The Dynamics of Mergers and Acquisitions in Oligopolistic Industries*

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Abstract

This paper develops a real options model to study the interaction between industry structure and takeover activity. In an asymmetric industry equilibrium, firms have an endogenous incentive to merge when restructuring decisions are motivated by operating and strategic benefits. The model predicts that (i) the likelihood of restructuring activities is greater in more concentrated industries or in industries that are more exposed to industry shocks, (ii) the magnitude of returns arising from restructuring to both merger firms and rival firms is higher in more concentrated industries, (iii) increased product market competition delays the timing of mergers, (iv) when the industry is sufficiently concentrated, bidder competition induces a bid premium, and this premium decreases with product market competition.

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1 Introduction

Many aspects of mergers and acquisitions have been studied extensively in economics and finance.[1] Despite the substantial development of this literature, some important issues of the takeover process are still not deeply understood. For example, while existing models largely focus on firm characteristics to explain why firms should merge or restructure, they have not been entirely successful at uncovering the relation between industry structure and takeovers. A drawback shared by many dynamic models of mergers and acquisitions is that they abstract away product market competition. The objective of this paper is to develop an industry equilibrium model that analyzes how product market competition affects the gains from mergers, the joint determination of the timing and terms of mergers, and the returns to both merger and rival firms. We also examine interactions between bidder competition and industry competition.

Recent empirical studies have documented ample evidence on the effect of industry characteristics on mergers and acquisitions. Analyzing a large number of industries during the 1980s and 1990s, Andrade, Mitchell and Stafford (2001), Harford (2005), and Mitchell and Mulherin (1996) find that mergers occur in waves driven by industry-wide shocks, and that mergers strongly cluster by industry within a wave. For the airline industry, Borenstein (1990) and Kim and Singal (1993) document that airfares on routes affected by a merger increase significantly relative to routes not affected by a merger. Similarly, Kim and Singal (1993) and Singal (1996) identify a positive relation between airfares and industry concentration. Eckbo (1985) documents that acquirers and targets earn abnormal returns, while Singal (1996) finds, in addition, that abnormal returns to rival firms are positively related to changes in industry concentration. In sum, these studies support the view that mergers not only affect industry equilibrium but also have substantial anticompetitive effects.[2]

While the preceding empirical evidence demonstrates that industry characteristics play an important role in the takeover process, the extant theoretical literature does not thoroughly analyze this issue.[3] Our model attempts to fill this gap. We build a dynamic industry equilibrium model of mergers and acquisitions in a real options framework in the spirit of Lambrecht

[1] See Weston et al. (1990) and Weston et al. (1998) for a comprehensive review of the literature.

[2] Based upon data from 1970s, empirical studies by Eckbo (1983), Eckbo (1985), and Eckbo and Wier (1985) find little evidence that challenged horizontal mergers are anticompetitive when using an event study methodology. While Singal (1996) offers a lucid discussion of the early evidence, McAfee and Williams (1988) cast doubt on the ability of event studies to detect anticompetitive mergers. In addition, Holmstrom and Kaplan (2001), Shleifer and Vishny (2003), and Andrade and Stafford (2004) point out a number of differences between the takeover activity in the 1960s or 1970s and the more recent merger waves of the 1980s or 1990s.

[3] The welfare implications of mergers in oligopolistic industries have been extensively analyzed in the industrial organization literature; see, e.g., Farrell and Shapiro (1990). However, this literature does not study the issues of merger timing, merger terms, and merger returns, which are the focus of our paper.

(2004). Unlike Lambrecht (2004), however, we consider an asymmetric oligopolistic industry structure similar to that in Perry and Porter (1985). We study mergers with a single bidder and with multiple bidders. We do not assume exogenously specified operating synergies, such as economies of scale. Instead, we specify a tangible asset that helps increase output for a given average cost. The new entity operates the assets of the two merging partners. Thus, the merged firm is larger, and its average and marginal costs are lower.[4] In addition to a different cost structure for the merged firm, all firms face a different industry structure after a merger. We derive a closed-form solution to an industry equilibrium in which the benefits from merging are determined endogenously by *both* cost reduction and product market competition.

Unlike Perry and Porter's (1985) static model, which analyzes incentives to merge for two small firms only, our dynamic model allows us to derive a number of predictions that are in line with existing empirical evidence. In addition, our dynamic model has some novel testable implications regarding merger returns, timing and terms of mergers, and bid premia. First, we introduce exogenous shocks to industry demand and demonstrate that a merger occurs at the first time the demand shock hits a trigger value from below. That is, mergers occur in a rising product market. Moreover, we show that the likelihood of mergers is greater for industries that are more exposed to industry shocks. These results are consistent with empirical evidence discussed earlier; see, e.g., Mitchell and Mulherin (1996).

Second, increased product market competition delays the timing of mergers. This result is in contrast to that derived in recent real options models, such as Grenadier (2002). Grenadier (2002) emphasizes that industry competition erodes the option value of waiting and thus accelerates option exercise. He presents a symmetric industry model in which anticompetitive profits result from exogenously reducing the number of identical firms that compete in the industry. Unlike his model, firms are asymmetric in our model, and anticompetitive profits result from a merger. These anticompetitive profits are larger for more concentrated industries because the effect of price increase after a merger is larger in these industries. Thus, firms in less competitive industries optimally exercise their option to merge earlier.

Third, our model's closed-form solutions show that the magnitude of cumulative returns is larger for small firms than for large firms because small firms benfit relatively more from a merger. Moreover, the cumulative returns to merger and rival firms are positively associated with industry concentration. The intuition is simple. Returns are under certain conditions positively related to anticompetitive profit gains, which are positively related to industry concentration.

[4] That is, we do not consider other forms of mergers in which the two entities remain independently operated, such as a formation of a conglomerate or a merger by a holding company.

Finally, we analyze a merger when a large firm bidder and a small firm bidder compete for a small firm target. We show that the large firm bidder wins the takeover contest, which is consistent with the stylized fact that targets are on average smaller than acquirers. In addition, competition with the small firm bidder may speed up the takeover process and may lead the large firm bidder to pay a bid premium that deters the competing small firm bidder. Notably, this equilibrium outcome arises when the industry is sufficiently concentrated. In this case, the bid premium increases with industry concentration. By contrast, if the industry is sufficiently competitive, then the small firm bidder may not matter and the equilibrium outcome is the same as that in the case without bidder competition.

Our paper contributes to a growing body of research using the real options approach to analyze the dynamics of mergers and acquisitions. Morellec and Zhdanov (2005) generalize Shleifer and Vishny's (2003) static model to incorporate imperfect information and uncertainty into a dynamic framework. Margsiri, Mello and Ruckes (2006) analyze the decision to grow internally or externally by an acquisition, while Smith and Triantis (1995) argue that acquisitions may enable internal growth. Hackbarth and Morellec (2006) examine the risk dynamics throughout the merger episode. Lambrecht and Myers (2007) study takeovers in declining industries motivated by greater efficiency through layoffs, consolidation, and disinvestment. Leland (2007) considers purely financial synergies in motivating acquisitions when timing is exogenous, while Morellec and Zhdanov (2007) explore interactions between financial leverage and takeover activity with endogenous timing. All these papers do not consider the impact of product market competition on takeover activity.[5] The papers closest to ours are Lambrecht (2004) and Bernile, Lyandres, and Zhdanov (2006). Bernile, Lyandres, and Zhdanov (2006) present an industry model with two incumbents and a potential new entrant. They focus on merger waves, which are under certain conditions deterred by the threat of entry. In contrast to their model, we study merger dynamics subject to product market competition with a large number of heterogeneous firms. Lambrecht (2004) examines mergers motivated by economies of scale. Although he does not focus on industry structure, he does analyze the case of a duopolistic industry and shows that market power strengthens symmetric firms' incentives to merge. Unlike his model, we consider an oligopolistic Cournot–Nash equilibrium with asymmetric firms. In addition, to focus on the role of industry structure, we abstract from economies of scale.

The reminder of the paper proceeds as follows. Section 2 presents the model. Section 3 analyzes the timing and terms of a merger with a single bidder. Section 4 studies mergers with

[5] Using the real options approach, Fries, Miller, and Perraudin (1997), Lambrecht (2001) and, Miao (2005) develop industry equilibrium models to analyze the implications of industry structure for firm investment, financing, entry and exit decisions.

multiple bidders and analyze the interaction between bidder competition and product market competition. Section 5 concludes. Proofs are relegated to an appendix.

2 The Model

We incorporate an asymmetric oligopolistic industry structure into a real options framework. After outlining the framework's assumptions, we characterize industry equilibrium when asymmetric firms play Cournot–Nash strategies. We then examine the incentive to merge when restructuring decisions are motivated by operating and strategic benefits. We finally provide further discussions on some of our model's assumptions.

2.1 Assumptions

Time is continuous, and uncertainty is modeled by a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. To construct a dynamic equilibrium model of firms' restructuring decisions, we consider an industry populated by infinitely-lived firms whose assets generate a continuous stream of cash flows. The industry consists of N heterogeneous firms that produce a single homogeneous product, where $N \geq 2$ is an integer. Each firm i initially owns an amount $k_i > 0$ of physical capital. In order to focus on the issue of mergers and acquisitions and to keep the analysis tractable, we follow many other papers in the literature, such as Lambrecht (2004) and Perry and Porter (1985), to assume that firms can grow only through a takeover, and that internal investment or new entry is not allowed.[6] In addition, we assume that capital does not depreciate over time. The industry's total capital stock is in fixed supply and equal to K. Thus, the industry's capital stock at each point in time satisfies

$$\sum_{i=1}^{N} k_i = K. \tag{1}$$

The cost structure is important in the model. We denote by $C(q, \kappa)$ the cost function of a firm that owns an amount κ of the capital stock and produces output q. The output q is produced with a combination of the fixed capital input, κ, and a vector of variable inputs, z, according to a smooth concave production function, $q = F(z, \kappa)$. Then the cost function $C(q, \kappa)$ is obtained from the cost minimization problem. Unlike Lambrecht (2004), we assume that the production function F has constant returns to scale.[7] This implies that $C(q, \kappa)$ is linearly homogenous in (q, κ). For analytical tractability, we adopt the quadratic specification

[6] See Section 2.4 for a discussion of this assumption.

[7] To complement existing models, we do not allow for synergies due to economies of scale (Lambrecht, 2004) or to an efficiency-enhancing capital reallocation (Morellec and Zhdanov, 2005).

of the cost function, $C(q,\kappa) = q^2/(2\kappa)$. This cost function may result from the Cobb-Douglas production function $q = \sqrt{\kappa z}$, where z may represent labor input. It is important to note that both the average and marginal costs decrease with the capital asset κ. Salant et al. (1983) show that if average cost is constant and independent of firm size, a merger may be unprofitable in a Cournot oligopoly with linear demand. It is profitable if and only if duopolists merge into monopoly. As pointed out by Perry and Porter (1985), the constant average cost assumption does not provide a sensible description of mergers.

The capital asset plays an important role in the model. It allows us to address the industry asymmetries caused by mergers of a subset of firms. A merged firm combines the capital assets of the two entities to produce output. It faces a different optimization problem immediately after restructuring because of its altered cost function and because of new strategic considerations that arise from the change in industry structure.

