

# Mispricing of S&P 500 Index Options

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## Abstract

We document widespread violations of stochastic dominance in the one-month S&P 500 index options market over the period 1986-2002. These violations imply that a trader can improve her expected utility by engaging in a zero-net-cost trade. We allow the market to be incomplete and also imperfect by introducing transaction costs and bid-ask spreads. There is higher incidence of violations by OTM than by ITM calls, contradicting the common inference drawn from the observed implied volatility smile that the problem lies with the left-hand tail of the index return distribution. Even though pre-crash option prices conform to the Black and Scholes (1973) and Merton (1973) model reasonably well, they are incorrectly priced. Over 1997-2002, many options, particularly OTM calls, are overpriced irrespective of which time period is used to determine the index return distribution. These results do not support the hypothesis that the options market is becoming more rational over time. Finally, our results dispel another common misconception, that the observed smile is too steep after the crash: most of the violations by post-crash options are due to the options being either underpriced over 1988-1995, or overpriced over 1997-2002.

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# 1 Introduction

A robust prediction of the celebrated Black and Scholes (1973) and Merton (1973) (BSM) option pricing model is that the volatility implied by market prices of options is constant across strike prices. Rubinstein (1994) tested this prediction on the S&P 500 index options (SPX), traded on the Chicago Board Options Exchange, an exchange that comes close to the dynamically complete and perfect market assumptions underlying the BSM model. From the start of the exchange-based trading in April 1986 until the October 1987 stock market crash, the implied volatility is a moderately downward-sloping or u-shaped function of the strike price, a pattern referred to as the “volatility smile”, also observed in international markets and to a lesser extent in the prices of individual-stock options. Following the crash, the volatility smile is typically more pronounced and downward sloping, often called a “volatility skew”.

An equivalent statement of the above prediction of the BSM model, that the volatility implied by market prices of options is constant across strike prices, is that the *risk-neutral* stock price distribution is lognormal. Jackwerth and Rubinstein (1996), Ait-Sahalia and Lo (1998), and Jackwerth (2000) estimated the risk-neutral stock price distribution from the cross section of option prices.<sup>1</sup> Jackwerth and Rubinstein (1996) confirmed that, prior to the October 1987 crash, the risk-neutral stock price distribution is close to lognormal, consistent with a moderate implied volatility smile. Thereafter, the distribution is systematically skewed to the left, consistent with a more pronounced skew.

These findings raise several important questions. Does the BSM model work well prior to the crash? If it does, is it because the risk-neutral probability of a stock market crash was low and consistent with a lognormal distribution? Or, is it because the risk-neutral probability of a stock market crash was erroneously perceived to be low by the market participants? Why does the BSM model

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<sup>1</sup> Jackwerth (2004) reviews the parametric and non-parametric methods for estimating the risk-neutral distribution.

typically fail after the crash? Is it because the risk-neutral probability of a stock market crash increased after the crash and became inconsistent with a lognormal distribution? Or, is it because the risk-neutral probability of a stock market crash was erroneously perceived to do so? Is the options market rational before and after the crash?

In this paper, we address the shortcomings of the BSM model in the context of an *equilibrium model* that links option prices to preferences.<sup>2</sup> In *no-arbitrage* models of option prices, the risk premia are *exogenous*. By contrast, in equilibrium models, the risk premia are *endogenous*.

Whereas downward sloping implied volatility is inconsistent with the BSM model, it is well understood that this pattern is not inconsistent with economic theory in general. Two fundamental assumptions of the BSM model are that the market is *dynamically complete* and *frictionless*. We empirically investigate whether the observed cross sections of one-month S&P 500 index option prices over 1986-2002 are consistent with various economic models that explicitly allow for a dynamically incomplete market and also an imperfect market that recognizes trading costs and bid-ask spreads. To our knowledge, this is the first large empirical study with transaction costs to address this issue.

Absence of arbitrage in a frictionless market implies the existence of a risk-neutral probability measure, not necessarily unique, such that the price of any asset equals the expectation of its payoff under the risk-neutral measure, discounted at the risk free rate. If a risk-neutral measure exists, the ratio of the risk-neutral probability density and the real probability density, discounted at the risk free rate, is referred to as the *pricing kernel* or *stochastic discount factor*. Thus, absence of arbitrage implies the existence of a strictly positive pricing kernel.

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<sup>2</sup> In another strand of the literature, the shortcomings of the BSM model are addressed in the context of *no-arbitrage* models that generalize the stock price process by including stock price jumps and stochastic volatility and also generalize the processes for the risk premia. Many of these models are critically discussed in Whaley (2003), Brown and Jackwerth (2004), Jackwerth (2004), Shefrin (2005), and Singleton (2006). By now, they also appear in standard textbooks such as McDonald (2005) and Hull (2006).

Economic theory imposes restrictions on equilibrium models beyond merely ruling out arbitrage. In a frictionless representative-agent economy, the pricing kernel equals the representative agent's intertemporal marginal rate of substitution over each trading period. If the representative agent has *state independent* (derived) utility of wealth, then the concavity of the utility function implies that the pricing kernel is a decreasing function of wealth.

The monotonicity restriction on the pricing kernel does not critically depend on the existence of a representative agent. If there does not exist at least one pricing kernel that is a decreasing function of wealth over each trading period, then there does not exist even *one* economic agent with state independent (derived) utility of wealth that is a marginal investor in the market. Therefore, *any* economic agent can increase her expected utility by trading in these assets. Hereafter, we employ the term *stochastic dominance violation* to connote the nonexistence of *even one* economic agent with increasing and concave utility that is a marginal investor in the market.<sup>3</sup> This means that the return of any agent's current portfolio is stochastically dominated (in the second degree) by the return of another feasible portfolio.

Under the two maintained hypotheses that the marginal investor's (derived) utility of wealth is state independent and wealth is monotone increasing in the market index level, the pricing kernel is a decreasing function of the market index level. Ait-Sahalia and Lo (2000), Jackwerth (2000), and Rosenberg and Engle (2002) estimated the pricing kernel implied by the observed cross section of prices of S&P 500 index options as a function of wealth, where wealth is proxied by the S&P 500 index level. Jackwerth (2000) reported that the pricing kernel is everywhere decreasing during the pre-crash period 1986-1987 but widespread violations occur over the post-crash period 1987-1995. Ait-Sahalia and Lo (2000) reported violations in 1993 and Rosenberg and Engle (2002) reported violations over the period 1991-1995.<sup>4</sup>

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<sup>3</sup> This line of research was initiated by Perrakis and Ryan (1984), Levy (1985), and Ritchken (1985). For more recent related contributions, see Perrakis (1986, 1993), Ritchken and Kuo (1988), and Ryan (2000, 2003).

<sup>4</sup> Rosenberg and Engle (2002) found violations when they used an orthogonal polynomial pricing

Several extant models addressed the inconsistencies with the BSM model and the violations of monotonicity of the pricing kernel. While not all of these models explicitly addressed the monotonicity of the pricing kernel, they did address the problem of reconciling the option prices with the historical index record. Essentially, these models introduced additional state variables and/or explored alternative specifications of preferences.<sup>5</sup> These models are suggestive but stop short of demonstrating that an estimated model with additional state variables and/or alternative specification of preferences explains the stochastic dominance violations on a month-by-month basis in the cross section of S&P 500 options. In this paper, we investigate stochastic dominance violations on a month-by-month basis in the cross section of S&P 500 option taking into account transaction costs, albeit without additional state variables or alternative specifications of preferences.

In estimating the statistical distribution of the S&P 500 index returns, we refrain from adopting the BSM assumption that the index price is a Brownian motion and, therefore, that the arithmetic returns on the S&P 500 index are

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kernel but not when they used a power pricing kernel which, by construction, is decreasing in wealth.

<sup>5</sup> These models are critically discussed in Singleton (2006). Bates (2001) introduced heterogeneous agents with utility functions that explicitly depend on the number of stock market crashes, over and above their dependence on the agent's terminal wealth. The calibrated economy exhibits the inconsistencies with the BSM model but fails to generate the non-monotonicity of the pricing kernel. Brown and Jackwerth (2004) suggested that the reported violations of the monotonicity of the pricing kernel may be an artifact of the maintained hypothesis that the pricing kernel is state independent but concluded that volatility cannot be the sole omitted state variable in the pricing kernel. Garcia, Luger and Renault (2003), and Santa-Clara and Yan (2004), among others, obtained plausible parameter estimates in models in which the pricing kernel is state dependent, using panel data on S&P 500 options. Others calibrated equilibrium models that generate a volatility smile pattern observed in option prices. Liu, Pan and Wang (2005) investigated rare-event premia driven by uncertainty aversion in the context of a calibrated equilibrium model and demonstrated that the model generates a volatility smile pattern observed in option prices. Benzoni, Collin-Dufresne, and Goldstein (2005) extended the above approach to show that uncertainty aversion is not a necessary ingredient of the model. More significantly, they demonstrated that the model can generate the stark regime shift that occurred at the time of the 1987 crash.

Alternative explanations include *buying pressure*, suggested by Bollen and Whaley (2004), and behavioral explanations based on *sentiment*, suggested by Han (2005) and Shefrin (2005).

lognormal. We do not impose a parametric form on the distribution of the index returns but proceed in four different ways. In the first approach, we estimate the unconditional distribution as the histograms extracted from two different *historical* index data samples covering the periods 1928-1986 and 1972-1986. In the second approach, we estimate the unconditional distribution as the histograms extracted from two different *forward-looking* samples, one that includes the October 1987 crash (1987-2002) and one that excludes it (1988-2002). In the third approach, we model the variance of the index returns as a GARCH (1, 1) process and estimate the *conditional* variance over the period 1972-2002 by the semiparametric method of Engle and Gonzalez-Rivera (1991) that does not impose the restriction that conditional returns are normally distributed. Finally, in the fourth approach, every month we estimate the variance as the squared Black-Scholes implied volatility of the closest ATM option and scale the unconditional distribution for that month to have the above variance.

We then test the compliance of option prices to the predictions of models that allow for market incompleteness, market imperfections and intermediate trading over the life of the options. Evidence of stochastic dominance violations means that any trader can increase her expected utility by engaging in a zero-net-cost trade. We consider a market with heterogeneous agents and investigate the restrictions on option prices imposed by a particular class of utility-maximizing traders that we simply refer to as traders. We do not make the restrictive assumption that all economic agents belong to the class of utility-maximizing traders. Thus, our results are robust and unaffected by the presence of agents with beliefs, endowments, preferences, trading restrictions, and transactions costs schedules that differ from those of the utility-maximizing traders modeled in this paper.

