

# Corporate Real Estate Holdings and the Cross Section of Stock Returns\*

Selale Tuzel<sup>†</sup>

January 11, 2005

## Abstract

This paper explores the link between the composition of firm's capital holdings and stock returns. I develop a general equilibrium production economy where firms use two factors, real estate capital and other capital, and investment is irreversible. Real estate depreciates slowly, this makes real estate investment riskier than investment in other capital. Firms with high real estate holdings are extremely vulnerable to bad productivity shocks. In equilibrium, investors demand a premium to hold such a firm. This prediction is supported empirically: I find that the returns of firms with a high share of real estate capital exceed that for low real estate firms by 4-7% annually adjusted for exposures to the market return, size, value and momentum factors. The model also predicts countercyclical variation in the *aggregate* share of real estate in total capital, which is a moment of the state variables. A cross sectional investigation of the conditional CAPM, where the change in aggregate share of real estate in total capital is used as the conditioning variable, delivers substantially improved results over its unconditional version.

---

\*I am truly indebted to Mark Grinblatt and Monika Piazzesi for their generous support, guidance and encouragement. I would like to thank Tony Bernardo, Michael Brennan, Harold Cole, Lars Hansen, Ayse Imrohoroglu, Selahattin Imrohoroglu, Oguzhan Ozbas, Richard Roll, Pedro Santa-Clara, Martin Schneider, Avanidhar Subrahmanyam, seminar participants at UCLA and macro working group at the University of Chicago for their helpful comments and discussions. Special thanks to my fellow Ph.D. students, especially Bruno Miranda, for many insights and invaluable discussions. I gratefully acknowledge financial support from the UCLA Dissertation Year Fellowship.

<sup>†</sup>Corresponding Address: The Anderson School of Management, University of California - Los Angeles; 110 Westwood Plaza C413, Los Angeles, CA, 90095-1481; Phone: (323)304-3363; Email: selale.tuzel@anderson.ucla.edu.

# 1 Introduction

Firms own and use many different capital goods. Capital is heterogeneous, a building is not a computer. Even if in some extreme cases one can be substituted with the other one in the firm's production (Barnes&Noble versus Barnes&Noble.com), yet other characteristics still distinguish them, such as the rates of depreciation. Commercial real estate and equipment naturally emerge as two major classes of capital goods. Their dollar values in the U.S. economy are comparable. The Bureau of Economic Analysis (BEA) estimates approximately 7.2 trillion dollars worth of nonresidential structures (value of buildings excluding the value of the land), and 4.5 trillion dollars worth of nonresidential equipment at the end of 2003. Most firms own and use both capital types in their operations, however, there is considerable variation in firms' capital composition. When firms with both types of capital are sorted on the share of buildings in their total physical capital (PPE) into quintile portfolios, the average share of buildings for the firms in the first and the fifth portfolios are 9% and 57%, respectively, while the average firm's buildings account for approximately 30% of the firm's total physical capital. In addition to the obvious differences in their roles in the firm's operations, structures and equipment are different in their durability. Structures, on the average, depreciate much more slowly than equipment. Bureau of Economic Analysis rates of depreciation for private nonresidential structures range between 1.5-3%, whereas the depreciation rates for private nonresidential equipment are in the range of 10-30% (Fraumeni, 1997). Glaeser and Gyourko (2004) point out the extremely durable nature of residential real estate. Therefore, structures require less replacement investment than equipment. This introduces significant heterogeneity into the capital stock of firms. The value of a firm depends on the underlying value of its assets, i.e. its capital stock. Therefore, the dynamics of a firm's value (return) is fundamentally linked to the changes in the firm's capital stock, both its size and composition.

In this paper, I study the link between the composition of the firms' capital holdings<sup>1</sup> and stock returns. I specifically explore the role of real estate holdings in the firm's investment decisions and capital. I develop a general equilibrium model, where a representative agent invests in the firms in the economy, and consumes their dividends. The firms use two factors, real estate capital (buildings/structures)<sup>2</sup> and other capital (equipment); stochastic productivity shocks lead to heterogeneity among firms. Investment in either form of capital is irreversible. Numerical solutions of the model predict a "real estate premium", i.e. in equilibrium, investors demand a premium to hold a firm that owns a lot of real estate as part of its capital. I consequently verify this prediction with firm level data. I find that the returns of firms with a high share of real estate exceed that for low real estate firms by 4-7% annually adjusted for exposures to the market return, size, value and momentum factors. The model also predicts countercyclical variation in

---

<sup>1</sup>The composition of the firm's capital is different from the *composition risk* in Piazzesi, Schneider and Tuzel (2004). The composition risk, measured by the changes in the expenditure share of housing in household's consumption, is part of the pricing kernel in that paper.

<sup>2</sup>Throughout this paper, I use the real estate/buildings/structures terms interchangeably. The BEA reports the quantity of structures, whereas Compustat reports the value of buildings.

the *aggregate* share of real estate in total capital. This prediction is also supported by the data. A cross sectional investigation of the conditional CAPM, where the change in aggregate share of real estate in total capital is used as the conditioning variable, delivers substantially improved results over its unconditional version.

Firms accumulate capital through real investment. In the presence of capital heterogeneity, the real investment decisions determine not only the size of the firm's capital, but also its composition. If the capital holdings can be costlessly adjusted at any time, then the composition of the firm's capital becomes trivial. The firm always holds the *optimal* capital mix for a given level of output, i.e. the mix of capital inputs that minimizes the firm's costs for a given level of output. Nevertheless, capital adjustment is rarely costless, and the frictions in capital adjustment can distort the firm's investment decisions together with its capital composition. Costless adjustment of the capital holdings allows firms to pay a smooth dividend stream; therefore reduces their risk. Firms can accommodate exogenous productivity shocks by increasing/decreasing their investments and capital holdings, keeping their dividends relatively smooth. Frictions in capital adjustment reduces the flexibility of the firms to accommodate exogenous shocks. If capital is heterogeneous, the implications of frictions can vary among capital types. For an extreme example, take an agricultural firm with two types of capital; land, which does not depreciate, and perishable seeds, which completely depreciate from one season to the next. If there are frictions to reduce the capital stock, this friction will have an impact on land investment, whereas it will have no impact on investment in seeds.<sup>3</sup>

I assume a particular form of friction in capital adjustment, that investment is irreversible. Irreversible investment implies infinite adjustment cost while adjusting the capital stock downward, whereas increasing the capital stock incurs no adjustment cost. For the agricultural firm portrayed above, the implications of irreversible investment are starkly asymmetric: The firm with a lot of land dreads bad exogenous shocks, and tries to mitigate their effect by decreasing the investment in perishable seeds. This change in investment policy distorts the capital composition of the firm, increasing the share of land in the firm's capital holdings. Positive exogenous shocks have the opposite effect, reducing the share of land in the firm's capital. As the share of land increases, the firm becomes more vulnerable to bad productivity shocks, therefore the investors demand a premium to hold them in equilibrium.

The presence of irreversibility constraints leads the firms, on average, to invest less and hold less capital than they would otherwise. The firm anticipates that the irreversibility constraint may bind in the future, therefore is more hesitant to invest (Bertola, 1988; Pindyck, 1988; Dixit, 1989; Dixit and Pindyck, 1994; Abel and Eberly, 1999). In addition, the asymmetric depreciation rates of the factors distort the composition of capital holdings in favor of the one that depreciates faster, i.e. non real estate capital. When a firm receives bad productivity shocks, its capital holdings go down; however, the composition of factors get closer to their "optimal" levels (optimal if factors of production can be costlessly adjusted). This unintentional move in factor composition actually makes

---

<sup>3</sup>Friction in land adjustment will impact the investment in seeds, but this is a second order effect.

the firm more productive, especially when the good shocks arrive. However, this firm is risky, because high real estate holdings make the firm extremely vulnerable to bad shocks. Investment in the slowly depreciating factor (real estate) becomes a sunk cost in bad times, and pays off well in good times; therefore investors would demand a premium to hold a firm with high real estate holdings.

The risks associated with investing in and holding real estate capital is well understood and frequently mentioned in the business press:

A number of analysts express concerns about Hilton and Starwood in particular, because the two companies' real estate poses additional recession risks ... Owning hotels is more risky than managing or franchising them because of the cost of carrying and maintaining property ... Hilton in particular could be hard hit by the economic slowdown. Hilton owns many of its hotels, unlike Marriott, which mostly franchises and manages properties owned by others.

- WSJ, 3/26/01

Different business cycle implications of investment in real estate capital and other, less durable capital types are also cited in the business press:

Yet the aftereffects of overinvestment in technology are likely to be less pronounced than those of previous investment busts. In the 1980s, a frenzy of real estate investment saddled the U.S. with commercial office space that took years to fill. During that time, new investment in such properties almost ground to a halt. By contrast, business equipment and software depreciate in just a few years, if not months. Rapid depreciation means that any excess capacity should be eliminated relatively quickly.

- WSJ, 1/5/01

The paper proceeds as follows. Section 2 discusses the key elements of the model and related work. Section 3 presents the model and derives the pricing equations. Section 4 briefly explains the computational solution, which is detailed in Appendix A. Section 5 explains the quantitative results. Section 6 ties the quantitative results to the data. The paper is concluded in Section 7.

## **2 Key Elements and Related Work**

### **2.1 Capital Heterogeneity**

Many different capital inputs enter the firm's production process. Nevertheless, capital is overwhelmingly modelled as homogeneous. Even though it is convenient to assume homogeneity of capital, this assumption implies that different capital goods are perfect

substitutes; i.e. workstations can be replaced by forklifts. The perfect substitutability assumption is typically rejected by the data, and the degree of substitutability is different across capital types (Denny and May, 1978).

There is a small literature on investment with capital heterogeneity. Samuelson (1961-1962), in a highly stylized economy where the same ratio of inputs are maintained across consumption and capital goods industries, finds that the heterogeneous capital goods can be reduced to a homogeneous capital good. Garegnani (1970) shows that equal proportions of inputs assumption is crucial for Samuelson's (1961-1962) results, and this assumption practically turns the heterogeneous capital economy to an economy with homogeneous capital and consumption good, the two being perfectly substitutable. Several papers study the aggregation problem of multiple capital goods in the presence of adjustment costs (Blackorby and Schworm, 1983; Epstein, 1983; Wildasin, 1984). Their common result is that, one has to impose very stringent set of assumptions in order to aggregate heterogeneous capital inputs into a homogeneous input as a weighted sum of multiple capital inputs. Wildasin (1984) extends the  $q$  theory of investment to the general case of multiple capital goods, and derives a relationship between  $q$  and the vector of investments in multiple capital goods. Chirinko (1993) uses Wildasin's (1994) result to estimate an investment equation relating  $q$  to investment in multiple capital goods. Epstein (1983) provides a model of optimal capital allocation with the assumption that the capital inputs are weakly separable, where multiple capital inputs can be aggregated into a capital aggregate in the form of a scalar index of multiple capital inputs. Hayashi and Inoue (1991) use this measure of capital aggregate, as opposed to the sum of nominal capital stocks, to estimate the relation between investment and  $q$  using a panel of data disaggregated to capital types from Japanese manufacturing firms.

Goolsbee and Gross (1997) use firm level data disaggregated to capital types from the airline industry to study the adjustment costs. They find that airlines have a significant region of inaction while adjusting their capital stocks, and aggregating at the firm level leads to disappearance of inaction region and an upward bias in adjustment costs. Doms and Dunne (1998) and Nilsen and Schiantarelli (2003), using plant level data disaggregated to capital types from a diverse set of industries from the U.S. and Norway, respectively, have similar conclusions with respect to the smoothing effects of aggregating data at the firm level. Cummins and Dey (1998), using firm level data from Compustat, also find that when capital heterogeneity is ignored, estimates of adjustment costs are upward biased and estimates of factor substitution in production are downward biased. Abel and Eberly (2002) also use panel data from Compustat to estimate several models of optimal investment, one of which incorporates capital heterogeneity and fixed adjustment costs. They find that firms do not choose to invest in all types of capital every period.

## 2.2 Irreversible Investment

Investment is frequently modelled as "irreversible" in the real investment literature (Pindyck, 1988; Dixit, 1989; Dixit and Pindyck, 1994; and many others). Recently,

several papers have studied the asset pricing implications of models with irreversible investment (Cooper, 2003; Gomes, Kogan and Zhang, 2003; Kogan, 2004; Berk, Green and Naik, 1999). Even though irreversible investment seems like an extreme assumption, Abel and Eberly (1994) show that disinvestment is never optimal if the resale price of capital is low enough relative to its purchase price;<sup>4</sup> therefore, investment is practically "irreversible". Cooper and Haltiwanger (2002) take it one step further, and report that even in the absence of frictions in the secondary markets, a modest amount of convex adjustment costs induce complete irreversibility of investment. Ramey and Shapiro (2001), collecting and analyzing data from aerospace industry auctions, find that reallocating capital entails substantial costs due to loss of value and time. They estimate that the average market value of equipment sold in auctions is 28 cents per dollar of replacement cost. Many factors contribute to the low resale prices for capital, including capital specificity, thin markets and adverse selection problems. Furthermore, Eisfeldt and Rampini (2003) find that capital reallocation is procyclical, even though the benefits to reallocation are countercyclical. Firms are stuck with excess capital when they most need to reverse their investments, which is during economic downturns.

Empirical evidence supports the hypothesis that investment is mostly irreversible. Cooper and Haltiwanger (2002), using a large panel of plant level data from Longitudinal Research Database, fit hybrid models of adjustment costs having both convex and non-convex cost components, including irreversible investment. They find that the models with the best fit imply complete or near complete investment irreversibility. Nilsen and Schiantarelli (2003), using plant level panel data from Norway, find evidence for irreversibility in both equipment and building investments. They also find that aggregating across different capital goods leads to a relatively smooth investment profile by shadowing the intermittent character of each type of investment. Leahey and Whited (1996) study the relationship between uncertainty and investment using panel data on individual firms. They find that increases in the uncertainty the firm faces decreases the firm's investment. They conclude that irreversible investment is the most likely explanation behind this stylized fact.

Real estate is different from equipment due to the presence of more established secondary markets<sup>5</sup> for real estate. Yet, investment in real estate can be highly irreversible. Ramey and Shapiro (2001), in one of the examples to motivate their study, report that the (now relocated) building of the Department of Economics at the University of California, Riverside has been converted from a motel. Complete bathrooms in each office and swimming pool in the courtyard certainly contributes less to the productivity of an educational institution than they would to the value of a lodging; the value of these amenities to the Economic department will naturally be lower than their replacement value. Abandoned industrial and commercial buildings, especially in downtowns around the Midwest, are examples of "irreversible" real estate investments. Furthermore, these

---

<sup>4</sup>Ramey and Shapiro (2001) find that the resale price of capital is frequently significantly lower than the replacement cost. Analyzing data from aerospace industry auctions, they estimate that the average market value of equipment sold in auctions is less than one third of their replacement cost.

