

# Overconfidence, Investment Policy, and Manager Welfare\*

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## Abstract

We use a simple capital budgeting problem to contrast the decisions of overconfident managers with those of rational managers. We reach the conclusion that managerial overconfidence can increase firm value. When making capital budgeting decisions, risk-averse managers sometimes choose to stay away from risky projects that would increase firm value. Overconfident managers overestimate their personal ability to reduce risk, and as a result may make capital budgeting decisions that are in the better interest of shareholders. An overconfident manager's biased decision-making does not necessarily make him worse off. Indeed, when compensation endogenously adjusts to reflect outside opportunities, managerial overconfidence enables the firm to offer the manager a flatter compensation contract. Depending on the level of competition for skilled labor, such flatter contracts translate into a higher firm value without affecting the manager's welfare, an improvement in the manager's welfare without affecting firm value, or improvements in both firm value and the manager's welfare. Overconfidence also has the added benefit of reducing moral hazard problems in the principal-agent relationship between the firm's shareholders and the manager, as overconfident managers overvalue the risk reduction afforded by their information acquisition efforts. Although optimism has the same benefits as overconfidence in affecting the manager's risk-taking behavior, it is not as effective at reducing moral hazard problems.

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# 1 Introduction

A vast experimental literature finds that individuals are usually overconfident, that is, they believe their knowledge is more precise than it actually is. Since overconfidence directly influences decision-making, it is natural to ask how overconfident managers will affect the value of the firm. Is managerial overconfidence sufficiently detrimental to firm value that shareholders should actively avoid hiring overconfident managers? What possible benefits might overconfident managers bring to the firm? Does overconfidence always make managers worse off as a result of poor decision-making, or can it have positive effects on their welfare?

We use a simple model of capital budgeting to contrast the decisions of overconfident managers with those of rational managers. We reach the conclusion that managerial overconfidence can often increase the value of the firm. More precisely, overconfidence can align managers' preferences for risky projects more closely with those of shareholders. Surprisingly, this increase in value for the shareholders does not have to come at a cost for the managers. Overconfident managers require less convexity in their compensation to take the risks that are desirable to shareholders. When contracts are set to match the managers' outside opportunities, shareholders can offer overconfident managers flatter compensation schedules, which are valued more highly by these risk-averse managers, but are cheaper for the firm. The benefits of overconfidence are even greater when the principal-agent relationship between the owners and the manager must also overcome a moral hazard problem. In that case, overconfidence has the added benefit of making the manager more willing to exert effort, as he overvalues the benefit of that effort.

Jensen and Meckling (1976) point out that due to agency issues, decisions of managers are not always in the best interests of shareholders. Treynor and Black (1976) write:

“If the corporation undertakes a risky new venture, the stockholders may not be very concerned, because they can balance this new risk against other risks that they hold in their portfolios. The managers, however, do not have a portfolio of employers. If the corporation does badly because the new venture fails, they do not have any risks except the others taken by the same corporation to balance against it. They are hurt by a failure more than the stockholders, who also hold stock in other corporations are hurt.”

Because of their greater risk aversion, rational managers will tend to undertake safe projects and forego risky investment opportunities to the detriment of the shareholders. Since overconfident managers believe that the information they gather about risky projects is better than it actually

is, they are less prone to playing it safe. Thus overconfident managers make decisions that are in the better interest of shareholders than do rational managers. While compensation contracts that increase the convexity of manager payoffs can be used to realign the decisions of a rational manager with those of shareholders, it is less expensive to simply hire an overconfident manager. The gains from overconfidence will at times be sufficient that shareholders actually prefer an overconfident manager with less ability to a rational manager with greater ability.

Because it is easier for the firm's owners to make an overconfident manager act in their best interest, they are also better equipped to offer him compensation contracts that he values more. In particular, in their effort to prevent the manager from leaving the firm and/or to compete for his services with other firms, the firm's owners will be able to offer an overconfident manager a flatter compensation schedule which may improve his welfare above that of an otherwise identical but rational manager. This is where our paper departs from much of the literature on behavioral finance and economics. In our model, the firm does not benefit simply from exploiting an overconfident manager's bias, i.e., our results do not emanate from a simple surplus transfer between an irrational manager and rational shareholders. Instead, the manager's overconfidence facilitates his commitment to do the right thing on behalf of shareholders who can then offer him a more advantageous compensation contract.

To understand our results, it is useful to think of the cheapest compensation contract that would satisfy a risk-averse agent's reservation utility in his relationship with a risk-neutral principal. Because of risk aversion, it is cheaper for the principal to pay the agent the same amount in all states of the world.<sup>1</sup> If the agent could commit to make all of his decisions in the best interest of the principal with this contract, total surplus would clearly be maximized, as the risk-neutral agent would then be absorbing all the risk. However, because the agent responds to incentives and thus cannot credibly commit to behaving in the best interest of the principal, a flat compensation schedule never achieves this maximum surplus. Only increasing, and possibly convex, compensation contracts can create incentives for the manager to act in the best interest of the principal. Of course, as soon as the compensation contract departs from a flat schedule, it becomes more costly to the risk-neutral principal, as the risk-averse agent is now bearing risk and requires additional total compensation for doing so.

