

Market Liquidity and Funding Liquidity*

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Abstract

We provide a model that links a assets' market liquidity — i.e., the ease of trading it — and traders' funding liquidity — i.e., their availability of funds. Traders provide market liquidity and their ability to do so depends on their funding. Conversely, traders' funding, i.e., their capital and the margins they are charged, depend on the assets' market liquidity. We show that under certain conditions margins are destabilizing and market liquidity and funding liquidity are mutually reinforcing, leading to liquidity spirals. The model explains the empirically documented features that market liquidity (i) is fragile, i.e. can suddenly dry up, (ii) has commonality across securities, (iii) is related to volatility, (iv) experiences “flight to liquidity” events, and (v) comoves with the market.

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Trading requires capital. When a trader — e.g. a dealer, hedge fund, or investment bank — buys a security, he can use the security as collateral and borrow against it, but he cannot borrow the entire price. The difference between the security’s price and collateral value, denoted the margin, must be financed with the trader’s own capital. Similarly, shortselling requires capital in the form of a margin; it does not free up capital. Hence, at any time the total margins on all positions cannot exceed the trader’s capital.

Our model shows that the funding of traders affects, and is affected by, market liquidity in a profound way. When funding liquidity is tight, traders become reluctant to take on positions, especially “capital-intensive” positions in high-margin securities. This lowers market liquidity. Further, under certain conditions, low future market liquidity increases the risk of financing a trade, thus increasing the margins.

Based on the links between funding and market liquidity, we provide a unified explanation for the main empirical features of market liquidity. In particular, the model implies that market liquidity (i) can suddenly dry up, (ii) has commonality across securities, (iii) is related to volatility, (iv) experiences “flight to liquidity” events, and (v) comoves with the market. The model further has several new testable implications

Our model is similar in spirit to Grossman and Miller (1988) with the added feature that speculators face the real-world funding constraint discussed above. In our model, different customers have offsetting demand shocks, but arrive sequentially to the market. This creates a temporary order imbalance. A group of speculators smooth price fluctuations and thus providing market liquidity. Speculators finance their trade through collateralized borrowing from financiers who set the margins to control their value-at-risk (VaR). We derive the competitive equilibrium of the model and explore its liquidity implications. We define market liquidity as the difference between the transaction price and the fundamental value, and funding liquidity as a speculator’s scarcity (or shadow cost) of capital.

We first analyze the properties of margins. We show that margins stabilize the price and decrease with market illiquidity if financiers know that prices diverge due to temporary market illiquidity and know that liquidity will be improved shortly as complementary customers arrive. This is because a current price divergence from fundamentals provide a “cushion” against future price moves, making the speculator’s position less risky in this case.

Margins can increase in market illiquidity and become destabilizing, on the other hand, when financiers’ are unsure whether price changes are due to fundamental news or liquidity shocks and fundamentals have time-varying volatility. This happens when a liquidity shock leads to price volatility which raises the financier expectations about future volatility. We predict that speculators face more destabilizing margins in specialized markets in which financiers cannot easily distinguish fundamental shock from liquidity shocks or cannot predict when a trade converges. Figure 1 shows an increase in margins for S&P 500 futures during the liquidity crises of 1987, 1990, and 1998,

and, also, anecdotal evidence from prime brokers suggest that destabilizing margins are empirically relevant.

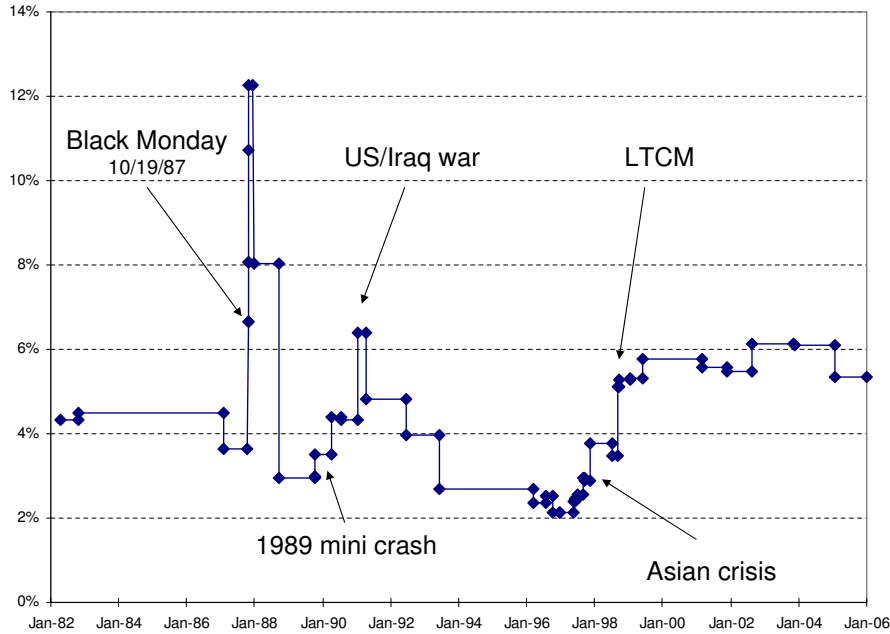


Figure 1: **Margins for S&P500 Futures.** The figure shows margin requirements on S&P500 futures for members of the Chicago Mercantile Exchange as a fraction of the value of the underlying S&P500 index multiplied by the size of the contract. (Initial or maintenance margins are the same for members.) Each dot represents a change in the dollar margin.

Turning to the liquidity implications, we first show that, as long as speculator capital is so abundant that there is no risk of hitting the funding constraint, market liquidity is naturally at its highest level and insensitive to marginal changes in capital and margins. However, when speculators hit their capital constraints — or risk hitting their capital constraints over the life of a trade — then they are forced to reduce their positions and market liquidity is reduced.

When margins are destabilizing or the speculators have large existing positions, there can be multiple equilibria and liquidity can be fragile. In one equilibrium markets are liquid, leading to favorable margin requirements for speculators, which in turn helps speculators make markets liquid. In another equilibrium, markets are illiquid, resulting in larger margin requirements (or speculator losses), thus restricting speculators from providing market liquidity. Importantly, any equilibrium selection has the property that small speculator losses can lead to a discontinuous drop of market liquidity. This “sudden dry-up” or fragility of market liquidity is due to the fact that with high speculator capital, markets must be in the liquid equilibrium, and, if speculator capital is reduced enough, the market must eventually switch to the low-liquidity/high-margin

equilibrium.¹ The events following the Russian default in 1998 is a vivid example of fragility of liquidity since a relatively small shock had a large impact. Indeed, compared to the total market capitalization of the US stock and bond market, the losses due to the Russian default were minuscule, but caused a shiver in world financial markets, see e.g. Figure 1.

Further, when markets are illiquid, market liquidity is highly sensitive to further changes in funding conditions. This is due to two liquidity spirals: first, a “margin spiral” emerges if margins are increasing in market illiquidity because a reduction in speculator wealth lowers market liquidity, leading to higher margins, tightening speculators’ funding constraint further, and so on. For instance, Figure 1 shows how margins gradually escalated within few days after the Black Monday in 1987. Second, a “loss spiral” arises if speculators hold a large initial position that is negatively correlated with customers’ demand shock. In this case, a funding shock increases market illiquidity, leading to speculator losses on their initial position, forcing speculators to sell more, causing a further price drop, and so on.² These liquidity spirals reinforce each other, implying a larger total effect than the sum of their separate effects. Paradoxically, the liquidity spirals imply that a larger shock to the customers’ demand for immediacy leads to a reduction in the provision of immediacy in such stress times.

Our model also provides a natural explanation for the commonality of liquidity across assets since shocks to the funding constraint of the speculator sector affect all securities. This may help explain why market liquidity is correlated across stocks (Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001) and Huberman and Halka (2001)), and across stocks and bonds (Chordia, Sarkar, and Subrahmanyam (2005)). In support of the idea that commonality is driven at least in part by our funding-liquidity mechanism, Chordia, Sarkar, and Subrahmanyam (2005) find that “money flows ... account for part of the commonality in stock and bond market liquidity.” Moreover, their finding that “during crisis periods, monetary expansions are associated with increased liquidity” is consistent with our model’s prediction that the effects are largest when traders are near their constraint. Coughenour and Saad (2004) provide further evidence of the funding-liquidity mechanism by showing that the comovement in liquidity among stocks handled by the same NYSE specialist firm is higher than for other stocks, commonality is higher for specialists with less capital, and decreases after a merger of specialists.

Next, our model predicts that market liquidity declines as fundamental volatility

¹Fragility can also be caused by asymmetric information on the amount of trading by portfolio insurance traders (Gennotte and Leland (1990)), and by losses on existing positions (Chowdhry and Nanda (1998)).

²The loss spiral is related to the multipliers that arise in Grossman (1988), Kiyotaki and Moore (1997), Shleifer and Vishny (1997), Chowdhry and Nanda (1998), Xiong (2001), Kyle and Xiong (2001), Gromb and Vayanos (2002), Morris and Shin (2004), Plantin, Sapra, and Shin (2005) and others. To our knowledge, our paper is the first to model the margin spiral and the interaction between the two multipliers.

increases, which is consistent with the empirical findings of Benston and Hagerman (1974) and Amihud and Mendelson (1989).³ The model implies that the liquidity differential between high-volatility and low-volatility securities increases as speculator capital deteriorates — a phenomenon often referred to as “flight to quality” or “flight to liquidity.” According to our model, this happens because a reduction in speculator capital induces traders to provide liquidity mostly in securities that do not use much capital (low volatility stocks since they have lower margins). Hence, illiquid securities are predicted to have more liquidity risk. Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) document empirical evidence consistent with flight to liquidity and the pricing of this liquidity risk.

Market-making firms are often net long the market. For instance, Ibbotson (1999) reports that security brokers and speculators have median market betas in excess of one. Therefore, capital constraints are more likely to be hit during market downturns, and this, together with mechanism outlined in our model, helps explain why sudden liquidity dry-ups occur more often when markets decline and liquidity comoves more during downturns. Following our model’s prediction, Hameed, Kang, and Viswanathan (2005) document that comovements in liquidity indeed are higher during large negative market moves.

Our paper is related to several literatures.⁴ Most directly related are the models with margin-constrained traders: Grossman and Vila (1992) and Liu and Longstaff (2004) derive optimal strategies in a partial equilibrium with a single security, Chowdhry and Nanda (1998) focus on fragility due to dealers losses, and Gromb and Vayanos (2002) derive a general equilibrium with one security and study welfare and liquidity provision. Our paper contributes to the literature by considering the simultaneous effect of margin constraints on multiple securities and by studying the nature of the margin constraint. Said simply, the existing theoretical literature uses a fixed or decreasing margin constraint — say \$5,000 per contract — and studies what happens when trading losses cause agents to hit this constraint, whereas we study how mar-

³The link between volatility and liquidity is shared by the models of Stoll (1978), Grossman and Miller (1988), and others. What sets our theory apart is that this link is connected with margin constraints. This leads to testable differences since, according to our model, the link is stronger when speculators are poorly financed, and high-volatility securities are more affected by speculator wealth shocks — our explanation of flight to quality.