<sup>5</sup>Some types of equipments, such as photocopy machines, laboratory equipments, microscopes, ... also have relatively established secondary markets.

abandoned buildings cannot be freely disposed, and they generally create hazardous environments for the communities around. In some cases, even the investment in "land" is considered irreversible. United States Environmental Protection Agency estimates that there are currently more than 450,000 brownfields in the U.S..<sup>6</sup> Recovering the land of these brownfields requires a careful and expensive cleanup effort, if it can be done at all.<sup>7</sup>

## 2.3 Related Work

To the best of my knowledge, there is no prior work investigating the implications of capital heterogeneity within the firm in the asset pricing context. A somewhat related line of literature is concerned with intangible capital (Hall, 2001; Hansen, Heaton and Li, 2004; Cummins, 2003; Li, 2004). Even though the existence and importance of intangible capital is widely agreed upon, interpreting and accounting for intangible capital is inherently difficult. Interpretations of intangible capital range from being a capital input in addition to physical capital to being a form of adjustment cost. Several papers attempt to measure the aggregate value of intangible capital in the U.S. economy (Atkeson and Kehoe, 2002; Hall, 2001; Li, 2004). Considering the difficulties with interpreting, measuring and modeling intangible capital, I choose to concentrate on heterogeneity in physical capital.

The interactions between business cycles and asset returns are studied in a strand of papers with production economies. The early studies (Danthine, Donaldson and Mehra, 1992; Rouwenhorst, 1995) find that standard one-sector business cycle models have counterfactual asset pricing implications, despite their relative success at explaining key business cycle facts. Jermann (1998) introduces capital adjustment costs to the standard business cycle model to mitigate the endogenous consumption smoothing mechanism inherent in production economies. Boldrin, Christiano and Fisher (2001) considers a two sector economy with limited labor mobility. The two sector model is designed to make the short term supply of capital completely inelastic, limiting the firm's ability to smooth its dividend stream. Both of these papers consider habit formation preferences, which generate a time varying risk premium.

The link between real investment and stock returns is explored by Cochrane (1991, 1996). Cochrane considers a production based asset pricing model with quadratic adjustment costs where the first order conditions of the producers describe the relationship between asset returns and real investment returns in a partial equilibrium framework. Cochrane (1991) predicts a contemporaneous relationship between asset returns and investment returns, acknowledging that if there are lags in the investment process, then

---

<sup>6</sup>A brownfield is a property, the expansion, redevelopment, or reuse of which may be complicated by the presence or potential presence of a hazardous substance, pollutant, or contaminant.

<http://www.epa.gov/brownfields/about.htm>

<sup>7</sup>A classic example is the Alcoa Plant on the New Jersey side of the Hudson River. Covering an area of more than one million square feet on a highly valued land, it has been closed since 1964 and the building remains contaminated with PCBs, highly toxic compounds that are now banned in the U.S..

<http://www.modern-ruins.com/alcoa/index.html>

the investment plans (rather than current investment returns) should covary with asset returns. Lamont (2000) reports that due to lags in investment, contemporaneous investment returns and stock returns have a strong negative covariation, however the investment *plans* strongly covary with asset returns. In their empirical analysis, Cochrane (1991) uses private domestic investment and Lamont (2000) uses private nonresidential investment data, and neither of them distinguishes between investment in different types of capital. Cochrane (1996) considers a two factor asset pricing model where the factors are returns to residential and nonresidential investment, tries to explain the cross sectional differences in asset returns. Based on Cochrane's production based model, Li, Vassalou and Xing (2003) perform an empirical investigation of asset returns with a four factor model, where the factors are investment growth rates in different sectors of the economy.

Recently, several papers investigate production based models with capital adjustment frictions in an attempt to link stock returns to a firm characteristics, the B/M (book value of equity/market value of equity) ratio (Cooper, 2003; Zhang, 2003; Kogan, 2004, Gourio, 2004). Their general idea is that, firms with high B/M ratios are burdened with excess capital in bad times. Frictions in capital adjustment mechanisms (irreversibilities, costly reversibility) prevent the firms from achieving their desired capital holdings, leading to discrepancies between the market and book values of assets and time varying stock returns. These papers mainly differ along the frictions they assume in capital adjustment mechanisms. Kogan (2004) assumes that investment is irreversible and subject to convex adjustment costs. Cooper (2003) considers the nonconvex (fixed) adjustment costs in addition to irreversible investment. Zhang (2003) assumes convex but asymmetric adjustment costs; firms face higher adjustment costs while cutting their capital compared to capital expansions. Gourio (2004) works with a putty clay model, where investment is irreversible, and the ratio of factors used in production is fixed once the capital is installed.

Even though real estate is an important component of aggregate wealth, it is generally omitted from the empirical and theoretical work in the asset pricing literature. There are a few notable exceptions, including Stambaugh (1982), Kullman (2003), Flavin and Yamashita (2002), Piazzesi, Schneider and Tuzel (2004) and Lustig and Nieuwerburgh (2004). Stambaugh (1982) constructs market portfolio as a combination of several asset groups, some of which includes proxies for residential real estate. Kullman (2003) includes measures of both residential real estate returns and commercial real estate returns (as measured from REITs) in addition to proxies for returns to human capital and stock market returns in the market portfolio. Stambaugh (1982) finds that the ability of the CAPM to explain the cross section of returns is insensitive to the construction of the market portfolio. Kullman (2003), on the other hand, finds that returns to residential real estate is significant in explaining the cross section of stock returns, whereas the returns to commercial real estate is insignificant. Flavin and Yamashita (2002) consider portfolio choice with exogenous returns in the presence of housing. Piazzesi, Schneider and Tuzel (2004) construct an equilibrium asset pricing model with housing, and show that the composition of the consumption bundle appears in the pricing kernel, and matters for

asset pricing. The expenditure share of housing predicts stock returns. Lustig and Nieuwerburgh (2004) find that the ratio of housing wealth to human wealth is related to the market price of risk, therefore has asset pricing implications.

Deng and Gyourko (1999) study the empirical relationship between real estate ownership by non-real estate firms and firm returns. They find that firms with high degrees of real estate concentration and high levels of risk as measured by beta experience lower returns. However, their measure of real estate concentration, PPE/Assets, does not measure the share of real estate in the firm's physical capital. Their ratio measures the ratio of physical assets in the firms total assets, this is what Braun (2003) calls the "tangibility" of firm. Braun (2003) finds that tangibility of firms is related to their financing possibilities and leverage; tangible firms find it easier to raise debt financing, and have higher leverage.

### 3 Setup

The economy is populated with many infinitely lived identical agents, who maximize expected discounted utility. There is a single consumption/investment good that is produced by two firms that use two types of capital. The investment is irreversible.

#### 3.1 Firms

There are two firms that produce a homogeneous good. The firms use two types of capital: real estate capital (such as buildings) and other capital. The firms are subject to different productivity shocks. The investment in either form of capital is irreversible. Real estate depreciates at rate  $\mu$  and other capital depreciates at rate  $\delta$ . I assume that real estate depreciates slower than other capital ( $\mu < \delta$ ). This assumption is motivated by the depreciation rates in the BEA tables (1-3% for nonresidential structures, 10-30% for nonresidential equipment, annually).

The production function for firm  $i = 1, 2$  is given by:

$$\begin{aligned} Y_{it} &= F(Z_{it}, K_{it}, H_{it}, L_{it}) \\ &= Z_{it}(K_{it}^{\alpha_1} H_{it}^{\alpha_2})^\alpha L_{it}^{1-\alpha} \end{aligned}$$

$H_{it}$  and  $K_{it}$  denote the beginning of period  $t$  stock of real estate (buildings) and other capital, respectively, where  $h_{it} = \log(H_{it}) \in [\underline{h}, \bar{h}]$  and  $k_{it} = \log(K_{it}) \in [\underline{k}, \bar{k}]$ ,  $\alpha, \alpha_1$  and  $\alpha_2 \in (0, 1)$ .  $L_{it}$  denotes the labor used in production, which is supplied inelastically to each firm and is normalized to 1. The firm productivity, denoted  $z_{it} = \log(Z_{it})$ , has a stationary and monotone Markov transition function, denoted  $p_{z_i}(z_{i,t+1}|z_{it})$ , as follows:

$$z_{i,t+1} = \rho_z z_{it} + \sigma_\varepsilon \varepsilon_{i,t+1}^z \tag{1}$$

where  $\varepsilon_{i,t+1}^z$  is IID normal shock and the correlation between  $\varepsilon_{i,t+1}^z$  and  $\varepsilon_{j,t+1}^z$  is  $\rho_\varepsilon$  for any pair  $(i, j)$  with  $i \neq j$ .  $\rho_\varepsilon > 0$  implies that there exists an aggregate productivity shock in the economy.

The investment is assumed to be irreversible, i.e. gross investment in either type of capital is non-negative:

$$\begin{aligned} K_{i,t+1} - (1 - \delta)K_{it} &\geq 0 \\ H_{i,t+1} - (1 - \mu)H_{it} &\geq 0 \end{aligned} \quad (2)$$

for  $i \in \{1, 2\}$ .

Firms are equity financed. The dividends to shareholders are equal to:

$$D_{it} = Y_{it} - [K_{i,t+1} - (1 - \delta)K_{it} + H_{i,t+1} - (1 - \mu)H_{it}] - w_{it}L_{it} \quad (3)$$

where  $w_{it}$  is the wage payments to labor services. Wage payments are determined by the marginal product of labor.

At each date  $t$ , firms choose  $\{K_{i,t+1}, H_{i,t+1}\}$  to maximize the net present value of their expected dividend stream:

$$E_t \left[ \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} D_{i,t+k} \right] \quad (4)$$

for  $i \in \{1, 2\}$ , subject to (Eq.1-2), where  $\frac{\beta^k \Lambda_{t+k}}{\Lambda_t}$  is the marginal rate of substitution of the firm's owners between time  $t$  and  $t + k$ .

One way to interpret the firms in this setting is to view them as regional economies. Since the agents do not derive utility from leisure, the people who live in one region inelastically supply labor services to firms operating in that region. It is also easier to interpret the irreversible investment assumption when the firms operate in different regions. The firms that operate in different regions would not be interested in trading capital, especially buildings, since it is not possible to make this capital productive to a firm that operates in another region.

Let  $\lambda_{it}$  and  $\nu_{it}$  denote the Lagrange multipliers on investment irreversibility constraints (Eq.2) for  $i = 1, 2$ . The Kuhn-Tucker conditions for the firm's optimization problem are:

$$\Lambda_t - \lambda_{it} = \int \int \beta [\Lambda_{t+1}(F_{K_{i,t+1}} + 1 - \delta) - (1 - \delta)\lambda_{i,t+1}] p_{z_1}(z_{1,t+1}|z_{1t}) p_{z_2}(z_{2,t+1}|z_{2t}) d_{z_1} d_{z_2} \quad (5)$$

$$\Lambda_t - \nu_{it} = \int \int \beta [\Lambda_{t+1}(F_{H_{i,t+1}} + 1 - \mu) - (1 - \mu)\nu_{i,t+1}] p_{z_1}(z_{1,t+1}|z_{1t}) p_{z_2}(z_{2,t+1}|z_{2t}) d_{z_1} d_{z_2}$$

$$\begin{aligned} \lambda_{it}[K_{i,t+1} - (1 - \delta)K_{it}] &= 0 \\ \nu_{it}[H_{i,t+1} - (1 - \mu)H_{it}] &= 0 \\ \lambda_{it}, \nu_{it} &\geq 0 \end{aligned} \quad (6)$$

and irreversibility constraints, Eq.2, where

$$\begin{aligned} F_{K_{it}} &= F_K(Z_{it}, K_{it}, H_{it}, L_{it}) \\ F_{H_{it}} &= F_H(Z_{it}, K_{it}, H_{it}, L_{it}) \end{aligned}$$

Tobin's  $q$  ( $q_{k_{it}}, q_{h_{it}}$ ), the consumption cost of capital, is defined as the marginal value of each type of capital ( $K_{i,t+1}, H_{i,t+1}$ ) to the firm,  $\Lambda_t - \lambda_{it}$  and  $\Lambda_t - \nu_{it}$ , divided by the marginal cost,  $\Lambda_t$ .

$$\begin{aligned} q_{k_{it}} &= 1 - \frac{\lambda_{it}}{\Lambda_t} \\ q_{h_{it}} &= 1 - \frac{\nu_{it}}{\Lambda_t} \end{aligned} \tag{7}$$

A little algebra on (5) leads us to:

$$\begin{aligned} 1 &= \int \int \frac{\beta \Lambda_{t+1}}{\Lambda_t} \frac{F_{K_{i,t+1}} + (1 - \delta)q_{k_{it+1}}}{q_{k_{it}}} p_{z_1}(z_{1,t+1}|z_{1t}) p_{z_2}(z_{2,t+1}|z_{2t}) d_{z_1} d_{z_2} \\ 1 &= \int \int \frac{\beta \Lambda_{t+1}}{\Lambda_t} \frac{F_{H_{i,t+1}} + (1 - \mu)q_{h_{it+1}}}{q_{h_{it}}} p_{z_1}(z_{1,t+1}|z_{1t}) p_{z_2}(z_{2,t+1}|z_{2t}) d_{z_1} d_{z_2} \end{aligned} \tag{8}$$

Multiplying both sides with  $K_{i,t+1}$  and  $H_{i,t+1}$ , respectively, rearranging, and adding the equations:

$$\begin{aligned} &q_{k_{it}} K_{i,t+1} + q_{h_{it}} H_{i,t+1} \\ &= \int \int \frac{\beta \Lambda_{t+1}}{\Lambda_t} [\alpha Y_{i,t+1} + (1 - \delta)K_{i,t+1} q_{k_{it+1}} + (1 - \mu)H_{i,t+1} q_{h_{it+1}}] p_{z_1}(z_{1,t+1}|z_{1t}) p_{z_2}(z_{2,t+1}|z_{2t}) d_{z_1} d_{z_2} \end{aligned} \tag{9}$$

The (end of period) value of a firm's equity ( $V_{it}$ ) is equal to the market value of its assets in place:

$$V_{it} = q_{k_{it}} K_{i,t+1} + q_{h_{it}} H_{i,t+1} \tag{10}$$

Replacing equations (10) and (3) in (9) gives the standard Euler equation:

$$1 = \int \int \frac{\beta \Lambda_{t+1}}{\Lambda_t} \frac{V_{i,t+1} + D_{i,t+1}}{V_{it}} p_{z_1}(z_{1,t+1}|z_{1t}) p_{z_2}(z_{2,t+1}|z_{2t}) d_{z_1} d_{z_2} \tag{11}$$

for  $i \in \{1, 2\}$ .