Our tests accommodate three implications associated with state dependence. First, each month we search for a pricing kernel to price the cross section of one-month options without imposing restrictions on the time series properties of the pricing kernel month by month. Thus we allow the pricing kernel to be state dependent. Second, in the second part of our investigation, we allow for intermediate trading; a trader's wealth on the expiration date of the options is

generally a function not only of the price of the market index on that date but also of the entire path of the index level, thereby rendering the pricing kernel state dependent. Third, we allow the variance of the index return to be state dependent and employ the forecasted conditional variance. Perrakis and Czerwonko (2006) extended the results in this paper to American options on S&P 500 index *futures*. They demonstrated corresponding violations and implemented trading strategies that exploit the violations.

The paper is organized as follows. In Section 2, we present a model for pricing options and state restrictions on the prices of options imposed by the absence of stochastic dominance violations. One form of these restrictions is a set of linear inequalities on the pricing kernel that can be tested by testing the feasibility of a linear program. The second form of these restrictions is an upper and lower bound on the prices of options. In Section 3, we test the compliance of bid and ask one-month index options to these restrictions and discuss the results. In the concluding Section 4, we summarize the empirical findings and suggest directions for future research.

## 2 Restrictions on Option Prices Imposed by Stochastic Dominance

### 2.1 The Market

We consider a market with heterogeneous agents and investigate the restrictions on option prices imposed by a particular class of utility-maximizing traders that we simply refer to as *traders*. We do not make the restrictive assumption that all agents belong to the class of the utility-maximizing traders. Thus our results are unaffected by the presence of agents with beliefs, endowments, preferences, trading

restrictions, and transactions cost schedules that differ from those of the utility-maximizing traders.

Trading occurs at a finite number of trading dates. The utility-maximizing traders are allowed to hold only two primary securities in the market, a bond and a stock. The stock has the natural interpretation as the market index. The bond is risk free and pays constant interest each period. The traders may buy and sell the bond without incurring transactions costs. We assume that the rate of return on the stock is identically and independently distributed over time.

Stock trades incur proportional transactions costs charged to the bond account. There is no presumption that all agents in the economy face the same schedule of transactions costs as the traders do. At each date, a trader chooses the investment in the bond and stock accounts to maximize the expected utility of net worth at the terminal date. We make the plausible assumption that the utility function is increasing and concave. Note that even this weak assumption of monotonicity and concavity of preferences is not imposed on all agents in the economy but only on the subset of agents that we refer to as traders.

In Appendix A, we formulate this problem as a dynamic program. As shown in Constantinides (1979), the value function is monotone increasing and concave in the dollar values in the bond and stock accounts, properties that it inherits from the monotonicity and concavity of the utility function. This implies that, at any date, the *marginal utility of wealth out of the bond account* is strictly positive and decreasing in the dollar value in the bond account; and the *marginal utility of wealth out of the stock account* is strictly positive and decreasing in the dollar value in the stock account. We search for marginal utilities with the above properties that support the prices of the bond, stock, and derivatives at a given point in time.

If we fail to find such a set marginal utilities, then any trader with increasing and concave utility can increase her expected utility by trading in the options, the index, and the risk free rate—hence equilibrium does not exist. These strategies are termed *stochastically dominant* for the purposes of this paper, insofar as they would be adopted by all traders with utility possessing the required properties, in the same

way that all risk averse investors would choose a dominant portfolio over a dominated one in conventional second degree stochastic dominance comparisons.

We emphasize that the restriction on option prices imposed by the criterion of the absence of stochastic dominance is motivated by the economically plausible assumption that there exists at least *one* agent in the economy with the properties that we assign to a trader. This is a substantially weaker assumption than requiring that *all* agents to have the properties that we assign to traders. Stochastic dominance then implies that at least one agent, *but not necessarily all agents*, increases her expected utility by trading.<sup>6</sup> In our empirical investigation, we report the percentage of months for which the problem is feasible. These are months for which stochastic dominance violations are ruled out.

## 2.2 Restrictions in the Single-Period Model

The single-period model does not rule out trading over the trader's horizon after the options expire; it just rules out trading over the one-month life of the options. In Section 2.3, we consider the more realistic case in which traders are allowed to trade the bond and stock at one intermediate date over the life of the options.

The stock market index has price  $S_0$  at the beginning of the period; *ex dividend* price  $S_{1i}$  with probability  $\pi_i$  in state  $i$ ,  $i = 1, \dots, I$  at the end of the period; and *cum dividend* price  $(1 + \delta)S_{1i}$  at the end of the period. We order the states such that  $S_{1i}$  is increasing in  $i$ .

We define  $M^B(0)$  as the marginal utility of wealth out of the bond account at the beginning of the period;  $M^S(0)$  as the marginal utility of wealth out of the stock account at the beginning of the period;  $M_i^B(1)$  as the marginal utility of wealth out of the bond account at the end of the period; and  $M_i^S(1)$  as the

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<sup>6</sup> We also emphasize that the restriction of the absence of stochastic dominance is weaker than the restriction that the capital asset pricing model (CAPM) holds. The CAPM requires that the pricing kernel be linearly decreasing in the index price. The absence of stochastic dominance merely imposes that the pricing kernel be monotone decreasing in the index price.

marginal utility of wealth out of the stock account at the end of the period.<sup>7</sup> The marginal utility of wealth out of the bond and stock accounts at the beginning of the period is strictly positive:

$$M^B(0) > 0 \tag{2.1}$$

and

$$M^S(0) > 0. \tag{2.2}$$

The marginal utility of wealth out of the bond account at the end of the period is strictly positive:<sup>8</sup>

$$M_i^B(1) > 0, \quad i = 1, \dots, I. \tag{2.3}$$

Historically, the sample mean of the premium of the market return over the risk free rate is positive. Under the assumption of positive expected premium, the trader is long in the stock. Since the assumption in the single-period model is that there is no trading between the bond and stock accounts over the life of the option, the trader's dollar value in the stock account at the end of the period is increasing in the stock return. Note that this conclusion critically depends on the assumption that there is no intermediate trading in the bond and stock. Since we employed the convention that the stock return is increasing in the state  $i$ , the dollar value in the stock account at the end of the period is increasing in the state  $i$ . Then the condition that the marginal utility of wealth out of the stock account at the end of the period is strictly positive and decreasing in the dollar value in the stock account is stated as follows:

$$M_1^S(1) \geq M_2^S(1) \geq \dots M_I^S(1) > 0. \tag{2.4}$$

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<sup>7</sup> The marginal utilities are formally defined in Appendix B.

<sup>8</sup> Since the value of the bond account at the end of the period is independent of the state  $i$ , we cannot impose the condition that the marginal utility of wealth out of the bond account is decreasing in the dollar value of the bond account.

On each date, the trader may transfer funds between the bond and stock accounts and incur transactions costs. Therefore, the marginal rate of substitution between the bond and stock accounts differs from unity by, at most, the transactions cost rate:

$$(1 - k) M^B(0) \leq M^S(0) \leq (1 + k) M^B(0) \quad (2.5)$$

and

$$(1 - k) M_i^B(1) \leq M_i^S(1) \leq (1 + k) M_i^B(1), \quad i = 1, \dots, I. \quad (2.6)$$

Marginal analysis on the bond holdings leads to the following condition on the marginal rate of substitution between the bond holdings at beginning and end of the period:

$$M^B(0) = R \sum_{i=1}^I \pi_i M_i^B(1). \quad (2.7)$$

Marginal analysis on the stock holdings leads to the following condition on the marginal rate of substitution between the stock holdings at the beginning of the period and the bond and stock holdings at the end of the period:

$$M^S(0) = \sum_{i=1}^I \pi_i \left[ \frac{S_{1i}}{S_0} M_i^S(1) + \frac{\delta S_{1i}}{S_0} M_i^B(1) \right]. \quad (2.8)$$

We consider  $J$  European call and put options on the index, with random *cash* payoff  $X_{ij}$  at the end of the period in state  $i$ . At the beginning of the period, the trader can buy the  $j^{\text{th}}$  derivative at price  $P_j + k_j$  and sell it at price  $P_j - k_j$ , net of transactions costs. Thus  $2k_j$  is the bid-ask spread plus the round-trip transactions cost that the trader incurs in trading the  $j^{\text{th}}$  derivative. Note that there is no presumption that all agents in the economy face the same bid-ask spreads and transactions costs as the traders do.

We assume that the traders are marginal in all the  $J$  derivatives. Furthermore, we assume that a trader has sufficiently small positions in the

derivatives relative to her holdings in the bond and stock that the monotonicity and concavity conditions on the value function remain valid. Marginal analysis leads to the following restrictions on the prices of options:

$$(P_j - k_j)M^B(0) \leq \sum_{i=1}^I \pi_i M_i^B(1) X_{ij} \leq (P_j + k_j)M^B(0), \quad j = 1, \dots, J. \quad (2.9)$$

Conditions (2.1)-(2.9) define a linear program. In our empirical analysis, each month we check for feasibility of conditions (2.1)-(2.9) by using the linear programming features of the optimization toolbox of MATLAB 7.0. We report the percentage of months in which the linear program is feasible and, therefore, stochastic dominance is ruled out.

A useful way to identify the options that cause infeasibility or near-infeasibility of the problem is to single out a “test” option, say the  $n^{\text{th}}$  option, and solve the problem

$$\max \left( or, \min \right) \sum_{i=1}^I \pi_i \frac{M_i^B(1)}{M^B(0)} X_{in}, \quad (2.10)$$

subject to conditions (2.1)-(2.8) and condition (2.11):

$$(P_j - k_j)M^B(0) \leq \sum_{i=1}^I \pi_i M_i^B(1) X_{ij} \leq (P_j + k_j)M^B(0), \quad j = 1, \dots, n-1, n+1, \dots, J. \quad (2.11)$$

If this problem is feasible, then the attained maximum and minimum have the following interpretation. If one can buy the test option for less than the minimum attained in this problem, then at least one investor, *but not necessarily all investors*, increases her expected utility by trading the test option. Likewise, if one can write the test option, for more than the maximum attained in this problem, then again at least one investor increases her expected utility by trading the test option.

## 2.3 Restrictions in the Two-Period Model

We relax the assumption of the single-period model that, over the one-month life of the options, markets for trading are open only at the beginning and end of the period; we allow for a third trading date in the middle of the month. We define the marginal utility of wealth out of the bond account and out of the stock account at each one of the three trading dates and set up the linear program as a direct extension of the program (2.1)-(2.9) in Section 2.2. The explicit program is given in Appendix B. In our empirical analysis, we report the percentage of months in which the linear program is feasible and, therefore, stochastic dominance is ruled out.

In principle, we may allow for more than one intermediate trading date over the one-month life of the options. However, the numerical implementation becomes tedious as both the number of constraints and variables in the linear program increase exponentially in the number of intermediate trading dates. This consideration motivates the development of bounds that are independent of the allowed frequency of trading of the stock and bond over the life of the option. These bounds are presented below.

## 2.4 The Constantinides and Perrakis (2002) Option Bounds

Constantinides and Perrakis (2002) recognized that it is possible to recursively apply the single-period approach and derive stochastic dominance bounds on option prices in a market with intermediate trading over the life of the options.<sup>9</sup> The significance of these bounds is that they are invariant to the allowed frequency of trading the bond and stock over the life of the options.

