

Liquidity and Congestion*

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Abstract

This paper studies the relationship between the arrival of potential investors and market liquidity in a search-based model of asset trading. The entry of investors into a specific market causes two contradictory effects. First, it reduces trading costs, which then attracts new investors (thick market externality effect). But secondly, as investors concentrate on one side of the market, the market becomes “congested”, decreasing the returns to participating in this market and discouraging new investors from entering (*congestion effect*). The equilibrium level of market liquidity depends on which of the two effects dominates. When congestion is the leading effect, some interesting results arise. In particular, we find that diminishing trading costs in our market can deteriorate liquidity and reduce welfare.

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1 INTRODUCTION

Liquidity is sometimes defined as a coordination phenomenon. In financial markets, as investors move into a specific market they facilitate trade for all investors by reducing the cost of participating in this market. At the same time, easier trade and lower trading costs attract potential investors. There is a thick market externality where new investors provide market liquidity and market liquidity attracts new investors. However, as investors prefer to join one side of a market, i.e. as they become buyers or sellers, this side of the market becomes “congested”, hindering trade. Congestion then discourages investors from entering this market.

One-sided markets arise during financial booms and, more drastically, during market crashes. When a market is in distress, liquidity typically vanishes playing a key role in the build-up of one-sided markets. The study of liquidity in one-sided markets is thus vital to understand the response of financial systems to the threat of market disruptions. Recent episodes of market distress include the LTCM crisis¹ in 1998, the September 11, 2001, events² and the turbulence in credit markets³ during the summer of 2007.

In this paper we present an alternative view of market liquidity. The main difference with the previous literature is that we consider not only a thick market externality but also a *congestion effect*. In our model, the arrival of new investors causes two opposite effects. First, it diminishes transaction costs and eases trade, which attracts potential investors. But secondly, as investors concentrate on one side of the market, trade becomes more difficult, reducing the returns to participating in this market and discouraging potential investors from entering. Market liquidity thus results from the tradeoff between thick market externalities and a congestion effect.

We assume an infinite-horizon steady-state market where agents can invest in one asset which can be traded only bilaterally. In this market, investors cannot trade instantaneously

¹For an analysis of the events surrounding the market turbulence in autumn 1998, see BIS (1999) and IMF (1998).

²Cohen and Remolona (2001) presents a summary of the September 11, 2001 episode in global financial markets. Also, McAndrews and Potter (2002) gives a detailed account of the consequences of the September 11, 2001, events on the US payment system and of the actions of the Federal Reserve System to provide liquidity to the financial system.

³See Fender and Hördahl (2007) for an overview of the key events over the period from end-May to end-August 2007.

but it takes some time to find a trading partner resulting in opportunity and other costs. Once an investor buys the asset, he holds it until his preference for the ownership change and he prefers to liquidate the investment and exit the market. To model the search process we adopt the framework introduced in Vayanos and Wang (2007). In our setting though, investors are heterogeneous in their investment opportunities in the sense that some investors have access to better investment options than others.

We compute explicitly the unique equilibrium allocations and the price at which investors trade with each other and show how they depend on the flow of new investors entering the market. Prices negotiated between investors are higher in the flow of potential investors. However, investors' entry decision is endogenous and thus depends on market, asset and investors characteristics. A change in investors' search abilities, for instance, affects both the rate of meetings between trading partners and the flow of investors entering the market, which then determines the distribution of potential partners with whom they can meet.

Moreover, the equilibrium flow of investors arises from a tradeoff between thick market complementarities and a congestion effect. When congestion is the dominating effect some interesting results come to light. First, reducing market frictions can *decrease* market attractiveness. Under some cases, one-sided markets can develop. A regulatory reform or the introduction of a technological advance, such as a new electronic trading system, can induce an adverse effect on the distribution of investors during upswings. Specifically, it would allow the few sellers present in the market to exit at faster rates leading to an even more unbalanced distribution of investors. Congestion then intensifies as the market becomes more one-sided, discouraging potential investors and thus dampening down the attractiveness of this market.

Second, *diminishing market frictions can deteriorate market liquidity and reduce welfare*. The reason for this counterintuitive result is the following. In a one-sided market with more sellers than buyers, for example during a fire sale, introducing a measure that improves the efficiency of the search process makes it easier for one of the few buyers present in the market to acquire the asset. But when the buyer purchases the asset (and a seller exits), the proportion of buyers to sellers falls further and the market becomes more one-sided. As investors cluster on the sell-side of this market, buyers gain a more favourable position in the bargaining process and try to lower the price they pay to acquire the asset. Reducing market frictions in a distressed market thus magnifies the effect of congestion and results in

a lower asset price (a higher price discount) and ultimately in a less liquid market. Investors who hold this asset and those trying to sell it are clearly worse-off as the market becomes more one-sided, leading to a decrease in overall welfare. From this point of view, this paper provides an example of the Theory of the Second Best. Improving the efficiency of the search process, when there are other imperfections in the market such as the ones arising from the congestion effect, is not necessarily welfare enhancing.

Third, market illiquidity measured by the price discount can increase while trading volume rises. Reducing search frictions during downswings amplifies the effect of congestion, resulting in a higher price discount and in a less liquid market. But a more efficient search process also increases the frequency of meetings between the investors already present in the market. More frequent meetings then translates into a higher trading volume. Thereby, a measure intended to shorten the waiting times needed to locate a trading partner in a market experiencing distressed selling can cause both higher price discount and higher trading volume. This third result joins the discussion on the measurement of the effect of liquidity on asset prices and shows how alternative measures capture different dimensions of market liquidity.

The outline of the paper is as follows. In the next section, we discuss the related literature. We introduce a theoretical framework to examine the relationship between market liquidity and the arrival of potential investors to this market in Section 3. Section 4 determines the population of investors, their expected utilities and the price of the asset, taking as given investors' decision to enter the market. Then, Section 5 endogenises the entering rule and characterises the study of the unique market equilibrium. Market liquidity and welfare are discussed in Section 6. Finally, Section 7 concludes. Some proofs and additional results are presented in the appendices.

2 RELATED LITERATURE

The notion of thick market complementarity is clearly captured in Diamond (1982a). He considers an economy where islanders face production opportunities and decide whether to remain unemployed or to climb a palm tree and retrieve coconuts. Trees differ in their heights (the cost of production). Islanders only climb trees shorter than a certain height and they cannot consume the coconuts they pick. They need to search for a trade to swap

the coconuts. The likelihood of meeting a trading partner in this economy increases in the number of potential traders available. This key feature constitutes the basis of the strategic complementarity in Diamond’s model. This is highlighted in Cooper and John (1988), where they discuss the economic relevance of strategic complementarities in agents’ payoffs and explain how they can lead to coordination failures. A related argument is presented in Milgrom and Roberts (1990). They show the Diamond-type search model is a supermodular⁴ game, where more production or participation activity by some islanders raises the returns to increased levels of activity by others.

Building on strategic complementarities Brunnermeier and Pedersen (2007) and Gromb and Vayanos (2002) analyse the link between capital and market liquidity. Also, Pagano (1989) focuses on the feedback loop between trading volume and liquidity to study concentration and fragmentation of trade across markets. In Dow (2004), multiple equilibria with different degrees of market liquidity result from informational asymmetries. Plantin (2004) assumes investors can learn privately about an issuer’s credit quality by holding an asset. This “learning by trading” also creates a thick market externality. From a broad perspective, this literature studies liquidity as a self-fulfilling phenomenon where both liquid and illiquid market equilibria may arise. Illiquid markets are thus a consequence of a coordination failure.

Our paper is also related to the search literature. The economics of search have their roots in Phelps (1972). Search-theoretic models such as the frameworks introduced in labour markets⁵ by Diamond (1982a), Diamond (1982b), Mortensen (1982) and Pissarides (1985) have been broadly used in different areas of economics. In asset pricing⁶, Duffie, Gârleanu and Pedersen introduce search and bargaining in models of asset market equilibrium to study the impact of these sources of illiquidity on asset prices. This paper is related to Duffie et al. (2005), which presents a theory of asset pricing and marketmaking in over-the-counter markets with search-based inefficiencies. They conclude that risk neutral investors receive narrower bid-ask spreads if they have easier access to other investors and marketmakers. Similarly to Duffie et al. (2005) we consider risk-neutral agents who can only invest in one asset. In our model though, investors can only trade with other investors and our focus,

⁴In the unidimensional case, a supermodular game is a game exhibiting strategic complementarities in which each agent’s strategy set is partially ordered. See Topkis (1979) and Cooper (1999) for a formal definition.

⁵See Pissarides (2001) for a review of the literature on search in labour markets.

⁶For an excellent review on liquidity and asset prices, see Amihud et al. (2005).

rather than on liquidity and marketmaking, lies on the endogenous relationship between market liquidity and the arrival of potential investors to this market.

Duffie et al. (2007) extends their setting to incorporate risk aversion and risk limits and finds that, under certain conditions, search frictions as well as risk aversion, volatility and hedging demand increase the illiquidity discount. Lagos and Rocheteau (2007) also generalises Duffie et al. (2005) to allow for general preferences, unrestricted long positions, idiosyncratic and aggregate uncertainty and entry of dealers. Our paper shares with theirs the existence of strategic complementarities and an endogenous entry decision. To define the entry of dealers, Lagos and Rocheteau (2007) specify that the contact rate between investors and dealers increases sublinearly in the number of dealers. In our framework, entry is the result of a decision problem where investors compare the benefits of this market to their best investment opportunities.

Weill (2007) and Vayanos and Wang (2007) extend the framework of Duffie, Gârleanu and Pedersen to allow investors to trade multiple assets⁷. They show that search frictions lead to cross-sectional variation in asset returns due to illiquidity differences. In Vayanos and Wang (2007) investors are heterogeneous in their trading horizons while in Weill (2007) investors are homogeneous, but there are differences in the assets' number of tradable shares. From a methodological point of view, our paper is closely related to Vayanos and Wang (2007). The main difference with their work is that we consider only one asset and focus on the analysis of the liquidity in the market for this asset rather than on the liquidity across two assets.

Our paper is close in spirit to Huang and Wang (2007). They also find that decreasing market frictions can diminish the level of market liquidity. However, their framework and the general mechanism that yields this result clearly differ from ours. Rather than a search-based model, they consider a centralised market where exogenous transaction costs take the form of participation costs. Agents can pay an ex-ante cost to trade constantly (and become market makers) or pay a spot cost to trade after observing their trading needs. Huang and Wang (2007) argues that, when there is insufficient supply of liquidity, lowering the cost to enter on the spot can decrease welfare because it reduces investors' incentives to become market makers. In our model, market liquidity results from a tradeoff between thick market

⁷See also Vayanos and Weill (2007) for an application to the on-the-run phenomenon, by which recently issued bonds have higher prices than older ones with the same cash flows. They develop a multi-asset model where both the spot market and the repo market operate through search.

externalities and congestion effects. We show that, when the congestion effect dominates, the market becomes one-sided and improving the efficiency of the search process can diminish market liquidity because it discourages agents from investing into our market.

This paper also relates to the literature on asset pricing with exogenous trading costs studied in Amihud and Mendelson (1986), Vayanos (1998) and Acharya and Pedersen (2005), among others. We complement this literature by endogenising transaction costs.

3 THE MODEL

Time is continuous and goes from zero to infinity. There is only one asset traded in the market with a total supply S . This asset pays a dividend flow d .

Consider risk-neutral agents, whom we will refer to as investors. By assuming risk neutrality, we aim to study the concentration of liquidity in a specific market without reference to investors' shifts in their attitudes towards risk. Investors are infinitely lived and have time preferences determined by a constant discount rate equal to $r > 0$. At some random time, investors decide to enter the market and aim to buy one unit of the asset. They become *buyers-to-be*. Once they purchase the asset, buyers-to-be become *non-searcher investors*. Non-searchers hold the asset and enjoy the full value d of its dividend flow until they receive a liquidity shock which makes them want to liquidate their portfolio and leave the market. We assume liquidity shocks arrive with a Poisson rate γ and reduce investors' valuation to a lower level $d - x$ of flow utility, where $x > 0$ captures the notion of a liquidity shock to the investors, for example, a sudden need for cash or the arrival of a good investment opportunity in another market. x could also be understood as the holding cost borne by the investor who receives a liquidity shock and is aiming to exit the market. At that time, non-searcher investors become *sellors-to-be* and seek to sell⁸. Upon selling, investors exit the market and join the initial group of outside investors.

The flow of investors entering the economy is defined by a function f . Investors are heterogeneous in their investment opportunities κ , i.e. we consider they differ on their outside

⁸Investors are risk neutral and thus have linear utility over the dividend flow d . Consequently, they optimally prefer to hold a maximum long position in the asset (which we can normalise to 1) or zero units of the asset (once they seek to exit the market).

options as some investors enjoy better investment possibilities than others. We assume f is a continuous and strictly positive function of the investor's investment opportunity class κ , such that the total flow of investors entering the economy is given by $\int_{\underline{\kappa}}^{\overline{\kappa}} f(\kappa) d\kappa$, where $[\underline{\kappa}, \overline{\kappa}]$ is the support of $f(\kappa)$. Only a fraction $\nu(\kappa)$ of the flow of investors entering the economy decides to invest in this market. At any point in time there is a non-negative flow of every class of investor from the outside investors' group into the market, and hence the total flow of investors entering the market is defined by $g = \int_{\underline{\kappa}}^{\overline{\kappa}} \nu(\kappa) f(\kappa) d\kappa$.

We assume markets operate through search, with buyers and sellers matched randomly over time in pairs. Search is characteristic of over-the-counter markets where investors need to locate trading partners and then bargain over prices. There is a cost associated to this search process. In a market where it is more likely to find a counterpart in a short time, the search cost is smaller and liquidity, measured by search costs, is higher. But we could think of a broader interpretation of the search friction. In a centralised market, it represents the cost of being forced to trade with an outside investor who does not understand the full value of the asset and requires an additional compensation for trading. These investors only buy the asset at a discount and sell it at a premium. This transaction cost decreases in the abundance of investors. In the market of a frequently traded asset, it is less likely that it is necessary to trade with an outside investor who "mis-values" the asset and hence the transaction cost linked to this asset is smaller and its liquidity higher. In this paper, we use the first intuition because of its more transparent interpretation.