### 3.2 Households

The households maximize expected discounted utility. Preferences over consumption take the standard form:

$$E_t \left[ \sum_{k=0}^{\infty} \beta^k u(C_{t+k}) \right] \text{ with } u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{12}$$

I assume that a complete set of contingent claims are marketed. Risk averse agents trade in claims to their individual labor income, so that their intertemporal marginal rate of substitutions are equated. Therefore, the households can be aggregated into a representative agent. The representative agent invests in a one-period riskless discount bond in zero net supply and two risky assets, the equity of firms. At every date  $t$ , the representative agent satisfies the following budget constraint:

$$b_{t+1} \frac{1}{1+r_t^f} + s_{1,t+1} V_{1t} + s_{2,t+1} V_{2t} + C_t \leq s_{1t}(V_{1t} + D_{1t}) + s_{2t}(V_{2t} + D_{2t}) + b_t + w_{1t} + w_{2t} \quad (13)$$

where  $b_{t+1}$ ,  $s_{1,t+1}$  and  $s_{2,t+1}$  denote period  $t$  acquisition of riskless bond and risky assets; and  $q_{r^f t}$ ,  $V_{1t}$ ,  $V_{2t}$  denote their prices, respectively.  $D_{1t}$  and  $D_{2t}$  denote period  $t$  dividends of the risky assets as defined in the previous section. At each date  $t$ , the agent chooses  $\{b_{t+1}, s_{1,t+1}, s_{2,t+1}, C_t\}$  to maximize (12) subject to (13).

The first order conditions for the representative agent's optimization problem are:

$$\begin{aligned} \frac{1}{1+r_t^f} &= E_t \left[ \frac{\beta u_C(C_{t+1})}{u_C(C_t)} \right] \\ 1 &= E_t \left[ \frac{\beta u_C(C_{t+1})}{u_C(C_t)} \frac{V_{i,t+1} + D_{i,t+1}}{V_{it}} \right] \end{aligned} \quad (14)$$

for  $i \in \{1, 2\}$ .

### 3.3 Equilibrium

The vector of endogenous state variables for the economy is denoted  $s = [k_1, k_2, h_1, h_2]$ . The vector of exogenous state variables is denoted  $z = [z_1, z_2]$ . A competitive equilibrium consists of consumption function,  $C(s, z)$ ; policy functions,  $k'_i(s, z)$ ,  $h'_i(s, z)$ ; Lagrange multiplier functions,  $\lambda_i(s, z)$ ,  $\nu_i(s, z)$ ; price functions for installed capital,  $q_{k_i}(s, z)$ ,  $q_{h_i}(s, z)$ ; price functions for firms,  $V_i(s, z)$ ; dividend functions,  $D_i(s, z)$ , for  $i = 1, 2$ ; risk free rate,  $r^f(s, z)$ , that solve the firms' optimization problems (maximize Eq.4 subject to Eq.1-2), solve the representative agent's optimization problem (maximize Eq.12 subject to Eq.13), and satisfy the aggregate resource constraint:

$$\begin{aligned} C(s, z) + \exp(k'_1(s, z)) + \exp(k'_2(s, z)) + \exp(h'_1(s, z)) + \exp(h'_2(s, z)) \\ \leq y_1 + y_2 + (1 - \delta) \exp(k_1) + (1 - \delta) \exp(k_2) + (1 - \mu) \exp(h_1) + (1 - \mu) \exp(h_2) \end{aligned}$$

## 4 Computational Solution

The model cannot be solved analytically. I therefore use numerical techniques. I solve the Euler equations (Eq.5) using the parameterized expectations algorithm (PEA) by Marcet

(1998). My computational solution closely follows the steps of the Chebyshev PEA algorithm in Christiano and Fisher (2000). The basic idea in the PEA is to substitute the conditional expectations that appear in the equilibrium conditions with parameterized functions of the state variables. The conditional expectation is parameterized using an exponentiated polynomial, where the exponential guarantees nonnegativity. I use Chebyshev polynomials as the basis functions for the polynomial. Once the conditional expectation function is approximated, the policy variables and the Lagrange multipliers can be expressed as functions of the approximated conditional expectations. The details of the solution are left to the Appendix.

PEA easily accommodates the irreversibility constraints in the model. Alternatively, one can parameterize the policy functions together with the Lagrange multiplier functions (which would lead to indirect parameterization of the conditional expectations); however, this would be a significantly more cumbersome computation. In addition, Christiano and Fisher (2000), studying a simple stochastic growth model with irreversible investment, find that the conditional expectation function is smoother than other functions characterizing the solution, such as the policy function, therefore parameterizing conditional expectations leads to more accurate approximations.<sup>8</sup> In the presence of irreversibility constraints, policy functions will have kinks due to occasionally binding constraints, which makes parameterizing the policy functions especially undesirable.

## 5 Quantitative Results

I consider asset pricing in a simple production economy with two types of capital (buildings and equipment) and irreversible investment. I am particularly interested in whether the *composition* of the capital bundle matters for asset pricing.

The presence of heterogeneous firms in the economy allows me to study the cross sectional implications. The firms receive different, but correlated productivity shocks, which, over time, lead to heterogeneity between them. Through simulations of the model economy, I show that the productivity shocks effect firms' investment decisions and lead to changes in firms' capital compositions. The composition of the capital bundle determines the *flexibility* of firms to accommodate future productivity shocks. As the share of buildings in the capital bundle of a firm increases, the firm gets less flexible to accommodate bad shocks in the future, i.e. gets *riskier*. I demonstrate that these riskier firms indeed earn higher returns in equilibrium.

Table 1 presents the parameters used in the simulations of the model economy. For the parameters that previous empirical studies guide us, I use the suggestions of those studies. The model is not calibrated to match any benchmark result. The capital share

---

<sup>8</sup>Christiano and Fisher (2000) solve a simple stochastic growth model with irreversibility constraints using six algorithms, including the version of the PEA I am using (which they call Chebyshev PEA). They find that Chebyshev PEA dominates other methods in terms of accuracy, speed and programmer time.

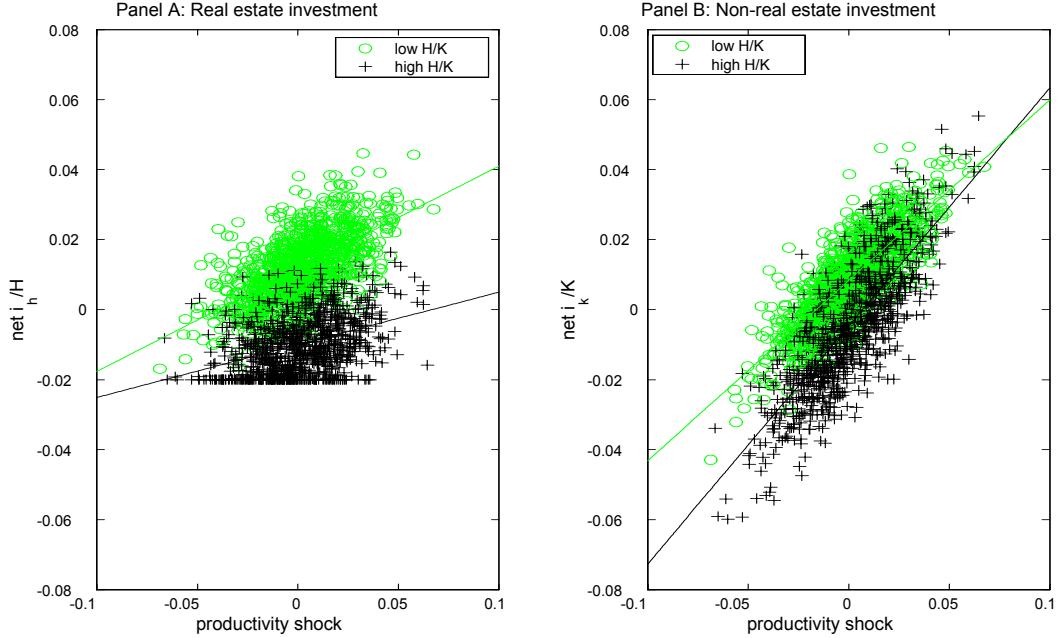


Figure 1: Net investment ratios vs. Productivity shock

$\alpha$  is set to 0.3, which is roughly in line with the values used in previous studies. Similarly, the time discount factor  $\beta$  is set to 0.99 (annually), and the coefficient of relative risk aversion  $\gamma$  is set to 1. The depreciation rates,  $\delta$  and  $\mu$ , are set to 0.12 and 0.02 for equipment and buildings, respectively. These are roughly the average BEA depreciation rates for equipment and buildings. I find that the average share of buildings in the firm's total plant, property and equipment in the firm's balance sheet is approximately 0.3. Therefore, I set the share of equipment  $\alpha_1$  to 0.7, and the share of buildings  $\alpha_2$  to 0.3. The coefficients regarding the productivity of firms are chosen somewhat arbitrarily.

Table 1: Parameter Values

| $\alpha$ | $\alpha_1$ | $\alpha_2$ | $\delta$ | $\mu$ | $\beta$ | $\rho_z$ | $\gamma$ | $\sigma_\varepsilon$ | $\rho_\varepsilon$ |
|----------|------------|------------|----------|-------|---------|----------|----------|----------------------|--------------------|
| 0.3      | 0.7        | 0.3        | 0.12     | 0.02  | 0.99    | 0.8      | 1        | 0.02                 | 0.5                |

Figure 1 plots the net investment rates for buildings and equipment,  $\frac{i_h}{H}$  and  $\frac{i_k}{K}$ , where  $i_{h_t} = H_{t+1} - H_t$  and  $i_{k_t} = K_{t+1} - K_t$ , as functions of the productivity shock. Every period, I compute the buildings/equipment ( $H/K$ ) ratios for firms. In the  $H/K$  ratio calculation I use the quantities of capital, which is equivalent to the book values in the model. In order to visually illustrate the differences between the high  $H/K$  and the low  $H/K$  firms, I only plot the firms in the highest and lowest 5 percentile with respect to their  $H/K$  ratios.

Figure 1 illustrates several differences between the policies of firms with high  $H/K$  and low  $H/K$  ratios. The high  $H/K$  firms occasionally hit the irreversibility constraint

in real estate investment when they receive bad productivity shocks. The investment in equipment is more responsive to the productivity shocks and fluctuates more than investment in buildings. The equipment investments of high  $H/K$  firms are even more responsive to the productivity shocks than the low  $H/K$  firms. When a firm receives bad productivity shocks, its equipment stock diminishes quickly, with little reduction in its stock of buildings, leading to a higher  $H/K$  ratio. Therefore, firms that experience a sequence of bad productivity shocks have high  $H/K$  ratios, and firms that experience a sequence of good productivity shocks have low  $H/K$  ratios.

I am interested in the rate of return  $r_i^s$  on each firm and the riskless borrowing rate  $r^f$  in the economy. They are defined as:

$$r_{i,t+1}^s = \log \left( \frac{V_{i,t+1} + D_{i,t+1}}{V_{it}} \right)$$

$$r_t^f = -\log E_t \left[ \frac{\beta u_C(C_{t+1})}{u_C(C_t)} \right]$$

In addition to the raw asset returns, I am interested in the "real estate premium" generated in this model economy. Every period, I compute the buildings/equipment ( $H/K$ ) ratios for firms, and label the firm with higher  $H/K$  ratio as high  $H/K$  and the firm with lower ratio as low  $H/K$  firm. At the beginning of each period, I form a synthetic "real estate" portfolio ( $HMK$ ) by buying the higher  $H/K$  firm, and selling the lower  $H/K$  firm. I present the first two moments of the asset returns in Table 2.

Table 2: Model Implied Moments of Asset Returns (% , annualized)

|          | $r^f$ | $r^s$ | $r^e$ | $r_{t \rightarrow t+1}^{HMK}$ | $r_{t \rightarrow t+3}^{HMK}$ | $r_{t \rightarrow t+5}^{HMK}$ |
|----------|-------|-------|-------|-------------------------------|-------------------------------|-------------------------------|
| $\mu$    | 1.03  | 1.64  | 0.61  | 0.04                          | 0.08                          | 0.09                          |
| $\sigma$ | 0.20  | 0.24  | 0.15  | 0.36                          | 0.27                          | 0.24                          |

Note: The table reports the means and standard deviations of the risk free rate ( $r^f$ ), the raw stock returns ( $r^s$ ), the excess stock returns ( $r^e$ ) and the real estate portfolio ( $r^{HMK}$ ). The stock returns are the average of the returns of the two firms.

The  $H/K$  spread is countercyclical, the difference between the real estate ratios of firms increase in bad times and decrease in good times. Figure 1 illustrates that, in good times, low  $H/K$  firms invest more in buildings than high  $H/K$  firms, decreasing the spread in  $H/K$ . In bad times, both type of firms have negative net investment. However, the irreversibility constraint limits the real estate disinvestment of high  $H/K$  firms, and the constrained firms disinvest a disproportionately large amount of non-real estate capital, widening the spread in  $H/K$ . The negative correlation coefficient between the  $H/K$  spread and the total output ( $\sim -0.3$ ) confirms that the  $H/K$  spread is countercyclical.

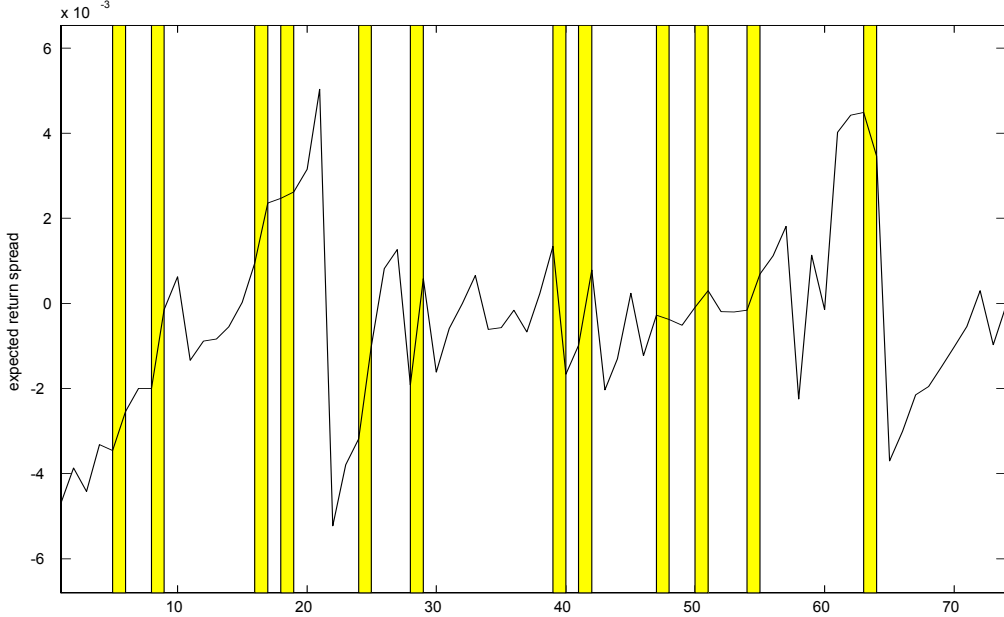


Figure 2: Expected Return Spread, sample 75 year period. The darker columns indicate periods in which output growth is more than one standard deviation below its mean.