We suppose the industry's inverse demand at time t is given by the following linear function

$$P(t) = a\,Y(t) - b\,Q(t), \tag{2}$$

where $Q(t)$ is the industry's output at time t, $Y(t)$ denotes the industry's demand shock at time t observed by all firms, and a and b are positive constants. Here, a represents exposure of demand to industry-wide shocks and b represents the price sensitivity of demand. We assume that the demand shock is governed by the following geometric Brownian motion process:

$$dY(t) = \mu\,Y(t)dt + \sigma\,Y(t)\,dW(t),\ \ Y(0) = y_0, \tag{3}$$

where μ and σ are positive constants and $(W_t)_{t\geq 0}$ is a standard Brownian motion defined on $(\Omega, \mathcal{F}, \mathcal{P})$.

We assume that all firms are Cournot–Nash players and that management acts in the best interests of shareholders. We also assume that shareholders are risk neutral and discount future cash flows by $r > 0$. Therefore, all corporate decisions are rational and value-maximizing choices. To ensure that the present value of profits is finite, we make the following assumption:

Assumption 1 *The parameters μ, σ, and r satisfy the condition $2\left(\mu + \sigma^2/2\right) < r$.*

Two firms may decide to merge if it is in their best interest. Mergers are costly in reality. We assume that each firm i incurs a fixed lump-sum cost $X_i > 0$ for $i = 1, ..., N$, if it engages in a merger. This cost captures fees to investment banks and lawyers as well as the cost of restructuring. Before analyzing the incentive to merge, we first characterize industry equilibrium.

2.2 Industry Equilibrium

Let $q_i(t)$ denote the quantity selected by firm i. Then firm i's instantaneous profit is given by

$$\pi_i(t) = [a\,Y(t) - b\,Q(t)]\,q_i(t) - q_i(t)^2/(2k_i)\,, \tag{4}$$

where

$$Q(t) = \sum_{i=1}^{N} q_i(t) \tag{5}$$

is the industry output at time t. Given the preceding instantaneous profits, we can compute firm value, or the present value of profits,

$$V_i(y) = \mathbb{E}^y\left[\int_0^\infty e^{-rt}\pi_i(t)\,dt\right], \tag{6}$$

where $\mathbb{E}^y[\cdot]$ denotes the conditional expectation operator given that the current shock takes the value $Y(0) = y$.

As in Grenadier (2002), we define strategies and industry equilibrium as follows. The strategy $\{(q_1^*(t), ..., q_N^*(t)) : t \geq 0\}$ constitutes an industry (Markov perfect Nash) equilibrium if, given information available at date t, $q_j^*(t)$ is optimal for firm $j = 1, ..., N$, when it takes other firms' strategies $q_i^*(t)$ for all $i \neq j$ as given. Because firms play a dynamic game, there could be multiple Markov perfect Nash equilibria, as is well known in game theory. Instead of finding all equilibria, we will focus on the equilibrium in which firms play static Cournot strategies. As is well known, the static Cournot strategies that firms play at each date constitute a Markov perfect Nash equilibrium. The following proposition characterizes this industry equilibrium.

Proposition 1 *The strategy*

$$q_i^*(t) = \frac{\theta_i}{1+B}\frac{aY(t)}{b}, \tag{7}$$

constitutes a Cournot–Nash industry equilibrium at time t for firms $i = 1, ..., N$, where

$$\theta_i = \frac{b}{b + k_i^{-1}}, \ \ \text{and} \ \ B = \sum_{i=1}^{N}\theta_i. \tag{8}$$

In this equilibrium, the industry output and price at time t are respectively given by

$$Q^*(t) = \frac{B}{1+B}\frac{aY(t)}{b}, \tag{9}$$

and

$$P^*(t) = \frac{aY(t)}{1+B}. \tag{10}$$

Note that this proposition also characterizes the equilibrium after a merger once we change the number of firms and the capital stock of the merged firm. A merger brings the capital of two firms under a single authority and thus reduces production cost. Without loss of generality, we assume that firms 1 and 2 merge. We use the subscript M to denote the merged firm. The merged firm has capital $k_M = k_1 + k_2$. By Proposition 1, the values of θ_i for the non-merging firms $i \geq 3$ are unaffected by the merger. We can show that the value of θ for the merged firm satisfies

$$\max\{\theta_1, \theta_2\} < \theta_M = \frac{\theta_1 + \theta_2 - 2\theta_1\theta_2}{1 - \theta_1\theta_2} < \theta_1 + \theta_2. \tag{11}$$

Thus, the value of B in Proposition 1, which determines total industry output, changes after the merger to

$$B_M = B + \theta_M - \theta_1 - \theta_2 < B. \tag{12}$$

By (9), (10), and (12), we conclude that the merger causes total output to fall and industry price to rise. In addition, we can use (12) and (7) to show that

$$\max\{q_1, q_2\} < q_M < q_1 + q_2. \tag{13}$$

This results implies that the merged firm produces more than either of its two merging firms does. However, this output level is less than the total output level of the two merging firms. The preceding analysis highlights the tension of a merger: After a merger, the industry price rises, but the merged firm restricts production. Thus, a merger may not generate a profit gain. We next turn to this issue.

2.3 Incentive to Merge

In order to analyze mergers tractably, we follow Perry and Porter (1985) and consider an oligopoly structure with small and large firms. Specifically, we assume that the industry initially consists of n identical large firms and m identical small firms. Each large firm owns an amount k of capital and incurs merger costs X_l if it engages in a merger. Each small firm owns an amount $k/2$ of capital and incurs merger costs X_s if it engages in a merger. In this case, equation (1) becomes $nk + mk/2 = K$. We can then use $m = 2Kk^{-1} - 2n$ to replace m in the analysis below. We assume that either two small firms can merge (symmetric merger) or a small and a large firm can merge (asymmetric merger).[8] In the former case, a merger preserves the two-type industry structure, but the number of small and large firms changes. In the latter case firm

[8] We do not consider a merger between two large firms because this case does not add any significant new insight. In addition, this merger is more likely to be challenged by antitrust authorities as discussed in Section 2.4. Solutions for this case are available from the authors upon request.

heterogeneity increases. That is, all non-merging small or large firms remain identical, but the merged firm is larger than each of these firms, destroying the two-type industry structure.

Symmetric merger. We first consider a symmetric merger. We use Proposition 1 to derive the equilibrium for the small-large oligopoly industry. In particular, the output levels of a small firm and a large firm are respectively given by

$$q_s^*(t;n) = \frac{a\left(b+k^{-1}\right)}{\Delta(n)}Y(t), \tag{14}$$

$$q_l^*(t;n) = \frac{a\left(b+2k^{-1}\right)}{\Delta(n)}Y(t), \tag{15}$$

where[9]

$$\Delta(n) \equiv \left(b+k^{-1}\right)\left(b+2\left(Kb+1\right)k^{-1}\right)-b^2n>0. \tag{16}$$

The industry output and price are given by

$$Q^*(t;n) = \frac{a}{b}\left[1-\frac{\left(b+k^{-1}\right)\left(b+2k^{-1}\right)}{\Delta(n)}\right]Y(t), \tag{17}$$

$$P^*(t;n) = \frac{a\left(b+k^{-1}\right)\left(b+2k^{-1}\right)}{\Delta(n)}Y(t). \tag{18}$$

Note that the argument n in the preceding equations indicates that there are n large firms in the industry. This notation is useful for our merger analysis below since the number of large firms changes after a restructuring option is exercised. We will use similar notation below.

Using the preceding expressions and equation (6), we can compute equilibrium firm value in the industry.

Proposition 2 *Consider a symmetric merger between two small firms in the small-large oligopoly industry. Suppose Assumption 1 holds. The equilibrium value of the type $f = s, l$ firm is given by*

$$V_f(y;n) = \frac{\Pi_f(n)\,y^2}{r-2\left(\mu+\sigma^2/2\right)}, \tag{19}$$

where

$$\Pi_s(n) = \frac{a^2\left(b+k^{-1}\right)^3}{\Delta(n)^2}, \tag{20}$$

$$\Pi_l(n) = \frac{a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right)}{\Delta(n)^2}. \tag{21}$$

[9] It follows from (1) that $Kk^{-1} > n$. Thus, we can show that both $\Delta(n)$ and $\Delta(n+1)$ are positive.

After a merger between two small firms, there are $n+1$ identical large firms and $m-2$ identical small firms in the industry. Thus, the value of the large firm after a merger is given by $V_l(y; n+1)$. It follows from Proposition 2 that the benefit from merging is given by

$$V_l(y; n+1) - 2V_s(y; n) = \frac{[\Pi_l(n+1) - 2\Pi_s(n)]\, y^2}{r - 2(\mu + \sigma^2/2)}. \tag{22}$$

To have takeover incentives, the term $\Pi_l(n+1) - 2\Pi_s(n)$ must be positive. This term represents the profitability of an anticompetitive merger. After a merger, the number of small firms in the industry is reduced and hence the industry's market structure is also changed. Using Proposition 1, it is straightforward to show that the output of two small firms prior to a merger exceeds the output of one merged firm. Thus, an incentive to merge requires that the increase in industry price be sufficient to offset the reduction in output of the merged firm. Based on these arguments, we can summarize the following conditions for takeover incentives.

Proposition 3 *Consider a symmetric merger between two small firms. Let $\Delta(n)$ be given in (16) and define the critical value*

$$\Delta^* \equiv \frac{b^2}{1-A}, \tag{23}$$

where A is given by

$$A \equiv \left(b + 2k^{-1}\right)\sqrt{\frac{b + k^{-1}/2}{2\left(b + k^{-1}\right)^3}}. \tag{24}$$

(i) If

$$\max_n \Delta(n) = \Delta(0) < \Delta^*, \tag{25}$$

then there will always be an incentive to merge. (ii) If

$$\min_n \Delta(n) = \Delta(K/k - 1) > \Delta^*, \tag{26}$$

then there will never be an incentive to merge. (iii) If

$$\Delta(0) > \Delta^* > \Delta(K/k - 1), \tag{27}$$

then when n is large enough so that $\Delta(n) < \Delta^*$, there will be an incentive to merge.

In this proposition, (25) and (27) provide two conditions for takeover incentives in our Cournot–Nash framework.[10] That is, when the increase in price outweighs the decrease in output such that the net effect leads to an increase in instantaneous profits, the two small firms

[10] Similar conditions have been derived by Perry and Porter (1985) in their static model.

have an incentive to form a large organization. These two conditions depend on the industry demand function through b, the size of a large firm k, and the industry structure through the number n of existing large firms prior to the restructuring. Moreover, the proposition shows that there could be no incentives to merge if the condition in (26) holds. Finally, observe that this oligopolistic industry also encompasses the situation in which mergers would not occur unless the industry was sufficiently concentrated to start with.

It is straightforward to show that there is always an incentive to merge when the industry consists of a small-firm duopoly.[11] A similar result is obtained by Salant et al. (1983) and Perry and Porter (1985) in a static industry model. More recently, Lambrecht (2004) analyzes a dynamic model in which duopolists merge to form a monopolist motivated by economies of scale, while Morellec and Zhdanov (2005) assume an exogenously specified synergy gain for a merger of two firms.