The task of computing these bounds is easy compared to the full-fledged investigation of the feasibility of conditions for large  $T$  for two reasons. First, the

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<sup>9</sup> Constantinides and Zariphopoulou (1999, 2001) and Constantinides and Perrakis (2002, 2004) derived bounds on option prices in the presence of transactions costs. For alternative ways to price options with transactions costs, see Leland (1985) and Bensaid *et al* (1992).

derivation of the bounds takes advantage of the special structure of the payoff of a call or put option, specifically the convexity of the payoff as a function of the stock price. Second, the set of assets is limited to three assets: the bond, stock and one option, the test option.

The upper and lower bounds on a test option have the following interpretation. If one can buy the test option for less than the lower option bound, then there is stochastic dominance violation between the bond, stock and the test option. Likewise, if one can write the test option for more than the upper option bound, then again there is stochastic dominance violation between the bond, stock and the test option. Below, we state these bounds without proof.<sup>10</sup>

At any time  $t$  prior to expiration, the following is an upper bound on the price of a call:

$$\bar{c}(S_t, t) = \frac{(1+k)}{(1-k)R_s^{T-t}} E\left[[S_T - K]^+ | S_t\right], \quad (2.11)$$

where  $R_s$  is the expected return on the stock per unit time.

A partition-independent lower bound for a call option can also be found, but only if it is additionally assumed that there exists at least one trader for whom the investment horizon coincides with the option expiration,  $T = T'$ . In such a case, transactions costs become irrelevant in the put-call parity and the following is a lower bound:<sup>11</sup>

$$\underline{c}(S_t, t) = (1+\delta)^{t-T} S_t - K / R^{T-t} + E[(K - S_T)^+ | S_t] / R_s^{T-t}, \quad (2.12)$$

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<sup>10</sup> These bounds may not be the tightest possible bounds for any given frequency of trading. However, they are presented here because of their universality in that they do not depend on the frequency of trading over the life of the option. For a comprehensive discussion and derivation of these and other possibly tighter bounds that are specific to the allowed frequency of trading, see Constantinides and Perrakis (2002). See also Constantinides and Perrakis (2004) for extensions to American-style options and futures options.

<sup>11</sup> In the special case of zero transactions costs, the assumption  $T = T'$  is redundant because the put-call parity holds.

where  $R$  is one plus the risk free rate per unit time.

Put option upper and lower bounds also exist that are independent of the frequency of trading. They are given as follows:

$$\bar{p}(S_t, t) = \frac{K}{R^{T-t}} + \frac{1-k}{1+k} (R_S^{T-t})^{-1} \left[ E[[K - S_T]^+ | S_t] - K \right], \quad (2.13)$$

and

$$\begin{aligned} \underline{p}(S_t, t) &= (R_S^{T-t})^{-1} \frac{1-k}{1+k} E[[K - S_T]^+ | S_t], \quad t \leq T-1 \\ &= [K - S_T]^+, \quad t = T. \end{aligned} \quad (2.14)$$

The call upper bound (2.11) provides a tighter upper bound on the implied volatility than the put upper bound (2.13). The call lower bound (2.12) and the put lower bound (2.14) provide similar lower bounds on the implied volatility. In figures 1-4, we present the upper bound on the implied volatility based on equation (2.11) and the lower bound based on equation (2.12). We discuss the violation of these bounds in Section 3.6.

## 3 Empirical Results

### 3.1 Data

We use the historical daily record of the S&P 500 index and its daily dividend record over the period 1928-2002. The monthly index return is based on 30 calendar day (21 trading day) returns. In order to avoid difficulties with the estimated historical mean of the returns, we demean all our samples and reintroduce a mean 4% annualized *premium* over the risk free rate.

We estimate both the *unconditional* and the *conditional* distribution of the index. The *unconditional* distribution is extracted from four alternative samples of thirty-day index returns: two *historical* returns samples over the periods 1928-1986

and 1972-1986; a *forward-looking* returns sample over the period 1987-2002 that includes the 1987 stock market crash; and a *forward-looking* returns sample over the period 1988-2002 that excludes the stock market crash.

We estimate the *conditional* distribution in two distinct ways. In the first way, we estimate the *conditional* distribution over the period 1972-2002 by the semiparametric GARCH (1, 1) model of Engle and Gonzalez-Rivera (1991), a model that does not impose the restriction that conditional returns are normally distributed, as explained in Appendix D.<sup>12</sup> In the second way, every month we estimate the variance as the squared Black-Scholes implied volatility of the closest ATM option and scale the unconditional distribution for that month to have the implied variance.

For the S&P 500 index options we use two data sources. For the period 1986-1995, we use the tick-by-tick Berkeley Options Database of all quotes and trades. We focus on the most liquid call options with K/S ratio (moneyness) in the range 0.90-1.05. For 108 months we retain only the call option quotes for the day corresponding to options thirty days to expiration.<sup>13</sup> For each day retained in the sample, we aggregate the quotes to the minute and pick the minute between 9:00-11:00 AM with the most quotes as our cross section for the month. We present these quotes in terms of their bid and ask implied volatilities. These are the volatilities which would be needed in the BSM formula to price the option exactly at the bid or ask quote, respectively. Details on this database are provided in Appendix C, Jackwerth and Rubinstein (1996), and Jackwerth (2000).

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<sup>12</sup> The index return sample and the option price sample do not align. We use the conditional volatility of the 30-day return period which starts *before* the option sample and covers it partly at the beginning. We recalculated the results by using the conditional volatility of the 30-day return period which starts *during* the option sample and covers it partly at the end and then continues beyond the option sample. The two sets of results are practically indistinguishable and thus, we do not report the latter results here.

<sup>13</sup> We lose some months for which we do not have sufficient data, i.e., months with less than five different strike prices, months after the crash of October 1987 until June 1988, and months before the introduction of S&P 500 index options in April 1986.

For the period 1997-2002, we obtain call *and* put option prices from the Option Metrics Database, described in Appendix C. Note that we do not have options data for 1996 from either data source. Only options with at least 100 traded contracts are included. We calculate a hypothetical noon option cross section from the closing cross section and the index observed at noon and the close. Here we assume that the implied volatilities do not change between noon and the close. We start out with 69 raw cross sections and are left with 68 final cross sections. The time to expiration is 29 days.

Since the Berkeley Options Database provides less noisy data than the Option Metrics Database, we expect a higher incidence of stochastic dominance violations over the 1997-2002 period than over the 1986-1995 period. Thus we are cautious in comparing results across these two periods.

### 3.2 Assumptions on Bid-Ask Spreads and Trading Fees

There is no presumption that all agents in the economy face the same bid-ask spreads and transactions costs as the traders do. We assume that the traders are subject to the following bid-ask spreads and trading fees. For the index, we model the combined one-half bid-ask spread and one-way trading fee as a one-way proportional transactions cost rate equal to 50 bps of the index price.

For the call options obtained from the Berkeley Options Database over the period 1986-1995, we proceed as follows. For the at-the-money call, we set the combined one-half bid-ask spread and one-way trading fee equal to 5, 10, or 20 bps of the index price. This corresponds to about 19, 38, or, 75 cents one-way fee per call, respectively. For any other call, the fee is proportional to the call price. Specifically, the combined one-half bid-ask spread and one-way trading fee is equal to the fee on the at-the-money call multiplied by the ratio of the price of the said call and the price of the at-the-money call. Only in Table 3 do we present results under the assumption that the fee is fixed: the combined one-half bid-ask spread and one-way trading fee is equal to the fee on the at-the-money call.

For the call and put options obtained from the Option Metrics Database over the period 1997-2002, we proceed as follows. For the at-the-money call, we set

the combined one-half bid-ask spread and one-way trading fee equal to 5, 10, or 20) bps of the index price. For any other option (call or put), the fee is proportional to the option price.

### 3.3 Stochastic Dominance Violations in the Single-Period Case

Each month we check for feasibility of conditions (2.1)-(2.9). Infeasibility of these conditions implies stochastic dominance: *any* trader can improve her utility by trading in these assets without incurring any out-of-pocket costs. If we rule out bid-ask spreads and trading fees, we find that these conditions are violated in all months.

We introduce bid-ask spreads and trading fees as described in Section 3.2. The one-way transactions cost rate (one-way trading fee plus half the bid-ask spread) on the index is 50 bps. For the at-the-money call, the one-way transactions cost rate is 10 bps of the index price, or about 38 cents. For any other option, the fee is proportional to the option price, as described in Section 3.2. The number of calls in each (filtered) monthly cross section fluctuates between 5 and 23 with median 10. The percentages of months without stochastic dominance violations are the bold entries displayed in Table 1. The bracketed numbers in the first row are bootstrap standard deviations of the first-row entries, based on 1,000 samples of the 1928-2002 historical returns. The standard deviations are high and, therefore, comparisons of the table entries across the rows and columns should be made with caution. In the top and bottom entries in each row, we display the non-violations in the cases where the one-way transactions costs on each call equal to 5 bps and 20 bps of the index price, respectively.

[TABLE 1]

The time series of option prices is divided into six periods and stochastic dominance violations in each period are reported in different columns, labeled as

panels A-F. The first period extends from May 1986 to October 16, 1987, just prior to the crash. The other five periods are all post-crash and span July 1988 to March 1991, April 1991 to August 1993, September 1993 to December 1995, February 1997 to December 1999 and February 2000 to December 2002. Note that we do not have options data for 1996 from either data source.

In the first four rows, we report stochastic dominance violations based on the unconditional distribution of index returns. For the first row, the index returns sample covers 1928-1986 and excludes the crash. Since there are too many observations, only every 6<sup>th</sup> return is recorded in building the empirical unconditional return distribution. For the second row, the index returns sample covers 1972-1986 and again excludes the crash. It is shorter than the first sample to control for the possibility of a regime shift in the return distribution. For the third row, the index returns sample covers 1987-2002, including the crash. For the fourth row, the index returns sample covers 1988-2002, excluding the crash.

In the last two rows, we report stochastic dominance violations based on the conditional distribution of index returns. For these two rows, the index returns sample covers 1972-2002. Every month we estimate the conditional volatility and scale the unconditional distribution to have the said volatility. For the fifth row, the conditional volatility is estimated as in Appendix D. For the last row, the conditional volatility is estimated as the Black-Scholes implied volatility of the closest ATM option.

In all five samples, the mean premium of the index return over the risk free return is adjusted to be 4% annually.<sup>14</sup>

Most table entries are well below 100%, indicating that there are a number of months in which the risk free rate, the price of the index, and the prices of the cross section of calls are inconsistent with a market in which there is even *one* risk-averse trader who is marginal in these securities, net of generous transactions costs.

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<sup>14</sup> We make this adjustment in order to eschew the issues of the predictability of the equity premium and its estimation from historical samples. Our results remain practically unchanged if we do not make this adjustment. Essentially, the prices of one-month options are insensitive to the expected return on the stock.

