsimonious way. We assume that after all the borrowings are accomplished, the banks that have successfully borrowed may be detected independently. Denote the probability of detecting borrowing from the discount window, borrowing from the auction, and the probability of verifying that a bank has not borrowed to be  $p_D$ ,  $p_A$ , and  $p_N$ , respectively. This penalty can be understood as a cost in bank's deteriorated reputation, a cost in a reduced chance to find counterparties, or a cost from a heightened chance of runs and increasing withdrawals by creditors. Let  $G_D$ ,  $G_A$ , and  $G_N$  be the type distributions of the banks

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<sup>16</sup>In contrast, in the first-price auction, banks shade their bids that depend on the liquidity supply and other participating banks.

that have borrowed from DW, from TAF, and have not borrowed, respectively. Let the stigma cost depend linearly on the expected financial condition of the bank. That is, for any detected borrowing decision  $\omega \in \{D, A, N\}$ ,

$$k_\omega \equiv k(G_\omega) = K - \kappa \int \theta dG_\omega(\theta).$$

Assume  $\kappa < \delta R$  so that the detection penalty is increasing in a slower rate than the real benefit of borrowing for worse banks. For expositional convenience, without loss of generality, let's normalize the stigma cost of a bank believed to have an unconditional average condition to be 0,  $k_\emptyset \equiv 0$ , that is,  $K \equiv \kappa \int_0^1 \theta dF(\theta)$ . In addition, purely for expositional ease, we present results as if  $p_N = 0$  in the main text (one interpretation of this assumption is that one can never know for sure that a bank did not borrow), and present proofs with a general  $p_N$  in the appendix.

### 3.4 Equilibrium

In summary, the setting is summarized by the return  $R$ , type distribution  $F$  of banks, discount rate  $r_D$  in DW, number  $m$  of units of liquidity auctioned, minimum bid  $r_A$  in TAF, and the penalty function  $k(G)$  attached to different belief distributions of bank's type.

Without loss of generality, we restrict each bank's strategy to be type-symmetric. Each bank  $\theta$ 's strategy can be succinctly described by  $\sigma(\theta) = (\sigma_{D1}(\theta), \sigma_A(\theta), \beta(\theta), \sigma_{D2}(\theta))$ , where  $\sigma_\omega(\theta)$  is the probability of borrowing from  $\omega \in \{D1, A, D2\}$ , and  $\beta(\theta)$  is its bid if it participates in the auction. Given strategies  $\sigma$ , beliefs about the financial situation can be inferred by Bayes' Rule; in this case, we say aggregate strategies  $\sigma$  generate posterior belief system  $G = (G_A, G_D, G_N)$ .

**Definition 1.** *Borrowing and bidding strategies  $\sigma^*$  and belief system  $G^*$  form an equilibrium if (i) each type- $\theta$  bank's strategy  $\sigma^*(\theta)$  maximizes its expected payoff given belief system  $G^*$ , and (ii) the belief system  $G^*$  is consistent with banks' aggregate strategies  $\sigma^*$ .*

Clearly, the best (i.e., type-1) bank has no intention to borrow at all, because it would only pay a price and stigma cost but has no benefit from borrowing. We assume that the borrowing benefit of the worst (i.e., type-0) bank is sufficiently high so that it has a strict incentive to borrow even given the most pessimistic belief about the banks who borrow:  $R - r_D - k(\underline{G}) > 0$ , where  $\underline{G}(\theta) = 1$  for all  $\theta > 0$ .

## 4 Equilibrium and Liquidity Provision

We present the solutions of two special cases (with only DW or only TAF) before presenting the solution of the general case of the model (with both DW and TAF). The two special cases will

provide not only demonstrations of the derivations of results in the general case but also results directly applicable in the general case.

## 4.1 Only DW

We start by examining the equilibrium when the government only sets up the discount window. The optimal borrowing decision can be characterized by one threshold: Weaker banks borrow from the discount window, and stronger banks do not borrow at all.

Note (again) that the best bank never borrows, because it knows that a liquidity shock could never affect it and therefore it never needs the liquidity, but borrowing incurs an interest cost as well as a stigma cost. The larger the probability a liquidity shock affects the bank, the more incentive the bank has in borrowing. If the assumption  $r_D < R - k(\underline{G})$  holds, the worst bank has a strict incentive to borrow from the discount window.

Furthermore, there is a unique equilibrium, which is guaranteed by the assumption of a log-concave cdf  $F$ .

**Theorem 1 (Equilibrium with only DW).** *Suppose  $m = 0$ . There exists a unique equilibrium characterizable by a threshold  $\theta^{DW} > 0$ : Banks  $\theta \in [0, \theta^{DW}]$  borrow from DW, and banks  $\theta \in (\theta^{DW}, 1]$  do not borrow. Equilibrium discount window stigma is*

$$k_D^{DW}(\theta^{DW}) = K - k \int_0^{\theta^{DW}} \theta dF(\theta) / F(\theta^{DW}),$$

where the threshold  $\theta^{DW}$  satisfies

$$(1 - \theta^{DW})R - r_D - p_D k_D^{DW}(\theta^{DW}) = 0. \quad (\text{DW})$$

The discount window provides liquidity to all the banks worse than  $\theta^{DW}$ , but the banks better than  $\theta^{DW}$  do not participate because the real economic benefits of borrowing to save the unrealized assets are dwarfed by the interest cost and the (expected) stigma cost. The change in the returns, interest rate, and stigma costs will affect liquidity provision as follows.

**Proposition 1 (Liquidity Provision with only DW).** *The expected total liquidity to be provided with only DW,  $L^{DW}$ , is  $nF(\theta^{DW})$ . It increases as (i) the return  $R$  increases, (ii) the discount rate  $r_D$  decreases, (iii) the probability of detection  $p_D$  decreases, and (iv) the stigma severity  $\kappa$  decreases.*

How total liquidity depends on the change in the distribution of banks' types is interesting, though: It may decrease when banks face higher liquidity risks overall.

**Proposition 2 (Market Condition and Liquidity Provision with only DW).** *Total liquidity with only DW,  $L^{DW}$ , (i) decreases when the distribution of banks  $\theta \leq \theta^{DW}$  shifts in a first-order stochastically dominated (FOSDed) way, and (ii) changes ambiguously when the type distribution  $F$  shifts in a FOSDed way.*

When the banks worse than  $\theta^{DW}$  face even higher liquidity risks than before, the banks who borrow from DW are perceived to be of even lower quality than before. As a result, the stigma cost rises, and the bank  $\theta^{DW}$  who was indifferent between borrowing from DW and not is no longer interested in borrowing. In other words, *the worsened conditions of the infra-marginal borrowing banks adversely affects the borrowing decision of the marginal borrowing bank.* As a result of the stigma cost, the discount window may not be effectively providing liquidity when the worst banks become worse. In general, when all banks face higher liquidity risks, not necessarily more banks would borrow because the heightened stigma cost dominates the worsened liquidity risks. The fact that banks were initially reluctant to borrow from DW before the introduction of TAF suggests that the worst banks in the economy were facing higher liquidity risks.

## 4.2 Only TAF

Next, we examine the equilibrium when the government only sets up the auction. The equilibrium can also be characterized by one threshold: Weaker banks bid their willingnesses to pay in the auction, and stronger banks do not participate in the auction and do not borrow at all.

**Theorem 2 (Equilibrium with only TAF).** *Suppose  $m > 0$  and  $r_D \geq R - k(\underline{G})$ . There exists a unique equilibrium characterized by a threshold  $\theta^{TAF}$ : (i) Banks  $\theta \in [0, \theta^{TAF}]$  bid  $\beta^{TAF}(\theta) = \delta(1 - \theta)R - p_A k_A$  in TAF, and (ii) banks  $\theta \in (\theta^{TAF}, 1]$  do not bid. Equilibrium auction stigma is*

$$k_A^{TAF}(\theta^{TAF}) = K - k \int_0^{\theta_s} \int_0^{\theta^{TAF}} \frac{\theta dF(\theta)}{F(\theta_s)} dH(\theta_s | \theta^{TAF}),$$

where  $H(\cdot | \theta^{TAF})$  is the distribution of the strongest winner given that banks  $\theta \in [0, \theta^{TAF}]$  participate in the auction, and the threshold  $\theta^{TAF}$  satisfies

$$\delta R(1 - \theta^{TAF}) - r_A - p_A k_A^{TAF}(\theta^{TAF}) = 0. \quad (\text{TAF})$$

TAF alone is not necessarily more effective than DW to provide liquidity. If the facilities are used alone, it is unclear which one will provide more liquidity. The combination of DW and TAF is needed to explain the increase in liquidity provision compared to DW only environment.