Agent overconfidence, by making the risk-averse agent more willing to bear risk, mitigates this problem. The agent thinks that he can control risk better than he really can, and so minimal extra compensation is necessary to make him take these risks. For the same risk-taking behavior,

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<sup>1</sup>Of course, bankruptcy is a constraint in doing this.

therefore, his compensation remains flatter than that of the rational agent; this makes the agent better off. Also, this compensation is cheaper for the principal, making him better off as well. In other words, the contract between the principal and the agent is closer to the flat wage contract that maximizes surplus, but still creates the right incentives for the agent. This closer proximity to first-best makes everyone better off (or, at the very least, someone better off and no one worse off).

Similar results on the Pareto optimality of overconfidence in firms are obtained by Gervais and Goldstein (2005), who show that agent overconfidence can make the firm and all agents, including the biased ones, better off. Better risk-sharing arrangements are not the source of their results, however, as their agents are assumed to be risk-neutral. Instead, they argue that overconfidence better commits an agent to the firm, and this has positive commitment externalities on the firm's other agents. This idea that the commitment of decision-making agents can be enhanced by their personal biases can also be found in the work of Bénabou and Tirole (2002) who show that self-deception improves welfare when the motivation gains from ignoring negative signals outweigh the losses from ignoring positive ones. This effect is also different from that in our paper, as the manager in our model uses all available information. In addition, our paper looks at the effect of the agent's bias on the principal, something that is not considered formally in this latter paper. Finally, Brunnermeier and Parker (2005) consider a model in which the distorted beliefs of agents can improve their welfare by allowing them to enjoy more anticipatory utility, something that is not considered nor possible in our one-period model.

Although many recent studies explore the implications of overconfidence for financial markets,<sup>2</sup> very few studies have looked at overconfidence in corporate settings. Roll (1986) suggests that overconfidence (hubris) may motivate many corporate takeovers. Kahneman and Lovallo (1993) argue that managerial overconfidence and optimism stem from managers taking an "inside view" of prospective projects. The inside view focuses on project specifics and readily anticipated scenarios while ignoring relevant statistical information such as "how often do projects like this usually succeed?" Bernardo and Welch (2001) provide an evolution rationale for the presence of overconfident entrepreneurs in a society. Heaton (2002) examines the implications of managerial optimism for

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<sup>2</sup>In Benos (1998), traders are overconfident about the precision of their own signals and their knowledge of the signals of others. De Long, Shleifer, Summers and Waldmann (1991) demonstrate that overconfident traders can survive in markets. Hirshleifer, Subrahmanyam and Titman (1994) argue that overconfidence can promote herding in securities markets. Odean (1998) examines how the overconfidence of different market participants affects markets differently. Daniel, Hirshleifer and Subrahmanyam (1998), and Gervais and Odean (2001) develop models in which, due to a self-attribution bias, overconfidence increases with success. Kyle and Wang (1997) argue that mutual funds may prefer to hire overconfident money managers, because overconfidence enables money managers to pre-commit to taking more than their share of duopoly profits.

the benefits and costs of free cash flow. He points out that in the corporate environment, irrational managers are not likely to be arbitrated away. Hackbarth (2003a,b) finds that managerial overconfidence leads to greater debt financing and that overconfidence, by acting as a commitment device, can also ameliorate bondholder and shareholder conflicts such as debt overhang. Adrian and Westerfield (2005) study optimal continuous-time contracting in a principal-agent problem with heterogeneous beliefs. Finally, Malmendier and Tate (2003) provide empirical evidence that optimistic managers invest more aggressively.

Most of our analysis concentrates on managerial overconfidence, but we also consider a closely related bias, namely managerial optimism. Optimistic managers believe that the expected payoffs of their firm's risky projects are greater than they actually are. Thus, like overconfident managers, optimistic managers are naturally less reluctant to choose risky projects. However, because their reason for doing so is fundamentally different, optimistic managers are not as easy to realign as overconfident managers in the presence of moral hazard. Indeed, optimistic managers undertake risky projects because these projects look better to them than they really are and, for the same reason, these managers do not feel as compelled as overconfident managers to gather effort-costly information. In essence, the over-valuation of one's skill to learn about a project (overconfidence) is more valuable to the firm than the over-valuation of the project itself (optimism). As a result, hiring an optimistic manager is not always in the best interest of the firm.

Our paper proceeds as follows. Section 2 introduces a simple capital budgeting problem that is used throughout the paper to analyze the effects of behavioral biases on the value of the firm. The same section presents the first-best solution for shareholders, which serves as a benchmark for later sections. Section 3 formally introduces the concept of overconfidence. It shows how this individual trait can affect the value of the firm when a manager's sole form of compensation is partial ownership of the firm. The principal-agent nature of the relationship between firm owners and managers is analyzed in section 4. This section shows how contracting with a manager is facilitated by his overconfidence, and how this can make the firm more valuable and the manager better off when the manager has opportunities outside the firm. In section 5, we show how our basic model can be extended to accommodate other factors, like optimism, that are likely to play a role in the capital budgeting decisions of firms and their managers. Finally, section 6 summarizes our findings, discusses how they relate to the existing literature on CEO compensation, and concludes. All the proofs are contained in the appendix.