In the top left box, the middle entry of 73% refers to the index return distribution over the period 1928-1986 and option prices over the pre-crash period from May 1986 to October 16, 1987. In 27% of these months, conditions (2.1)-(2.9) are infeasible and the prices imply stochastic dominance violations despite the generous allowance for transactions costs. The next three entries to the right, panels B-D, refer to call prices over the first three post-crash periods. There are fewer violations in the first three post-crash periods than in the pre-crash period.

Violations dramatically increase in the last two post-crash periods, panels E-F. However, these results should be interpreted with caution. Recall that the quality of the option data in the 1997-2002 period is inferior to the quality in the 1986-1995 period. However, the quality of data in the 1997-1999 and 2000-2002 periods is the same and comparisons are meaningful. There are more violations in the 2000-2002 period than in the 1997-1999 period. The later finding is reversed in the last row where we employ the conditional index return distribution.

We investigate the robustness of the historical estimate of the index return distribution over the period 1928-1986 by re-estimating the historical distribution of the index return over the more recent period 1972-1986. The results are displayed in the second row of Table 1. The results in panels A-D remain largely unchanged. However the incidence of violations substantially increases in the period 1997-2002.

When we use the forward-looking index sample 1987-2002 that includes the crash (third row) or the forward-looking index sample 1988-2002 that excludes it (fourth row), the pre-crash options exhibit more violations (panel A). Also, when we use the conditional index return distribution, the incidence of violations is higher than when we use the historical index samples and comparable to the incidence of violations when we use the forward-looking index sample.

Our interpretation is that, before the crash, option traders were unsophisticated and were extensively using the BSM pricing model. Recall the stylized observation that, from the start of the exchange-based trading until the

October 1987 stock market crash, the implied volatility is a moderately downward-sloping function of the strike price; following the crash, the volatility smile is typically more pronounced. This means that the BSM model typically fits the data better before the crash than after it, once the constant volatility input is judiciously chosen as an input to the BSM formula. This does not imply that investors were more rational before the crash than after it. In fact, our results in panels A-D suggest that options were priced more rationally after the crash than before it. The results contrast with the evidence in Jackwerth (2000), that the estimated pricing kernel is monotonically decreasing (corresponding to few, if any, violations) in the pre-crash period, but locally increasing (corresponding to several violations) during the post-crash period.<sup>15</sup>

Looking across rows, we observe that the pre-crash call prices are more consistent with the historical index distribution (1928-2002 and 1972-2002) than the post-crash distribution (with or without the crash event, conditional or unconditional). This result accords with intuition. The post-crash option prices in 1988-1995 (panels B-D) generally have few violations. However, violations by option prices in 1997-2002 (panels E-F) send a mixed message. The violations are lowest when using either the longer historical index sample 1972-1986 or the conditional index return distribution.

### 3.4 Robustness in the Single-Period Case

Floor traders, institutional investors and broker-assisted investors face different transactions cost schedules in trading options. Are the results robust under different transactions cost schedules? In Table 1, the number in the top row of each cell is the percentage of non-violations when the combined one-half bid-ask spread and one-way trading fee on one option is based on 5 bps of the index price. We observe a large percentage of violations for all index and option price

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<sup>15</sup> The pattern in Jackwerth (2000) does not match with Table 1 for two reasons. First, he applies a different technique, estimating separately the smoothed risk-neutral and actual distributions and then taking their ratio. Second, his option price sample ends in 1995.

periods. Consistent with the earlier observation, there are fewer violations in the first three post-crash periods than in the pre-crash period. The number in the third row of each cell is the percentage of non-violations when the combined one-half bid-ask spread and one-way trading fee on one option is based on 20 bps of the index price. Predictably, we observe fewer violations for all index and option price periods. There is no consistent pattern in these violations.

[TABLE 2]

Is the pattern of violation similar across the in-the-money and out-of-the-money options? Table 2 displays the percentage of months in which stochastic dominance is absent in the cross section of in-the-money calls (top entry) and out-of-the-money calls (bottom entry). In all cases, there is a higher percentage of violations by OTM calls than by ITM calls, suggesting that the mispricing is caused by the right-hand tail of the index return distribution and not by the left-hand tail.<sup>16</sup> Another way to see this is by comparing the first rows in Table 1 with the Table 2 OTM results: addition of the ITM calls does not decrease the feasibility. This observation is novel and contradicts the common inference drawn from the observed implied volatility smile that the problem lies with the left-hand tail of the index return distribution. We revisit these violation patterns in Section 3.6 where we discuss the violations of bounds on the prices of options.

[TABLE 3]

Table 3 displays the percentage of months in which stochastic dominance violations are absent in the cross section of option prices but now with fixed instead of proportional transactions costs. The one-way transactions costs rate (one-way trading fee plus half the bid-ask spread) on the index is 50 bps. The one-way transactions costs on each option is 20 bps of the index price. The

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<sup>16</sup> This inference is subject to the criticism that it may be an artifact of sample size. The sample of OTM calls is larger than the sample of ITM calls. Other things equal, the larger the sample, the harder it is to find a monotone decreasing pricing kernel that prices the calls.

pattern of violations is similar to the pattern displayed in Table 1 with variable transactions costs. With the exception of panel D, there are generally fewer violations when the transactions costs are fixed. Recall from Table 2 that OTM calls are responsible for more violations than ITM calls. Fixed transactions costs imply larger transactions costs for the troublesome OTM calls, provide greater leeway for the prices of these calls and, therefore, decrease the number of violations.

### 3.5 Stochastic Dominance Violations in the Two-Period Model

In the previous section, we considered feasibility in the context of the single-period model. We established that there are stochastic dominance violations in a significant percentage of the months. Does the percentage of stochastic dominance violations increase or decrease as the allowed frequency of trading in the stock and bond over the life of the option increases? In the special case of zero transactions costs, *i.i.d.* returns *and* constant relative risk aversion, it can be theoretically shown that the percentage of violations should increase as the allowed frequency of trading increases. However, we cannot provide a theoretical answer if we relax any of the above three assumptions. Therefore, we address the question empirically.

We compare the percentage of stochastic dominance violations in two models, one with one intermediate trading date over the life of the options and another with no intermediate trading dates over the life of the options. To this end, we partition the 30-day horizon into two 15-day intervals and approximate the 15-day return distribution by a 21-point kernel density estimate of the 15-day returns. We use the standard Gaussian kernel of Silverman (1986, pp. 15, 43, and 45). The assumed transactions costs are as in the base case presented in Table 1. The one-way transactions costs rate (one-way trading fee plus half the bid-ask spread) on the index is 50 bps. The one-way transactions cost on the at-the-money call is 20 bps of the index price. For any other call, the fee is proportional to the call price, as described in Section 3.2. The results are presented in Table 4.

[TABLE 4]

We may not investigate the effect of intermediate trading by directly comparing the results in Tables 1 and 4 because the return generating process differs in the two tables. Recall that the results in Table 1 are based on a 30-day stock return generating process that has as many different returns as the different observed realizations and frequency equal to the observed frequency.<sup>17</sup> By contrast, the results in Table 4 are based on a simplified 15-day 21-point kernel density estimate of the 15-day returns. The coarseness of the grid is dictated by the need to keep the problem computationally manageable. The 30-day return then is the product of two 15-day returns treated as i.i.d. With this process of the 30-day return, we calculate the percentage of months without stochastic dominance violations and report the results in Table 4 in brackets.

The effect of allowing for one intermediate trading date over the life of the one-month options is shown by the top entries in Table 4. These entries are contrasted with the bracketed entries which represent the percentage of months without stochastic dominance violations when intermediate trading is forbidden. The comparison shows that intermediate trading has an ambiguous effect on stochastic dominance violations in the option samples, that depends to a large extent on the distribution used to calculate the index returns. For the long historical sample intermediate trading slightly decreases the frequency of violations in all panels, while it increases it, sometimes dramatically so, for the shorter historical sample. The increase in the frequency of violations also predominates, but not always consistently, for the remaining three samples. We conclude that intermediate trading does not weaken, and possibly strengthens, the single-period systematic evidence of stochastic dominance violations. In the next section, we obtain further insights on the causes of infeasibility, by displaying the options that violate the upper and lower bounds on option prices.

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<sup>17</sup> For the long historical sample of stock returns, we take only every sixth monthly return.

### 3.6 Stochastic Dominance Bounds in the Single-Period and Multiperiod Cases

In Section 2.4, equations (2.11)-(2.14), we stated a set of stochastic dominance bounds on option prices that apply irrespective of the permitted frequency of trading in the bond and stock accounts over the life of the option. We calculate these bounds and translate them as bounds on the implied volatility of option prices. In figures 1-4, we present the upper implied volatility bound based on (2.11) and the lower bound based on (2.12). The bid-ask spread on the option price is taken into consideration, as we present both the bid and ask option prices, translated into implied volatilities. A violation occurs whenever an observed option bid price lies above the upper bound or an observed option ask price lies below the lower bound. The upper and lower option bounds are based on the index return distribution derived from the historical index samples 1928-1986 (figure 1) and 1972-1986 (figure 2), and the forward looking samples 1987-2002 (figure 3) and 1988-2002 (figure 4). In each figure, the six panels correspond to option prices (implied volatilities) over the pre-crash period (panel A) and the five post-crash periods (panels B-F). In all cases, the transactions costs rate on the index is 20 bps.

[FIGURES 1-4]

The downward-sloping shape of the bounds is similar across figures 1-4. However, the upper and lower bounds in figure 1 are higher than the bounds in figures 2-4 because the index volatility over 1928-1986 is 40% higher than the index volatility over the index periods corresponding to figures 2-4. The pattern of violations follows quite naturally. The flat pre-crash smile fits reasonably well within the bounds based on the index return over 1928-1986 even though these are downward sloping. The post-crash smiles over 1988-1995 (panels B-D) are too low for the rather high location of these bounds.

The bounds based on the index return over the historical 1972-1986 and forward-looking samples (figures 2-4) are located somewhat lower than the historical 1928-1986 sample bounds. Therefore, they match the also downward-sloping post-crash option prices in panels B-D rather well because they are located somewhat lower too. However, they do not match very well the higher horizontal smile of the pre-crash options.

Several option prices over the periods 1997-1999 and 2000-2002 (panels E-F) are way above the bounds in all the figures, irrespective of whether the bounds were calculated from historical or forward-looking index returns. This is an altogether different pattern of violations than in the earlier panels A-D. In interpreting the high incidence of violations of option prices over the period 1997-2002 in Tables 1-4, we were conservative because of concerns regarding the quality of the Option Metrics Database. The figures provide a clearer picture. If the violations were the result of low quality of the data, then we would observe roughly as many violations of the lower bound as we do of the upper bound. This is not the case. Most of the violations are violations of the upper bound. Simply put, over 1997-2002, many options, particularly OTM calls, were overpriced relative to the theoretical bounds, irrespective of which time period is used to determine the index return distribution. These results do not support the hypothesis that the options market is becoming more rational over time, particularly after the crash. The decrease in violations over the post-crash period 1988-1995 (panels B-D) is followed by a substantial increase in violations over 1997-2002 (panels E-F).

