Excess Returns of Companies with a Distinguished Player

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JOB MARKET PAPER[1]

Abstract

We study the share market of a public company with a *distinguished player*. A player is distinguished with respect to a company if he can do both, trade shares *and* enhance the value of the company by exerting costly effort. Due to these private effort costs, shares have a lower value to the distinguished player as compared to other investors. We show how these different valuations can lead to a trade price strictly below the equilibrium value of the company. This implies that buyers enjoy *excess returns* on their investment and is at odds with the efficient markets hypothesis. It further involves a substantial reinterpretation of traditional *no-arbitrage* towards a game-theoretic understanding. The empirical prediction that companies with a distinguished player yield excess-returns was confirmed for the sample of S&P500 firms and S&P1500 firms in a companion paper by von Lilienfeld-Toal and Rünzi (2007). Our results are robust with respect to trading rules, discrete versus continuous effort, trading costs, noise traders, and price taking behavior.
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[1]This is a job market paper for Ulf von Lilienfeld-Toal

1 Introduction

Consider a manager of a firm who can raise the value of a firm by exerting costly effort *and* trade shares of the firm on the stock market. Applying the standard no-arbitrage equilibrium concept of asset pricing to determine the value of this firm leads to a paradox. All relevant information should be priced, including the ownership and thereby the effort of the manager. If, however, the privately costly effort is already priced, the manager would be better off to sell his shares, not exerting effort, and saving the private effort costs instead.[2]

This paradox is mirrored in the empirical evidence provided by von Lilienfeld-Toal and Rünzi (2007) who show that standard arbitrage free asset pricing cannot explain the cross section of stock returns for firms with an owner-manager. For example, they show that a value-weighted portfolio consisting of all S&P 500 firms (1994-2005) in which the CEO holds more than 10% of the company's stocks significantly outperforms the total market portfolio by 13% p.a.

In this article we propose a solution to the theoretical paradox that is also consistent with the empirical evidence. We analyze incentives and interactions among completely and symmetrically informed players who can be rational or irrational traders of shares of a company. We introduce a *distinguished player* who can trade *and* increase the firm's value by exerting costly effort. This distinguished player reflects a basic concept of corporate finance – i.e. operates in a standard moral hazard context. For example, the distinguished player could be interpreted as the *agent* in the Grossman and Hart (1983) model.[3] While the information structure and production technology is similar, we let the distinguished player trade shares – and hence incentives – before exerting effort. The main task is to analyze the outcome of the ex-ante market game before the distinguished player's effort decision.

Two classes of trade equilibria are of interest: *true value* and *excess returns equilibria.* In a true value equilibrium, shares of the firm are traded at the price that equals the equilibrium value of the firm. In excess returns equilibria, shares of the firm are traded at a price strictly below the equilibrium value of the firm.

Our main results are (1) *true value equilibria* may not exist in rational call auction markets. This formalizes the aforementioned paradox of endogenous firm value and arbitrage free asset pricing. (2) *Excess returns equilibria* may exist – even if true value equilibria do not exist. Furthermore, excess returns equilibria do not exist without a distinguished player. This provides an explanation for the abnormal returns of owner-CEO firms as reported in von Lilienfeld-Toal and Rünzi (2007). (3) Excess returns equilibria are robust with respect to (i) trading costs, (ii) noise traders and price taking behavior, (iii) discrete vs. continuous effort and (iv) the specification of the market microstructure.

[2] A related paradox is discussed in Grossman and Stiglitz (1980) where information is costly.

[3] In this respect we follow the literature on asset pricing with large shareholders (see e.g. Bolton and von Thadden (1998), DeMarzo and Urosevic (2006), or Admati, Pfleiderer and Zechner (1994)).

The main ideas behind these results are now motivated in more detail.

Non-existence of true value equilibria. The result that true value equilibria may not exist constitutes the formalization of the paradox and provides a first insight as to why analyzing firms with a distinguished player is an important task that leads to surprising results. The intuition behind non-existence is as follows. Due to private effort costs, the valuation of the distinguished player and outside investors differ and the distinguished player values shares strictly lower than outside investors. As a result, if shares of the firm are traded at the true value the distinguished player wishes to sell his shares whereas outside investors are indifferent between trading and not trading. Two ingredients are required to show that this cannot be an equilibrium: Continuous effort and anonymous trading. Under continuous effort, the distinguished player always adjusts his effort and hence saves some effort costs if he manages to sell some shares, irrespective of how many shares he sells. This implies that the distinguished player always benefits if he manages to sell without having a strong impact on the share price. Further, anonymous trading makes sure that the distinguished player can indeed sell shares without affecting the price. Since trade is anonymous on real world stock exchanges, this implies that true value equilibria systematically fail to exist if effort is continuous.

The non-existence result is a negative result but our analysis does not stop here. Rather, we show that stock prices of firms with a distinguished player can still be analyzed within a standard market game since excess returns equilibria exist even in set-ups where true value equilibria do not exist.

Excess returns equilibria. In contrast to standard asset pricing theory "no-arbitrage" in our theory does not imply that the market price and the true equilibrium value coincide. In excess returns equilibria those traders who manage to buy below the equilibrium value do indeed realize *strictly positive gains* even without any informational advantage whereas rational sellers suffer a strict loss. In this sense, the excess returns phenomenon appears to contradict the traditional interpretation of *efficient markets* and *no-arbitrage* in equilibrium (see for example Fama, 1970 or Ross, 1976). The obvious question is, why does not everybody buy maximally at a price for which buying yields strictly positive gains? A game theoretic inspection reveals that in an excess returns equilibrium "no-arbitrage" is still valid in the sense that no investor can gain by buying or selling more or less. Rational traders are aware of the fact that shares are traded below their *true value*. But at the same time they acknowledge that the distinguished player wants to sell shares – or buy less – whenever the share price exceeds a certain threshold and this threshold is below the equilibrium value. Hence, trade at the equilibrium value would encourage the distinguished player to sell his shares in an anonymous market and save on effort costs instead. Therefore, in a fully rational environment excess returns equilibria are characterized by the property that any deviation that drives up the market price towards the true value triggers the distinguished player to withdraw instead of raising the company value to the anticipated level which in turn causes even bigger losses to everybody. This latter property of excess returns equilibria is called *pivotalness*. It can serve as a disciplining off-equilibrium coordination device. Put differently, a failure to coordinate on a sufficiently low market price below the true equilibrium value may destroy wealth for all shareholders by removing incentives for the distinguished player to work hard and generate positive externalities.

To analyze excess returns is fruitful for several reasons. First, they may be the *only* equilibria that exist. Moreover, excess returns equilibria have properties that differ from standard no-arbitrage asset pricing which allows to derive new predictions that can be tested in empirical work. For example, the theory developed here correctly predicted the hypothesis tested in von Lilienfeld-Toal and Rünzi (2007). Testing additional properties of excess returns equilibria promises to yield additional new empirical results that so far have not been investigated, simply for the fact that they are inconsistent with the traditional no-arbitrage asset pricing theory. Finally, excess returns equilibria are not only consistent with abnormal returns for owner-CEO firms but also with other phenomena that have traditionally been considered as "asset pricing anomalies" (e.g. equity premium, excess volatility, no trade theorems, lockup agreements, limited stock market participation, ...) as is discussed in more detail in von Lilienfeld-Toal (2005).

Robustness: Many small traders and irrationality. Our benchmark analysis is carried out in a fully rational world. Since irrationality and noise traders are considered to be important ingredients of real world stock markets it is unclear how robust our results are with respect to the introduction of noise. In particular, it is unclear whether or not excess returns equilibria survive if the number of investors gets arbitrarily large while the market is noisy. We show that within our *distinguished player environment* neither *full rationality and pivotalness* nor *strategic behavior of small traders* are necessary ingredients for the existence of an excess returns equilibrium. We show this by analyzing a continuum-trader-version of the model with noise traders. Within the so defined stochastic environment we demonstrate existence of excess returns equilibria for several reasonable market mechanisms. The basic idea is the following. With noise final allocations and prices are random variables. As before, the distinguished player plans to sell shares whenever the share price exceeds a certain threshold which now occurs with positive probability. This implies that rational outside investors do not want to buy shares above this threshold because the distinguished player will not exert costly effort in that case. Outside investors only want to buy shares if the share price is below the threshold price. To make sure not to buy from the distinguished player, outside investors do not submit buy orders with a limit price above the threshold price of the distinguished player. As a result, outside investors do not bid up the share price even though shares are undervalued on average and even though bidding up the share price would be possible.

The existence of these equilibria shows that a distinguished player in combination with irrational noise traders provides an alternative explanation for excess returns equilibria besides pivotalness in fully rational models.

Market microstructure. Our formulation and results establish progress in another direction. It is well known that results in the market microstructure literature hinge critically on the exact specification of the market mechanism. For example, O'Hara (1995) discusses different market microstructure models and states (p. ix) that the "generality of their results, and hence their applicability, is not well understood". This observation is bothering since any specific market mechanism is only an approximation of real world trading systems and it is not known what constitutes a good approximation. While our existence results, as the previous literature, depend on the specific market mechanism our characterization results in section 4 implying the crucial predictions on asset pricing are general and *do not* depend on the market mechanism. Moreover, our various existence and non-existence results in section 3 are formulated for pricing and allocation rules that turn up in the real world. They demonstrate to which extent other details of the relevant equilibria beyond our characterization results indeed do depend on the market mechanism. In particular, it is shown by comparing relevant real world exchange rules how the variation of these rules can influence possible equilibrium behavior.

Institutional or contractual clauses. It is difficult to argue that in anonymous markets distinguished players differ from other traders in their ability to commit not to trade. On the one hand, neither SEC regulations nor privately stipulated contracts can fully rule out anonymous trading by distinguished players. The majority of shares held by executives are common shares that are not subject to a non-selling clause. Furthermore, SEC regulations force insiders to report trade ex post and sometimes forces insiders to reveal plans to trade ex ante. Nevertheless, these shares are then traded anonymously on the market and outside investors do not know whether they buy from insiders or from other outsiders. On the other hand, in anonymous markets already the perception of some strategic outside investors about the role of a distinguished player is sufficient to support excess returns equilibria due to this theory.

The multiplicity and rich equilibrium structure of this theory is less surprising from the perspective of *large games* since our model in fact establishes a semi-anonymous game. Semi-anonymity of a game means that players' payoffs only depend on aggregated actions of player-types rather than on the individual action profile. In this context we only consider two types of players, regular investors and the distinguished player. Here, *semi-anonymity* is a crucial property since it means that traders do not care about the composition of bids among regular shareholders. However, being of a different type, the distinguished player's actions distinctly enter the payoffs of traders. This circumstance brings about the pattern of true value versus excess returns equilibria. Equilibria of semi-anonymous games in general have been characterized in Blonski (2005). Strong theoretical support for the so identified equilibrium structure in these games comes from Kalai (2004) who has shown that equilibria of semi-anonymous games are surprisingly robust in various senses when the number of players gets large.

The paper proceeds as follows. Section 2 establishes the reference model, introduces formally the idea of a distinguished player, and sets up the notation for general market mechanisms. Any particular market mechanism specifies a corresponding market game. In section 3 we show non-existence of true value equilibria and existence of excess returns equilibria for several real world call auctions and the Kyle (1989) market mechanism. In section 4 we characterize excess returns and true value equilibria for general market mechanisms. Section 5 derives robustness results and establishes existence of the excess returns equilibrium with a continuum of traders and noise traders. Section 6 discusses in more detail the relationship of this theory to the most relevant existing literature. Section 7 discusses further extensions like multiple distinguished players or a market maker while section 8 concludes. Appendix A explains more rigorously the rich and general strategy space of the market game, Appendix B introduces the language for stochastic market mechanisms while Appendix C contains all remaining proofs.

2 Market Game with a Distinguished Player

Distinguished player. Denote by $i = 0$ a *distinguished player* being interpreted as a "manager-owner", raider, activist shareholder, or founder of a firm. Further, denote by $i = 1, ..., N$ *outside investors* with (weakly) positive stakes in this firm[4]. *Distinguishedness* of a player-investor is defined as the ability to enhance the value of the firm. The firm either has value $\underline{v}$ or $\bar{v} \equiv \underline{v} + \Delta v \geq \underline{v}$ and the realization of this value depends on the distinguished player's effort decision which can be discrete $e \in \{0, 1\}$ or continuous $e \in \mathbb{R}_+$. If the distinguished player exerts effort e the firm yields value $v = \underline{v} + e(\bar{v} - \underline{v})$. Exerting effort causes private effort costs $c(e) = c \cdot e^2$. We assume that $\Delta v > c > 0$ so that the efficient or first best effort choice is $e^{FB} > 0$, in both the continuous and the discrete effort case. To compare with known models in the literature, we are also interested in the case $\Delta v = \bar{v} - \underline{v} = 0$ where no player is a distinguished player.

Firm ownership. Our object of investigation is a market game where stakes of the firm can be traded before the distinguished player decides on his effort decision. The *initial ownership structure* of the firm before the market game takes place is exogenously

[4]To study the role of small investors and price taking behavior we study a continuum of investors in section 5.

given. It is defined by an element α of the simplex

$$\Delta = \left\{ \xi = (\xi_0, ..., \xi_N) \middle| \xi_i \in Q \text{ and } \sum_{i=0}^{N} \xi_i = 1 \right\}$$

where quantities Q can be either continuous $Q = [0,1]$ or discrete $Q = \left\{0, \frac{1}{M}, \frac{2}{M}, ..., \frac{M}{M}\right\} \subset [0,1]$ with M indivisible shares. Within the latter interpretation, initially player i owns $\alpha_i M$ shares of the firm. The market game to be described subsequently endogenously results in the final ownership denoted by $\omega = (\omega_0, \omega_1, ..., \omega_N) \in \Delta$ with $\omega_i \in Q$ and $\sum_{i=0}^{N} \omega_i = 1$.

Effort choice. In the market game yet to be defined, stakes of the firm are traded before the distinguished player decides about his effort. Once the market game is over the distinguished player chooses effort to maximize the net value

$$\omega_0 \left(\underline{v} + e\Delta v \right) - c(e)$$

of his final stake ω_0 in the firm. Let

$$e(\omega) \in \underset{e}{\operatorname{argmax}}\, \omega_0 \left(\underline{v} + e\Delta v \right) - c(e). \tag{1}$$

For discrete effort the distinguished player may be indifferent between 0 and 1. For this case we suppose as tie breaking rule that effort is 1. Hence, for discrete effort it follows that

$$e(\omega) = \begin{cases} 1 & \text{for } \omega_0 \Delta v \geq c \\ 0 & \text{otherwise.} \end{cases}$$

whereas $e(\omega) = (\omega_0 \Delta v)/(2c)$ in the continuous case. Similarly, the payoff of any outside investor $i = 1, \ldots, N$ after the market game is given as $\omega_i \left(\underline{v} + e\Delta v \right)$, i.e. the final value of his stake after the distinguished player's effort choice.

We assume effort to be non-contractible and that no additional contractual provisions are used. This approach is also used in the related literature, see e.g. Admati, Pfleiderer and Zechner (1994) or DeMarzo and Urosevic (2006) and is supported by empirical evidence.[5] We believe that as long as the market price enters the distinguished player's incentives, the problems we discuss here cannot be perfectly alleviated by appropriately chosen contractual clauses. We therefore simply assume that no contractual provisions are used at all.

[5] According to the execucomp database, there are 5106 officer year observations, where an officer owns more than 5% of unrestricted shares within all S&P 500 or S&P 1500 firms between 1992-2004. In contrast, only in 26 officer years an officer holds more than 5% of restricted shares. For a description of the data, see von Lilienfeld-Toal and Rünzi (2007).

Prices and strategies. To allow for models with discrete or continuous prices we introduce $P \subset \mathbb{R} \cup \{-\infty, \infty\}$ as the set of feasible prices. Similarly as for quantities the relevant examples are a continuous $P = \mathbb{R} \cup \{-\infty, \infty\}$ or a discrete price range $P = P_\delta := \{-\infty, ..., \underline{v}, \underline{v} + \delta, \underline{v} + 2\delta, ..., \bar{v}, ..., \infty\}$ with some exogenous tick size δ. Real world market mechanisms distinguish between buy and sell prices p_b, p_s. The difference $\gamma := p_b - p_s \geq 0$ is called *bid ask spread*[6]. If prices $p \in P_\delta$ are discrete clearly the bid ask spread γ is supposed to be a non negative integer multiple of the tick size, i.e. $\frac{\gamma}{\delta} \in \mathbb{N}$. From here $p \equiv p_s$ always denotes sell prices whereas the corresponding buy price is $p_b = p + \gamma$. Clearly, all buy orders depend on p_b and all sell orders depend on p_s.

Strategies or *market actions* $a_i \in A_i$ of an investor i are collections of buy and sell orders. Mathematically, a strategy a_i can be described by a pair $\{D_i(p), S_i(p)\}$ of set-valued demand and supply functions or correspondences. For example, $q \in S_i(p)$ represents a quantity of shares trader i is willing to sell at price p. Together, $Z_i(p) = D_i(p) - S_i(p) \subset Q$ is a set of positive or negative net quantities composed by demand and supply quantities that would be acceptable for investor i at price p.[7] In appendix A we describe in more detail how the rich mathematical object of an excess demand correspondence is the result of a general and realistic set of possible orders such as buy and sell limit orders, market orders, stop orders and all or nothing orders (fill or kill orders). Further, appendix C contains existence proofs for specific market mechanisms and displays pictures with aggregated excess demand correspondences that illustrate the structure of relevant strategy spaces.