## 2 The Model

### 2.1 The Firm

An all-equity firm initially consists of half a dollar in cash, and is considering the possibility to invest that money in a risky project. At the beginning of the period, one such project becomes available. Its type  $\tilde{\phi}$ , which we assume to be uniformly distributed on  $[0, \frac{1}{2}]$ , is then revealed to the firm's decision-maker, whom we shall refer to as the *manager*.<sup>3</sup> We assume that  $\tilde{\phi}$  is not verifiable by the firm even after it becomes known by the manager.<sup>4</sup> All risky projects return one or zero dollar one period from now with probabilities  $\tilde{\phi}$  and  $1 - \tilde{\phi}$  respectively; we denote this end-of-period cash flow by  $\tilde{v}$ . For simplicity, we assume that the risk of these projects is completely idiosyncratic, and that the correct discount rate, the riskfree rate, is zero. Given this, the net present value of any risky project is negative, and so the initial value of the firm, one half, comes entirely from its cash.

The potential value from a risky project comes from the information that can be acquired about it. In particular, after learning the project's type, the manager acquires a signal

$$\tilde{s} = \begin{cases} \tilde{v}, & \text{prob. } \frac{1+a}{2} \\ 1 - \tilde{v}, & \text{prob. } \frac{1-a}{2}, \end{cases}$$

where  $a \in [0, 1]$ . This signal is more informative for larger values of  $a$ , as the correct value of the project is then observed more often. As such,  $a$  effectively measures information quality. We can also think of  $a$  as the ability of the manager, as only he possesses  $\tilde{s}$ . For now, we assume that acquiring the signal is costless for the manager; we consider the addition of an effort cost later in section 5.

In this framework, the potential impact of the project on firm value depends on the manager's conditional assessment of the project being successful and his decision as to whether or not to undertake the project. This in turn depends on the manager's incentives. In this and the next sections, we assume that the manager derives his utility exclusively from partial ownership of the firm, which is equivalent to assuming that the manager is compensated with firm stock. In section 4, we take a closer look at the manager's incentives and analyze how more general compensation contracts can align or inadvertently misalign the manager's decisions with the interests of shareholders. This two-step approach allows us to disentangle the effects of risk aversion, behavioral biases, and compensation on decision-making.

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<sup>3</sup>The assumption for the distribution of  $\tilde{\phi}$  is not important for our results, but the exposition is greatly simplified with the uniform distribution, as closed-form solutions then become possible.

<sup>4</sup>This assumption will start playing a role only in section 4 in which we tackle contracting issues.

## 2.2 Updating and First-Best

Given that the risk of the project available to the firm is purely idiosyncratic and that the riskfree rate is zero, we know that the value of the firm, to well-diversified or risk-neutral shareholders, is simply the expected value of its end-of-period cash flows. Clearly, the expected value of the risky project changes after information is gathered about it. It is easy to verify that

$$\Pr \left\{ \tilde{v} = 1 \mid \tilde{s} = 1, \tilde{\phi} = \phi \right\} = \frac{(1+a)\phi}{1-a+2a\phi} \equiv \phi_1, \quad \text{and} \quad (1)$$

$$\Pr \left\{ \tilde{v} = 1 \mid \tilde{s} = 0, \tilde{\phi} = \phi \right\} = \frac{(1-a)\phi}{1+a-2a\phi} \equiv \phi_0, \quad (2)$$

so that

$$E \left[ \tilde{v} \mid \tilde{s} = 1, \tilde{\phi} = \phi \right] = \frac{(1+a)\phi}{1-a+2a\phi} \geq \phi, \quad \text{and} \quad (3)$$

$$E \left[ \tilde{v} \mid \tilde{s} = 0, \tilde{\phi} = \phi \right] = \frac{(1-a)\phi}{1+a-2a\phi} \leq \phi. \quad (4)$$

Notice that  $\phi_1$  ( $\phi_0$ ) gets closer to one (zero) as  $a$  increases: more weight is put on the information when its precision is large. This also translates into more extreme assessments of the risky project's value, as shown in (3) and (4). Because the project requires an initial investment of half a dollar and  $\phi \in [0, \frac{1}{2}]$ , the project can only have a positive net present value after  $\tilde{s} = 1$  is observed. Even then, the project's net present value is positive if and only if  $\phi_1 > \frac{1}{2}$ , that is, if  $\phi > \frac{1-a}{2}$ . Clearly, the firm's owners would like the project to be undertaken only in that scenario, as this will generate the first-best value of the firm. Assuming that the manager indeed follows this policy,<sup>5</sup> the following proposition derives the ex ante value of the firm, before the project's type is observed and before any decision is made by the manager.