Asymmetric merger. We now turn to an asymmetric merger. After a merger between a large firm and a small firm, the industry consists of $n-1$ identical large firms, $m-1$ identical small firms, and a huge merged firm. The merged firm owns capital $k_M = k + k/2 = 3k/2$. Using Proposition 1, we can show that the post-merger output levels of a small firm and a large firm are respectively given by

$$q_s^a(t; n-1) = \frac{a\left(b + k^{-1}\right)}{\Delta(n) - b^2\left(1 + \frac{2}{2+3bk}\right)} Y(t), \tag{28}$$

$$q_l^a(t; n-1) = \frac{a\left(b + 2k^{-1}\right)}{\Delta(n) - b^2\left(1 + \frac{2}{2+3bk}\right)} Y(t), \tag{29}$$

and the merged firm produces output at the level:

$$q_M^a(t; n-1) = \frac{\frac{3a}{3b+2k^{-1}}\left(b + k^{-1}\right)\left(b + 2k^{-1}\right)}{\Delta(n) - b^2\left(1 + \frac{2}{2+3bk}\right)} Y(t). \tag{30}$$

In addition, the post-merger industry output and price are given by

$$Q^a(t; n-1) = \frac{a}{b}\left[1 - \frac{\left(b + k^{-1}\right)\left(b + 2k^{-1}\right)}{\Delta(n) - b^2\left(1 + \frac{2}{2+3bk}\right)}\right] Y(t), \tag{31}$$

$$P^a(t; n-1) = \frac{a\left(b + k^{-1}\right)\left(b + 2k^{-1}\right)}{\Delta(n) - b^2\left(1 + \frac{2}{2+3bk}\right)} Y(t). \tag{32}$$

[11] One can immediately verify that condition (25) is satisfied for $k = K$, $n = 0$, and $m = 2$, which is a limiting case of our model and the one considered e.g. by Lambrecht (2004).

Using the preceding expressions, we can derive equilibrium firm value after an asymmetric merger in the following proposition:

Proposition 4 *Consider an asymmetric merger between a large firm and a small firm in the small-large oligopoly industry. Suppose Assumption 1 holds. After this merger, the equilibrium firm value is given by*

$$V_f^a(y; n-1) = \frac{\Pi_f^a(n-1)\, y^2}{r - 2(\mu + \sigma^2/2)}, \text{ for } f = s, l, M, \tag{33}$$

where

$$\Pi_s^a(n-1) = \frac{a^2\left(b + k^{-1}\right)^3}{\left[\Delta(n) - b^2\left(1 + \frac{2}{2+3bk}\right)\right]^2}, \tag{34}$$

$$\Pi_l^a(n-1) = \frac{a^2\left(b + 2k^{-1}\right)^2\left(b + k^{-1}/2\right)}{\left[\Delta(n) - b^2\left(1 + \frac{2}{2+3bk}\right)\right]^2}, \tag{35}$$

$$\Pi_M^a(n-1) = \frac{3a^2\left(b + 2k^{-1}\right)^2\left(b + k^{-1}\right)^2\left(3b + k^{-1}\right) / \left(3b + 2k^{-1}\right)^2}{\left[\Delta(n) - b^2\left(1 + \frac{2}{2+3bk}\right)\right]^2}. \tag{36}$$

The benefit from an asymmetric merger is given by

$$V_M^a(y; n-1) - V_s(y; n) - V_l(y; n) = \frac{\left[\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)\right] y^2}{r - 2(\mu + \sigma^2/2)}. \tag{37}$$

To have an incentive to merge, the preceding term must be positive. Analogous to Proposition 3, we have the following result.

Proposition 5 *Consider an asymmetric merger between a small and a large firm in the small-large oligopoly industry. Let $\Delta(n)$ be given in (16) and let the critical value Δ^* takes the value*

$$\Delta^a \equiv \frac{b^2}{1-D}\left(1 + \frac{2}{2+3bk}\right), \tag{38}$$

where D is given by

$$D \equiv \frac{\left(b + k^{-1}\right)\left(b + 2k^{-1}\right)}{3b + 2k^{-1}} \sqrt{\frac{3\left(3b + k^{-1}\right)}{\left(b + k^{-1}\right)^3 + \left(b + 2k^{-1}\right)^2\left(b + k^{-1}/2\right)}}. \tag{39}$$

Then parts (i)-(iii) in Proposition 3 apply here.

For the analysis in Section 3, we make the following assumption.

Assumption 2 *Suppose* $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n) > 0$ *and* $\Pi_l(n+1) - 2\,\Pi_s(n) < 0$ *so that there is an incentive to merge between a large firm and a small firm, but no incentive to merge between two small firms.*

A sufficient condition for this assumption is that case (i) or (iii) holds in Proposition 5, but both cases are violated in Proposition 3. For the analysis in Section 4, we make the following assumption.

Assumption 3 *Suppose* $\Pi_l(n+1) - 2\,\Pi_s(n) > 0$ *and* $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n) > 0$ *so that there is an incentive to merge between a large and a small firm and between two small firms.*

A sufficient condition for this assumption is that case (i) or (iii) holds in both Propositions 3 and 5. The following lemma is useful for our later merger analysis.

Lemma 1 *Under Assumption 2 or 3, the profit differentials* $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)$ *increases with the parameters* a *and* n*, and the profit ratios* $[\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)]/\Pi_s(n)$ *and* $[\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)]/\Pi_l(n)$ *increase with the parameter* n*.*

To interpret this lemma, recall that the parameter n represents the number of large firms in the industry prior to a merger. This parameter proxies for industry concentration. A higher value of n represents a higher level of industry concentration. The parameter a represents the exposure of industry demand to the industry-wide shock. A larger value of a implies that the increase in industry price is higher in response to an increase in the exogenous demand shock. Lemma 1 then demonstrates that under certain conditions the anticompetitive gains from a merger measured in terms of either profit differential or profit ratio are larger in more concentrated industries. In addition, when measured in terms of profit differential, these gains are also larger in industries that are more exposed to the industry-wide shock.

Example 1. The conditions in Assumption 2 or 3 are not easy to check analytically. We thus use a numerical example to illustrate them and Propositions 3 and 5. We set the baseline parameter values: $k = 0.2$, $b = 0.5$, and $K = 1$. Figure 1 depicts equations (23) and (38) by a dashed and a dotted lines, respectively, as a function of the price sensitivity of demand b and the size of a large firm k. To determine the incentive to merge, we need to compare the critical threshold values with the values of $\Delta(0)$ and $\Delta(1/k-1)$. The figure shows that asymmetric mergers are always more profitable than symmetric mergers given that the threshold function Δ^* always lies below Δ^a. Moreover, the top-left panel illustrates that a sufficiently low value of b leads to an incentive to merge for both symmetric and asymmetric mergers, as implied by condition (25). This incentive is determined by the parameters b, k, and K only, and is independent of industry concentration. But for very high values of b, there is no incentive to merge for both symmetric and asymmetric mergers, as implied by condition (26). In an intermediate region, asymmetric mergers are still profitable, while symmetric mergers are not as implied by condition (27). The bottom-left panel illustrates that there is an incentive to have an asymmetric merger, but there is never an incentive to have a symmetric merger for $k \in [0.17, 0.23]$. The panels on the right illustrate that, in an industry with $n \in [1, 5]$ firms, only an incentive for asymmetric mergers exists when $b = 0.5$, but an incentive for both types of mergers exists when $b = 0.4$. Thus, a small change in the price sensitivity can have a large impact on the incentive to merge even when the number of large firms remains unchanged.

[Insert Figure 1 Here]

Figure 2 presents the profit differentials of symmetric and asymmetric mergers for a wide range of parameter values of b and n. Both profit differentials are monotonically increasing with the number of large firms n, but non-monotonic with the price sensitivity parameter b. Intuitively, this non-monotonicity results from two opposing effects of a decrease in b. Specifically, it raises price, but lowers output. Thus, the economic impact of changes in the price sensitivity parameter on the profit differentials is ambiguous because it depends on whether the price effect or the quantity effect dominates. Finally, on the lower left end of the surfaces we find again that asymmetric mergers can be profitable, while symmetric mergers are not so that Assumption 2 is satisfied. For example, this happens when $b = 0.5$ and n takes values from 1 to 5. By contrast, when $b = 0.4$ and n takes values from 1 to 5, both symmetric and asymmetric mergers are profitable so that Assumption 3 is satisfied.

[Insert Figure 2 Here]

2.4 Discussion

In this section, we provide further discussions on two of our main modeling choices. First, we assume that firms can grow only through a takeover and that internal investment or new entry is not allowed. From a technical point of view, if firms can make internal investment by adjusting capital continuously over time, then the capital stock is a state variable. Because the industry shock must be another state variable, our model would become a two dimensional combined stopping and control problem, which is very hard to solve. From an economic point of view, Lambrecht (2004) elaborates on the choice between internal investment and external acquisitions. As the discussion in his paper also applies to our setting, we do not repeat it here.

Introducing continuous entry into our model would be interesting, but may complicate our analysis significantly. Intuitively, entry may erode anticompetitive profits. However, when sunk entry costs are sufficiently high, the opportunity for entry created by an anticompetitive merger plausibly is too small to induce entry, as shown in Werden and Froeb (1998). More recently, Pesendorfer (2005) analyzes dynamic mergers under continuous entry. Unlike our model with heterogeneous firms, his model assumes that firms are identical and operate at a constant average cost, as in Salant et al. (1983). In contrast to Salant et al. (1983), he shows that a merger for monopoly may not be profitable and a merger in a non-concentrated industry can be profitable.

Second, we assume that mergers between two large firms or further mergers over time are not allowed. Instead, we only allow two small firms to merge or a large firm and a small firm to merge. Of course, this assumption makes our analysis tractable and permits us to focus on the key questions as to how product market competition interacts with bidder competition and how product market competition influences the timing and terms of mergers as well as merger returns. We can justify this assumption by invoking anti-trust law. As described in White (1987, p. 16), "the Guidelines use the Herfindahl-Hirschman Index (HHI) as their primary market concentration guide, with concentration levels of 1000 and 1800 as their two key levels. Any merger in a market with a post-merger HHI below 1000 is unlikely to be challenged; a merger in a market with a post-merger HHI above 1800 is likely to be challenged (if the merger partners have market shares that cause the HHI to increase by more than 100), unless other mitigating circumstances exist, like easy entry. Mergers in markets with post-concentration HHI levels between 1000 and 1800 require further analysis before a decision is made whether to challenge." In our model, a merger of two large firms would raise industry concentration to a higher level than a merger of two small firms or a merger between a small firm and a large firm. The industry concentration level following a merger of two large firms is more likely to cross the regulatory threshold, and thus such a merger is more likely to be challenged by anti-trust authorities.[12]

[12] In our model for an industry with two large firms and a price sensitivity of $b = 0.5$, the pre-merger HHI equals 1378 when $k = 0.2$, $K = 1$, and $a = 100$. This concentration measure rises to 1578 (1734) following a merger of two small firms (a merger of a small and a large firm). In contrast, the post-merger HHI is 2022 after a merger of two large firms.

3 Mergers with a single bidder

In this section, we analyze the timing and terms of a merger between a small firm target and a large firm bidder. We suppose Assumption 2 holds so that two identical small firms do not have an incentive to merge, but a large firm and a small firm have an incentive to merge. This case is interesting because mergers typically involve a larger firm acquiring a smaller firm in corporate practice. For example, in Andrade, Mitchell, and Stafford's (2001) sample of 4,256 deals from 1973 to 1998, the target's median relative size is 11.7% of the acquirer. Moreover, Moeller, Schlingemann, and Stulz (2004) measure relative size as the transaction value divided by the acquirer's equity value and report averages of 19.2% (50.2%) for 5,503 small (6,520 large) acquirers between 1980 and 2001. Finally, Rhodes-Kropf and Robinson (2007) also document positive and significant differences in market equity valuations of bidders and targets.