⁴Market liquidity is studied specifically in market microstructure (Kyle (1985), Glosten and Milgrom (1985)), Stoll (1978), Ho and Stoll (1981, 1983), Grossman and Miller (1988)), and related to the limits of arbitrage (DeLong, Shleifer, Summers, and Waldmann (1990), Shleifer and Vishny (1997), Abreu and Brunnermeier (2002)), funding liquidity is examined in corporate finance (Shleifer and Vishny (1992), Holmström and Tirole (1998,2001)) and banking (Diamond and Dybvig (1983), Allen and Gale (1998, 2004, 2005)), funding and collateral constraints are also studied in macro economics (Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Lustig (2004)), and general equilibrium with incomplete markets (Geanakoplos (1997, 2003)), and, finally recent papers consider illiquidity with constrained traders (Attari, Mello, and Ruckes (2005), Bernardo and Welch (2004), Brunnermeier and Pedersen (2005), Morris and Shin (2004), and Weill (2004)).

ket conditions lead to changes in the margin requirement itself — e.g. an increase from \$5,000 to \$15,000 per futures contract as happened in October 1987 — and the feedback effects between the margin and the market conditions.

Our four main new results are that: (i) margins increase with market illiquidity when financiers cannot distinguish fundamental shocks and liquidity shocks and fundamentals have time-varying volatility (Section 2), (ii) this makes margins destabilizing, leading to sudden liquidity dry ups and margin spirals (Section 3), (iii) liquidity crisis simultaneously affect many securities, mostly risky high-margin securities, resulting in commonality of liquidity and flight to quality (Section 4), and (iv) liquidity risk matters even before speculators hit their capital constraints with (Section 5). We describe the institutional features associated with the funding of trading activity for the main liquidity providers, namely market makers, banks, and hedge funds and how funding changed during important crises periods (Section 6). Finally, we outline how our model’s new testable predictions may be helpful for a novel line of empirical work that links measures of speculators’ funding conditions to measures of market liquidity (Section 7). We first describe the model.

1 Model

Setup. The economy has J risky assets, traded at times $t = 0, 1, 2, 3$. At time $t = 3$, each security j pays off v^j , a random variable defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$. There is no aggregate risk and the risk-free interest rate is normalized to zero, so the fundamental value of each stock is its conditional expected value of the final payoff $v_t^j = E_t[v^j]$. Fundamental volatility has an autoregressive conditional heteroskedasticity (ARCH) structure. Specifically, v_t^j evolves according to

$$v_{t+1}^j = v_t^j + \Delta v_{t+1}^j = v_t^j + \sigma_{t+1}^j \varepsilon_{t+1}^j, \quad (1)$$

where ε_t^j is i.i.d. across time and assets with a cumulative distribution function Φ , symmetric around zero, and the volatility σ_t^j has dynamics

$$\sigma_{t+1}^j = \underline{\sigma}^j + \theta^j |\Delta v_t^j|, \quad (2)$$

where $\underline{\sigma}^j, \theta \geq 0$. A positive θ implies that shocks to fundamentals increase future volatility.

There are three groups of market participants: “customers” and “speculators” trade assets while “financiers” finance speculators’ positions.

There are three risk-averse customers. At time 0, customer $k = 0, 1, 2$ has a cash holding of W_0^k bonds and zero shares, but finds out that he will experience an endowment shock of $\mathbf{z}^k = \{z^{1,k}, \dots, z^{J,k}\}$ shares at time $t = 3$, where \mathbf{z} is a random variables such that the aggregate endowment shock is zero $\sum_{k=0}^2 z^{j,k} = 0$. We denote

the aggregate endowment shock by $Z_t := \sum_{k=0}^t \mathbf{z}^k$.

The basic liquidity problem arises because customers may arrive sequentially giving rise to order imbalance. In particular, customer k only begins trading at time $t = k$ with probability a , while all customers start trading at once at $t = 0$ with probability $(1-a)$. Before customers arrive at the market place, their demand is $\mathbf{y}_t^k = 0$. After they arrive, they choose their security position \mathbf{y}_t^k in order to maximize their exponential utility function $U(W_3^k) = -\exp\{-\rho W_3^k\}$ over final wealth. Wealth W_t^k , including the value of the \mathbf{z}^k shares, evolves according to

$$W_{t+1}^k = W_t^k + (\mathbf{p}_{t+1} - \mathbf{p}_t)' (\mathbf{y}_t^k + \mathbf{z}^k) \quad (3)$$

The early customers trading need is accommodated by speculators who provide liquidity/immediacy. Speculators are risk-neutral and maximize expected final wealth W_3 . They face the constraint that the total margin on their position \mathbf{x}_t cannot exceed their capital W_t :

$$\sum_j (x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-}) \leq W_t \quad (4)$$

where $x_t^{j+} \geq 0$ and $x_t^{j-} \geq 0$ are the positive and negative parts of $x_t^j = x_t^{j+} - x_t^{j-}$, respectively, and $m_t^{j+} \geq 0$ and $m_t^{j-} \geq 0$ are the dollar margin on long and short positions, respectively. Section 6 is devoted to a detailed discussion of the institutional features related to this key constraint.

Speculators start out with an initial cash position of W_0 and zero shares, and their wealth evolves according to

$$W_t = W_{t-1} + (\mathbf{p}_t - \mathbf{p}_{t-1})' \mathbf{x}_{t-1} + \eta_t, \quad (5)$$

where η_t is a wealth shock arising from other activities, e.g. the speculators' investment banking arm.⁵

The financier sets the margins to limit his counterparty credit risk. Specifically, the financier ensures that the margin is large enough to cover the position's π -value-at-risk (where π is a nonnegative number close to zero, e.g. 1%):

$$\begin{aligned} \pi &= Pr(-\Delta p_{t+1}^j > m_t^{j+} | \mathcal{F}_t) \\ \pi &= Pr(\Delta p_{t+1}^j > m_t^{j-} | \mathcal{F}_t) \end{aligned} \quad (6)$$

Clearly, the margin is larger for more volatile assets. The margin depends on financiers' information set \mathcal{F}_t . We consider two important benchmarks: "informed financiers" who know the fundamental value and the liquidity shocks z , $\mathcal{F}_t = \sigma\{\mathbf{z}, \mathbf{v}_0, \dots, \mathbf{v}_t, \mathbf{p}_0, \dots, \mathbf{p}_t\}$, and "uninformed financiers" who only observes prices, $\mathcal{F}_t = \sigma\{\mathbf{p}_0, \dots, \mathbf{p}_t\}$. This margin

⁵Note that if we assume that speculators are able to raise additional funds but only with a lag at time $t = 2$, we do not need to assume that the customers' endowment shocks z^j aggregate to zero.

specification is motivated by the real-world institutional features described in Section 6. Theoretically, Stiglitz and Weiss (1981) show how credit rationing can be due to adverse selection and moral hazard in the lending market, and Geanakoplos (2003) considers endogenous contracts in a general-equilibrium framework of imperfect commitment.

We let Λ_t^j be the (signed) deviation of the price from fundamental value

$$\Lambda_t^j = p_t^j - v_t^j \quad (7)$$

and we define our measure of market illiquidity as the absolute amount of this deviation, $|\Lambda_t^j|$. We consider competitive equilibria of the economy:

Definition 1 *An equilibrium is a price process \mathbf{p}_t such that (i) \mathbf{x}_t maximizes the speculators' expected final profit subject to the margin constraint (4), (ii) each \mathbf{y}_t^k maximizes k -customers expected utility after his arrival at the marketplace and is zero beforehand, (iii) margins are set according to the VaR specification (6), and (iv) markets clear, $\mathbf{x}_t + \sum_{k=0}^2 \mathbf{y}_t^k = 0$.*

Outline of Equilibrium. We derive the optimal strategies for customers and speculators using dynamic programming, starting from time 2, and working backwards. The customers' value function is denoted Γ and the speculators' is denoted J . At time 2, customers k 's problem is

$$\Gamma_2(W_2^k, p_2, v_2) = \max_{y_2^k} -E_2[e^{-\gamma W_3^k}] \quad (8)$$

$$= \max_{y_2^k} -e^{-\gamma(E_2[W_3^k] - \frac{\gamma}{2} \text{Var}_2[W_3^k])} \quad (9)$$

which has the solution

$$y_2^{j,k} = \frac{v_2^j - p_2^j}{\gamma(\sigma_3^j)^2} - z^{j,k} \quad (10)$$

Clearly, since all customers are present in the market at time 2, the unique equilibrium is $p_2 = v_2$. Indeed, when the prices are equal to fundamentals the aggregate customer demand is zero, $\sum_k y_2^{j,k} = 0$, and the speculator also has a zero demand. We get the customer's value function $\Gamma_2(W_2^k, p_2 = v_2, v_2) = -e^{-\gamma W_2^k}$, and the speculator's value function $J_2(W_2, p_2 = v_2, v_2) = W_2$.

The equilibrium before time 2 depends on whether the customers arrive sequentially or all at time 0. If all customers arrive at time 0, then the simple arguments above show that $p_t = v_t$ at any time $t = 0, 1, 2$.

We are interested in the case with sequential arrival of the customers such that the speculators' liquidity provision is needed. At time 1, customers 0 and 1 are present in the market, but customer 2 has not arrived yet. As above, customer $k = 0, 1$ has a

demand and value function of

$$y_1^{j,k} = \frac{v_1^j - p_1^j}{\gamma(\sigma_2^j)^2} - z^{j,k} \quad (11)$$

$$\Gamma_1(W_1^k, p_1, v_1) = -\exp \left\{ -\gamma \left[W_1^k + \sum_j \frac{(v_1^j - p_1^j)^2}{2\gamma(\sigma_2^j)^2} \right] \right\} \quad (12)$$

At time 0, customer 0 arrives in the market and maximizes $E_0[\Gamma_1(W_1^k, p_1, v_1)]$.