We adopt the search framework presented in Vayanos and Wang (2007). To define the search process, we first need to describe the rate at which investors willing to buy meet those willing to sell and once they meet we need to specify how the asset price is determined. The ease in finding a trading partner depends on the availability of potential partners. Let us consider that an investor seeking to buy or sell meets other investors according to a Poisson process with a fixed intensity. Thus, for each investor the arrival of a trading partner occurs at a Poisson rate proportional to the measure of the partner's group. Denote by η_b the measure of buyers-to-be and by η_s the measure of investors seeking to sell (sellers-to-be). Then, a buyer-to-be meets sellers-to-be with a Poisson intensity $\lambda\eta_s$ and a seller-to-be meets buyers-to-be at a rate $\lambda\eta_b$, where λ measures the efficiency of the search and a high λ represents an

efficient search process. The overall flow of meetings⁹ between trading partners is then given by $\lambda\eta_b\eta_s$.

Once investors meet they bargain over the price p of the asset. These meetings always result in trade as Proposition 5 shows. For simplicity we assume that either the investor willing to buy or the one willing to sell is chosen randomly to make a take-it-or-leave-it offer to his trading partner. Denoting by $\frac{z}{1+z}$ the probability of the buyer-to-be being selected to make the offer and thus by $\frac{1}{1+z}$ the probability that the seller-to-be makes the offer, $z \in (0, \infty)$ captures the buyer's-to-be bargaining power.

Figure 1 describes our market, specifying the different types of investors and the flows between types. η_0 denotes the measure of non-searcher investors.

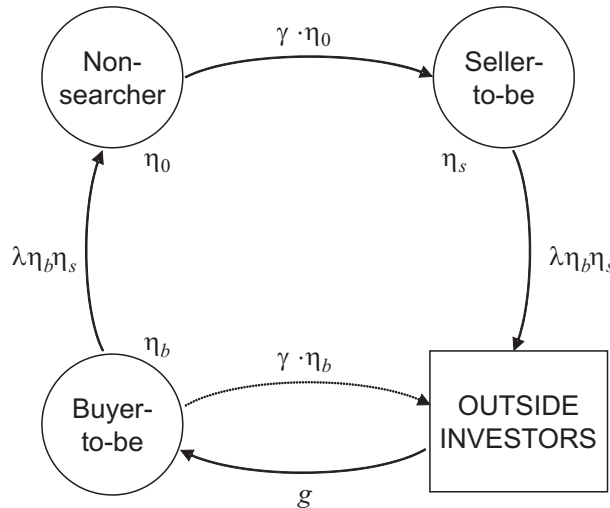


Figure 1: An outside investor enters the market and becomes a buyer-to-be aiming to meet a seller-to-be. If he suffers a liquidity shock before meeting a trading partner, he exits the market. On the contrary, if he meets a seller-to-be, he bargains over the price, buys the asset (pays p) and becomes a non-searcher. He holds the asset until he receives a liquidity shock. At that time, he becomes a seller-to-be seeking a buyer-to-be. When he meets a buyer-to-be, he bargains over the price, sells the asset (receives p) and exits the market returning to the group of outside investors.

⁹See Duffie and Sun (2007) for a formal proof of this result. This application of the exact law of large numbers for random search and matching has previously been used in Duffie et al. (2005), Duffie et al. (2007) and Vayanos and Wang (2007) among others.

4 ANALYSIS

In this section we first solve for the steady-state measure of every type of investor in the market. Next, in Subsection 4.2, we describe investors' flow utilities. We show in Subsection 4.3 that every meeting between trading partners results in trade and we discuss thick market externalities in Subsection 4.4.

4.1 MEASURE OF INVESTORS

In this subsection we determine the measure of buyers-to-be (η_b), non-searcher investors (η_0) and sellers-to-be (η_s). Although investors are heterogeneous in their investment opportunities κ , once they enter the market their class does not alter their behaviour in this market. Investors develop sudden needs for cash at the same Poisson rate γ , independently of their outside investment opportunities κ . In consequence, we do not need to consider the distribution of investment opportunities within each population but the aggregate measure of buyers-to-be, non-searcher investors and sellers-to-be. This assumption could be generalised by considering γ a function of the outside option κ . The analysis would be similar but the notation more complicated, as we would need to take into account the distribution of investment opportunities κ within each group of investors rather than the aggregate measures¹⁰.

In equilibrium, the market needs to clear and thus the supply of the asset equals the measure of investors holding the asset, each of whom holds one unit of the asset. Specifically, the sum of the measures of non-searchers and sellers-to-be is equal to the total supply of the asset:

$$\eta_0 + \eta_s = S \quad \Rightarrow \quad \eta_s = S - \eta_0 \quad (1)$$

In a steady state, the inflow of investors joining a group matches the outflow such that the rate of change of the group's population is zero. The inflow and outflow of the different types of investors are summarised in Figure 1. Let us first consider the non-searcher investors. In this case, inflows are given by the buyers-to-be who meet a trading partner and buy the asset ($\lambda\eta_b\eta_s$), while non-searchers receiving a liquidity shock ($\gamma\eta_0$) constitute the outflow. Setting

¹⁰See Section 3 in Vayanos and Wang (2007) for a particular case.

inflow equal to outflow and using equation (1) yields:

$$\eta_b = \frac{\gamma}{\lambda} \frac{\eta_0}{S - \eta_0} \quad (2)$$

We now analyse the population of buyers-to-be. The flows of investors coming from the outside group are defined by g . The outflow is comprised of the buyers-to-be who receive a liquidity shock before meeting a trading partner ($\gamma\eta_b$) and of those who meet sellers-to-be and buy the asset ($\lambda\eta_b\eta_s$). Then,

$$g = \gamma\eta_b + \lambda\eta_b\eta_s$$

Using equations (1) and (2) we can rewrite the previous equation as:

$$g = \gamma \left(1 + \frac{\gamma}{\lambda} \frac{1}{S - \eta_0} \right) \eta_0 \quad (3)$$

Equation (3) determines η_0 as a function of g . Then, substituting η_0 in equations (1) and (2) specifies η_s and η_b respectively. Let us first assume the flow of investors entering the market g is constant. We generalise our results in Subsection 5.1.

Proposition 1. *Given g constant, there is a unique solution to the system (1) - (3) given by:*

$$\eta_0 = \frac{1}{2\gamma} A \quad (4)$$

$$\eta_s = S - \frac{1}{2\gamma} A \quad (5)$$

$$\eta_b = \frac{\gamma}{\lambda} \frac{A}{2\gamma S - A} \quad (6)$$

where $A = (g + \gamma S + \frac{\gamma^2}{\lambda}) - \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}$.

The proof is presented in Appendix A. It is interesting to note how the different measures of investors respond to changes in the parameters of our model. For instance, as the flow of investors g entering the market rises, the measure of investors willing to buy (buyers-to-be) and of those passively holding the asset (non-searchers) increase. However, given that there are more investors seeking to buy the asset, it is now easier for a seller-to-be to find a

trading partner and hence the measure of investors seeking to sell falls. This is summarised in Proposition 2 and proven in Appendix A.

Proposition 2. *The measure of buyers-to-be and non-searcher investors is increasing in g ($\frac{\partial \eta_b}{\partial g}, \frac{\partial \eta_0}{\partial g} > 0$) while the measure of sellers-to-be decreases in g ($\frac{\partial \eta_s}{\partial g} < 0$).*

Given the measures of investors η_b seeking to buy and those η_s seeking to sell, the efficiency of the search process λ defines the overall flow of meetings (and transactions, according to Proposition 5) in our market. However, the measures of the different types of investors also depend on the efficiency of the search process. In particular, for the same level of investors entering the market, if the search process is more efficient, there will be a lower measure of investors “waiting” to meet a potential seller ($\frac{\partial \eta_b}{\partial \lambda} < 0$). Thus, outside investors, who enter the market, meet a trading partner and become non-searcher investors at a faster rate if the search process is more efficient ($\frac{\partial \eta_0}{\partial \lambda} > 0$). A proportion of non-searcher investors then joins the pool of sellers-to-be and hence there is a higher flow of investors coming from the non-searchers to the group of sellers-to-be. And, although there are more inflows of investors and less investors seeking to buy, if the search process is more efficient, the measure of sellers-to-be “waiting” to sell is reduced ($\frac{\partial \eta_s}{\partial \lambda} < 0$). Proposition 3 presents these results:

Proposition 3. *Given g constant, the measure of buyers-to-be and sellers-to-be is decreasing in λ ($\frac{\partial \eta_b}{\partial \lambda}, \frac{\partial \eta_s}{\partial \lambda} < 0$) while the measure of non-searcher investors increases in λ ($\frac{\partial \eta_0}{\partial \lambda} > 0$).*

The proof is in Appendix A.

4.2 EXPECTED UTILITIES AND PRICE

We now determine the expected utility of the buyers-to-be (v_b), the non-searcher investors (v_0) and the sellers-to-be (v_s), as well as the price p . Investors exit this market because of a need for cash. We assume that the expected utility of outside investors is zero. Once they are out of the market, investors have different investment opportunities and decide where to invest next. They could even choose to re-enter this market again.

To derive the expected utility of every type of investor we analyse the possible transitions between types. For example, a buyer-to-be can leave the market if he receives a liquidity

shock, remain a potential buyer or meet a seller-to-be and become a non-searcher. This is summarised in Figure 2:

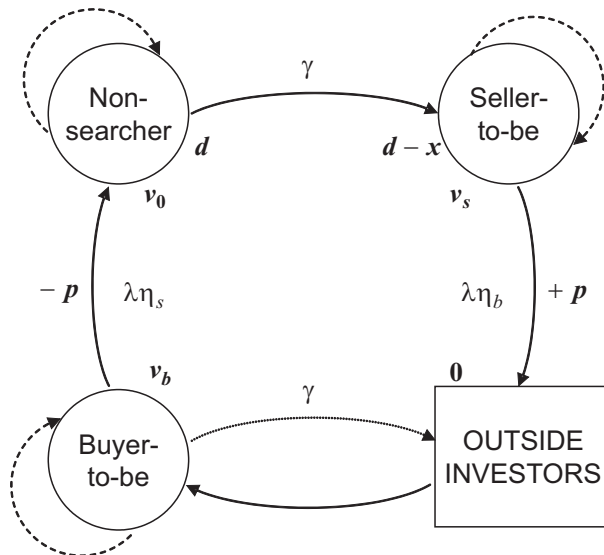


Figure 2: Groups of investors and transitions between groups.

The utility flow rv_b of buyers-to-be is thus equal to the expected flow of exiting the market ($(0 - v_b)\gamma$) and becoming an outside investor plus the expected flow derived from meeting a trading partner seeking to sell (which occurs at rate $\lambda\eta_s$), buying the asset (paying p) and becoming a non-searcher investor ($\lambda\eta_s(v_0 - v_b - p)$). Then,

$$rv_b = -\gamma v_b + \lambda\eta_s(v_0 - v_b - p) \quad (7)$$

Non-searcher investors can either remain non-searchers enjoying the full value d of the asset's dividend flow or receive a liquidity shock with probability γ and become a seller-to-be. In this case, the flow of utility of being a non-searcher is

$$rv_0 = d + \gamma(v_s - v_0) \quad (8)$$

Sellers-to-be exit the market as soon as they meet a trading partner, i.e., with intensity $\lambda\eta_b$ they sell the asset (receiving p) and become outside investors with zero expected utility.

Meanwhile, they enjoy a low level $d - x$ of utility. Thus,

$$rv_s = (d - x) + \lambda\eta_b(p + 0 - v_s) \quad (9)$$

The asset price is determined by bilateral bargaining between a buyer-to-be and a seller-to-be. We have assumed that with probability $\frac{z}{1+z}$ the buyer-to-be makes a take-it-or-leave-it offer to his trading partner and offers him his reservation value v_s . With probability $\frac{1}{1+z}$, the seller-to-be is chosen to offer the buyer-to-be his reservation value $v_0 - v_b$. As a result,

$$p = \frac{z}{1+z}v_s + \frac{1}{1+z}(v_0 - v_b) \quad (10)$$

where z measures the buyer's-to-be bargaining power which we treat as exogenous. Proposition 4 summarises this subsection's main result. The proof is in Appendix A.

Proposition 4. *Given g constant, the system of equations (7)-(10) has a unique solution given by:*

$$v_b = k \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \frac{\lambda\eta_s z}{r + \gamma} \quad (11)$$

$$v_0 = \frac{d}{r} - k \left(\frac{x}{r} + \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \right) \frac{\gamma}{r + \gamma} \quad (12)$$

$$v_s = \frac{d}{r} - k \left(\frac{x}{r} + \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \right) \quad (13)$$

$$p = \frac{d}{r} - k \frac{x}{r} \quad (14)$$

where $k = \frac{(r + \gamma + \lambda\eta_s)z + \gamma}{(r + \gamma + \lambda\eta_s)z + (r + \gamma + \lambda\eta_b)}$.

The price of the asset as given by equation (14) is thus equal to the present value of all future dividend flows d , discounted at the rate r , minus a price discount due to illiquidity. The second term is the product of present value of the holding cost x borne by investors seeking to exit the market and a function k . $k \in (0, 1)$ measures the severity or intensity of the illiquidity discount¹¹.