We consider a merger mechanism in which the bidder submits the bid in the form of an ownership share of the merged firm's equity.[13] Given this bid, both the bidder and the target select their value-maximizing merger timing. The equilibrium bid is equal to the value such that the merger timing selected by the bidder and the target agrees. The merger offers participants in the deal an option to exchange one asset for another. That is, they can exchange their shares in the initial firm for a fraction of the shares of the merged firm. Thus, the merger opportunity is analogous to an exchange option (Margrabe, 1978). The equilibrium timing and terms of the merger are the outcome of an option exercise game in which each participant determines an exercise strategy for its exchange option (see Grenadier, 2002, Lambrecht, 2004, and Morellec and Zhdanov, 2005). We will first solve for the equilibrium and then show that the equilibrium timing is globally optimal.

3.1 Equilibrium

To solve for the equilibrium, we first consider the exercise strategy of the large firm bidder. Let ξ_l denote the ownership share of the large firm in the merged entity. Then $1-\xi_l$ is the ownership share of the small firm target. The merger surplus accruing to the large firm is given by the positive part of the (net) payoff from the merger:

$$\left[\xi_l V_M^a\left(y;n-1\right)-V_l\left(y;n\right)-X_l\right]^+, \tag{40}$$

where $V_M^a\left(y;n-1\right)$ and $V_l\left(y;n\right)$ are given in Propositions 2 and 4. When considering a merger, the large firm trades off the stochastic benefit from merging against the fixed cost X_l of merging.

[13] This merger mechanism is similar to a friendly merger analyzed in Lambrecht (2004). In an earlier working paper version of our paper, we also analyze a hostile takeover as in Lambrecht (2004). We do not consider this case here since it does not bring a significant new insight.

Since firms have the option but not the obligation to merge, the surplus from merging has a call option feature.

Let y_l^* denote the merger threshold selected by the large firm. The value of this firm's option to merge, denoted by $OM_l\big(y, y_l^*, \xi_l; n\big)$ for $y \leq y_l^*$, is given by

$$OM_l\left(y, y_l^*, \xi_l; n\right) = \mathbb{E}^y\left\{e^{-r\tau_{y_l^*}}\left[\xi_l V_M^a\left(Y(\tau_{y_l^*}); n-1\right) - V_l\left(Y(\tau_{y_l^*}); n\right) - X_l\right]\right\}, \qquad (41)$$

where $\tau_{y_l^*}$ denotes the first passage time of the process (Y_t) starting from the value y to the merger threshold y_l^* selected by the large firm. By a standard argument (e.g. Karatzas and Shreve, 1999), we can show that

$$OM_l\left(y, y_l^*, \xi_l; n\right) = \left[\xi_l V_M^a\left(y_l^*; n-1\right) - V_l\left(y_l^*; n\right) - X_l\right]\left(\frac{y}{y_l^*}\right)^{\beta}, \qquad (42)$$

where β denotes the positive root of the characteristic equation

$$0.5\,\sigma^2\,\beta\,(\beta-1) + \mu\,\beta - r = 0. \qquad (43)$$

Note that it is straightforward to prove that $\beta > 2$ under Assumption 1. Equation (42) admits an intuitive interpretation. The value of the option to merge is equal to the surplus, $\left[\xi_l V_M^a\left(y_l^*; n-1\right) - V_l\left(y_l^*; n\right) - X_l\right]$, generated at the time of the merger multiplied by a discount factor $(y/y_l^*)^{\beta}$. This discount factor can be interpreted as the Arrow-Debreu price of a primary claim that delivers \$1 at the time and in the state when the merger occurs. It can also be regarded as the probability of the demand shock $Y\left(t\right)$ reaching the merger threshold y_l^* for the first time from below, given that the current level of the demand shock is y.

The optimal threshold y_l^* selected by the large firm maximizes the value of the merger option (42). Thus, it satisfies the following first-order condition:

$$\frac{\partial OM_l(y, y_l^*, \xi_l; n)}{\partial y_l^*} = 0. \qquad (44)$$

Solving this equation yields:

$$y_l^* = \sqrt{\frac{\beta X_l}{\beta - 2}\frac{r - 2\left(\mu + \sigma^2/2\right)}{\xi_l \Pi_M^a\left(n-1\right) - \Pi_l\left(n\right)}}. \qquad (45)$$

It follows that, as a function of ξ_l, y_l^* decreases with ξ_l. This function gives the merger threshold for a given value of the ownership share ξ_l.

We next turn to the exercise strategy of the small firm target, which can be solved in a similar fashion. Akin to equation (42), the value of the small firm target's option to merge is given by

$$OM_s\left(y, y_s^*, \xi_l; n\right) = \left[\left(1-\xi_l\right) V_M^a\left(y_s^*; n-1\right) - V_s\left(y_s^*; n\right) - X_s\right]\left(\frac{y}{y_s^*}\right)^{\beta}. \qquad (46)$$

The optimal exercise strategy y_s^* selected by the small firm target satisfies the first-order condition

$$\frac{\partial OM_s(y, y_s^*, \xi_l; n)}{\partial y_s^*} = 0. \tag{47}$$

Solving this equation yields:

$$y_s^* = \sqrt{\frac{\beta X_s}{\beta - 2} \frac{r - 2\left(\mu + \sigma^2/2\right)}{\left(1 - \xi_l\right) \Pi_M^a \left(n - 1\right) - \Pi_s \left(n\right)}}. \tag{48}$$

This equation implies that as a function of ξ_l, the merger threshold y_s^* selected by the small firm target increases with ξ_l.

In equilibrium, the negotiated ownership share must be such that the merger timing selected by the bidder and the target agrees in that $y_l^* = y_s^*$. Using this condition, we can solve for the equilibrium timing and terms of the merger.

Proposition 6 *Consider an asymmetric merger between a large firm and a small firm. Suppose that Assumptions 1 and 2 hold. (i) The value-maximizing restructuring policy is to merge when the industry shock* (Y_t) *reaches the threshold value*

$$y^* = \sqrt{\frac{\beta \left(X_s + X_l\right)}{\beta - 2} \frac{r - 2\left(\mu + \sigma^2/2\right)}{\Pi_M^a \left(n - 1\right) - \Pi_l \left(n\right) - \Pi_s \left(n\right)}}. \tag{49}$$

(ii) The merger threshold y^* *decreases with* a *and* n*. (iii) The share of the merged firm accruing to the large firm is given by*

$$\xi_l^* = \frac{X_l}{X_s + X_l} + \frac{\Pi_l \left(n\right) X_s - \Pi_s \left(n\right) X_l}{\left(X_s + X_l\right) \Pi_M^a \left(n - 1\right)}. \tag{50}$$

(iv) The ownership share ξ_l^* *decreases with* n*.*

Parts (i) and (ii) of this proposition characterize the merger timing. Since one can verify that the values of the option to merge for both firms are increasing with the realization y of the industry shock, a merger occurs in a rising product market. Thus, consistent with empirical evidence documented by Maksimovic and Phillips (2001), cyclical product markets generate procyclical mergers. This result is also consistent with Mitchell and Mulherin's (1996) empirical finding that industry shocks contribute to the merger and restructuring activities observed during the 1980s.

As is well known, the merger threshold y^* given in (49) determines the merger timing and merger likelihood. A higher value of the merger threshold implies a larger value of the expected time of the merger and a lower probability of merger within a given time horizon.[14] To interpret equation (49), we use Propositions 2 and 4 to rewrite it as

$$V_M^a\left(y^*, n-1\right) - V_l\left(y^*; n\right) - V_s\left(y^*; n\right) = \frac{\beta\left(X_s + X_l\right)}{\beta - 2} > X_s + X_l. \tag{51}$$

This equation implies that, at the time of the merger, the benefit from the merger exceeds the sum of the merger costs $X_s + X_l$. This reflects the option value of waiting because the merger is irreversible. Since mergers and acquisitions are analogous to an irreversible investment, the standard comparative statics results regarding the merger timing are well known in the real options literature (e.g., Dixit and Pindyck, 1994). For example, an increase in the industry's demand uncertainty delays the timing of mergers, and an increase in the drift of the industry's demand shock speeds up the timing of mergers. So we suppress this discussion.

The novel comparative statics results in our paper are related to industry characteristics. First, part (ii) of Proposition 6 implies that the optimal merger threshold declines with the parameter a. Since the parameter a represents the exposure of the industry demand to the exogenous shock, one should expect to observe more mergers and acquisitions in industries whose demand is more exposed or more sensitive to exogenous shocks. This result is consistent with the empirical evidence documented by Mitchell and Mulherin (1996). They find that industries that experienced the greatest amount of merger and restructuring activity in the 1980s are those that were exposed the most to industry shocks. The intuition behind the preceding result is that an increase in the parameter a raises industry demand for a given positive shock. Thus, it raises anticompetitive gains $\Pi_M^a\left(n-1\right) - \Pi_l\left(n\right) - \Pi_s\left(n\right)$ as shown in Lemma 1, thereby raising the benefits from merging.

We next turn to the effect of industry concentration. Part (ii) of Proposition 6 implies that one should expect to see more mergers and acquisitions taking place in more concentrated (i.e. less competitive) industries. The economic intuition behind this result is simple. A relatively higher level of pre-merger industry concentration is associated with relatively larger anticompetitive profits by Lemma 1, which raises the incentive to merge, *ceteris paribus*. In particular, a larger magnitude of anticompetitive profits leads to an increase in restructuring benefits and thus firms in *less* competitive industries will optimally exercise their option to merge, in expectation, earlier.

[14] The probability of a merger taking place over a time interval $[0, T]$ is given by

$$\Pr\left(\sup_{0 \le \tau \le T} Y\left(\tau\right) \ge y^*\right) = \mathcal{N}\left[\frac{\ln\left(y_0/y^*\right) + \left(\mu - \sigma^2/2\right)T}{\sigma\sqrt{T}}\right] + \left(\frac{y_0}{y^*}\right)^{-\left(2\mu - \sigma^2\right)/\sigma^2} \mathcal{N}\left[\frac{\ln\left(y_0/y^*\right) - \left(\mu - \sigma^2/2\right)T}{\sigma\sqrt{T}}\right],$$

where $\mathcal{N}$ is the standard normal cumulative distribution function. This probability decreases with the merger threshold y^*.

This implication for the optimal timing of mergers in our Cournot–Nash framework is in sharp contrast to most of the earlier findings in the real options literature. Notably, Grenadier (2002) demonstrates that firms in more competitive industries will optimally exercise their investment options earlier in a symmetric industry equilibrium model with irreversible investment. We attribute the startling difference in results to differences in the economic modeling of industry competition and structure. In our asymmetric industry equilibrium model, anticompetitive profits result from merging two firms to form a new firm, which endogenously alters product market competition. In Grenadier's (2002) model, anticompetitive profits result from exogenously reducing the number of identical firms that compete for an investment opportunity in the industry. In addition, Grenadier (2002) studies an incremental investment problem, while we analyze a single discrete option exercise decision. We will consider scarcity of targets that can lead to competition among multiple bidders in Section 4. This extension will hurt the acquirer, as also shown by Morellec and Zhdanov (2005), and hence will potentially attenuate the delayed option exercise due to the product market effects that are central to our model.