At time $t = 1$, if the market is perfectly liquid so that $p_1^j = v_1^j$ for all j , then the speculator is indifferent between all possible positions x_1 . If some securities have $p_1 \neq v_1$, then the risk neutral speculator invests all his capital such that his margin constraint binds. The speculator optimally trades only in securities with the highest expected profit per dollar used. The profit per dollar used is $(v_1^j - p_1^j)/m_1^{j+}$ on a long position and $-(v_1^j - p_1^j)/m_1^{j-}$ on a short position. Each (active) speculator's shadow cost of capital, denoted ϕ_1 , is 1 plus the maximum profit per dollar used:

$$\phi_1 = 1 + \max_j \left(\max \left\{ \frac{v_1^j - p_1^j}{m_1^{j+}}, \frac{-(v_1^j - p_1^j)}{m_1^{j-}} \right\} \right) \quad (13)$$

where the margins for long and short positions are set by the financier as described in the next section. If the speculators are (strictly) bankrupt, i.e. if $W_1 < 0$, then $\phi_1 = 0$. Each speculator's value function is therefore

$$J_1(W_1, p_1, v_1, p_0, v_0) = W_1 \phi_1 \quad (14)$$

At time $t = 0$, the speculator maximizes $E_0[W_1 \phi_1]$ subject to his capital constraint. His time-0 shadow cost of capital is

$$\phi_0 = 1 + \max_j \left(\max \left\{ \frac{\frac{E_0[\phi_1 p_1^j]}{E_0[\phi_1]} - p_0^j}{m_0^{j+}}, \frac{-(\frac{E_0[\phi_1 p_1^j]}{E_0[\phi_1]} - p_0^j)}{m_0^{j-}} \right\} \right) \quad (15)$$

and the speculator invests only in securities with the highest expected profit per capital use, where profit is calculated using the pricing kernel $\phi_1/E_0[\phi_1]$. The pricing kernel is best understood by noting that if the speculator is unconstrained then he is indifferent between (i) investing p_0^j dollars at time $t = 0$ and receiving in expectation in "marginal value terms" $E_0[\phi_1 p_1^j]$ at time $t = 1$ and (ii) holding p_0^j in cash on the side-line which gives him in "marginal value terms" $p_0^j E_0[\phi_1]$. Consequently, if speculators are unconstrained, $\phi_0 = 0$. If on the other hand speculators are constrained from investing in asset j , $E_0[\phi_1 p_1^j] > p_0^j E_0[\phi_1]$ and hence, $\phi_0 > 0$.

The equilibrium prices at times 1 and 0 do not have simple expressions. However,

after having derived the margin conditions, we characterize several important properties of these prices, which illuminates the connection between market liquidity, $|\Lambda|$, and speculators' funding situation.

2 Margin Setting and Liquidity (Time 1)

We want to determine the financiers' margin at time 1, m_1 , both in the case informed and uninformed financiers. The financier sets the margin such that it covers the position's value-at-risk, knowing that the prices equals the fundamental in the next period, $p_2 = v_2$.

If informed financiers know the fundamental values v_1 (or, equivalently, know the demand shocks z_0, z_1), they are aware of Λ_1 . Since $\Lambda_2 = 0$, margins on long positions at $t = 1$ are set according to

$$\begin{aligned}\pi &= Pr(-\Delta p_2^j > m_1^{j+} | \mathcal{F}_1^i) \\ &= Pr(-\Delta v_2^j + \Lambda_1^j > m_1^{j+}) \\ &= 1 - \Phi\left(\frac{m_1^{j+} - \Lambda_1^j}{\sigma_2^j}\right)\end{aligned}\tag{16}$$

which implies that

$$\begin{aligned}m_1^{j+} &= \Phi^{-1}(1 - \pi) \sigma_2^j + \Lambda_1^j \\ &= \bar{\sigma}^j + \bar{\theta} |\Delta v_1^j| + \Lambda_1^j\end{aligned}\tag{17}$$

where we define

$$\bar{\sigma}^j = \underline{\sigma}^j \Phi^{-1}(1 - \pi)\tag{18}$$

$$\bar{\theta} = \theta \Phi^{-1}(1 - \pi)\tag{19}$$

The margin on a short position can be derived similarly:

Proposition 1 (Stabilizing Margins and the Cushioning Effect) *When the financier is informed about the fundamental value and knows that prices will equal fundamentals next period $t = 2$, then the margins on long and short positions are, respectively,*

$$m_1^{j+} = \max\{\bar{\sigma}^j + \bar{\theta} |\Delta v_1^j| + \Lambda_1^j, 0\}\tag{20}$$

$$m_1^{j-} = \max\{\bar{\sigma}^j + \bar{\theta} |\Delta v_1^j| - \Lambda_1^j, 0\}\tag{21}$$

The more prices are below fundamentals $\Lambda_1^j < 0$, the lower is the margin on a long position m_1^{j+} , and the more prices are above fundamentals $\Lambda_1^j > 0$, the lower is the

margin on a short position m_1^{j-} . Hence, in this case illiquidity reduces margins for speculators who buy low and sell high.

The margins are reduced by illiquidity because the speculator is expected to profit when prices return to fundamentals at time 2, and this profit “cushions” the speculators from losses due to fundamental volatility. Thus, we denote the margins set by informed financiers at $t = 1$ as *stabilizing margins*.

Stabilizing margins are an interesting benchmark, and they are hard to escape in a theoretical model. However, real-world liquidity crisis are often associated with increases in margins, not decreases. To capture this, we turn to the case of a financier who is uninformed about the current fundamental so that he must set his margin based on the observed prices p_0 and p_1 . This is in general a complicated problem since the financier needs to filter out the probability that a liquidity shock occurred and the values of z_0 and z_1 . The expression becomes simple, however, if the financiers’ prior probability of a liquidity shock is small so that he finds it likely that $p_t^j = v_t^j$, implying a common margin $m_1^j = m_1^{j+} = m_1^{j-}$ for long and short positions in the limit:

Proposition 2 (Destabilizing Margins) *When the financier is uninformed about the fundamental value, then as $a \rightarrow 0$ the margins on long and short positions approach*

$$m_1^j = \bar{\sigma}^j + \bar{\theta}|\Delta p_1^j| = \bar{\sigma}^j + \bar{\theta}|\Delta v_1^j + \Delta\Lambda_1^j| \quad (22)$$

Margins are increasing in price volatility and illiquidity shocks can increase margins.

Intuitively, since liquidity risk tends to increase the price volatility, and since an uninformed financier may interpret price volatility as fundamental volatility, this increases margins.⁶ Equation (22) shows that illiquidity increases margins when the liquidity shock $\Delta\Lambda_1^j$ has the same sign as the fundamental shock Δv_1^j or is greater in magnitude, but margins are reduced if the liquidity shocks counterbalances a fundamental move. We denote the phenomenon that margins can increase as illiquidity rises by *destabilizing margins*. As we will see next, the information of the financier — i.e. whether margins are stabilizing or destabilizing — has important implications for the equilibrium.

3 Fragility and Liquidity Spirals (Time 1)

Fragility. This section discusses liquidity spirals and fragility — the property that a small change in fundamentals can lead to a large jump in liquidity. For simplicity we illustrate this with a single security $J = 1$.

⁶In the analysis of time 0, we shall see that margins can also be destabilizing when price volatility signals future liquidity risk (not necessarily fundamental risk).

We say that liquidity is fragile if the equilibrium price $p_t(\eta_t, v_t)$ cannot be chosen to be continuous in the exogenous shocks, namely η_t and Δv_t . Fragility arises when the excess demand for shares $x_t + \sum_{k=0}^1 \mathbf{y}_1$ can be non-monotonic in the price. While under “normal” circumstances a high price leads to a low total demand – excess demand is decreasing – binding funding constraints along with destabilizing margins (margin effect) or speculators’ losses (loss effect) can lead to an increasing demand curve.

It is natural to focus on *stable* equilibria. An equilibrium is stable if a small negative (positive) price perturbation leads to excess demand (supply), which, intuitively, “pushes” the price back up (down) to its equilibrium level.

Proposition 3 (Fragility) *(i) With informed financiers, the market is fragile at time 1 if x_0 is large enough.*

(ii) With uninformed financiers, the market is fragile at time 1 if x_0 large enough or if the ARCH parameter, θ , is large enough and the financiers’ prior on customer shock z -shock, a , is small enough.

We discuss the margin effect and loss effect in connection with a numerical example for the more realistic case in which the financiers are uninformed.

Numerical Examples. We set $z_0 = 0$, $z_1 = 40$, $\gamma = .01$, $v_0 = 140$, $p_0 = 130$, $\bar{\sigma} = 10$, and $\eta_1 = 0$. First, we focus on the margin effect. We assume that fundamentals follow an ARCH process by setting $\theta = .3$ and switch off the loss-effect by assuming initially that speculators’ time zero position is $x_0 = 0$.

In Figure 2 we set $v_1 = 120 < p_0 = 130$. Customers’ supply is given by the upward sloping line. If the $p_1 = v_1$, the market is perfectly liquid and aware customers sell all 40 shares they anticipate to receive in time $t = 3$. For lower prices they throw fewer shares on the market.

The speculators’ demand function is primarily determined by margin constraints and the realization of v_1 . The speculators’ margin constraints $|x_1| \leq W_1/(\bar{\sigma} + |\Delta p_1|)$ are given by four hyperbola forming a figure that we refer to as a “hyperbolic star” as depicted in Figure 2. The vertexes are at $p_1 = p_0 = 130$. At this price level, the margin is smallest and hence the constraint is most relaxed. As p_1 departs from $p_0 = 130$, margins increase and speculators become more constrained – the horizontal distance between two hyperbola shrinks. The horizontal line $p_1 = v_1$ determines which constraints are binding or said differently, which parts of the hyperbolic star form the speculators’ demand curve. For price levels above v_1 risk neutral speculators want to go short the asset, $x_1 < 0$ and their demand is limited by the left hyperbola, while for $p_1 < v_1$ their demand is positive, $x_1 > 0$ and constrained by the right hyperbola. The speculators’ demand curve is given by the dark blue curve in Figure 2. Note that the shape of the speculators’ demand curve depends crucially on the realization of v_1 , in particular on whether v_1 is larger or smaller than p_0 .

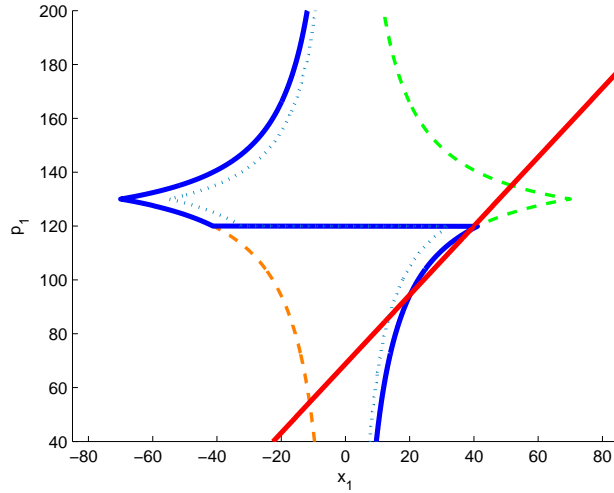


Figure 2: **“Hyperbolic Star”**. **Fragility due to Destabilizing Margins**. When financiers are uninformed and $\theta = 0.3$, liquidity is fragile due to destabilizing margins. Speculators’ constraint is given by the “hyperbolic star” for $x_0 = 0$. The realization of v_1 determines which part of it form speculators’ demand function. Customers’ supply curve is linearly increasing.