It is interesting to highlight that the asset price will be higher when fundamentals are

¹¹See Section 6 for a discussion of market liquidity.

stronger (i.e. if the asset pays a higher dividend flow d) and whenever the demand for the asset increases ($\frac{\partial p}{\partial d}, \frac{\partial p}{\partial \eta_b} > 0$). On the contrary, the price decreases with investors trying to sell the asset and in the buyer's-to-be bargaining power ($\frac{\partial p}{\partial z}, \frac{\partial p}{\partial \eta_s} < 0$). If during the bargaining process the buyer-to-be holds a more favourable position, he would try to lower the price paid to acquire the asset. The proof of this set of comparative statics is presented in Appendix B.

4.3 TRADE AMONG INVESTORS

In this subsection we prove a result we have assumed so far in our analysis:

Proposition 5. *All meetings between buyers-to-be and sellers-to-be result in trade.*

Proof. Trade between buyers-to-be and sellers-to-be occurs if the gain from trade is strictly positive, i.e., if the buyers'-to-be reservation value $v_0 - v_b$ exceeds the sellers'-to-be reservation value v_s . Let us see if $(v_0 - v_b) - v_s > 0$. Subtracting equations (13) and (11) from (12), we get:

$$(v_0 - v_b) - v_s = \frac{x(1+z)}{(r+\gamma)(1+z) + \lambda\eta_s z + \lambda\eta_b}$$

which is always strictly greater than zero since $x, r, \gamma, \lambda, \eta_s, \eta_b, z > 0$.

Therefore, once investors meet, trade among partners always occurs. □

4.4 THICK MARKET EXTERNALITY

In financial markets, thick market externalities arise when the gains from investing in a market depend on the number of investors who decide to come to the market. In this case, the more traders move into a market, the easier become the transactions and as a result the bigger is the gain derived from participating in this market. In our framework, the price of the asset is higher as the flow of investors moving into the market increases¹² ($\frac{\partial p}{\partial g} > 0$). As investors arrive to this market, the costs of search are reduced and hence the illiquidity discount is diminished. This increases the returns to investing in this market, making it more

¹²The proof is presented in Appendix B.

attractive to new investors. To understand how higher participation may encourage further participation we need to endogenise investors' entry decisions.

5 EQUILIBRIUM

In our setting, market equilibrium is determined by the fraction of investors entering the market, a measure of each group of investors, their expected utilities and the price of the asset. We centre our study on the steady-state analysis. In the previous section we take as given investors' decision to enter the market, and we now endogenise the entering rule in Subsection 5.1. A formal definition of the market equilibrium is then presented in Subsection 5.2. Subsection 5.3 introduces the congestion effect.

5.1 ENTERING RULE

In this subsection we endogenise the entering rule. In our framework, outside investors can choose between entering the market, which we will refer to as *our market*, and investing in an alternative market. Investors are heterogeneous in their outside investment opportunities κ , i.e. each class of investor has access to different investment opportunities. However, once they enter our market, their type no longer influences their decisions in the sense that every buyer-to-be, for instance, enjoys the same expected utility independently of his original outside opportunity. Interestingly, a buyer's-to-be expected utility does depend on the flow of investors who entered this market before him.

Let us refer to the investor who is deciding between moving or not into our market as the marginal investor. And, let us denote by κ' and by $v_{alt}(\kappa')$, respectively, the best outside investment opportunity of the marginal investor and his expected utility from investing in that alternative market. For simplicity, we assume $v_{alt}(\kappa') = \kappa'$, such that an investor with a better outside option (higher κ) enjoys a higher level of expected utility.

When an investor faces the decision to choose a market, he prefers to enter our market if the expected utility v_b of being a buyer-to-be in this market is higher than the expected utility v_{alt} derived from his best outside option. Then, if our market represents the best

opportunity for the marginal investor, it is also preferred by any other investor with a worse investment opportunity, i.e. any investor whose type $\kappa < \kappa'$ moves into our market too. As a result, when our market is chosen by a marginal investor with a high type, a high flow of investors enters our market. A high flow of investors implies an increase in the measure of buyers-to-be, which then affects the expected utility of being a buyer-to-be. Thus, even though each investor's type does not alter his expected utility, the type of the last investor who enters does. The type of this last investor defines the total flow who prefers our market and hence determines how concentrated the population of buyers-to-be is.

Let us define the fraction $\nu(\kappa)$ of investors with outside investment opportunity κ who enters the market as follows:

$$\nu(\kappa) = \begin{cases} 0 & \text{if } \kappa > \kappa' \\ [0, 1] & \text{if } \kappa = \kappa' \\ 1 & \text{if } \kappa < \kappa' \end{cases}$$

where $1 - \nu(\kappa)$ represents the fraction of investors with outside option κ who invests in alternative markets. The total flow of investors moving into our market is thus given by: $g(\kappa') = \int_{\underline{\kappa}}^{\bar{\kappa}} \nu(\kappa) f(\kappa) d\kappa$, where f defines the total flow of investors entering the economy. In equilibrium, as we discuss in more detail in the next subsection, the total flow g^* depends on the equilibrium fraction of investors ν^* entering our market. But the equilibrium fraction of investors is determined by the marginal investor who is indifferent between our market and his best outside option. We refer to this investor as the indifferent investor. For the indifferent investor, the expected utility of being a buyer-to-be equals the expected utility of his best outside option:

$$v_b \left(g^* = \int_{\underline{\kappa}}^{\bar{\kappa}} \nu^*(\kappa) f(\kappa) d\kappa \right) = v_{alt}(\kappa^*) \quad (15)$$

Before we proceed, let us introduce the formal definition of market equilibrium.

5.2 EQUILIBRIUM DEFINITION AND CHARACTERISATION

Definition 1. A *market equilibrium* consists of a fraction $\nu(\kappa)$ of investors entering the market, measures (η_s, η_b, η_0) of investors and expected utilities and prices (v_b, v_0, v_s, p) such that:

- $(\eta_s^*, \eta_b^*, \eta_0^*)$ solve the market-clearing condition and inflow-outflow equations given by the system (1) - (3),
- $(v_b^*, v_0^*, v_s^*, p^*)$ solve the flow-value equations for the expected utilities and the pricing condition given by the system (7) - (10),
- $\nu^*(\kappa)$ solves the entering condition given by the system (15).

To analyse the equilibria in this market, we need to solve for the fixed points of the system of equations (1) - (3), (7) - (10) and (15). There are two types of possible scenarios depending on the behaviour of the expected utility v_{alt} of investing in an alternative market and the expected utility v_b of being a buyer-to-be in our market. There is an equilibrium where all investors clearly prefer one market (either all enter or no one enters) or an equilibrium where a fraction of investors is better off by investing in our market while others prefer not to enter. Theorem 1 summarises a key result:

Theorem 1. *There is a unique market equilibrium.*

The proof is in Appendix C. To gain some intuition for this result, let us introduce Figure 3. Figure 3 represents the expected utility v_{alt} of investing in an alternative market and the expected utility of being a buyer-to-be of the marginal investor, i.e. the one deciding whether or not to enter our market. Consider, for example, the marginal investor with outside investment opportunity κ'_1 . He compares the utility of his outside option, $v_{alt}(\kappa'_1) = \kappa'_1$, to the utility of being a buyer-to-be, $v_b(g(\kappa'_1))$, given that investors with outside opportunities $\kappa < \kappa'_1$ have already entered our market. He enters since $v_b(g(\kappa'_1)) > v_{alt}(\kappa'_1)$, as shown in Figure 3. Now, let us focus on the marginal investor with investment opportunity κ'_2 . The expected utility of being a buyer-to-be has decreased because now all investors with $\kappa < \kappa'_2$ are in the market. Still he is better-off by moving into our market. Suppose marginal investor

κ^* is now facing the entry decision. For him, $v_b(g(\kappa^*)) = v_{alt}(\kappa^*)$ and he is indifferent between markets. Any investor with a better outside opportunity prefers not to enter.

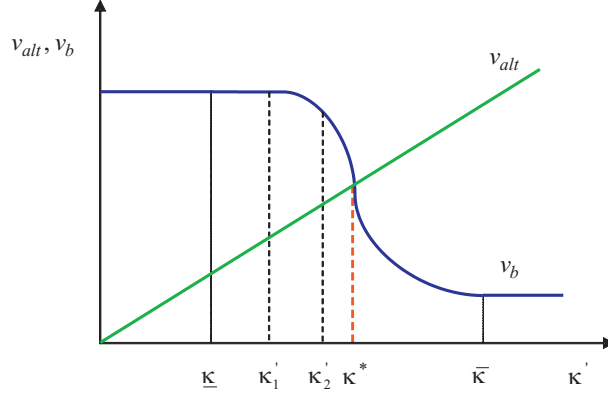


Figure 3: UNIQUE MARKET EQUILIBRIUM - Investors compare expected utilities v_b and v_{alt} and decide to move into our market if $v_b > v_{alt}$. κ^* defines the outside investment opportunity which makes investors indifferent between entering or not our market. $\underline{\kappa}$ and $\bar{\kappa}$ determine the support of the flow of investors who enter the economy.

Let us see why the equilibrium is unique. Given non-negative expected utilities, if $v_b(\kappa' = 0) > v_{alt}(\kappa' = 0)$ and v_b decreases in κ' while v_{alt} is strictly increasing, then by continuity there exists a unique threshold κ^* such that expected utilities are equal and investors indifferent between markets. A unique threshold κ^* then defines a unique flow of investors $g^* \equiv g(\kappa' = \kappa^*)$ entering our market. And given a unique flow of investors g^* , steady-state measures, expected utilities and the asset price can be determined uniquely as stated in Propositions 1 and 4. Consequently, market equilibrium is unique. It is interesting to note that the expected utility of buyers-to-be decreases as more investors enter our market. We discuss this result in the following subsection.

5.3 MARKET CONGESTION

Why is the expected utility of a buyer-to-be reduced as the flow of investors entering the market rises? Because buyers-to-be suffer from a *congestion effect*. In our market, an increase in the flow of investors g affects differently the steady-state measures of investors. Every investor who enters our market becomes a buyer-to-be first. Then, only a proportion of

buyers-to-be meets a trading partner, purchases the asset and becomes a non-searcher. Only a fraction of non-searcher receives a liquidity shock becoming a seller-to-be. But, given that the measure of buyers-to-be has increased, it is now easier for a seller-to-be to meet a trading partner and hence the steady-state measure of investors seeking to sell is reduced as the flow of investors g increases¹³. As a result, in our framework buyers-to-be are worse off when g rises because it is now more difficult for them to meet a seller-to-be and purchase the asset. There is a congestion effect as investors move into our market in the sense that buyers-to-be face a crowded market where there is increasing competition among buyers-to-be for the fewer sellers-to-be.

6 LIQUIDITY, MARKET EFFICIENCY AND WELFARE

In this section we first discuss the relationship between market liquidity and the equilibrium flow of investors who move into our market. In our model, the equilibrium flow is endogenously determined and depends on the characteristics defining the market, the asset and the investors. To analyse, for instance, the consequences on market liquidity of a change in market efficiency we need to understand both the direct effect of this change on the asset price, and hence on liquidity, and also the indirect effect through the equilibrium flow of investors. Subsection 6.2 examines the introduction of a new electronic system in our market to provide some intuition for the interaction between search costs and the equilibrium flow of investors and thus to better understand this indirect effect. The general relationship between the flow of investors and the parameters of the model, including search efficiency, is derived in Subsection 6.3. Finally, in Subsection 6.4 we introduce welfare and study the implications on welfare and market liquidity of an improvement in the efficiency of the search process when our market experiences a fire sale.

6.1 MARKET LIQUIDITY

In our model, an investor willing to buy or sell needs to find a trading partner and bargain over the asset price before the transaction takes place. Investors cannot trade instantaneously

¹³Comparative statics of the steady-state measures of investors in the market were introduced in Section 4.1 (See Proposition 2).

but there is a time delay due to this search process. This search cost can be identified with the expected time required to locate a trading partner and, as a result, liquidity can be viewed as inversely related to this time delay. In Subsection 4.2 we define illiquidity as measured by the illiquidity discount

$$k \frac{x}{r}$$

where $\frac{x}{r}$ is the present value of the holding cost x and $k = \frac{(r+\gamma+\lambda\eta_s)z+\gamma}{(r+\gamma+\lambda\eta_s)z+(r+\gamma+\lambda\eta_b)}$. Let us denote by $\tau^s \equiv \frac{1}{\lambda\eta_b}$ the expected time required to locate a buyer-to-be and by $\tau^b \equiv \frac{1}{\lambda\eta_s}$ the expected time it takes for a buyer-to-be to meet a seller-to-be. The function k is increasing in τ^s and decreasing in τ^b . Then, as the time a seller-to-be needs to wait before he can leave the market (τ^s) increases, k rises and the effect of the illiquidity discount is more severe. In contrast, if a buyer-to-be needs to wait longer to locate a seller-to-be, the effect of illiquidity discount is diminished. The equilibrium level of market liquidity thus rises in η_b^* but diminishes in η_s^* . In our market, an increase in the equilibrium measure of buyers-to-be and a reduction in the equilibrium measure of sellers-to-be occurs whenever the equilibrium flow of investors, g^* , moving into our market increases¹⁴. We formalise this result in the following proposition, which we prove in Appendix D:

Proposition 6. *Liquidity increases in the flow of investors entering the market.*

Understanding the relationship between market liquidity and investors' decision to enter a market constitutes one of the main motivations of our analysis. In our model, there is a trading externality as the arrival of new investors facilitates the search process for every investor in the market. If the flow of potential traders increases, trade becomes easier and liquidity rises.

However, as the flow of investors moving into a market increases, the congestion effect makes it more difficult for a buyer-to-be to locate a trading partner. Consequently, as the market gets crowded, it becomes less attractive to investors. This translates into a lower flow of investors entering the market and as a result into a less liquid market.