Parts (iii) and (iv) of Proposition 6 characterize the ownership share. Part (iii) shows that the large firm bidder demands a larger ownership share than the small firm target if they incur identical merger costs $X_l = X_s$. Intuitively, because the large firm has a larger pre-merger firm value, it demands a larger ownership share in order for it to give up its old firm in exchange for a share in the merged firm. This result is consistent with the finding of Lambrecht (2004) in which mergers are motivated by economies of scale. Unlike Lambrecht (2004), we show that the pre-merger industry concentration level also influences the ownership share. In particular, part (iv) of Proposition 6 shows that a large merging firm demands a smaller ownership share in more concentrated industries, *ceteris paribus*. The intuition is that the pre-merger profit differential between the large and the small merging partners relative to the value of the merged firm decreases with industry concentration. Thus, the large firm does not need to demand a larger share in more concentrated industries.

Example 2. Figure 3 illustrates Proposition 6. The input parameter values are $a = 100$, $b = 0.5$, $k = 0.2$, $K = 1$, $n = 2$, $r = 8\%$, $X_l = 2$, $X_s = 2$, $\mu = 1\%$, and $\sigma = 20\%$. As discussed earlier, the non-monotonicity of the profit differential with respect to the price sensitivity results from a tradeoff between a price effect and a quantity effect. The top panel of the figure displays the impact of this economic phenomenon on the merger threshold. Moreover, because the merger gains increase with industry concentration, the merger threshold decreases with the number of large firms in the industry, as shown in the bottom panel. In particular, when n rises from 2 to 5, the merger threshold y^* declines from 2.30 to 1.84. If we set the initial shock

$y_0 = 1$, these threshold values imply that the likelihood of a merger over a five-year horizon rises from 5.0% to 14.7%.

[Insert Figure 3 Here]

3.2 Cumulative merger returns

We now turn to cumulative returns resulting from an asymmetric merger. The equity value of a large merging firm before the merger, denoted by $E_l\left(Y(t);n\right)$, is equal to the value of its assets in place plus the value of the merger option:

$$E_l\left(Y(t);n\right) = V_l\left(Y(t);n\right) + OM_l\left(Y(t),y^*,\xi_l^*;n\right), \tag{52}$$

where $OM_l\left(Y(t),y^*,\xi_l^*;n\right)$ is given by (42) with y^* and ξ_l^* given in Proposition 6. Similarly, the equity value of a small merging firm before the merger is given by

$$E_s\left(Y(t);n\right) = V_s\left(Y(t);n\right) + OM_s\left(Y(t),y^*,\xi_l^*;n\right), \tag{53}$$

where $OM_s\left(\cdot;n\right)$ is defined in (46).

We may express the cumulative merger returns as a fraction of the stand-alone equity value $V_f\left(Y(t);n\right)$ of the small firm $f = s$ and the large firm $f = l$. That is, the cumulative returns to the small and large merging firms at time $t \leq \tau_{y^*}$ is given by

$$R_{f,M}\left(Y(t),n\right) = \frac{E_f\left(Y(t);n\right) - V_f\left(Y(t);n\right)}{V_f\left(Y(t);n\right)} = \frac{OM_f\left(Y(t),y^*,\xi_l^*;n\right)}{V_f\left(Y(t);n\right)}, \tag{54}$$

for $f = s, l$. The cumulative return to the merging firm f at the time of the merger announcement is equal to the preceding expression evaluated at $t = \tau_{y^*}$ or $Y\left(t\right) = y^*$.

Similarly, we can compute the cumulative merger return to a rival firm at the time of the merger announcement. The stock value of a rival firm $f = s, l$ prior to the announcement of a merger at date $t \leq \tau_{y^*}$ is given by

$$V_f\left(Y(t);n\right) + \mathbb{E}\left[e^{-r(\tau_{y^*}-t)}\left(V_f^a\left(Y(\tau_{y^*});n-1\right) - V_f\left(Y(\tau_{y^*});n\right)\right)|Y\left(t\right)\right]. \tag{55}$$

That is, it is equal to the firm value before the merger plus an option value from the merger. This option value results from the fact that the value of the rival firm becomes $V_f^a\left(Y(\tau_{y^*});n-1\right)$ after the asymmetric merger since there are $n-1$ large firms, $m-1$ small firms, and a huge merged firm in the industry. The cumulative return to a small or large rival firm before the merger is given by

$$R_{f,R}\left(Y\left(t\right);n\right) = \frac{\mathbb{E}\left[e^{-r(\tau_{y^*}-t)}\left(V_f^a\left(Y(\tau_{y^*});n-1\right) - V_f\left(Y(\tau_{y^*});n\right)\right)|Y\left(t\right)\right]}{V_f\left(Y(t);n\right)} \tag{56}$$

for $f = s, l$. We will focus on the cumulative return at the time of the merger announcement, when $t = \tau_{y^*}$ or $Y(\tau_{y^*}) = y^*$. The following proposition characterizes the cumulative returns at the time of the merger announcement.

Proposition 7 *Consider an asymmetric merger between a large firm and a small firm. Suppose that Assumptions 1 and 2 hold. (i) The cumulative merger returns to the small and large merging firms at the time of restructuring are given by*

$$R_{f,M}\left(y^*; n\right) = \frac{2}{\beta} \frac{\Pi_M^a\left(n-1\right) - \Pi_l\left(n\right) - \Pi_s\left(n\right)}{\Pi_f\left(n\right)} \frac{X_f}{X_s + X_l}, \tag{57}$$

for $f = s, l$. (ii) The cumulative merger returns to a small and a large rival firm at the time of restructuring are given by

$$R_{f,R}\left(y^*; n\right) = \left[1 - \frac{b^2}{\Delta\left(n\right)}\left(1 + \frac{2}{2+3bk}\right)\right]^{-2} - 1, \tag{58}$$

for $f = s, l$. (iii) All the above returns are positive and increase with n.

Proposition 7 highlights several interesting aspects of cumulative merger returns in our Cournot–Nash framework. First, equation (57) reveals that the cumulative returns to the two merging firms have three determinants including an anticompetitive effect, a size effect, and a hysteresis effect. The hysteresis effect is represented by β. It implies that more uncertainty leads to higher cumulative returns to both the acquirer and the target. With more uncertainty, the merger option is exercised when it is deeper in the money, resulting in higher cumulative merger returns. The size effect reflects the fact that the large merging firm or the firm with a smaller merger cost has a smaller return than the small merging firm or the firm with a larger merger cost. The preceding size and hysteresis effects are also derived by Lambrecht (2004) in the presence of economies of scale. In contrast to Lambrecht (2004), our model with constant returns to scale does not have a synergy effect discussed by Lambrecht (2004). Instead, we have the anticompetitive and cost reduction effect represented by the term $\Pi_M^a\left(n-1\right) - \Pi_l\left(n\right) - \Pi_s\left(n\right)$, which characterizes the benefit from merging. This effect reflects the fact that after a merger, the number of small and large firms in the industry decreases and a huge merged firm emerges. Hence, the market structure and competitive landscape of the industry change. In addition, the huge merged firm combines the assets of the two merging firms, thereby reducing the production cost. The term $[\Pi_M^a\left(n-1\right) - \Pi_l\left(n\right) - \Pi_s\left(n\right)]/\Pi_f\left(n\right)$ therefore indicates the percentage gain in profits. Note that this percentage gain differs for a small and a large merging firm as it depends on the firm's status quo and its relative contribution to the merger benefits.

Second, merging firms have higher cumulative returns in more concentrated industries. The intuition is that the anticompetitive effect on the industry's equilibrium price after a merger is stronger for those industries. As a consequence, merging firms derive higher restructuring benefits. Third, equation (58) reveals that the cumulative return to a small or a large rival firm at the time of a merger announcement is positive, too. Like the cumulative returns to merging firms, the cumulative returns to rival firms increase with industry concentration. The intuition is that the industry's equilibrium price rises after the merger, and rival firms also benefit from this price increase. This benefit increases with industry concentration. Finally, the magnitude of rival returns does not depend on firm size as rival firms do not change their capital stock.

Example 3. We illustrate Proposition 7 by a numerical example in which we set the parameter values $a = 100$, $b = 0.5$, $k = 0.2$, $n = 2$, $r = 8\%$, $X_l = 2$, $X_s = 2$, $\mu = 1\%$, and $\sigma = 20\%$. As in the previous example, we set $X_l = X_s$ to focus on the portion of cumulative returns that is not simply due to differences in mergers costs. Figure 4 presents the returns of a small merging firm (dashed line) and a large merging firm (dotted line) given in equation (57) as well as the return to a rival firm (solid line) given in equation (58) as functions of the price sensitivity of demand b and the number of large firms n. The figure reveals several interesting aspects of the fundamental determinants of merger and rival returns, such as firm size, profitability, and the firm's contribution to the creation of the merger benefit. First, the cumulative return for a small merging firm always exceeds that of a large merging firm. Second, the cumulative merger returns vary non-monotonically with the price sensitivity of demand b. Interestingly, rival returns however are increasing with b. Industry rivals benefit more from the price effect than they suffer from the quantity effect because they do not change their firm size and hence their negative quantity adjustment is less severe than that of the merging firms. Third, a higher level of industry concentration leads unambiguously to higher returns for all firms in the industry.

[Insert Figure 4 Here]

3.3 Global optimality

As in Lambrecht (2004) and Morellec and Zhdanov (2005), the equilibrium merger timing analyzed in Section 3.1 is globally optimal. Formally, consider a central planner who selects the merger timing to maximize the total surplus from the merger. This globally optimal equilibrium solves the following problem

$$\max_{\tau} \mathbb{E}^y \left\{ e^{-r\tau} \left[V_M^a \left(Y_\tau; n-1 \right) - V_s \left(Y_\tau; n \right) - V_l \left(Y_\tau; n \right) - X_s - X_l \right] \right\}. \tag{59}$$

We can easily check that the surplus-maximizing policy is characterized by the trigger policy whereby the two firms merge the first time the process reaches the threshold y^* given in Proposition 6. This result suggests that to solve for the equilibrium we can first solve for the globally optimal merger timing and then solve for the sharing rule such that the globally optimal merger timing is also optimal to each of the two merging partners.

4 Bidder competition and industry competition

So far, we have focused on an asymmetric merger with a single bidder. In this section, we consider two bidders who compete for a small firm target. One of the bidders is a small firm and the other one is a large firm. We suppose Assumption 3 holds so that both bidders have incentives to merge with the target. As argued by Fishman (1988) and Morellec and Zhdanov (2005), bidder competition puts the target in an advantageous position and allows target shareholders to extract a higher premium from the bidding firms. Unlike these two papers, we do not consider asymmetric information. In addition, merger benefits are, in our model, endogenously derived from an industry equilibrium instead of exogenously specified. Thus, product market competition plays an important role.

We now analyze the equilibrium in a takeover contest, following similar steps as in Morellec and Zhdanov (2005). Once the takeover contest is initiated, the two bidding firms submit their bids to the target in the form of a fraction of the merged firm's equity to be owned by target shareholders upon the takeover consummation. The bidder who offers the highest value to the target shareholders wins the contest. Given the winner's ownership share, the winning bidder and the target select their merger timing independently. In equilibrium, they must agree on the merger timing.