Market mechanism. By adding up individual behavior an action profile $a \in A$ induces the market excess demand correspondence

$$Z(p) = \sum_{i=0,...,N} Z_i(p)$$

which decomposes into aggregated buy and sell offers

$$D(p) = \sum_{i=0,...,N} D_i(p) \text{ and } S(p) = \sum_{i=0,...,N} S_i(p)$$

called the *market demand* and *market supply correspondences.* They define sets of quantities the market as a whole is willing to buy or to sell at a given sell price p. Relevant for many real and theoretical market mechanisms is the *limit order trade volume* $\tau(p)$ for

[6]We explicitly want to include the case of bid ask spread $\gamma = 0$ to be able to compare this model with models without trading costs.

[7]To allow traders as in reality to choose demand and supply rather than just the sum of both – i.e. excess demand – opens the possibility for "beller strategies" in which a trader might, for example, try to bid up the stock price by submitting buy orders and simultaneously selling stocks. It turns out that these strategies complicate existence proofs but we want to consider them since they are not ruled out in many real world trading systems.

$p \in P$ defined by the maximum tradable quantity

$$\tau(p) = \min\left\{\sup \sum_{i=0,\dots,N} d_i(p), \sup \sum_{i=0,\dots,N} s_i(p)\right\}$$

of the short side of the market restricted to limit orders and stop orders[8] but excluding all-or-nothing orders.

Denote by $\phi = (x, y) = ((x_0, ..., x_N), (y_0, ..., y_N))$ a buy-sell-transaction vector and

$$\Phi = \left\{\phi = (x, y) \middle| x_i \in Q, y_i \in Q, \sum x_i - y_i = 0, \alpha + x - y \in \Delta\right\}$$

the set of feasible buy-sell transaction vectors and $x-y$ a corresponding net trade vector. For initial allocation α and net trade vector $x-y$ the final allocation is $\omega = \alpha+x-y \in \Delta$.

We call player i *strictly wealth constrained* iff i can only submit sell orders[9]. For a wealth constrained player i a bid strategy consists only of selling bids and therefore takes the form $a_i = \left\{(\sigma_i^1, \sigma_i^2, ...), (\mathring{\sigma}_i^1, \mathring{\sigma}_i^2, ...), (\psi_i^1, \psi_i^2, ...), (\mathring{\psi}_i^1, \mathring{\psi}_i^2, ...)\right\}$ or $z_i(p) = -S_i(p)$.

Definition 1 For any initial ownership $\alpha \in \Delta$ and any strategy profile $a \in A$ a deterministic market mechanism μ with bid ask spread γ is a mapping

$$\mu : \Delta \times A \to P \times \Phi \text{ with } \mu(\alpha, a) = (p^\mu(a), x^\mu(a), y^\mu(a))$$

where for initial ownership α and strategy profile a the market mechanism μ picks a sell price $p^\mu(a) \in P$, a buy price $p^\mu(a) + \gamma$ and for any player a subset of submitted orders being executed, i.e.

$$\begin{aligned} x_i^\mu(a) &\in D_i(p) \cup \{0\} \text{ and} \\ y_i^\mu(a) &\in S_i(p) \cup \{0\}. \end{aligned}$$

We call this latter property "voluntary trade". Hence, net trades $x_i^\mu(a) - y_i^\mu(a) \in Z_i(p) \cup \{0\}$ are composed by submitted orders. By specifying the trade vector the market mechanism μ thereby determines the ex post ownership structure given as

$$\omega^\mu(a) := \alpha + x^\mu(a) - y^\mu(a). \qquad \square$$

Voluntary trade $x_i \in D_i(p) \cup \{0\}, y_i \in S_i(p) \cup \{0\}$ means that *only* submitted orders or nothing are executed, i.e. nobody can be forced to trade and conversely nobody can enforce trade. Market mechanism μ is said to *maximize the trade volume* if μ picks a price that maximizes the limit order and stop order trade volume $\tau(p)$. Of course, market mechanisms can be specified that do not maximize the trade volume. For example, price

[8]The small letters d_i and s_i indicate convex valued demand and supply correspondences composed only by limit orders and stop orders. For details see appendix A on page 28.

[9]For example, this is likely to be the very reason why the distinguished player needs funding by outsiders. Otherwise he would prefer to run the firm himself.

could be determined by maximizing the total trade volume including fill or kill orders. Alternatively the distinguished player or other players could be treated with priority if they submit orders. However, we are not aware of a real world market mechanism not using maximal trade volume with top priority. Nevertheless, our characterization results in section 4 do not rely on this property of maximal trade volume. Conversely, we will see in Proposition 2 that a property like maximal trade volume may be sufficient to guarantee existence of trade equilibria. Moreover, a market mechanism is called *anonymous* if permuting players' names does not affect price and final allocation.

Market game Γ_μ. Any market mechanism μ with bid ask spread γ together with an initial ownership α induces a *market game* Γ_μ with strategy space A and *payoff functions* given by

$$u_i(a) = \omega_i^\mu(a)\left(\underline{v} + e(\omega^\mu(a))\Delta v\right) - \left(p^\mu(a) + \gamma\right) x_i^\mu(a) + p^\mu(a) \cdot y_i^\mu(a)$$

for $i = 1, ..., N$ and

$$u_0(a) = \omega_0^\mu(a)\left(\underline{v} + e(\omega^\mu(a)) \cdot \Delta v\right) - \left(p^\mu(a) + \gamma\right) x_0^\mu(a) + p^\mu(a) \cdot y_0^\mu(a) - c(e(\omega^\mu(a)))$$

for the distinguished player $i = 0$. Any action profile a in a deterministic market mechanism generates a unique final allocation $\omega^\mu(a)$ and thereby induces a unique optimal effort decision $e(\omega^\mu(a))$ as derived in equation (1). Hence, company value $v(a)$ is given as

$$v^\mu(a) \quad = \quad \underline{v} + e(\omega^\mu(a))\Delta v$$

No arbitrage. A strategy profile a of the market game Γ_μ is said to satisfy the traditional *no-arbitrage condition* iff there exists no trader i who realizes strictly positive gains under the price $p^\mu(a)$ and buy-sell transactions $x_i^\mu(a), y_i^\mu(a)$ determined by market mechanism μ:

$$\left[p^\mu(a) - v(a)\right] y_i^\mu(a) + \left[v(a) - p^\mu(a) - \gamma\right] x_i^\mu(a) \leq 0$$

for all $i = 0, \ldots, N$.

Generalized No-arbitrage and Equilibrium. We use the pure strategy Nash equilibrium as our solution concept. A strategy profile a^* is said to satisfy *generalized no-arbitrage* if it is a *Nash equilibrium* or just *equilibrium* of market game Γ_μ, i.e. no player can strictly improve or in the language of game theory every player plays a best response a_i^* to other players strategy profiles a_{-i}^*. Correspondingly, $(p^*, x^*, y^*) = \mu(\alpha, a^*)$ and $\omega^* = \alpha + x^* - y^*$ are called *equilibrium price*, *equilibrium trades* and *equilibrium ex post allocation* of market game equilibrium a^* under market mechanism μ. Furthermore, we will call $e(\omega(a^*))$ equilibrium effort denoted by e^*. While any equilibrium by definition satisfies generalized no arbitrage hitherto it is not clear if the same holds for traditional no-arbitrage.

No Trade Equilibrium. If no player submits an order no player can gain anything by submitting orders. This simple observation together with the *voluntary trade property* guarantees that for any market mechanism there always exists a no-trade equilibrium where no player submits orders or trades. In the remainder of this article we concentrate on more interesting equilibria where we can observe a price such that trade occurs.

True value and excess returns equilibria. An equilibrium a^* with $\omega^* \neq \alpha$ is called a *trade equilibrium* of Γ_μ. *Excess returns* for a firm are defined as

$$R^\mu(a^*) := v^\mu(a^*) - p^\mu(a^*),$$

i.e. the difference between equilibrium firm value and equilibrium price.

Definition 2 A trade equilibrium in which shares are traded at their equilibrium value is called a *true value equilibrium*, i.e.

$$v^\mu(a^*) = p^\mu(a^*) \Leftrightarrow R^\mu(a^*) = 0 \ \texttt{and} \ \omega^* \neq \alpha.$$

A trade equilibrium in which shares are traded below their equilibrium value is called an *excess returns equilibrium*, i.e.

$$v^\mu(a^*) > p^\mu(a^*) \Leftrightarrow R^\mu(a^*) > 0 \ \texttt{and} \ \omega^* \neq \alpha.$$

A trade equilibrium with high equilibrium effort $e^* > 0$ and equilibrium price $p^* > \underline{v}$ is called a *high* true value equilibrium and a true value equilibrium with low effort $e^* = 0$ and equilibrium price $p^* = \underline{v}$ is called *low* true value equilibrium. □

Excess returns equilibria. In an excess returns equilibrium it appears more attractive to be a buyer than a seller. A net equilibrium buyer i who buys enough to strictly overcome transaction cost losses gains

$$(v^* - p^* - \gamma)x_i^* - (v^* - p^*)y_i^* > 0$$

and is called *equilibrium winner*. Although the role of its counterpart – the net equilibrium seller – is less pleasant it can well be rational if the alternative is low effort of the distinguished player triggering a lower value for all.

3 Trading Rules and Existence.

In this section we show existence of excess returns equilibria and non-existence of true value equilibria. To show existence and non-existence, we need to consider any potential deviation for every trader. Since market rules determine how the market price and/or the allocation changes due to a change of submitted orders, we must become very explicit concerning market rules. In contrast to this, our characterization results hold for general market micro-structures. These characterization results show that the economic intuition behind our results are robust and carry over to a significantly more general class of market microstructures.

To show existence, we concentrate here on electronic call auctions, because 1) we can use the exact and fully specified rules taken from real world trading mechanisms, 2) we do not need to specify the timing as to who trades when and knows what, and 3) these rules are used in the real world and in the literature.[10]

While we limit our analysis of existence on real world call auctions, there is still considerable degree of freedom in the details. Price setting rules are less problematic since the same rules are used in almost all call auction mechanisms on stock exchanges around the world. In contrast, rationing rules regulating allocation and the decision which orders are executed must be treated with great care. Rationing rules differ across exchanges and are crucial for our analysis, in particular rules concerning *size priority* and *fill-or-kill orders*.[11]

Size priority. Size priority is used in different exchanges[12] and in one example we use an opening auction motivated by the Tokyo stock exchange. The trading period in Tokyo starts with an opening auction and orders valid for the opening auction can be submitted prior to the trading period. Furthermore,

> "...all limit orders received prior to the start of trading have equal time priority. Price, time, and size priority hold in this order for limit orders placed during the trading sessions." (Lehmann and Modest, 1994, p. 954)

Breaking up orders. Different priority rules are applied at NYSE.

> "The NYSE does not follow a strict time priority rule. To minimize the breaking up of large orders, the time priority rule applies only to the first limit order. The

[10] Trading rules can be ordered by continuous trading vs. call auctions and electronic market places vs. dealer markets. For existence and non-existence results, one cannot use dealer markets since they are not sufficiently explicit. For example, as a rule specialists on the NYSE "*... have an exchange mandated obligation to maintain fair and orderly markets.*" (Lehmann and Modest (1994, p. 952)). To show existence, it is necessary to overcome the lack of preciseness in the regulation of specialists. In the literature, this problem is often solved by assuming that there is perfect competition between market makers and hence equilibrium price equals equilibrium value. Since a major goal of this paper is to derive the equilibrium trade price endogenously and to allow for a deviation from this assumption, this approach is not feasible here. Moreover, continuous trading is more involved as we would have to specify the timing of orders and the information set for every agent.

[11] Reny and Perry (2006) put similar emphasis on the exact specification of rationing rules.

[12] There are also several exchanges where size priority is not used, for example at the Paris Bourse (see Biais et al. (1995) or Australia stock exchange (see Aitken et al. (1998)).

remaining limit orders follow a size priority rule; namely limit orders that match the size of the market order at the best price are given priority over other limit orders ..." (Huang and Stoll, 2001, p. 506)

Accordingly, the objective of not breaking up orders is applied in the market mechanism we call NYSE.[13]

Fill-or-kill orders. Similar to size priority, fill-or-kill orders are allowed at some – e.g. Amsterdam or XETRA – but not all real world call auctions. Note, however, that they can be used in most continuous trading settings, e.g. NYSE or Paris[14].

To establish a realistic setting we define four different market micro-structures which are all call auctions. All four call auctions use the same price setting rules and set price priority as the most important allocation rule. Rationing rules apply if price priority does not already lead to a unique allocation. It is at this point, i.e. the second rule concerning allocation, that these market mechanisms differ.

Example 1: Call auctions.

A: Price setting.

1. The price is set to maximize the trade volume $\tau(p)$.[15]
2. Should there be more than one such price, surplus $|s^*(p) - d^*(p)|$ is minimized, not counting fill-or-kill orders.
3. Should there still be more than one potential price, the minimal price will be taken if there is excess supply. For excess demand, the maximum price is taken.
4. Should there still be more than one price, the price closest to a reference price will be chosen and we choose $\bar{v}$ to be the reference price.[16]

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[13] It can be argued that the upstairs market used at NYSE and many other exchanges – e.g. Paris Bourse or XETRA – also gives priority to large orders since only large orders can be traded upstairs (and also downstairs).

[14] See Venkataraman (2001, p. 1450).

[15] Fill or kill orders can be submitted. However, they do not have an impact on price setting. This means that the price and the corresponding executable trading volume or excess demand are calculated as if the fill-or-kill order was not present. A description of the Amsterdam stock exchange (AON are all or nothing orders which is another word for fill or kill orders) as taken from http://www.keytradebank.com/form.html?level=form&option=rul&market=aex is similar: " on the segment of the double auction, ... the fixing price is calculated without the AON orders. Just before the fixing, the AON orders are added to the orderbook."

[16] The reference price in real world trading systems is the last traded price (XETRA, p. 27). Since our model only allows for one round of trading, we cannot use the last traded stock price as the reference price.

B: Allocation rules:

Rule	Amsterdam μ_A	Tokyo μ_T	NYSE μ_N	Absolute Size Priority μ_S
i.)	Orders are executed according to price priority. This rule does not apply to fill-or-kill orders. Stop orders are not executed.			
ii.)	Fill or kill orders are only matched against each other if they cannot be executed against normal bids. The allocation of fill-or-kill orders maximizes executable trading volume.	Fill-or-kill orders are not executed. Orders with limit price p^* are executed using size priority.	Fill-or-kill orders are not executed. Fully executable orders using p^* as limit price are executed first.[17]	Orders with limit price p^* and all fill-or-kill orders are executed according to size priority.
iii.)	Orders with the same priority are executed in a random order.			

The second set of rules, our reinterpretation of the Kyle (1989) market microstructure[18] was chosen because this market microstructure is prominent in the literature. This allows to compare our results with the results of other papers building on the Kyle market microstructure.

Example 2: Kyle market mechanism μ_K. Our reformulation of the Kyle microstructure is as follows:

1. The price is set to maximize the trading volume and
2. the quantity allocation maximizes trading volume.
3. Among those prices and quantity allocations obeying (i) and (ii), Kyle's (1989) market mechanism picks a price p that minimizes absolute value. If both p and $-p$ satisfy this property the market mechanism picks the positive price.
4. Market transaction vector ϕ minimizes $\sum_i (x_i)^2 + (y_i)^2$.
5. Market transaction vector ϕ does not execute fill or kill orders[19].

[17] Think of this rule as follows: Every order on the short side of the market and every order on the long side of the market which does not use p^* as limit price are matched first. From the remaining orders on the long side of the market, an order is drawn from the subset of all executable orders. After this draw has been matched, another order is drawn from the (new) subset of fully executable orders. This procedure is continued until no fully executable order exists on the long side of the market. Then, a draw is taken from all remaining orders that use p^* as the limit price and this order is broken up.

[18] Actually, Kyle (1989) did not mention trade volume maximization. However, without this latent assumption and market transactions minimization the market mechanism would always implement no trade.

[19] Actually Kyle does not allow fill or kill orders which is equivalent to not being executed. This assumption is necessary to guarantee convex valued excess demand correspondences. Our results, however, do not rely on this assumption.

Existence. We are now in a position to formulate our existence and non-existence results.[20]

Our first result is negative. True value equilibria do not exist in a fully rational market if effort is continuous and trade is organized in anonymous call markets. Our second result is positive. In the same set-up excess returns equilibria exist and we can thereby analyze stock price behavior of firms with a distinguished player using the standard equilibrium concept in a fully rational framework. Excess returns equilibria exist for both, continuous or discrete effort.

Our interpretation of existence and non-existence results at this point is as follows. If we are interested in a rational theory of asset pricing with distinguished players, it is crucial to understand the properties of excess returns equilibria as they may be the only rational equilibria that exist. Furthermore, the pricing predictions from our existence results have been taken to the data by von Lilienfeld-Toal and Rünzi (2007) who find evidence for excess returns equilibria for firms with owner-CEOs. This provides a second motivation for the analysis of excess returns equilibria.