The expected real estate premium is also countercyclical. During recessions, firms with high real estate holdings are hit particularly bad, therefore are riskier than the firms with low real estate holdings. Therefore, investors expect higher premium from such firms during recessions. Figure 2 demonstrates this feature of the economy. In Figure 2, I simulate the economy for 75 periods and plot model generated expected returns of the  $HMK$  portfolio (expected return spread) against time. The length of simulation corresponds to 1929-2003 period, for which I plot several empirical and model generated graphs below. The darker columns in the figure represent periods in which the output growth is more than one standard deviation below its mean (to proxy for recessions). The expected return spread generally rises during *recessions*. The negative correlation coefficient between the expected real estate premium and the total output ( $\sim -0.4$ ) confirms that the expected real estate premium is countercyclical.

In this framework, aggregate  $H/K$  ratio  $\left(\frac{\sum_i H_i}{\sum_i K_i}\right)$  arises as an important variable. It is already established that the  $H/K$  ratios of firms increase with bad productivity shocks and decrease with good productivity shocks. Figure 3 indicates that the aggregate  $H/K$  ratio is highly countercyclical. Aggregate  $H/K$  ratio is a moment of state variables in the economy, and it is informative about the state of the economy. Later, empirical results will confirm that aggregate  $H/K$  ratio indeed captures important information about the economy.

In Figure 4, I simulate the economy for 75 periods and plot model generated aggregate  $H/K$  ratio against time. Similar to Figure 2, the length of simulation corresponds to 1929-2003 period, for which empirical  $H/K$  ratio is calculated and plotted in Section 6.

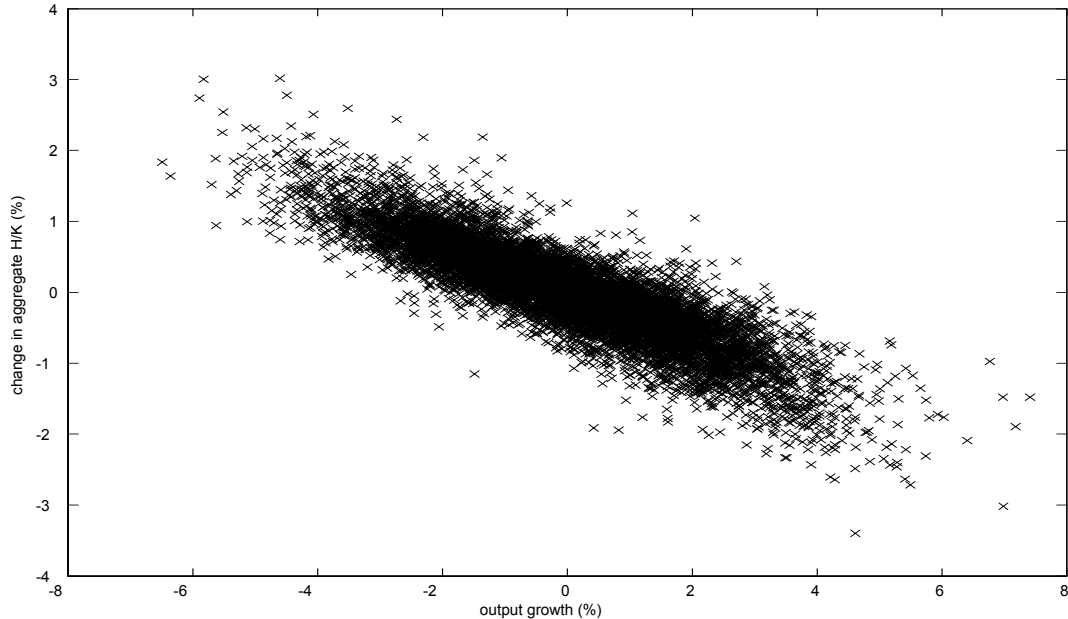


Figure 3: Change in aggregate H/K vs. Output growth

The darker columns in the figure represent periods in which the output growth is more than one standard deviation below its mean (to proxy for recessions). The aggregate  $H/K$  ratio rises during *recessions*, since the firms find it difficult to reduce their real estate holdings, whereas the other capital depreciates, hence decreases faster.

In this model economy, Tobin's  $q$  differs from 1 only when the irreversibility constraint binds. If an irreversibility constraint binds at any time, the market value of the installed capital for that firm goes below its book value, pushing Tobin's  $q$  below 1. Since there is no adjustment cost or limitation for adjusting the capital level upward, Tobin's  $q$  never exceeds 1 in this economy (i.e. market value of installed capital never exceeds its book value). A variation of this model economy, which incorporates frictions/adjustment costs while adjusting the capital level upward, would generate richer results in terms of  $q$ . Such a model could also be used to study the "value premium", where value firms would have  $q$  values below 1 and growth firms would have  $q$  values above 1.

## 6 Empirical Results

In this section, I examine the empirical relationship between the composition of the firms' capital holdings, specifically buildings and equipment, and stock returns. In the first part, I study the relationship at the firm level. I look at the capital composition of individual firms, and try to understand whether there are any cross sectional differences in firm returns with respect to their capital composition. In the second part, I consider the composition of the aggregate capital in the economy. This variable comes out as a moment

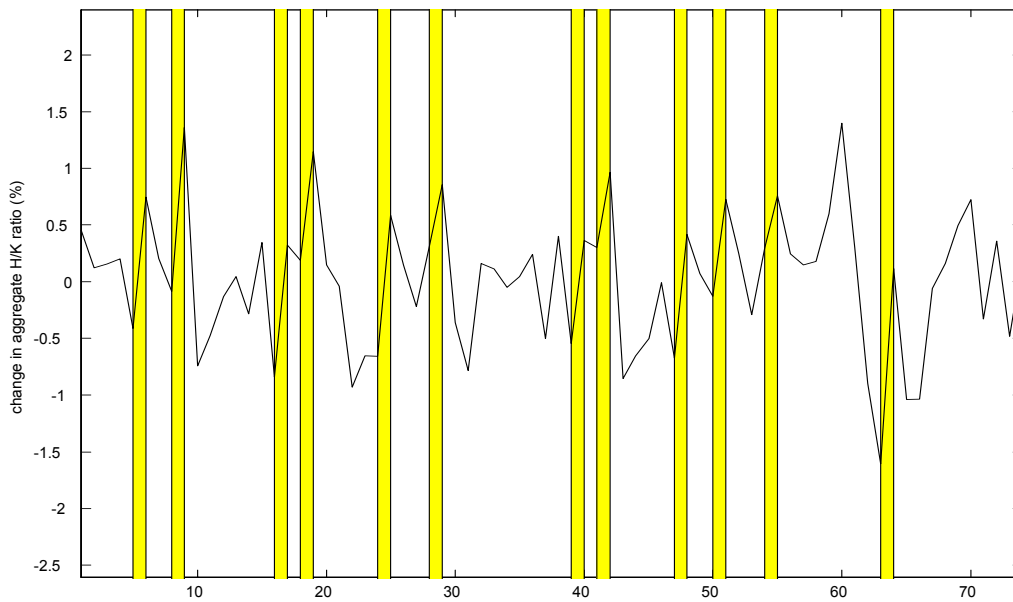


Figure 4: Annual Change in Aggregate  $H/K$  Ratio, sample 75 year period. The darker columns indicate periods in which output growth is more than one standard deviation below its mean.

of the state variables in my model economy, therefore is economically meaningful. I use the aggregate  $H/K$  ratio as a conditioning variable and try to explain the cross sectional differences in the returns of size and B/M sorted portfolios via the conditional CAPM.

## 6.1 Data

In order to measure the real estate holdings of firms, I use real estate related accounting variables. Compustat Industrial Annual provides a breakdown of plant, property and equipment (PPE) into buildings, machinery and equipment, natural resources, land and improvements, construction in progress and capitalized leases. Among these items, I identified buildings, land and improvements and construction in progress as items related to real estate. Buildings is the single biggest component of real estate for most firms. Compustat provides net<sup>9</sup> values of these items over 1969-1997 and historical cost values over 1984-2002. These values are the book values of the assets. In order to make the capital compositions comparable between firms, I calculated a real estate ratio ( $RER$ ) for each firm in every year by dividing the real estate components of PPE by total PPE. Since neither net, nor historical cost series span throughout the whole 1969-2002 period, I used net values until 1984, and switched to historical cost values starting 1984.<sup>10</sup> My choice of using net versus historical cost values over 1984-1997 is somewhat arbitrary, but the results are insensitive to the method of choice. I repeated the analyses

<sup>9</sup>"Net" is "at cost" - "accumulated depreciation".

<sup>10</sup>If I have net (gross) real estate holdings in the nominator, I used net (gross) PPE in the denominator.

using several combinations of real estate components (buildings, buildings+land, buildings+construction, buildings+land+construction), and the results are not sensitive to the choice of real estate components, either. The value of buildings dominates in the *RER* for all combinations. Since the data for buildings are available for more firms than the data for land or construction, I used only buildings in the nominator of the *RER*.

I use all firms with positive holdings of buildings and other capital, as reported in Compustat Industrial Annual, and stock return data from CRSP. A considerable number of mostly small firms do not own any real estate. I excluded them from the sample since their capital structure clearly is not compatible with the simple firm dynamics in the model economy. Following Fama and French (1992 and others), to ensure that the accounting variables are known before the returns they are used to analyze, I match the accounting data for all fiscal yearends in calendar year  $t-1$  with the returns for July of year  $t$  to June of year  $t+1$ , allowing for a minimum of 6 months gap between fiscal yearend and return tests.

In addition to my original sample, I consider two subsamples for empirical tests. The empirical studies in investment literature typically use firm or plant level data from manufacturing firms (Hayashi and Inoue, 1991, Caballero, Engel and Haltiwanger, 1995, and many others). In order to be consistent with this literature, I report my results for manufacturing firms separately. The other subsample that I consider is related to the secured debt holdings of firms. My simple model overlooks the financing decisions of firms. The firms with lower flexibility, i.e. the high real estate firms, are riskier. However, real estate ownership generally gives the firm the option to raise secured debt at a lower nominal cost, and limits the firm's liability in case of default. Therefore, secured debt ownership may alleviate the risk of holding real estate. Compustat Industrial Annual reports mortgages and other secured debt holdings of firms starting in 1981. This secured debt series includes capitalized lease obligations, which is also separately reported. I construct a secured debt ratio (*SDR*) for each firm by dividing the secured debt excluding capitalized lease obligations by total assets.

The aggregate H/K is constructed using the chain type quantity indexes for non-residential equipment and structures by Bureau of Economic Analysis, by dividing the quantity index for nonresidential structures by the quantity index for equipment. The data for 1929-1995 period is taken from the Table 2 of the May 1997 issue of Survey of Current Business (Katz and Herman, 1997). More recent data is taken from the BEA Fixed Assets Table 4, chain type quantity indexes for net stock of private fixed assets. The ratio is trending downward over time (i.e. the quantity of structures relative to equipment is decreasing), therefore I use the annual percentage change in the ratio as the scaling (conditioning) variable. The test assets in Fama-MacBeth regressions are the 25 value weighted size and B/M sorted portfolios (FF portfolios). FF portfolio returns, FF factors and the momentum factor are taken from Kenneth French's website.

## 6.2 Firm Level Analysis

This section investigates whether a stock's expected return is related to its capital composition; the share of its buildings in its total capital. I follow a straightforward portfolio based approach by sorting the firms in my sample every year according to their *RER* and grouping them into quintile portfolios. Table 3 reports the descriptive statistics for *RER* sorted portfolios. On the average, buildings make up 30% of a firm's physical capital. However, there is significant dispersion in the capital composition among firms. The lowest *RER* group, on the average, has around 10% *RER*, whereas the buildings make up more than 50% of the physical capital of the highest *RER* group. The returns of the portfolios are dispersed as well. With the exception of the highest *RER* portfolio of all firms, excess raw returns increase monotonically as the *RER* increases. The monotonic relationship between *RER* and excess returns is maintained for the manufacturing firms even for the highest *RER* portfolio.

The secured debt ratio (*SDR*) of firms have a different pattern. Since secured debt data are not available before 1981, *SDR* can only be computed starting in 1981. *SDR* has a slight U-shaped pattern with respect to the share of buildings in the firm's capital. The ratio makes a big spike for the highest *RER* group of all firms, whereas it is relatively flat for the manufacturing firms.

The excess returns of *RER* sorted portfolios are regressed on well known risk factors such as excess market returns (*MKT*), *SMB* (returns of portfolio that is long in small, short in big firms), *HML* (returns of portfolio that is long in high B/M, short in low B/M firms) and *MOM* (momentum portfolio returns, long in short term winners, short in short term losers). The intercepts of the regressions (alphas) represent the pricing errors. If these well known factors can account for all the risk in *RER* sorted portfolios, the alphas should be indistinguishable from zero. Table 4 presents the alphas and betas of *RER* sorted portfolios with respect to market, *SBM*, *HML* and *MOM* factors for the whole sample using value and equally weighted portfolios. Table 5 reports the same for the sample of manufacturing firms, Tables 6 and 7 present them starting in 1983 for the sample of firms that has necessary data for the computation of the secured debt ratio. Betas are estimated by regressing the portfolio excess returns on the 4 factors. The alphas are estimated as intercepts from the regressions of excess portfolio returns on the same factors. Monthly alphas are annualized by multiplying with 12. If real estate risk is priced, risk-adjusted returns (i.e. alphas) should exhibit systematic differences. This is indeed the case in the data. Like the raw returns, alphas increase monotonically as the *RER* increases, except the portfolio with the highest *RER* (which also has higher *SDR*). The value weighted portfolios that are long in high *RER* portfolios and short in low *RER* portfolio (5-1 and 4-1 portfolios) have alphas varying between 4% and 7% over the 1971-2003 period. The equally weighted portfolios produce smaller, but still significant alphas.