In equilibrium, the flow of investors and hence the level of market liquidity result from a tradeoff between thick market complementarities and congestion effects. The equilibrium

¹⁴See Proposition 2.

flow of investors though is determined endogenously in our framework and depends on the market, investors and asset characteristics such as search efficiency, frequency of the liquidity shocks and dividend flow, among others. If we were interested in the consequences on market liquidity of a change in any of these characteristics, we would need to consider two different type of implications. Assume, for instance, an improvement in the efficiency of the search process. It would not only increase the rate at which investors meet but it would also affect the flow of investors who enter our market. Specifically, it raises the frequency of meetings between buyers-to-be and sellers-to-be, favouring market liquidity, and induces two opposite effects on the equilibrium flow of investors who move into our market. First, trading externalities attract potential investors, increasing the flow. But, secondly, congestion deters investors from entering our market, diminishing the flow. The overall level of market liquidity thus depends on this tradeoff and on the effect of the improvement in efficiency on the trading frequency. We discuss the aggregate effect on market liquidity in Subsection 6.4, but we first introduce the following example to better understand the interaction between trading costs and flow of investors.

6.2 AN EXAMPLE OF A TECHNOLOGICAL INNOVATION

We consider a search-based market of asset trading as the one described in the previous sections. We are interested in understanding the consequences of a technological innovation intended to increase the efficiency of the search process, such as the introduction of a new electronic trading system. The efficiency of the search process in our model is defined by the parameter λ . A high value of λ represents an efficient search process and corresponds to a market where the rate at which investors meet trading partners is high and hence the friction introduced by the search process and its associated cost are low.

We assume the flow of investors f entering the economy is uniformly distributed¹⁵ with support $[0, 5]$, where $\underline{\kappa} = 0$ and $\bar{\kappa} = 5$. Investors have time preferences with discount rate equal to 1% ($r = 0.01$). The asset pays a dividend flow $d = 2$ and is in total supply

¹⁵Formally, we assume a beta distribution defined on the interval $[0, 5]$ with shape parameters $a = 1$ and $b = 1$, which is identical to a uniform distribution with support $[0, 5]$. The beta distribution is a flexible class of distributions defined on the unit interval $[0, 1]$, whose density function may take on different shapes depending on the choice of the two parameters. These include the uniform density function and hump-shaped densities (See Evans et al. (1993)). We introduce the beta distribution to facilitate the comparison between settings when we later discuss the second example.

$S = 2$. The holding cost is defined as a 40% of the dividend flow to indicate that once an investor receives a liquidity shock his valuation of the asset drops to a 60% of the initial value. Liquidity shocks arrive at a Poisson rate $\gamma = 0.2$ and hence the expected time between shocks is 5. The value of z is chosen such that buyers-to-be and sellers-to-be have the same bargaining power, i.e. $z = 1$. We refer to this example as the baseline setting.

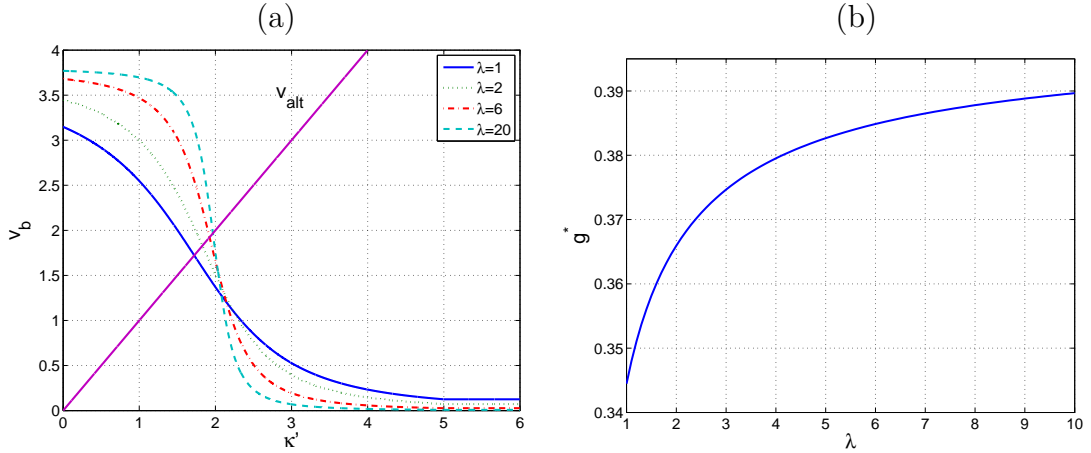


Figure 4: BASELINE SETTING - Improving efficiency (higher value of λ) attracts more investors to our market (higher g^*). The value of the model parameters is set at the following: $r = 0.01$, $S = 2$, $x = 0.8$, $z = 1$, $\gamma = 0.2$, $a = 1$, $b = 1$, $\underline{\kappa} = 0$ and $\bar{\kappa} = 5$.

Figure 4(a) represents the expected utility v_{alt} of investing in an alternative market and the expected utility v_b of being a buyer-to-be as a function of the marginal investor's outside investment opportunity κ' . The expected utility v_b of buyers-to-be is plotted for four different values of the market efficiency λ , where a higher λ indicates a more efficient search process. The intersection between v_b and v_{alt} gives, for each level of search efficiency, the threshold κ^* that defines the indifferent investor. κ^* is hence the solution to our fixed point problem. In equilibrium, investors whose best outside investment opportunity κ' is below the threshold value κ^* enter our market, while those with $\kappa' > \kappa^*$ prefer the alternative market. Figure 4(a) shows that an improvement in the efficiency of the search process (higher value of λ) makes our market attractive to more investors (higher κ^*). A higher threshold κ^* then corresponds to an increase in the flow of investors g^* who prefer our market. Figure 4(b) depicts the equilibrium flow of investors g^* entering our market, which is strictly increasing in the efficiency of the search process.

An increase in the equilibrium flow of investors g^* causes a rise in the equilibrium measures of buyers-to-be η_b^* and non-searchers η_0^* and a reduction in the equilibrium measure of sellers-to-be¹⁶ η_s^* . But the equilibrium measures of investors in our market also depend on the efficiency of the search process¹⁷. In particular, as the search process becomes more efficient (higher value of λ), the measures of investors “waiting” to buy or sell (η_b^* and η_s^*) decrease while the measure of non-searchers rises. The overall effect on the equilibrium measures is presented in the top panel of Figure 5(a):

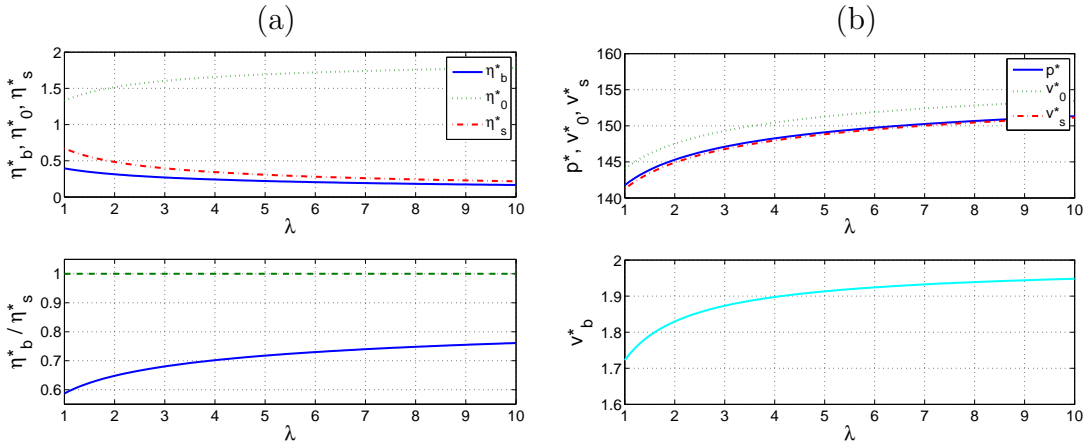


Figure 5: BASELINE SETTING - Equilibrium measures of investors in our market and ratio of buyers-to-be to sellers-to-be (a), expected utilities and price (b) as a function of the market efficiency λ . Other parameters are set at the following values: $r = 0.01$, $S = 2$, $x = 0.8$, $z = 1$, $\gamma = 0.2$, $a = 1$, $b = 1$, $\underline{\kappa} = 0$ and $\bar{\kappa} = 5$.

More interestingly, the bottom panel of Figure 5(a) illustrates the ratio between buyers-to-be and sellers-to-be as a function of the efficiency of the search process. This ratio captures the notion of congestion in our market. A high value of the proportion of buyers-to-be to sellers-to-be ($\gg 1$) describes a market where there is strong competition among buyers-to-be for the few sellers-to-be. There is congestion on the buyer-side in this market. On the contrary, a very low value of this ratio corresponds to a market where there is congestion on the sell-side (more sellers-to-be than buyers-to-be). The effect of congestion gets attenuated as the ratio between buyers-to-be and sellers-to-be tends to 1 as in our baseline setting.

Figure 5(b) depicts equilibrium price and expected utility of sellers-to-be and non-searchers

¹⁶See Proposition 2.

¹⁷See Proposition 3.

(top panel) and buyers-to-be (bottom panel) as a function of λ . Price and expected utilities increase in the efficiency of the search process.

In this baseline setting a new electronic trading system, which improves search efficiency, enhances the attractiveness of our market. But this is not always the case. Let us introduce the following example.

6.2.1 MARKET BOOM OR MARKET CRASH IN AN OUTSIDE MARKET

Assume a scenario similar to the one we have just discussed in the baseline setting and let us now consider a severe adverse shock which affects investors' outside investment opportunities. The worsening of investors' outside options could correspond to a boom in our market or to a market crash in another market¹⁸ and would affect the distribution of investors f entering the economy as a function of their outside investment opportunities κ . It would lead to a shift to the left of the mass of the distribution of investors f . In particular, we consider a beta distribution with support $[0, 5]$ and parameters $a = 2$ and $b = 15$, which is a right-skewed hump-shaped density function.

We present our results in Figures 6 and 7, where the value of all parameters (but the distribution parameters) remains as in the baseline setting, i.e., $r = 0.01$, $d = 2$, $S = 2$, $x = 0.4d$, $\gamma = 0.2$ and $z = 1$.

Figure 6(a) illustrates the expected utility v_{alt} of investors' outside options and the expected utility v_b of being a buyer-to-be in our market for the same four values of market efficiency considered in the baseline setting. It is interesting to highlight that the equilibrium threshold κ^* now decreases in the search efficiency, such that to a market with a more efficient search process corresponds a lower cutoff value κ^* of the outside option, which then defines a lower equilibrium flow of investors g^* entering the market. As Figure 6(b) clearly shows, the equilibrium flow of investors entering our market strictly decreases in the search efficiency.

Why is the equilibrium flow of investors decreasing as the search process becomes more efficient? Let us see why this is the case. Our market is now attractive to more investors because of the worsening of conditions in another market. This is indicated in Figures 4(b)

¹⁸In either case, market conditions improve significantly in our market compared to those in alternative markets. For the ease of exposition, we consider the market crash interpretation.

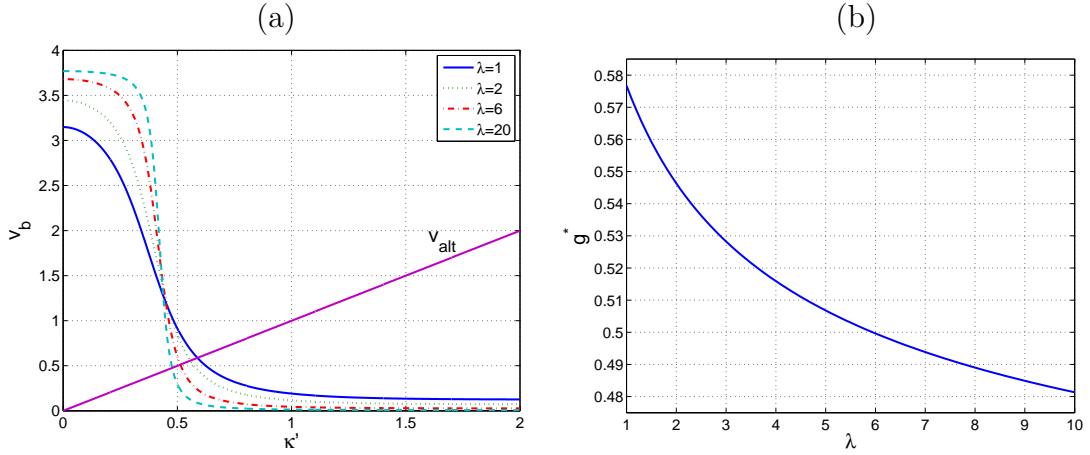


Figure 6: MARKET CRASH SETTING - Improving efficiency (higher value of λ) discourages investors from entering our market (lower equilibrium flow of investors g^*). Model parameters are set at the following values: $r = 0.01$, $S = 2$, $x = 0.8$, $z = 1$, $\gamma = 0.2$, $a = 2$, $b = 15$, $\underline{\kappa} = 0$ and $\bar{\kappa} = 5$.

and 6(b), which show that for any given level of market efficiency (fixing λ), the equilibrium flow of investors now entering our market is higher than in the baseline setting. Then, from the buyers'-to-be perspective, our market has become crowded in the sense that there are too many buyers-to-be for each investor seeking to sell and hence it is now more difficult to meet a trading partner and purchase the asset. If search frictions were then reduced in this market (higher values of λ), the effect of congestion would be amplified. Investors would meet at faster rates, which reduces the measures of buyers-to-be and sellers-to-be as shown in the top panel of Figure 7(a) but, most importantly, it would allow sellers-to-be to exit faster leading to an even more unbalanced distribution of investors (bottom panel of Figure 7(a)).

As the bottom panel of Figure 7(b) illustrates, buyers-to-be are worse-off as the search process becomes more efficient and congestion intensifies. This discourages potential investors from moving into our market, reducing the equilibrium flow of investors g^* .

The reason for this counterintuitive result is that lower trading frictions in a one-sided market magnify the effect of congestion, discouraging investors from entering this market. In this case, congestion dominates thick market externalities and hence the introduction of a measure intended to improve market efficiency results in a less attractive market.