We can show that the value of the merged firm is higher when the small firm target merges with a large firm bidder than that when it merges with a small firm bidder. Formally, we can use equations (21) and (36) to show that

$$V_M^a\left(y; n-1\right) > V_l\left(y; n+1\right). \tag{60}$$

It follows that the large firm is the stronger of the two bidders and wins the takeover contest because the large firm bidder can always copy the small firm bidder's bid and can also deliver a larger value to target shareholders. Even though the large firm bidder wins the contest, the losing small firm bidder may influence the equilibrium terms and timing of the merger. To analyze this point, we define the breakeven share for the small firm bidder as

$$\xi_s^{BE} V_l\left(y; n+1\right) - V_s\left(y; n\right) - X_s = 0. \tag{61}$$

If the small firm bidder places a bid higher than $1-\xi_s^{BE}$, then it will realize a negative value by entering the deal. Thus, the small firm bidder would be better off losing the takeover contest. We consider two cases.

In the first case, the value associated with the ownership share offered to target shareholders if the large firm bidder wins is greater than that associated with the breakeven share of the small firm bidder if it wins:

$$(1-\xi_l^*)\, V_M^a\,(y; n-1) > \left(1-\xi_s^{BE}\right) V_l\,(y; n+1)\,. \tag{62}$$

In this case, for the small firm bidder to win, it must offer an ownership share of at least $\left(1-\xi_s^{BE}\right)$ to target shareholders. But this implies a negative value for the small firm bidder. Thus, it would rather drop out of the takeover contest. The losing bidder is too weak to matter and the equilibrium is the same as in the case of a single bidder analyzed in Section 3.

In the second case, the losing bidder is strong so that bidder competition matters. This case happens when the value associated with the ownership share offered to target shareholders if the large firm bidder wins is lower than that associated with the breakeven share of the small firm bidder if it wins:

$$(1-\xi_l^*)\, V_M^a\,(y; n-1) < \left(1-\xi_s^{BE}\right) V_l\,(y; n+1)\,. \tag{63}$$

In this case, the small firm bidder has an incentive to bid an amount slightly less than $\left(1-\xi_s^{BE}\right)$ in order to win the contest. Anticipating this bidder's competition, the large firm bidder will place a bid higher than $(1-\xi_l^*)$ until equality holds in (63). Therefore, we define $\xi_{\max}$ as the value satisfying the equation

$$(1-\xi_{\max})\, V_M^a\,(y; n-1) = \left(1-\xi_s^{BE}\right) V_l\,(y; n+1)\,. \tag{64}$$

The value $\xi_{\max}$ is the maximal share that the large firm bidder can extract from the merged firm such that it still wins the takeover contest. If it demands a share higher than $\xi_{\max}$ or it places a bid lower than $(1-\xi_{\max})$, then the small firm will outbid the large firm and win the contest. Thus, the best response of the large firm is to place a bid $(1-\xi_{\max})$.

From equation (64), we can solve for the merger threshold selected by the winning large firm bidder:

$$y = y_B\,(\xi_{\max}) \equiv \sqrt{\frac{X_s\,(r-2\,(\mu+\sigma^2/2))}{\Pi_l\,(n+1)-\Pi_s\,(n)-(1-\xi_{\max})\,\Pi_M^a\,(n-1)}}. \tag{65}$$

Using equation (48), we can derive the merger threshold selected by the small firm target's given the winning bidder's demanded ownership share $\xi_{\max}$:

$$y = y_T(\xi_{\max}) \equiv \sqrt{\frac{\beta X_s}{\beta - 2} \frac{r - 2(\mu + \sigma^2/2)}{(1 - \xi_{\max}) \Pi_M^a (n-1) - \Pi_s (n)}}. \tag{66}$$

In equilibrium, the winning bidder and the target must agree on the merger timing in that $y_B(\xi_{\max}) = y_T(\xi_{\max})$. We can then solve the system of equations (65) and (66) for the winning bidder's ownership share when the losing bidder is strong:

$$\xi_{\max}^* = 1 - \frac{\beta \Pi_l (n+1) - 2\Pi_s (n)}{2(\beta - 1) \Pi_M^a (n-1)}. \tag{67}$$

Combining the above two cases, we have the following result:

Proposition 8 *Consider a small firm bidder and a large firm bidder competing for a small firm target. Suppose Assumptions 1 and 3 hold. Then the large firm wins the contest. (i) If condition (62) holds, then the losing bidder is weak and the equilibrium is described in Proposition 6. (ii) If condition (63) holds, then the losing bidder is strong so that bidder competition speeds up the takeover process and erodes the ownership stake of bidding shareholders. The share of the merged firm accruing to the winning large firm bidder is given by $\xi_{\max}^*$ defined in (67). In addition, the takeover takes place the first time the industry shock (Y_t) reaches the threshold value $y_{bc}^* \equiv y_B(\xi_{\max}^*)$.*

To interpret this result further, we define the bid premium resulting from bidder competition as the percentage increase in the target equity value, compared to the case without bidder competition. These equity values are evaluated at the equilibrium time of the merger. Formally, the bid premium resulting from bidder competition is given by

$$\frac{[(1-\xi_{\max}^*)V_M^a(y_{bc}^*; n-1) - (1-\xi_l^*)V_M^a(y_{bc}^*; n-1)]}{(1-\xi_l^*)V_M^a(y_{bc}^*; n-1)} = \frac{\xi_l^* - \xi_{\max}^*}{1-\xi_l^*}. \tag{68}$$

This bid premium is positive only when the losing small firm bidder is strong. In this case, condition (63) holds. Using (63) and (64), we can then show that $\xi_l^* > \xi_{\max}^*$. To make this condition more transparent, we use (50) and (67) to derive that $\xi_l^* > \xi_{\max}^*$ if and only if

$$\Pi_l (n+1) - 2\Pi_s (n) > \frac{2(\beta - 1) X_s}{\beta (X_s + X_l)} [\Pi_M^a (n-1) - \Pi_l (n) - \Pi_s (n)]. \tag{69}$$

The preceding condition says that the small bidder is strong if the benefit from its merger with the small firm target is larger than the benefit from the merger between the large firm bidder and the small firm target, multiplied by a constant factor. This factor depends on the merger costs and the uncertainty surrounding industry shocks. It reveals that condition (69) is more likely to hold if the small bidder's merger cost is relatively smaller. Importantly, both this condition and the bid premium depend on the industry characteristics such as the industry concentration level n and the price sensitivity of demand parameter b. As a result, there is interaction between product market competition and bidder competition in our model.

Example 4. We now use a numerical example to illustrate Proposition 8 and condition (69). This example also illustrates the interaction between bidder competition and industry competition. We set the baseline parameter values $a = 100$, $k = 0.2$, $K = 1$, $b = 0.4$, $n = 4$, $r = 8\%$, $\mu = 1\%$, and $\sigma = 20\%$. To ensure that the small bidder can be potentially strong in the second case of Proposition 8, we set $X_l = 20$ and $X_s = 1$. The two panels on the left of Figure 5 present the ratio of the expression on the left-hand side of condition (69) to the expression on the right-hard side as a function of the number of large firms n and the price sensitivity of demand b. The panels on the right present the bid premium in equation (68) as a function of the number of large firms n and the price sensitivity of demand b. The figure highlights several key elements of the industry equilibrium with bidder competition. First, the top-right panel shows that the bid premium is first an increasing and then a decreasing function of the price sensitivity parameter b. When b is sufficiently high, condition (69) fails and the bid premium is negative. Second, note that negative values for the bid premium are not a discount, but rather indicate that in this case the small bidder is too weak to matter so that the target shareholders only receive $(1 - \xi_l^*)$ as in Section 3. Third, low product market competition mutes bidder competition because the small firm bidder is too weak. But as n increases the small bidder may become strong so that target shareholders can extract a bid premium from the winning bidder. The panels on the bottom reveal that when the industry is sufficiently concentrated, bidder competition matters. In the numerical example, bidder competition induces an additional bid premium if there are at least three large firms in the industry. Fourth, if the industry is sufficiently concentrated, then the bid premium increases with industry concentration. Finally, we should point out that our model is stylized and cannot be calibrated precisely to the data. Yet the purpose of this example is to emphasize the interesting case of a sign change in the bid premium, which is naturally in a region of small values of the bid premium. While the bid premium resulting from bidder competition in our setting can be larger for different paramter values, it is only one component of the combined run-up and markup, which empirically may exceed 20%, as documented by Schwert (1996).

[Insert Figure 5 Here]

5 Conclusion

This paper develops a real options model of mergers and acquisitions that jointly determines the industry's product market equilibrium and the timing and terms of takeovers. The analysis in the paper explicitly recognizes the role of the strategic product market interaction resulting from the deal and derives equilibrium restructuring strategies by solving an option exercise game between bidding and target shareholders.

The model's predictions are generally consistent with the available empirical evidence. Importantly, the model also generates some new predictions. In addition, our analysis provides some novel testable implications. First, while increased product market competition among heterogeneous firms lowers cumulative returns in takeover deals, it does not speed up the acquisition process. Unlike other real options models, anticompetitive profits from a merger in our model are larger for less competitive industries due to a higher price adjustment at the time of the restructuring. As a result, we arrive at the surprising conclusion that firms in less competitive industries optimally exercise their real options earlier. Second, we show that the likelihood of restructuring activities and the magnitude of cumulative returns to both the merging and rival firms are positively associated with industry concentration. In addition, the merger likelihood is greater in industries that are more exposed to demand shocks. Third, we show that when the industry is sufficiently concentrated, bidder competition induces an additional bid premium and this premium increases with industry concentration. The preceding predictions are empirically testable, and such an empirical study is left for future research.

One limitation of our analysis is that we follow most papers in the literature to assume exogenous mergers in the sense that the merger structure (who merges with whom and who remains independent) is exogenously imposed in the absence of anti-trust law. It would be interesting to consider endogenous mergers when each firm makes individual merger decisions and responds to mergers by other firms. In this case, multiple mergers may arise, and the order of mergers may be endogenous. This extension is nontrivial even in a static model; see, e.g., Qiu and Zhou (2007). Incorporating endogenous mergers into a dynamic model would be a fruitful topic for future research.