Table 3: Descriptive Statistics for *RER* Sorted Portfolios (% , annualized)

| <i>RER</i> quintile                         | low   | 2     | 3     | 4     | high  | all  |
|---|-------|-------|-------|-------|-------|------|
| All firms (July 1971 - June 2003)           |       |       |       |       |       |      |
| <i>RER</i>                                  | 0.09  | 0.20  | 0.28  | 0.37  | 0.57  | 0.30 |
| $r_{EW}^e$                                  | 7.92  | 8.52  | 9.48  | 10.80 | 9.72  |      |
| $\sigma_{EW}^e$                             | 22.07 | 20.44 | 19.70 | 20.34 | 20.99 |      |
| $r_{VW}^e$                                  | 2.40  | 4.08  | 3.60  | 6.60  | 5.04  |      |
| $\sigma_{VW}^e$                             | 19.70 | 17.07 | 17.41 | 17.96 | 18.34 |      |
| # of firms                                  |       |       |       |       |       | 1966 |
| Manufacturing firms (July 1971 - June 2003) |       |       |       |       |       |      |
| <i>RER</i>                                  | 0.12  | 0.22  | 0.29  | 0.36  | 0.53  | 0.30 |
| $r_{EW}^e$                                  | 8.52  | 8.52  | 9.60  | 10.56 | 11.40 |      |
| $\sigma_{EW}^e$                             | 22.08 | 20.14 | 19.58 | 20.28 | 21.80 |      |
| $r_{VW}^e$                                  | 2.76  | 3.24  | 5.40  | 5.52  | 6.60  |      |
| $\sigma_{VW}^e$                             | 19.38 | 17.29 | 16.93 | 17.53 | 18.34 |      |
| # of firms                                  |       |       |       |       |       | 1255 |
| All firms (July 1983 - June 2003)           |       |       |       |       |       |      |
| <i>RER</i>                                  | 0.08  | 0.18  | 0.25  | 0.34  | 0.56  | 0.28 |
| <i>SDR</i>                                  | 0.09  | 0.07  | 0.07  | 0.08  | 0.13  |      |
| $r_{EW}^e$                                  | 6.12  | 6.96  | 7.44  | 9.12  | 6.60  |      |
| $\sigma_{EW}^e$                             | 20.29 | 18.73 | 17.60 | 18.20 | 18.63 |      |
| $r_{VW}^e$                                  | 2.40  | 4.92  | 2.16  | 8.88  | 6.00  |      |
| $\sigma_{VW}^e$                             | 20.66 | 16.52 | 16.91 | 18.27 | 19.00 |      |
| # of firms                                  |       |       |       |       |       | 1591 |
| Manufacturing firms (July 1983 - June 2003) |       |       |       |       |       |      |
| <i>RER</i>                                  | 0.11  | 0.20  | 0.26  | 0.33  | 0.50  | 0.28 |
| <i>SDR</i>                                  | 0.07  | 0.06  | 0.06  | 0.07  | 0.08  |      |
| $r_{EW}^e$                                  | 7.20  | 6.72  | 7.56  | 8.88  | 9.24  |      |
| $\sigma_{EW}^e$                             | 20.84 | 18.40 | 17.74 | 18.21 | 19.89 |      |
| $r_{VW}^e$                                  | 3.84  | 4.80  | 3.96  | 6.96  | 6.84  |      |
| $\sigma_{VW}^e$                             | 19.83 | 16.11 | 16.80 | 17.62 | 18.35 |      |
| # of firms                                  |       |       |       |       |       | 994  |

Note: For *RER* and *SDR*, equal-weighted averages are first taken over all firms in that portfolio, then over years. *RER* is buildings/PPE; *SDR* is (secured debt - capitalized leases)/assets.  $r_{EW}^e$  is equal-weighted monthly excess returns,  $r_{VW}^e$  is value-weighted monthly excess returns, annualized, averages are taken over time (%).  $\sigma_{EW}^e$  and  $\sigma_{VW}^e$  are the corresponding standard deviations. Excess returns are measured in the year following the portfolio formation.

The alphas for the manufacturing firms are bigger and more significant. Furthermore, the irregularity of the highest *RER* portfolio disappears when the sample of firms are limited to the manufacturing firms.

One plausible concern with *RER* sorted portfolios is that firms in some industries may naturally have high / low *RERs*. For example, health services (hospitals) and hotels tend to have very high *RERs*, whereas transportation companies tend to have low *RERs*. Consequently, the returns of extreme *RER* portfolios may reflect industry specific factors. Looking at the sample of manufacturing firms to a large extent mitigates this problem. Alternatively, I calculate industry adjusted real estate ratios for firms (*adjRER*). I group firms based on 2 digit SIC codes and calculate the average of the *RERs* within each group. I subtract these industry *RERs* from the firm *RERs* to get industry adjusted real estate ratios for firms (*adjRER*).

Table 8 presents the descriptive statistics, alphas and betas of *adjRER* sorted portfolios for the whole sample using value and equally weighted portfolios. There is significant variation in industry adjusted real estate ratios. For the firms in the first quintile, the share of real estate in the firms' total physical capital is 20% lower than the average firm in that industry, and it is 23% higher for the firms in the fifth quintile. Variation in industry adjusted real estate ratios (*adjRERs*) implies that there is considerable heterogeneity in capital composition within industries. The value and equal weighted portfolios that are long in high *adjRER* portfolios and short in low *adjRER* portfolio (5-1 portfolio) have alphas in the range of 2.5-3.5% over the 1971-2003 period. Both alphas are statistically significant.

Table 4: Alphas and Betas of Portfolios Sorted on *RER*, All Firms

July 1971 - June 2003

| <i>RER</i> quintile | low                       | 2                | 3                | 4                | high             | 5-1              | 4-1              |
|---------------------|---------------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                     | Value weighted portfolios |                  |                  |                  |                  |                  |                  |
| <i>alpha</i>        | -4.08<br>(-3.31)          | -1.01<br>(-0.90) | -0.28<br>(-0.22) | 2.81<br>(2.54)   | -0.2<br>(-0.15)  | 3.88<br>(2.08)   | 6.89<br>(4.01)   |
| <i>MKT</i>          | 1.10<br>(48.89)           | 0.99<br>(43.44)  | 0.97<br>(40.10)  | 0.96<br>(38.68)  | 1.01<br>(27.92)  | -0.09<br>(-1.96) | -0.15<br>(-4.74) |
| <i>SMB</i>          | 0.19<br>(5.00)            | -0.02<br>(-0.45) | -0.01<br>(-0.26) | -0.05<br>(-1.46) | 0.05<br>(0.88)   | -0.13<br>(-1.70) | -0.24<br>(-4.60) |
| <i>HML</i>          | -0.02<br>(-0.51)          | 0.03<br>(0.70)   | -0.07<br>(-1.87) | -0.30<br>(-6.35) | -0.07<br>(-1.10) | -0.05<br>(-0.64) | -0.28<br>(-5.06) |
| <i>MOM</i>          | -0.02<br>(-0.48)          | -0.07<br>(-2.41) | -0.12<br>(-3.99) | -0.03<br>(-0.97) | -0.04<br>(-1.13) | -0.03<br>(-0.50) | -0.02<br>(-0.34) |
|                     | Equal weighted portfolios |                  |                  |                  |                  |                  |                  |
| <i>alpha</i>        | -0.82<br>(-0.83)          | -0.47<br>(-0.50) | 0.89<br>(0.95)   | 2.35<br>(2.21)   | 0.67<br>(0.53)   | 1.49<br>(1.55)   | 3.18<br>(3.50)   |
| <i>MKT</i>          | 1.04<br>(52.55)           | 0.99<br>(47.81)  | 0.96<br>(47.71)  | 0.94<br>(39.41)  | 0.93<br>(35.40)  | -0.11<br>(-4.90) | -0.10<br>(-4.84) |
| <i>SMB</i>          | 1.00<br>(25.89)           | 0.90<br>(24.89)  | 0.85<br>(22.60)  | 0.93<br>(23.07)  | 1.04<br>(24.14)  | 0.04<br>(1.35)   | -0.07<br>(-2.01) |
| <i>HML</i>          | 0.35<br>(9.95)            | 0.40<br>(11.00)  | 0.36<br>(9.97)   | 0.26<br>(6.23)   | 0.36<br>(7.92)   | 0.02<br>(0.50)   | -0.09<br>(-2.40) |
| <i>MOM</i>          | -0.08<br>(-3.10)          | -0.04<br>(-1.34) | -0.03<br>(-1.17) | -0.02<br>(-0.64) | -0.01<br>(-0.38) | 0.07<br>(2.83)   | 0.06<br>(2.79)   |

Note: Regressions of value and equal weighted excess portfolio returns on FF and momentum factor returns. Alphas are annualized (%). *t*-statistics are in parentheses.

Table 5: Alphas and Betas of Portfolios Sorted on *RER*, Manufacturing Firms

July 1971 - June 2003

| <i>RER</i> quintile       | low              | 2                | 3                | 4                | high             | 5-1              | 4-1              |
|---------------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Value weighted portfolios |                  |                  |                  |                  |                  |                  |                  |
| <i>alpha</i>              | -3.85<br>(-2.90) | -1.92<br>(-1.52) | 1.00<br>(0.72)   | 1.83<br>(1.37)   | 0.99<br>(0.69)   | 4.84<br>(2.59)   | 5.68<br>(3.10)   |
| <i>MKT</i>                | 1.07<br>(34.82)  | 0.98<br>(36.19)  | 0.94<br>(30.07)  | 0.94<br>(32.98)  | 1.00<br>(33.69)  | -0.07<br>(-1.86) | -0.12<br>(-3.29) |
| <i>SMB</i>                | 0.20<br>(3.92)   | 0.03<br>(0.75)   | -0.02<br>(-0.56) | -0.15<br>(-3.73) | 0.02<br>(0.54)   | -0.18<br>(-3.12) | -0.35<br>(-5.24) |
| <i>HML</i>                | 0.04<br>(0.74)   | 0.00<br>(-0.03)  | 0.00<br>(0.03)   | -0.24<br>(-4.59) | -0.09<br>(-1.48) | -0.13<br>(-1.73) | -0.28<br>(-3.77) |
| <i>MOM</i>                | -0.01<br>(-0.35) | -0.05<br>(-1.47) | -0.09<br>(-2.22) | -0.04<br>(-1.05) | 0.01<br>(0.22)   | 0.02<br>(0.49)   | -0.02<br>(-0.44) |
| Equal weighted portfolios |                  |                  |                  |                  |                  |                  |                  |
| <i>alpha</i>              | -0.22<br>(-0.20) | -0.67<br>(-0.66) | 1.08<br>(1.05)   | 2.08<br>(1.93)   | 2.42<br>(1.84)   | 2.64<br>(2.42)   | 2.29<br>(2.47)   |
| <i>MKT</i>                | 1.03<br>(46.71)  | 1.00<br>(43.56)  | 0.96<br>(42.69)  | 0.96<br>(38.93)  | 0.95<br>(32.76)  | -0.08<br>(-3.51) | -0.07<br>(-3.95) |
| <i>SMB</i>                | 0.99<br>(27.06)  | 0.83<br>(19.06)  | 0.82<br>(21.74)  | 0.89<br>(22.99)  | 1.08<br>(23.51)  | 0.09<br>(2.48)   | -0.10<br>(-3.70) |
| <i>HML</i>                | 0.33<br>(8.76)   | 0.42<br>(9.95)   | 0.36<br>(9.03)   | 0.26<br>(5.85)   | 0.31<br>(6.03)   | -0.02<br>(-0.48) | -0.07<br>(-2.03) |
| <i>MOM</i>                | -0.07<br>(-2.67) | -0.03<br>(-0.89) | -0.04<br>(-1.26) | -0.01<br>(-0.38) | -0.02<br>(-0.63) | 0.05<br>(1.89)   | 0.06<br>(2.36)   |

Note: Regressions of value and equal weighted excess portfolio returns on FF and momentum factor returns. Alphas are annualized (%). *t*-statistics are in parentheses.

Table 6: Alphas and Betas of Portfolios Sorted on *RER*, All Firms

July 1983 - June 2003

| <i>RER</i> quintile | low                       | 2                | 3                | 4                | high             | 5-1              | 4-1              |
|---------------------|---------------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                     | Value weighted portfolios |                  |                  |                  |                  |                  |                  |
| <i>alpha</i>        | -4.88<br>(-3.22)          | -1.94<br>(-1.65) | -2.21<br>(-1.33) | 3.06<br>(2.36)   | -0.74<br>(-0.38) | 4.15<br>(1.46)   | 7.94<br>(3.98)   |
| <i>MKT</i>          | 1.14<br>(38.84)           | 0.98<br>(36.91)  | 0.92<br>(27.96)  | 0.98<br>(34.92)  | 1.04<br>(20.18)  | -0.10<br>(-1.50) | -0.16<br>(-3.77) |
| <i>SMB</i>          | 0.44<br>(9.55)            | -0.01<br>(-0.19) | -0.01<br>(-0.27) | -0.04<br>(-0.91) | 0.04<br>(0.50)   | -0.40<br>(-3.57) | -0.48<br>(-8.29) |
| <i>HML</i>          | 0.01<br>(0.32)            | 0.01<br>(0.28)   | -0.11<br>(-2.05) | -0.30<br>(-5.39) | -0.11<br>(-1.35) | -0.13<br>(-1.30) | -0.32<br>(-4.91) |
| <i>MOM</i>          | -0.08<br>(-2.33)          | -0.01<br>(-0.44) | -0.18<br>(-4.92) | -0.01<br>(-0.40) | -0.03<br>(-0.53) | 0.05<br>(0.71)   | 0.06<br>(1.39)   |
|                     | Equal weighted portfolios |                  |                  |                  |                  |                  |                  |
| <i>alpha</i>        | -1.10<br>(-0.83)          | -0.80<br>(-0.58) | 0.01<br>(0.01)   | 1.99<br>(1.29)   | -0.62<br>(-0.34) | 0.49<br>(0.37)   | 3.10<br>(2.31)   |
| <i>MKT</i>          | 1.01<br>(41.17)           | 0.96<br>(30.45)  | 0.93<br>(30.79)  | 0.90<br>(30.27)  | 0.87<br>(24.03)  | -0.14<br>(-4.57) | -0.12<br>(-4.85) |
| <i>SMB</i>          | 0.92<br>(18.26)           | 0.86<br>(16.80)  | 0.77<br>(15.68)  | 0.84<br>(16.09)  | 0.94<br>(15.32)  | 0.03<br>(0.65)   | -0.08<br>(-1.87) |
| <i>HML</i>          | 0.31<br>(6.07)            | 0.40<br>(6.84)   | 0.34<br>(6.33)   | 0.23<br>(4.18)   | 0.34<br>(5.23)   | -0.03<br>(0.75)  | -0.08<br>(-1.74) |
| <i>MOM</i>          | -0.10<br>(-2.40)          | -0.03<br>(-0.69) | -0.02<br>(-0.55) | 0.00<br>(-0.01)  | -0.01<br>(-0.10) | 0.09<br>(2.45)   | 0.10<br>(3.33)   |

Note: Regressions of value and equal weighted excess portfolio returns on FF and momentum factor returns. Alphas are annualized (%). *t*-statistics are in parentheses.