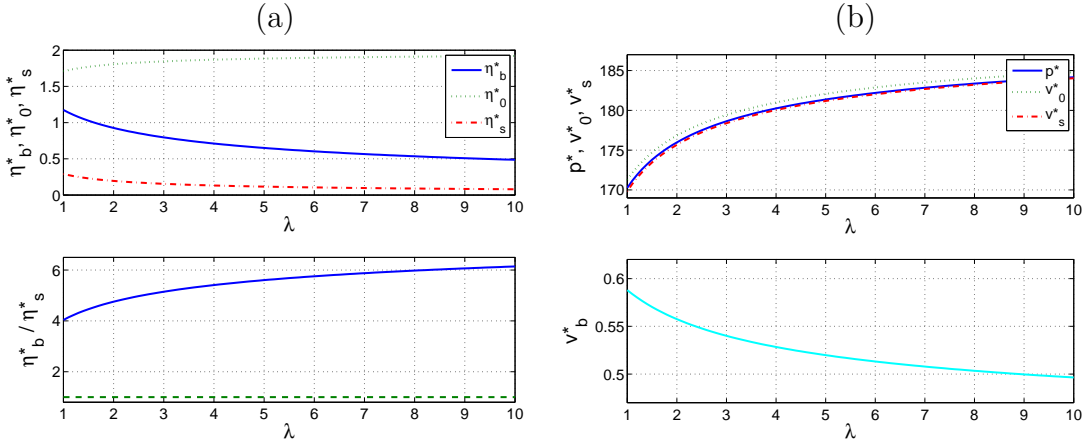


Figure 7: MARKET CRASH SETTING - Equilibrium measures of investors in our market and ratio of buyers-to-be to sellers-to-be (a), expected utilities and price (b) as a function of the market efficiency λ . Other parameters are set at the following values: $r = 0.01$, $S = 2$, $x = 0.8$, $z = 1$, $\gamma = 0.2$, $a = 2$, $b = 15$, $\underline{\kappa} = 0$ and $\bar{\kappa} = 5$.

6.3 FLOW OF INVESTORS AND MARKET EFFICIENCY

In this subsection we determine the general relationship between the equilibrium flow of investors g^* entering the market and the efficiency λ of the search process. To simplify the analysis we first derive the equilibrium measure of sellers-to-be (η_s^*) as a function of the market efficiency λ and the other nine parameters of the model (γ , r , S , x , z , a , b , $\underline{\kappa}$ and $\bar{\kappa}$). There is a one-to-one relationship between g^* and η_s^* . Hence, once we compute η_s^* , we can then determine the equilibrium flow of investors g^* who enter our market.

In our setting, market equilibrium is the solution to the system of equations (1)-(3), (7)-(10) and (15). We thus need to solve for the fixed point of this system, which is reduced to solving the indifference condition that defines investors' entry rule. Investors, in our framework, compare the expected utility v_{alt} of investing in an alternative market to the expected utility v_b derived from being a buyer-to-be in our market and they decide to move in whenever $v_b > \kappa' \equiv v_{alt}$. To present this indifference condition ($v_b = \kappa'$) as a function of the measure of sellers-to-be (η_s), let us first redefine the measure of buyers-to-be η_b as a function of η_s . Using equations (5) and (6) we find:

$$\eta_b = \frac{\gamma}{\lambda} \frac{A}{2\gamma S - A} = \frac{\gamma}{\lambda} \frac{2\gamma(S - \eta_s)}{2\gamma S - 2\gamma(S - \eta_s)} \quad \Rightarrow \quad \eta_b = \frac{\gamma}{\lambda} \frac{S - \eta_s}{\eta_s} \quad (16)$$

We can now express the expected utility v_b of buyers-to-be as a function of η_s by substituting equation (16) into equation (11):

$$v_b = \frac{x}{r + \gamma} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \quad (17)$$

Next we write κ' as a function of η_s . In this model, the flow of investors g who move into our market is determined by the proportion of the total flow of investors f whose expected utility v_b of being a buyer-to-be exceeds their best outside option κ' . We assume the flow of investors f follows a beta distribution with support $[\underline{\kappa}, \bar{\kappa}]$ and shape parameters¹⁹ a and b . For notational convenience we omit reference to the shape parameters. Then,

$$g(\kappa') = \int_{\underline{\kappa}}^{\kappa'} f_{beta}(\kappa) d\kappa = F_{beta}(\kappa') \quad \Rightarrow \quad \kappa' = F_{beta}^{-1}(g) \quad (18)$$

where f_{beta} and F_{beta} denote respectively the probability density function (*pdf*) and the cumulative distribution function (*cdf*) of a beta distribution. F_{beta}^{-1} is the inverse cumulative distribution function. Using equation (5) and the definition of A in Page 11 we can express the flow of investors g as a function of the measure of sellers-to-be η_s :

$$g = \gamma \left(1 + \frac{\gamma}{\lambda \eta_s} \right) (S - \eta_s) \quad (19)$$

Substituting equation (19) in equation (18) yields:

$$\kappa' = F_{beta}^{-1} \left(\gamma \left(1 + \frac{\gamma}{\lambda \eta_s} \right) (S - \eta_s) \right) \quad (20)$$

¹⁹The probability density function of the beta distribution defined over the interval $[0, 1]$ with shape parameters a and b is:

$$f_{beta}(y; a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} y^{a-1} (1-y)^{b-1}$$

where $a, b > 0$ and $\Gamma(\cdot)$ is the gamma function. For integer values of a and b , the cumulative distribution function of the beta distribution is given by:

$$F_{beta}(y; a, b) = \sum_{j=a}^{a+b-1} \binom{a+b-1}{j} y^j (1-y)^{a+b-1-j}$$

where $\binom{a+b-1}{j} = \frac{(a+b-1)!}{j!(a+b-1-j)!}$.

The indifference condition results from equating the expected utility v_b of buyers-to-be (equation (17)) to the marginal investor outside option κ' (equation (20)):

$$\frac{x}{r + \gamma \lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} = F_{beta}^{-1} \left(\gamma \left(1 + \frac{\gamma}{\lambda \eta_s} \right) (S - \eta_s) \right)$$

Rearranging, we get

$$\gamma \left(1 + \frac{\gamma}{\lambda \eta_s} \right) (S - \eta_s) = F_{beta} \left(\frac{x}{r + \gamma \lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \right)$$

Then,

$$\begin{aligned} \gamma \left(1 + \frac{\gamma}{\lambda \eta_s} \right) (S - \eta_s) &= \sum_{j=a}^{a+b-1} \binom{a+b-1}{j} \left(\frac{x}{r + \gamma \lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \right)^j \\ &\quad \left(1 - \frac{x}{r + \gamma \lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \right)^{a+b-1-j} \end{aligned} \quad (21)$$

Equation (21) is a polynomial of degree $2(a+b)$ in the measure of sellers-to-be²⁰. To solve for η_s^* we use the bisection method²¹. Once we compute η_s^* , we can derive g^* :

$$g^* = g^*(\lambda, \gamma, r, S, x, z, a, b, \underline{\kappa}, \bar{\kappa})$$

²⁰In the simple case of shape parameters of the beta distribution both equal to 1 ($a = 1 = b$), which corresponds to a uniform distribution with support $[\underline{\kappa}, \bar{\kappa}]$, the indifference condition ($v_b = \kappa'$) is:

$$\frac{x}{r + \gamma \lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} = \underline{\kappa} + (\bar{\kappa} - \underline{\kappa}) \gamma \left(1 + \frac{\gamma}{\lambda \eta_s} \right) (S - \eta_s)$$

Reorganising terms yields the following polynomial of degree four in the measure of sellers-to-be η_s :

$$\begin{aligned} \lambda^2 z (\bar{\kappa} - \underline{\kappa}) \gamma \eta_s^4 &+ \left[\lambda (\bar{\kappa} - \underline{\kappa}) \gamma C - \lambda z D + \frac{x}{r + \gamma} \lambda^2 z \right] \eta_s^3 + \left[\lambda (\bar{\kappa} - \underline{\kappa}) \gamma^2 S (1 - z) - CD \right] \eta_s^2 - \\ &- \left[\gamma S D + (\bar{\kappa} - \underline{\kappa}) \gamma^2 S C \right] \eta_s - (\bar{\kappa} - \underline{\kappa}) \gamma^3 S^2 = 0 \end{aligned}$$

where $C = (r + \gamma)(1 + z) - \gamma$ and $D = \lambda \underline{\kappa} + \lambda (\bar{\kappa} - \underline{\kappa}) \gamma S - (\bar{\kappa} - \underline{\kappa}) \gamma^2$. There exists closed-form solution to this equation. In particular, there are at most four solutions but only one, η_s^* , (as proved in Subsection 5.2) lies in the interval $(0, S)$, the set of possible values of the measure of sellers-to-be. Unfortunately, the solution is intractable. We use the bisection method over the interval $[0, S]$ to determine the zero of this equation.

²¹The bisection algorithm is a numerical method for finding the root of a function. It recursively divides an interval in half and selects the subinterval containing the root, until the interval is sufficiently small. Burden and Faires (1993) presents a clear description of this algorithm as well as other numerical methods for solving root-finding problems.

where g^* is a function of the efficiency of the search process λ , the rate γ at which investors receive liquidity shocks, the discount rate r , the supply of the asset S , the holding cost x , buyer's-to-be bargaining power z , the shape parameters a and b of the beta distribution and the support $[\underline{\kappa}, \bar{\kappa}]$ of the flow of investors f entering the economy. To gain some intuition for how the model parameters affect the equilibrium flow of investors g^* , we set the value of those defining the distribution of f and vary the other parameters of the model. We assume $a = 2$, $b = 15$, $\underline{\kappa} = 0$ and $\bar{\kappa} = 5$ as in the market crash setting in Subsection 6.2.1. The first set of results is depicted in Figure 8:

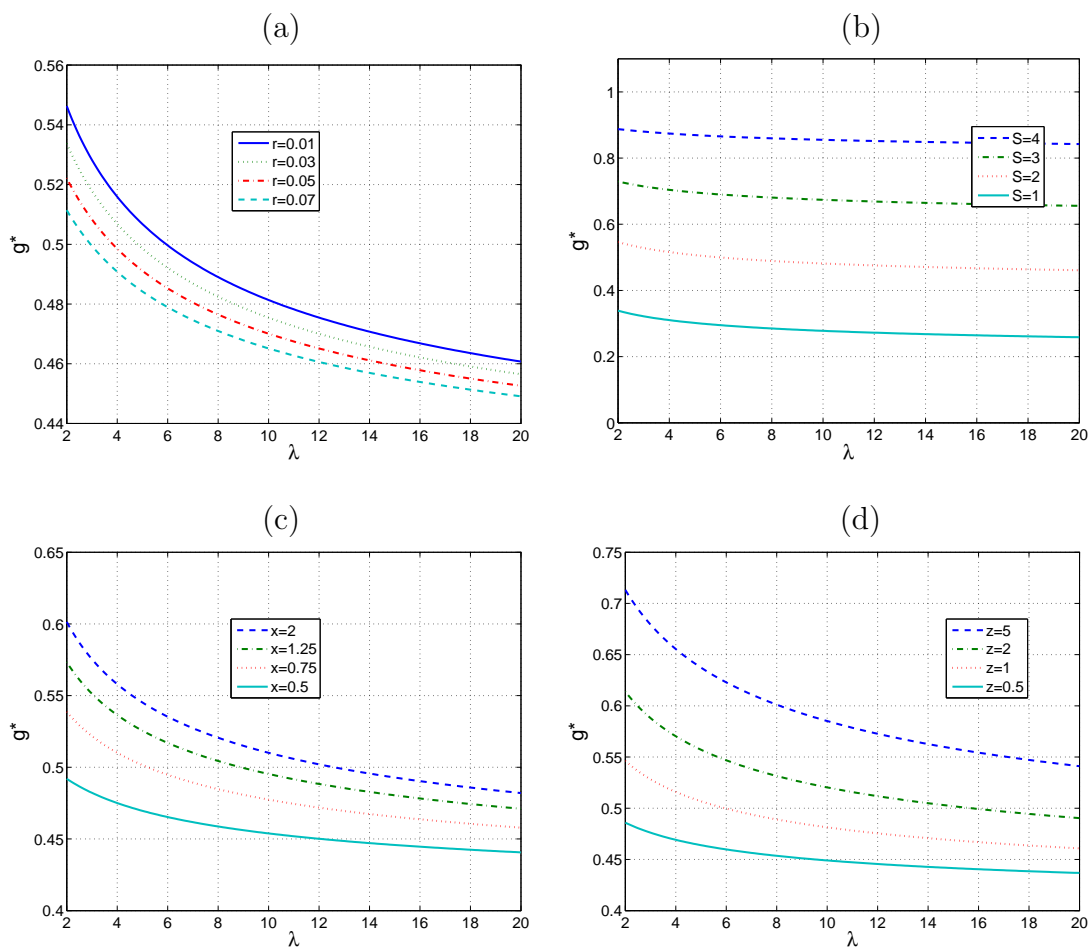


Figure 8: Equilibrium flow of investors g^* entering our market as a function of the market efficiency λ for different values of r (a), S (b), x (c) and z (d). Other parameters are set at the following values: $r = 0.01$, $S = 2$, $x = 0.8$, $z = 1$, $\gamma = 0.2$, $a = 2$, $b = 15$, $\underline{\kappa} = 0$ and $\bar{\kappa} = 5$.

Figure 8 represents g^* as a function of the efficiency of the search process λ , where g^*

is plotted for four different values of the discount rate r (a), the supply of the asset S (b), the holding cost x (c) and the buyers'-to-be bargaining power z (d). The distribution of parameters underlying these graphs corresponds to a one-sided market scenario discussed in Subsection 6.2.1. Then, in all four cases, increasing market efficiency (higher values of λ) translates into a lower equilibrium flow of investors entering the market. Also, for a given level of market efficiency (fixed λ), more investors move into our market as we increase the total supply of the asset, the holding cost or the buyers'-to-be bargaining power. The equilibrium flow of investors decreases as they become more impatient (higher r).