Across all figures, we observe that both upper and lower bounds exhibit a clear smile pattern. Thus the theory states that option prices should exhibit a smile both before and after the crash. The observed pre-crash option prices (panel A) approximately conform to the BSM model with a horizontal smile. By contrast, the post-crash observed option prices (panels B-D) progressively show more marked departures from horizontality, which still lie within the bounds in panel B but violate strongly the bounds in panels C and D, even around at-the-money. This conforms closely to the observation originally made by Rubinstein (1994) that option prices behave differently before and after the crash, with the former following the BSM model and the latter not. Over the period 1997-2002 (panels E-

F), option prices exhibit a mild smile. However, their predominant feature is that they are overpriced, particularly the OTM calls.

In all figures, panel A, several pre-crash ask prices of OTM calls in panel A fall below the lower bound. Even though pre-crash option prices follow the BSM model reasonably well, it does not follow that these options are correctly priced. Our novel finding is that pre-crash option prices are incorrectly priced, if the distribution of the index return is based on the historical experience. Furthermore, some of these prices are below the bounds, contrary to received wisdom that historical volatility generally underprices options in the BSM model.

All figures dispel another common misconception, that the observed smile is too steep after the crash. Our novel finding is that most of the bound violations by post-crash options are due to the options being either underpriced (over 1988-1995, panels B-D) or overpriced (over 1997-2002, panels E-F).

## 4 Concluding Remarks

We document widespread violations of stochastic dominance in the one-month S&P 500 index options market over the period 1986-2002, before and after the October 1987 stock market crash. We do not impose a parametric model on the index return distribution but estimate it as the histogram of the sample distribution, using five different index return samples: long and short samples before the crash; two forward-looking samples, one that includes the crash and one that excludes it; and a sample with forecasted conditional volatility. We allow the market to be incomplete and also be imperfect by introducing generous transactions costs in trading the index and options.

Evidence of stochastic dominance violations means that any trader can increase her expected utility by engaging in a zero-net-cost trade. We consider a market with heterogeneous agents and investigate the restrictions on option prices imposed by a particular class of utility-maximizing traders that we simply refer to

as traders. We do not make the restrictive assumption that all economic agents belong to the class of the utility-maximizing traders. Thus our results are robust and unaffected by the presence of agents with beliefs, endowments, preferences, trading restrictions, and transactions cost schedules that differ from those of the utility-maximizing traders modeled in this paper.

Our empirical design allows for three implications associated with state dependence. First, each month we search for a pricing kernel to price the cross section of one-month options without imposing restrictions on the time series properties of the pricing kernel month by month. Thus we allow the pricing kernel to be state dependent. Second, we allow for intermediate trading; a trader's wealth on the expiration date of the options is generally a function not only of the price of the market index on that date but also of the entire path of the index level thereby rendering the pricing kernel state dependent. Third, we allow the variance of the index return to be state dependent and employ the estimated conditional variance.

The pre-crash call prices are more consistent with the historical index distribution than the post-crash distribution. This result accords with intuition. The post-crash option prices in 1988-1995 generally have few violations. However, violations by option prices in 1997-2002 send a mixed message. The violations are lowest when using either the longer historical index sample 1972-1986 or the conditional index return distribution. Thus, there is no systematic evidence that investors are more rational after the crash than before it.

In all cases, there is a higher percentage of violations by OTM calls than by ITM calls, suggesting that the right-hand tail of the index return distribution is at least as problematic as the left-hand tail. This observation is novel and contradicts the common inference drawn from the observed implied volatility smile that the problem lies with the left-hand tail of the index return distribution.

Over 1997-2002, many options, particularly OTM calls, are overpriced relative to the theoretical bounds, irrespective of which time period is used to determine the index return distribution. One possible explanation is the poor quality of the data over this period compared to the data over the 1986-1995 period. In any case, these results do not support the hypothesis that the options market is becoming more rational over time.

Even though pre-crash option prices conform to the BSM model reasonably well, it does not follow that these options are correctly priced. Our novel finding is that pre-crash options are incorrectly priced, if the distribution of the index return is based on the historical experience. Our interpretation of these results is that, before the crash, option traders were extensively using the BSM pricing model. Recall the stylized observation that, from the start of the exchange-based trading until the October 1987 stock market crash, the implied volatility is a moderately downward-sloping function of the strike price; following the crash, the volatility smile is typically more pronounced. This means that the BSM model typically fits the data better before the crash than after it, once the constant volatility input is judiciously chosen as an input to the BSM formula. However, the fit of the BSM model or lack of it does not speak on the rationality of option prices.

Our results dispel another common misconception, that the observed smile is too steep after the crash. Our novel finding is that most of the bound violations by post-crash options are due to the options being either underpriced over 1988-1995, or overpriced over 1997-2002.

Finally, in many of the violations, option prices are below the bounds, contrary to received wisdom that historical volatility generally underprices options in the BSM model.

By providing an integrated approach to the pricing of options that allows for incomplete and imperfect markets, we provide testable restrictions on option prices that include the BSM model as a special case. We reviewed the empirical evidence on the prices of S&P 500 index options. The economic restrictions are violated surprisingly often, suggesting that the mispricing of these options cannot be entirely attributed to the fact that the BSM model does not allow for market incompleteness and realistic transaction costs.

In this paper, we allowed for a number of implications associated with state variables. Whereas several extant models addressed the inconsistencies with the BSM model and the violations of monotonicity of the pricing kernel by introducing additional state variables and/or exploring alternative specifications of preferences (see, footnote 5), it remains an open and challenging topic for future research to investigate whether these particular state variables and alternative specifications of

preferences explain on a month-by-month basis the cross section of S&P 500 option prices.

# Appendix A

Trading occurs at a finite number of trading dates,  $t = 0, 1, \dots, T, \dots, T'$ .<sup>18</sup> The utility-maximizing traders are allowed to hold only two primary securities in the market, a bond and a stock. The bond is risk free and pays constant interest  $R - 1$  each period. The traders may buy and sell the bond without incurring transactions costs. At date  $t$ , the *cum dividend* stock price is  $(1 + \delta_t)S_t$ , the cash dividend is  $\delta_t S_t$ , and the *ex dividend* stock price is  $S_t$ , where  $\delta_t$  is the dividend yield. We assume that the rate of return on the stock,  $(1 + \delta_{t+1})S_{t+1}/S_t$ , is identically and independently distributed over time.

Stock trades incur proportional transactions costs charged to the bond account as follows. At each date  $t$ , the trader pays  $(1 + k)S_t$  out of the bond account to purchase one *ex dividend* share of stock and is credited  $(1 - k)S_t$  in the bond account to sell (or, sell short) one *ex dividend* share of stock. We assume that the transactions cost rate satisfies the restriction  $0 \leq k < 1$ .

A trader enters the market at date  $t$  with dollar holdings  $x_t$  in the bond account and  $y_t/S_t$  *ex dividend* shares of stock. The endowments are stated net of any dividend payable on the stock at time  $t$ .<sup>19</sup> The trader increases (or, decreases) the dollar holdings in the stock account from  $y_t$  to  $y_t' = y_t + v_t$  by decreasing (or, increasing) the bond account from  $x_t$  to  $x_t' = x_t - v_t - k|v_t|$ . The decision variable  $v_t$  is constrained to be measurable with respect to the information at date  $t$ . The bond account dynamics is

$$x_{t+1} = \{x_t - v_t - k|v_t|\}R + (y_t + v_t)\frac{\delta_t S_{t+1}}{S_t}, \quad t \leq T' - 1 \quad (\text{A.1})$$

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<sup>18</sup> The calendar length of the trading horizon is  $N$  years and the calendar length between trading dates is  $N/T'$  years. Later on we vary  $T'$  and consider the mispricing of options under different assumptions regarding the calendar length between trading dates.

<sup>19</sup> We elaborate on the precise sequence of events. The trader enters the market at date  $t$  with dollar holdings  $x_t - \delta_t y_t$  in the bond account and  $y_t/S_t$  *cum dividend* shares of stock. Then the stock pays cash dividend  $\delta_t y_t$  and the dollar holdings in the bond account become  $x_t$ . Thus, the trader has dollar holdings  $x_t$  in the bond account and  $y_t/S_t$  *ex dividend* shares of stock.

and the stock account dynamics is

$$y_{t+1} = (y_t + \nu_t) \frac{S_{t+1}}{S_t}, \quad t \leq T'-1. \quad (\text{A.2})$$

At the terminal date, the stock account is liquidated,  $\nu_{T'} = -y_{T'}$ , and the net worth is  $x_{T'} + y_{T'} - k |y_{T'}|$ . At each date  $t$ , the trader chooses investment  $\nu_t$  to maximize the expected utility of net worth,  $E[u(x_{T'} + y_{T'} - k |y_{T'}|) | S_t]$ .<sup>20</sup> We make the plausible assumption that the utility function,  $u(\cdot)$ , is increasing and concave, and is defined for both positive and negative terminal net worth.<sup>21</sup> Note that even this weak assumption of monotonicity and concavity of preferences is not imposed on all agents in the economy but only on the subset of agents that we refer to as traders.

We recursively define the value function  $V(t) \equiv V(x_t, y_t, t)$  as

$$V(x_t, y_t, t) = \max_{\nu} E \left[ V \left( \{x_t - \nu - k | \nu | \} R + (y_t + \nu) \frac{\delta_t S_{t+1}}{S_t}, (y_t + \nu) \frac{S_{t+1}}{S_t}, t+1 \right) | S_t \right] \quad (\text{A.3})$$

for  $t \leq T'-1$  and

$$V(x_{T'}, y_{T'}, T') = u(x_{T'} + y_{T'} - k |y_{T'}|). \quad (\text{A.4})$$

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<sup>20</sup> The results extend routinely to the case that consumption occurs at each trading date and utility is defined over consumption at each of the trading dates and over the net worth at the terminal date. See Constantinides (1979) for details. The model with utility defined over terminal net worth alone is a more realistic representation of the objective function of financial institutions.

<sup>21</sup> If utility is defined only for non-negative net worth, then the decision variable is constrained to be a member of a convex set that ensures the non-negativity of net worth. See, Constantinides (1979) for details. However, the derivation of bounds on the prices of derivatives requires an entirely different approach and yields weaker bounds. This problem is studied in Constantinides and Zariphopoulou (1999, 2001).

We assume that the parameters satisfy appropriate technical conditions such that the value function exists and is once differentiable.