In Figure 2 demand is upward sloping for prices below $v_1 = 120$ because a larger price drop Δp_1 increases the financiers’ estimate of fundamental volatility and consequently margins. There are two stable equilibria, a perfect liquidity equilibrium with price $p_1 = v_1 = 120$ and an illiquidity equilibrium with a price of about 87.5. We ignore the unstable equilibrium in between the two stable ones.

As one reduces the speculators’ funding the perfect liquidity equilibrium $p_1 = v_1$ eventually breaks away and price has to drop. The dashed line corresponds to the speculators’ demand function after a wealth shock $\eta_1 = -150$. For this level of funding only the “illiquid equilibrium” remains.

Figure 3 highlights the “disconnect” between the perfect-liquidity equilibrium and the (stable) funding-constrained illiquid equilibrium from a different angle. Panel A plots the equilibrium price correspondence for different exogenous funding shocks η_1 and shows that a marginal reduction in funding cannot always lead to a smooth reduction in market liquidity. There must be a level of funding such that an infinitesimal drop in funding leads to a discontinuous drop in market liquidity. Panel B plots it for different realizations of Δv_1 (holding $W_0 = 700$ fixed) and shows the same form of discontinuity for adverse fundamental shocks v_1 .

This discontinuity in prices can help explain the sudden market liquidity dry-ups,

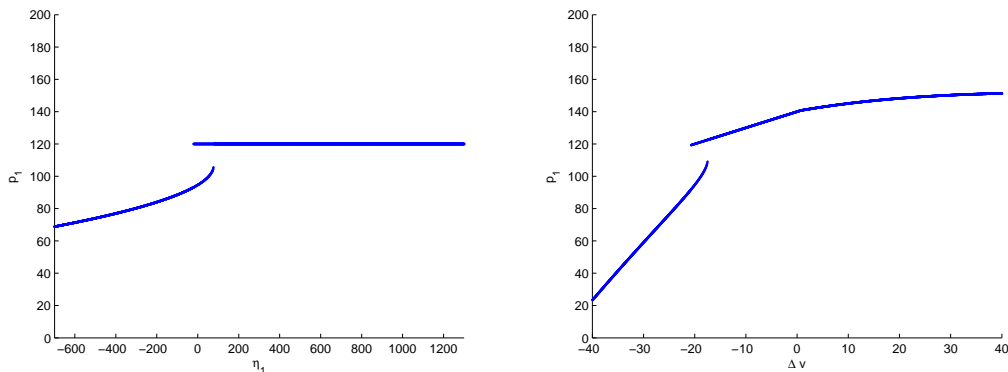


Figure 3: **Equilibrium Price Correspondence for Different Levels of Funding W_0 and Realization of Δv_1** when financiers are uninformed and $\theta = .3$

that is, the fragility of liquidity. For instance, when Russia defaulted on some of its debt in 1998, global arbitrageurs lost a small fraction of overall arbitrage capital. This relatively trivial wealth shock had, however, a large effect on liquidity in global financial markets, consistent with our fragility result when a small wealth shock just pushes the equilibrium over the edge.

While such a discontinuous jump must occur at some level of funding W_0 for any equilibrium selection criterion, it is natural to consider the equilibrium selection criterion based on “hysteresis”. This means that, if initially speculators are well financed and we are in the perfect-liquidity equilibrium, then we will stay on this perfect-liquidity equilibrium branch as long as we can. Hence, we move into the illiquid equilibrium only when speculators wealth drops below $W_0 = 680$. When that happens, illiquidity jumps up by a large amount. As the funding level returns to 680 and beyond, the market stays at the illiquid equilibrium and only jumps back to the perfect funding liquidity equilibrium at a funding level of around $W_0 = 777$. At that point, liquidity improves more modestly compared to the drop at $W_0 = 680$.

Panel B of Figure 3 depicts the equilibrium correspondence for different realizations of Δv_1 . This graph is more easily understood in conjunction of Figure 2. The realization of v_1 determines which parts of “hyperbolic star” of Figure 2 form the speculators’ demand curve. For lower v_1 , the horizontal line of speculators’ demand is shifted downwards. Even though customers’ supply also moves with v_1 , the perfect liquidity equilibrium vanishes and only an illiquid equilibrium remains. This is the reason for the discontinuous jump in Panel B of Figure 3. For an intermediate range of v_1 -realizations the p_1 is linear in v_1 . In this range the margin constraint is not binding and hence prices move one-to-one with fundamentals. For high realizations of v_1 , Δp_1 is high and financiers’ volatility estimate and consequently the margin increase so much that the

margin constraint becomes binding. This is the reason why the price increase is slightly depressed above the kink. For low realizations of v_1 , the perfect liquidity equilibrium vanishes and only an illiquid equilibrium remains. Panel B reveals a very interesting asymmetry. Negative fundamental shocks leads to a much larger price movement than corresponding positive shocks for $Z_1 > 0$. It is important to note that this asymmetry arises even though we switched off loss effect by setting $x_0 = 0$. If x_0 were positive instead, this asymmetry would be more pronounced, since a low realization of v_1 also erodes the speculators' mark-to-market wealth $b_0 + x_0(p_1 - p_0)$. Put differently, the loss effects of Panel A would add to the effects of Panel B.

The price correspondence p_1 is also discontinuous in the size the customers' shock Z_1 . A larger Z_1 -shock simply shifts customers' supply in Figure 2 down. It is easy to see that for slightly larger Z_1 -shock the perfect liquidity equilibrium vanishes/breaks away. When this happens, marginally larger demand for liquidity by customers leads to a drastic reduction of liquidity supply by the speculators.

Next, we focus on the loss effect and switch off the margin effect by setting the ARCH parameter θ equal to zero. For $\theta = 0$, margins m_1 are constant.

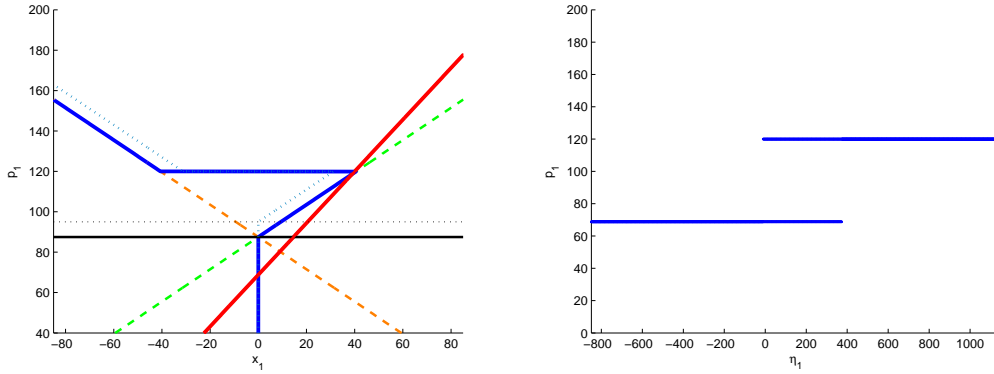


Figure 4: **Fragility due to Loss-Effect.** Speculators' demand and customers' supply if $x_0 = 20$ and no ARCH effect, $\theta = 0$. $W_0 = 850$

Note that if x_0 were equal zero, the hyperbolic star would reduce itself to two vertical line through the vertexes at $p_1 = 130$. That is, as the price p_1 moves away from p_0 , the funding constraint does not become more binding since for $\theta = 0$ margins are constant. For $x_0 > 0$ however, the two lines reflecting the margin constraints are tilted outwards as shown by the two dashed lines in Figure 4 (Panel A). This is because, as the price p_1 rises (falls), so do speculators' capital gains. Hence, the funding constraint becomes more relaxed (tight). The demand curve is determined by two factors: first, as before the realization of v_1 determines which part of the two dashed lines is relevant; second, because of speculators' initial leverage there is level of p_1 below which speculators are

bankrupt. This is reflected in the lower horizontal line in Figure 4 where we assume that speculators hold 20 shares and have negative cash holdings of $b_0 = -1750$.

With customers' supply given as before, there are two stable equilibria. One perfect liquidity equilibrium and a second equilibrium in which speculators provide no liquidity since they are bankrupt. In this example, liquidity is fragile, since a small negative η_1 -shock makes the perfect liquidity equilibrium with $p_1 = v_1 = 120$ vanish. The price has to drop to the "bankruptcy equilibrium price" of about $p_1 = 68.8$.

Panel B of Figure 4 shows equilibrium price correspondence for different levels of speculators' funding shocks η_1 . It shows that for speculators' funding shocks η_1 between -10 to $+373$ there are multiple equilibria and that independent of the equilibrium selection criterion there has to be at least one discontinuous jump in prices as η_1 changes. Combining the ARCH-effect ($\theta > 0$) with the wealth loss effect ($x_0 \neq 0$) increases the severity of fragility.

Note that the loss effect emerges independently whether the financiers are informed or not, while the margin effect only arises when margins are destabilizing, that is financiers are uninformed.