**Proposition 2.1 (First-Best)** *The value of the firm is maximized when the manager undertakes only the risky projects of type  $\tilde{\phi} > \frac{1-a}{2} \equiv \bar{\phi}^{\text{FB}}$  for which he has favorable information ( $\tilde{s} = 1$ ). With this first-best strategy, the initial value of the firm is*

$$F^{\text{FB}} \equiv \frac{1}{2} + \frac{a^2}{8}. \quad (5)$$

The value of the firm is improved from its cash value of  $\frac{1}{2}$  by the presence and decisions of the manager. In fact,  $F^{\text{FB}}$  is clearly increasing in  $a$ : the firm directly benefits from the ability of its manager. This increase in value comes from two sources. First, a high-ability manager is able to

<sup>5</sup>This may not be the case when the manager is risk-averse, or when compensation ends up tilting his preferences towards either the risky project or the safe investment. We will come back to these issues shortly.

consider a larger range of project types. Indeed, because his information allows him to learn a lot about a project (i.e., because his posterior beliefs about a project are far from his priors), he is sometimes able to learn that a project with low ex ante prospects (i.e., low  $\tilde{\phi}$ ) is in fact worthwhile for the firm to undertake because of the positive information ( $\tilde{s} = 1$ ) he has about it. This can be seen through a first-best threshold  $\bar{\phi}^{\text{FB}}$  for  $\tilde{\phi}$  that is decreasing in  $a$ . For example, when  $a = 1$ , the manager undertakes all projects about which he has positive information, as  $\bar{\phi}^{\text{FB}}$  is then zero. Second, conditional on observing a positive signal about a project of a given type  $\tilde{\phi} = \phi$ , the conditional value of this project is strictly increasing in  $a$ . Thus undertaking such a project creates more value for the firm. In other words, the odds that the manager is right are simply better for any given project that is undertaken. So, not only is a high-ability manager able to consider a larger set of projects, but he is also right about them more often.

### 2.3 The Effect of Risk Aversion

The first-best outcome maximizes the current value of the firm to risk-neutral shareholders. To attain it, the firm's manager must not care about risk when making his capital budgeting decisions. However, capital budgeting decisions will be made by agents whose human capital is tied to the firm, e.g., the CEO of the firm. As pointed out by Jensen and Meckling (1976), this agent's risk aversion is likely to affect his decisions. Compensation contracts can be used to reduce these agency costs by realigning the objective of the firm's manager with those of the shareholders. We discuss these in section 4. For now, we concentrate on the problem of a risk-averse manager whose sole form of compensation still comes from his partial ownership of the firm. We show how the risk aversion of this decision-maker will affect his capital budgeting decisions and in turn the value of his firm.

To keep the analysis of this and later sections tractable, we model risk aversion as a constant reduction  $r \in [0, 1)$  in marginal utility between the medium outcome (i.e., dropping the risky project in favor of riskfree cash) and the high outcome (i.e., undertaking a successful risky project). More precisely, for this section, the manager is assumed to get a boost in utility of only  $(1 - r)\frac{1}{2}$  when he chooses a successful risky project instead of the riskfree investment for his firm. This effectively makes the firm's manager risk-averse: the three potential outcomes of the capital budgeting decisions,  $\{0, \frac{1}{2}, 1\}$ , will respectively yield  $\{0, \frac{1}{2}, 1 - \frac{r}{2}\}$  in utility to the manager, making his utility function a concave function of the firm's end-of-period value. Note that, with this three-outcome specification, assuming more traditional utility functions would not change any of our results. Our specification, however, has the advantage of keeping the analysis tractable.

The risk-averse manager has the same information technology as the risk-neutral manager of section 2.2. Like the risk-neutral manager, the risk-averse manager never finds it optimal to undertake a risky project of any type after he observes a negative signal ( $\tilde{s} = 0$ ) about it.<sup>6</sup> After observing  $\tilde{\phi} = \phi$  and  $\tilde{s} = 1$ , the manager's tradeoff is as follows: his expected utility from undertaking the risky project is  $\phi_1 [\frac{1}{2} + (1-r)\frac{1}{2}]$ , whereas his expected utility from dropping it is  $\frac{1}{2}$ . Thus he undertakes the project if and only if  $\phi_1 > \frac{1}{2-r}$  or, equivalently,  $\phi > \frac{1-a}{2-r(1+a)} \equiv \bar{\phi}^{\text{RA}}$ . Because this threshold is increasing in  $r$ , we find that the manager undertakes fewer projects as he becomes more risk-averse, keeping his ability fixed. This makes sense, as the signal that the manager receives is not perfect and so some uncertainty about  $\tilde{v}$  remains. Risk-averse managers will be more tempted to avoid this uncertainty by holding onto cash, especially when their information does not carry their posteriors far above the positive net present value threshold. Indeed, the risk-averse manager avoids all projects of type  $\tilde{\phi} \in (\bar{\phi}^{\text{FB}}, \bar{\phi}^{\text{RA}}]$ , even though they would increase firm value when  $\tilde{s} = 1$  is observed. The following proposition derives the precise effect that this has on firm value.

**Proposition 2.2** *Suppose that the firm is managed and partially owned by a single risk-averse individual with risk aversion  $r \geq 0$ . This manager undertakes only the risky projects of type  $\tilde{\phi} > \frac{1-a}{2-r(1+a)}$  for which he has favorable information ( $\tilde{s} = 1$ ). With this strategy, the initial value of the firm is*

$$F = F^{\text{FB}} - \frac{1}{8} \left[ \frac{(1-a^2)r}{2-r(1+a)} \right]^2. \quad (6)$$

As anticipated, the firm's value is decreasing in  $r$ . The manager's utility gain from reducing risk does not transfer to the firm's shareholders. Instead, as seen in (6), the shareholders suffer an agency cost of  $\frac{1}{8} \left[ \frac{(1-a^2)r}{2-r(1+a)} \right]^2$ , which is increasing in  $r$ .