More interesting is the interaction between market efficiency λ and the arrival rate of liquidity shocks γ . Figure 9(a) demonstrates how the equilibrium flow of investors g^* , who enter our market, varies with the market efficiency λ and the frequency of liquidity shocks γ . Contours are depicted in Figure 9(b). Now, the relationship between g^* and λ is non-monotonic. It is first decreasing in market efficiency, corresponding to a one-sided market scenario, but then it becomes increasing in λ for higher values of the liquidity shock rate γ .

If liquidity shocks arrive at very low rates (low values of γ), investors hold the asset, on average, for a long time. As a result, there are few investors trying to sell and exit the market. Increasing the efficiency of this market (raising λ) attracts new investors, amplifying the effect of congestion. The market becomes one-sided because there are more buyers-to-be and few sellers-to-be. In this case, reducing market frictions diminishes the flow of investors. This is shown in Figure 9(c) for values of $\gamma \leq 0.3$. This phenomenon is attenuated as investors need to exit at a faster rate. Then, for intermediate values of γ , there are enough sellers-to-be in our market and improving market efficiency attracts new investors ($\gamma = 0.4$ and $\gamma = 0.5$ in Figure 9(c)). Thick market externalities dominate congestion. Also, as Figure 9(d) indicates, if investors need for cash is very frequent (values of γ above 0.5), they prefer not to invest and the flow of investors g^* who enter our market is reduced. Still, for a given frequency of the liquidity shocks γ , diminishing search frictions improves the attractiveness of our market.

6.4 LIQUIDITY AND WELFARE

In this subsection we discuss market liquidity and present the welfare analysis. In particular, we are interested in the implications of potential policies designed to improve the efficiency

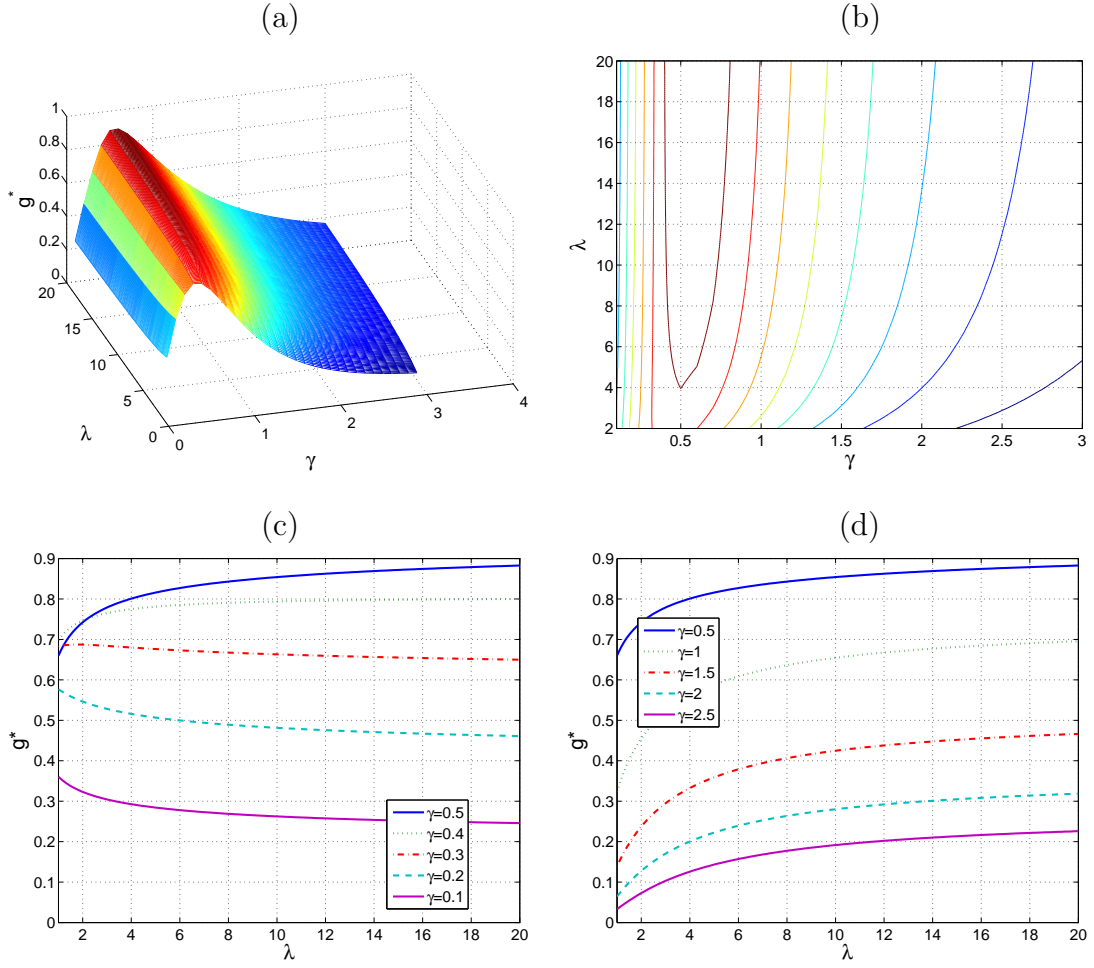


Figure 9: Equilibrium flow of investors g^* entering our market as a function of the market efficiency λ and the frequency of liquidity shocks γ . The values of other parameters of the model are set at the following: $r = 0.01$, $S = 2$, $x = 0.8$, $z = 1$, $a = 2$, $b = 15$, $\underline{\kappa} = 0$ and $\bar{\kappa} = 5$.

of the search process and to thus reduce market frictions.

We measure welfare by the weighted sum of investors' expected utilities. Weights are determined by the measure of every type of investors in our economy, including the outside investors. Then, our measure of welfare is:

$$W = \eta_b v_b + \eta_0 v_0 + \eta_s v_s + \int_{\kappa^*}^{\bar{\kappa}} \kappa f(\kappa) d\kappa \quad (22)$$

where the first three terms represent the welfare of the investors who prefer to enter our market ($W_{\text{inside investors}}$) and the last term reflects the welfare of outside investors ($W_{\text{outside investors}}$).

Outside investors (those with investment opportunities above the threshold value κ^*) enjoy the expected utility derived from investing in an alternative market v_{alt} , which for simplicity we assume equal to κ' , the outside investment opportunity. Substituting equations (11)-(13) and the *pdf* of a beta distribution into equation (22), we get:

$$W_{\text{inside investors}} = \frac{d}{r}S - kx \frac{1}{r + \gamma} \frac{\frac{\gamma}{r}S[(\gamma + \lambda\eta_s)z + (r + \gamma)] + \eta_s[(r + 2\gamma + \lambda\eta_s)z + (r + \gamma)]}{(r + \gamma + \lambda\eta_s)z + \gamma}$$

$$W_{\text{outside investors}} = \frac{a}{a + b} \left[1 - F_{\text{beta}}(\kappa^*; a + 1, b) \right] = \frac{a}{a + b} \left[1 - \sum_{j=a+1}^{a+b} \binom{a+b}{j} (\kappa^*)^j (1 - \kappa^*)^{a+b-j} \right]$$

To gain some intuition for how changes in market efficiency affect welfare we introduce the last example.

6.4.1 FIRE SALES IN OUR MARKET

Consider a search-based market similar to the baseline setting described in Subsection 6.2 and assume investors need for cash is now more frequent. Specifically, we assume liquidity shocks arrive at a Poisson rate $\gamma = 0.4$. The value of all other parameters remains as in the baseline case: $r = 0.01$, $d = 2$, $S = 2$, $x = 0.4d$, $z = 1$, $a = 1$, $b = 1$, $\underline{\kappa} = 0$ and $\bar{\kappa} = 5$.

Investors now prefer to hold the asset, on average, for a shorter period of time and they are willing to sell and exit our market at faster rates. Then, for any given value of the search efficiency, the equilibrium measure of sellers-to-be has increased significantly (top panel of Figure 10(a)) compared to the baseline setting (top panel of Figure 5(a)), while the equilibrium measure of non-searchers has decreased. Given that there are now more sellers-to-be in our market, it is easier for an investor seeking to purchase the asset to meet a trading partner. As a result, the equilibrium measure of buyers-to-be has diminished compared to the baseline case. Most importantly, the proportion of buyers-to-be to sellers-to-be has fallen drastically. This is depicted in the bottom panel of Figure 10(a). Our market is now one-sided and there is severe congestion on the sell-side of the market. This scenario could correspond to a market experiencing a fire sale.

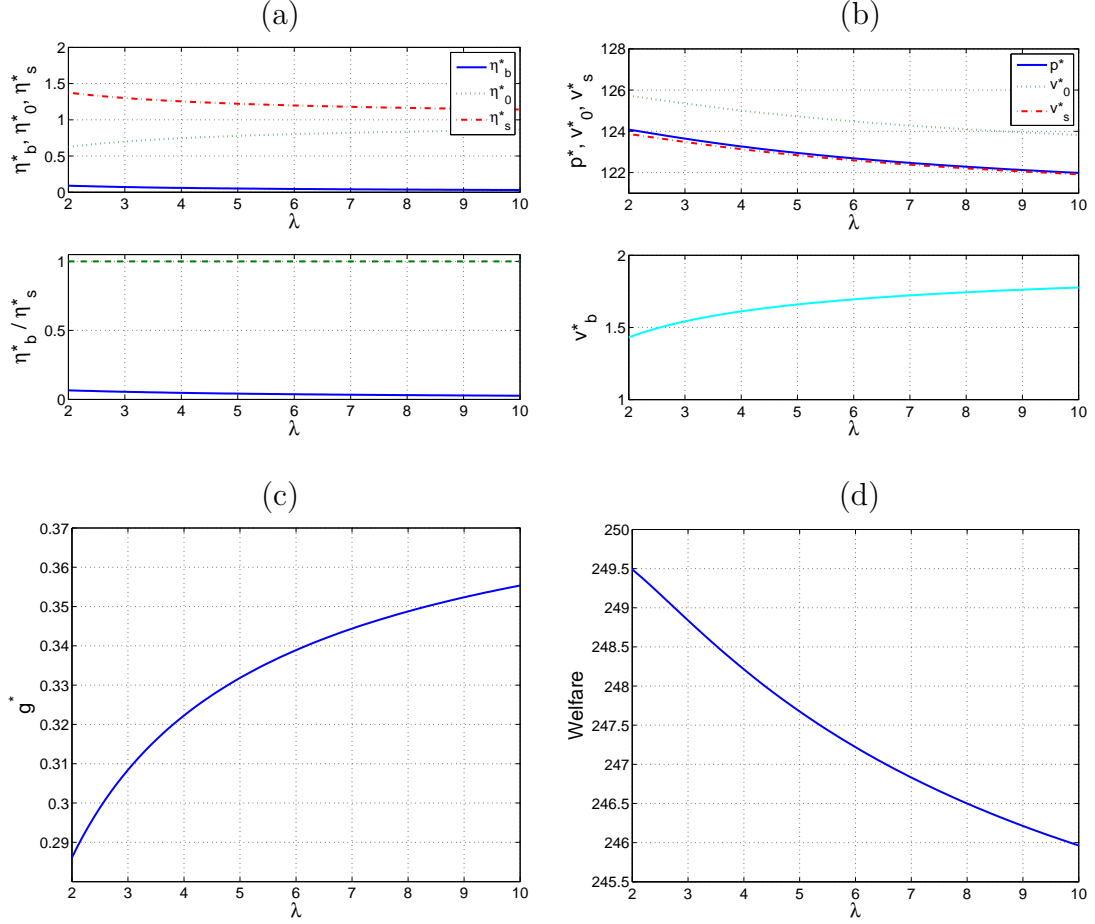


Figure 10: FIRE SALES SETTING - Equilibrium measures of investors in our market and ratio of buyers-to-be to sellers-to-be (a), expected utilities and price (b), flow of investors g^* (c) and welfare (d) as a function of the market efficiency λ . Other parameters are set at the following values: $r = 0.01$, $S = 2$, $x = 0.8$, $z = 1$, $\gamma = 0.4$, $a = 1$, $b = 1$, $\underline{\kappa} = 0$ and $\bar{\kappa} = 5$.

Increasing the efficiency of the search process (higher value of λ) in this market causes two effects. First, it raises the flow of investors who enter our market as plotted in Figure 10(c). Secondly, investors meet at faster rates reducing the equilibrium measure of buyers-to-be and sellers-to-be (top panel of Figure 10(a)). The overall effect on the ratio of buyers-to-be to sellers-to-be is presented in the bottom panel of Figure 10(a). As the market becomes more efficient, the proportion of buyers-to-be to sellers-to-be falls further and from the sellers'-to-be perspective the market gets even more crowded. Congestion intensifies as it is now more difficult to meet a buyer-to-be and exit the market. Hence, as the top panel of Figure 10(b) illustrates, sellers-to-be and non-searchers (who become sellers-to-be at rate γ) are worse-off

as efficiency rises. The expected utility of buyers-to-be increases in λ because they can now acquire the asset at faster rates (bottom panel of Figure 10(b)).

A very interesting result is presented in Figure 10(d). We find that as search frictions are reduced, welfare decreases. In this market, improving the efficiency of the search process amplifies the effect of congestion. There are then fewer buyers-to-be per each seller-to-be and the expected utilities of investors holding the asset fall. This induces an adverse effect on welfare.

Figure 11(a) represents our measure of illiquidity as a function of the efficiency of the search process λ , where illiquidity is defined as the price discount. As market efficiency increases and the population of investors gets saturated with sellers-to-be, the price of the asset falls as shown in the top panel of Figure 10(b). This leads to the rise in illiquidity depicted in Figure 11(a). Intuitively, given that there are few buyers-to-be compared to sellers-to-be, the price of the asset behaves as if buyers-to-be would hold a more favourable position in the bargaining process. The effect is equivalent to an increase in the buyers'-to-be bargaining power z , which is exogenous in our model. If we were to endogenise z , the effect on the price (and hence on market liquidity) would be amplified.