### 4.3 Both DW and TAF

We now solve for the equilibrium when both discount window and term auction facility are available. We will first describe a bank's bidding strategy in TAF, followed by its incentives in choosing between DW and TAF. Our result shows that relatively stronger banks have more incentives to bid in TAF rather than borrow immediately from DW, which is the key force behind the separation of types in equilibrium.

**Lemma 1.** *Only banks  $\theta \leq \theta_D$  would borrow from the discount window if they have lost in the auction, where  $\theta_D = 1 - (r_D + p_D k_D)/R$ .*

**Lemma 2.** *Banks  $\theta \in (\theta_1, \theta_A]$  participate in the auction, where*

$$\theta_1 = 1 - \frac{r_D - r_A + p_D k_D - p_A k_A}{(1 - \delta)R}, \quad \theta_A = 1 - \frac{r_A + p_A k_A}{\delta R}.$$

and bid

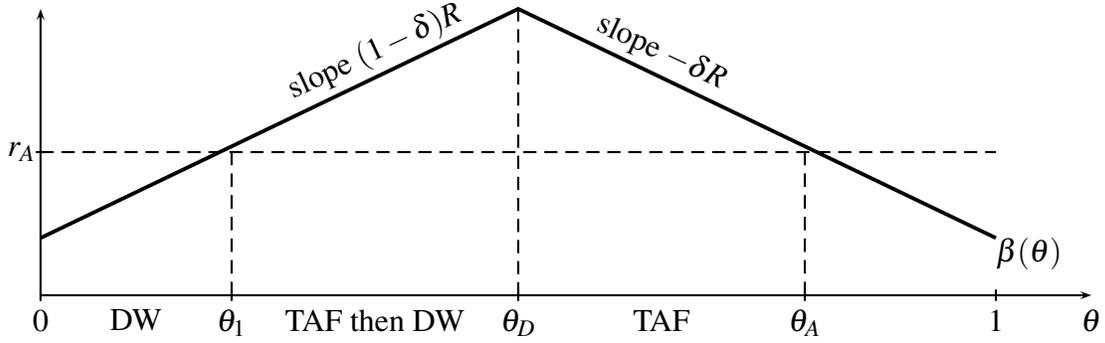
$$\beta(\theta) = \begin{cases} r_D + p_D k_D - p_A k_A - (1 - \delta)R(1 - \theta) & \text{if } \theta < \theta_D \\ \delta R(1 - \theta) - p_A k_A & \text{if } \theta \geq \theta_D \end{cases}.$$

Note that bids are increasing in  $\theta$  when  $\theta < \theta_D$  and decreasing in  $\theta$  when  $\theta \geq \theta_D$ . Therefore, bank  $\theta_D$  has the highest willingness to pay, and banks farther away from  $\theta_D$  have lower willingness to pay. The bids are decreasing at the rate of  $(1 - \delta)R$  for the banks worse than  $\theta_D$  and are decreasing at the rate of  $\delta R$  for the banks better than  $\theta_D$ . The winners in the auctions are going to be the banks that are the closest to  $\theta_D$ . In other words, banks  $\theta < \theta_D$  that would be willing to borrow in the discount window were attracted to participate in the auction, and banks  $\theta > \theta_D$  that would not have borrowed from the discount window were also attracted to participate in the auction. For any bank, as long as its willingness to pay is above  $r_A$ , it will participate in the auction by submitting a bid higher than  $r_A$ . Figure 2 shows the bids in TAF (if there is no minimum bid) and the optimal facility choice of different banks.

The bids may be higher than the concurrent discount rate, because of the difference in the stigma costs between the two borrowing facilities. Banks close to  $\theta_D$  are willing to bid more than  $r_D$ , up to  $p_D k_D - p_A k_A$  more to be exact, to avoid the stigma cost.

**Lemma 3 (Equilibrium with both DW and TAF: High Chance of Early Liquidity Shock).** *Suppose  $m > 0$ ,  $r_D < R - k(\underline{G})$ , and  $\delta \leq [r_A + k(\theta^{DW})] / [r_D + p_D k_D^{DW}(\theta^{DW})]$ . In the unique equilibrium, banks  $\theta \in [0, \theta^{DW}]$  borrow from the discount window, and banks  $\theta \in (\theta^{DW}, 1]$  do not borrow.*

Therefore, delaying the release of the funds from TAF for too long will make the program ineffective.



**Figure 2: Facility Choice and TAF Bids**

**Theorem 3 (Equilibrium with both DW and TAF: Low Chance of Early Liquidity Shock).**

Suppose  $m > 0$ ,  $r_D < R - k(\underline{G})$ , and  $\delta > [r_A + k(\theta^{DW})] / [r_D + p_D k_D^{DW}(\theta^{DW})]$ . There exists a unique equilibrium characterized by three thresholds,  $\theta_1$ ,  $\theta_D$ , and  $\theta_A$ : (i) banks  $\theta \in [0, \theta_1]$  borrow from the discount window before the auction, (ii) banks  $\theta \in (\theta_1, \theta_D]$  bid in the auction and borrow from the discount window if they lose in the auction, (iii) banks  $\theta \in (\theta_D, \theta_A]$  bid in the auction and do not borrow if they lose in the auction, and (iv) banks  $\theta \in (\theta_A, 1]$  neither borrow from the discount window nor participate in the auction.

Theorem 3 immediately implies:

**Corollary 1.** *Equilibrium discount window stigma  $k_D^*$  is larger than auction stigma  $k_A^*$ .*

There are three forces to separate the banks participating in DW and those in TAF. First, the possibility of early liquidation as a result of delayed released funds in TAF deters the worst banks from borrowing in TAF. Second, the exclusion of the worst banks from the auction increases the discount window stigma and decreases the auction stigma, thus further pushing more banks to borrow from TAF. Finally, the competitive nature of the auction attracts banks that would not have borrowed with only DW by offering them a chance to borrow cheaper than the discount rate. TAF serves as a substitute for DW for the banks that are close to and worse than  $\theta^{DW}$ . They substitute into borrowing in the auction from borrowing in the discount window. TAF serves as a complement for DW in terms of total lending. Banks that are close to and better than  $\theta^{DW}$  substitute into borrowing in the auction from borrowing in the discount window.

For total liquidity, consider the expected marginal borrower. The expected marginal borrower is better than  $\theta^{DW}$ , because they borrow from the auction, and the distribution of the types of the banks participating in the auction in DW and TAF setting first-order stochastically dominates the distributions of the types of the banks borrowing from the discount window.

**Proposition 3 (Liquidity Provision with both DW and TAF).** *The combination of TAF and DW provides more total liquidity in expectation than does DW alone:  $L^* > L^{DW}$ . The liquidity provided by DW decreases when TAF is introduced.*

## 5 Empirical Analysis

Our theory predicts that banks that borrowed from DW (DW banks) were fundamentally weaker than the banks that borrowed from TAF (TAF banks). Equivalently, the marginal value of liquidity and therefore the need to borrow were also higher for DW banks. In this section, we examine this hypothesis using data from various sources, including the bank regulatory database, the equity returns, and the CDS spreads. We will also study whether borrowing from DW and TAF have led to abnormal changes in equity returns and CDS spreads. Throughout this section, all analysis is conducted at the bank holding company level (BHC). Under Section 23A of the Federal Reserve Act, it is illegal for a member bank to channel funds borrowed from LOLR to other affiliates within the same BHC. In late 2007, however, temporary exemptions of Section 23A were granted (Bernanke, 2015). Therefore, by conducting the analysis at the BHC level, we implicitly assume an efficient internal capital market within a BHC, which is consistent with the evidence in Cetorelli and Goldberg (2012) and Ben-David et al. (2017).

### 5.1 Description of DW and TAF Borrowing During the Crisis

Let us start by describing BHC's borrowing behaviors from DW and TAF auction during the Great Recession. The main dataset we use is obtained through Bloomberg and includes 407 institutions that borrowed from the Federal Reserve between August 1, 2007 and April 30, 2010. These data were released by the Fed on March 31, 2011, under a court order, after Bloomberg filed a lawsuit against Fed Board for information disclosure.<sup>17</sup> The data contain information on each institution's daily outstanding balance of its borrowing from the discount window, the Term Auction Facility as well as five other related programs. Later on, we will merge this dataset with the bank regulatory database, equity returns, and CDS spreads to study how financial conditions affected BHC's borrowing decisions.