Theorem 1 *Consider the market game Γ_μ with sufficiently small tick size δ. Suppose effort is continuous $e \in \mathbb{R}_+$ and tradable quantities are discrete. Then, the following is true.*

(I) There exists no true value equilibrium under the Amsterdam, Tokyo, and NYSE market microstructure for zero bid ask spread $\gamma = 0$.[21]

(II) However, there exists an excess returns equilibrium under Amsterdam, Tokyo, NYSE, absolute size priority, and Kyle market microstructure for zero bid ask spread $\gamma = 0$. □

Theorem 2 *Consider the market game Γ_μ with sufficiently small tick size δ. Suppose effort is discrete $e \in \{0, 1\}$ and consider a sufficiently small bid ask spread γ and let prices be discrete.*

(A) Small initial ownership α_0. Let $i = 0$ be a distinguished player with initial stake $\alpha_0 < \frac{c}{\Delta v}$. Then, an excess returns equilibrium exists under any trade volume maximizing market mechanism. Furthermore, an excess returns equilibrium with an equilibrium winner exists under any call auction mechanism (Amsterdam, Tokyo, NYSE, absolute size priority, and Kyle) if $(1 - \frac{1}{M}) \cdot \Delta v \geq c$.

[20] Note that the call auctions we use here are stochastic market mechanisms. The rigorous formulation of stochastic market mechanisms and the implied stochastic market game is postponed to appendix B and will be used extensively in the proofs in appendix C.

[21] If $\gamma > 0$ true value equilibria do not exist for a more general class of market microstructures unless investors are pivotal which is shown in section 4.

(B) High initial ownership α_0. Let $i = 0$ be a distinguished player with initial stake $\alpha_0 \geq \frac{c}{\Delta v}$. Then, there are initial ownership structures α such that an excess returns equilibrium with an equilibrium winner

(i) exists in the Amsterdam Market mechanism μ_A if the distinguished player $i = 0$ is strictly wealth constrained and

(ii) exists for the Kyle Market Microstructure μ_K. □

All proofs are to be found in appendix C, page 30ff with the exception of the proof for discrete effort and small initial ownership which follows below in the main text.

Theorem 1 shows that trade at the true value is inherently instable. The distinguished player always wants to sell shares. If effort is continuous, selling one single share already yields an improvement for the distinguished player.

Under the same conditions, however, there exist excess returns equilibria. In this respect, trading strictly below equilibrium value is more robust since excess returns equilibria are the only to exist if effort is continuous and all investors are rational. Note that it is straightforward to construct equilibria with substantial excess returns under the Amsterdam, or absolute size priority market microstructure. Under the Amsterdam market microstructures, for example, it suffices if the distinguished player is the only buyer who submits a fill or kill order buying a large number of shares.

The non-existence proof proceeds by characterizing any potential candidate true value equilibrium and finding a contradiction. In every candidate true value equilibrium the following is true. Either the distinguished player has an incentive to change his ex post holding and adjust effort accordingly which is feasible if effort is *continuous*. Or outside investors have an incentive to change their ex post holdings to trade less against the distinguished player. The driving force is that outside investors and the distinguished players can never be indifferent at the same time between buying and selling due to different valuations of shares of the firm. Anonymity of our call auctions makes sure that outside investors or the distinguished player can indeed deviate and change ex post ownership ω.

The intuition behind the existence result of excess returns equilibria is similar for discrete and continuous effort choice. To build up this intuition we proceed with the existence proof for small initial ownership under discrete effort choice.

PROOF (OF THEOREM 2, PART (A)) Suppose discrete quantities $Q = \{0, \frac{1}{M}, \frac{2}{M}, ..., \frac{M}{M}\} \subset [0, 1]$ with M indivisible shares. Let $\hat{\tau} \equiv \min\{x_0 | x_0 \geq \frac{c}{\Delta v} - \alpha_0, x_0 + \alpha_0 \in \Delta\}$. Equilibrium strategies are given as follows. The distinguished player submits a buy order to buy $M\hat{\tau}$ shares using $\underline{v}$ as a price limit, i.e. submits a limit buy order $\beta = (\hat{\tau}, \underline{v})$. Furthermore, $M\hat{\tau} + 1$ equilibrium sellers submit sell limit orders $\sigma = (\frac{1}{M}, \underline{v})$. One outside investor submits a market buy order for one share, i.e. an order with $\beta = (\frac{1}{M}, \infty)$ if $(1 - \frac{1}{M}) \cdot \Delta v \geq c$. All other investors do not submit any orders.

Clearly, $p^* = \underline{v}$ because trade does not occur at any other price. Further, note that the distinguished player exerts effort if his ex post ownership stake is $\alpha_0 + \hat{\tau}$ but not for smaller ex post ownership stakes by construction of $\hat{\tau}$. Moreover, the equilibrium buyer is an equilibrium winner if $\Delta v \geq \gamma$.

To show that this is indeed an equilibrium consider potential deviations.

1. Sellers cannot benefit from not selling since they are pivotal and would trigger the distinguished player to cut back on his effort. As a result, unsold shares would be worth $\underline{v}$ which yields equilibrium utility – or eventually even less if they own shares they do not sell in equilibrium.

2. A similar argument applies to an outside investor who wants to increase buy orders. To buy more is only feasible if the distinguished player withdraws effort afterwards; hence more buying is not a beneficial deviation.

3. The distinguished player cannot benefit from deviating. Increasing his ex post ownership stake ω_0 is not feasible since there are no other sell offers on the market and buying at a higher price is not a beneficial deviation.

   Reducing his ownership stake is not beneficial either since exerting effort is beneficial and hence $(\alpha_0 + x_0) \cdot \bar{v} - c \geq (\alpha_0 + x_0) \cdot \underline{v} \geq (\alpha_0 + x_0 - \epsilon) \cdot \underline{v} - (x_0 + \epsilon) \cdot p^*$ if $\epsilon \in M$ and if $\epsilon \leq \hat{\tau}$. ■

4 Trade Equilibrium Characterization

Having shown existence for some specific but relevant trading rules, we continue with the more general results, the characterization of excess returns equilibria.

Theorem 3 *Let a^* be an excess returns equilibrium. Then, for any market mechanism μ the following properties are satisfied.*

1. *In equilibrium a^* the distinguished player exerts effort $e^* > 0$.*

2. *In equilibrium a^* the distinguished player $i = 0$ is not a seller $\omega_0 \geq \alpha_0$.*

3. *Each investor is pivotal in the sense that selling less than specified or buying more than specified by equilibrium strategies triggers the distinguished player $i = 0$ to sell more or buy less and to reduce effort subsequently or it triggers a price increase.*

4. *The distinguished player submits a buy or sell order.*

5. *In an excess returns equilibrium with an equilibrium winner traditional no arbitrage does not hold while generalized no arbitrage holds by definition.* □

Proof to be found on page 38.

No distinguished player. This formulation of the model contains the special case $\Delta v = 0$ with no distinguished player. The following proposition shows that models without distinguished players have no excess returns equilibria and therefore are not robust with respect to the introduction of arbitrarily small distinguished players $\Delta v > 0$ if excess returns equilibria exist.

Proposition 1 *For a model without a distinguished player $\Delta v = 0$ excess returns equilibria do not exist and therefore traditional no arbitrage is always satisfied.* □

Proof to be found on page 38.

Our interpretation at this point is, first, that the presence of a distinguished player should turn our attention to both, true value equilibria and to excess returns equilibria. Second, market models assuming that distinguished players cannot trade and influence the firm's value at the same time are not robust with respect to the introduction of a distinguished player.

True Value Equilibria. We have seen in Theorem 1 that true value equilibria do not exist in standard call auctions. Nevertheless, to improve our understanding we are interested in a positive result and ask the question under which conditions they can exist in rational markets and if they exist, how they look like. It turns out that a positive bid ask spread is another obstacle for the existence of true value equilibria; at least in a standard world of price taking agents.

Proposition 2 *Consider market game Γ_μ defined by market mechanism μ, bid ask spread γ, and initial ownership α.*

1. *For strictly positive bid ask spread $\gamma > 0$ high true value equilibria do not exist in which no investor $k \in \{1, ..., N\}$ is pivotal in the sense that selling less or buying more than specified by equilibrium strategies triggers the distinguished player $i = 0$ to sell more or buy less and to reduce effort subsequently.*

2. *For strictly positive bid ask spread $\gamma > 0$, low true value equilibria do not exist, and true value equilibria do not exist if effort is discrete and $\alpha_0 < c/\Delta v$.*

3. *Any true value equilibrium satisfies traditional no arbitrage.*

4. *Assume from here bid ask spread $\gamma = 0$ and effort to be discrete. For $\alpha_0 \Delta v > c$ a high true value equilibrium exists if the market mechanism μ maximizes the trade volume and if either quantities are continuous or the number of shares M is sufficiently large – i.e. $M > \frac{\Delta v}{\alpha_0 \cdot \Delta v - c}$. If conversely $\alpha_0 \Delta v < c$ and $\underline{v} > 0$ a low true value equilibria exists if the market mechanism μ maximizes the trade volume and if either quantities are continuous or if M is sufficiently large.*

Proof to be found on page 38.

Clearly, to be an equilibrium buyer for strictly positive bid ask spread $\gamma > 0$ in the high true value equilibrium is undesirable since equilibrium buyers must pay $\bar{v} + \gamma$ for something that is known to be worth $\bar{v}$ in equilibrium. The only way to keep buyers happy in a true value equilibrium is pivotalness, i.e. the off equilibrium threat of the distinguished player to decrease their payoff even further.

Traditionally noise traders had to be introduced to initiate trade as long as transaction cost was strictly positive. This observation dates back to the work on no-trade theorems, as for example in Milgrom and Stokey (1982). Our proposition 2 shows that the presence of a distinguished player adds another explanation to initiate trade without noise. More intriguingly, proposition 2 rules out trade equilibria with bid ask spread $\gamma > 0$ unless some traders are pivotal which means that they are 'forced' to sell by the off-equilibrium threat of distinguished player not to exert effort otherwise. Since in true value equilibria the distinguished player has no reason to enforce trade at the true price without noise traders no trade seems to be as realistic.

Proposition 2 together with Theorem 1 indicate that true value equilibria may exist, but only under very restrictive assumptions. True value equilibria only exist for certain combinations of bid-ask spread, ownership structures, and effort specifications. This is a lack of robustness that does not occur for excess returns equilibria. They exist, for example, under continuous effort and zero bid-ask spreads, or for positive bid ask spread, discrete effort and $\alpha_0 < c/\Delta v$ whereas true value equilibria do not exist in these settings. This is not to say that excess returns equilibria may not be plagued by other robustness issues, for example the implied high rationality of outside investors. Rather, both, excess returns equilibria *and* true value equilibria may be relevant, and we think it is an important task to understand excess returns equilibria.

In the following section we undertake a first step and analyze the robustness of excess returns equilibria when irrational noise traders are introduced and the number of outside investors is allowed to be arbitrarily large.

5 Noise and Small Price Takers

We have seen in proposition 1 that excess returns equilibria do not exist without a distinguished player. In this section we show that other seemingly salient ingredients of the previous fully rational model in fact are not crucial. In particular, the existence of an excess returns equilibrium in a model with a distinguished player does neither rely on traders' rationality and pivotalness nor on their strategic influence on the price. Mathematically, the least form of strategic influence on prices by rational traders is modelled by a continuum of traders. It turns out that if there are irrational noise traders an excess returns equilibrium can be established even in an environment with a continuum of traders. The logic is the following. By some traders' irrationality the price and final allocation become random variables. Moreover, irrationality raises the chances for rational investors to gain by buying from other traders than the distinguished player. This notably includes the possibility that in this equilibrium small rational investors can behave as price takers and still gain in expectation. These traders face the following trade-off. To raise the buying price limit above the equilibrium trade price raises the chances to buy for an attractive price below the equilibrium value. However, the downside is that this strategy makes it also more likely to trade against the distinguished player at some higher price that in turn happens to exceed equilibrium value.

Specifications. The set of investors in this section is given by

$$i \in I = [0,1] = DP \cup RI \cup NT = \{0\} \cup (0,1) \cup \{1\}$$

consisting of three types of investors. As before the distinguished player is $i = 0$ and small[22] rational outside investors are $i \in (0,1)$. The distinguished player initially owns proportion $\alpha_0 \geq 0$ of shares and rational outside investors together own $\alpha_r < 1-\alpha_0$ shares. In this section moreover we suppose the presence of irrational noise traders trading for exogenous reasons.[23] Since only their aggregated behavior matters for rational investors they are treated from here as if they were a single irrational investor $i = 1$. These noise traders initially own together the remaining $\alpha_1 = 1 - \alpha_0 - \alpha_r$ shares.[24]

The distinguished player is assumed to be strictly wealth constrained and faces again a binary effort choice $e \in \{0,1\}$. To make sure that best responses are well defined we further assume that every rational investor is budget constrained with a finite budget. The aggregated budget constraint across all rational investors is non-binding and larger than $\bar{v}$ meaning that jointly outside investors can afford to buy the entire firm even at the highest reasonable price.

Prices are discrete $P = P_\delta := \{..., \underline{v}, \underline{v} + \delta, ..., \bar{v}, ...\}$ with tick size δ and quantities $Q = [0,1]$ are continuous.

Noise. Suppose noise traders only submit market orders and only the excess demand correspondence of noise traders denoted by $\tilde{Z}_1$ matters for the rest of the market[25]. We further suppose that $\tilde{Z}_1$ is a random variable with support $[-\alpha_\theta, b] \subset \mathbb{R}$ where $-\alpha_\theta < 0 < b \leq \alpha_\rho$. The assumption $b \leq \alpha_\rho$ means that the event $\tilde{Z}_1 > \alpha_\rho$ that noise traders want to buy more than rational investors own has probability 0. We introduce this assumption to

[22] In the finance literature continuum traders are often called "atomistic".

[23] The presence of noise traders is often motivated by exogenous liquidity shocks.

[24] More formally, initial ownership structure $\alpha \in \Delta$ in this section is a (probability) measure with $\int_I \alpha_i di = 1$. Among rational investors we allow here for a finite set of block owners, in particular the distinguished player $i = 0$ typically is a block owner with $\alpha_0 > 0$. The big majority of investors of Lebesgue measure 1 are small investors who individually own 0 and only jointly own a strictly positive fracion of shares.

[25] This is the case anyway if the market mechanism executes market orders against each other.

make sure that existence of excess returns equilibria is not driven by the specification of noise. The distribution function F is allowed to be discontinuous. $F_-(z) := \Pr(\tilde{Z}_1 < z)$ and $F_+(z) := \Pr(\tilde{Z}_1 \leq z)$ of random variable $\tilde{Z}_1$ then satisfies $F_-(-\alpha_\theta) = 0$ and $F_+(b) = 1$. In particular, in contrast to the previous sections we suppose non-degenerate noise, i.e. $\Pr(\tilde{Z}_1 = 0) = F_+(0) - F_-(0) = 0$ and $F(0) \equiv F_+(0) = F_-(0) \in (0, 1)$ meaning that the events $\Pr(\tilde{Z}_1 > 0) = 1 - F(0) > 0$ and $\Pr(\tilde{Z}_1 < 0) = F(0) > 0$ have strictly positive probability. Furthermore, we will say that noise increases from low noise l to high noise h represented by distributions F_l and F_h if

1. $\Pr_h(\tilde{Z}_1 > \tilde{x}) \geq \Pr_l(\tilde{Z}_1 > \tilde{x})$ for all $\tilde{x} \in (0, b]$ and $\Pr_h(\tilde{Z}_1 > \tilde{x}) > \Pr_l(\tilde{Z}_1 > \tilde{x})$ for some $\tilde{x} \in (0, b]$

2. $\Pr_h(\tilde{Z}_1 < \tilde{x}) \geq \Pr_l(\tilde{Z}_1 < \tilde{x})$ for all $\tilde{x} \in [-\alpha_\theta, 0)$ and $\Pr_h(\tilde{Z}_1 < \tilde{x}) > \Pr_l(\tilde{Z}_1 < \tilde{x})$ for some $\tilde{x} \in [-\alpha_\theta, 0)$.

Theorem 4 *There exist initial ownership structures α and effort cost c such that for a sufficiently small tick size $\delta > 0$*

(i) an excess returns equilibrium exists under the NYSE market microstructure μ_N for any non-degenerate distribution F,

(ii) an excess returns equilibrium exists under the absolute size priority market microstructure μ_S for any non-degenerate distribution F,

(iii) and an excess returns equilibrium exists under the Tokyo market microstructure μ_T for some non-degenerate distribution F.

(iv) Suppose noise increases from low noise l to high noise h and the market microstructure is either NYSE μ_N or absolute size priority μ_S. Then, there always exist α_0, c, a^* such that a^* is an excess returns equilibrium under high noise h while a^* is not an equilibrium under low noise l.

(v) Without noise, an excess returns equilibrium cannot exist with a continuum of traders under any call auction mechanism.

(vi) Without a distinguished player ($\bar{v} = \underline{v}$) excess returns equilibria do not exist under the NYSE, Tokyo or absolute size priority market microstructure under any non-degenerate distribution F. □

Proof to be found on page 39.

It should be noted that parameters can easily be specified such that excess returns can be substantial. Furthermore, for any noise, excess returns equilibria exist for a whole range of cost parameters and for every cost parameter it holds for a whole range of ownership structures.