Appendix

Proof of Proposition 1: In the Cournot–Nash industry equilibrium, each firm $i = 1, ...N$ has the objective to

$$\max_{q_i(t)} \pi_i(t) \tag{A.1}$$

while taking other firms' output strategies as given, where $\pi_i(t)$ is reported in equation (4). This maximization problem has the following first-order condition for each firm i:

$$a\,Y(t) - b\,Q(t) = \left(b + 2k_i^{-1}\right) q_i(t). \tag{A.2}$$

Using (1) and (5), we can solve the system of first-order conditions (A.2) to obtain the equilibrium expression for $q_i^*(t)$ in equation (7). Aggregating individual firms' output choices yields the industry's optimal output level $Q^*(t)$ in equation (9). Based on this result, the industry's equilibrium price process $P^*(t)$ in equation (10) immediately follows from substituting $Q^*(t)$ into equation (2) and simplifying. ∎

Proof of Proposition 2: Using Proposition 1, firm i's profits are given by

$$\pi_i(t) = Pq_i - q_i^2/(2k_i) = \left(\frac{aY(t)}{1+B}\right)^2 \left(1 - \frac{\theta_i}{2bk_i}\right) \frac{\theta_i}{b}. \tag{A.3}$$

We need to substitute the values of θ_i and k_i for a small, a large or a merged firm to compute their profits and firm values. Substituting the equilibrium output choice $q_s^*(t)$ and $q_l^*(t)$ in equations (14) and (15) with $k_s = k/2$ and $k_l = k$, $\theta_s = b/\left(b + 2k^{-1}\right)$, and $\theta_l = b/\left(b + k^{-1}\right)$, back into the expression of instantaneous operating profits in equations (A.3) produces the closed-form solutions in equations (20) and (21). Finally, using these equations to evaluate equation (6), we can derive the expression for equilibrium firm value $V_i(y; n)$ for $i = s, l$ that is reported in equation (19). ∎

Proof of Proposition 3: We first find the critical value Δ^* in equation (23) by solving the equation

$$\Pi_l(n+1) - 2\,\Pi_s(n) = 0. \tag{A.4}$$

Economically, equation (A.4) represents a break-even condition for the incentive to merge. To begin, notice that the functional form of $\Delta(n)$ in equation (16) has the following useful property:

$$\Delta(n) = \Delta(n+1) + b^2. \tag{A.5}$$

Thus, using (20), and (21), we can write

$$\Pi_l(n+1) - 2\,\Pi_s(n) = \frac{a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right)}{\Delta(n+1)^2} - \frac{2\,a^2\left(b+k^{-1}\right)^3}{\Delta(n)^2} \tag{A.6}$$

By inserting equations (A.5) and (A.6) into the break-even condition in equation (A.4), rearranging, and simplifying, we obtain:

$$\frac{2\,a^2\left(b+k^{-1}\right)^3}{\Delta(n)^2}\left[\frac{A^2}{\left(1-b^2/\Delta(n)\right)^2} - 1\right] = 0, \tag{A.7}$$

where the positive constant A is given in equation (24). By solving equation (A.7) for $\Delta(n)$, we can determine the critical value Δ^* in equation (23). Since one can easily verify that the term $\Pi_l(n+1) - 2\,\Pi_s(n)$ is increasing in the number of large firms, we can therefore derive the three conditions for the incentive to merge in equations (25), (26), and (27). ▮

Proof of Proposition 4: We use similar arguments as in the proof of Proposition 2. Substituting the equilibrium output choice $q_s^a(t)$, $q_l^a(t)$, and $q_M^a(t)$ in equations (28), (29), and (30) with $k_M = 3k/2$ and $\theta_M = b/\left(b+\frac{2}{3}k^{-1}\right)$ back into the expression of instantaneous operating profits in equations (A.3) produces the closed-form solutions in equations (34), (35), and (36). Finally, using these equations to evaluate equation (6), we can derive the expression for equilibrium firm value $V_i(y;n)$ for $i = s, l$ that is reported in equation (33). ▮

Proof of Proposition 5: We use similar arguments as in the proof of Proposition 3. We first find the critical value Δ^a in equation (38) by solving the equation

$$\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n) = 0. \tag{A.8}$$

Using (20), (21), and (36), we write the break-even condition (A.8) as follows:

$$\frac{3\left(b+2k^{-1}\right)^2\left(b+k^{-1}\right)^2\left(3b+k^{-1}\right)/\left(3b+2k^{-1}\right)^2}{\left[\Delta(n) - b^2\left(1+\frac{2}{2+3bk}\right)\right]^2} - \frac{\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right) + \left(b+k^{-1}\right)^3}{\Delta(n)^2} = 0. \tag{A.9}$$

After rearranging and simplifying, we rewrite the preceding equation as

$$\frac{\left(b+k^{-1}/2\right)\left(b+2k^{-1}\right)^2\left(b+k^{-1}\right)^3}{\Delta(n)^2}\left[\frac{D^2}{\left(1-b^2\left(1+\frac{2}{2+3bk}\right)/\Delta(n)\right)^2} - 1\right] = 0, \tag{A.10}$$

where the positive constant D is given in equation (39). By solving equation (A.10) for $\Delta(n)$, we can determine the critical value Δ^a in equation (38). Since one can easily verify that the term $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)$ is increasing in the number of large firms, we can therefore derive the three conditions for the incentive to merge that correspond to in equations (25), (26), and (27). ▮

Proof of Lemma 1: We use Propositions 2 and 4 to show that

$$\begin{aligned} & \Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n) \\ = & \frac{3a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}\right)^2\left(3b+k^{-1}\right)/\left(3b+2k^{-1}\right)^2}{\left[\Delta(n) - b^2\left(1+\frac{2}{2+3bk}\right)\right]^2} \\ & -\frac{a^2\left(b+k^{-1}\right)^3}{\Delta(n)^2} - \frac{a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right)}{\Delta(n)^2} \\ = & \frac{a^2\left(b+k^{-1}\right)^3 + a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right)}{\left[\Delta(n) - b^2\left(1+\frac{2}{2+3bk}\right)\right]^2} \\ & \times\left\{D^2 - \left[1-\frac{b^2\left(1+\frac{2}{2+3bk}\right)}{\Delta(n)}\right]^2\right\}, \end{aligned} \tag{A.11}$$

where D is defined in (39). Under Assumption 2 or 3, the expression in the above curly bracket is positive. In addition, since $\Delta(n)$ decreases with n, we know that $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)$ increases with n.

Now for the profit ratio $\left[\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)\right]/\Pi_f(n)$. We can show it is equal to a positive constant times the expression

$$\left[1-\frac{b^2\left(1+\frac{2}{2+3bk}\right)}{\Delta(n)}\right]^{-2}\left\{D^2 - \left[1-\frac{b^2\left(1+\frac{2}{2+3bk}\right)}{\Delta(n)}\right]^2\right\}. \tag{A.12}$$

Under Assumption 2 or 3, we can show that this expression increases with n. ▮

Proof of Proposition 6: Using equation (41), we can derive

$$OM_l\left(y, y_l^*, \xi_l; n\right) = \left[\xi_l V_M^a\left(y_l^*; n-1\right) - V_l\left(y_l^*; n\right) - X_l\right]\mathbb{E}^y\left[e^{-r\tau_{y_l^*}}\right], \tag{A.13}$$

for some undetermined threshold $y_l^* \geq y$. By Karatzas and Shreve (1999), we know that

$$\mathbb{E}^y\left[e^{-r\tau_{y_l^*}}\right] = \left(\frac{y}{y_l^*}\right)^\beta, \tag{A.14}$$

where β is the positive root of the characteristic equation (43). Thus, we obtain equation (42). Solving the first-order condition (44), we obtain equation (45). Similarly, we can derive equation

(48). We can then use equation (45) and (48) to determine the equilibrium sharing rule ξ_l^* in (50) by solving the equation $y_l^* = y_s^*$ for ξ_l. Substituting ξ_l^* into either (45) or (48) yields the option value-maximizing merger threshold reported in equation (49). Finally, the comparative statics results in part (iii) of the proposition follow from Lemma 1 and Propositions 2 and 4. ∎

Proof of Proposition 7: Evaluating equations (54) and (56) at the merger threshold $Y(t) = y^*$ and using the results from Propositions 4 and 6, we find the cumulative merger returns in equations (57) and (58). Finally, part (iii) follows immediately from Lemma 1. ∎

Proof of Proposition 8: It follows from straightforward algebra given in Section 4. We thus omit the detailed derivations here. ∎

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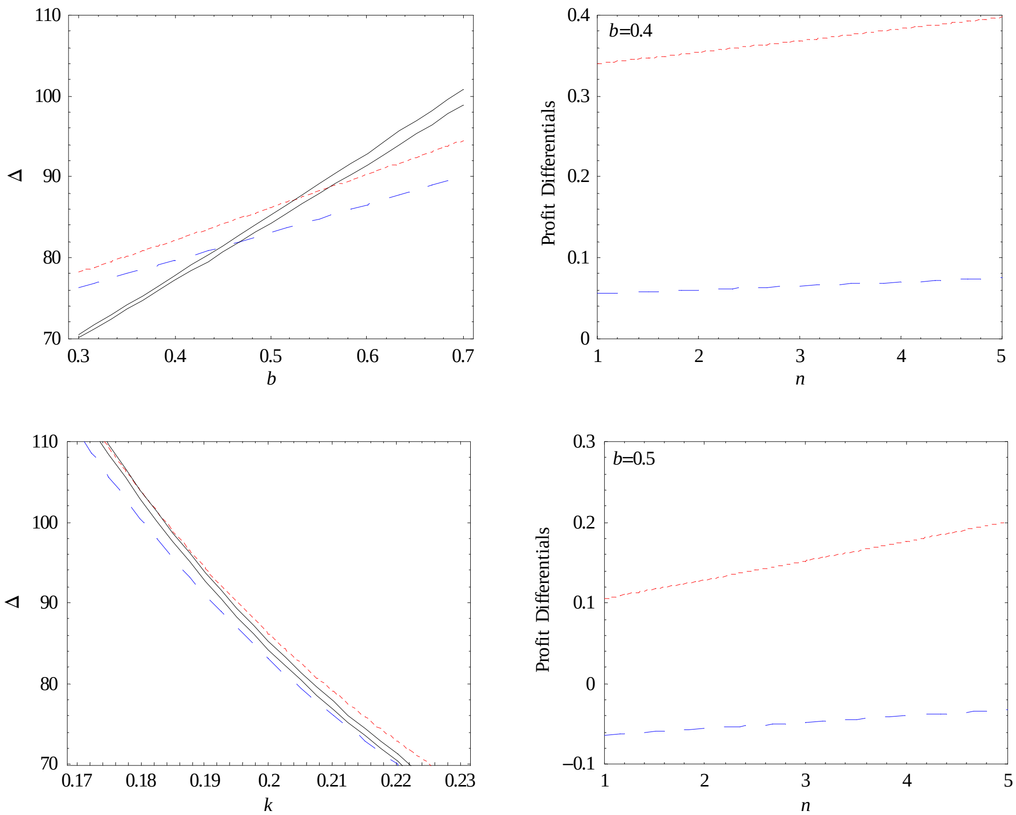

Figure 1. Incentive to Merge in a Symmetric Merger and an Asymmetric Merger. This figure plots $\Delta(0)$ and $\Delta(1/k-1)$ (solid lines) along with the critical thresholds Δ^* (dashed line) and Δ^a (dotted line) as a function of the price sensitivity of demand b and the size of a large firm k in the panels on the left. The panels on the right depict the profit differentials $\Pi_l(n+1) - 2\Pi_s(n)$ (dashed line) and $\Pi_M(n+1) - \Pi_s(n) - \Pi_l(n)$ (dotted line) as a function of the number of large firms n. We set $a = 100$, $b = 0.5$, $K = 1$, and $k = 0.2$.