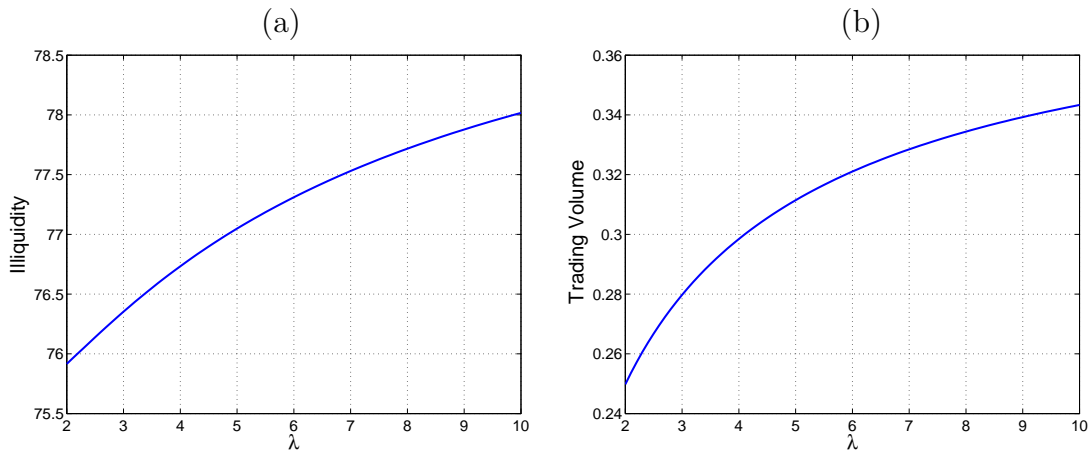


Figure 11: FIRE SALES SETTING - Illiquidity measured by price discount (a) and trading volume (b) as a function of the market efficiency λ . The value of the model parameters is set at the following: $r = 0.01$, $S = 2$, $x = 0.8$, $z = 1$, $\gamma = 0.4$, $a = 1$, $b = 1$, $\underline{\kappa} = 0$ and $\bar{\kappa} = 5$.

Our market becomes less liquid as the search frictions are reduced. However, as Figure

11(b) indicates, trading volume increases. The reason for this counterintuitive result is the following. Facilitating search in our market has two consequences. First, it magnifies the effect of congestion. There are fewer buyers-to-be relative to the measure of investors trying to exit. Buyers-to-be prefer to pay less to purchase the asset, which translates into a lower price and hence into a less liquid market (higher price discount). Second, it raises the frequency of meeting between trading partners. Investors in our market now meet at a faster rate, increasing the trading volume. Consequently, even though our market is less liquid, investors meet faster and trading volume increases.

7 CONCLUSIONS

This paper proposes a search-based model to study the relationship between market liquidity and the endogenous arrival of potential investors to a specific market. As investors enter a market, they make trade easier, attracting new investors. This gives rise to a thick market externality. Interestingly, as investors get attracted to this market, the market becomes crowded and congestion reduces the returns to investing. This paper aims to complement the literature on self-fulfilling liquidity by incorporating a second effect: the congestion effect.

In our market traders can invest in one asset which can be traded only when a pair of investors meet and bargain over the terms of trade. Finding a trading partner takes time and introduces opportunity and other costs. Investors' ability to trade thus affects the illiquidity discount and ultimately, the equilibrium price. We present a numerical example of an advance in trading technology to illustrate the link between the flow of new investors and market liquidity, and to discuss the implications of search frictions on market liquidity.

We then derive the general relationship between the equilibrium flow of investors moving into a market and the efficiency of the search process and highlight the tradeoff between the thick market complementarity and the congestion effect. The equilibrium outcome depends on which of these two effects dominates. In particular, we find that diminishing trade frictions in a market with many buyers and too few sellers leads to a lower equilibrium flow of investors into this market. Less search frictions would allow sellers to exit faster amplifying the effect of congestion (even more buyers per seller) and further discouraging investors from entering this market. We also show that reducing market frictions, in a "congested" market experiencing

a fire sale, induces an adverse effect on both market liquidity and welfare. Improving search efficiency (to facilitate coordination and enhance liquidity), magnifies the effect of congestion (less buyers per seller trying to exit) to the detriment of the overall level of market liquidity and social welfare. From this perspective, this paper presents an example of the Theory of the Second Best, where eliminating one but not all market imperfections does not necessary increase efficiency as it may amplify the effect of the remaining distortions.

Appendix

A Proofs of Propositions 1 - 4

Proof of Proposition 1

Proof. Rearranging equation (3), we get

$$h(\eta_0) \equiv \gamma\eta_0^2 - \left(g + \gamma S + \frac{\gamma^2}{\lambda}\right)\eta_0 + Sg = 0$$

where $\eta_0 \in \mathbb{R}_+$. This quadratic function takes positive values as $\eta_0 \rightarrow \infty$, is non-negative at $\eta_0 = 0$ and negative at $\eta_0 = S$. Then, by continuity, the polynomial equation has a root in the interval $[0, S)$ and another one in the interval (S, ∞) . The two solutions $\eta_0^{(1)}$ and $\eta_0^{(2)}$ are given by:

$$\begin{aligned}\eta_0^{(1)} &= \frac{1}{2\gamma} \left[\left(g + \gamma S + \frac{\gamma^2}{\lambda}\right) - \sqrt{\left(g + \gamma S + \frac{\gamma^2}{\lambda}\right)^2 - 4\gamma g S} \right] \\ \eta_0^{(2)} &= \frac{1}{2\gamma} \left[\left(g + \gamma S + \frac{\gamma^2}{\lambda}\right) + \sqrt{\left(g + \gamma S + \frac{\gamma^2}{\lambda}\right)^2 - 4\gamma g S} \right]\end{aligned}$$

where $0 \leq \eta_0^{(1)} < S < \eta_0^{(2)} < \infty$. $\eta_0^{(2)}$ is thus not a valid solution since the total supply of the asset is held either by the non-searchers or by the sellers-to-be and as a result the measure of non-searchers cannot exceed the supply of the asset. Then, there is unique solution to

equation (3) given by:

$$\eta_0 = \frac{1}{2\gamma}A \quad (\text{A.1})$$

where $A = (g + \gamma S + \frac{\gamma^2}{\lambda}) - \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}$. Plugging equation (A.1) into equations (1) and (2), we find

$$\begin{aligned} \eta_s &= S - \frac{1}{2\gamma}A \\ \eta_b &= \frac{\gamma}{\lambda} \frac{A}{2\gamma S - A} \end{aligned}$$

which proves Proposition 1. □

Proof of Proposition 2

Proof. Let us compute the partial derivatives of the measures given by the system of equations (4) - (6) with respect to g :

$$\frac{\partial \eta_0}{\partial g} = \frac{\partial \eta_0}{\partial A} \frac{\partial A}{\partial g} = \frac{1}{2\gamma} \frac{\partial A}{\partial g} \quad (\text{A.2})$$

$$\frac{\partial \eta_s}{\partial g} = \frac{\partial \eta_s}{\partial A} \frac{\partial A}{\partial g} = -\frac{1}{2\gamma} \frac{\partial A}{\partial g} \quad (\text{A.3})$$

$$\frac{\partial \eta_b}{\partial g} = \frac{\partial \eta_b}{\partial A} \frac{\partial A}{\partial g} = \frac{2\gamma^2}{\lambda} \frac{S}{(2\gamma S - A)^2} \frac{\partial A}{\partial g} \quad (\text{A.4})$$

where

$$\frac{\partial A}{\partial g} = 1 - \frac{(g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} \quad (\text{A.5})$$

To determine the sign of $\frac{\partial A}{\partial g}$, we check if the second term on the right-hand-side of equation (A.5) is greater than 1:

$$\begin{aligned} \frac{(g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} &> 1 \quad ; \\ (g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S &> \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} \quad ; \end{aligned} \quad (\text{A.6})$$

where the right-hand-side of equation (A.6) is strictly positive since

$$\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} = \sqrt{(g - \gamma S)^2 + 2(g + \gamma S)\frac{\gamma^2}{\lambda} + \frac{\gamma^4}{\lambda^2}} > 0 \quad (\text{A.7})$$

We analyse two cases. If $(g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S \leq 0$, then equation (A.6) is not satisfied. On the contrary, if $(g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S > 0$,

$$\begin{aligned} \left[(g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S \right]^2 &> \left[\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} \right]^2; \\ (g - \gamma S + \frac{\gamma^2}{\lambda})^2 &> (g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S; \end{aligned}$$

Simplifying we arrive to:

$$4\frac{\gamma^3}{\lambda}S < 0$$

a contradiction, since γ, λ and $S > 0$. Therefore, the second term in equation (A.5) is strictly lower than 1 and as a result:

$$\frac{\partial A}{\partial g} > 0 \quad (\text{A.8})$$

Thus, substituting the previous equation into equations (A.2) - (A.4) yields:

$$\begin{aligned} \frac{\partial \eta_0}{\partial g} &> 0 \\ \frac{\partial \eta_s}{\partial g} &< 0 \\ \frac{\partial \eta_b}{\partial g} &> 0 \end{aligned}$$

since γ, λ and $S > 0$. □

Proof of Proposition 3

Proof. Using equations (4) - (6) we can compute the partial derivatives of the measures of every type of investor with respect to the efficiency of the search process λ :

$$\frac{\partial \eta_0}{\partial \lambda} = \frac{\partial \eta_0}{\partial A} \frac{\partial A}{\partial \lambda} = \frac{1}{2\gamma} \frac{\partial A}{\partial \lambda} \quad (\text{A.9})$$

$$\frac{\partial \eta_s}{\partial \lambda} = \frac{\partial \eta_s}{\partial A} \frac{\partial A}{\partial \lambda} = -\frac{1}{2\gamma} \frac{\partial A}{\partial \lambda} \quad (\text{A.10})$$

$$\frac{\partial \eta_b}{\partial \lambda} = \frac{\gamma}{\lambda} \frac{1}{2\gamma S - A} \left[\frac{2\gamma S}{2\gamma S - A} \frac{\partial A}{\partial \lambda} - \frac{1}{\lambda} A \right] \quad (\text{A.11})$$

where

$$\frac{\partial A}{\partial \lambda} = -\frac{\gamma^2}{\lambda^2} \left(1 - \frac{g + \gamma S + \frac{\gamma^2}{\lambda}}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} \right) \quad (\text{A.12})$$

We verify whether the second term in the expression in parenthesis is greater than 1 to determine the sign of $\frac{\partial A}{\partial \lambda}$,

$$\begin{aligned} \frac{(g + \gamma S + \frac{\gamma^2}{\lambda})}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} &> 1; \\ \left(g + \gamma S + \frac{\gamma^2}{\lambda}\right)^2 &> \left[\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}\right]^2; \\ \left(g + \gamma S + \frac{\gamma^2}{\lambda}\right)^2 &> (g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S; \end{aligned} \quad (\text{A.13})$$

where we can square both sides of the expression because, using equation (A.7) and g, γ, S and $\lambda > 0$, the numerator and denominator are strictly positive. Rearranging equation (A.13) we get:

$$4\gamma g S > 0$$

which is true since γ, g and $S > 0$. As a result, the second term in the expression in parenthesis in equation (A.12) is strictly greater than 1 and

$$\frac{\partial A}{\partial \lambda} > 0 \quad (\text{A.14})$$

Thus, plugging the previous equation into equations (A.9) - (A.10) we find:

$$\begin{aligned}\frac{\partial \eta_0}{\partial \lambda} &> 0 \\ \frac{\partial \eta_s}{\partial \lambda} &< 0\end{aligned}$$

The proof that $\frac{\partial \eta_b}{\partial \lambda} < 0$ is not so straightforward. Let us first rearrange equation (A.12) as follows

$$\frac{\partial A}{\partial \lambda} = \frac{\gamma^2}{\lambda^2} \frac{A}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} \quad (\text{A.15})$$

Now, substituting equation (A.15) in equation (A.11) we get:

$$\frac{\partial \eta_b}{\partial \lambda} = \frac{\gamma}{\lambda^2} \frac{A}{2\gamma S - A} \left[\frac{2\gamma S}{2\gamma S - A} \frac{\gamma^2}{\lambda} \frac{1}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} - 1 \right] \quad (\text{A.16})$$

where we need to derive the sign of the expression in brackets to determine the sign of $\frac{\partial \eta_b}{\partial \lambda}$. Let us then verify if the first term of the expression in brackets in equation (A.16) is strictly lower than 1:

$$\begin{aligned}\frac{2\gamma S}{2\gamma S - A} \frac{\gamma^2}{\lambda} \frac{1}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} &< 1; \\ (2\gamma S - A)\lambda \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} - 2\gamma^3 S &> 0; \\ \lambda \left[(2\gamma S - A)\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} - \frac{2\gamma^3 S}{\lambda} \right] &> 0;\end{aligned}$$

Given that $\lambda > 0$ and $A = (g + \gamma S + \frac{\gamma^2}{\lambda}) - \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}$, then

$$\begin{aligned}\left[2\gamma S - (g + \gamma S + \frac{\gamma^2}{\lambda}) \right] \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} + \left[\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} \right]^2 - \frac{2\gamma^3 S}{\lambda} &> 0; \\ \left(g - \gamma S + \frac{\gamma^2}{\lambda} \right)^2 - \left(g - \gamma S + \frac{\gamma^2}{\lambda} \right) \sqrt{(g - \gamma S + \frac{\gamma^2}{\lambda})^2 + \frac{4\gamma^3 S}{\lambda}} + \frac{2\gamma^3 S}{\lambda} &> 0;\end{aligned}$$

To simplify the exposition of the proof, let us define $D \equiv g - \gamma S + \frac{\gamma^2}{\lambda}$. Therefore,

$$D^2 - D\sqrt{D^2 + \frac{4\gamma^3 S}{\lambda}} + \frac{2\gamma^3 S}{\lambda} > 0 \quad (\text{A.17})$$