The following line of arguments provides the main intuition of the proof and shows why investors have no incentive to bid up the share price. In our excess returns equilibria with noise, the distinguished player sells his shares with strictly positive probability whenever $p \geq p_0$ and shares are overvalued at these high prices which implies that $p_0 \in (\underline{v}, \bar{v})$. However, for any price $p \leq p_0 - \delta$, the distinguished player does not sell his shares. As a result, shares are undervalued if the resulting price is $p \leq p_0 - \delta$. Hence, rational investors want to sell if $p \geq p_0$ and to buy if $p \leq p_0 - \delta$. Hence, the value of the firm is price dependent and investors are always rationed: There is excess demand if $p \leq p_0 - \delta$ and excess supply if $p \geq p_0$. Principally, rational investors can overcome the rationing by increasing their buy limits. The downside from this strategy is that they always buy shares, which are sometimes undervalued and sometimes overvalued. Since the rationing factor is determined by comparing noise against rational investors, an increase of noise facilitates the existence of excess returns equilibria for two reasons. First, liquidity is increased which makes it more likely that the distinguished player can sell his shares on the market. Secondly, the rationing problem is reduced which implies that rational investors have a smaller incentive to increase their limit price used in their buy orders.

It is interesting to compare the noisy environment with our earlier analysis. First of all and most important, our voluntary trade property of market mechanisms no longer holds. Noise traders are irrational and are forced to trade for exogenous reasons.

Since the voluntary trade property does not hold for all investors our characterization results of section 4 are not valid any more. In particular, *pivotalness* ceases to hold. In the voluntary trade framework, pivotalness was the key to produce liquidity. Investors had an incentive to trade in equilibrium to guarantee that the distinguished player does not sell his shares. In the present context, the role of providing liquidity is taken by the noise traders. Therefore, excess returns equilibria exist even if investors are small price takers who are not pivotal.

6 Related Literature

The paper relates to empirical and theoretical contributions, in particular those jointly addressing corporate governance and asset pricing. We will first discuss the empirical literature and argue that there is evidence for i) excess returns equilibria, ii) the existence of distinguished players and iii) non-atomistic, pivotal investors with price impact. Second, we discuss related theoretical contributions.

In order to support the excess returns equilibrium phenomenon formulated by this theory, it is first necessary to identify a distinguished player. As potential candidates, von Lilienfeld-Toal and Rünzi (2007) investigate owner-CEOs. They show that the presence of an owner-CEO is not priced. They consider the universe of S&P 1500 (S&P 500) firms from 1996-2005 (1994-2005) and find that a portfolio consisting of S&P 1500 firms in which the CEO owns more than 10% of shares of the company produces statistically significant annualized abnormal returns of approximately 12%. These results carry over to several robustness checks including a formulation of industry adjusted returns and splitting the sample into boom and burst years. Moreover, von Lilienfeld-Toal and Rünzi (2007) investigate to which extent excess returns differ if effort of the CEO is important. To do so, they sort firms with regard to managerial discretion. They find that excess returns tend to be more important for firms in industries where the CEO has a strong influence on firm performance, younger firms, and growth firms. Other potential candidates for distinguished players are founder-CEOs. Fahlenbrach (2007) finds that founder-CEO firms outperform the market by approximately 10% and these results are again robust to various specifications. While these results are consistent with excess returns equilibria, they are inconsistent with true value equilibria. Hence, true value equilibria may not only fail to exist in theory, empirical evidence also suggests that excess returns equilibria are more relevant, provided good candidates for distinguished players are found.

Aforementioned papers also document the empirical importance of distinguished players and value increasing shareholders for listed US firms. For example, within the S&P 1500 firm universe, according to von Lilienfeld-Toal and Rünzi (2007) more than 10% of firms have an officer who owns more than 10% of outstanding stocks. Fahlenbrach (2007) reports that founder-CEOs are present in 11% of the largest US firms (founders hold on average 11% of shares of a firm). A similar emphasis is put forward in the recent paper by Holderness (2006) with the title "The Myth of Diffuse Ownership in the United States". In a representative sample of 375 US firms he reports that 96% of US firms have at least one blockholder who owns more than 5% of shares of the firm. Average ownership of all blockholders, directors, and officers is 43% (median 43%), average ownership of the largest shareholder is 26% (median 17%), and average ownership of officers and directors is 24% (median 17%). The latter finding is consistent with results in Fahlenbrach and Stulz (2007) who look at the much larger universe of US firms covered in the compact disclosure discs. They analyze 27,636 firm years from 1988-2003 and report mean ownership of officers and directors to be 22.4% (median 15.8%).

One group of shareholders that are reasonable candidates for non-price taking, pivotal outside investors are institutional investors who control more than 100 million US dollars. It is well documented that trades of this group of investors have an influence on the price. Chan and Lakonishok (1995, p. 1147) argue that "*For many institutional investors, however, even a moderately-sized position in a stock may represent a large fraction of the stock's trading volume*". They document an average price impact of 1% for buy orders or $-.35\%$ for sell orders. Their sample consists of NYSE and AMEX trades of 37 large investment management firms from July 1986 until the end of 1988. Noteworthy, the trades of these 37 institutional investors accounted for approximately 5% of trading volume on NYSE and AMEX in this time period.

Apart from the importance of trading volume of institutional investors, it is also known that the ownership of institutional investors is economically significant. Gompers and Metrick (2001) consider the holdings of institutional investors from 1980-1996. Shareholdings of institutions is increasing over time and in December 1996, the last quarter of their sample, institutional investors hold more than 50% of the market capitalization of US firms. We interpret these observations that there are only a few important institutional investors as supportive for the assumption that outside investors can act strategically rather than as pure price takers. In December 1996, there are only 1303 institutions. In particular, the largest 100 institutions hold approximately *one third* (37.1%) of the entire market capitalization and the largest 10 institutions hold 14.6% of market capitalization of all US firms.

Most of the theoretical literature about large shareholders and trading games are almost exclusively focussed on what we call true value equilibria. Prominent examples include Shleifer and Vishny (1986), Admati, Pfleiderer and Zechner (1994), Maug (1998), DeMarzo and Urosevic (2006), Kahn and Whinton (1998), or Magill and Quinzii (2002). All these papers study a large and value increasing shareholder who may increase a firm's value while increasing a firm's value causes private effort costs. Some of them are more general in other important respects (asymmetric information, dynamic framework, ...) while our paper is more general with respect to the market microstructure and is the only one to investigate and characterize excess returns equilibria.

With respect to excess returns equilibria closest to our paper is von Lilienfeld-Toal (2005), who identifies an excess returns equilibrium for a very specific market microstructure and then turns attention to the empirical implications of excess returns. In particular, it argues that excess returns equilibria are consistent with i) negative abnormal returns around unlock days and ii) positive abnormal returns for firms with a distinguished player. In contrast to our present theory it contains no general theory, no characterization, no statements on non-existence of the true-value equilibrium or on irrational traders.

Note that excess returns equilibria may occur in the model of Bolton and von Thadden (1993) which also is concerned with corporate control issues. It does not focus, however, on (asset) pricing implications of excess returns equilibria and in particular does not relate excess returns to no-arbitrage in asset pricing. Rather, they are mainly interested in the question when blocks of shares remain, vanish or are newly created. The reason as to why excess returns equilibria may exist in the model of Bolton and von Thadden (1993) is similar to our notion of pivotalness. A related explanation of takeovers is given by Bagnoli and Lipman (1985) or Holmstrom and Nalebuff (1992). The latter papers analyze potential solutions to the free rider problem first mentioned by Grossman and Hart (1980).[26]

All above mentioned papers derive the results using a particular market microstruc-

[26] In fact, in a sense our model could be interpreted as a generalization of Bagnoli and Lipman (1985) and Holmstrom and Nalebuff (1992) if the distinguished player's value-enhancing capability only unfolds for $\alpha_0 \geq \frac{1}{2}$, the strategy space of the distinguished player is limited to a takeover bid, and other shareholders can only submit sell orders.

ture. We are not aware of other papers deriving characterization results that are valid for a broad class of market micro-structures. Conversely, our assumptions on complete symmetric information in a static market game are clearly more specific in other respects than a large number of the above mentioned contributions.

Our setting can be viewed as double sided auctions with strategic trading and the paper relates to this branch of market microstructure theory. Papers falling within our framework are for example Kyle (1985), Kyle (1989), Rochet and Vila (1994), or Reny and Perry (2006). While their exact specification of price setting and quantity allocation rules is within our class of market mechanisms, the economic environment we are interested in is distinct as compared to these papers. Moreover, market microstructure theory is also interested in the price impact of individual trades which is aptly pointed out by O'Hara (2003): "... asset pricing ignores the central fact that market microstructure focuses on: Asset prices evolve in markets".

Our paper is also related to the literature on no trade theorems, for example Milgrom and Stokey (1982) or Tirole (1982). The driving force behind no trade theorems is the fact that there are no gains from trade or negative gains from trade in the presence of transaction costs. In the class of models we are interested in, gains from trade are zero for true value equilibria and consequently, true value equilibria in the traditional sense fail to exist for positive bid ask spreads. In excess returns equilibria, in contrast, gains from trade are no longer zero sum since the owner manager's threat to sell is viable and trade at a low price prevents the owner manager from selling.

Finally, the paper relates to the literature on agency problems as in Holmstrom (1979) or Grossman and Hart (1983). In particular, models with bilateral contracting and non-exclusive contracts are concerned with externalities among trading partners. Examples are Bisin and Guaitoli (2004), Bizer and DeMarzo (2004), Kahn and Mookherjee (1998) and Segal and Whinston (2004). In these papers, a distinguished player can write contracts with many players while in our model, the distinguished player can anonymously trade with many outside investors.

7 Extensions

It is straightforward to set up a similar theory with more than one distinguished player $j = 0, ..., M$ each of them characterized by his stake, effort cost and potential value $\alpha_j, c_j, \Delta v_j$. In an efficient equilibrium those for which $c_j < \Delta v_j$ should exert effort. Clearly, since distinguished players can behave as regular investors as well the presence of a single distinguished player is sufficient to raise the possibility of excess returns equilibria. For empirical testing we expect that the significance of the most important distinguished player is a better predictor for excess returns than the number of distinguished players since triggering this player to sell causes the biggest potential payoff losses. If the market price offers the right incentives for this most important distinguished player it should as well do for minor distinguished players.

We have not discussed the potential role of market makers in our model even though our market microstructure allows for an analysis of market makers. Analyzing the role of market makers can be done as follows. Suppose agent $N+1$ is the market maker. Moreover, agent $N+1$ submits buy and sell orders $(1, \infty)$ and $(1, -\infty)$. In other words, the agent $N+1$ submits to buy and sell the entire firm. Hence, any market imbalance can be bought by agent $N+1$ or sold to agent $N+1$ without violating our "voluntary trade" assumption. Moreover, the market maker is hardwired such that he maximizes u_{N+1}, potentially subject to institutional rules such as market clearing, i.e. offering a clean up price. The resulting allocations and prices will be the same as in a market mechanism in which all agents $i = 0, 1, ..., N$ submit their shares, a market maker observes these orders and sets a price to maximize his utility, subject to obeying institutional rules.

Since our model in principle exhibits multiple equilibria, empirical observations might help to judge which equilibria are most relevant in reality. Building on this observation, the following questions could be addressed in future empirical research: How general is the occurrence of excess returns equilibria for smaller firms than S&P1500? How do abnormal returns increase or decrease with firm size and with the initial stake of the distinguished player? Can we detect the importance and magnitude of private costs of control from these excess returns equilibria? Are there any differences trading volume for firms with and without a distinguished player, especially if we control for transaction costs? Moreover, it might be possible to test more general aspects of game theory using stock market data and the methodology of games in aggregated form developed in Blonski (2005) which allows robust predictions on the structure of large semianonymous games without specific knowledge about individual preferences. The numerous implications of excess returns equilibria promise further interesting combinations of theoretical and empirical work on incentives, game theory and asset pricing.

Our model is a static model which does not incorporate any dynamic aspects of real world trading. Therefore, it is important to see how our model fares in a dynamic version. Finally, as emphasized in the introduction information is typically asymmetric if distinguished players trade in stock markets. It is an important, interesting and challenging project to study the interplay between the structure of excess returns equilibria with informational asymmetries. As the notion of no-arbitrage equilibrium in traditional capital market valuation theory generalizes from the complete information case to the case of information asymmetries we conjecture that the notion of generalized no-arbitrage as formulated in this article generalizes as well to the case with informational asymmetries.

8 Conclusions

To summarize our main findings, we show that shares of a firm with a distinguished player can often not be priced correctly. True value equilibria do not exist under positive trading costs or if effort is continuous and trading anonymous. In contrast, excess returns equilibria exist in both, a fully rational world and a world with noise and a continuum of traders. We characterize excess returns equilibria for a general class of market mechanisms. It turns out that the existence of a distinguished player is necessary for excess returns equilibria to exist.

Our theory is general in the sense that it contains the benchmark case of a frictionless efficient market without distinguished player and without transaction costs and with the usual true value equilibria as a special case. In this sense our results show that pricing predictions of models with frictionless markets are not robust with respect to the introduction of an arbitrarily small[27] distinguished player.

The main intuition behind the existence of excess returns equilibria is as follows. Whenever share prices of a firm exceed a certain threshold, the distinguished player prefers to sell his shares – or does not want to buy shares – and reduce effort subsequently. As a result, shares are traded below this threshold price. Due to the private effort costs, this threshold price is below the equilibrium value.

Now, trade can occur for two reasons. Equilibrium sellers – selling shares below the equilibrium value – can be pivotal and highly rational. They then know that not selling shares will trigger the distinguished player to sell shares instead, cut back on the costly effort and reduce firm value. This renders everyone worse off, including deviating equilibrium sellers. In the absence of pivotal traders, noise traders may fill the liquidity gap. Noise traders may sell for exogenous and stochastic reasons. Then, the equilibrium share price is stochastic and the distinguished player has an incentive to sell at the high realizations of the share price but not at the low realizations of the share price. Rational buyers may then prefer not to buy at high share prices but only buy at low share prices. As a consequence, they are not increasing demand – by submitting bids with higher buy limits – and hence they are not increasing the share price even though on average, shares are traded below the equilibrium value.

Additional research questions arise naturally. For example, what happens in a dynamic formulation or under asymmetric information or with risk aversion? While the economic intuition behind our results is quite strong, it is also apparent that a rigorous formulation of such questions is not straightforward. All our proofs turn out to be involved and full of details.

Since there are two different explanations for trade to occur in an excess returns equilibrium (fully rational, pivotal players or irrational noise traders), it would be interesting to empirically account for the importance of each explanation. Even though there exists some evidence for excess returns equilibria (von Lilienfeld-Toal and Rünzi (2007) and Fahlenbrach (2007)), it is a worthwhile task to identify excess returns equilibria in other circumstances or conversely to identify circumstances where excess returns do not exist.

[27] The size of the distinguished player is defined by his maximal contribution to the company's fundamental value.

Investigating different aspects and puzzles of asset pricing, both in theoretical and empirical work also promises to be fruitful. The importance to carefully investigate asset pricing phenomena in light of our theory becomes clear when it comes to judging the empirically observed excess returns. Observing abnormal returns due to a certain investment strategy, as documented by von Lilienfeld-Toal and Rünzi (2007), need not be a sign of irrational behavior but might be the result of excess returns equilibria and highly rational behavior.

9 Appendix A: Orders and Strategies

Orders. Demand and supply correspondences can be composed by sets of orders. The following description of the relevant market actions denoted as *orders* is chosen to model real world market mechanisms as closely as possible. First, denote by $B = Q \times P$ the space of *buy limit orders* with typical element $\beta = (b, p)$ where $p_b = p + \gamma \in P$ denotes the limit price up to which a player is willing to buy any quantity $q \leq b \in Q$. Conversely, for a *sell limit order* $\sigma = (s, p) \in S = Q \times P$ the price p is the minimal price from which the submitting trader is willing to sell q up to quantity s. Buy and sell orders can be interpreted as downward sloping step correspondences. For example, the buy limit order $\beta = (b', p')$ is precisely defined by the correspondence $\beta : P \to Q$ where

$$\beta(p) = \begin{cases} \{q \in Q \,|q \leq b'\} & \text{for } p \leq p' + \gamma \\ 0 & \text{otherwise} \end{cases}.$$

A trader who submits $\beta = (b, \infty)$ or $\sigma = (s, -\infty)$ is said to submit a *market order* since a certain quantity is ordered for buy or sell independently of price. Market order correspondences are bounded by vertical lines. Similarly, the graphs of *buy stop orders* denoted by $\chi = (b_{st}, p)$ and *sell stop orders* denoted by $\psi = (s_{st}, p)$ are upward-sloping step correspondences. The interpretation is that any quantity up to b_{st} is bought above price p or quantity s_{st} is sold below price p. We also allow so called *fill or kill orders* or *all or nothing orders* that specify that a certain quantity is to be bought or sold entirely or not at all. A fill or kill order is denoted by $\mathring{\beta} = (b, p)$, $\mathring{\sigma}$, $\mathring{\chi}$, or $\mathring{\psi}$. More precisely, say for $\mathring{\beta} = (b', p')$, the related correspondence $\mathring{\beta} : P \to Q$ has a non-convex graph and is defined as

$$\mathring{\beta}(p) = \begin{cases} \{0, b'\} & \text{for } p \leq p' + \gamma \\ 0 & \text{otherwise} \end{cases}.$$

Strategies. A *market game strategy* a_i of player $i = 0, ..., N$ is a collection of orders

$$\begin{aligned} a_i &= \{(\beta_i^1, \beta_i^2 ...), (\mathring{\beta}_i^1, \mathring{\beta}_i^2, ...), (\sigma_i^1, \sigma_i^2, ...), (\mathring{\sigma}_i^1, \mathring{\sigma}_i^2, ...), \\ &\quad (\chi_i^1, \chi_i^2 ...), (\mathring{\chi}_i^1, \mathring{\chi}_i^2, ...), (\psi_i^1, \psi_i^2, ...), (\mathring{\psi}_i^1, \mathring{\psi}_i^2, ...)\}. \end{aligned}$$

Denote by A_i the corresponding *strategy space* of player i and by $A = A_0 \times \cdots \times A_N$ the strategy profiles. Adding up buy and sell orders for some player i yields the individual excess demand correspondence

$$\begin{aligned} Z_i(p) &= z_i(p) + \mathring{z}_i(p) \text{ composed by} \\ z_i(p) &= \sum_{\beta,\sigma,\chi,\psi \in a_i} \beta(p) + \chi(p) - \sigma(p) - \psi(p) \text{ and} \\ \mathring{z}_i(p) &= \sum_{\mathring{\beta},\mathring{\sigma},\mathring{\chi},\mathring{\psi} \in a_i} \mathring{\beta}(p) + \mathring{\chi}(p) - \mathring{\sigma}(p) - \mathring{\psi}(p) \end{aligned}$$

adding up buy and sell orders[28] of player i. A market game strategy can be decomposed into buy orders and sell orders. Denote by

$$\begin{aligned} D_i(p) &= d_i(p) + \mathring{d}_i(p) = \sum_{\beta \in a_i} \beta(p) + \sum_{\chi \in a_i} \chi(p) + \sum_{\mathring{\beta} \in a_i} \mathring{\beta}(p) + \sum_{\mathring{\chi} \in a_i} \mathring{\chi}(p) \\ S_i(p) &= s_i(p) + \mathring{s}_i(p) = \sum_{\sigma \in a_i} \sigma(p) + \sum_{\psi \in a_i} \psi(p) + \sum_{\mathring{\sigma} \in a_i} \mathring{\sigma}(p) + \sum_{\mathring{\psi} \in a_i} \mathring{\psi}(p) \end{aligned}$$

player i's individual demand and supply correspondences given as quantities player i is willing to buy or to sell at a given price p. In particular $d_i(p)$ and $s_i(p)$ specify individual demand and supply excluding fill or kill orders.