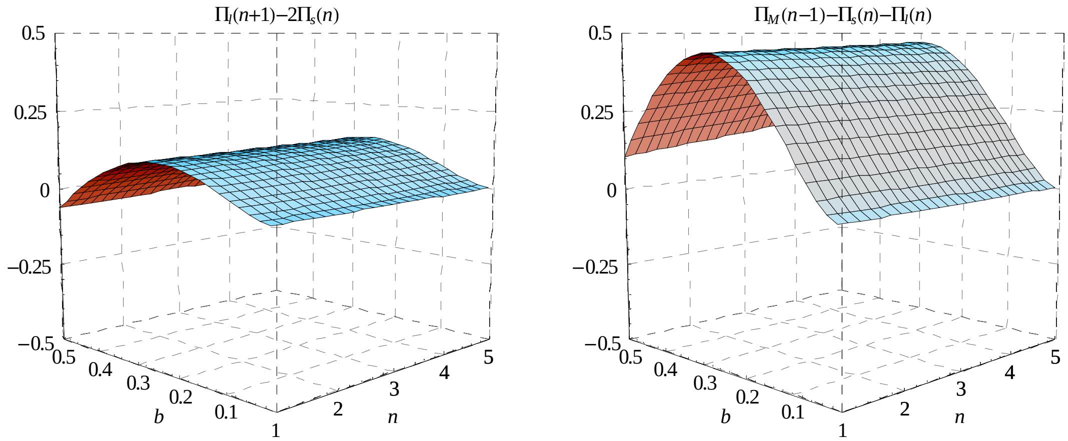

Figure 2. Profit Differentials in Symmetric and Asymmetric Mergers.

This figure depicts the profit differentials $\Pi_l\left(n+1\right)-2\Pi_s\left(n\right)$ and $\Pi_M\left(n+1\right)-\Pi_s\left(n\right)-\Pi_l\left(n\right)$ as functions of the price sensitivity b and the number of large firms n when $a=100$, $K=1$, and $k=0.2$.

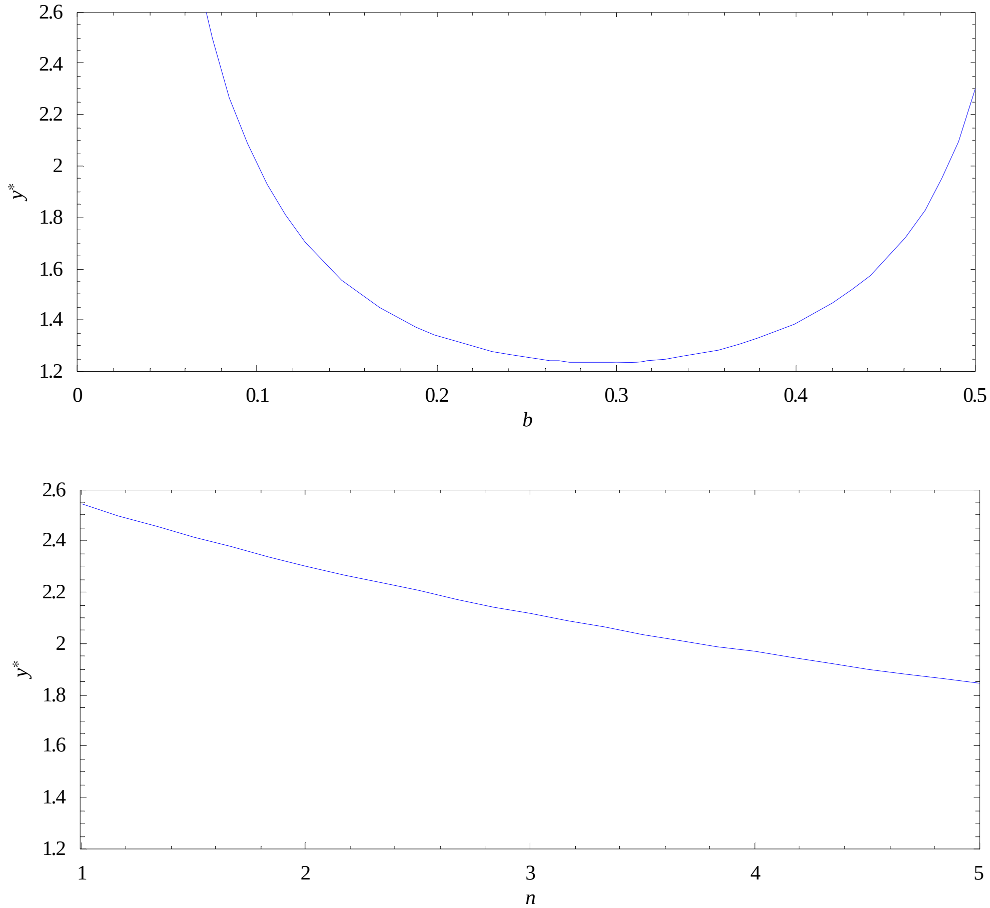

Figure 3. Merger Threshold.

This figure plots the merger threshold y^* as a function of the price sensitivity of demand b and the number of large firms n. We set $a = 100$, $b = 0.5$, $K = 1$, $k = 0.2$, $n = 2$, $r = 8\%$, $X_l = 2$, $X_s = 2$, $\mu = 1\%$, and $\sigma = 20\%$.

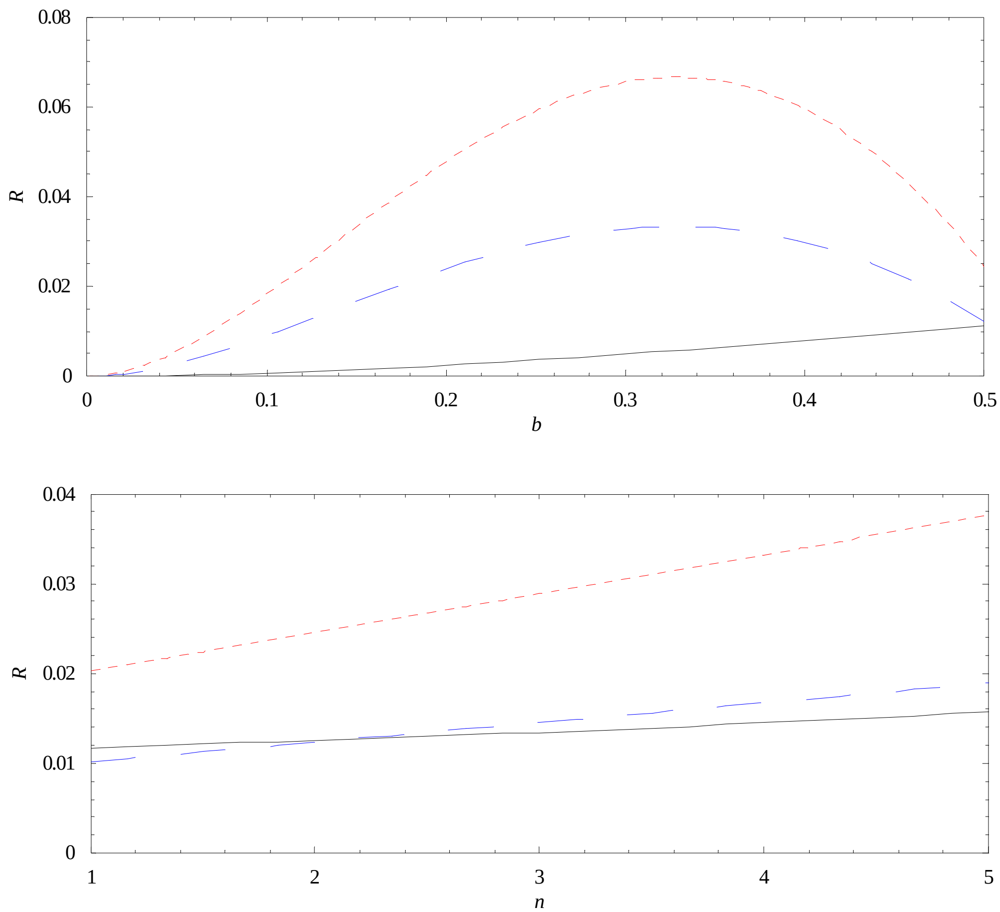

Figure 4. Cumulative Merger Returns.

This figure depicts the cumulative returns of a small merging firm (dotted line), a large merging firm (dashed line), and a rival firm (solid line) as a function of the price sensitivity of demand b and the number of large firms n. We set $a = 100$, $b = 0.5$, $K = 1$, $k = 0.2$, $n = 2$, $r = 8\%$, $X_l = 2$, $X_s = 2$, $\mu = 1\%$, and $\sigma = 20\%$.

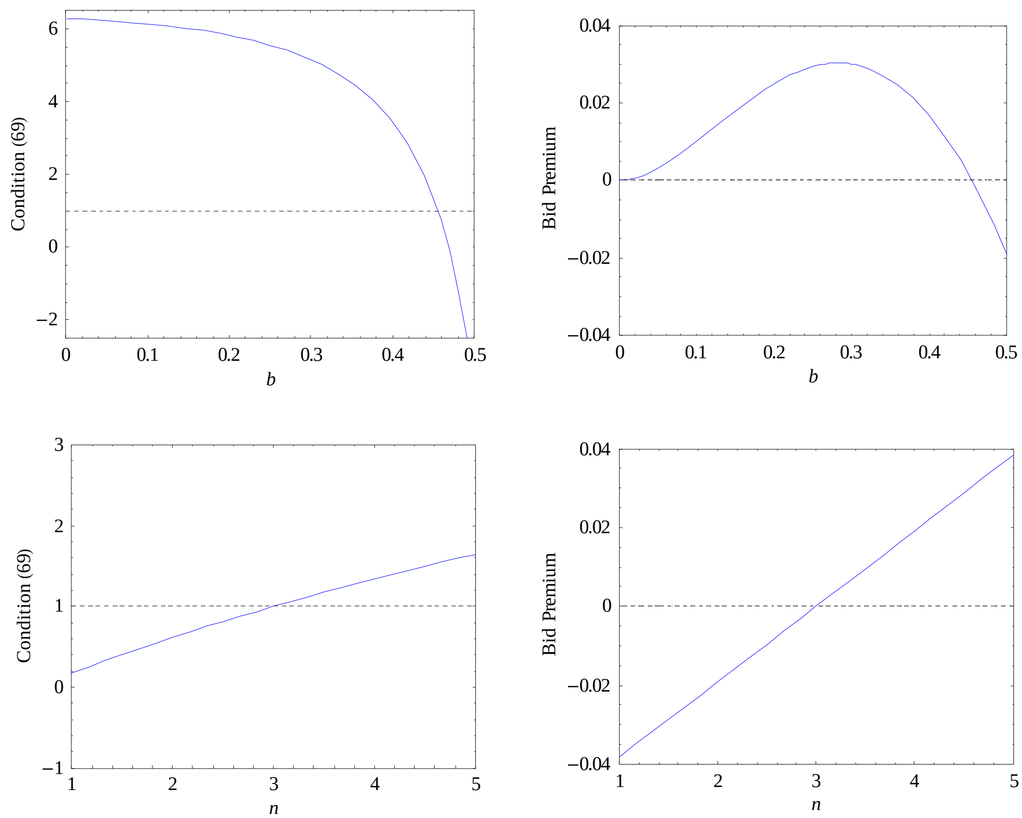

Figure 5. Bidder Competition and Industry Competition.

The figure presents the ratio of the expression on the left-hand side of condition (69) to the expression on the right-hard side as a function of the number of large firms n and the price sensitivity of demand b in the panels on the left. Th panels on the right present the bid premium in equation (68) as a function of the number of large firms n and the price sensitivity of demand b. It is assumed that $a = 100$, $b = 0.4$, $K = 1$, $k = 0.2$, $n = 4$, $r = 8\%$, $X_l = 20$, $X_s = 1$, $\mu = 1\%$, and $\sigma = 20\%$.