### 3 The Role of Overconfidence

The firm's value to risk-neutral shareholders is negatively affected by managerial risk aversion. In this section, we show how managerial overconfidence may help restore firm value. In some cases, as we show, the first-best outcome may even be restored. Following the work of Daniel, Hirshleifer and Subrahmanyam (1998), Odean (1998), and Gervais and Odean (2001), we define overconfidence to be the perception that private information is more precise and more reliable than it really is. More precisely, we assume that the overconfident manager thinks that  $a$  is equal

<sup>6</sup>To see this, notice that the manager's expected utility from undertaking a risky project of type  $\tilde{\phi} = \phi$  after observing  $\tilde{s} = 0$  is  $\phi_0 [\frac{1}{2} + (1-r)\frac{1}{2}] < \phi [\frac{1}{2} + \frac{1}{2}] = \phi$ , which is smaller than  $\frac{1}{2}$ , the utility he gets from dropping the risky project.

to  $a + \delta$ , where the difference  $\delta \in [0, 1 - a]$  measures the degree of overconfidence. As a result, the overconfident manager puts too much weight on his information, and over-adjusts his beliefs towards his information. He therefore thinks that the project is better (worse) than it really is after observing  $\tilde{s} = 1$  ( $\tilde{s} = 0$ ), as can be seen from (3) (from (4)) which is increasing (decreasing) in  $a$ .

As before, the manager never considers undertaking the project after he receives negative information about it. Thus the decision of the manager still amounts to deciding whether or not to undertake the project after he learns positive information about it. However, because the overconfident manager thinks that he receives a more precise signal than he actually does, and so thinks that the residual risk of the project is not as great as it really is, he is more inclined to undertake a risky project with imperfect information. In particular, he is now willing to accept projects of lower types.

Were the incentives of the manager perfectly aligned with those of the firm's owners (i.e., if the manager were risk-neutral), overconfidence would be detrimental as it would make the manager undertake projects of types lower than  $\bar{\phi}^{\text{FB}}$ . However, we know from section 2.3 that a risk-averse manager is reluctant to undertake projects of type  $\tilde{\phi} \in (\bar{\phi}^{\text{FB}}, \bar{\phi}^{\text{RA}}]$ . In these circumstances, the manager's overconfidence can have a positive effect by offsetting his risk aversion in such a way that his decisions become similar to that of a rational value-maximizing owner. This positive role of overconfidence is stated more precisely in the following proposition.

**Proposition 3.1** *For any manager with risk aversion  $r \in (0, 1)$ , there is a level of overconfidence,*

$$\delta^{\text{FB}} \equiv \frac{r(1 - a^2)}{2 - r(1 - a)}, \quad (7)$$

*such that the value of the firm is equal to the first-best value  $F^{\text{FB}}$ . The value of the firm is strictly increasing (decreasing) in  $\delta$  for  $\delta < \delta^{\text{FB}}$  ( $\delta > \delta^{\text{FB}}$ ).*

The last part of this proposition has important implications. In particular, for a given level of risk aversion, it is always the case that some overconfidence helps restore some of the value that is lost to decisions made to reduce risk. This process is not monotonic. Too much overconfidence distorts the decision-making process in that the manager may over-rely on his imperfect information. Still, it is possible for the decisions of a risk-averse manager to be close to profit-maximizing, even when he does not suspect it. This will be the case when the manager's overconfidence correctly counterbalances his risk aversion. This is the reason why we analyze compensation contracts separately: in some cases, changing the compensation of the decision-maker is not needed to restore the value lost to his risk aversion and may even be harmful to the firm.

Another implication of Proposition 3.1 is the fact that the ability to identify successful projects is not the only factor contributing to firm value. As the risk aversion of the decision-maker affects firm value, the behavioral traits of this decision-maker also possibly affect firm value. More than that, as we show next, a firm managed by an overconfident individual can be worth more than that managed by a rational individual with equal or even higher ability.

**Proposition 3.2** *Suppose that the value of a firm managed and partially owned by a risk-averse rational individual with ability  $a$  is  $F < F^{\text{FB}}$ . Then there exists a value  $\bar{a} < a$  such that the firm with a manager of any ability  $a' \in (\bar{a}, a)$  can be worth more than  $F$ .*

Note that this proposition does not say that the firm is *always* worth more with a lower-ability manager; the firm will be worth more only if the lower-ability manager has the right overconfidence. Again, this is because moderate levels of overconfidence effectively make the manager act as a profit maximizer.

## 4 Compensation Issues

### 4.1 Managers and Compensation

So far, we have assumed that the capital budgeting decisions are made by a manager whose utility is derived solely from partial ownership of the firm, that is, from compensation that only includes company stock. As we have seen, such compensation may cause risk-averse managers to behave more cautiously than is in the best interests of risk-neutral shareholders. To better align the incentives of managers with the interests of shareholders, firms often offer managers compensation packages that include stock options, which effectively “convexify” the managers’ preferences.