Liquidity Spirals. Once the economy enters into the illiquid equilibrium, market liquidity becomes highly sensitive to shocks. We identify two amplification mechanisms, a "margin spiral" due to increasing margins as speculator financing worsens, and a "loss spiral" due to escalating speculator losses. Figure 5 illustrates these "liquidity

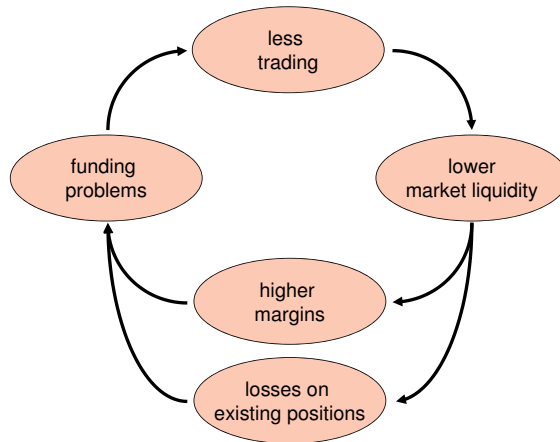


Figure 5: Liquidity Spirals

spirals": A shock to speculators' capital, $\eta_1 < 0$, forces speculators to provide less market liquidity, which increases the price impact of the customers' demand pressure. With uninformed financiers and a positive ARCH effect $\theta > 0$, the resulting price swing increases financiers' estimate of the fundamental volatility and, hence, increases

the margin, thereby worsening the speculators' funding problems even further, and so on, leading to a "margin spiral." Similarly, increased market illiquidity can lead to losses on the speculators' existing positions, worsening their funding problem and so on, leading to a loss spiral. Mathematically, the spirals are seen as follows:

Proposition 4

(a) *If speculators' capital constraint is slack then the price p_1 is equal to v_1 and insensitive to local changes in speculator wealth.*

(b) **(Liquidity Spirals)** *In a stable illiquid equilibrium with selling pressure from customers, $Z_1, x_1 > 0$, the price sensitivity to speculator wealth shocks η_1 is*

$$\frac{\partial p_1}{\partial \eta_1} = \frac{1}{\frac{2}{\gamma(\sigma_2)^2}m_1^+ + \frac{\partial m_1^+}{\partial p_1}x_1 - x_0} \quad (23)$$

and with buying pressure from customers, $Z_1, x_1 < 0$,

$$\frac{\partial p_1}{\partial \eta_1} = \frac{-1}{\frac{2}{\gamma(\sigma_2)^2}m_1^- + \frac{\partial m_1^-}{\partial p_1}x_1 + x_0}. \quad (24)$$

A margin spiral arises if $\frac{\partial m_1^+}{\partial p_1} < 0$ or $\frac{\partial m_1^-}{\partial p_1} > 0$, which happens with positive probability if speculators are uninformed and a is small enough. A loss spiral arises if speculators' previous position is in the opposite direction as the demand pressure $x_0 Z_1 > 0$.

This proposition is intuitive. Imagine first what happens if the speculators face a wealth shock of \$1, margins are constant, and speculator has no inventory $x_0 = 0$. In this case, the speculator must reduce his position by $1/m$. Since the slope of each of the two customers' demand curve is (see (11)) is $1/(\gamma(\sigma_2)^2)$, we get a total price effect of $1/(\frac{2}{\gamma(\sigma_2)^2}m_1)$.

The two additional terms in the denominator imply amplification or dampening effects due to changes in the margin requirement and to profit/losses on the speculators' existing positions. To see that, recall that for any $k > 0$ and l with $|l| < k$ it holds that $\frac{1}{k-l} = \frac{1}{k} + \frac{l}{k^2} + \frac{l^2}{k^3} + \dots$ so with $k = \frac{2}{\gamma(\sigma_2)^2}m_1$ and $l = \frac{\partial m_1^\pm}{\partial p_1}x_1 \pm x_0$ such that each term in this infinite series corresponds to one loop around the circle in Figure 5. The total effect of the changing margin and the speculators' position amplifies the effect if $l > 0$. Intuitively, if e.g. $Z_1 > 0$ then customers' selling pressure is pushing down the price, and $\frac{\partial m_1^+}{\partial p_1} < 0$ means that as prices go down, margins increase, making speculators' funding tighter and thus destabilizing the system. Similarly, when customers are buying, $\frac{\partial m_1^-}{\partial p_1} > 0$ implies that increasing prices leads to increased margins, making it harder for speculators to shortsell, thus destabilizing the system. The system is also destabilized if speculators' losses money on their previous position as prices move away from fundamentals similar to e.g. Shleifer and Vishny (1997).

Interestingly, the total effect of a margin spiral and a loss spiral is greater than the sum of their separate effects. This can be seen mathematically by using simple convexity arguments, and it can be seen intuitively from the flow diagram of Figure 5.

Note that spirals can also be “started” by a shock in liquidity demand Z_1 , fundamentals, v_1 etc. It is straightforward to compute the price sensitivity with respect to endowment shock Z_1 or fundamental shocks Δv_1 . Indeed, they are just multiples of $\frac{\partial p_1}{\partial \eta_1}$. More specifically, $\frac{\partial p_1}{\partial Z_1} = -m_1^\pm \frac{\partial p_1}{\partial \eta_1}$. In contrast to a change in η_1 , a Z_1 shock is not leveraged by $1/m_1$. The multiplier w.r.t. changes in fundamental v_1 is $\frac{\partial p_1}{\partial v_1} = \left[\frac{2}{\gamma(\sigma_2)^2} m_1 - \frac{4\theta \text{sign}\{\Delta v_1\}}{\gamma(\sigma_2)^3} \right] \frac{\partial p_1}{\partial \eta_1}$. Changes in v_1 affect the intercept of customers’ supply curve causing a price impact that depends on the supply curve’s slope $2/\gamma(\sigma_2)^2$. The second term is due to the ARCH structure. As v_1 varies so does σ_2^2 and with it the slope of the customers’ supply curve.

In summary, fragility and liquidity spirals both imply that, during “bad” times, small changes in underlying funding conditions (or liquidity demand) can lead to sharp reductions in liquidity. We next turn to the cross-sectional implications of illiquidity.

4 Commonality and Flight to Quality (Time 1)

Since speculators are risk-neutral, they optimally invest all their capital in securities that have the greatest expected profit $|\Lambda^j|$ per capital use, i.e. per dollar margin m^j , as expressed in Equation (13). That equation also introduces the shadow cost of capital ϕ_1 as the marginal value of an extra dollar. The speculators’ shadow cost of capital ϕ_1 captures well the notion of funding liquidity. Indeed, a high ϕ means that the available funding — from capital W_1 and from collateralized financing with margins m_1^j — is low relative to the needed funding, which depends on the investment opportunities deriving from demand shocks z^j .

The market liquidity of all assets depend on the speculators’ funding liquidity, especially high-margin assets, and this has several interesting implications:

Proposition 5 *Suppose θ^j close enough to zero for all j , and either financiers are informed or uninformed with probability a small enough. Then we have:*

(i) (Commonality of Market Liquidity) *The market illiquidity $|\Lambda|$ of any two securities k and l comove,*

$$\text{Cov}_0 (|\Lambda_1^k|, |\Lambda_1^l|) \geq 0 \quad (25)$$

and market illiquidity comoves with funding illiquidity as measured by the speculators’ shadow cost of capital ϕ_1

$$\text{Cov}_0 [|\Lambda_1^k|, \phi_1] \geq 0 \quad (26)$$

(ii) (Commonality of Fragility) *Jumps in market liquidity occurs simultaneously for all assets for which speculators are marginal investors.*

(iii) **(Quality=Liquidity)** If asset l has lower fundamental volatility than asset k , $\underline{\sigma}^l < \underline{\sigma}^k$, then l also has lower market illiquidity,

$$|\Lambda_1^l| \leq |\Lambda_1^k| \quad (27)$$

(iv) **(Flight to Quality)** The market liquidity differential between high- and low-fundamental-volatility securities is bigger when speculator funding is tight, that is, $\underline{\sigma}^l < \underline{\sigma}^k$ implies that $|\Lambda_1^k|$ increases more with a negative wealth shock to the speculator,

$$\frac{\partial |\Lambda_1^l|}{\partial(-\eta_1)} \leq \frac{\partial |\Lambda_1^k|}{\partial(-\eta_1)} \quad (28)$$

if $x_1^k \neq 0$. Further, if with large enough probability $x^k \neq 0$, then

$$\text{Cov}_0(|\Lambda_1^l|, \phi_1) \leq \text{Cov}_0(|\Lambda_1^k|, \phi_1). \quad (29)$$

Numerical Example, Continued. To illustrate these findings we extend our numerical example of Section 3. We now consider a setting with two assets that only differ in their long-run fundamental volatility. The long-run fundamental volatility of asset 1 is $\bar{\sigma}^1 = 10$, while asset 2 has long-run fundamental volatility $\bar{\sigma}^2 = 15$.

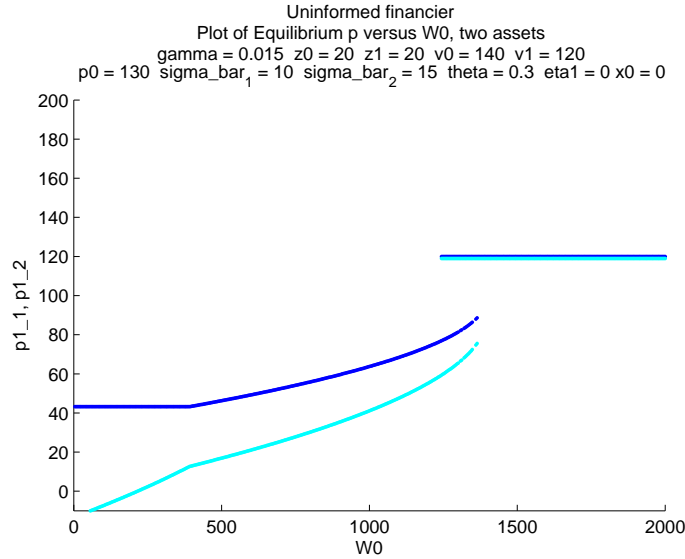


Figure 6: **Flight to Quality and Commonality in Liquidity.** The figure plots that price p_1^j of assets 1 and 2 as functions of speculators' funding W_0 . Asset 1 has lower long-run fundamental risk than asset 2, $\bar{\sigma}^1 = 10 < 15 = \bar{\sigma}^2$.

Figure 6 depicts the assets' equilibrium price for different funding levels W_0 . First note that as speculators' funding tightens and our funding illiquidity measure ϕ_1 rises, the market illiquidity measure $|\Lambda_1^j|$ rises for both assets. Hence, for random $\eta_1 \text{Cov}_0 [|\Lambda_1^k|, |\Lambda_1^l|] > 0$, our commonality in liquidity result.

“Commonality in fragility” can not directly be seen from Figure 6, but the fact that the range of W_0 for which there are multiple prices p_1^j coincides for both assets is indicative of it. The intuition for this result is the following. Whenever funding is unconstrained then there is perfect market liquidity provision for all assets. If funding is constrained, then it cannot be the case that speculators provide perfect liquidity for one asset but not for the other one, since they always would have an incentive to shift funds towards the asset with non-perfect market liquidity. Hence, market illiquidity jumps for both assets exactly at the same funding level.

Our result (“Quality=Liquidity”) that relates fundamental volatility to market liquidity is reflected by the fact that for any given funding level, p_1^2 is always below p_1^1 . That is, the high-fundamental-volatility asset 2 is always less liquid than the low-fundamental-volatility asset 1.