10 Appendix B: Stochastic Market Mechanisms

Stochastic trade equilibria. Real world market mechanisms often are specified by a list of rules with decreasing order of priority. Sometimes there remains some ambiguity with respect to equilibrium price or allocation if all rules are satisfied by more than one price and/or set of executed orders such that a random choice may be implemented. In this section we consider risk neutral investors facing a stochastic market mechanism.

A stochastic ownership structure $\tilde{\xi} \in \tilde{\Delta}$ is an element of the space of probability measures $\tilde{\Delta}$ on simplex Δ. Accordingly, define stochastic market prices $\tilde{p} \in \tilde{P}$ and stochastic trade vectors $(\tilde{x}, \tilde{y}) \in \tilde{\Phi}$.

Definition 3 For any initial ownership $\alpha \in \Delta$ and strategy profile $a \in A$ a stochastic market mechanism $\tilde{\mu}$ with bid ask spread γ is a mapping

$$\tilde{\mu} : \Delta \times A \to \tilde{P} \times \tilde{\Phi}$$

where for initial ownership α and strategy profile a the market mechanism $\tilde{\mu}(\alpha, a) = (\tilde{p}, \tilde{x}, \tilde{y})$ picks a stochastic sell price $\tilde{p}$ (and buy price $\tilde{p} + \gamma$) and for any player trade is voluntary. This means that only submitted orders can be executed, i.e. for any state of nature $(\tilde{x}_i, \tilde{y}_i) \in D_i(p) \times S_i(p)$ and therefore $\tilde{\omega}_i - \alpha_i \in Z_i(p)$. Again, the stochastic trading volume is

$$\tilde{\tau}_{\mu} : \Delta \times A \to [0, 1]$$

and $\tilde{\tau}_{\tilde{\mu}}(\alpha, a) = \sum_{i=0}^{N} \tilde{x}_i (\alpha, \gamma, a)$. □

The distinguished player picks his effort decision e after the stochastic market game is over and the realizations of all random variables are known. Denote by $\tilde{e}$ the random effort decision induced by the realization of $(\tilde{x}, \tilde{y})$ which determines the final stake of the distinguished player. Similar as before, a stochastic market mechanism $\tilde{\mu}$ together with an initial ownership α and bid ask spread γ induces a *stochastic market game* Γ_{μ} with strategy space A and risk neutral *payoff functions* given by

$$\begin{aligned} u_0(a) &= E\left[\tilde{\omega}_0 \underline{v} + \tilde{e}\left(\tilde{\omega}_0 \Delta v - c\right) - \left(\tilde{p} + \gamma\right) \tilde{x}_0 + \tilde{p}\tilde{y}_0\right] \text{ and} \\ u_i(a) &= E\left[\tilde{\omega}_i \left(\underline{v} + \tilde{e}\Delta v\right) - \left(\tilde{p} + \gamma\right) \tilde{x}_i + \tilde{p}\tilde{y}_i\right] \text{ for } i = 1, ...N \end{aligned}$$

where E means expectation value.

[28]It is necessary to differentiate in our notation the cases including and excluding fill or kill orders since in most real world market mechanisms kill or fill orders are treated differently. For example, they are not written in the order book and thereby have no direct influence on the market price.

Stochastic true value and excess returns equilibria. A *stochastic true value equilibrium* is an equilibrium where $E(p^*|\alpha \neq \omega) = E(\tilde{v}|\alpha \neq \omega)$ and a *stochastic excess returns equilibrium* is defined as an equilibrium where $E(p^*|\alpha \neq \omega) < E(\tilde{v}|\alpha \neq \omega)$.

11 Appendix C: Proofs

PROOF (OF THEOREM 1, PAGE 15) (I) non-existence, (II) existence.

(I) Suppose a^* were a true value equilibrium with $Ev(\omega_0^\mu(a^*)) = p^*$. The proof proceeds by showing that the distinguished player $i = 0$ or some outside investor $i > 0$ can always improve which is a contradiction to a^* being an equilibrium. The logic of the proof is always as follows: Whenever shares are traded in a true value equilibrium, shares are overvalued from the perspective of the distinguished player and he wants to sell. Whenever shares are priced correctly from the perspective of the distinguished player they are undervalued from the perspective of outside investors.

The proof uses the following auxiliary result that the distinguished player wants to change his ex post holdings at the equilibrium price. This is shown by proving the following lemma.

Lemma 1 *Consider a candidate true value equilibrium a^* with corresponding ω^*, x^*, y^* and deterministic ω_0^*. Then, strategy a_0' generating profile $a' = (a_0', a_1^*, a_2^*, \ldots, a_N^*)$ is a profitable deviation for the distinguished player – i.e. $u_0(a') > u_0(a^*)$ – if the price never changes $p(a') = p(a^*)$ but new ex-post allocation $\omega_0' \equiv \omega_0^\mu(a') \neq \omega_0^*$ occurs with positive probability. Further, if $\omega_0^* \neq \alpha_0$, strategy $a_0' = 0$ (not trading) is a beneficial deviation.* □

PROOF Since for a stochastic market mechanism ex-post ownership is a random variable we show the first claim of the lemma for any ex-post realization ω_0' with $\omega_0' \equiv \omega_0^\mu(a') \neq \omega_0^*$.

For $\omega_0' \neq \omega_0^*$ we can rewrite $\omega_0' = \alpha_0 + x_0^* + \epsilon_x - y_0^* - \epsilon_y$ with $\epsilon_x \neq \epsilon_y$. Then, ex post utility is given as

$$\begin{aligned}
u_0(a') &= \omega_0' Ev(e(\omega_0')) - p^* \cdot (x_0^* + \epsilon_x) + p^* \cdot (y_0^* + \epsilon_y) - c(e(\omega_0')) \\
&> \omega_0' Ev(e(\omega_0^*)) - p^* \cdot (x_0^* + \epsilon_x) + p^* \cdot (y_0^* + \epsilon_y) - c(e(\omega_0^*)) \\
&= \underbrace{(\omega_0' - \epsilon_x + \epsilon_y)}_{\omega_0^*} Ev(e(\omega_0^*)) - p^* \cdot (x_0^* - y_0^*) - c(e(\omega_0^*)) \\
&= u_0(a^*).
\end{aligned}$$

for any $\omega_0' \equiv \omega_0^\mu(a') \neq \omega_0^*$. The strict inequality $>$ follows since $e(\omega_0^*) \neq e(\omega_0') = \operatorname{argmax}_e \omega_0' E(v(e)) - c(e)$ and the subsequent equations make use of $p^* = Ev(e(\omega_0^*))$ in a true value equilibrium. This implies that the distinguished player improves if $\omega_0' \equiv \omega_0^\mu(a') \neq \omega_0^*$ and is unaffected if $\omega_0' \equiv \omega_0^\mu(a') = \omega_0^*$.

For the special case $\omega_0^* > \alpha_0$, strategy $a_0' = 0$ (not trading) is a beneficial deviation because then

$$\begin{aligned}
u_0(a') &= \alpha_0 Ev(e(\alpha_0)) - c(e(\alpha_0)) \\
&> \alpha_0 Ev(e(\omega_0^*)) - c(e(\omega_0^*)) \\
&= [\omega_0^* - x_0^* + y_0^*] Ev(\omega_0^*) - c(e(\omega_0^*)) \\
&= \omega_0^* Ev(\omega_0^*) - p^* \cdot (x_0^* - y_0^*) - c(e(\omega_0^*)) \\
&= u_0(a^*)
\end{aligned}$$

for the same reasons as in the first part of the lemma. ■

Now we proceed by considering the following 3 cases. (i) ω_0 is deterministic and the distinguished player always trades, (ii) ω_0 is deterministic and the distinguished player never trades, or (iii) ω_0 is stochastic.

(i) Suppose first ω_0 is deterministic and $\omega_0^* \neq \alpha_0$. From lemma 1 it follows that $a_0' = 0$ is a beneficial deviation and hence a^* cannot be a true value equilibrium.

(ii) Now, suppose ex post ownership ω_0^* is deterministic and $\omega_0^\mu(a^*) = \alpha_0$. Consider first the case where $d(p^*) = 0$ where nobody submits limit buy orders. Since we are looking at a trade equilibrium, there must exist fill-or-kill orders that are executed. In this case the distinguished player can mimic one fill-or-kill buy order that is executed with positive probability. By definition of the market mechanisms this will not have a price impact and the deviating fill-or-kill order of the distinguished player will be executed with positive probability. This would constitute a beneficial deviation due to lemma 1. Therefore, in a true value equilibrium holds $d(p^*) > 0$ if ω_0^* is deterministic.

We next show non-negative excess limit-order-demand $d^*(p^*) \geq s^*(p^*)$. If to the contrary $d^*(p^*) < s^*(p^*)$ the distinguished player can improve buy submitting an order $a_0' = \beta_0' = (s^*(p^*) - d^*(p^*), p^*)$. As a result, his order will be served and the price will not change. Again, by virtue of lemma 1, a_0' is a beneficial deviation and hence a^* not an equilibrium.

Next we claim that if $d^*(p^*) \geq s^*(p^*)$ there must exist a buy order $\beta_i^* = (x, p^*)$ with price limit p^* at the equilibrium price for some positive quantity $x > 0$ which is partially or fully executed with positive probability. Suppose not. Then, $d(p^* + \delta) \geq s(p^*)$. Since $s(p^*) \leq s(p^*+\delta)$ it follows that $\tau(p^*) \leq \tau(p^*+\delta)$. Clearly, $\tau(p^*) < \tau(p^*+\delta)$ is a contradiction of rule 1 from the market mechanism to maximize trade volume. Hence, $\tau(p^*) = \tau(p^* + \delta)$. But then, p^* is picked because the surplus at p^* (the number of unexecuted orders given, price setting rule 2) is (weakly) smaller at p^* than at $p^* + \delta$. The surplus at $p^* + \delta$ is given as $s(p^* + \delta) - d(p^* + \delta)$. This implies that an equilibrium seller can submit a deviating order $a_i' \cup \beta_i'$ with $\beta_i' = (x, p^*)$ where $x > s(p^* + \delta) - d(p^* + \delta)$. This order results in a price increase from p^* to $p^* + \delta$ because now the surplus is greater at p^*. This leads to an improvement for the seller since he can now sell at a higher price (his deviating buy order will not be executed and his equilibrium sell order will be executed due to price priority). This shows that there exists a buy order $\beta_i^* = (x, p^*)$ for some $x > 0$ which is partially or fully executed with positive probability.

This implies that $\tau^*(p^* + \delta) < \tau^*(p^*)$. Otherwise, no buy order using p^* as a price limit will be executed: All buy orders using $p^* + \delta$ as price limit are executed due to price priority.

Now, we claim that the distinguished player can construct a deviating buy order $a_0' = a_0^* \cup \beta_0'$ with $\beta_0' = (x, p^*)$ where x is chosen by some other outside investor who is served in the candidate equilibrium with positive probability. As a result, the price will not change (note that $\tau(p^* + \delta | a') < \tau^*(p^* | a')$ continues to hold since the LHS of this equation is not affected by the additional order). Since both orders have the same priority, the distinguished player will now be served with positive probability. Again, lemma 1 implies that the distinguished player improves. Hence, it cannot be an equilibrium that the distinguished player never trades.

(iii) Next, suppose ex post ownership ω_0^* is stochastic. We argue that this cannot be an equilibrium in a sequence of steps. First, we note that there must exist some outside investors whose ex post ownership structure is also stochastic. Then, we show that the ex post ownership of the distinguished player must be a two point distribution (i.e. either he buys a block or he sells a block). Finally, if the distinguished player buys shares, he must be the only buyer (among the set of players with stochastic ownership) who buys and if he sells he must be the only seller (among the set of investors with stochastic ex post ownership). This implies that all agents with stochastic ownership can improve: Buyers only buy if the distinguished player does not buy, hence they buy only overvalued shares. For sellers the same argument apply and they only do not sell if shares are overvalued and they are better off always selling which can be implemented by a deviating strategy.

It is helpful to start with the following lemma.

Lemma 2 *Consider any candidate true value equilibrium a^* with stochastic $\omega_0^* \in [\omega_0^{min}, \omega_0^{max}]$ where $\omega_0^{min} < \omega_0^{max}$.*

(a) Then, for any $\omega_0' \in (\omega_0^{min}, \omega_0^{max})$ with $\Pr(\omega_0^* = \omega_0') > 0$ it is true that $u(a^*|\omega_0^* = \omega_0') < \max\{u(a^*|\omega_0^* = \omega_0^{max}), u(a^*|\omega_0^* = \omega_0^{min})\}$.

(b) The distinguished player can find strategies a_0' for which $\lim_{\delta\to 0} u(a_0') \geq u(a^*|\omega_0 = \omega_0^{min})$ if $\omega_0^{min} \leq \alpha_0$ and $\lim_{\delta\to 0} u(a_0') \geq u(a^*|\omega_0 = \omega_0^{min})$ if $\omega_0^{max} \geq \alpha_0$.

(c) For small enough tick size δ, ex post ownership of the distinguished player follows a two point distribution, i.e. $\omega_0^* \in \{\omega_0^{min}, \omega_0^{max}\}$ with $\omega_0^{min} \leq \alpha_0 \leq \omega_0^{max}$. Furthermore, the distinguished player is indifferent between ω_0^{min} and ω_0^{max}: $u(a^*|\omega_0^* = \omega_0^{max}) = u(a^*|\omega_0^* = \omega_0^{min})$.

(d) It is true that $\omega_j^* = \omega_j^{max} \Rightarrow \omega_0^* = \omega_0^{min}$ for all outside investors $j \neq 0$ who have stochastic ex post ownership.

Proof (a) Suppose first that $p^* \leq E(v(\omega_0'))$, i.e. shares are undervalued if $\omega_0 = \omega_0'$. Then, $u(a^*|\omega_0^* = \omega_0') \leq \omega_0^{max} \cdot E(v(e(\omega_0'))) - p^*\{\omega_0^{max} - \alpha_0\} < u(a^*|\omega_0^* = \omega_0^{max})$ because $e(\omega_0') \neq e(\omega_0^{max})$. If $p^* \geq E(v(\omega_0'))$ we get $u(a^*|\omega_0^* = \omega_0') \leq \omega_0^{min} \cdot E(v(e(\omega_0'))) - p^*\{\omega_0^{min} - \alpha_0\} < u(a^*|\omega_0^* = \omega_0^{min})$ for the same reasons.

(b) Our claim is trivially true if $\omega_0^{min} = \alpha_0$ or $\omega_0^{max} = \alpha_0$. Then, not submitting an order $a_0' = 0$ yields $u(a') = u(a^*|\omega_0 = \alpha_0)$.

The distinguished player submits a limit order using p^* as limit price which is randomly executed. Suppose first an order mimicking $\omega_0^{max} > \alpha_0$. Since a limit order is rationed, it follows that $d(p^*) > s(p^*)$ and $\tau^*(p^*) \geq \tau^*(p^*+\delta)$. (using similar arguments as applied in the proof of part(ii) of this proposition.) The distinguished player can now submit a buy order $\beta_0' = (x, p^*+\delta)$ where x is appropriately chosen to guarantee that $\alpha_0 + x - y^* = \omega_0^{max}$. Note that ex post ownership $\omega_0' = \omega_0^{max}$ and hence $u(a') = \omega_0^{max} \cdot E(v(\omega_0^{max})) - p(a')(\alpha_0 - \omega_0^{max})$. Note also that $p(a') \in \{p^*, p^*+\delta\}$ because $\tau(p^*+\delta|a')$ (weakly) increases and $\tau^*(p^*)$ does not decrease. Hence, either $u(a') = \omega_0^{max} \cdot E(v(\omega_0^{max})) - p^*(\alpha_0 - \omega_0^{max})$ and the claim holds for any element of the sequence or $u(a') = \omega_0^{max} \cdot E(v(\omega_0^{max})) - (p^*+\delta)(\alpha_0 - \omega_0^{max})$ and for the limit of the sequence $\delta \to 0$ the claim is true.