Table 7: Alphas and Betas of Portfolios Sorted on *RER*, Manufacturing Firms

July 1983 - June 2003

| <i>RER</i> quintile       | low              | 2                | 3                | 4                | high             | 5-1              | 4-1              |
|---------------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Value weighted portfolios |                  |                  |                  |                  |                  |                  |                  |
| <i>alpha</i>              | -4.05<br>(-2.51) | -2.24<br>(-1.46) | -1.79<br>(-1.06) | 0.64<br>(0.42)   | 0.66<br>(0.33)   | 4.71<br>(1.78)   | 4.69<br>(2.18)   |
| <i>MKT</i>                | 1.09<br>(28.81)  | 0.95<br>(22.69)  | 0.94<br>(26.21)  | 0.98<br>(33.30)  | 0.98<br>(24.88)  | -0.11<br>(-1.93) | -0.11<br>(-2.57) |
| <i>SMB</i>                | 0.32<br>(5.07)   | -0.01<br>(-0.17) | 0.00<br>(0.08)   | -0.10<br>(-1.90) | 0.05<br>(0.81)   | -0.27<br>(-3.84) | -0.41<br>(-6.93) |
| <i>HML</i>                | -0.02<br>(-0.39) | 0.00<br>(-0.03)  | 0.01<br>(0.11)   | -0.19<br>(-3.44) | -0.13<br>(-1.62) | -0.10<br>(-1.14) | -0.17<br>(-2.45) |
| <i>MOM</i>                | 0.02<br>(0.42)   | 0.03<br>(0.85)   | -0.09<br>(-1.77) | 0.00<br>(-0.08)  | -0.04<br>(-0.62) | -0.06<br>(-0.86) | -0.02<br>(-0.44) |
| Equal weighted portfolios |                  |                  |                  |                  |                  |                  |                  |
| <i>alpha</i>              | 0.04<br>(0.03)   | -1.05<br>(-0.72) | -0.08<br>(-0.05) | 1.67<br>(1.08)   | 2.74<br>(1.37)   | 2.70<br>(1.81)   | 1.62<br>(1.33)   |
| <i>MKT</i>                | 1.02<br>(35.26)  | 0.96<br>(28.64)  | 0.94<br>(28.38)  | 0.92<br>(30.48)  | 0.88<br>(22.21)  | -0.14<br>(-4.62) | -0.10<br>(-4.49) |
| <i>SMB</i>                | 0.94<br>(17.88)  | 0.79<br>(13.02)  | 0.76<br>(15.42)  | 0.78<br>(15.85)  | 1.00<br>(14.07)  | 0.07<br>(1.45)   | -0.15<br>(-4.97) |
| <i>HML</i>                | 0.26<br>(4.56)   | 0.41<br>(6.53)   | 0.37<br>(6.38)   | 0.21<br>(3.96)   | 0.24<br>(3.34)   | -0.02<br>(-0.38) | -0.05<br>(-1.31) |
| <i>MOM</i>                | -0.09<br>(-2.33) | -0.03<br>(-0.70) | -0.02<br>(-0.36) | 0.01<br>(0.21)   | -0.06<br>(-1.06) | 0.04<br>(1.03)   | 0.1<br>(3.13)    |

Note: Regressions of value and equal weighted excess portfolio returns on FF and momentum factor returns. Alphas are annualized (%). *t*-statistics are in parentheses.

Table 8: Descriptive Statistics, Alphas and Betas of Portfolios Sorted on  $adjRER$ , All Firms

July 1971 - June 2003

| $adjRER$ quintile         | low       | 2         | 3         | 4         | high      | 5-1       |
|---------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| Descriptive statistics    |           |           |           |           |           |           |
| $adjRER$                  | -0.20     | -0.08     | -0.01     | 0.06      | 0.23      |           |
| $r_{EW}^e$                | 8.42      | 8.12      | 9.66      | 10.18     | 10.18     |           |
| $\sigma_{EW}^e$           | 21.93     | 20.27     | 19.96     | 20.34     | 21.16     |           |
| $r_{VW}^e$                | 3.32      | 3.78      | 5.16      | 4.13      | 5.44      |           |
| $\sigma_{VW}^e$           | 18.99     | 17.02     | 17.23     | 17.41     | 18.85     |           |
| Value weighted portfolios |           |           |           |           |           |           |
| $alpha$                   | -2.69     | -1.42     | -0.03     | 0.49      | 0.60      | 3.28      |
|                           | ( -2.74 ) | ( -1.19 ) | ( -0.03 ) | ( 0.44 )  | ( 0.49 )  | ( 2.24 )  |
| $MKT$                     | 1.06      | 0.98      | 0.99      | 0.97      | 1.02      | -0.04     |
|                           | ( 50.53 ) | ( 38.00 ) | ( 38.75 ) | ( 39.22 ) | ( 37.01 ) | ( -1.03 ) |
| $SMB$                     | 0.21      | -0.03     | -0.03     | -0.07     | 0.12      | -0.09     |
|                           | ( 7.02 )  | ( -1.09 ) | ( -0.90 ) | ( -1.91 ) | ( 2.57 )  | ( -1.5 )  |
| $HML$                     | -0.03     | -0.01     | -0.04     | -0.18     | -0.15     | -0.12     |
|                           | ( -0.81 ) | ( -0.16 ) | ( -0.94 ) | ( -4.11 ) | ( -3.27 ) | ( -2.1 )  |
| $MOM$                     | -0.04     | -0.03     | -0.03     | -0.09     | -0.06     | -0.02     |
|                           | ( -1.49 ) | ( -0.98 ) | ( -0.81 ) | ( -3.33 ) | ( -2.17 ) | ( -0.54 ) |
| Equal weighted portfolios |           |           |           |           |           |           |
| $alpha$                   | -0.48     | -0.71     | 0.83      | 1.58      | 1.60      | 2.08      |
|                           | ( -0.48 ) | ( -0.75 ) | ( 0.89 )  | ( 1.51 )  | ( 1.32 )  | ( 2.49 )  |
| $MKT$                     | 1.01      | 0.99      | 0.97      | 0.96      | 0.92      | -0.09     |
|                           | ( 49.58 ) | ( 47.83 ) | ( 47.25 ) | ( 42.13 ) | ( 39.49 ) | ( -4.52 ) |
| $SMB$                     | 1.02      | 0.87      | 0.87      | 0.90      | 1.05      | 0.03      |
|                           | ( 24.62 ) | ( 24.27 ) | ( 24.32 ) | ( 21.96 ) | ( 25.74 ) | ( 0.92 )  |
| $HML$                     | 0.36      | 0.38      | 0.36      | 0.31      | 0.32      | -0.05     |
|                           | ( 9.23 )  | ( 10.08 ) | ( 9.36 )  | ( 8.02 )  | ( 7.50 )  | ( -1.18 ) |
| $MOM$                     | -0.07     | -0.04     | -0.02     | -0.03     | -0.04     | 0.03      |
|                           | ( -2.32 ) | ( -1.32 ) | ( -0.83 ) | ( -0.88 ) | ( -1.08 ) | ( 1.06 )  |

Note: Descriptive statistics and regressions of value and equal weighted excess portfolio returns on FF and momentum factor returns. For  $adjRER$ , equal-weighted averages are first taken over all firms in that portfolio, then over years.  $adjRER$  is  $buildings/PPE - (buildings/PPE)_{industry}$ .  $r_{EW}^e$  is equal-weighted monthly excess returns,  $r_{VW}^e$  is value-weighted monthly excess returns, annualized, averages are taken over time (%).  $\sigma_{EW}^e$  and  $\sigma_{VW}^e$  are the corresponding standard deviations. Excess returns are measured in the year following the portfolio formation. Alphas are annualized (%).  $t$ -statistics are in parentheses.

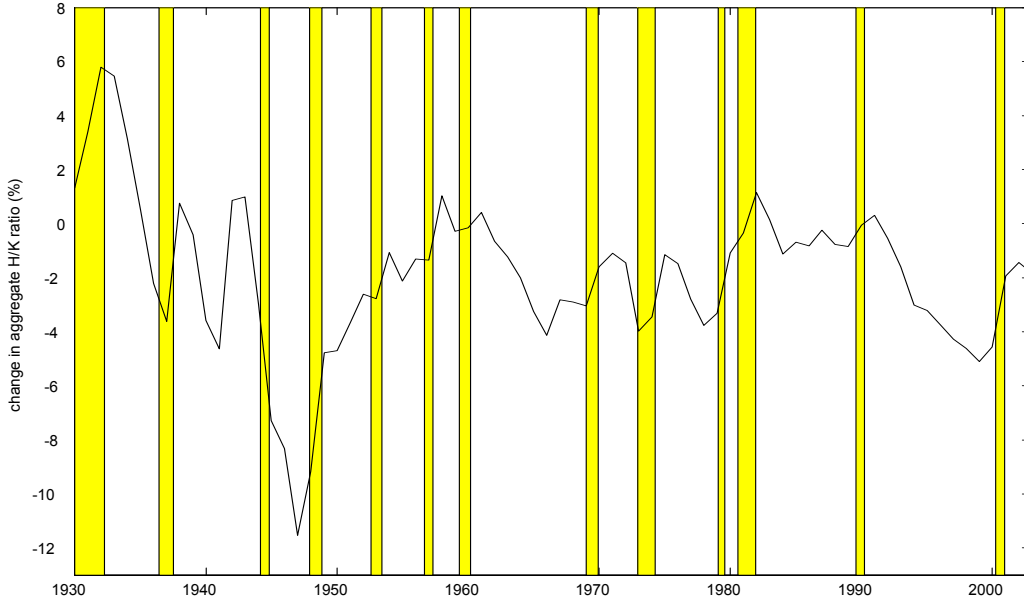


Figure 5: Annual Change in Aggregate  $H/K$  Ratio, 1930-2003. The darker columns indicate NBER recession periods.

The results of the firm level analysis are interesting. I find that the firms with higher  $RER$  indeed earn higher returns after adjusting for common risk factors, suggesting that the owners of these firms are compensated for their real estate risk exposure. This empirical result is consistent with the predictions of the model economy (Table 2).

### 6.3 Implications of Aggregate Capital Composition

I construct an aggregate measure of capital composition, aggregate  $H/K$ , using quantity indexes of aggregate capital by dividing the quantity index for nonresidential structures by the quantity index for equipment. Aggregate  $H/K$  is a moment of the state variables in the model economy, and is therefore economically meaningful.

Figure 5 plots the annual change in aggregate  $H/K$  ratio over the 1930-2003 period. The darker columns in the figure indicate NBER recession periods. The aggregate  $H/K$  ratio increased during all but one recession, which coincides with the second world war. The correlation coefficient between the GDP growth and the change in aggregate  $H/K$  ratio is around -0.4 throughout the postwar era. This empirical result is consistent with the predictions of the model economy (Figure 3), the aggregate  $H/K$  ratio increases in bad times.

I utilize the information content of the aggregate  $H/K$  ratio by using it as a conditioning variable, and undertake a cross sectional investigation of the conditional CAPM using the 25 size and B/M sorted Fama-French portfolios. The unconditional (static) version of CAPM has been unable to explain the cross sectional differences in firm re-

turns. The first column of figure 6 plots the average realized returns of the 25 portfolios against their CAPM fitted counterparts. The plots are almost flat, indicating that there is virtually no relationship between the realized and expected mean returns based on CAPM.

In standard applications of CAPM, the parameters of the discount factor are constant, i.e. the discount factor is of the form  $m_{t+1} = a - br_{vw,t+1}$ . By scaling CAPM, I model the dependence of the parameters in the discount factor on variables in time  $t$  information set,  $a_t = a(z_t)$ ,  $b_t = b(z_t)$ . My choice of conditioning variable ( $z_t$ ) is the change in the aggregate  $H/K$  ratio,  $\Delta H/K_t$ , which is a moment of the state variables in the model economy described above. The idea behind conditioning the CAPM with the aggregate  $H/K$  is that, the risk of an asset is determined by the covariance of the asset's return with the market return conditional on the time  $t$  information set of investors, which is approximated by the aggregate  $H/K$  ratio. The assets with returns that are more tightly correlated with market returns when the aggregate  $H/K$  ratio goes up trade at a discount. The discount factor takes the form

$$\begin{aligned} m_{t+1} &= a_0 + a_1 \Delta H/K_t + (b_0 + b_1 \Delta H/K_t) r_{vw,t+1} \\ &= a_0 + a_1 \Delta H/K_t + b_0 r_{vw,t+1} + b_1 (\Delta H/K_t \cdot r_{vw,t+1}). \end{aligned}$$

My estimation is in the spirit of Cochrane (1996) and Lettau and Ludvigson (2001). Cochrane (1996) estimates unconditional and conditional factor models where he uses returns on physical investment as factors and term premium and dividend/price ratio as conditioning variables. Lettau and Ludvigson (2001) estimate conditional versions of CAPM and consumption CAPM. Their conditioning variable *cay* is a cointegrating residual between the logarithms of consumption, asset wealth and labor income. Cochrane (1996) uses GMM, whereas Lettau and Ludvigson (2001) use Fama-MacBeth regressions to estimate the models.

$E(m_{t+1} r_{i,t+1}^e) = 0$  implies

$$\begin{aligned} 0 &= E((a_0 + a_1 \Delta H/K_t + b_0 r_{vw,t+1} + b_1 (\Delta H/K_t \cdot r_{vw,t+1})) r_{i,t+1}^e) \\ 0 &= a_0 E(r_{i,t+1}^e) + a_1 \text{cov}(\Delta H/K_t, r_{i,t+1}^e) + a_1 E(r_{i,t+1}^e) E(\Delta H/K_t) \\ &\quad + b_0 \text{cov}(r_{vw,t+1}, r_{i,t+1}^e) + b_0 E(r_{i,t+1}^e) E(r_{vw,t+1}) \\ &\quad + b_1 \text{cov}(\Delta H/K_t \cdot r_{vw,t+1}, r_{i,t+1}^e) + b_1 E(r_{i,t+1}^e) E(\Delta H/K_t \cdot r_{vw,t+1}) \\ E(r_{i,t+1}^e) &= -\frac{a_1 \text{var}(\Delta H/K_t)}{E(m_{t+1})} \beta_{\Delta H/K_t} - \frac{b_0 \text{var}(r_{vw,t+1})}{E(m_{t+1})} \beta_{r_{vw,t+1}} - \frac{b_1 \text{var}(\Delta H/K_t \cdot r_{vw,t+1})}{E(m_{t+1})} \beta_{\Delta H/K_t \cdot r_{vw,t+1}} \end{aligned}$$

where  $\beta_x = \frac{\text{cov}(x, r_{i,t+1}^e)}{\text{var}(x)}$ .