Since the Bloomberg dataset were collected by scraping over 29,000 pages of PDF files released from the Fed board following the FOIA request, the process of data collection could be potentially compromised. To evaluate the data's quality, we calculate the aggregate weekly outstanding balance in DW and TAF programs from the Bloomberg dataset and compare these num-

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<sup>17</sup>For details, see <https://www.bloomberg.com/news/articles/2011-03-31/federal-reserve-releases-discount-window-loan-records-under-court-order>. In May 2008, Bloomberg News' reporter Mark Pittman filed a FOIA request with the Fed, requesting data about details of discount window lending and collateral. Unsurprisingly, it was stonewalled by the central bank. In Nov 2008, Bloomberg LP's Bloomberg News filed a lawsuit challenging the Fed, with Fox News Network later filing a similar lawsuits. Other news organization also showed support by filing legal briefs. In March 2011, the U.S. Supreme Court ruled that the Fed to release the discount loans in response to the lawsuits. Later that month, the Fed released the data, in the form of 894 PDF files with more than 29,000 pages on two CD-ROMS. Bloomberg News later published an exhaustive analysis that included the detailed data.

bers with the official ones released by the Board of the Federal Reserve.<sup>18</sup> Figure 3 shows the comparison. Clearly, the Bloomberg data managed to capture the vast majority of borrowing behaviors in both DW and TAF auction.

[Figure 3 about here.]

Table 1 provides the basic summary statistics of BHCs' borrowing behavior during the crisis. Approximately 73 percent of borrowing institutions (313 out of 407) are banks, together with diversified financial services (mostly asset management firms), insurance companies, savings and loans, and other financial service firms. Foreign banks who borrowed through their U.S. subsidiaries were also included. Banks' choices of borrowing facilities were quite heterogeneous. While a majority (260 out of 407) tapped both facilities, some only used one throughout the period. Borrowing frequencies in both programs exhibit large skewness. While the median bank tapped the discount window twice throughout the sample period, the Alaska USA Federal Credit Union used it a total of 242 times. Similarly, among the 60 TAF auctions, Mitsubishi UFJ Financial Group borrowed a total of 28 times, whereas the median bank borrowed only a total of three times. On average, TAF lent more liquidity (\$3174 million) than DW (\$1529 million), consistent with the evidence in Figure 1a that TAF was relatively more successful. However, the Dexia Group, the bank that borrowed the most from DW, took out a total of approximately \$190 million over the three-year period, exceeding its counterpart in TAF ( $\approx$  \$100 million by Bank of America Corp). This evidence suggests that DW banks were in more need of liquidity than TAF banks.

[Table 1 about here.]

## 5.2 Evidence from Bank Regulatory Database

Were DW banks fundamentally different from TAF banks? We attempt to answer this question by first exploring the data at the quarterly frequency. To do so, we link the Bloomberg data to FR Y-9C, the Consolidated Financial Statements for Holding Companies. The Y-9C reports collect financial-statement data from BHCs on a quarterly basis, which are then published in the Federal Reserve Bulletin. These reports are required to submit by all domestic BHCs, within 40 or 45 calendar days following the end of a quarter. While this merge allows us to use proxies for banks' financial conditions, it unfortunately eliminates all the foreign banks from the borrowing sample, which took out about 60% of TAF loans (Benmelech, 2012). Among 289 U.S. banks that borrowed from either DW or TAF, we manage to merge Y-9C reports to 135 of them. These banks constitute of 42.2% of borrowings from DW, and 81.8% from TAF.<sup>19</sup>

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<sup>18</sup>The data can be downloaded from <https://www.federalreserve.gov/datadownload/>

<sup>19</sup>The largest three unmatched banks were NB Holdings Corporation, Citigroup Holdings Company, and Wachovia Corporation.

To explore how BHCs' financial conditions affect their borrowings from DW and TAF, we estimate the following specification

$$\begin{aligned} \log(1 + LOLR_{it}) = & \alpha + \beta_1 x_{it} + \beta_2 \mathbb{1}_{DW_{it} > 0} + \beta_3 \mathbb{1}_{TAF_{it} > 0} \\ & + \beta_4 \mathbb{1}_{DW_{it} > 0} \times x_{it} + \beta_5 \mathbb{1}_{TAF_{it} > 0} \times x_{it} + \Gamma \cdot [\text{Size}_{it}, \text{ROA}_{it}] + Q_t + \varepsilon_{it}. \end{aligned} \quad (1)$$

The left hand side of Equation 1 is the logarithm of BHC  $i$ 's total borrowing from DW and TAF in quarter  $t$ . On the right hand side,  $x_{it}$  is one of the proxies for BHC  $i$ 's financial condition in quarter  $t$ , including its Tier-1 capital to risk-weighted asset ratio (TIRWA), leverage, unused loan commitment over asset, and liquid asset over asset.<sup>20</sup> The two dummy variables,  $\mathbb{1}_{DW_{it} > 0}$  and  $\mathbb{1}_{TAF_{it} > 0}$ , indicate whether BHC  $i$  borrowed from DW and TAF in quarter  $t$ , where  $DW_{it}$  and  $TAF_{it}$  are the total amount of borrowing from DW and TAF by bank  $i$  in quarter  $t$ .  $Q_t$  is the quarter fixed effect to take into account variations in the aggregate economy conditions, and banks size and ROAs are included as additional controls. Note that we do not include BHC fixed effects in these regressions, since as shown in Table 1, a majority of the BHCs only borrow from DW or TAF once throughout the sample period. BHC fixed effects will effectively absorb all the explanatory power. Moreover, consistent with the theory that bank fundamentals are unobservable to the public, we use the *contemporaneous* measurement of tier-1 capital ratio and book leverage, as opposed to those lagged by one period. Our results stay unchanged if we lag all the  $x_{it}$  variables by one period.

We are mainly interested in the interaction variables  $\mathbb{1}_{DW_{it} > 0} \times x_{it}$  and  $\mathbb{1}_{TAF_{it} > 0} \times x_{it}$ . The coefficient  $\beta_4$  ( $\beta_5$ ) measures how changes in BHC  $i$ 's financial condition affects its borrowing amount from the LOLR, conditional on BHC  $i$  borrowing from DW (TAF) respectively. Our theory predicts that  $\beta_5 > \beta_4$  if  $x_{it}$  stands for TIRWA and liquid asset/asset, and  $\beta_5 < \beta_4$  otherwise.

[Tables 2 and 3 about here.]

### 5.2.1 Addressing Endogeneity

The specifications in Equation 1 suffers from potential endogeneity issues such as omitted variables, reverse causality, and selection biases. Table 3 presents the results when we lag all the proxies for BHCs' financial strengths by one quarter, which are qualitatively the same as the one without lags, eliminating concerns for reverse causality. To address issues as omitted variables, we further employ a difference in difference (DID) approach and explore the international aspects of borrowing banks. Notably, following the bankruptcy of Lehman Brothers and the increasing pressure in the financial market, several countries undertook interventions to combat the potential crisis. The implementation dates of country-specific policies, however, were staggered as these

<sup>20</sup>In Y-9C report, Tier-1 capital to risk-weighted assets is defined as  $\text{bhck8274}/\text{bhcka223}$ , and book Leverage is defined as  $1 - (\text{bhck3210}/\text{bhck2170})$ .

policies could be largely driven by the political bargaining and renegotiation. The staggered policy structure offers us an ideal setup to study the difference in these countries' banks' borrowing decisions from the lender of the last resort in the U.S. In early October 2008, leaders from the G7 countries met and established a plan of action that aimed to stabilize financial markets, restore the flow of credit, and support global economic growth. Following the meeting, all the G7 countries except for Japan immediately launched credit guarantee programs that effectively reduced the liquidity risk faced by domestic financial institutions.<sup>21</sup> In this subsection, we compare the decisions to borrow from the LOLR by banks in the G7 countries versus the U.S. and study whether they have switched more from DW borrowing to TAF borrowing following the credit guarantee programs. In particular, we estimate the following equations on a bi-weekly basis using data from 2008 Q3:

$$\log(1 + DW_{iw}) = \alpha + T_i + \lambda_w + \delta_{DW} (T_i \times \lambda_w) + \varepsilon_{iw} \quad (2)$$

$$\log(1 + TAF_{iw}) = \alpha + T_i + \lambda_w + \delta_{TAF} (T_i \times \lambda_w) + \varepsilon_{iw}. \quad (3)$$

In the specification,  $\log(1 + DW_{iw})$  and  $\log(1 + TAF_{iw})$  are bank  $i$ 's total amount of borrowing within the total of two weeks  $w$ .  $T_i$  is a dummy variable indicating whether bank  $i$  falls into the treated country (Canada/Germany/France/Italy), and  $\lambda_w$  is a time dummy variable that equals 1 following the credit guarantee programs and 0 otherwise. The coefficients we are interested in are  $\delta_{DW}$  and  $\delta_{TAF}$ . Our theory predicts that  $\delta_{TAF} > \delta_{DW}$  since these credit guarantee programs effectively reduce the riskiness of domestic banks. Note that since we restrict the sample to 2008 Q3 and thus do not control for any additional variables that measure banks' health.