Next, consider mimicking ($\omega_0^{min} < \alpha_0$). Then, the distinguished player can now submit a sell order $\sigma_0' = (y, p^* - \delta)$ with y appropriately chosen to guarantee that $\alpha_0 + x^* - y = \omega_0^{min}$. Similar arguments now imply that the distinguished player now always has $\omega_0 = \omega_0^{min}$ and the price decreases by at most δ.

The distinguished player does not submit a limit order using p^* as limit price which is randomly executed. If a limit order is not rationed, the distinguished player only submits fill or kill orders that are executed stochastically. Now, the distinguished player can replace any set of fill or kill orders that are executed if $\omega_0 = \omega_0^{min}$ (resp. $\omega_0 = \omega_0^{max}$) by limit orders without the fill or kill provision that use p^* as the price limit. If the price does not change ($p(a') = p^*$), these limit orders will always be executed because they have higher priority than fill or kill orders and the claim is shown.

If the price changes due to the proposed deviating strategy, a slightly more complicated strategy must be used to guarantee ω_0^{min} or ω_0^{max}. Consider first $\omega_0^{min} < \alpha_0$. Note first that a change in price implies that $\tau^*(p^*) = \tau^*(p^* - \delta)$ due to trade volume maximizing. Furthermore, $|d^*(p^*) - s^*(p^*)| \leq |d^*(p^* - \delta) - s^*(p^* - \delta)|$. Now, consider any set of fill or kill orders submitted by the distinguished player that leads to ω_0^{min}. This set of fill or kill orders is now replaced by limit orders without the fill or kill provision using p^* as the limit price (for example, $\sigma_0' = (y, p^*)$ with y chosen appropriately to guarantee ω_0^{min}). Clearly, $\tau(p^*|a') \geq \tau^*(p^*)$. This simple strategy is complemented by a buy order $\beta_0'' = (x'', p^* - \delta)$ with x'' sufficiently large. Since, $\tau(p^*|a') \geq \tau^*(p^*) \geq \tau^*(p^*)$, sufficiently large x'' now guarantees that the surplus is also smaller at p^* under the deviating strategy a'. Then, the price does not change and indeed $u_0(a') = u_0(a * |\omega_0^{min})$.

Next, consider mimicking $\omega_0^{max} > \alpha_0$. Note first if $d^*(p^*) < s^*(p^*)$, the distinguished player can pick any set of executed fill or kill orders executed that leads to ω_0^{max} and replace them by limit orders with the same quantities that use p^* as the limit price. Since this will increase demand $d(p^*|a') > d^*(p^*)$ a price change does not follow. Hence, we are now concerned with the case that $d^*(p^*) \geq s^*(p^*)$. Suppose first that $d^*(p^*) > s^*(p^*)$. Then, the distinguished player can submit buy orders using $p^* + \delta$ as price limit. As a result, the price will increase by at most δ and $\omega_0 \geq \omega_0^{max}$ (the inequality may occur if demand of fill-or-kill orders is reduced at $p^* + \delta$ and the distinguished player has to submit large enough buy orders to guarantee that fill or kill sell orders can be executed.) If $\omega_0(a') > \omega_0^{max}$ note that $p^* < E(v(e(\omega_0^{max})))$ and the distinguished player can buy even more undervalued shares.

Finally, suppose that $d^*(p^*) = s^*(p^*)$ (this can in particular occur if $\tau^*(p^*) = 0$). Since $s^*(p^*) \leq s^*(p^* + \delta)$ it follows that an increase in the share price can only occur due to an increase of demand at p^* if $d^*(p^*) = d^*(p^* + \delta)$. In that case, however, it must be the case that the surplus at $|s^*(p^* + \delta) - d^*(p^* + \delta)| > 0 = |d^*(p^*) - s^*(p^*)|$ due to the last price setting rule since $p^* < \bar{v}$ for a true value equilibrium if ex post ownership $\omega_0 < 1$ with positive probability. From this, it follows $s^*(p^* + \delta) > d^*(p^* + \delta)$ and $\tau(p^* + \delta|a') > \tau(\tilde{p})$ for all $\tilde{p} \notin \{p^*, p^* + \delta\}$. Hence, a buy order using $p^* + \delta$ as a price limit leads to a price increase of at most δ and it will be served since limit orders have higher priority than fill or kill orders.

(c) If the claim is not true, combining (a) and (b) implies that the distinguished player can always find a deviating strategy by mimicking ω_0^{min} or ω_0^{max}. Also, if the distinguished player is not indifferent between ω_0^{min} and ω_0^{max} he can pick a mimicking strategy that approximate the ex post ownership that leads to a higher utility. Note that the distinguished player cannot be indifferent between ω_0^{min} and ω_0^{max} if either $\omega_0^{min} > \alpha_0$ or $\alpha_0 > \omega_0^{max}$.

(d) Suppose that the claim $\omega_j^* = \omega_j^{max} \Rightarrow \omega_0^* = \omega_0^{min}$ for all $j \neq 0$ does not hold. Then, there exist a $k \neq 0$ for which $\omega_k^* = \omega_k^{max}$ *and* $\omega_0^* = \omega_0^{max}$. This implies that outside investor k submits a buy order which is served while $\omega_0^* = \omega_0^{max}$ or outside investor k submits a sell order which is not served while $\omega_0^* = \omega_0^{max}$. Then, the distinguished player can choose the deviating strategy a_0' which mimicks ω_0^{max} as described under (b) and complement this with a buy order $\beta_0'' = (x'', p')$ where p' is the price used to guarantee ω_0^{max} as derived in (b) and $x'' = \omega_k^{max} - \omega_k^{min}$. This increases utility from the distinguished player because he can now buy more undervalued shares.

This completes the proof of the lemma. ■

From lemma 2 it follows that outside investors who submit a buy order which leads to stochastic ex post ownership can benefit from not buying. Their buy order is only executed if $\omega_0^* = \omega_0^{min}$. However, in a true value equilibrium $p^* = \Pr(\omega_0^* = \omega_0^{min}) \cdot E(v(e(\omega_0^{min}))) +$

$\Pr(\omega_0^* = \omega_0^{max}) \cdot E(v(e(\omega_0^{max})))$. Since $E(v(e(\omega_0^{max}))) > E(v(e(\omega_0^{min})))$ this implies $p^* > E(v(e(\omega_0^{min})))$ and buying outside investors only buy overvalued shares. Hence, not buying constitutes an improvement.

Outside investors who sell stochastically can improve by employing the following deviating strategies. If the distinguished player submits fill or kill orders that are executed randomly, selling outside investors can deviate from submitting fill or kill orders themselves and submit a limit sell order using p^* as the price limit. This limit sell order is accompanied by a large enough (unexecuted) buy order using $p^* - \delta$ as price limit which then guarantees that the price does not decrease. Then, the limit sell order will always be executed before the distinguished player sells his shares. If the distinguished player submits limit orders, outside investors can reduce the price limit of their limit sell orders to $p^* - \delta$. As a result, their order will be executed due to price priority and the price decrease is at most δ. This completes the proof of non-existence of true value equilibria if ex post ownership is stochastic.

Ex post ownership of the distinguished player can neither be deterministic nor stochastic in a true value equilibrium which proves non-existence of a true value equilibrium.

(II) To prove existence of the excess returns equilibrium we construct equilibrium strategies as follows. The distinguished player submits a buy order for one share $a_0^* = \beta_0^* = (1, \hat{p})$ for any $\hat{p} \in]Ev(e(\alpha_0)), \bar{p}[$ with $\bar{p}$ sufficiently close to $Ev(e(\alpha_0))$ and for small enough δ there exist such $\hat{p}$. It turns out that $\bar{p} \in]Ev(e(\alpha_0)), Ev(e(\alpha_0 + 1/M))[$. Furthermore, one outside investor submits a sell order for one share using $\hat{p}$ as the price limit $a_i^* = \sigma_i^* = (1, \hat{p})$.

Every market mechanism sets $p^* = \hat{p}$ since all other prices lead to zero trade volume and we are looking at an excess returns equilibrium because $\omega_0^* = \alpha_0 + 1/M$, hence $p^* < E(v^*)$ and $\omega \neq \alpha$. Outside investors cannot benefit from increasing demand as this is only feasible if they buy instead of the distinguished player. Then, shares are worth $(Ev(e(\alpha_0))) < \hat{p}$ which is thus not a profitable deviation. The equilibrium seller cannot benefit from not selling since then the share not sold will be worth $(Ev(e(\alpha_0))) < \hat{p}$.

What remains to be shown is that the distinguished player cannot benefit from not trading. This is true if

$$\begin{aligned} &\alpha_0 \cdot E(v(e(\alpha_0))) - c(e(\alpha_0)) \\ < &(\alpha_0 + 1/M) \cdot E(v(e(\alpha_0 + 1/M))) - p^* \cdot (1/M) - c(e(\alpha_0 + 1/M)) \end{aligned}$$

This inequality holds for $p^* = E(v(e(\alpha_0)))$ because $e(\alpha_0) \neq e(\alpha_0 + 1/M)$ and it therefore also holds for a p^* sufficiently close to $E(v(e(\alpha_0)))$. ■

Proof (of Theorem 2, part (B), page 15) (i) Amsterdam market mechanism μ_A, (ii) Kyle market mechanism μ_K.

(i) Amsterdam market mechanism μ_A: Specify $\gamma < \frac{c}{\alpha_0}, \delta < \frac{1}{2}\left(\frac{c}{\alpha_0} - \gamma\right)$ and $\alpha_0 \geq c/\Delta v$. To prove existence of an excess returns equilibrium we have to specify a strategy profile a^* such that the price and net trade $(p^*, x, y) = \mu_A(\alpha, a^*)$ induced by the Amsterdam market mechanism satisfy (1) $p^* < \bar{v} - \gamma$, (2) $\omega_0 \Delta v \geq c$ high effort of the distinguished player $i = 0$, (3) which is a best response for all players $i = 0, ..., N$ and (4) there is an equilibrium winner. Before proving existence under μ_A, note that there exists a unique price $p_0 \in \left[\bar{v} - \frac{c}{\alpha_0}, \bar{v} - \frac{c}{\alpha_0} + \delta\right) \cap P$. The crucial property of this price is that p_0 is the maximal price in P below which the distinguished player prefers not to sell and exerting effort $e = 1$ to selling all his shares and exerting low effort $e = 0$ since

$$p_0 < \bar{v} - \frac{c}{\alpha_0} + \delta \Leftrightarrow \alpha_0 \bar{v} - c > \alpha_0 (p_0 - \delta) \qquad (*)$$

We call this property (*). Now, consider strategy profile a^* specified by the following conditions.

(1) $a_0^* = \mathring{\sigma}_0$ with $\mathring{\sigma}_0(p_0) = (\alpha_0, p_0)$: The distinguished player $i = 0$ submits a single fill-or-kill order using p_0 as the limit price to sell his entire α_0 stocks.

(2) Further at least 2 investors $i \neq 0$ submit a fill-or-kill order to buy α_0 stocks using a market order, i.e. for these investors $\mathring{\beta}_i = (\alpha_0, \infty) \in a_i^*$.

(3) Buying shareholders submit market buy orders (b_i, ∞) with $\sum_{i=1}^N b_i = q$, hence the entire market quantity is bought by investors who use market orders. Assume that at least one of the buyers is not a seller at the same time.

(4) Selling shareholders use limit orders $(s_i, p_0 - \delta)$ to sell their shares. Each seller keeps at least a proportion $\frac{\delta}{\Delta v + \delta}$ of his initial stake, i.e. $s_i \leq \frac{\Delta v}{\Delta v + \delta} \alpha_i$. Selling bids sum up to the traded quantity $\sum_{i=1}^N s_i (p_0 - \delta) = q$.

(5) Finally, some shareholder $j \neq 0$ submits a limit sell order $\sigma_j(s_j, p_0)$ using a price limit p_0 and a quantity s_j satisfying $\alpha_0 > s_j \geq \max_i s_i$, i.e. between α_0 and the largest equilibrium seller quantity $\max_i s_i$.

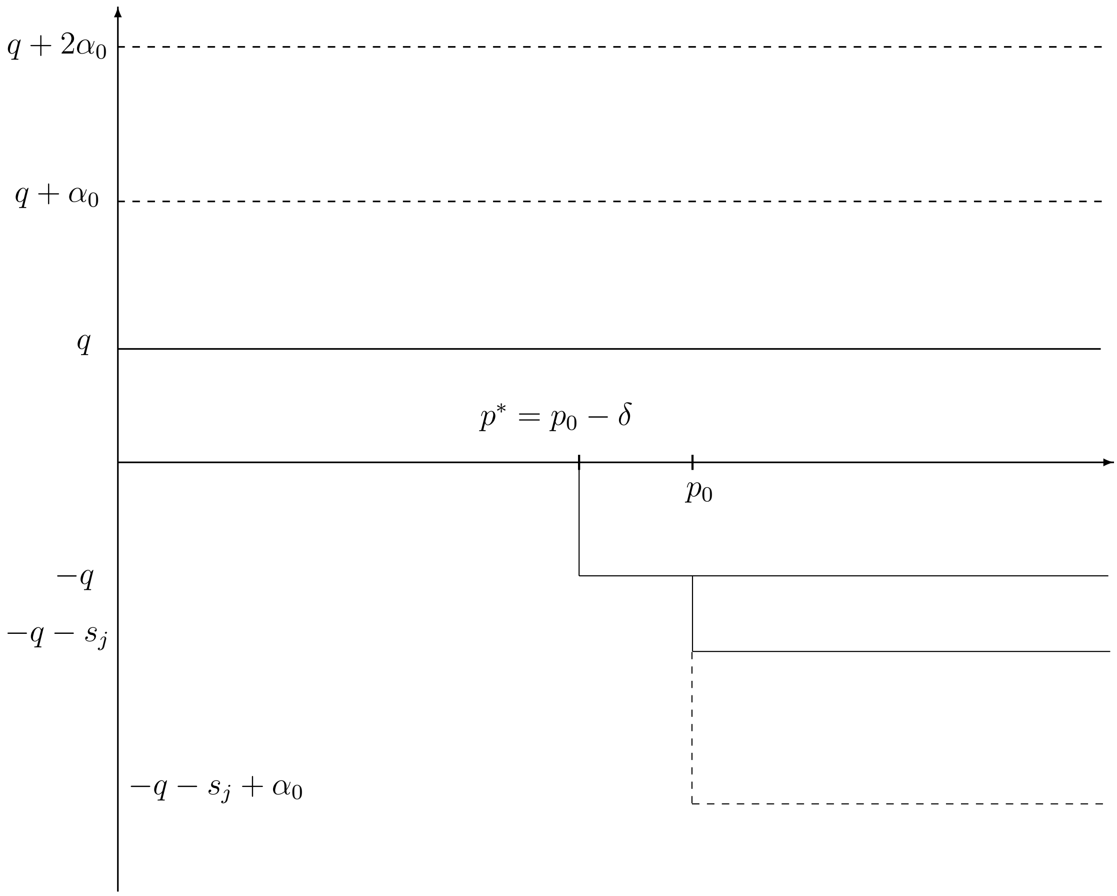

Figure 1: Excess returns equilibrium aggregated excess demand correspondence for the Amsterdam rules

First, we observe that applying the Amsterdam market mechanism μ_A to strategy profile a^* implements market price $p^* = p_0 - \delta$ and trade volume $\tau(p^*) = q$. To see this note that the trade volume is maximized and the market clears at $p^* = p_0 - \delta$. For higher prices, there is excess supply, lower prices have trade volume of zero. Inspection of figure 1 yields that the trade volume at $p^* = p_0 - \delta$ is $\tau(p^*) = q$. It remains to show that strategy profile a^* satisfies conditions (1), (2), (3) and (4).

Since by specification the bid ask spread satisfies $\gamma < \frac{c}{\alpha_0}$ we have $p_0 < \bar{v} - \frac{c}{\alpha_0} + \delta < \bar{v} - \gamma + \delta$ and hence $p^* = p_0 - \delta < \bar{v} - \gamma$ which shows that condition (1) holds. Since the distinguished player does not trade at p^* and therefore $\omega_0 \Delta v = \alpha_0 \Delta v \geq c$ and condition (2) holds. The presence of an equilibrium buyer who is not a seller guarantees (4). Not surprisingly, the most work is condition (3) to show that nobody can gain by deviating. Instead of tediously checking the universe of all possible deviations we proceed by discussing only optimal deviations. Moreover, any particular deviation will lead to a certain price and allocation given the behavior of all other players. We organize possible deviations first by the resulting price level denoted by $\hat{p}$. Second, among all deviations implementing price $\hat{p}$ we only consider an optimal deviation for any player i in the sense that no other deviation of this player which implement $\hat{p}$ can yield more.