I estimate the following cross sectional regression:<sup>11</sup>

$$E(r_{i,t+1}^e) = \beta_{\Delta H/K_t} \lambda_{\Delta H/K_t} + \beta_{r_{vw,t+1}} \lambda_{r_{vw,t+1}} + \beta_{\Delta H/K_t \cdot r_{vw,t+1}} \lambda_{\Delta H/K_t \cdot r_{vw,t+1}}$$

---

<sup>11</sup>I demean the scaling variable, change in aggregate  $H/K_t$ , in my empirical analysis.

Table 9 reports the estimates for factor loadings ( $\lambda$ ) from cross sectional Fama-MacBeth regressions using the returns of 25 size and B/M sorted Fama-French portfolios, together with  $t$ -statistics, Shanken corrected  $t$ -statistics and  $R^2$ s. I report the results for the unconditional CAPM, FF 3-factor model and the conditional CAPM, where CAPM is scaled by the change in aggregate  $H/K$  ratio. Figure 6 plots the average realized returns of the 25 portfolios against their model fitted counterparts.

The unconditional CAPM has virtually no power in explaining the cross section of average returns over this sample. The adjusted  $R^2$ s of the cross sectional regressions are negative in the postwar subperiods. By contrast, the conditional CAPM performs relatively well in all subperiods. The adjusted  $R^2$  of the cross sectional regression ranges between 50-70%. The loadings on the market return scaled by the aggregate  $H/K$  ratio ( $H/K_t \cdot r_{vw,t+1}$ ) and the aggregate  $H/K$  ratio are positive and statistically significant. The fit of the model is comparable to that of the Fama-French 3 factor model, even though the latter model produces slightly higher adjusted  $R^2$ s.

Table 9: Fama-MacBeth Regression Results Using 25 Size and B/M Sorted Portfolios

| Row         | Model       | <i>Constant</i>             | $\lambda_{H/K_t}$        | $\lambda_{r_{vw,t+1}}$      | $\lambda_{SMB_{t+1}}$    | $\lambda_{HML_{t+1}}$    | $\lambda_{H/K_t \cdot r_{vw,t+1}}$ | $R^2$           |
|-------------|-------------|-----------------------------|--------------------------|-----------------------------|--------------------------|--------------------------|------------------------------------|-----------------|
| 1931 – 2003 |             |                             |                          |                             |                          |                          |                                    |                 |
| 1           | CAPM        | -2.29<br>(-0.55)<br>(-0.47) |                          | 12.01<br>(2.51)<br>(1.99)   |                          |                          |                                    | 0.35<br>(0.33)  |
| 2           | Scaled CAPM | 3.98<br>(0.93)<br>(0.77)    | 0.01<br>(2.45)<br>(1.85) | 4.87<br>(1.00)<br>(0.77)    |                          |                          | 0.43<br>(3.69)<br>(2.65)           | 0.74<br>(0.70)  |
| 3           | FF          | 16.05<br>(3.15)<br>(2.48)   |                          | -7.70<br>(-1.37)<br>(-1.02) | 4.21<br>(2.40)<br>(1.51) | 6.35<br>(3.55)<br>(2.25) |                                    | 0.86<br>(0.84)  |
| 1948 – 2003 |             |                             |                          |                             |                          |                          |                                    |                 |
| 4           | CAPM        | 12.94<br>(2.44)<br>(2.43)   |                          | -1.77<br>(-0.30)<br>(-0.28) |                          |                          |                                    | 0.01<br>(-0.04) |
| 5           | Scaled CAPM | 6.84<br>(1.32)<br>(0.70)    | 0.03<br>(3.53)<br>(1.85) | 1.96<br>(0.34)<br>(0.18)    |                          |                          | 0.49<br>(3.44)<br>(1.80)           | 0.58<br>(0.52)  |
| 6           | FF          | 11.70<br>(2.99)<br>(2.66)   |                          | -3.76<br>(-0.82)<br>(-0.66) | 2.37<br>(1.30)<br>(0.87) | 5.80<br>(3.17)<br>(2.12) |                                    | 0.80<br>(0.77)  |
| 1964 – 2003 |             |                             |                          |                             |                          |                          |                                    |                 |
| 7           | CAPM        | 11.55<br>(2.31)<br>(2.30)   |                          | -1.74<br>(-0.30)<br>(-0.27) |                          |                          |                                    | 0.01<br>(-0.03) |
| 8           | Scaled CAPM | -2.20<br>(-0.38)<br>(-0.20) | 0.01<br>(2.09)<br>(1.04) | 7.91<br>(1.25)<br>(0.64)    |                          |                          | 0.38<br>(3.14)<br>(1.63)           | 0.59<br>(0.53)  |
| 9           | FF          | 11.06<br>(2.27)<br>(1.97)   |                          | -5.49<br>(-0.98)<br>(-0.78) | 4.13<br>(1.70)<br>(1.12) | 6.11<br>(2.61)<br>(1.72) |                                    | 0.80<br>(0.77)  |

Note: The table reports the cross sectional regression coefficients,  $t$ -statistics and  $R^2$ s. In each row, the first line reports the regression coefficients and the  $R^2$ s, the second line reports the  $t$ -statistics and adjusted  $R^2$ s, the third line reports the  $t$ -statistics after Shanken (1992) correction.

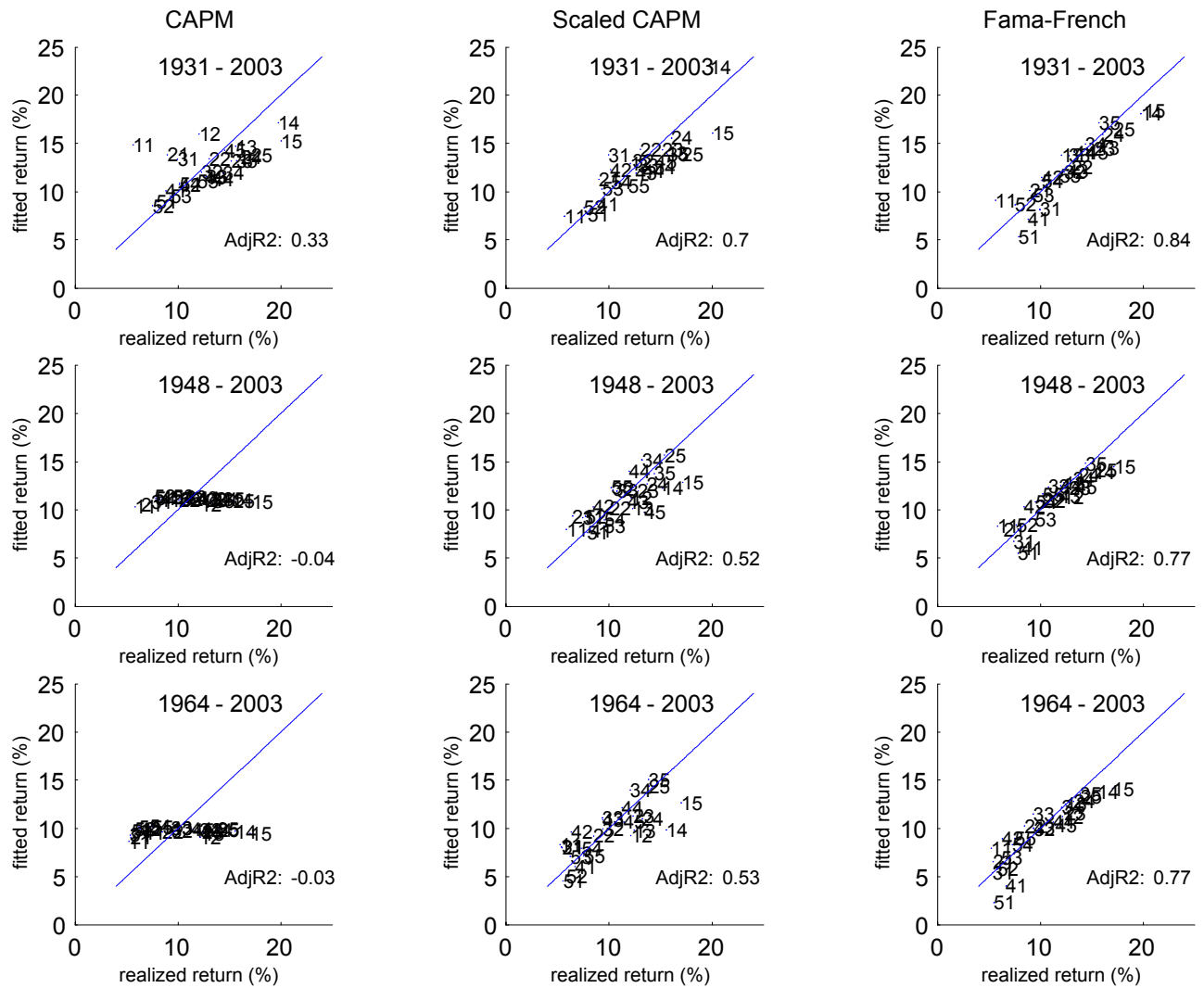


Figure 6: Actual vs. predicted mean excess returns of 25 size and B/M sorted portfolios

## 7 Conclusion

I introduce a general equilibrium production model where the firms use two factors, real estate capital and other capital, and investment is irreversible. Slow depreciation of real estate capital makes real estate investment riskier than investment in other capital. Due to the irreversibility of investment, the firm will find it difficult to reduce its real estate holdings when it would like to do so, whereas the other capital depreciates, hence decreases faster. Therefore, recessions hurt firms with high real estate holdings particularly bad. In equilibrium, investors demand a premium to hold such a firm. This prediction is also empirically supported. Using a portfolio based approach, I find that the returns of firms with a high share of real estate capital exceed that for low real estate firms by 4-7% annually adjusted for exposures to the market return, size, value and momentum factors. The model also predicts countercyclical variation in the *aggregate* share of real estate in total capital, which is a moment of the state variables. The empirical evidence is also consistent with this prediction. I undertake a cross sectional investigation of the conditional CAPM, where the change in aggregate share of real estate in total capital is used as the conditioning variable. The conditional CAPM delivers substantially improved results over its unconditional version.

The capital breakdown I consider is limited to real estate and other physical capital. Nevertheless, this breakdown excludes at least one big class of capital, which is referred to as intangible or organizational capital. Accounting for intangible capital is inherently difficult; neither there is a consensus on how intangible capital is defined, nor on how it is measured. Its definition involves a variety of concepts, such as organizational culture, copyrights and research and development. Further research integrating intangible capital into the firm's capital composition would provide a more realistic and comprehensive view of the firm than what this simple model portrays.

## References

- [1] Abel, Andrew B. and Janice C. Eberly (1994). "A unified model of investment under uncertainty." *American Economic Review* 84, pages 1369-1384.
- [2] Abel, Andrew B. and Janice C. Eberly (1999). "The effects of irreversibility and uncertainty on capital accumulation." *Journal of Monetary Economics* 44, pages 339-377.
- [3] Atkeson, Andrew and Patrick J. Kehoe (2002). "Measuring organization capital." NBER Working paper 8722.
- [4] Berk, Jonathan B., Green, Richard C. and Vasant Naik (1999). "Optimal investment, growth options, and security returns." *Journal of Finance* 54, pages 1553-1607.
- [5] Bertola, Giuseppe (1988). "Adjustment costs and dynamic factor demands: investment and employment under uncertainty." Doctoral dissertation, MIT.
- [6] Blackorby, Charles and William Schworm (1983). "Aggregating heterogeneous capital goods in adjustment-cost technologies." *Scandinavian Journal of Economics* 85, pages 207-222.
- [7] Boldrin, Michele, Christiano, Lawrence J. and Jonas D. M. Fisher (2001). "Habit persistence, asset returns, and the business cycle." *American Economic Review* 91, pages 149-166.
- [8] Braun, Matias (2003). "Financial contractibility and asset hardness." Working paper, UCLA.
- [9] Caballero, Ricardo J., Engel, Eduardo M. R. A. and John Haltiwanger (1995) "Plant Level Adjustment and Aggregate Investment Dynamics," *Brookings Papers on Economic Activity* 2, pages. 1-54.
- [10] Chirinko, Robert S. (1993). "Multiple capital inputs,  $q$ , and investment spending." *Journal of Economic Dynamics and Control* 17, pages 907-928.
- [11] Christiano, Lawrence J. and Jonas D. M. Fisher (2000). "Algorithms for solving dynamic models with occasionally binding constraints." *Journal of Economic Dynamics and Control* 24, pages 1179-1232.
- [12] Cochrane, John H. (1991). "Production-based asset pricing and the link between stock returns and economic fluctuations." *Journal of Finance* 46, pages 209-37.
- [13] Cochrane, John H. (1996). "A cross-sectional test of an investment-based asset pricing model." *Journal of Political Economy* 104, pages 572-621.
- [14] Cooper, Ilan (2003). "Asset pricing implications of non-convex adjustment costs and irreversibility of investment." Working paper, BI Norwegian School of Management.

- [15] Cooper, Russell W. and John C. Haltiwanger (2002). "On the nature of capital adjustment costs." Working paper, Boston University.
- [16] Cummins, Jason G. (2003). "A new approach to the valuation of intangible capital." NBER Working paper 9924.
- [17] Cummins, Jason G. and Matthew Dey (1998). "Taxation, investment, and firm growth with heterogeneous capital." Working paper, NYU.
- [18] Danthine, Jean-Pierre, Donaldson, John B. and Rajnish Mehra (1992). "The equity premium and the allocation of income risk." *Journal of Economic Dynamics and Control* 16, pages 509-532.
- [19] Deng, Yongheng and Joseph Gyourko (1999). "Real estate ownership by non-real estate firms: The impact on firm returns." Working paper, University of Pennsylvania.
- [20] Denny, Michael, Douglas J. May (1998). "Homotheticity and real value-added in Canadian manufacturing." *Production Economics: A Dual Approach to Theory and Applications* (Editors, M. Fuss and D. McFadden), North Holland Publishing Company.
- [21] Dixit, Anivash K. (1989). "Entry and exit decisions under uncertainty." *Journal of Political Economy* 97, pages 620-638.
- [22] Dixit, Avinash K. and Robert S. Pindyck (1994). *Investment under uncertainty*, Princeton University Press.
- [23] Doms, Mark and Timothy Dunne (1998). "Capital adjustment patterns in manufacturing plants." *Review of Economic Dynamics* 1, pages 409-429.
- [24] Eisfeldt, Andrea L. and Adriano A. Rampini (2003). "Capital reallocation and liquidity." Working paper, Northwestern University.
- [25] Epstein, Larry G. (1983). "Aggregating quasi-fixed factors." *Scandinavian Journal of Economics* 85, pages 191-205.
- [26] Fama, Eugene F. and Kenneth R. French (1992). "The cross section of expected stock returns." *The Journal of Finance* 47, pages 427-465.
- [27] Flavin, Marjorie and Takashi Yamashita (2002). "Owner-occupied housing and the composition of the household portfolio." *American Economic Review* 92, pages 345-62.
- [28] Fraumeni, Barbara M. (1997). "The measurement of depreciation in the U.S. national income and product accounts." *Survey of Current Business* 77:7, pages 7-23.
- [29] Garegnani, Pierangelo (1970). "Heterogeneous capital, the production function and the theory of distribution." *Review of Economic Studies* 37, pages 407-36.