[Table 4 about here.]

Table 4 presents the results across all countries.

[Figures 4, 5, and 6 about here.]

### 5.2.2 Bank Failure and LOLR Borrowing

Next, we study whether banks that borrowed more from DW were also more likely to fail subsequently. To do so, we manually collect data on whether a bank failed, was acquired, or got nationalized by the government by December 31, 2011. The choice of this particular date was slightly arbitrary, but our results also hold if applying an earlier date such as June 30, 2010.

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<sup>21</sup>These policies are collected by the New Bagehot Project offered by the Yale Program on Financial Stability (<https://newbagehot.yale.edu/>). We are grateful to the YPFS team for sharing the data.

In total, 36 financial institutions failed by Decemeber 31, 2011. Among them, 11 failed in 2008, 8 in 2009, 7 in 2010, and 10 in 2011. We study whether the banks that borrowed relatively more from DW than TAF were more likely to fail.

[Table 5 about here.]

Table 5 reports the results. The first column shows that compared to a bank that only borrowed from TAF, a bank that solely borrowed from DW was more likely to fail within the same quarter by an additional probability of 0.7%. Column (2) shows this additional probability of a bank failing eventually is 12.5%. Both results are significant and economically statistically, implying that DW banks were more riskier than TAF banks.

### 5.3 Evidence from Bank and CDS Spreads

In this subsection, we take advantage of the relative high frequency of the Bloomberg data and match borrowing banks with their CDS spreads in the Markit database. Since only very large banks have CDS contracts outstanding, we could match 70 of them, which accounts for 24.8% of DW borrowing and 79.4% of TAF borrowing.

Figure 7 plots the level of 5-year CDS spreads around borrowing dates, after removing fixed effects of BHC, month and CDS ratings. Two observations are prominent. First, prior to the borrowing events, DW banks have persistently higher CDS spreads than TAF banks. The difference (about 0.05) is significant relative to the standard deviation (less than 0.002), implying that prior to the borrowing, DW banks have higher probability of default as acknowledged by the CDS price. Second, following both borrowing events, BHCs' CDS spreads drop within the next five days, even though it seems TAF banks drop slightly more than DW banks. Two reasons can potentially explain the difference in drop. First, TAF banks in general take out loans with bigger sizes, and therefore, their funding constraint is more relaxed. Second, if the borrowing from DW and TAF have the identical probability of being detected, TAF borrowing suffers a lower level of stigma costs.

[Figure 7 about here.]

Formally, we estimate the following specification(s)

$$y_{it} = \alpha + \beta CDS_{it-1} + \gamma CDS \text{ rating}_{it-1} + Q_m + \gamma_i + \varepsilon_{it}, \quad (4)$$

where  $y_{it}$  is a dummy variable that takes one if a bank borrows from DW or TAF. Table 6 reports the results. In the first column,  $y_{it} = 1$  if BHC  $i$  borrows from DW on date  $t$  and 0 if BHC  $i$  borrows from TAF on date  $t$ . The coefficient shows that if BHC's 5-year CDS spreads on date  $t - 1$

increases by 100 basis points, its probability to borrow from DW as opposed to TAF increases by 0.1%. In Column (2) and (3),  $y_{it} = 0$  if BHC  $i$  does not borrow from either DW or TAF. In (2),  $y_{it} = 1$  if it borrows from DW, whereas in (3),  $y_{it} = 1$  if it borrows from TAF. Clearly, results show that lagged CDS can predict DW borrowing but not TAF borrowing.

[Table 6 about here.]

## 6 Conclusion

In this paper, we investigated how the Term Auction Facility mitigates the stigma associated with borrowing from the Discount Window, when it was used in accordance with the Discount Window. We constructed an auction model with endogenous participation and showed optimal auction bidding strategies that internalized any stigma associated with the auction naturally increased participation and consequently mitigated the borrowing stigma.

We showed the following results consistent with the empirical observations. First, banks with strong financial health were reluctant to borrow from the Discount Window due to the standard adverse selection logic à la [Akerlof \(1970\)](#). Second, when both DW and TAF are available, the weakest banks borrowed from DW, and relatively strong ones participated in TAF. Among those who lost in the auction, relatively weak ones moved on to borrow from DW. Third, we show the introduction of TAF may or may not expand the set of banks who obtained liquidity. Lastly, stop-out rate of TAF may be higher or lower than the primary discount rate.

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# A Appendix

## A.1 Omitted Proofs

**Proof of Theorem 1.** A type- $\theta$  bank prefers borrowing from the discount window over not borrowing at all if and only if

$$u_D(\theta) = (1 - \theta)R - r_D - p_D k_D \geq u_N(\theta) = -p_N k_N.^{22}$$

Clearly, the incentive to borrow from discount window decreases with  $\theta$ . Therefore, for any given  $k_D$  and  $k_N$ , a bank borrows from the discount window if and only if

$$\theta \leq \frac{R - r_D - p_D k_D + p_N k_N}{R}.$$

Therefore, there exists a threshold, let's denote it by  $\theta^{DW}$ , such that banks worse than  $\theta^{DW}$  borrow from the discount window and banks better than  $\theta^{DW}$  do not borrow at all. Bank  $\theta^{DW}$  is indifferent between borrowing and not borrowing, implying that

$$(1 - \theta^{DW})R - r_D - p_D k_D + p_N k_N = 0.$$

In equilibrium,  $k_D$  and  $k_N$  depend on  $\theta^{DW}$ , as follows,

$$k_D = K - k \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})}.$$

$$k_N = K - k \int_{\theta^{DW}}^1 \frac{\theta dF(\theta)}{1 - F(\theta^{DW})}.$$

Plugging equilibrium  $k_D$  and  $k_N$  into the equilibrium condition above, we see that  $\theta^{DW}$  is determined by

$$(1 - \theta^{DW})R - r_D - p_D \left[ K - k \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})} \right] + p_N \left[ K - k \int_{\theta^{DW}}^1 \frac{\theta dF(\theta)}{1 - F(\theta^{DW})} \right] = 0,$$

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<sup>22</sup>The complete form of the condition should be

$$(1 - \theta)R - r_D - p_D k_D - (1 - p_D)k_0 \geq -p_N k_N - (1 - p_N)k_0.$$

which simplifies to

$$(1 - \theta)R - r_D - p_D(k_D - k_0) \geq -p_N(k_N - k_0).$$

Since we are normalizing  $k_0$  to be 0, we can write the condition in its simpler form.

which is rearranged to be

$$R - r_D - p_D K + p_N K - \theta^{DW} R + p_D k \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})} - p_N k \int_{\theta^{DW}}^1 \frac{\theta dF(\theta)}{1 - F(\theta^{DW})} = 0. \quad (DW)$$

The terms involving  $\theta^{DW}$  can be rearranged to be

$$-\theta^{DW}(R - p_D k) - p_D k \left[ \theta^{DW} - \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})} \right] - p_N k \int_{\theta^{DW}}^1 \frac{\theta dF(\theta)}{1 - F(\theta^{DW})}.$$