To study how compensation and managerial biases interact, we use the same framework as in previous sections, but assume that the firm chooses the compensation package from which the manager derives his utility. As before, the end-of-period payoff of the firm’s investment is 0,  $\frac{1}{2}$  or 1 in the low, medium and high states. Let us denote the manager’s compensation in each of these states by  $\{\Delta_L, \Delta_L + \Delta_M, \Delta_L + \Delta_M + \Delta_H\}$ ; that is,  $\Delta_M$  is the increment in compensation from the low to the medium state, and  $\Delta_H$  is the increment in compensation from the medium to the high state. Because the firm is worthless after a failed risky project, we assume in what follows that  $\Delta_L = 0$ . This assumption that the manager does not get paid after a failed project should not be taken literally; it is meant to capture the idea that, in a multi-period contracting environment, the manager is likely to get dismissed after (repeated) failure and/or his bankrupt firm may simply

cease to operate. In this context,  $\Delta_M$  represents the ongoing manager's salary, and  $\Delta_H$  represents promotions, bonuses and the exercise of stock options following successful periods. As before, we assume that the manager's risk aversion is captured by a constant reduction  $r \in [0, 1)$  in marginal utility between the medium outcome and the high outcome. As a result of the above assumptions, the manager's utility in each of the three states is  $\{0, \Delta_M, \Delta_M + (1 - r)\Delta_H\}$ .

## 4.2 Perfect Realignment

We are initially interested in characterizing the compensation package that will make the manager act like a maximizer of firm value. For now, we ignore the fact that the manager's compensation is paid by the firm and so effectively reduces the firm's value. This is equivalent to assuming that the reservation utility of the manager is arbitrarily close to zero, as only the relative size of  $\Delta_M$  and  $\Delta_H$  matters but both are set close to zero. Later in this section, we consider the possibility that the manager's reservation utility is greater than zero, and how this comes to affect firm value.

**Proposition 4.1** *Suppose that the manager hired by the firm is characterized by a risk aversion of  $r$ , ability  $a$ , and overconfidence  $\delta$ . Then the compensation package  $\{\Delta_M, \Delta_H\}$  that realigns his incentives with those of the risk-neutral shareholders must satisfy*

$$\Delta_H = \frac{(1 + a)(1 - a - \delta)}{(1 - a)(1 + a + \delta)} \frac{\Delta_M}{1 - r}. \quad (8)$$

Notice that the high-state compensation  $\Delta_H$  required to make the manager act in the best interest of the shareholders is increasing in  $r$ . For example, when  $\delta = 0$  (no overconfidence), equation (8) reduces to  $\Delta_H = \frac{\Delta_M}{1 - r}$ , which implies that the compensation package that perfectly realigns the incentives of the risk-averse manager is convex (since  $\Delta_H$  is then larger than  $\Delta_M$  for any  $r < 1$ ). This makes sense: convexity of the compensation contract is required to make the manager less subject to the conservatism brought about by his risk aversion.

The interesting aspect of (8) is the fact that the right-hand side of the equation is decreasing in  $\delta$ . This means that less convexity is required to realign the incentives of an overconfident risk-averse manager than to realign those of an otherwise identical rational manager. The intuition is simple: biased managers have a natural tendency to overcome the effects of their risk aversion, and so outside incentives are not needed quite as much. This observation may have important implications if including stock options in the compensation of a firm's top managers is expensive. If firms base the option compensation they offer managers on the assumption that the managers are rational, the firms will end up paying overconfident managers more than is in the best interests of the firm, while misaligning managers' incentives.

More importantly, as we show next, overconfident managers may end up better off *because* of their bias. Indeed, when an overconfident manager has outside opportunities that provide him with strictly positive reservation utility, the firm’s ability to align his incentives with a less convex compensation contract makes it possible to offer him more compensation in the other states of the world. Because the risk-averse manager values these other states more, he is sometimes better off than if he had been rational. More than that, in some cases, such flatter compensation is also cheaper for the firm, which is then more profitable and more valuable.

To develop these results, let us assume that the manager’s reservation utility is given by  $\bar{u} > 0$ , and that this number is small compared to the firm’s value.<sup>7</sup> The participation constraint of the manager can be analyzed from two perspectives in this model: that of the biased manager or that of the firm. Because the overconfident manager subjectively thinks it more likely than warranted that he will undertake a successful project on behalf of the firm, he tends to overvalue the extra compensation  $\Delta_H$  that he will receive in that state. This makes it easier and cheaper for the firm to satisfy the manager’s participation constraint, as less compensation has to be offered in the medium state to meet his reservation utility. This amounts to the firm taking advantage of the manager’s bias when setting his compensation. As we show in section 4.4, competition across firms for talented managers makes such abuse impossible. Indeed, any firm offering a compensation package that effectively transfers surplus from the manager to the firm will be undercut by some other firm, and this alone will ensure that managers are not vulnerable despite their biases.

As such, a more interesting approach is to calculate the participation constraint from the perspective of the firm, which can compute the correct likelihood of each state, given the manager’s bias. Because little extra compensation is required in the high state in order to make the manager behave like a profit maximizer, the firm realizes that it will have to better compensate him in the other states if, on average, his reservation utility has to be met. In this case, the participation constraint is more like a *retention constraint*, as the firm ensures that on average, the manager will be as satisfied ex post as he thinks he can be elsewhere.<sup>8</sup> In other words, the firm fully anticipates its manager to leave if he does not experience an average utility of  $\bar{u}$  at the end of the period. In what follows, we favor this second interpretation of the manager’s participation constraint, as any surplus created by the manager’s bias can then only originate from his decisions, and not to

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<sup>7</sup>To be more precise, because the firm starts out with half a dollar and can never be worth more than (5), and because marginal compensation in the high state is valued with a factor of  $1 - r$  by the manager, a condition that keeps  $\bar{u}$  sufficiently small for the firm to always want to hire the manager is that  $\bar{u} < (1 - r) \frac{a^2}{8}$ .