We consider two possible scenarios. If $D \leq 0$, then equation (A.17) is satisfied since λ, γ and $S > 0$. On the contrary, if $D > 0$, then we need to prove that

$$D^2 + \frac{2\gamma^3 S}{\lambda} > D\sqrt{D^2 + \frac{4\gamma^3 S}{\lambda}}$$

Squaring both sides and rearranging, we find

$$D^4 + \frac{4\gamma^3 S}{\lambda} D^2 + \frac{4\gamma^6 S^2}{\lambda^2} > D^2 \left(D^2 + \frac{4\gamma^3 S}{\lambda} \right)$$

Simplifying,

$$\frac{4\gamma^6 S^2}{\lambda^2} > 0$$

and this is always satisfied. Then, we have shown that the first term in the expression in brackets in equation (A.16) is strictly lower than 1 and as a result

$$\frac{\partial \eta_b}{\partial \lambda} < 0$$

which completes the proof of Proposition 3. □

Proof of Proposition 4

Proof. Using equation (10), we can rewrite equations (7) and (9) as:

$$rv_b = -\gamma v_b + \lambda \eta_s \frac{z}{1+z} (v_0 - v_b - v_s) \quad (\text{A.18})$$

$$rv_s = d - x + \lambda \eta_b \frac{1}{1+z} (v_0 - v_b - v_s) \quad (\text{A.19})$$

Subtracting equation (A.18) from equation (8) yields:

$$\begin{aligned}
r(v_0 - v_b) &= d + \gamma(v_s - v_0) - \left[-\gamma v_b + \lambda\eta_s \frac{z}{1+z}(v_0 - v_b - v_s) \right] = \\
&= d + \gamma(v_s - v_0 + v_b) - \lambda\eta_s \frac{z}{1+z}(v_0 - v_b - v_s) \Rightarrow \\
\Rightarrow v_0 - v_b &= \frac{d + (\gamma + \lambda\eta_s \frac{z}{1+z})v_s}{r + \gamma + \lambda\eta_s \frac{z}{1+z}} \tag{A.20}
\end{aligned}$$

We can solve for v_s by plugging equation (A.20) into equation (A.19):

$$\begin{aligned}
rv_s &= d - x + \lambda\eta_b \frac{1}{1+z} \left[\frac{d + (\gamma + \lambda\eta_s \frac{z}{1+z})v_s}{r + \gamma + \lambda\eta_s \frac{z}{1+z}} - v_s \right] = \\
&= d - x + \lambda\eta_b \frac{d - rv_s}{(r + \gamma + \lambda\eta_s)z + r + \gamma} \Rightarrow \\
\Rightarrow \left[1 + \frac{\lambda\eta_b}{(r + \gamma + \lambda\eta_s)z + r + \gamma} \right] rv_s &= \left[1 + \frac{\lambda\eta_b}{(r + \gamma + \lambda\eta_s)z + r + \gamma} \right] d - x \Rightarrow \\
\Rightarrow v_s &= \frac{d}{r} - k \frac{x}{r} - k \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \tag{A.21}
\end{aligned}$$

where

$$k \equiv \frac{(r + \gamma + \lambda\eta_s)z + \gamma}{(r + \gamma + \lambda\eta_s)z + (r + \gamma + \lambda\eta_b)}$$

Given v_s , we can determine v_0 , v_b and p uniquely from equations (8), (A.18) and (10) respectively. Let us compute them. We can solve for v_0 by plugging equation (A.21) into equation (8):

$$\begin{aligned}
rv_0 &= d + \gamma \left[\frac{d}{r} - k \frac{x}{r} - k \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \right] - \gamma v_0 \Rightarrow \\
\Rightarrow v_0 &= \frac{d}{r} - k \frac{x}{r} \frac{\gamma}{r + \gamma} - k \frac{\gamma}{r + \gamma} \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \tag{A.22}
\end{aligned}$$

We now compute v_b by substituting equations (A.21) and (A.22) into equation (A.18):

$$\begin{aligned}
rv_b &= -\gamma v_b + \lambda \eta_s \frac{z}{1+z} \left[\frac{d}{r} - k \frac{x}{r} \frac{\gamma}{r+\gamma} - k \frac{\gamma}{r+\gamma} \frac{x}{(r+\gamma+\lambda \eta_s)z+\gamma} - \right. \\
&\quad \left. - v_b - \frac{d}{r} + k \frac{x}{r} + k \frac{x}{(r+\gamma+\lambda \eta_s)z+\gamma} \right] \Rightarrow \\
&\Rightarrow \left[r + \gamma + \lambda \eta_s \frac{z}{1+z} \right] v_b = \lambda \eta_s \frac{z}{1+z} \left[k \frac{x}{r+\gamma} + k \frac{x}{(r+\gamma+\lambda \eta_s)z+\gamma} \frac{r}{r+\gamma} \right] \Rightarrow \\
\Rightarrow v_b &= k \frac{x}{r+\gamma} \frac{\lambda \eta_s z}{(r+\gamma+\lambda \eta_s)z+\gamma} \tag{A.23}
\end{aligned}$$

We now solve for the price. Plugging equations (A.21) - (A.23) into equation (10) we get:

$$\begin{aligned}
p &= \frac{1}{1+z} \left[\left(\frac{d}{r} - k \frac{x}{r} - k \frac{x}{(r+\gamma+\lambda \eta_s)z+\gamma} \right) z + \right. \\
&\quad \left. + \frac{d}{r} - k \frac{x}{r} \frac{\gamma}{r+\gamma} - k \frac{\gamma}{r+\gamma} \frac{x}{(r+\gamma+\lambda \eta_s)z+\gamma} - k \frac{x}{r+\gamma} \frac{\lambda \eta_s z}{(r+\gamma+\lambda \eta_s)z+\gamma} \right] = \\
&= \frac{d}{r} - \frac{1}{1+z} \left[k \frac{x}{r} \left(\frac{\gamma}{r+\gamma} + z \right) + k \frac{x}{r+\gamma} \right] \Rightarrow \\
\Rightarrow p &= \frac{d}{r} - k \frac{x}{r} \tag{A.24}
\end{aligned}$$

This concludes the proof of Proposition 4. □

B Additional Proofs

B.1 Proof of $\frac{\partial p}{\partial d}, \frac{\partial p}{\partial \eta_b} > 0$ and $\frac{\partial p}{\partial z}, \frac{\partial p}{\partial \eta_s} < 0$

Proof. Using equation (14), the partial derivative of the price with respect to the dividend flow d is

$$\frac{\partial p}{\partial d} = \frac{1}{r} > 0 \quad \Rightarrow \quad \frac{\partial p}{\partial d} > 0 \quad \forall d$$

Let us now compute the partial derivative of the price with respect to the measure of buyers-to-be η_b :

$$\frac{\partial p}{\partial \eta_b} = \frac{\partial p}{\partial k} \frac{\partial k}{\partial \eta_b} = -\frac{x}{r} \frac{\partial k}{\partial \eta_b}$$

where:

$$\frac{\partial k}{\partial \eta_b} = -\lambda \frac{(r + \gamma + \lambda \eta_s)z + \gamma}{[(r + \gamma + \lambda \eta_s)z + (r + \gamma + \lambda \eta_b)]^2}$$

which is strictly lower than zero since $r, \gamma, \lambda, \eta_s, z > 0$. Therefore,

$$\frac{\partial p}{\partial \eta_b} > 0 \quad \forall \eta_b$$

Next, we obtain the partial derivative of the price with respect to the buyer's-to-be bargaining power z :

$$\frac{\partial p}{\partial z} = \frac{\partial p}{\partial k} \frac{\partial k}{\partial z} = -\frac{x}{r} \frac{\partial k}{\partial z}$$

where:

$$\frac{\partial k}{\partial z} = \frac{r \lambda \eta_s \eta_b}{[(r + \gamma + \lambda \eta_s)z + (r + \gamma + \lambda \eta_b)]^2}$$

which is strictly greater than zero since $r, \lambda, \eta_s, \eta_b > 0$. Then,

$$\frac{\partial p}{\partial z} < 0 \quad \forall z$$

To complete the proof, we calculate the partial derivative of the asset price with respect to the measure of sellers-to-be:

$$\frac{\partial p}{\partial \eta_s} = \frac{\partial p}{\partial k} \frac{\partial k}{\partial \eta_s} = -\frac{x}{r} \frac{\partial k}{\partial \eta_s}$$

where:

$$\frac{\partial k}{\partial \eta_s} = \frac{\lambda z (r + \lambda \eta_b)}{[(r + \gamma + \lambda \eta_s)z + (r + \gamma + \lambda \eta_b)]^2}$$

which is strictly greater than zero since $r, \lambda, \eta_b, z > 0$. Thus,

$$\frac{\partial p}{\partial \eta_s} < 0 \quad \forall \eta_s$$

□

B.2 Proof of $\frac{\partial p}{\partial g} > 0$

Proof. Using equation (14), the partial derivative of the asset price with respect to the flow of investors g entering the market is

$$\frac{\partial p}{\partial g} = -\frac{x}{r} \frac{\partial k}{\partial g}$$

where $k = \frac{(r+\gamma+\lambda\eta_s)z+\gamma}{(r+\gamma+\lambda\eta_s)z+(r+\gamma+\lambda\eta_b)}$. Let us derive the partial derivative of k with respect to the flow of investors g :

$$\frac{\partial k}{\partial g} = \frac{1}{[(r+\gamma+\lambda\eta_s)z+(r+\gamma+\lambda\eta_b)]^2} \left\{ (r+\lambda\eta_b)\lambda z \frac{\partial \eta_s}{\partial g} - [(r+\gamma+\lambda\eta_s)z+\gamma] \lambda \frac{\partial \eta_b}{\partial g} \right\}$$

which is strictly lower than zero since $r, \gamma, \lambda, \eta_s, \eta_b, z > 0$ and, as shown in Proposition 2, $\frac{\partial \eta_s}{\partial g} < 0$ and $\frac{\partial \eta_b}{\partial g} > 0$. Then,

$$\frac{\partial p}{\partial g} = -\frac{x}{r} \frac{\partial k}{\partial g} > 0$$

which proves the price increases in the flow of investors entering the market. \square

C Proof of Theorem 1

Proof. In our framework, the marginal investor decides whether to enter or not after comparing the expected utility v_{alt} of investing in an alternative market to the expected utility v_b of a buyer-to-be in our market. The expected utility of the marginal investor $v_{alt} = \kappa'$ is a non-negative and strictly increasing function of his outside investment opportunity κ' . Also, $v_b(\kappa' = 0) > v_{alt}(\kappa' = 0) = 0$. Hence, if v_b were decreasing in the outside investment opportunity of the marginal investor, κ' , then there would be a unique threshold κ^* satisfying the indifference condition $v_b(g(\kappa^*)) = v_{alt}(\kappa^*)$. Let us show this is the case.

The expected utility v_b of a buyer-to-be is a function of the flow of investors g entering the market. Let us compute the partial derivative of v_b , defined in equation (11), with respect to g :

$$\frac{\partial v_b}{\partial g} = \frac{\lambda z x}{r + \gamma} \frac{1}{[(1+z)(r+\gamma) + \lambda(z\eta_s + \eta_b)]^2} \left\{ [(1+z)(r+\gamma) + \lambda\eta_b] \frac{\partial \eta_s}{\partial g} - \lambda\eta_s \frac{\partial \eta_b}{\partial g} \right\}$$

which is strictly negative since $r, \gamma, x, z, \lambda, \eta_b, \eta_s > 0$ and, as shown in Proposition 2, $\frac{\partial \eta_s}{\partial g} < 0$ and $\frac{\partial \eta_b}{\partial g} > 0$. Hence, the expected utility v_b of a buyer-to-be strictly decreases in the flow of investors g entering the market. However g , as given by $g(\kappa') = \int_{\underline{\kappa}}^{\bar{\kappa}} \nu(\kappa) f(\kappa) d\kappa = \int_{\underline{\kappa}}^{\kappa'} f(\kappa) d\kappa$, is increasing in κ' . As a result,

$$\frac{\partial v_b}{\partial \kappa'} = \frac{\partial v_b}{\partial g} \frac{\partial g}{\partial \kappa'} \leq 0$$

where $\frac{\partial v_b}{\partial g} < 0$ and $\frac{\partial g}{\partial \kappa'} \geq 0$.

Then, by continuity, there exists a unique value of κ' satisfying the indifference condition: $v_b(g(\kappa^*)) = v_{alt}(\kappa^*)$. A unique threshold κ^* thus defines a unique flow of investors $g^* = g(\kappa^*)$ entering the market. But given a flow of investors entering the market, there exists unique equilibrium measures $(\eta_b^*, \eta_0^*, \eta_s^*)$ of each type of investor, expected utilities (v_b^*, v_0^*, v_s^*) and price of the asset, p^* , as proved in Propositions 1 and 4. Consequently, market equilibrium, as presented in Definition 1, is unique. This proves Theorem 1. \square

D Proof of Proposition 6

Proof. The illiquidity discount is defined as:

$$k \frac{x}{r}$$

where $\frac{x}{r}$ is the present value of the holding cost x , $k = \frac{(r+\gamma+\lambda\eta_s)z+\gamma}{(r+\gamma+\lambda\eta_s)z+(r+\gamma+\lambda\eta_b)}$ and η_s and η_b , as given by equations (5) and (6), are functions of the flow of investors g . Let us compute the partial derivative with respect to the flow of investors g entering the market:

$$\frac{\partial}{\partial g} \left(k \frac{x}{r} \right) = \frac{x}{r} \frac{\partial k}{\partial g} < 0$$

since $\frac{\partial k}{\partial g} < 0$ (as shown in subsection B.2) and $x, r > 0$. Hence, illiquidity decreases in g or equivalently, market liquidity increases in the flow of investors entering our market. \square

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