The first term,  $-\theta^{DW}(R - p_D k)$ , is decreasing in  $\theta^{DW}$ , because  $R > 1 > p_D k$ . Because the distribution is assumed to be log-concave, the term in the square bracket (mean advantage over inferior, as [Bagnoli and Bergstrom \(2005\)](#) name it) is increasing ([Bagnoli and Bergstrom, 2005](#), Theorem 5), so the second term,  $-p_D k \left[ \theta^{DW} - \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})} \right]$ , is decreasing in  $\theta^{DW}$ . The third term,  $-p_N k \int_{\theta^{DW}}^1 \frac{\theta dF(\theta)}{1 - F(\theta^{DW})}$ , is decreasing in  $\theta^{DW}$ , because the integral represents the expected bank type conditional on the bank is better than  $\theta^{DW}$ . The left hand side of Equation (DW) is strictly decreasing in  $\theta^{DW}$ , so there is a unique  $\theta^{DW}$ .  $\square$

**Proof of Proposition 1.** The LHS of Equation (DW) strictly increases when (i)  $R$  increases, (ii)  $r_D$  decreases, (iii)  $p_D$  increases, or (iv)  $\kappa$  increases (when  $p_N = 0$ ). Since the LHS is strictly decreasing in  $\theta^{DW}$ , the equilibrium  $\theta^{DW}$  increases as a result of any of the changes (i)-(iv).  $\square$

**Proof of Proposition 2.** The LHS of Equation (DW) strictly decreases when  $F$  for  $\theta < \theta^{DW}$  shifts in a FOSDed way, because the only term affected by the change,  $\int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})}$ , strictly decreases. Hence, the new threshold  $\tilde{\theta}^{DW}$  is strictly smaller than  $\theta^{DW}$ . The expected total liquidity to be provided,  $nF(\tilde{\theta}^{DW})$ , is also smaller.  $\square$

**Proof of Theorem 2.** Bank  $\theta$  bids (gross) interest rate  $\beta(\theta)$  such that the bank's payoff from winning in the auction with this rate is the same as the payoff from not borrowing,

$$\delta(1 - \theta)R - c - \beta(\theta) - p_A k_A = -p_N k_N.$$

In other words, the bid is the bank's maximum willingness to pay (WTP) for the loan:

$$\beta(\theta) = \delta(1 - \theta)R - c - (p_A k_A - p_N k_N).$$

Note that the bid is strictly decreasing in  $\theta$ . Therefore, worse banks are willing to bid higher interest rates. Consequently, given any stigma costs, there exists a threshold bank  $\theta^{TAF}$  such that banks worse than  $\theta^{TAF}$  are willing to bid more than the minimum bid  $r_A$  and all banks better than

$\theta^{TAF}$  are not willing to bid more than  $r_A$ . Bank  $\theta^{TAF}$  bids exactly the pre-specified minimum bid  $r_A$ :

$$\theta^{TAF} = 1 - \frac{p_A k_A - p_N k_N + r_A + c}{\delta R}.$$

Now, consider the equilibrium stigma costs.

$$k_A(\theta^{TAF}) = K - k \int_0^{\theta_s} \int_0^{\theta^{TAF}} \frac{\theta dF(\theta)}{F(\theta_s)} dH(\theta_s | \theta^{TAF}),$$

$$k_N(\theta^{TAF}) = K - k \int_0^{\theta_s} \int_{\theta_s}^1 \frac{\theta dF(\theta)}{1 - F(\theta_s)} dH(\theta_s | \theta^{TAF}),$$

where  $H(\theta_s | \theta^{TAF})$  is the distribution of the stop-out rate given banks  $\theta \in [0, \theta^{TAF}]$  participate in the auction.

Rearranging the equation above, we have that in equilibrium  $\theta^{TAF}$  satisfies

$$[\delta R - r_A - c] - [\delta R \theta^{TAF} + p_A k_A(\theta^{TAF})] + [p_N k_N(\theta^{TAF})] = 0. \quad (\text{TAF})$$

The term in the first square bracket does not depend on  $\theta^{TAF}$ . The term in the second square bracket can be rearranged to be

$$\delta R \int_0^{\theta_s} [\theta^{TAF} - \theta_s] dH(\theta_s | \theta^{TAF}) + \int_0^{\theta_s} \left[ \delta R \theta_s - p_A k \int_0^{\theta_s} \frac{\theta dF(\theta)}{F(\theta_s)} \right] dH(\theta_s | \theta^{TAF}).$$

The square bracket in the integral is increasing in  $\theta^{TAF}$ , and the second term is also increasing in  $\theta$  because each term in the integral (Bagnoli and Bergstrom, 2005, mean advantage over inferior) is positive, as long as  $\delta R > p_A k$ . The term in the third square bracket in (TAF) is decreasing in  $\theta^{TAF}$ . Therefore, the left hand side of Equation (TAF) is strictly decreasing in  $\theta^{TAF}$ . Hence, there is a unique equilibrium. □

**Proof of Lemma 1.** Bank  $\theta$  would borrow in the auction if and only if  $(1 - \theta)R - r_D - p_D k_D \geq -p_N k_N$ , which simplifies to  $\theta \geq \theta_D \equiv 1 - (r_D + p_D k_D - p_N k_N)/R$ . □

**Proof of Lemma 2.** For the banks that could still get a positive payoff from borrowing in the discount window if they lose in the auction, they are willing to pay up to  $\beta^D(\theta)$  such that

$$R(1 - \theta) - r_D - p_D k_D = \delta R(1 - \theta) - c - \beta^D(\theta) - p_A k_A.$$

Rearrange, we get

$$\beta^D(\theta) = r_D + p_D k_D - p_A k_A - (1 - \delta)R(1 - \theta) - c.$$

Note that the bid is increasing in  $\theta$ , for  $\theta < \theta_D$ .

On the other hand, for the banks that could not get a positive payoff from borrowing in the discount window, they are willing to pay up to  $\beta^N(\theta)$  such that

$$-p_N k_N = \delta R(1 - \theta) - c - \beta^N(\theta) - p_A k_A.$$

Rearrange, we get

$$\beta^N(\theta) = \delta R(1 - \theta) - c + p_N k_N - p_A k_A.$$

Note that the bid is decreasing in  $\theta$ , for  $\theta > \theta_D$ .

Altogether, the maximum willingness to pay for the auction is

$$\beta(\theta) = \begin{cases} \beta^D(\theta) = r_D + p_D k_D - p_A k_A - (1 - \delta)R(1 - \theta) - c & \text{if } \theta < \theta_D \\ \beta^N(\theta) = \delta R(1 - \theta) - c + p_N k_N - p_A k_A & \text{if } \theta \geq \theta_D \end{cases}$$

Bank  $\theta$  participates in the auction if its maximum willingness to pay in the auction is greater the minimum required bid  $r_A$ . That is, if the bank's type is between  $\theta_1$  and  $\theta_A$ , where  $\beta^D(\theta_1) = r_A$  and  $\beta^N(\theta_A) = r_A$ . Solving for those conditions and simplifying, we get

$$\theta_1 = 1 - \frac{r_D - r_A + p_D k_D - p_A k_A - c}{(1 - \delta)R}, \quad \theta_A = 1 - \frac{r_A + c + p_A k_A - p_N k_N}{\delta R}.$$

□

**Proof of Lemma 3.** By Lemma 1, banks borrow from the discount window if and only if

$$\theta \leq \theta_D = 1 - \frac{r_D + p_D k_D}{R}.$$

Among these banks, they are willing to wait for the auction if and only if

$$\theta > \theta_1 = 1 - \frac{r_D - r_A + p_D k_D - p_A k_A}{(1 - \delta)R}.$$

The banks that borrow from the discount window would not participate in the auction if and only if  $\theta_1 \geq \theta_D$ , which is

$$1 - \frac{r_D - r_A + p_D k_D - p_A k_A}{(1 - \delta)R} \geq 1 - \frac{r_D + p_D k_D}{R}.$$

The inequality can be simplified to

$$r_D + p_D k_D \geq \frac{r_D - r_A + p_D k_D - p_A k_A}{1 - \delta},$$

which further simplifies to

$$r_D + p_D k_D - \delta(r_D + p_D k_D) \geq r_D + p_D k_D - r_A - p_A k_A,$$

which can be further simplified to  $\delta \leq (r_A + p_A k_A)/(r_D + p_D k_D)$ . Hence, in equilibrium, if  $\delta \leq r_A/(r_D + p_D k_D^*)$ , banks that would borrow from the discount window if they have lost in the auction would not participate in the auction in the first place.