1. $\hat{p} = p^*$: To increase demand or or reduce supply at p^* is not feasible without triggering a price increase for outside investors. The distinguished player $i = 0$ cannot gain from deviating by property (*) because $p^* < p_0$. The 2 or more shareholders who offer to buy a block of stocks cannot gain from cancelling the fill or kill order since their orders are not executed in equilibrium. Reducing demand or not trading at all is not beneficial neither. To see this note that not buying yields 0 instead of $\bar{v} - p_b = \bar{v} - p_0 - \delta - \gamma$ per share. But our specification $\delta < \frac{1}{2}\left(\frac{c}{\alpha_0} - \gamma\right)$ implies $\bar{v} - p_0 - \delta - \gamma > 0$. Increasing supply is not beneficial because it would imply selling undervalued shares.

2. $\hat{p} < p^*$: This can only be achieved by increasing supply. Since all shares not sold are worth $1 > p^*$ in equilibrium, this is not a profitable deviation and the distinguished player does not benefit by construction of p^*.

3. $\hat{p} > p^*$: An equilibrium seller i could deviate by using a beller strategy, buying and selling at a price $\hat{p} > p^*$. Although buying and selling some amount at the same time cancels out seller i gains $s_i(p^*) \cdot (\hat{p} - p^*)$ for his original equilibrium sell quantity by raising the sell price from p^* to $\hat{p}$. The downside is that this deviation of an equilibrium seller only works if he buys the additional supply s_j. This deviation triggers the distinguished player to sell and to reduce effort. Thereby player i looses $s_j(\hat{p} - \underline{v})$ since $\hat{p}$ is what he has to pay for any such share and $\underline{v}$ is what it is worth later on. Comparing gains and losses yields

$$\text{gains} = s_i(p^*) \cdot (\hat{p} - p^*) \leq \max_i s_i(p^*) \cdot (\hat{p} - p^*) \leq s_j(\hat{p} - \underline{v}) = \text{losses}$$

since $s_j \geq \max_i s_i$ by specification of a^* and $\frac{\hat{p} - p^*}{\hat{p} - \underline{v}} < 1$ since $p^* > \underline{v}$. Alternatively an equilibrium seller i could increase the stock price just by δ by reducing supply to $s_i' < s_i$ and not buying stocks. By this deviation seller i gains less than δs_i. It is not profitable, however, since sellers loose Δv on each share of their remaining stock $\alpha_i - s_i' > \alpha_i - s_i$. In our specification of a^* for every equilibrium seller holds $s_i \leq \frac{\Delta v}{\Delta v + \delta} \alpha_i \Leftrightarrow \delta s_i < \Delta v (\alpha_i - s_i)$. Thus, comparing gains and losses yields

$$\text{gains} < \delta s_i < \Delta v (\alpha_i - s_i) < \Delta v (\alpha_i - s_i') = \text{losses}.$$

Furthermore, a price $\hat{p} > p^*$ could be induced by other investors who increase demand. This is not beneficial since this triggers the fill or kill order to be executed and decrease effort of the distinguished player. This is also true for the k players who submit fill or kill orders. The distinguished player's threat to sell his shares will still be viable because more than one player submits a fill or kill order.

This proves existence of an excess returns equilibrium for the Amsterdam market structure.

(ii) Kyle market mechanism μ_K: To show existence for the Kyle market structure specify $\delta < \frac{1}{2}\left(\frac{c}{\alpha_0} - \gamma\right)$ and $\alpha_0 \geq c/\Delta v$. Since the strategy of this proof is similar to the existence proof of excess returns equilibria for the Amsterdam market rules we provide a less detailed proof. Consider strategy profile a^* specified by the following conditions.

(1) $a_0^* = \sigma_0$ with $\sigma_0(p_0) = (\alpha_0, p_0)$: The distinguished player $i = 0$ submits an order using p_0 defined as in the existence proof of the Amsterdam trading rule as the limit price to sell his entire α_0 stocks.

(2) An investor $j \neq 0$ submits a limit order to sell less than half of what he owns, i.e. quantity $q < \frac{\alpha_j}{2}$ at limit price $p_0 - \delta$.

(3) Each investor $i = 0, 1, ..., N$ submits a market buy order with quantity $b_i = \frac{q}{N+1}$. Together these orders add up to $\sum_{i=0}^{N} b_i = q$, hence the entire market quantity is bought by investors who use market orders.

(4) Finally, it holds that $\alpha_0 - c/\Delta v < q$. Investor k submits an additional buy order to buy (at least) $b_k \geq \frac{q}{N+1}$ using a price limit $p_0 - \delta$.

The aggregated excess demand correspondence induced by a^* is shown in figure 2.

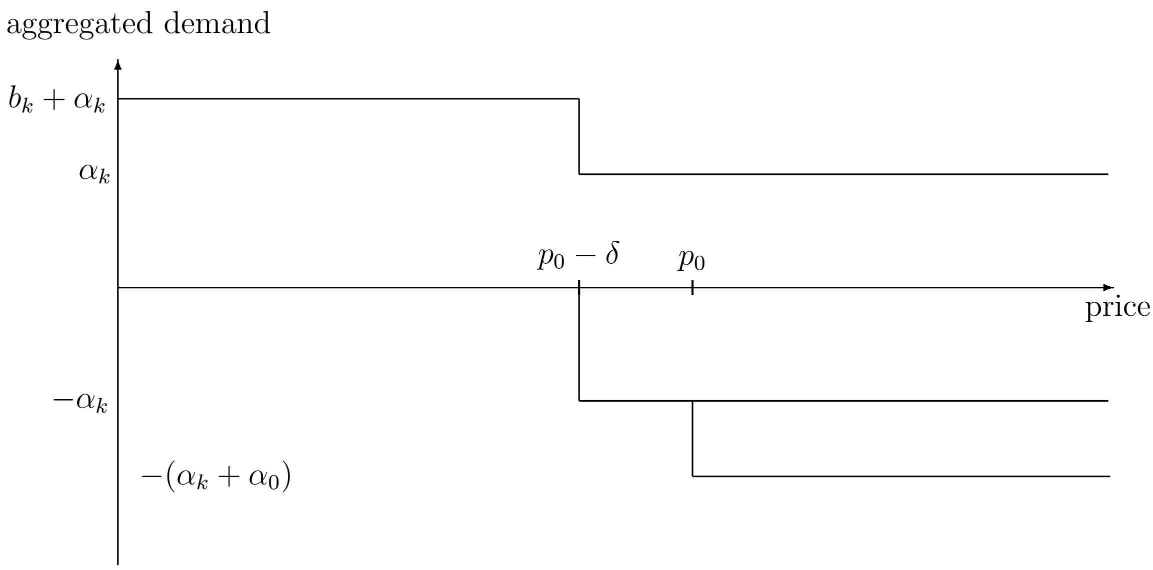

Figure 2: Excess returns equilibrium aggregated excess demand correspondence for the Kyle market rules

Applying the Kyle market rules to strategy profile a^* yields price $p^* = p_0 - \delta$ and $\omega_0 \Delta v = \alpha_0 \Delta v \geq c$ and in turn high effort $e = 1$. All equilibrium buyers except j are not sellers at the same time and therefore equilibrium winners. It remains to show that there is no strictly improving deviation for some investor. As before we organize possible deviations first by the resulting price level denoted by $\hat{p}$ and second, among all deviations implementing price $\hat{p}$ only consider an optimal deviation for any player i in the sense that no other deviation of this player can yield more.

1. $\hat{p} = p^*$: Deviations leading to the same price $\hat{p} = p^*$ are not beneficial. All $N + 1$ investors submit buy orders to buy stocks at p^*. Since the Kyle trading mechanism picks the allocation that minimizes $\sum_i (x_i)^2 + (y_i)^2$, no investor can increase the executed order size beyond $\frac{q}{N+1}$ by increasing demand at p^*. To reduce supply at p^* is not possible without triggering an increase in the stock price. Finally, the distinguished player $i = 0$ cannot gain from selling

at p^* by construction of p_0. Again, as for the Amsterdam rule not buying yields 0 instead of $\bar{v}-p_b = \bar{v}-p_0-\delta-\gamma$ per share. But our specification $\delta < \frac{1}{2}\left(\frac{c}{\alpha_0} - \gamma\right)$ implies $\bar{v}-p_0-\delta-\gamma > 0$.

2. $\hat{p} < p^*$: Picking a stock price at $\hat{p} < p^*$ is not possible for buying shareholders and not profitable for the selling shareholder.

3. $\hat{p} > p^*$: Deviations leading to stock prices $\hat{p} > p^*$ are clearly not beneficial for shareholders buying stocks but could potentially be beneficial for the distinguished player $i = 0$ or the selling shareholder $i = j$ and achievable using a beller strategy. Since $D(p) = q = \tau(p^*)$ for all $\hat{p} > p^*$, the distinguished player cannot increase ω_0 above ω_0^* and hence to drive up the stock price $\hat{p} > p^*$ only makes buying stocks more expensive. The selling shareholder j cannot gain from picking a stock price $\hat{p} > p^*$ since j can at most sell q stocks at a price $\hat{p} > p^*$. This price increase, however, triggers the distinguished player to reduce effort to $e = 0$ and thus comparing j's deviation payoff with his equilibrium payoff yields

$$\hat{p} \cdot q + 0 \cdot (q - \alpha_j) \leq 1 \cdot q + 0 \cdot (q - \alpha_j) < p^* \cdot q + 1 \cdot (\alpha_j - q)$$

while the last inequality holds by specification of a^*, in particular by $q < \frac{\alpha_j}{2}$. The low effort is triggered even if the selling shareholder withdraws his buy order because then investor's k buy order will be served. ■

Proof (of Theorem 3, page 17) 1. Suppose conversely that the distinguished player $i = 0$ would exert low effort $e^* = 0$. Then, any equilibrium seller can improve by not selling.

2. For the distinguished player $i = 0$ to exert effort $e^* > 0$ and to be a net seller $\omega_0^\mu(a^*) < \alpha_0$ is not optimal, since by not selling and exerting effort $e^* > 0$ he can improve. However, we have seen in 1. that $i = 0$ exerts effort $e^* > 0$. This proves 2. and shows that there must be other players being equilibrium sellers. Otherwise the trade volume would be 0.

3. Suppose not. Then, any non-pivotal equilibrium seller could benefit from not selling shares since they are worth more than the equilibrium price. All non-pivotal investors could benefit from increasing demand if they could buy at the equilibrium price.

4. Sellers only sell in equilibrium if they are pivotal with respect to the distinguished player's $i = 0$ effort choice. Since the market mechanism satisfies voluntary trade, this can only happen if the distinguished player submits an order.

5. An equilibrium winner satisfies $[\bar{v} - p^* - \gamma]\, x_i^* - [\bar{v} - p^*]\, y_i^* > 0$ and violates traditional no-arbitrage by definition:

$$\begin{aligned} [\bar{v} - p^* - \gamma]\, x_i^* - [\bar{v} - p^*]\, y_i^* &> 0 \Leftrightarrow \\ [p^\mu(a^*) - v(a^*)]\, y_i^\mu(a^*) + [v(a^*) - p^\mu(a^*) - \gamma]\, x_i^\mu(a^*) &> 0. \end{aligned}$$ ■

Proof (of Proposition 1, page 18) Let $\Delta v = 0$. This means that $e^* = 0$ which is a contradiction for a^* to be an excess returns equilibrium due to Theorem 3 part 1. ■

Proof (of Proposition 2, page 18) 1. Consider the converse case of a high true value equilibrium with bid ask spread $\gamma > 0$ where no outside investor $k \in \{1, ..., N\}$ is pivotal in the sense that selling less or buying more than specified by equilibrium strategies triggers the distinguished player $i = 0$ to sell and to reduce effort subsequently. In this case any equilibrium buyer can improve by not submitting buy orders and thereby saving $\gamma > 0$.

2. In a low true value trade equilibrium with $p^* = \underline{v}$ with bid ask spread $\gamma > 0$ a buyer could improve by not buying since he pays $p_b^* = \underline{v} + \gamma$. In contrast to the high true value equilibrium pivotalness does not play a role since by definition players payoff cannot decrease below $\underline{v}$ per share. Since a low true value equilibrium does not exist, every equilibrium involves $e^* > 0$. For this to occur, the distinguished player must buy shares if $\alpha_0 < c/\Delta v$. But then, the distinguished player can improve by not buying (and saving effort costs and bid ask spread), a contradiction.

3. For a true value equilibrium a^* holds $p^\mu(a^*) = v(a^*)$ and thereby

$$\begin{aligned}[p^\mu(a^*) - v(a^*)]\, y_i^\mu(a^*) + [v(a^*) - p^\mu(a^*) - \gamma]\, x_i^\mu(a^*) &= \\ 0 \cdot y_i^* - \gamma \cdot x_i^* &\leq 0 \end{aligned} \qquad \blacksquare$$

since $\gamma \cdot x_i^* \geq 0$ by definition.

4. Construct a strategy profile with resulting net trade $x - y \neq 0$ such that no player can strictly improve by deviating. To do this let a player j with positive initial stake $\alpha_j > 0$ just submit a single limit sell order $a_j = \{\sigma_j\} = \{(s_j, \bar{v})\}$. Moreover, the constraints $s_j \leq \alpha_j$ and $s_j \leq \alpha_0 - \frac{c}{\Delta v}$ must be satisfied. Since either quantities are continuous or M is large enough, we can find quantities that satisfy the latter inequality. Some other player $-j$ submits a single limit buy order $a_{-j} = \{\beta_{-j}\} = \{(s_j, \bar{v})\}$. And all other players submit nothing. If μ maximizes the trade volume at equilibrium price $p^* = \bar{v}$ then quantity s_j is traded between player j and player $-j$ and therefore we observe a non-zero net transaction $x - y \neq 0$. No player can improve since for any other price players can only trade with themselves and at true value equilibrium price $p^* = \bar{v}$ they are indifferent between trading and not trading. Especially, the distinguished player $i = 0$ cannot sell enough to choose $e = 0$ subsequently. This proves existence for a trade volume maximizing market mechanism μ.

   Consider now $\alpha_0 \Delta v < c$ and $\underline{v} > 0$. This implies $\omega_0 \Delta v < c$ and a low effort level $e = 0$. This implies that if a true value equilibrium exists then it yields low price $p^* = \underline{v}$. Existence is constructed similarly as before by picking a pair of players submitting one buy order and one sell order at this price but nothing else. Again, a trade volume maximizing market mechanism guarantees that the pair of orders is executed at this price and nobody can improve. If quantities are continuous or M is sufficiently large, we can again find traded quantities that do not allow the distinguished player to buy shares and exert effort subsequently.

Proof (of Theorem 4, page 21) Structure of the proof. We proceed by defining an economy with ownership structure and budget constraints. For this economy, we propose a candidate equilibrium strategy profile a^* for the distinguished player and rational investors. The task is to specify everything appropriately such that one finds a strategy profile a^* that establishes an excess returns equilibrium. The strategies, ownership structure, and budget constraints we propose are supposed to satisfy conditions (2)-(4). The proof contains three auxiliary results lemmata 3 to 5. While lemma 3 states some preparatory results lemma 4 shows that there exist parameters fulfilling conditions (2)-(4). Finally, lemma 5 shows that for candidate equilibrium strategy profile a^* there are no strictly improving deviations and thereby that indeed it forms a Nash equilibrium given the parameters satisfy conditions (2)-(4). The proofs in part (i) for μ_N and parts (ii) and (iii) for μ_S and μ_T only differ in the specification of the distinguished player's equilibrium strategy and in the budget constraints.

(i) 1. **The economy.** Small rational outside investors[29] are specified with initial stake α_i $\forall i \in$

[29] When we talk about the stakes of small continuum investors we mean infinitesimal stakes. For example, a measurable subset J of small investors jointly owns $\int_J \alpha_i di = \int_J 1 d\alpha$ stakes. For example, if J has Lebesgue measure $\lambda(J)$ and every investor owns the same α_r infinitesimal stakes then they jointly own $\lambda(J) \cdot \alpha_r$ stakes. The same holds for infinitesimal budgets.

(0, 1) with aggregated stake $\int_{(0,1)} \alpha_i di = \alpha_r$ and budget constraint B_i with aggregated constraint $\int_{(0,1)} B_i di = B > \bar{v}$. This specification implies that the aggregated budget constraint is never binding since together small investors can afford to buy strictly more than the entire firm at the highest value. However for any single investor there is a finite upper bound B_i up to which buy orders can be submitted.[30]

2. **Equilibrium strategies.**

   DP Distinguished player
   $a_0^* = \sigma_0$ with $\sigma_0(p_0) = (\alpha_0, p_0)$ where

$$p_0 := \min_{p \in P} \left\{ p | p > \bar{v} - \frac{c}{\alpha_0} \right\}. \tag{2}$$

   The distinguished player $i = 0$ submits a single order using p_0 as the limit price to sell his shares. This first condition specifies the price limit of the distinguished player as the lowest price at which the distinguished player has a strict incentive to sell instead of exerting effort.

   ROI Rational outside investors
   $a_i^* = \{\beta_i^*, \sigma_i^*\} = \{(\rho_i, p_0 - \delta), (\alpha_i, p_0)\}$ for $i \neq 0$ with $\rho_i := \frac{B_i}{p_0 - \delta}$.
   Here, ρ_i is the maximal buy order at price $p_0 - \delta$ any rational buyer can afford according to his budget constraint. Outside investors submit maximal buy orders for ρ_i shares using $p_0 - \delta$ as a price limit and sell orders for all their shares using p_0 as the limit price.

3. **Allocation and prices**

   The market mechanism together with the action profile specifies the expected equilibrium price, the value of the firm – implicitly determined by the ex post stakes owned by the distinguished player – and which outside investors' orders are executed determined by stochastic rationing.