Equations (A.1)-(A.4) define a dynamic program that can be numerically solved for given utility function and stock return distribution. We shall not solve this dynamic program because our goal is to derive restrictions on the prices of options that are independent of the specific functional form of the utility function but solely depend on the plausible assumption that the traders' utility function is monotone increasing and concave in the terminal wealth.

The value function is increasing and concave in  $(x_t, y_t)$ , properties that it inherits from the assumed monotonicity and concavity of the utility function, as proven in Constantinides (1979):

$$V_x(t) > 0, \quad V_y(t) > 0, \quad t = 0, \dots, T, \dots, T'. \quad (\text{A.5})$$

and

$$\begin{aligned} V(\alpha x_t + (1 - \alpha)x_t', \alpha y_t + (1 - \alpha)y_t', t) &> \alpha V(x_t, y_t, t) + (1 - \alpha)V(x_t', y_t', t), \\ 0 < \alpha < 1, \quad t = 0, \dots, T, \dots, T'. \end{aligned} \quad (\text{A.6})$$

On each date, the trader may transfer funds between the bond and stock accounts and incur transactions costs. Therefore, the marginal rate of substitution between the bond and stock accounts differs from unity by, at most, the transactions cost rate:

$$(1 - k)V_x(t) \leq V_y(t) \leq (1 + k)V_x(t), \quad t = 0, \dots, T, \dots, T'. \quad (\text{A.7})$$

Marginal analysis on the bond holdings leads to the following condition on the marginal rate of substitution between the bond holdings at dates  $t$  and  $t+1$ :

$$V_x(t) = R E_t[V_x(t+1)], \quad t = 0, \dots, T, \dots, T'-1. \quad (\text{A.8})$$

Finally, marginal analysis on the stock holdings leads to the following condition on the marginal rate of substitution between the stock holdings at date  $t$  and the bond and stock holdings at date  $t+1$ :

$$V_y(t) = E_t \left[ \frac{S_{t+1}}{S_t} V_y(t+1) + \frac{\delta_t S_{t+1}}{S_t} V_x(t+1) \right], \quad t = 0, \dots, T, \dots, T'-1. \quad (\text{A.9})$$

# Appendix B

We allow for three trading dates,  $t=0,1,2$ , at the beginning, middle and end of the month. We define the stock returns over the first sub-period as  $z_{1i} \equiv (1 + \delta)S_{1i} / S_0$ , corresponding to the  $I, i = 1, \dots, I$  states on date one. We assume that the returns over the two sub-periods are independent. Thus, the stock returns over the second sub-period,  $z_{2k} \equiv (1 + \delta)S_{2ik} / S_{1i}$ ,  $k = 1, \dots, I$ , are independent of  $i$ . There are  $I^2$ ,  $i = 1, \dots, I, k = 1, \dots, I$ , states on date two.

We define the state-dependent marginal utility of wealth out of the bond account on each one of the three trading dates as  $M^B(0) \equiv V_x(0)$ ,  $M_i^B(1) \equiv V_x(1)$  and  $M_{ik}^B(2) \equiv V_x(2)$ . Likewise, we define the state-dependent marginal utility of wealth out of the stock account on each of the three trading dates as  $M^S(0) \equiv V_y(0)$ ,  $M_i^S(1) \equiv V_y(1)$  and  $M_{ik}^S(2) \equiv V_y(2)$ . The conditions on positivity and monotonicity of the marginal utility of wealth out of the bond and stock accounts at  $t=0,1$  are given by equations (2.1)-(2.4). The corresponding conditions at  $t=2$  are:

$$M_{ik}^B(2) > 0, \quad i, k = 1, \dots, I \quad (\text{B.1})$$

and

$$M_{i1}^S(2) \geq M_{i2}^S(2) \geq \dots M_{ik}^S(2) \geq \dots M_{iI}^S(2) > 0, \quad i = 1, \dots, I. \quad (\text{B.2})$$

On each date, the trader may transfer funds between the bond and stock accounts and incur transactions costs. Conditions (2.5) and (2.6) hold. The corresponding condition at  $t=2$  is:

$$(1 - k)M_{ik}^B(2) \leq M_{ik}^S(2) \leq (1 + k)M_{ik}^B(2), \quad i, k = 1, \dots, I. \quad (\text{B.3})$$

Conditions (2.7) and (2.8) on the marginal rate of substitution between dates zero and one hold. The corresponding conditions between dates one and two are as follows:

$$M_i^B(1) = R \sum_{k=1}^I \pi_k M_{ik}^B(2), \quad i = 1, \dots, I \quad (\text{B.4})$$

and

$$M_i^S(1) = \sum_{k=1}^I \pi_k [z_{2k} M_{ik}^S(2) + \delta z_{2k} M_{ik}^B(2)], \quad i = 1, \dots, I. \quad (\text{B.5})$$

We consider  $J$  European call and put options on the index, with random *cash* payoff  $X_{ikj}$  at  $t = 2$  in state  $ik$ . Condition (2.9) is replaced by:

$$(P_j - k_j) M^B(0) \leq \sum_{i=1}^I \sum_{k=1}^I \pi_i \pi_k M_{ik}^B(2) X_{ikj} \leq (P_j + k_j) M^B(0), \quad j = 1, \dots, J. \quad (\text{B.6})$$

The probability of state  $ik$  is  $\pi_i \pi_k$  because, by assumption, the stock returns are independent over the two sub-periods.

Conditions (2.1)-(2.8) and (B.1)-(B.6) jointly define a linear program. In our empirical analysis, we report the percentage of months in which the linear program is feasible and, therefore, stochastic dominance is ruled out.

# Appendix C

## 1. Berkeley Options Database

The Berkeley Options Database contains all minute-by-minute quotes and trades of the European options and futures on the S&P 500 index from April 2, 1986 to December 29, 1995. Details on this database are found in Jackwerth and Rubinstein (1996), Jackwerth (2000) and below.

**Index Level.** Traders typically use the index futures market rather than the cash market to hedge their option positions. The reason is that the cash market prices lag futures prices by a few minutes due to lags in reporting transactions of the constituent stocks in the index. We check this claim by regressing the index on each of the first twenty minute lags of the futures price. The single regression with the highest adjusted  $R^2$  is assumed to indicate the lag for a given day. The median lag of the index over the 1542 days from 1986 to 1992 is seven minutes. Because the index is stale, we compute a futures-based index for each minute from the futures market as  $S_0 = (1 + \delta)^{-1} RF$ , where  $F$  is the futures price at the option expiration. For each day, we use the median interest rate  $R$  implied by all futures quotes and trades and the index level at that time. We approximate the dividend yield  $\delta$  by assuming that the dividend amount and timing expected by the market were identical to the dividends actually paid on the S&P 500 index. However, some limited tests indicate that the choice of the index does not seem to affect the results of this paper.

**Interest Rate.** We compute implied interest rates embedded in the European put-call parity relation. Armed with option quotes, we calculate separate lending and borrowing interest returns from put-call parity where we use the above future-based index. For each expiration date, we assign a single lending and borrowing rate to each day, which is the median of all daily observations across all strike prices. We then use the average of these two interest rates as our daily spot rate for the particular time to expiration. Finally, we obtain the interpolated interest rates from the implied forward curve. If there is data missing, we assume

that the spot rate curve can be extrapolated horizontally for the shorter and longer times-to-expiration. Again, some limited tests indicate that the results are not affected by the exact choice of the interest rate.

**Option Prices.** We use only bid and ask prices on call options. For each day retained in the sample, we aggregate the quotes to the minute and pick the minute between 9:00-11:00 AM with the most quotes as our cross section for the month.

We use only call options with 30 days to expiration which occur once every month during our sample. We also trim the sample to allow for moneyness levels between 0.90 and 1.05. Cross sections with fewer than 5 option quotes are discarded. We also eliminate the cross sections right after the crash of 1987 as the data is noisy and restart the sample with the cross section expiring on July 15, 1988.

**Arbitrage Violations.** In the process of setting up the database, we check for a number of errors which might have been contained in the original minute-by-minute transaction level data. We eliminate a few obvious data-entry errors as well as a few quotes with excessive spreads—more than 200 cents for options and 20 cents for futures. General arbitrage violations are eliminated from the data set. We also check for violations of vertical and butterfly spreads. Within each minute, we keep the largest set of option quotes which satisfies the restriction  $S(1 + \delta) \geq C_i \geq \max[0, S(1 + \delta) - K_i R]$ .

Early exercise is not an issue as the S&P 500 options are European and the discreteness of quotes and trades only introduces a stronger upward bias in the midpoint implied volatilities for deep-out-of-the-money puts (moneyness less than 0.6) which we do not use in our empirical work. We start out with 107 raw cross sections and are left with 98 final cross sections.

## 2. Option Metrics Database

The Option Metrics Database contains indicative end-of-day European call and put option quotes on the S&P 500 index from January 2, 1997 to December 31, 2002. In merging the Option Metrics Database with the Berkeley Options Database, we

follow the above procedure as much as possible, given the closing prices data that the Option Metrics Database provides. Therefore, only departures and innovations from the above procedure are noted.

**Index Level.** As the closing (noon) index price, we use the price implied by the closing (noon) futures price.

**Interest Rate.** As we cannot arrive at consistently positive interest rates implied by option prices, we use T-bill rates instead, obtained from Federal Reserve Bank of St. Louis Economic Research Database (FRED<sup>®</sup>).

**Option Prices.** In the final sample, only call and put options with at least 100 traded contracts are included. We calculate a hypothetical noon option cross section from the closing cross section and the index observed at noon and the close. Here we assume that the implied volatilities do not change between noon and the close. We start out with 69 raw cross sections and are left with 68 final cross sections. The time to expiration is 29 days.

### 3. S&P 500 Information Bulletin

We obtain the historical daily record of the S&P 500 index and its daily dividend record over the period 1928-2002 from the S&P 500 Information Bulletin. Before April 1982, dividends are estimated from monthly dividend yields.

# Appendix D

The GARCH (1,1) special case of the Engle and Gonzalez-Rivera (1991) semiparametric model applied to the monthly S&P 500 index return,  $y_t$ , is described by equations (D.1)-(D.3):

$$y_t = \bar{y} + \varepsilon_t \quad (\text{D.1})$$

$$h^{-1/2} \varepsilon_t \sim i.i.d. g(0,1) \quad (\text{D.2})$$

and

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}, \quad (\text{D.3})$$

where  $g(0,1)$  is an unknown distribution with zero mean and unit variance.

The parameters  $(\omega, \alpha, \beta)$  are estimated by maximum likelihood under the (false) assumption that  $h^{-1/2} \varepsilon_t \sim i.i.d. N(0,1)$ . Then the time series  $\{h_t^{-1/2} \varepsilon_t\}$  is calculated and the true density  $g(0,1)$  is estimated as the histogram of all the time series observations. The histogram may be smoothed by kernel methods but we do not undertake this step in order to keep the procedure comparable to that followed in estimating the unconditional distribution.