The graph also illustrates our result on “Flight to Quality.” To see this, let us look at the relative sensitivity of p_1 with respect to changes in W_0 : For funding levels above 1244, market liquidity is perfect for both assets, i.e. $p_1^1 = p_1^2 = v_1 = 120$. In this high range of funding, market liquidity is insensitive to marginal changes in funding. As funding falls below 1244, market illiquidity of both assets increases since speculators must take smaller stakes in both asset. Importantly, as funding decreases, $p_1^2(W_0)$ decreases more steeply than $p_1^1(W_0)$, that is, asset 2 is more sensitive to funding declines than asset 1. This is because speculators cut back more on the “funding-intensive” asset 2 with high margin requirement. The speculators want to maximize their profit per dollar margin, $|\Lambda^j|/m^j$ and therefore $|\Lambda^2|$ must be higher than $|\Lambda^1|$ to compensate speculators for using more capital for margin.

For sufficiently low funding levels speculators put all their funds only into asset 2. Since asset 2 is more volatile, risk averse customers are more eager to sell this stock. Hence, the price is lower and it is more attractive to provide market liquidity for asset 2 for very low levels of funding.

5 Liquidity Risk (Time 0)

In this section, we demonstrate that funding liquidity risk matters even before margin requirements actually bind. Indeed, speculators trade more cautiously if there is a risk of binding margin constraints over the life of the trade. To see this, consider a security that time-0 customers are selling and the speculators are buying. Equation (15) implies

that

$$p_0^j = \frac{E_0[\phi_1 p_1^j]}{E_0[\phi_1]} - m_0^{j+}(\phi_0 - 1) \quad (30)$$

Hence, if margins requirement are binding already at time 0, then $\phi_0 > 1$ and this reduces the price of the asset proportionally to the margin requirement m_0^{j+} . Since we have studied this effect extensively in our analysis of time 1, we now focus on the case in which margins are not binding at time 0, implying $\phi_0 = 1$. In this case,

$$p_0^j = \frac{E_0[\phi_1 p_1^j]}{E_0[\phi_1]} \quad (31)$$

$$= E_0[p_1^j] + \frac{Cov_0[\phi_1, p_1^j]}{E_0[\phi_1]} \quad (32)$$

This shows that the price at time 0 is the expected time-1 price — which already depends on the liquidity issues at time-1 — further adjusted for liquidity risk in the form of a covariance term. The liquidity risk term is intuitive: The time-0 price is lower if the covariance is negative, that is, if the security has a low payoff during future funding liquidity crisis when ϕ_1 is high.

Why are speculators reluctant to provide liquidity even when they are not constrained? An adverse shock lowers speculators' wealth but creates a profitable investment opportunity in $t = 1$. The latter could provide a natural “dynamic hedge”. Hence one could expect an substantial increase in their $t = 0$ -hedging demand that lowers illiquidity in $t = 0$. However, exactly the opposite occurs in our setting with capital constraints. Capital constraints prevent speculators from taking advantage from this investment opportunities in $t = 1$. Hence, speculators are reluctant to trade away the illiquidity in $t = 0$. To the contrary, they keep cash on the side-line to exploit potential future investment opportunities and they dislike assets that have a relatively low price in the states of nature when capital is tight. In this sense, our mechanism based on capital constraints is different from one that is driven “risk-aversion” driven mechanism.

Moreover, margins can also be destabilizing at time $t = 0$ even when the financiers' are fully informed. To see this note that if we reduce the speculators initial wealth W_0 while keeping fixed the other parameters, then the market becomes less liquid in the sense that the price is further from the fundamental value. At the same time, the equilibrium price in $t = 1$ is more volatile and thus equilibrium margin at time zero actually increases. Thus margin requirement can be increasing in Λ_0 .

Numerical Example, Continued. We illustrate these effect with a numerical example for the case of informed financiers.

TO BE INCLUDED

6 Institutional Background on Funding Liquidity

It is important to understand the institutional features related to the capital constraints of the main providers of market liquidity. We review these features for securities firms such as market makers, banks' proprietary traders, and hedge funds.

6.1 Funding Requirements for Hedge Funds

We first consider the funding issues faced by hedge funds since they have relatively simple balance sheets and face little regulation. A hedge fund's capital consists of its equity capital supplied by the partners and of possible long-term debt financing that can be relied upon during a potential funding crisis. Since a hedge fund is a partnership, the equity is not locked into the firm indefinitely as in a corporation. The investors (that is, the partners) can withdraw their capital at certain times, but — to ensure funding — the withdrawal is subject to so-called lock-up periods (typically at least a month, often several months or even years). A hedge fund usually cannot issue long-term unsecured bonds, but some (large) hedge funds manage to obtain debt financing in the form of medium-term bank loans or in the form of a guaranteed line of credit.⁷ Hedge funds lever their capital using collateralized borrowing financed by the hedge fund's prime broker(s). The prime brokerage business is opaque since the terms of the financing are subject to negotiation and are secret to outsiders. We describe stylized financing terms and, later, we discuss caveats.

If a hedge fund buys at time t a long position of $x_t^j > 0$ shares of a security j with price p_t^j , then this requires the hedge fund to come up with $x_t^j p_t^j$ dollars. The security can, however, be used as collateral for a new loan of, say, l_t^j dollars. The difference between the price of the security and the collateral value is denoted the margin requirement $m_t^{j+} = p_t^j - l_t^j$. Hence, this position uses $x_t^j m_t^{j+}$ dollars of the fund's capital. The collateralized funding implies that the capital use depends on margins, not notional amounts.

The margins on fixed income securities and over-the-counter (OTC) derivatives are set through a negotiation between the hedge fund and the broker that finances the trade, often the hedge funds' prime broker. The margins are typically set such as to make the loan almost risk free for the broker, that is, such that it covers the largest possible adverse price move with a certain confidence as captured by (6).⁸

⁷A line of credit may have a "material adverse change" clause or other covenants subject to discretionary interpretation of the lender. Such covenants imply that the line of credit may not be a reliable source of funding during a crisis.

⁸Often brokers also take into account the time it takes between a fail by the hedge fund is noticed and the security is actually sold. Hence, the margin of a one-day collateralized loan depends on the

If the hedge fund wants to sell short a security, then the fund asks one of its brokers to locate a security that can be borrowed, and then the fund sells the borrowed security. Duffie, Gârleanu, and Pedersen (2002) describe in detail the institutional arrangements of shorting. The broker keeps the proceeds of the short sale to be able to repurchase the security if the hedge fund fails and, additionally, requires that the hedge fund posts a margin m_t^{j-} that covers the largest possible adverse price move with a certain confidence as in Equation (6).

In the U.S., margins on equities are subject to Regulation T, which stipulates that non-brokers/speculators must have an initial margin (downpayment) of 50% of the market value of the underlying stock, both for new long and short positions. Hedge funds can circumvent Regulation T for instance by organizing the transaction as a total return swap, which is a derivative that is functionally equivalent to buying the stock. Stock exchanges and self-regulatory organizations (e.g. NASD) impose maintenance margins for existing stock positions. For example, the NYSE and the NASD require that investors maintain a minimum margin of 25% for long stock positions and of 30% for short stock positions.

The margin on exchange traded futures (or options) is also set by the exchange. The principle for setting the margin for futures or options is the same as that described above. The margin is set such as to make the exchange almost immune to default risk of the counterparty, and hence riskier contracts have larger margins.

At the end of the financing period, time $t + 1$, the position is “marked-to-market,” which means that the hedge fund receives any gains (or pays any losses) that have occurred between t and $t + 1$, that is, the fund receives $x_t^j(p_{t+1}^j - p_t^j)$ and pays interest on the loan at the funding rate. If the trade is kept on, the broker keeps the margin to protect against losses going forward from time $t + 1$. The margin can be adjusted if the risk of the collateral has changed unless the counterparties have contractually fixed the margin for a certain period. Since a hedge fund must be able to finance all its security positions at any point of time, they face the capital constraint (4) at any point in time.

So far, we focused on situations in which margins are covered using risk-free assets (cash). A hedge fund can also post risky assets to cover margins. If the margins are covered with risky assets, the market value of these assets must be higher, since a so-called *haircut* is subtracted. For example, a fund who bought x_t^j shares of stock j and has to come up of with margins of $x_t^j m_t^j$, can cover it with $x_t^{j'}$ of his uncollateralized bonds j' . Since the bond is risky, a haircut $h_t^{j'}$ is subtracted and his funding constraint becomes $x_t^j m_t^j \leq W_t - x_t^{j'} h_t^{j'}$. Moving the haircut term to the left hand side reveals that the haircut is equivalent to a margin. Hence, the fund essentially still faces funding constraint (4). Indeed, the fund could have alternatively used the bonds j' to raise cash and then use this cash to cover the margins for asset j . We therefore use the

estimated risk of holding the asset over a time period that is often set to be five to ten days.

terms margins and haircuts interchangeably.

We have described how funding constraints work when margins and haircuts are set separately for each security position. It is, however, sometimes possible to “cross-margin”, i.e. to jointly finance several trades that are part of the same strategy. This leads to a lower total margin if the risks of the various positions are partially offsetting. For instance, much of the interest rate risk is eliminated in a “spread trade” with a long position in one bond and a short position in a similar bond. Hence, the margin/haircut of a jointly financed spread trade is smaller than the sum of the margins of the long and short bonds. For a strategy that is financed jointly, we can reinterpret security j as such a strategy. Prime brokers compete by, among other things, offering low margins and haircuts — a key consideration for hedge funds — which means that it is becoming increasingly easy to finance more and more strategies jointly. In the extreme, one can imagine a joint financing of a hedge fund’s total position such that the funding constraint becomes

$$M_t(x_t^1, \dots, x_t^J) \leq W_t, \quad (33)$$

where M_t is the margin of the whole portfolio (x_t^1, \dots, x_t^J) , that is, the most the portfolio can lose with a certain confidence $Pr(-\sum_j \Delta p_{t+1}^j x_t^j > M_t | \mathcal{F}_t) = \pi$. Currently, it is often not practical to jointly finance a large portfolio. This is because a large hedge fund finances its trades using several brokers, both a hedge fund and a broker can consist of several legal entities (possibly located in different jurisdictions), certain trades need separate margins paid to exchanges (e.g. futures and options) or to other counterparties of the prime broker (e.g. securities lenders), prime brokers may not have sufficiently sophisticated models to evaluate the diversification benefits (e.g. because they don’t have enough data on the historical performance of newer products such as CDOs), and because of other practical difficulties in providing joint financing. Further, if the margin requirement relies on assumed stress scenarios in which the securities are perfectly correlated (e.g. due to predatory trading as in Brunnermeier and Pedersen (2005)), then (33) coincides with (4).