<sup>8</sup>Of course, with just one period and one project, this interpretation is outside the model, but one could easily consider a large number of small identical projects in each period, and the realized outcome would then always be close to the average outcome.

his irrational willingness to accept a contract that underpays him. As we discuss in section 4.4, this approach also has the benefit of being consistent with the endogenous reservation utility of managers in competitive labor markets.

As before, suppose that the firm is considering hiring a manager whose risk aversion, ability and overconfidence are  $r$ ,  $a$  and  $\delta$  respectively. Denote the compensation package  $\{\Delta_M, \Delta_H\}$  that realigns this manager's incentives with those of the risk-neutral shareholders and satisfies the firm's retention constraint by  $\{\bar{\Delta}_M, \bar{\Delta}_H\}$ . Clearly, this contract must satisfy (8) so that the manager picks the same threshold for  $\tilde{\phi}$  as the firm's shareholders would. Furthermore,  $\bar{\Delta}_M$  must be large enough that the manager's expected utility, as calculated by the firm, is at least  $\bar{u}$ . The following proposition characterizes this contract.

**Proposition 4.2** *The compensation package  $\{\bar{\Delta}_M, \bar{\Delta}_H\}$  that realigns the manager's incentives with those of the risk-neutral shareholders and satisfies the firm's retention constraint is given by*

$$\bar{\Delta}_M = \frac{4(1-a)(1+a+\delta)\bar{u}}{(1-a^2)(4+a^2)+2\delta(2-3a)}, \quad (9)$$

$$\bar{\Delta}_H = \frac{4(1+a)(1-a-\delta)\bar{u}}{[(1-a^2)(4+a^2)+2\delta(2-3a)](1-r)}. \quad (10)$$

*The resulting value of the firm (shown in the proof) is decreasing in  $r$  and increasing in  $\delta$ .*

It is easy to verify that  $\bar{\Delta}_H$  increases as the manager's risk aversion  $r$  increases. This is the classic agency problem between the firm's shareholders and the risk-averse manager. When hiring a risk-averse (and under-diversified) individual to manage their firm, shareholders must set up a compensation contract that will make this manager undertake all risky projects that have a positive net present value. This means that rewards must be offered to the manager in good but risky states of the world. To retain the manager's services, the firm still needs to pay him the same base salary, as can be seen from the fact that  $\bar{\Delta}_M$  does not depend on  $r$ . Indeed, the increase in  $\bar{\Delta}_H$  does not make the manager better off; it only ensures that his expected utility is the same from undertaking the risky project as it is from holding onto cash when  $\tilde{\phi} = \bar{\phi}^{\text{FB}}$  and  $\tilde{s} = 1$ . In other words, the increase in  $\bar{\Delta}_H$  only serves to remove the effect of the manager's risk aversion, and so the same base compensation is still required to retain his services. Of course, increasing  $\bar{\Delta}_H$  as its manager becomes more risk-averse is costly to the firm, and this leads to a reduction in firm value.

The last part of Proposition 4.2 shows that an easy solution to this problem exists when managers are not fully rational. In particular, it is easy to verify that  $\bar{\Delta}_H$  decreases with  $\delta$ : overconfident managers do not require as large of a reward for taking on risk, as they irrationally think that they

control these risks. Convincing the manager to undertake all positive net present value projects is therefore easier and cheaper. Retaining this manager, however, does require that more compensation be offered to him in safer states of the world. Indeed,  $\bar{\Delta}_M$  increases with  $\delta$ . Such compensation is valued more by the risk-averse manager, and so proves to be cheaper for the firm. As a result, the manager is equally well-off (his expected utility is  $\bar{u}$  in all cases), but the firm saves money and is worth more. Of course, at the time that he joins the firm, the manager expects more than  $\bar{u}$  in utility but, given that he only requires  $\bar{u}$  to accept the contract, this is “free money” to him.

An alternative way to look at this result is to think of the firm’s tradeoff between the cost of retaining the manager’s services in the long run and the cost of creating an incentive for him to take on the right risks for the firm. For example, the contract  $\{\Delta_M, \Delta_H\} = \{\bar{u}, 0\}$  would satisfy the retention constraint but would create no incentive for the manager to take risks; indeed, the manager would then simply avoid any kind of risk and collect  $\bar{u}$  for sure. Unfortunately for the firm, this is the cheapest contract that satisfies the retention constraint. Any other contract giving the manager at least  $\bar{u}$  in expected utility requires the firm to set  $\Delta_H$  to a value greater than zero and makes the expected cost of compensation higher for the firm. To see this, suppose that  $\Delta_M$  is reduced to a value below  $\bar{u}$ . To retain the manager, the firm must now offer him extra compensation in the high state ( $\Delta_H > 0$ ), which creates an incentive for him to undertake risky projects with high enough  $\tilde{\phi}$  after a positive signal. If we denote the (endogenous) probability of the low, medium and high states by  $p_L$ ,  $p_M$  and  $p_H$  respectively, the resulting contract must satisfy

$$p_L(0) + p_M\Delta_M + p_H[\Delta_M + (1-r)\Delta_H] = \bar{u}. \quad (11)$$

This contract costs the firm

$$p_L(0) + p_M\Delta_M + p_H(\Delta_M + \Delta_H) = \bar{u} + rp_H\Delta_H, \quad (12)$$

which is greater than  $\bar{u}$ . The last term in (12) represents the cost of realigning the manager’s incentives with the shareholders’. Because it raises the contract’s total cost, it also reduces firm value.