One may consider re-estimating the parameters  $(\omega, \alpha, \beta)$  by maximum likelihood, replacing the assumption that  $h^{-1/2} \varepsilon_t \sim i.i.d. N(0,1)$  with the assumption that  $h^{-1/2} \varepsilon_t \sim i.i.d. \hat{g}(0,1)$ , where  $\hat{g}(0,1)$  is the estimated density in the last step above. Engle and Gonzalez-Rivera (1991) showed by simulation that this additional step is unnecessary in practice.

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**Table 1. Percentage of Months without Stochastic Dominance Violations in the Single-Period Case**

The table displays the percentage of months in which stochastic dominance violations are absent in the cross-section of option prices. The bracketed numbers are the bootstrap standard errors of these percentages. The one-way transaction costs rate (one-way trading fees plus half the bid-ask spread) on the index is 50 bps. The one-way transaction costs on each option are proportional to the index price, as explained in Section 3.2. In each row, the one-way transaction costs on each option, as a proportion of the index price, are 5 bps (top entry), 10 bps (middle entry), and 20 bps (bottom entry).

	Panel A: 860516- 871016	Panel B: 880715- 910315	Panel C: 910419- 930820	Panel D: 930917- 951215	Panel E: 970221- 991217	Panel F: 000218- 021220
Number of Months	15	29	28	26	35	33
Unconditional Index Return distribution, 1928-1986	60 <b>73</b> (25) 73	59 <b>76</b> (23) 90	46 <b>89</b> (27) 100	0 <b>46</b> (27) 92	14 <b>37</b> (23) 60	21 <b>39</b> (19) 48
Unconditional Index Return distribution, 1972-1986	33 <b>47</b> 73	52 <b>62</b> 76	50 <b>89</b> 100	4 <b>46</b> 92	6 <b>14</b> 34	9 <b>18</b> 27
Unconditional Index Return distribution, 1987-2002	13 <b>13</b> 40	48 <b>69</b> 76	61 <b>86</b> 100	12 <b>54</b> 96	14 <b>20</b> 26	3 <b>12</b> 15
Unconditional Index Return distribution, 1988-2002	20 <b>33</b> 47	48 <b>66</b> 76	54 <b>89</b> 100	4 <b>50</b> 92	3 <b>17</b> 31	9 <b>12</b> 24
Conditional Index Return distribution, 1972-2002, Based on GARCH (1,1)	27 <b>27</b> 47	48 <b>76</b> 86	75 <b>89</b> 89	65 <b>88</b> 100	9 <b>23</b> 40	18 <b>42</b> 58
Conditional Index Return distribution, 1972-2002, Based on Implied Vol	53 <b>87</b> 100	76 <b>93</b> 100	82 <b>96</b> 100	77 <b>85</b> 100	23 <b>57</b> 80	45 <b>79</b> 91

**Table 2. Percentage of Months without Stochastic Dominance Violations in the Single-Period Case—ITM and OTM Calls Separately**

The table displays the percentage of months in which stochastic dominance violations are absent in the cross-section of in-the-money calls (top entry) and out-of-the-money calls (bottom entry). The one-way transaction costs rate (one-way trading fees plus half the bid-ask spread) on the index is 50 bps. The one-way transaction costs on each option, as a proportion of the index price, are 10 bps, as explained in Section 3.2.

	Panel A: 860516- 871016	Panel B: 880715- 910315	Panel C: 910419- 930820	Panel D: 930917- 951215	Panel E: 970221- 991217	Panel F: 000218- 021220
Number of Months	15	29	28	26	35	33
Unconditional Index Return distribution, 1928-1986	80 73	83 76	100 96	85 69	63 37	45 39
Unconditional Index Return distribution, 1972-1986	73 53	76 66	100 96	92 62	34 14	27 18
Unconditional Index Return distribution, 1987-2002	67 20	79 72	100 93	92 88	40 23	27 12
Unconditional Index Return distribution, 1988-2002	73 40	76 72	96 100	88 92	34 23	27 12
Conditional Index Return distribution, 1972-2002, Based on GARCH (1,1)	53 33	90 79	96 89	100 88	37 26	58 48
Conditional Index Return distribution, 1972-2002, Based on Implied Vol	100 87	100 93	100 96	100 85	97 57	97 82

**Table 3. Percentage of Months without Stochastic Dominance Violations in the Single-Period Case and Fixed Transaction Costs**

The table displays the percentage of months in which stochastic dominance violations are absent in the cross-section of option prices. The one-way transaction costs rate (one-way trading fees plus half the bid-ask spread) on the index is 50 bps. The one-way transaction costs on each option, as a proportion of the index price, are 10 bps, as explained in Section 3.2.

	Panel A: 860516- 871016	Panel B: 880715- 910315	Panel C: 910419- 930820	Panel D: 930917- 951215	Panel E: 970221- 991217	Panel F: 000218- 021220
Number of Months	15	29	28	26	35	33
Unconditional Index Return distribution, 1928-1986	73	76	54	4	51	42
Unconditional Index Return distribution, 1972-1986	53	66	57	8	26	18
Unconditional Index Return distribution, 1987-2002	40	69	68	15	26	15
Unconditional Index Return distribution, 1988-2002	47	59	68	8	23	15
Conditional Index Return distribution, 1972-2002, Based on GARCH (1,1)	40	76	86	92	31	39
Conditional Index Return distribution, 1972-2002, Based on Implied Vol	87	93	96	96	74	79

**Table 4. Percentage of Months without Stochastic Dominance Violations in the Two-Period Case**

The table displays the percentage of months in which stochastic dominance violations are absent in the cross section of option prices *when one intermediate trading date is allowed* over the life of the one-month options. The one-way transactions costs rate on the index is 50 bps. The one-way transaction costs on each option, as a proportion of the index price, are 10 bps, as explained in Section 3.2. In parentheses, the table displays the percentage of months in which stochastic dominance violations are absent in the case when *no intermediate trading is allowed* over the life of the one-month options. Two periods of 15 days and a kernel density of 15-day returns are used (discretized to 21 values from  $e^{-0.20}$  to  $e^{0.20}$ , spaced 0.02 apart in log spacing).

	Panel A: 860516- 871016	Panel B: 880715- 910315	Panel C: 910419- 930820	Panel D: 930917- 951215	Panel E: 970221- 991217	Panel F: 000218- 021220
Number of Months	15	29	28	26	35	33
Unconditional Index Return distribution, 1928-1986	60 (47)	17 (14)	18 (14)	0 (0)	3 (30)	3 (0)
Unconditional Index Return distribution, 1972-1986	33 (53)	41 (41)	39 (54)	15 (15)	9 (9)	3 (12)
Unconditional Index Return distribution, 1987-2002	33 (47)	41 (45)	36 (54)	19 (15)	3 (6)	3 (9)
Unconditional Index Return distribution, 1988-2002	33 (40)	48 (52)	61 (61)	27 (27)	6 (9)	6 (12)
Conditional Index Return distribution, 1972-2002, Based on GARCH (1,1)	40 (47)	41 (48)	57 (79)	42 (54)	6 (6)	3 (15)
Conditional Index Return distribution, 1972-2002, Based on Implied Vol	73 (93)	59 (55)	61 (82)	54 (58)	9 (17)	21 (33)

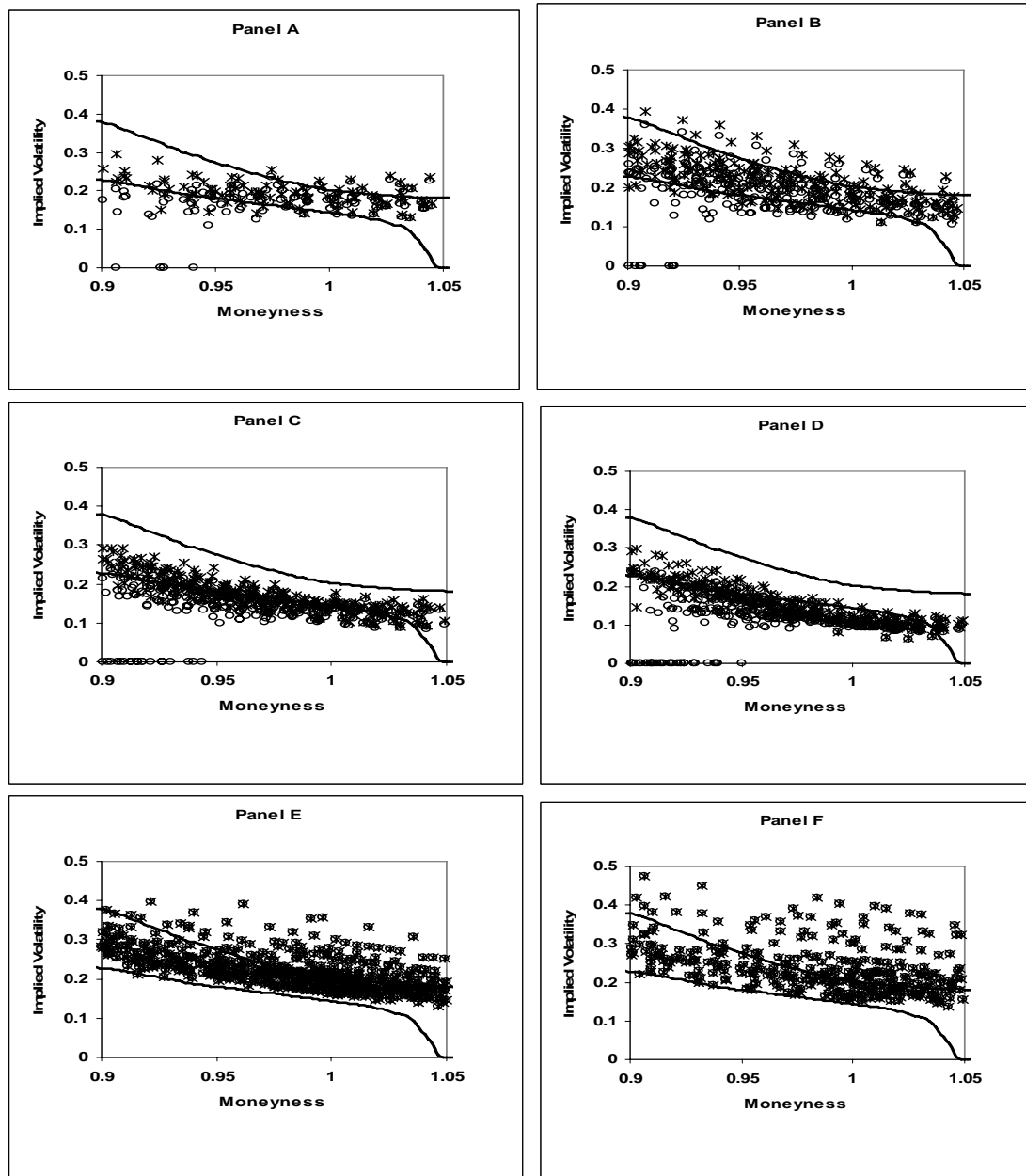
**Table 5. Percentage of Months without Stochastic Dominance Violations Using Conditional Implied-Volatility-Based Index Return Distributions with  $\pm 2\%$  Offset**

The table displays the percentage of months in which stochastic dominance violation is absent in the cross-section of option prices. The one-way transactions cost rate (one-way trading fees plus half the bid-ask spread) on the index is 50 bps. The one-way transactions cost on each option is proportional to the index price, as explained in Section 3.2, and 10 bps. All results use the conditional implied-volatility-based index return distributions based on the index sample 1972-2002. The four offsets are used to change the implied at-the-money volatility by -2, -1, 1, or 2%, annualized. The bold results “Best of Above” count a monthly cross section as feasible if feasibility is established either without implied vol offset or with any of the four offsets.