Knowing the condition derived above, we can directly verify that banks  $\theta \in [0, \theta^{DW}]$  borrowing from the discount window immediately is part of an equilibrium. When banks  $\theta \in [0, \theta^{DW}]$  borrow from the discount window, the equilibrium discount window stigma is  $k_D^* = k_D^{DW}(\theta^{DW})$ , and since we have the assumption  $\delta \leq r_A/[r_D + p_D k_D^{DW}(\theta^{DW})]$ , by the condition derived above, we have that no discount window bank would be willing to participate in the auction. Furthermore, since bank  $\theta^{DW}$ , who should have the highest willingness to pay for the auction, is not willing to participate in the auction, no bank will participate in the auction.  $\square$

**Proof of Theorem 3.** For notational simplicity, let  $p_A = p_D = 1$ ,  $p_N = 0$ , and  $c = 0$ . An equilibrium is determined by three thresholds,  $\theta_1$ ,  $\theta_D$ , and  $\theta_A$ , where

$$\begin{aligned}\theta_D &= 1 - \frac{r_D + k_D}{R} \\ \theta_1 &= 1 - \frac{r_D + k_D - r_A - k_A}{(1 - \delta)R} \\ \theta_A &= 1 - \frac{r_A + k_A}{\delta R}\end{aligned}$$

Rearranging the three equations, we have

$$(1 - \theta_D)R - r_D - k_D = 0 \tag{DW2}$$

$$(1 - \theta_1)(1 - \delta)R - r_D - k_D = r_A + k_A \tag{DW1}$$

$$(1 - \theta_A)\delta R = r_A + k_A \tag{A}$$

The stigma costs  $k_D$  and  $k_A$  are

$$\begin{aligned}k_D(\theta_D, \theta_1, \theta_A) &= K - \kappa \frac{\int_0^{\theta_1} \theta dF(\theta) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)}{F(\theta_1) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)} \\ k_A(\theta_1, \theta_A) &= K - \kappa \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_{s2}(s)} \frac{\theta dF(\theta)}{F(\theta_{s2}(s)) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)\end{aligned}$$

where  $[\theta_{s1}(s), \theta_{s2}(s)]$  is the interval of the types of the banks winning the auction when  $s$  is the stop-out rate.

Plugging  $k_A(\theta_1, \theta_A)$  into Equation (A), we have

$$\delta R - r_A - K - (\delta R - \kappa)\theta_A - \kappa \left[ \theta_A - \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_{s2}(s)} \frac{\theta dF(\theta)}{F(\theta_{s2}(s)) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A) \right] = 0.$$

The term in the square bracket is the advantage over mean inferior for an order statistics distribution. Then by [Chen et al. \(2009\)](#), the order statistics distribution is log-concave. Hence, by ([Bagnoli and Bergstrom, 2005](#), Theorem 5), the term in the square bracket is increasing in  $\theta_A$ . If  $\delta R > k$ , then the left hand of the equation above is strictly decreasing in  $\theta_A$ . For each fixed  $\theta_1$ , there is a unique  $\theta_A$  that satisfies the equation. Let  $\tilde{\theta}_A(\theta_1)$  represent this function, and note that  $\tilde{\theta}_A(\theta_1)$  is increasing in  $\theta_1$ .

Plugging  $k_D$  into Equation (DW2) and rearranging, we have

$$R - r_D - K - \theta_D R + \kappa \frac{1}{\Delta} \int_0^{\theta_1} \theta dF(\theta) - \kappa \frac{1}{\Delta} \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \tilde{\theta}_A(\theta_1)) = 0,$$

where  $\Delta$  represents the denominator in the fractional part of the expression of  $k_D$ . The terms that include  $\theta_D$  can be rearranged as

$$-\theta_D(R - \kappa) - \kappa \left[ \theta_D - \frac{\int_0^{\theta_1} \theta dF(\theta) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \tilde{\theta}_A(\theta_1))}{F(\theta_1) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \tilde{\theta}_A(\theta_1))} \right].$$

Again, the term in the square bracket is mean advantage over inferior for a truncated order statistics distribution, which continues to be log-concave, and therefore it is increasing in  $\theta_D$ . Therefore, for each  $\theta_1$ , there is a unique  $\theta_D$  that satisfies Equation (DW2). Let  $\tilde{\theta}_D(\theta_1)$  represent this function.

Plugging  $\tilde{\theta}_D(\theta_1)$ ,  $\tilde{\theta}_A(\theta_1)$ ,  $k_D$ , and  $k_A$  into Equation (DW1), we have

$$-r_D - r_A + (1 - \delta)R - \theta_1(1 - \delta)R - k_D(\theta_1, \tilde{\theta}_D(\theta_1), \tilde{\theta}_A(\theta_1)) - k_A(\theta_1, \tilde{\theta}_A(\theta_1)) = 0.$$

Using the same trick as before, we rearrange all the terms that include  $\theta_1$ :

$$-\theta_1[(1 - \delta)R - \kappa] - \kappa \left[ \theta_1 - \frac{\int_0^{\theta_1} \theta dF(\theta) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)}{F(\theta_1) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)} \right] - k_A(\theta_1, \tilde{\theta}_A(\theta_1)).$$

The expression is strictly decreasing for the same reason as the previous argument, as long as  $(1 - \delta)R > \kappa$ . Therefore, there is a unique  $\theta_1$ .

□

**Remark.** If the two discount windows are separately detected, then the stigma costs are

$$k_{D2} = K - \kappa \frac{\int_{r_A}^{\infty} \int_{\theta_1}^{\theta_D} \theta \frac{dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)}{\int_{r_A}^{\infty} \int_{\theta_1}^{\theta_D} \frac{dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)},$$

$$k_{D1} = K - \kappa \int_0^{\theta_1} \frac{\theta dF(\theta)}{F(\theta_1)},$$

and the  $k_D$  in Equation (DW2) is  $k_{D2}$ , and the  $k_D$  in Equation (DW1) is  $k_{D1}$ . The proof above will proceed in a similar fashion.

**Proof of Proposition 3.** From the previous proof we see that the equilibrium condition for the banks who borrow from the discount window in DW and TAF setting is

$$(1 - \theta_D^*)R - r_D - k_D^* = 0.$$

Compare to the equilibrium condition for the banks who borrow from the discount window in DW only setting:

$$(1 - \theta^{DW})R - r_D - k^{DW} = 0.$$

As long as  $k^{DW} < k_D^*$ , there are fewer banks willing to borrow from DW. This is the case, because the strongest banks among the banks worse  $\theta_D$  win in the auction.

For total liquidity, consider the expected marginal borrower. The expected marginal borrower is better than  $\theta^{DW}$ , because they borrow from the auction, and the distribution of the types of the banks participating in the auction in DW and TAF setting first-order stochastically dominates the distributions of the types of the banks borrowing from the discount window.  $\square$

**Interbank Market.** Suppose banks can borrow from the interbank market at a rate of  $r > r_D$ . The borrowing benefit is then  $(1 - \theta)R - r - p_I k_I$ , where now  $I$  denotes borrowing from the interbank market. A bank borrows from the interbank market if and only if

$$(1 - \theta)R - r - p_N k_N > (1 - \theta)R - r_D - p_D k_D.$$

The condition is simplified to  $r < r_D + p_D k_D - p_N k_N$ . Hence, when  $p_N k_N < p_D k_D$ , banks are willing to pay a higher interest rate in the interbank market to avoid discount window, consistent with empirical evidence. But, for sufficiently large  $r$ , interbank market borrowing is not optimal even if it is available.

## References in the Appendix

**Bagnoli, Mark and Theodore C. Bergstrom**, “Log-Concave Probability and Its Applications,” *Economic Theory*, 2005, 26, 445–469.

**Chen, Huaihou, Hongmei Xie, and Taizhong Hu**, “Log-Concavity of Generalized Order Statistics,” *Statistics and Probability Letters*, 2009, 79, 396–399.