6.2 Funding Requirements for Banks

A bank’s capital consists of equity capital plus its long-term borrowings (including credit lines secured from individual or syndicates of commercial banks), reduced by assets that cannot be readily employed (e.g. goodwill, intangible assets, property, equipment, and capital needed for daily operations), and further reduced by uncollateralized loans extended by the bank to others (see e.g. Goldman Sachs 2003 Annual Report). Banks also raise money using short-term uncollateralized loans such as commercial papers and promissory notes, and, in the case of commercial banks, demand deposits. These sources of financing cannot, however, be relied on in times of funding crises since lenders may be unwilling to continue lending, and therefore this short-term

funding is often not included in measures of capital.

The financing of a bank's trading activity is largely based on *collateralized* borrowing. Banks can finance long positions using collateralized borrowing from corporations with excess cash, other banks, insurance companies, and the Federal Reserve Bank, and can borrow securities to shortsell from, for instance, mutual funds and pension funds. These transactions typically require margins which must be financed by the bank's capital as captured by the funding constraint (4).

The financing of a bank's proprietary trading is more complicated than that of a hedge fund, however. For instance, banks may negotiate zero margins with certain counterparties, and banks can often sell short shares held in house, that is, held in a customer's margin account (in "street name") such that the bank does not need to use capital to borrow the share externally. Further, a bank receives margins when financing hedge funds (i.e. the margin is negative from the point of view of the bank). However, often the bank wants to pass on the trade to an exchange or another counterparty and hence has to pay a margin to the exchange. In spite of these caveats, we believe that in times of stress, banks face margins and are ultimately subject to a funding constraint in the spirit of (4). For instance, Lehman Brothers, 2001 Annual Report (page 46) states that

"the following must be funded with cash capital: Secured funding 'haircuts,' to reflect the estimated value of cash that would be advanced to the Company by counterparties against available inventory, Fixed assets and goodwill, [and] Operational cash ... ,"

and Goldman Sachs, 2003 Annual Report (page 62) makes a similar statement.

In addition, banks have to satisfy certain regulatory requirements. Commercial banks are subject to the Basel accord, supervised by the Federal Reserve system for US banks. In short, the Basel accord of 1988 requires that a bank's "eligible capital" exceeds 8% of the "risk-weighted asset holdings," which is the sum of each asset holding multiplied by its risk weight. The risk weight is 0% for cash and government securities, 50% for mortgage-backed loans, and 100% for all other assets. The requirement posed by the 1988 Basel accord corresponds to Equation (4) with margins of 0%, 4%, and 8%, respectively. In 1996, the accord was amended, allowing banks to measure market risk using an internal model similar to (33) rather than using standardized risk weights.

U.S. broker-speculators, including banks acting as such, are subject to the Securities and Exchange Commission's (SEC's) "net capital rule" (SEC Rule 15c3-1). This rule stipulates, among other things, that a broker must have a minimum "net capital," which is defined as equity capital plus approved subordinate liabilities minus "securities haircuts" and operational charges. The haircuts are set as security-dependent percentages of the market value. The standard rule requires that the net capital exceeds at least $6\frac{2}{3}\%$ (15:1 leverage) of aggregate indebtedness (broker's total money liabilities) or

alternatively 2% of aggregate debit items arising from customer transactions. This constraint is similar in spirit to (4).⁹ As of August 20, 2004, SEC amended (SEC Release No. 34-49830) the net capital rule for Consolidated Supervised Entities (CSE) such that CSE's may, under certain circumstances, use their internal risk models similar to (33) to determine whether they fulfill their capital requirement.

6.3 Funding Requirements for Market Makers

There are various types of market-making firms. Some are small partnerships, whereas others are parts of large investment banks. The small firms are financed in a similar way to hedge funds in that they rely primarily on collateralized financing; the funding of banks was described in Section 6.2.

Certain market makers, such as NYSE specialists, have an obligation to make a market and a binding funding constraint means that they cannot fulfill this requirement. Hence, avoiding the funding constraint is especially crucial for such market makers.

Market makers are in principle subject to the SEC's net capital rule (described in Section 6.2), but the rule has special exceptions for market makers. Hence, market makers' main regulatory requirements are those imposed by the exchange on which they operate. These constraints are often similar in spirit to (4).

6.4 Lessons from Past Funding Liquidity Crises

In this section we revisit several real world examples that illustrate that funding liquidity risk is not purely a theoretical possibility. In the 1987 stock market crash numerous market makers hit (or violated) their funding constraint:

“By the end of trading on October 19, [1987] thirteen [NYSE specialist] units had no buying power”
— SEC (1988), page 4-58

Several of these firms managed to reduce their positions and continue their operations. Others did not. For instance, Tompane was so illiquid that it was taken over by Merrill Lynch Specialists and Beauchamp was taken over by Spear, Leeds & Kellogg (Beauchamp's clearing broker).

Also, market makers outside the NYSE experienced funding troubles: the Amex market makers Damm Frank and Santangelo were taken over; at least 12 OTC market makers ceased operations; and several trading firms went bankrupt.

These funding problems were due to (i) reductions in capital arising from trading losses and default on unsecured customer debt, and (ii) an increased funding need

⁹Let L be the lower of $6\frac{2}{3}\%$ of total indebtedness or 2% of debit items and h^j the haircut for security j ; then the rule requires that $L \leq W - \sum_j h^j x^j$, that is, $\sum_j h^j x^j \leq W - L$.

stemming from increased inventory and, consistent with our model, increased margins. One New York City bank, for instance, increased margins/haircuts from 20% to 25% for certain borrowers, and another bank increased margins from 25% to 30% for all specialists (SEC (1988) page 5-27 and 5-28). Other banks reduced the funding period by making intra-day margin calls, and at least two banks made intra-day margin calls based on assumed 15% and 25% losses, thus effectively increasing the haircut by 15% and 25%. Also, some broker-speculators experienced a reduction in their line of credit and – as Figure 1 shows – margins at the futures exchanges also drastically increased.

The events following the Russian default in 1998 is another stark example of funding liquidity risk. The hedge fund Long Term Capital Management (LTCM) had been aware of funding liquidity risk. Indeed, they estimated that in times of severe stress, haircuts on AAA-rated commercial mortgages would increase from 2% to 10%, and similarly for other securities (HBS Case N9-200-007(A)). In response to this, LTCM had negotiated long-term financing with margins fixed for several weeks on many of their collateralized loans. Other firms with similar strategies, however, experienced increased margins. Due to an escalating liquidity spiral, LTCM could ultimately not fund its positions in spite of their numerous measures to control funding risk and was taken over by 14 banks in September 1998.

Naturally, financial institutions try to manage their funding liquidity risk. For instance, Goldman Sachs (2003 Annual Report, page 62) states that it seeks to maintain net capital in excess of total margins and haircuts that it would face in periods of market stress plus the total draws on unfunded commitments at such times. Hence, Goldman Sachs recognizes that it may not have access to short-term borrowing during a crisis, that margins and haircuts may increase during such a crisis, and that counterparties may withdraw funds at such times.

Finally, our analysis suggests that central banks can help mitigate market liquidity problems by controlling funding liquidity. If a central bank is better than typical financiers of speculators at distinguishing liquidity shocks from fundamental shocks, then the central bank can convey this information and urge financiers to relax their funding requirements — as the Federal Reserve Bank of New York did during the 1987 stock market crash. Central banks can also improve market liquidity by boosting speculators' funding conditions during a liquidity crisis, or by simply stating the intention to provide extra funding during times of crisis, which will loosen margin requirements immediately as financiers' worst-case scenarios improve.

7 Conclusion and New Testable Predictions

As discussed in the introduction, our model's market liquidity predictions are consistent with a host of stylized empirical facts. Our analysis further suggests a novel line of empirical work that tests the model at a deeper level: namely its prediction that

speculators' funding is a driving force underlying these market liquidity effects.

First, it would be of interest to empirically study the determinants of margin requirements, e.g. using data from futures markets or from prime brokers. Our model suggests that both fundamental volatility and liquidity-driven volatility affects margins. Empirically, fundamental volatility can be captured using price changes over a longer time period, and the total fundamental and liquidity-based volatility is captured by short-term price changes as in the literature on variance ratios (see e.g. Campbell, Lo, and MacKinlay (1997)). Our model predicts that, in markets where it is harder for financiers to be informed, margins depend on the total fundamental and liquidity-based volatility. In particular, in times of liquidity crisis, margins increase in such markets, and, more generally, margins should comove with illiquidity in the time series and in the cross section.¹⁰

Second, our model suggests that an exogenous shock to the capital of the speculator sector should lead to a reduction in market liquidity. Hence, a clean test of the model would be to identify exogenous capital shocks such an unrelated decision to close down a trading desk, a merger leading to reduced total trading capital, or a loss in one market unrelated to the fundamentals of another market, and then study the market liquidity and margin around such events.

Third, the model implies that the effect of speculator capital on market liquidity is highly non-linear: a marginal change in capital has a small effect when speculators are far from their constraints, but a large effect when speculators are close from their constraints.

Fourth, the model suggests that a cause of the commonality in liquidity is that the speculator sector's shadow cost of capital is a driving state variable. Hence, a measure of speculator capital tightness should help explain the empirical comovement of market liquidity. Further our result "commonality of fragility" suggest that especially sharp liquidity reductions occur simultaneously across several assets.

Fifth, the model predicts that the sensitivity of margins and market liquidity to speculator capital is larger for securities that are risky and illiquid on average. Hence, the model suggests that a shock to speculator capital would lead to a reduction in market liquidity through a spiral effect that is stronger for illiquid securities.

We believe that the model's new theoretical predictions suggest a line of empirical work that seeks to link measures of speculators' funding conditions (margin requirements and capital) to measures of market liquidity.

¹⁰One must be cautious with the interpretation of the empirical results related to changes in Regulation T since this regulation may not affect speculators but affects the demanders of liquidity, namely the customers.

A Appendix: Proofs

Proof of Propositions 1 and 2. These results follow from the calculations in the text. ■

Proof of Proposition 3. We prove the proposition for $Z_1 > 0$, implying $p_1 \leq v_1$ and $x_1 \geq 0$. The complementary case is analogous. To see how the equilibrium depends on the exogenous shocks, we first combine the equilibrium condition $x_1 = -\sum_{k=0}^1 y_1^k$ with the speculators' funding constraint to get

$$m_1^+ \left(Z_1 - \frac{2}{\gamma(\sigma_2)^2} (v_1 - p_1) \right) \leq b_0 + p_1 x_0 + \eta_1 \quad (34)$$

that is,

$$m_1^+ \left(Z_1 - \frac{2}{\gamma(\sigma_2)^2} (v_1 - p_1) \right) - p_1 x_0 - b_0 \leq \eta_1 \quad (35)$$

For η_1 large enough, this inequality is satisfied for $p_1 = v_1$, that is, it is a stable equilibrium that the market is perfectly liquid. For η_1 low enough, the inequality is violated for $p_1 = \frac{2v_1}{\gamma(\sigma_2)^2} - Z_1$, that is, it is an equilibrium that the speculator is in default. We are interested in intermediate values of η_1 . If the left-hand-side of (35) is increasing in p_1 then p_1 is a continuous increasing function of η_1 , implying no fragility with respect to η_1 .