	Panel A: 860516- 871016	Panel B: 880715- 910315	Panel C: 910419- 930820	Panel D: 930917- 951215	Panel E: 970221- 991217	Panel F: 000218- 021220
Number of Months	15	29	28	26	35	33
Implied Vol - 2%	13	66	57	58	26	42
Implied Vol -1%	40	90	93	81	57	73
Implied Vol	87	93	96	85	57	79
Implied Vol + 1%	100	90	100	96	54	79
Implied Vol + 2%	100	86	100	96	49	73
<b>Best of Above</b>	<b>100</b>	<b>93</b>	<b>100</b>	<b>96</b>	<b>57</b>	<b>85</b>

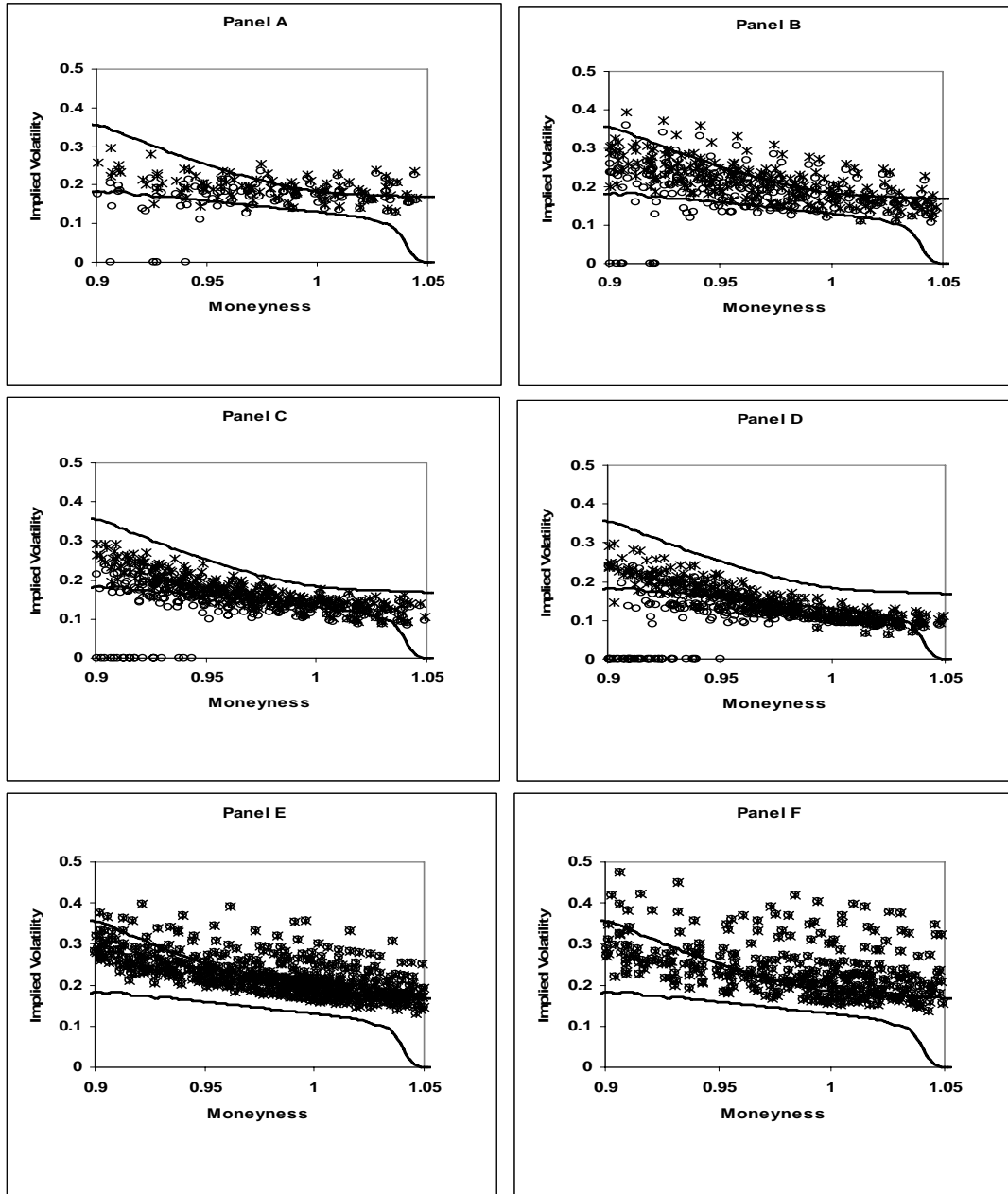
**Figure 1. Bound Violations Based on the Historical Index Sample 1928-1986**

The six panels display the upper and lower option bounds (implied volatilities) calculated with the index return distribution based on the historical index sample 1928-1986, as a function of the moneyness ( $K/S$ ). The figures also display the observed bid (circles) and ask (crosses) option implied volatilities over the pre-crash period (panel A) and the five post-crash periods (panels B-F). The transactions costs rate on the index is 20 bps.



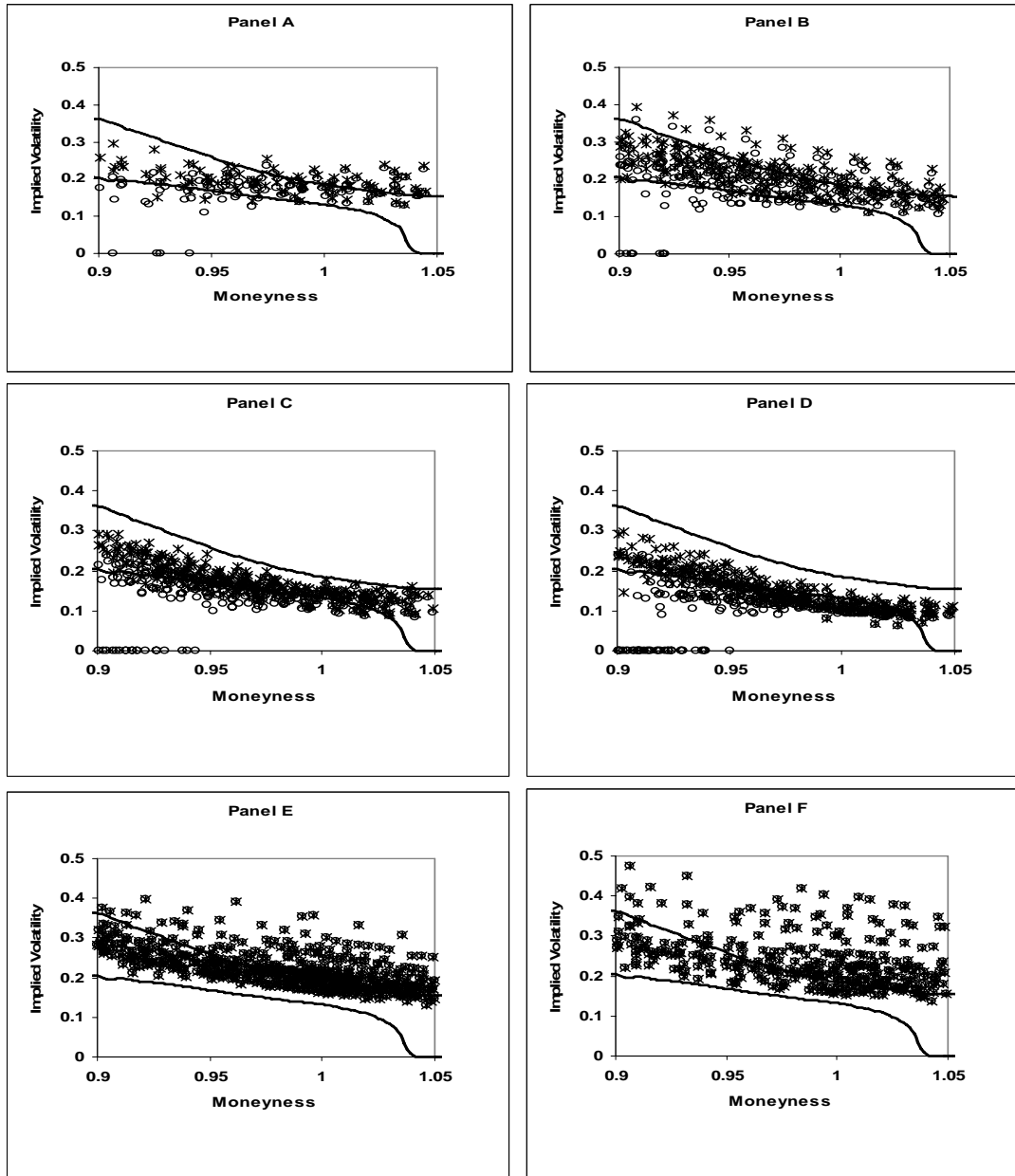
**Figure 2. Bound Violations Based on the Historical Index Sample 1972-1986**

The six panels display the upper and lower option bounds (implied volatilities) calculated with the index return distribution based on the historical index sample 1972-1986, as a function of the moneyness ( $K/S$ ). The figures also display the observed bid (circles) and ask (crosses) option implied volatilities over the pre-crash period (panel A) and the five post-crash periods (panels B-F). The transactions costs rate on the index is 20 bps.



**Figure 3. Bound Violations Based on the Forward-Looking Index Sample 1987-2002**

The six panels display the upper and lower option bounds (implied volatilities) calculated with the index return distribution based on the forward-looking index sample 1987-2002, as a function of the moneyness ( $K/S$ ). The figures also display the observed bid (circles) and ask (crosses) option implied volatilities over the pre-crash period (panel A) and the five post-crash periods (panels B-F). The transactions costs rate on the index is 20 bps.



**Figure 4. Bound Violations Based on the Forward-Looking Index Sample 1988-2002**

The six panels display the upper and lower option bounds (implied volatilities) calculated with the index return distribution based on the forward-looking index sample 1988-2002, as a function of the moneyness ( $K/S$ ). The figures also display the observed bid (circles) and ask (crosses) option implied volatilities over the pre-crash period (panel A) and the five post-crash periods (panels B-F). The transactions costs rate on the index is 20 bps.