## A.2 Tables

**Table 1: Summary Statistics of Bloomberg**

	N	Mean	Max	Min	SD	10 <sup>th</sup>	50 <sup>th</sup>	90 <sup>th</sup>
No. of Borrowers	407							
Banks	313							
Diversified Financial Services	24							
Insurance Companies	12							
Savings and Loans	30							
Market Cap on Aug 1, 2007 (MM)		28525	399089	11	49876.8	107	7331	81813
Foreign Banks	92							
DW-only banks	18							
TAF-only banks	86							
borrow both	260							
Total DW events		12	242	0	28.7	0	2	35
Total TAF events		5	28	0	5.1	0	3	13
Total DW amount (MM)		1529	190155	0	10393.8	0	20	1809
Total TAF amount (MM)		3174	100167	0	10727.5	0	58	7250
Number of days in debt to Fed		323	814	28	196.8	85	306	606

**Table 2: LOLR Borrowing and Bank Fundamentals**

	T1RWA	Lev	Unused Com/Asset	ST WS /Asset	Liquid Asset/Asset
DW × Condition	-9.743 (13.930)	5.732 (18.382)	12.084** (5.654)	0.117 (5.710)	0.163 (4.236)
TAF × Condition	33.706** (16.292)	-60.817*** (19.692)	17.724*** (6.436)	-0.962 (5.739)	4.652 (6.356)
Condition	-2.514 (3.137)	7.708* (4.262)	-0.508 (1.711)	3.070*** (1.155)	-4.926*** (1.269)
$\mathbb{1}\{\text{DW}\}$	11.986*** (1.578)	5.851 (16.797)	8.819*** (1.156)	10.833*** (1.430)	10.833*** (0.718)
$\mathbb{1}\{\text{TAF}\}$	8.629*** (1.933)	67.494*** (17.796)	9.013*** (1.357)	12.430*** (1.534)	11.622*** (0.931)
log(Size)	0.440*** (0.067)	0.426*** (0.073)	0.388*** (0.083)	0.419*** (0.069)	0.456*** (0.058)
ROA	-27.877 (22.937)	-29.703 (23.021)	-17.516 (23.751)	-21.334 (22.408)	-13.928 (22.557)
Constant	-5.670*** (1.020)	-12.705*** (4.575)	-5.197*** (1.076)	-6.301*** (1.052)	-5.626*** (0.842)
Observations	1514	1514	1253	1514	1514
Adjusted $R^2$	0.852	0.855	0.848	0.851	0.852

**Table 3: LOLR Borrowing and Lagged Bank Fundamentals**

	T1RWA	Lev	Unused Com/Asset	ST WS /Asset	Liquid Asset/Asset
DW× Condition	3.212 (14.894)	-0.989 (19.860)	10.948** (5.413)	-2.146 (5.705)	1.562 (4.031)
TAF× Condition	26.516 (18.201)	-58.952*** (20.867)	16.474** (6.452)	2.008 (5.719)	1.844 (6.831)
Condition	-4.144 (3.069)	9.579** (4.320)	-0.596 (1.543)	3.334*** (1.102)	-4.329*** (1.314)
$\mathbb{1}\{DW\}$	10.588*** (1.644)	11.941 (18.168)	9.019*** (1.122)	11.354*** (1.447)	10.680*** (0.722)
$\mathbb{1}\{TAF\}$	9.433*** (2.056)	65.783*** (18.882)	9.148*** (1.397)	11.671*** (1.579)	11.901*** (0.963)
log(Size)	0.462*** (0.068)	0.452*** (0.073)	0.370*** (0.076)	0.436*** (0.068)	0.467*** (0.058)
ROA	-25.099 (23.165)	-23.964 (23.301)	-12.198 (23.592)	-21.440 (22.020)	-12.579 (22.788)
Constant	-5.827*** (1.010)	-14.833*** (4.630)	-4.924*** (0.972)	-6.590*** (1.046)	-5.864*** (0.845)
Observations	1493	1493	1364	1493	1493
Adjusted $R^2$	0.851	0.855	0.851	0.852	0.852

**Table 4: DID and LOLR Borrowing**

	All DW	All TAF	CAN DW	CAN TAF	DEU DW	DEU TAF	FRA DW	FRA TAF
DID	-0.423 (1.161)	1.281 (1.577)	-2.059** (1.043)	5.224** (2.222)	0.595 (1.813)	1.713 (2.334)	-3.195 (2.998)	-1.127 (3.115)
Post	0.040 (0.255)	1.779*** (0.290)	0.051 (0.230)	2.048*** (0.274)	0.040 (0.255)	1.779*** (0.290)	0.040 (0.255)	1.779*** (0.290)
Treat	1.354 (0.968)	10.890*** (1.359)	-0.323 (1.034)	12.524*** (2.208)	2.401 (1.501)	8.893*** (2.046)	2.417 (2.817)	14.780*** (2.667)
Constant	2.333*** (0.220)	1.996*** (0.231)	2.332*** (0.181)	2.050*** (0.191)	2.333*** (0.220)	1.996*** (0.231)	2.333*** (0.220)	1.996*** (0.231)
Observations	2728	2728	2560	2560	2632	2632	2560	2560
Adjusted $R^2$	0.001	0.159	0.001	0.089	0.009	0.082	-0.000	0.063

**Table 5: LOLR Borrowing and Bank Failure**

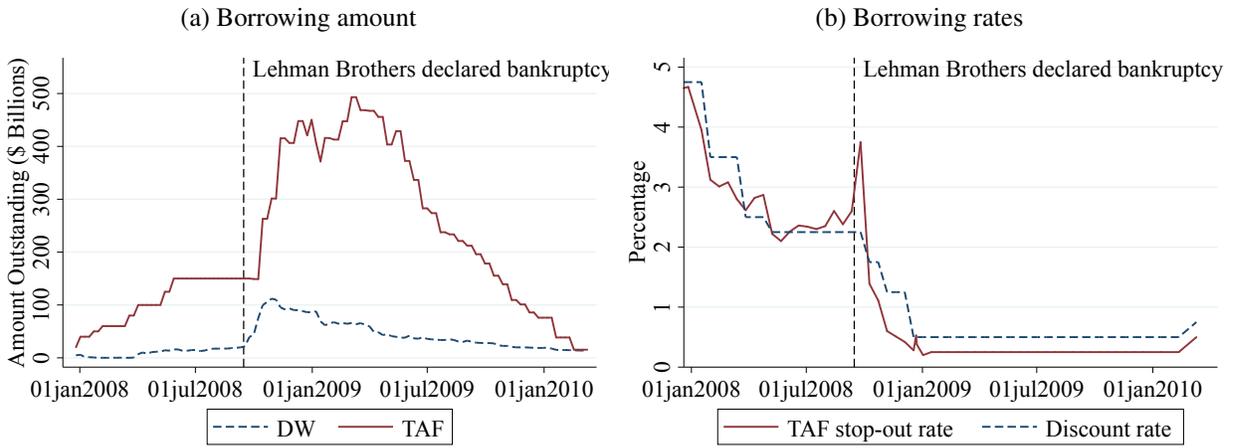
	Fail this quarter	Fail during Crisis
DW/(DW+TAF)	0.007* (0.004)	0.125** (0.050)
Constant	0.003 (0.002)	0.050*** (0.019)
Observations	1586	364
Adjusted $R^2$	0.001	0.020

**Table 6: CDS Spreads and Borrowing Events**

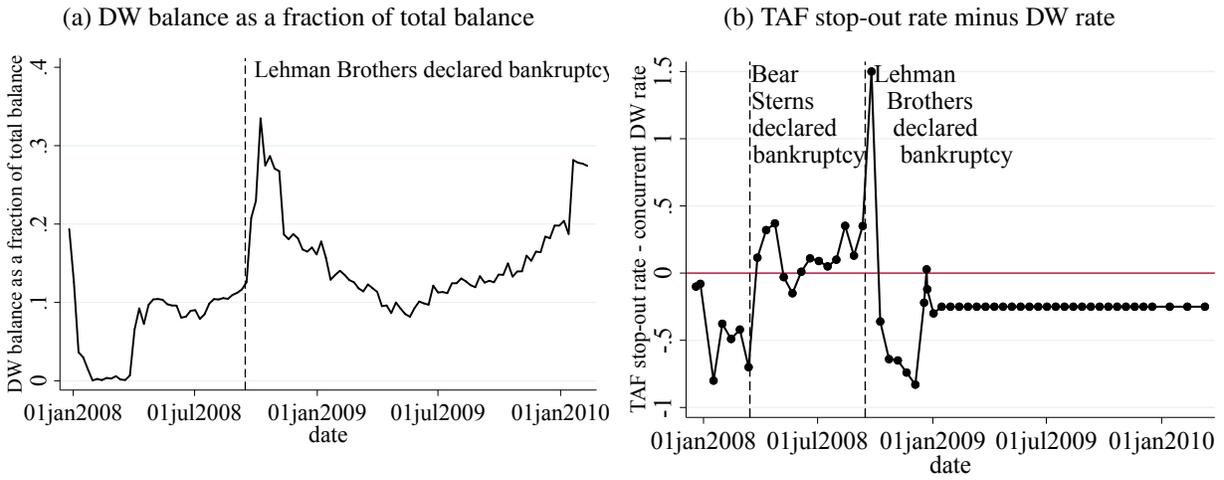
	(1) DW/TAF	(2) DW/None	(3) TAF/None
Lagged 5y CDS spread	0.129** (0.058)	0.004*** (0.001)	0.001 (0.002)
Constant	1.257*** (0.279)	0.030*** (0.007)	0.004 (0.010)
N	707	33440	33617
R <sup>2</sup>	0.466	0.043	0.016