- [30] Glaeser, Edward L. and Joseph Gyourko (2004). "Urban decline and durable housing." *Journal of Political Economy*, forthcoming.
- [31] Gomes, João F., Kogan, Leonid and Lu Zhang (2003). "Equilibrium cross-section of returns." *Journal of Political Economy* 111, pages 693-732.
- [32] Goolsbee, Austan and David B. Gross (1997). "Estimating adjustment costs with data on heterogeneous capital goods." NBER Working paper 6342.
- [33] Gourio, Francois (2004). "Operating leverage, stock market cyclicity and the cross section of returns." Working paper, University of Chicago.
- [34] Hall, Robert E. (2001). "The stock market and capital accumulation." *American Economic Review* 91, pages 1185-1202.
- [35] Hansen Lars P., John Heaton and Nan Li (2004). "Intangible Risk?" *Measuring Capital in the New Economy* (Editors, Carol Corrado, John Haltiwanger and Dan Sichel), The University of Chicago Press, forthcoming.
- [36] Hayashi, Fumio and Tohru Inoue (1991). "The Relation between firm growth and  $q$  with multiple capital goods: theory and evidence from panel data on Japanese firms." *Econometrica* 59, pages 731-753.
- [37] Jermann, Urban J. (1998). "Asset pricing in production economies." *Journal of Monetary Economics* 41, pages 257-275.
- [38] Katz, Arnold J. and Shelby W. Herman (1997). "Improved estimates of fixed reproducible tangible wealth, 1929-95." *Survey of Current Business* 77:5, pages 69-92.
- [39] Kogan, Leonid (2004). "Asset prices and real investment." *Journal of Financial Economics*, forthcoming.
- [40] Kullman, Cornelia (2003) ."Real Estate and its Role in Asset Pricing." Working paper, UBC.
- [41] Lamont, Owen (2000). "Investment plans and stock returns." *Journal of Finance* 55, pages 2719-2745.
- [42] Leahy, John V. and Toni M. Whited (1996). "The effect of uncertainty on investment: some stylized facts." *Journal of Money, Credit and Banking* 28, pages 64-83.
- [43] Lettau, Martin, and Sydney Ludvigson (2001b) "Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying." *Journal of Political Economy* 109, pages 1238–1287.
- [44] Li, Nan (2004) "Intangible capital and investment returns." Working Paper, University of Chicago.
- [45] Li, Qing, Vassalou, Maria and Yuhang Xing (2003). "Sector investment growth rates and the cross-section of equity returns." Working paper, Columbia University.

- [46] Lustig, Hanno and Stijn Van Nieuwerburgh (2004). "A theory of housing collateral, consumption insurance and risk premia." Working paper, NYU.
- [47] Marcet, Albert (1988). "Solving nonlinear stochastic growth models by parameterizing expectations." Working paper, Carnegie-Mellon University.
- [48] Marcet, Albert and Guido Lorenzoni (1998). "Parameterized expectations approach; some practical issues." *Computational Methods for the Study of Dynamic Economies* (Editors, Ramon Marimon and Andrew Scott), Oxford University Press.
- [49] Nilsen, Oivind A. and Fabio Schiantarelli (2003). "Zeros and lumps in investment: empirical evidence on irreversibilities and nonconvexities." *The Review of Economics and Statistics* 85, pages 1021-1037.
- [50] Piazzesi, Monika, Schneider, Martin and Selale Tuzel (2004). "Housing, consumption, and asset pricing." Working paper, UCLA.
- [51] Pindyck, Robert S. (1988). "Irreversible investment, capacity choice, and the value of the firm." *American Economic Review* 78, pages 969-985.
- [52] Ramey, Valerie and Matthew Shapiro (2001). "Displaced capital: A study of aerospace plant closings." *Journal of Political Economy* 109, pages. 958-992.
- [53] Rouwenhorst, Geert K. (1995). "Asset Pricing Implications of Equilibrium Business Cycle Models." *Frontiers of Business Cycle Research* ( Editor, Thomas F. Cooley), Princeton University Press.
- [54] Shanken, Jay (1992). "On the estimation of beta pricing models." *Review of Financial Studies* 5, 1-34, pages 1-33.
- [55] Stambaugh, Robert F. (1982) ."On the Exclusion of Assets from Tests of the Two Parameter Model: A Sensitivity Analysis." *Journal of Financial Economics* 10, pages 237-268.
- [56] Wildasin, David E. (1984). "The  $q$  theory of investment with many capital goods." *American Economic Review* 74, pages 203-10.
- [57] Zhang, Lu (2003). "Value premium." *Journal of Finance*, forthcoming.

## Appendix A: Computational Solution

I solve the Euler equations (Eq.5) using the Chebyshev PEA algorithm in Christiano and Fisher (2000). In order to use the PEA, the system should be "invertible", i.e., once the parameterized expectation is substituted in the equilibrium condition, one should be able to construct the policy function. The Euler equations that will be solved (Eq.5) are not "invertible" in the sense that the policy functions,  $[k'_1, k'_2, h'_1, h'_2]$ , cannot be retrieved by substituting the parameterized expectations in the right hand side. A value for  $C$  can be found from any one of the four Euler equations, but there is no way to compute individual policy functions. In order to make the system invertible, following Marcat and Lorenzoni (1988), I slightly modify the Euler equations by premultiplying both sides of the 4 Euler equations by  $k'_1, k'_2, h'_1, h'_2$ , respectively. Since capital levels are never zero in equilibrium, the new equations are satisfied if and only if the original Euler equations are satisfied. The modified Euler equations are as follows, where the primes denote next period's values<sup>12</sup>:

$$(u_C - \lambda_i) k'_i = \int \int \beta(u_{C'}(F_{K'_i} + 1 - \delta) - \lambda'_i(1 - \delta)) k'_i p_{z_1}(z'_1|z_1) p_{z_2}(z'_2|z_2) d_{z_1} d_{z_2}$$

$$(u_C - \nu_i) h'_i = \int \int \beta(u_{C'}(F_{H'_i} + 1 - \mu) - \nu'_i(1 - \mu)) h'_i p_{z_1}(z'_1|z_1) p_{z_2}(z'_2|z_2) d_{z_1} d_{z_2}$$

for  $i \in \{1, 2\}$ ; where  $u_{C'} = u_C(C')$ ,  $F_{K'_i} = F_K(K'_i)$ ,  $F_{H'_i} = F_H(H'_i)$ .

The solution is a function  $\hat{e}_a(s, z)$ , with a finite set of parameters,  $a$  :

$$\exp(\hat{e}_a) \approx \left[ \begin{array}{l} \int \int \beta(u_{C'}(F_{K'_1} + 1 - \delta) - \lambda'_1(1 - \delta)) k'_1 p_{z_1}(z'_1|z_1) p_{z_2}(z'_2|z_2) d_{z_1} d_{z_2} \\ \int \int \beta(u_{C'}(F_{K'_2} + 1 - \delta) - \lambda'_2(1 - \delta)) k'_2 p_{z_1}(z'_1|z_1) p_{z_2}(z'_2|z_2) d_{z_1} d_{z_2} \\ \int \int \beta(u_{C'}(F_{H'_1} + 1 - \mu) - \nu'_1(1 - \mu)) h'_1 p_{z_1}(z'_1|z_1) p_{z_2}(z'_2|z_2) d_{z_1} d_{z_2} \\ \int \int \beta(u_{C'}(F_{H'_2} + 1 - \mu) - \nu'_2(1 - \mu)) h'_2 p_{z_1}(z'_1|z_1) p_{z_2}(z'_2|z_2) d_{z_1} d_{z_2} \end{array} \right]'$$

where  $\hat{e}_a$  is constructed using Chebyshev polynomials as basis functions:  $\hat{e}_a(s, z) = a' [T(\varphi(k_1)) \otimes T(\varphi(k_2)) \otimes T(\varphi(h_1)) \otimes T(\varphi(h_2)) \otimes T(\varphi(z_1)) \otimes T(\varphi(z_2))]$ , where the basis functions,  $T(x) = [T_0(x) T_1(x) \dots T_{N-1}(x)]'$ , are Chebyshev polynomials, and  $\varphi(x) = 2 \frac{x-x}{x-x} - 1$ .  $a$  denote the  $(N_{k_1} \times N_{k_2} \times N_{h_1} \times N_{h_2} \times N_{z_1} \times N_{z_2}) \times 4$  dimensional vector of parameters for  $\hat{e}_a$ .

The relations linking the policy functions,  $\hat{s}'_a(s, z) = [\hat{k}'_{1a}, \hat{k}'_{2a}, \hat{h}'_{1a}, \hat{h}'_{2a}]$  and the multiplier functions,  $\hat{m}_a(s, z) = [\hat{\lambda}_{1a}, \hat{\lambda}_{2a}, \hat{\nu}_{1a}, \hat{\nu}_{2a}]$ , to  $\hat{e}_a(s, z)$  are as follows:

$$\hat{s}'_a = \max\{\log(1 - \delta) + k_1, \log(1 - \delta) + k_2, \log(1 - \mu) + h_1, \log(1 - \mu) + h_2\}, \hat{e}_a / u_{\hat{C}_a}$$

$$\hat{C}_a = Y_1 + Y_2 - \exp(\hat{k}'_{1a}) - \exp(\hat{k}'_{2a}) - \exp(\hat{h}'_{1a}) - \exp(\hat{h}'_{2a})$$

$$\hat{m}_a = u_{\hat{C}_a} - \hat{e}_a / \hat{s}'_a$$

<sup>12</sup>When the time subscript is dropped, primes denote next period's values.

Let's define  $\hat{e}_a(s, z)$ ,  $RHS_a(s'_a(s, z), z')$  and  $X$  as follows:

$$\hat{e}_a = \begin{pmatrix} \hat{e}_a(k_{11}, k_{21}, h_{11}, h_{21}, z_{11}, z_{21}) \\ \dots \\ \hat{e}_a(k_{1M_{k_1}}, k_{2M_{k_2}}, h_{1M_{h_1}}, h_{2M_{h_2}}, z_{1M_{z_1}}, z_{2M_{z_2}}) \end{pmatrix}$$

$$RHS_a = \begin{pmatrix} \int \int \beta(u_{\hat{C}'_a}(F_{\hat{K}'_{1a}} + 1 - \delta) - \hat{\lambda}'_{1a}(1 - \delta)) \hat{k}'_{1a} p_{z_1}(z'_1|z_1) p_{z_2}(z'_2|z_2) d_{z_1} d_{z_2} \\ \int \int \beta(u_{\hat{C}'_a}(F_{\hat{K}'_{2a}} + 1 - \delta) - \hat{\lambda}'_{2a}(1 - \delta)) \hat{k}'_{2a} p_{z_1}(z'_1|z_1) p_{z_2}(z'_2|z_2) d_{z_1} d_{z_2} \\ \int \int \beta(u_{\hat{C}'_a}(F_{\hat{H}'_{1a}} + 1 - \mu) - \hat{v}'_{1a}(1 - \mu)) \hat{h}'_{1a} p_{z_1}(z'_1|z_1) p_{z_2}(z'_2|z_2) d_{z_1} d_{z_2} \\ \int \int \beta(u_{\hat{C}'_a}(F_{\hat{H}'_{2a}} + 1 - \mu) - \hat{v}'_{2a}(1 - \mu)) \hat{h}'_{2a} p_{z_1}(z'_1|z_1) p_{z_2}(z'_2|z_2) d_{z_1} d_{z_2} \end{pmatrix}'$$

$X = [T(k_{11}) \dots T(k_{1M_{k_1}})]' \otimes [T(k_{21}) \dots T(k_{2M_{k_2}})]' \otimes [T(h_{11}) \dots T(h_{1M_{h_1}})]' \otimes [T(h_{21}) \dots T(h_{2M_{h_2}})]' \otimes [T(z_{11}) \dots T(z_{1M_{z_1}})]' \otimes [T(z_{21}) \dots T(z_{2M_{z_2}})]'$  is an  $(M_{k_1} \times M_{k_2} \times M_{h_1} \times M_{h_2} \times M_{z_1} \times M_{z_2}) \times (N_{k_1} \times N_{k_2} \times N_{h_1} \times N_{h_2} \times N_{z_1} \times N_{z_2})$  matrix, where  $k_{ij}$ 's are  $M_{k_i}$  roots of the  $M_{k_i}$ th order Chebyshev polynomial,  $T_{M_{k_i}}(x)$ , and  $h_{ij}$ 's and  $z_{ij}$ 's are defined accordingly.

The procedure to estimate the parameter set,  $a$ , starts with computing a fixed set of grid points,  $k_{1j_{k_1}}, k_{2j_{k_2}}, h_{1j_{h_1}}, h_{2j_{h_2}}, j_{k_1} = 1, \dots, M_{k_1}, j_{k_2} = 1, \dots, M_{k_2}, j_{h_1} = 1, \dots, M_{h_1}, j_{h_2} = 1, \dots, M_{h_2}$ , and coming up with an initial guess for  $a$ <sup>13</sup>. The new value for  $a$  is computed as follows:

1. compute  $RHS_a$ , which is a  $(M_{k_1} \times M_{k_2} \times M_{h_1} \times M_{h_2} \times M_{z_1} \times M_{z_2}) \times 1$  vector,
2. retrieve  $a$  by regressing  $RHS_a$  on  $X$ ,  $\tilde{a} = (X'X)^{-1}X'RHS_a$ . If  $\tilde{a}$  is close enough to the initial guess, convergence is achieved. Otherwise, the initial guess is updated, and the procedure is repeated until the convergence is achieved.

---

<sup>13</sup>Following Christiano and Fisher (2000), for the initial guess of the policy function, I use a log linear approximation, truncated to ensure that gross investment is nonnegative. I use zero function for the Lagrange multipliers.