   Expected price $E(p)$.

   In this strategy profile rational investors and the distinguished player never trade with each other. Therefore, market mechanism $\mu = \mu_N$ picks $p^\mu = p_0 - \delta$ if $\tilde{Z}_1 < 0$ and $p^\mu = p_0$ if $\tilde{Z}_1 > 0$. This yields an expected price

$$E(p) = \Pr(\tilde{Z}_1 < 0) \cdot (p_0 - \delta) + \Pr(\tilde{Z}_1 \geq 0) \cdot p_0.$$

   Expected company value $E(v)$.

   The expected value of the company depends only on the distinguished player's final stake ω_0. Since the market mechanism guarantees fully executed orders, the distinguished player either sells all his shares or none at all. Consequently, the distinguished player exerts effort if $\omega_0 > 0$. This implies an expected value

$$E(v) = \Pr(\omega_0 = \alpha_0) \cdot \bar{\nu} + \Pr(\omega_0 \neq \alpha_0) \cdot \underline{v}.$$

[30]The following interpretation of the budget constraint matters for the proof. Though B_i can be arbitrarily large, we suppose that the market mechanism does not execute buy orders $\beta_i = (b, p)$ with $b \cdot p > B_i$ of small investors since they could not not afford them unless they submit sell orders that are executed. Since traders don't know in advance if and for which price their sell order is executed – which could be random – buy orders are not allowed to be based on them. This assumption excludes equilibrium deviations where traders behave as bellers – i.e. buyers and sellers at the same time in order to increase their budget.

It is important to note that $\Pr(\omega_0 \neq \alpha_0) > 0$ as long as $\alpha_0 < b$, i.e. α_0 is small enough such that the distinguished player can sell all his shares to noise traders. To guarantee an *excess returns equilibrium* we state the second condition

$$\begin{aligned} E(v^*) - E(p^*) &= \\ R(p_0) &> 0 \end{aligned} \tag{3}$$

where

$$\begin{aligned} R(p) \quad : &= \Pr(\tilde{Z}_1 < 0) \cdot (\bar{\nu} - p + \delta) + \Pr(\tilde{Z}_1 \geq 0) \cdot \{\Pr(\omega_0 = \alpha_0 | \tilde{Z}_1 > 0) \cdot (\bar{\nu} - p) \\ &+ \Pr\left(\omega_0 \neq \alpha_0 | \tilde{Z}_1 > 0\right) \cdot (\underline{v} - p)\}. \end{aligned}$$

Note that $R(p)$ is a measure for excess returns.

Rationing.

To determine the rationing parameters consider the 3 relevant events:

(a) Noise traders sell in aggregate with probability $\Pr(\tilde{Z}_1 < 0)$: In this case the market mechanism realizes price $p_0 - \delta$ and aggregated demand is $\rho = \int_{(0,1)} \rho_i$ while expected aggregated supply is $E(\|\tilde{Z}_1\| | \tilde{Z}_1 < 0)$. This implies that every rational buyer will be served with probability

$$\lambda_1 := \frac{E(\|\tilde{Z}_1\| | \tilde{Z}_1 < 0)}{\rho}.$$

Since $\rho > 1$ and $0 < E(\|\tilde{Z}_1\| | \tilde{Z}_1 < 0) < 1$ there will always be rationing $0 < \lambda_1 < 1$ in this event.

(b) Noise traders buy in aggregate and the distinguished player does not sell his shares which occurs with probability $\Pr(\tilde{Z}_1 \geq 0) \cdot \Pr(\omega_0 = \alpha_0 | \tilde{Z}_1 \geq 0)$: In this case price p_0 is realized and expected aggregated demand is $E(\|\tilde{Z}_1\| | \tilde{Z}_1 \geq 0)$ while aggregated supply is α_r. Every small outside investor is served with probability

$$\lambda_2 := \frac{E(\|\tilde{Z}_1\| | \tilde{Z}_1 \geq 0 \texttt{ and } \omega_0 = \alpha_0)}{\alpha_r}.$$

(c) Noise traders buy in aggregate and the distinguished player sells his shares which occurs with probability $\Pr(\tilde{Z}_1 \geq 0) \cdot \Pr(\omega_0 \neq \alpha_0 | \tilde{Z}_1 \geq 0)$: In this case price p_0 is realized and expected aggregated demand is $E(\|\tilde{Z}_1\| | \tilde{Z}_1 \geq 0)$ which has to be reduced by α_0 since the distinguished player is assumed to sell his shares. Again, aggregated supply is α_r from rational investors. Every small outside investor is served with probability

$$\lambda_3 := \frac{E(\|\tilde{Z}_1\| | \tilde{Z}_1 \geq 0 \texttt{ and } \omega_0 \neq \alpha_0) - \alpha_0}{\alpha_r}.$$

The third condition reflects the critical trade-off for outside investors. It makes sure that they do not want to deviate from their equilibrium strategy and is given as

$$\Pr\left(\tilde{Z}_1 < 0\right) \cdot \lambda_1 \cdot (\bar{\nu} - p_0 + \delta) \geq R(p_0). \tag{4}$$

Investors compare being rationed in their buying order but only buying for the beneficial lower price $p_0 - \delta$ against always being served but including situations where they can trade against the distinguished player. The crucial difference here to the remainder of the article is that small traders can neither influence the price – i.e. they are *price takers* – nor the final allocation of other traders or the firm value and hence they are *not pivotal*.

The following claims are preparatory results that help to show existence in lemma 4.

Lemma 3 *Part A: Let* $\Pi(p) := R(p) - \Pr\left(\tilde{Z}_1 < 0\right) \cdot \lambda_1 \cdot (\bar{v} - p + \delta)$. *For small enough* δ, $R(\underline{v}) > 0 > R(\bar{v})$ *and* $\Pi(\underline{v}) > 0 > \Pi(\bar{v})$ *if* $\Pr\left(\omega_0 \neq \alpha_0 | \tilde{Z}_1 > 0\right) > 0$.
Part B: Conditions (2), (3), and (4) imply the following:

1. $p_0 > \underline{v}$.
2. *For small enough* δ *the IC constraint* $c < \alpha_0 \Delta v$ *is fulfilled*
3. $p_0 > E(v|\tilde{Z}_1 > 0)$*: Shares are overvalued if* $p^* = p_0$. □

Proof (of lemma 3) Part A: $R(\underline{v}) > 0$ follow from plugging $p = \underline{v}$ into the definition of $R(p)$. $R(\bar{v}) < 0$: This is true for small enough δ if $\Pr\left(\omega_0 \neq \alpha_0 | \tilde{Z}_1 > 0\right) > 0$ because

$$R(\bar{v}) = \Pr(\tilde{Z}_1 < 0) \cdot \delta + \Pr\left(\omega_0 \neq \alpha_0 | \tilde{Z}_1 > 0\right) \cdot (\underline{v} - \bar{v}) < 0$$

holds for small enough δ. To prove the second set of inequalities, replace $R(\cdot)$ in above arguments by $\Pi(\cdot)$. This proves the claim.
Part B:
Claim 1: Suppose not. Then, it follows from the definition of $R(p_0)$ that $R(p_0) > Pr(\tilde{Z}_1 < 0) \cdot (\bar{v} - p_0 + \delta) > Pr(\tilde{Z}_1 < 0) \cdot \lambda_1 \cdot (\bar{v} - p_0 + \delta)$ where the latter inequality is a contradiction of (4).
Claim 2: From equation (2) it follows that $\bar{v} - \frac{c}{\alpha_0} > p_0 - \delta$. Hence, if $p_0 - \delta > \underline{v} \Leftrightarrow p_0 - \underline{v} > \delta$ (which is possible due to lemma 3 claim 1) it is true that $\bar{v} - \frac{c}{\alpha_0} > \underline{v} \Leftrightarrow \alpha_0 \geq c/\Delta v$. This proves the claim.
Claim 3. Suppose not $\Rightarrow$

$$\begin{aligned} E\left(v|\tilde{Z}_1 > 0\right) - p_0 &= \\ \Pr(\omega_0 = \alpha_0 | \tilde{Z}_1 > 0) \cdot (\bar{v} - p_0) + \Pr\left(\omega_0 \neq \alpha_0 | \tilde{Z}_1 > 0\right) \cdot (\underline{v} - p_0) \geq 0 \Leftrightarrow & \\ R(p_0) - \Pr\left(\tilde{Z}_1 < 0\right) \cdot (\bar{v} - p_0 + \delta) &\geq 0 \end{aligned}$$

which by $\lambda_1 < 1$ implies $R(p_0) > \Pr(\tilde{Z}_1 < 0) \cdot \lambda_1 \cdot (\bar{v} - p_0 + \delta)$, a contradiction to inequality (3).■

With these observations we can now show existence:

Lemma 4 *For small enough* δ*, there always exist* p_0, α_0*, and* c *such that constraints (2), (3), and (4) hold for any non-degenerate noise.* □

Proof (of lemma 4) In a first step we choose α_0 small enough to ensure $\Pr(\omega_0 \neq \alpha_0 | \tilde{Z}_1 > 0) > 0$. We can now find a p_0 to guarantee that $R(p_0)$ is positive and sufficiently small since $R(\underline{v}) > 0 > R(\bar{v})$ (lemma 3) and R is continuous in p. Note that $p_0 < \bar{v}$ (since otherwise $R(p_0) < 0$). Hence, it is true that $\Pr\left(\tilde{Z}_1 < 0\right) \cdot \lambda_1 \cdot (\bar{v} - p_0 + \delta) > 0$. Small enough but positive excess returns $R(p_0)$ together with the observation that $\Pr\left(\tilde{Z}_1 < 0\right) \cdot \lambda_1 \cdot (\bar{v} - p_0 + \delta) > 0$ guarantee equations (3) and (4). Constraint (2) can be guaranteed by choosing c appropriately. ■

Lemma 5 *Strategy* $a^* = \{a_0^*, \{a_i^*\}_{i \in (0,1)}\}$ *with* $a_0^* = \sigma_0(p_0) = (\alpha_0, p_0)$ *and* $a_i^* = \{\beta_i^*, \sigma_i^*\} = \{(\rho_i, p_0 - \delta), (\alpha_i, p_0)\}$ *for* $i \neq 0$ *is an equilibrium strategy if constraints (2), (3), and (4) are met and* $\mu = \mu_N$. □

Proof (of lemma 5) We first derive equilibrium utility and then check possible deviations.

1. Equilibrium utility.

   Equilibrium utility of a risk neutral rational outside investor $i \in (0,1)$ is given by $u_i(a^*) = E\left[\omega_i \tilde{v} - p\tilde{x}_i + p\tilde{y}_i\right]$ where now the company value $\tilde{v}$ and the executed trade vector $(\tilde{x}_i, \tilde{y}_i)$ are random variables depending on the realization of noise $\tilde{Z}_1$. For the given strategy profile

$$\begin{aligned} u_i(a^*) &= E\left[\omega_i \tilde{v} - p\tilde{x}_i + p\tilde{y}_i\right] \\ &= \Pr(\tilde{Z}_1 < 0) \cdot \left[(\alpha_i + \lambda_1 \rho_i)\,\bar{v} - \lambda_1 \rho_i (p_0 - \delta)\right] \\ &\quad + \Pr(\omega_0 = \alpha_0 | \tilde{Z}_1 > 0) \cdot \alpha_i \cdot \left[(1-\lambda_2) \cdot \bar{\nu} + \lambda_2 \cdot p_0\right] \\ &\quad + \Pr\left(\omega_0 \neq \alpha_0 | \tilde{Z}_1 > 0\right) \cdot \alpha_i \cdot \left[(1-\lambda_3) \cdot \underline{\nu} + \lambda_3 \cdot p_0\right]. \end{aligned}$$

   In the first expression $\lambda_1 \rho_i(\bar{v} - p_0 + \delta)$ are the expected benefits from buying shares at price $p_0 - \delta$ if $\tilde{Z}_1 < 0$. The second and third expressions are utility from selling shares which can happen if $\tilde{Z}_1 \geq 0$. The second term is utility from selling if the distinguished player exerts high effort. Finally, if the distinguished player sells his shares he will not exert effort. Again, there is rationing and only a fraction λ_3 of all sell orders can be served in that case.

2. Deviations.

   We discuss deviations at any equilibrium price that can occur with positive probability.

   (a) Price $p \geq p_0$.

      i. Outside investor $i \in (0,1)$: No outside investor has an impact on equilibrium price and we can restrict our analysis of deviations to the two realizations of equilibrium prices. Outside investors could increase their demand at $p \geq p_0$ by submitting a buy orders $\beta_i' = (b, p_0 + \epsilon)$ with $b > 0$ and $\epsilon \geq 0$.
      As a consequence, the buy order would not be rationed due to price priority which is beneficial for the low price and adverse if the distinguished player sells his shares. It is not a beneficial deviation if

$$\begin{aligned} \Pr(\tilde{Z}_1 < 0) \cdot \lambda_1 \cdot (\bar{\nu} - p_0 + \delta) \quad \geq \quad & \Pr(\tilde{Z}_1 < 0) \cdot (\bar{\nu} - p_0 + \delta) \\ & + \Pr(\omega_0 = \alpha_0 | \tilde{Z}_1 > 0) \cdot (\bar{\nu} - p_0) \\ & + \Pr\left(\omega_0 \neq \alpha_0 | \tilde{Z}_1 > 0\right) \cdot (\underline{v} - p_0) \end{aligned}$$

      which holds by equation (4).
      Decreasing sell orders is not beneficial since shares are overvalued if $p \geq p_0$ by virtue of lemma 3 part 3.

      ii. Distinguished Player: Offering to sell at higher prices is not beneficial since these orders will not be executed. This is true because we assume that $b \leq \alpha_\rho$ and hence $\tilde{Z}_1 > \alpha_\rho$ is not possible and orders from rational outside investors can always match the noise. Offering to sell only a fraction is not beneficial by construction of p_0.

   (b) Price $p < p_0$.

      i. Outside investor $i \in (0,1)$: Clearly, increasing supply or decreasing demand at $p < p_0$ is not a beneficial deviation since shares are undervalued for $p < p_0$.

      ii. Distinguished Player: Selling at $p < p_0$ is not beneficial by construction of p_0. ■

This completes the proof of part (i).

(ii) $\mu = \mu_S$: Specify similarly as in (i). However, the equilibrium strategy of the distinguished player is a fill-or-kill order: $a_0^* = \mathring{\sigma}_0$ with $\mathring{\sigma}_0\,(p_0) = (\alpha_0, p_0)$. The distinguished player $i = 0$ submits a single fill-or-kill order using the same p_0 as the limit price to sell all his shares. Furthermore, budget constraints are symmetric: $b_i = b_j$ for all $i \neq j$ and $i \in (0,1)$ and $j \in (0,1)$.

(iii) $\mu = \mu_T$: Specify noise $\Pr(\tilde{Z}_1 \in (0,t) = 0)$ for some $t > 0$ and $\alpha_0 \leq t$. Furthermore, budget constraints are as in (ii). Finally, the distinguished player now submits the same limit order as in (i). With these re-specifications the proof is the same as in (i).

(iv) To save on notation, we will use above defined functions with the subscript l and h for low and high noise. For example, $R_l(p, \alpha_0) := \Pr_l(\tilde{Z}_1 < 0) \cdot (\bar{\nu} - p + \delta) + \Pr_l(0 \leq \tilde{Z}_1 < \alpha_0) \cdot (\bar{\nu} - p) + \Pr_l\left(\tilde{Z}_1 \geq \alpha_0\right) \cdot (\underline{v} - p)$.

The proof makes use of the following observations. Condition (2) is not affected by a change of the distribution of noise. However, conditions (3) and (4) are harder to meet if noise decreases. Hence, we construct a strategy profile a^* together with p_0 and c such that condition (4) satisfied under high noise but not under low noise.

Let p_h be implicitly defined $\Pi_l(p_h) = 0$. Clearly, p_h exists since $\Pi(\underline{v}) > 0 > \Pi(\bar{v})$ and $\Pi(p)$ is continuous in p. Now, pick an $\{\alpha_0, c, a^*\}$ for the economy specified in part (i) of this proof with p_0 sufficiently close to p_h which satisfies conditions (2), (3), and (4). Now, from above we know that there exist such parameters and that a^* is an equilibrium under noise h. However, it ceases to be an equilibrium under noise l since rational investors have an incentive to deviate from their proposed equilibrium strategies because condition (4) no longer holds if p_0 is sufficiently close to p_h. To see this, note that λ_1 strictly decreases if noise is reduced from high noise h to low noise l. Hence, the LHS of condition (4) strictly decreases. Furthermore, the RHS of condition (4) cannot decrease and may even increase.

This completes the proof of part (iv).

(v) Without noise, our call auction market mechanisms obey the voluntary trade property. Hence, we can again apply our characterization result. However, since agents cannot be pivotal in the continuum case, an excess returns equilibrium cannot exist.

(vi) The proof is by contradiction. Suppose to the contrary that there exists an excess returns equilibrium. This implies that shares are traded at a price $p < v = \bar{v} = \underline{v}$. Note first that not every investor can be served with probability one since $\alpha_1 < 1$ but aggregate budget constraint allows rational investors to buy more than the entire firm at prices $p < v$. Therefore, the strictly positive measure of traders who are not served with probability one have an incentive to increase their price limit. This would increase their equilibrium utility since they will now always be served at p by virtue of the price priority property of the market microstructures studied in this section.

This completes the proof of theorem 4. ∎

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