Fragile arises if the left-hand-side of (35) can be decreasing p_1 . Intuitively, this expression measures the speculators' funding need at the equilibrium position, and fragility arises if the funding need is greater when prices are lower, that is, further from fundamentals. (This can be shown to be equivalent to a non-monotonic excess demand function.)

When the financier is informed, the left-hand-side of (35) is

$$(\bar{\sigma} + \bar{\theta} |\Delta v_1| + p_1 - v_1) \left(Z_1 + \frac{2}{\gamma(\sigma_2)^2} (p_1 - v_1) \right) - p_1 x_0 - b_0 \quad (36)$$

Since the first product is a product of two positive increasing functions of p_1 , this is decreasing in p_1 only if x_0 is large enough.

When the financier is uninformed and $a = 0$, the left-hand-side of (35) is

$$(\bar{\sigma} + \bar{\theta} |\Delta p_1|) \left(Z_1 + \frac{2}{\gamma(\sigma_2)^2} (p_1 - v_1) \right) - p_1 x_0 - b_0 \quad (37)$$

When $p_1 < p_0$, $|\Delta p_1| = p_0 - p_1$ is decreasing in p_1 and, if $\bar{\theta}$ is large enough, this can make the entire expression decreasing. Also, the expression is decreasing if x_0 is large

enough.

It can be shown that the price cannot be chosen continuous in η_1 when the left-hand-side of (35) can be decreasing. ■

Proof of Proposition 4. When the funding constraint binds, we use the implicit function theorem to compute the derivatives. Using that y_1 is given by (11, the equilibrium condition $x_1 = -y_1$, the fact that the speculators' funding constraint binds in an illiquid equilibrium, and that $v_1 - p_1 > 0$ when $Z_1 > 0$, we have

$$m_1^+ \left(Z_1 - \frac{2}{\gamma(\sigma_2)^2} (v_1 - p_1) \right) = b_0 + p_1 x_0 + \eta_1 \quad (38)$$

We differentiate this expression to get

$$\frac{\partial m_1^+}{\partial p_1} \frac{\partial p_1}{\partial \eta_1} \left(Z_1 - \frac{2}{\gamma(\sigma_2)^2} (v_1 - p_1) \right) + m_1^+ \frac{2}{\gamma(\sigma_2)^2} \frac{\partial p_1}{\partial \eta_1} = \frac{\partial p_1}{\partial \eta_1} x_0 + 1 \quad (39)$$

which leads to Equation (23) after rearranging. The case of $Z_1 < 0$ and Equation (24) is analogous.

Finally, spiral effects happen if one of the last two terms in the denominator of the right-hand-side of Equations (23)-(24) is negative. When the speculator is informed, $\frac{\partial m_1^+}{\partial p_1} = 1$ and $\frac{\partial m_1^-}{\partial p_1} = -1$ using Proposition 1. Hence, in this case margins are stabilizing.

If the speculators are uninformed and a approaches 0, then using Proposition 2 we have that $\frac{\partial m_1^+}{\partial p_1} = \frac{\partial m_1^+}{\partial \Lambda_1}$ approaches $-\bar{\theta} < 0$ for $v_1 - v_0 + \Lambda_1 - \Lambda_0 < 0$ and $\frac{\partial m_1^-}{\partial p_1} = \frac{\partial m_1^-}{\partial \Lambda_1}$ approaches $\bar{\theta} > 0$ for $v_1 - v_0 + \Lambda_1 - \Lambda_0 > 0$. This means that there is a margin spiral with positive probability. The case of a loss spiral is immediately seen to depend on the sign on x_0 . ■

Proof of Proposition 5. We first consider the equation that characterizes a constrained equilibrium. When there is selling pressure from customers, $Z_1^j > 0$, it holds that

$$|\Lambda_1^j| = -\Lambda_1^j = v_1^j - p_1^j = \min \left\{ \phi_1 m_1^{j+}, \frac{\gamma(\sigma_2^j)^2}{2} Z_1^j \right\} \quad (40)$$

and if customers are buying, $Z_1^j < 0$, we have

$$|\Lambda_1^j| = \Lambda_1^j = p_1^j - v_1^j = \min \left\{ \phi_1 m_1^{j-}, \frac{\gamma(\sigma_2^j)^2}{2} (-Z_1^j) \right\} \quad (41)$$

Using the equilibrium condition $x_1^j = -\sum_k y_1^{j,k}$, Equation (11) for $y_1^{j,k}$, the speculators'

funding condition becomes

$$\sum_{Z_1^j > \frac{2\phi_1 m_1^{j+}}{\gamma(\sigma_2^j)^2}} m_1^{j+} \left(Z_1^j - \frac{2\phi_1 m_1^{j+}}{\gamma(\sigma_2^j)^2} \right) + \sum_{-Z_1^j > \frac{2\phi_1 m_1^{j-}}{\gamma(\sigma_2^j)^2}} m_1^{j-} \left(-Z_1^j - \frac{2\phi_1 m_1^{j-}}{\gamma(\sigma_2^j)^2} \right) = \sum_j x_0^j p_1^j + b_0 + \eta_1 \quad (42)$$

where the margins are evaluated at the prices solving (40)–(41). When ϕ_1 approaches infinity, the left-hand-side of (42) becomes zero, and when ϕ_1 approaches zero, the left-hand-side approaches the capital needed to make the market perfectly liquid. As in the case of one security, there can be multiple equilibria and fragility (Proposition 3). On a stable equilibrium branch, ϕ_1 increases as η_1 decreases.

Of course, the equilibrium shadow cost of capital ϕ_1 is random since $\eta_1, \Delta v_1^1, \dots, \Delta v_1^J$ are random. To see the commonality in liquidity, we note that $|\Lambda^j|$ is increasing in ϕ_1 for each $j = k, l$. To see this, consider first the case $Z_1^j > 0$. When the financiers are uninformed, $a = 0$, and $\theta^j = 0$, then $m_1^{j+} = \bar{\sigma}^k$, and, therefore, Equation (40) shows directly that $|\Lambda_1^j|$ increases in ϕ_1 (since the minimum of increasing functions is increasing). When financiers are informed and $\theta^j = 0$ then $m_1^{j+} = \bar{\sigma}^k + \Lambda_1^j$, and, therefore, Equation (40) can be solved to be $|\Lambda_1^j| = \min\{\frac{\phi_1}{1+\phi_1}\bar{\sigma}^j, \frac{\gamma(\sigma_2^k)^2}{2}Z_1^k\}$, which increases in ϕ_1 . Similarly, Equation (41) shows that $|\Lambda^j|$ is increasing in ϕ_1 when $Z_1^j < 0$.

Now, since $|\Lambda^j|$ is increasing in ϕ_1 and does not depend on other state variables under these conditions, $\text{Cov}[\Lambda^k(\phi), \Lambda^l(\phi)] \geq 0$ because any two functions which are both increasing in the same random variable are positively correlated.

To see part (ii) of the proposition, note that, for all j , $|\Lambda^j|$ is a continuous function of ϕ_1 , which is locally insensitive to ϕ_1 if and only if the speculator is not marginal on security j (i.e. if the second term in Equation (40) or (41) attains the minimum). Hence, $|\Lambda^j|$ jumps if and only if ϕ_1 jumps.

To see part (iii), we write illiquidity using Equations (40)–(41) as

$$|\Lambda_1^j| = \min\left\{ \phi_1 m_1^{j, \text{sign}(Z_1^j)}, \frac{\gamma(\sigma_2^j)^2}{2} |Z_1^j| \right\} \quad (43)$$

Hence, using the expression for the margin, if the financier is uninformed and $\theta^j = a = 0$ then

$$|\Lambda_1^j| = \min\left\{ \phi_1 \bar{\sigma}_1^j, \frac{\gamma(\sigma_2^j)^2}{2} |Z_1^j| \right\} \quad (44)$$

and, if the financier is informed and $\theta^j = 0$, then

$$|\Lambda_1^j| = \min\left\{ \frac{\phi_1}{1+\phi_1} \bar{\sigma}_1^j, \frac{\gamma(\sigma_2^j)^2}{2} |Z_1^j| \right\} \quad (45)$$

In case of an uninformed financier as in (44), we have that if $x_1^k \neq 0$

$$|\Lambda_1^k| = \phi_1 \bar{\sigma}_1^k \geq \phi_1 \bar{\sigma}_1^l \geq |\Lambda_1^l| \quad (46)$$

and if $|Z_1^k| \geq |Z_1^l|$

$$|\Lambda_1^k| = \min\left\{\phi_1 \bar{\sigma}_1^k, \frac{\gamma(\sigma_2^i)^2}{2}|Z_1^k|\right\} \geq \min\left\{\phi_1 \bar{\sigma}_1^l, \frac{\gamma(\sigma_2^i)^2}{2}|Z_1^l|\right\} = |\Lambda_1^l| \quad (47)$$

With an informed financier, if it is seen that $|\Lambda_1^k| \geq |\Lambda_1^l|$ using the same arguments.

For part (iv) of the proposition, we use that

$$\frac{\partial |\Lambda_1^j|}{\partial(-\eta_1)} = \frac{\partial |\Lambda_1^j|}{\partial \phi_1} \frac{\partial \phi_1}{\partial(-\eta_1)} \quad (48)$$

Further, $\frac{\partial \phi_1}{\partial(-\eta_1)} \geq 0$ and, from Equations (44)–(45) we see that $\frac{\partial |\Lambda_1^k|}{\partial \phi_1} \geq \frac{\partial |\Lambda_1^l|}{\partial \phi_1}$. The result that $\text{Cov}(\Lambda^k, \phi) \geq \text{Cov}(\Lambda^l, \phi)$ now follows from Lemma 1 below. ■

Lemma 1 *Let X be a random variable and g_i , $i = 1, 2$, be weakly increasing functions X , where g_1 has a larger derivative than g_2 , that is, $g_1'(x) \geq g_2'(x)$ for all x . Then,*

$$\text{Cov}[X, g_1(X)] \geq \text{Cov}[X, g_2(X)] \quad (49)$$

Proof. For $i = 1, 2$ we have

$$\text{Cov}[X, g_i(X)] = E[(X - E[X])g_i(X)] \quad (50)$$

$$= E\left[(X - E[X])\left(\int_{E[X]}^X g_i'(y) dy\right)\right] \quad (51)$$

The latter expression is a product of two terms that always have the same sign. Hence, this is higher if g_i' is larger. ■

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