### A.3 Figures

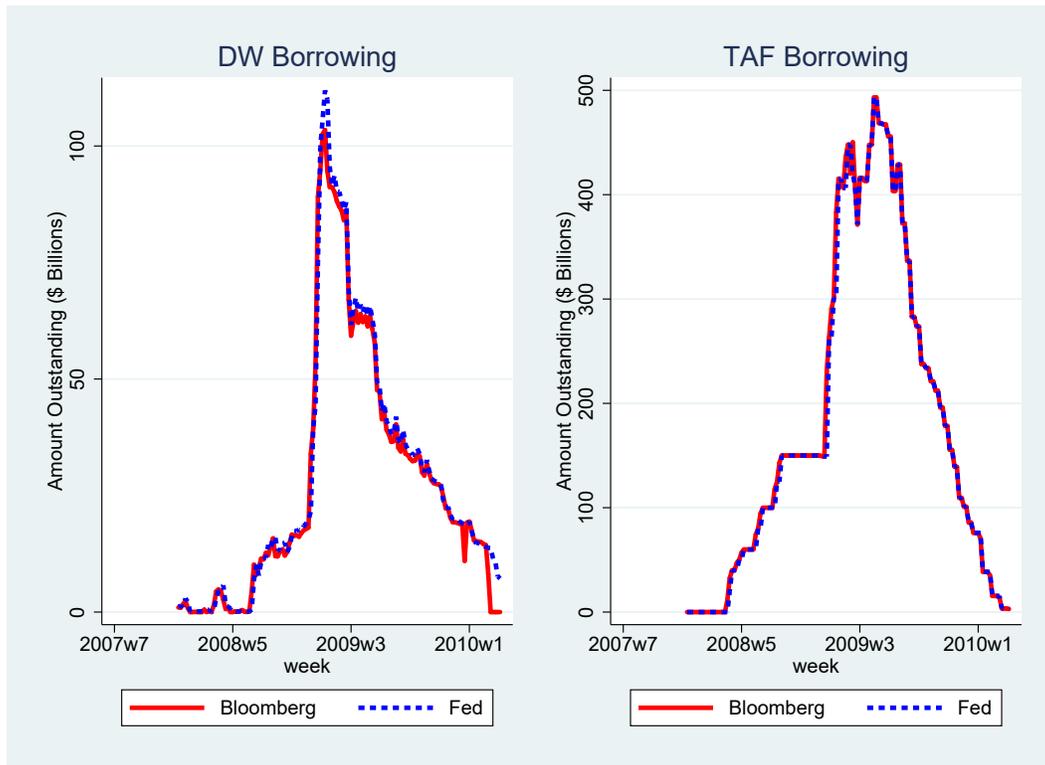
**Figure 1: Borrowing amounts and rates in DW and TAF from 2008 to 2010**



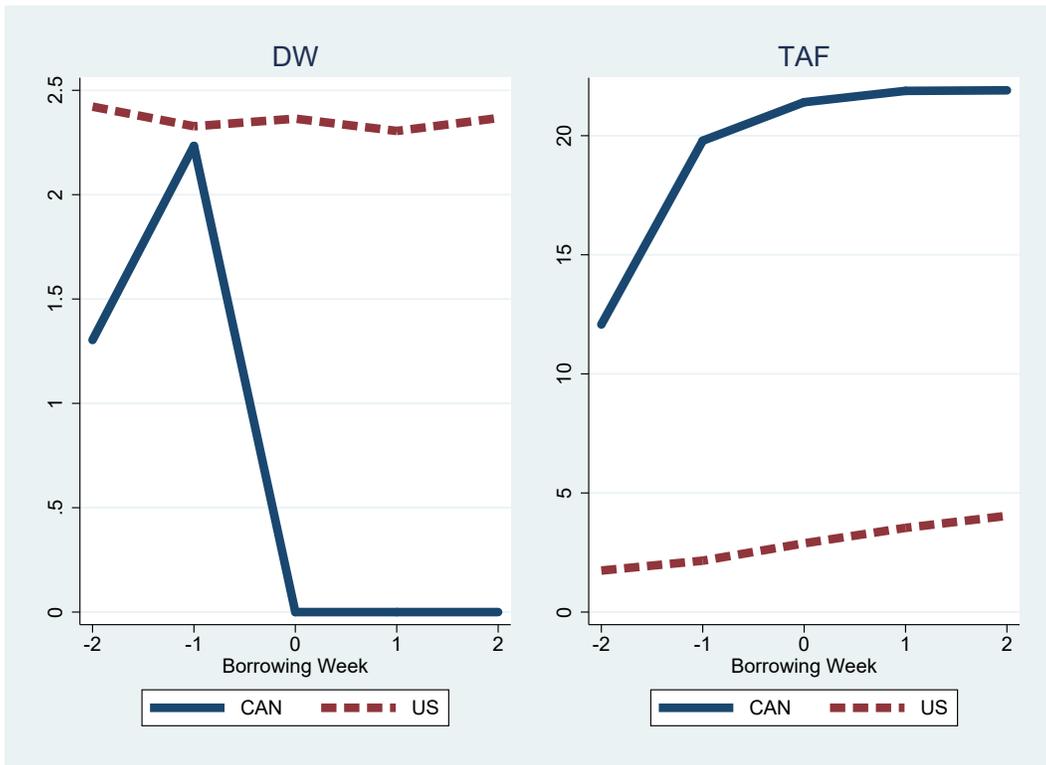
**Figure 2: Borrowing amounts and rates in DW versus TAF from 2008 to 2010**



**Figure 3: Comparison between Bloomberg Data and Fed Data**



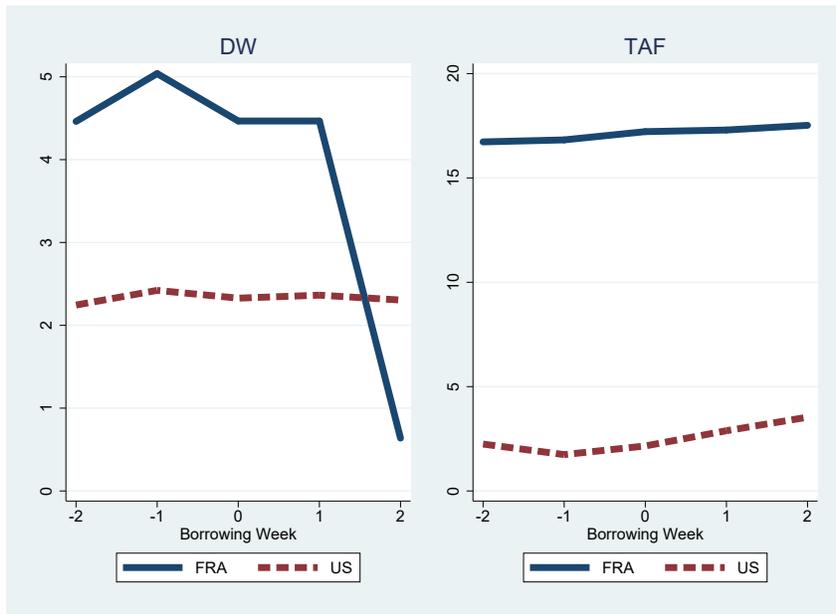
**Figure 4: DID: Canada v.s. U.S.**



**Figure 5: DID: Germany v.s. U.S.**



**Figure 6: DID: France v.s. U.S.**



**Figure 7: CDS Spreads around Borrowing Events**