Notice that realigning the manager’s incentives with those of the firm’s shareholders fixes  $p_H$ , as the first-best threshold for  $\tilde{\phi}$  will endogenously correspond to a certain probability,  $p_H^{\text{FB}}$  say, of undertaking a successful project.<sup>9</sup> We see from (12), therefore, that the amount of extra compensation  $\Delta_H$  necessary to realign the manager’s incentives is the key determinant of firm value. In

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<sup>9</sup>As shown in the proof of Proposition 4.2, this probability is given by  $p_H^{\text{FB}} = \frac{1}{8}a(1+a)(2-a)$ . In fact, the same proof shows that  $p_M^{\text{FB}} = 1 - \frac{a(2-a^2)}{4}$  and  $p_L^{\text{FB}} = \frac{1}{8}a(1-a)(2+a)$ .

particular, realigning the manager with as small a  $\Delta_H$  as possible is critical. This, it turns out, is precisely what managerial overconfidence does. Indeed, Proposition 4.1 shows that less extra compensation is needed in good states to realign the incentives of an overconfident manager. In other words, a manager's overconfidence allows the firm to compensate him with a flatter schedule, which is cheaper for the firm. Also, because the retention constraint is satisfied, the manager is equally well-off. In effect, therefore, the manager's overconfidence reduces the size of the dead-weight loss that his risk aversion creates.

### 4.3 Value Maximization

The fact that the manager's incentives can be realigned perfectly with those of the shareholders is important, but not necessarily optimal for the shareholders. After all, the shareholders seek to maximize firm value *after* taking into account the compensation that is paid to the firm's employees. In this principal-agent framework therefore, it may not be optimal for the firm to perfectly realign the incentives of the manager (i.e., to make sure that he will pick the first-best threshold for  $\tilde{\phi}$ ), as this might prove too costly. In other words, when choosing the manager's compensation, the firm must determine whether the cost of creating further incentives towards first-best are worth the extra revenues that these incentives generate.

This section tackles this issue by looking at the firm's implicit choice of  $\bar{\phi}$ , the  $\tilde{\phi}$ -threshold above which the manager's signal determines his capital budgeting decision, through its choice of compensation for the manager. As before, we assume that the firm must meet the manager's retention constraint, that is, the manager's expected utility must be at least  $\bar{u}$ . Let us denote the compensation package that satisfies the firm's retention constraint and makes the manager choose a given  $\bar{\phi}$  by  $\{\Delta_M(\bar{\phi}), \Delta_H(\bar{\phi})\}$ . It is easy to show that  $\Delta_M(\bar{\phi})$  is decreasing in  $r$  and increasing in  $\delta$ , while  $\Delta_H(\bar{\phi})$  is increasing in  $r$  and decreasing in  $\delta$ . In other words, the result that overconfidence allows for a flattening of the manager's compensation schedule rendered steep by his risk aversion holds for any choice of  $\bar{\phi}$ , not just  $\bar{\phi} = \bar{\phi}^{\text{FB}}$  as in section 4.2.

As such, we can use the same arguments as in (11) and (12) to conclude that the expected cost of the retention-compatible contract that makes the manager choose any  $\bar{\phi}$  is decreasing in  $\delta$ . This, as the following proposition shows, implies that the firm can afford to get closer to first-best decision-making when setting its compensation for the overconfident manager, and this results in a larger firm value.

**Proposition 4.3** *The maximized value of the firm, subject to a retention constraint of  $\bar{u}$  with its manager, is increasing in  $\delta$ .*

This result is illustrated in Figure 4.3, which shows the value of the firm as a function of the threshold  $\bar{\phi}$  that it implicitly sets through the manager's compensation contract. In this figure, the continuous line shows that firm value is maximized with a threshold of  $\bar{\phi} = \bar{\phi}^{\text{FB}}$  with a manager who is risk-neutral and rational. In that case, it is easy to show that the optimal contract is a stock contract (i.e.,  $\Delta_{\text{M}} = \Delta_{\text{H}}$ ) and firm value is equal to  $\frac{1}{2} + \frac{a^2}{8} - \bar{u}$ . When the manager is risk-averse, however, the extra compensation offered by the firm in the high state is not valued as much by the manager, and so more of it is necessary to create an incentive for the manager to make the same decisions as the risk-neutral manager. Because this is expensive, the firm scales back its effort to realign the manager perfectly and ends up with a compensation that makes the manager choose  $\bar{\phi} = \bar{\phi}'$  which is higher than  $\bar{\phi}^{\text{FB}}$ , as illustrated by the dotted line in Figure 4.3. In other words, the gains from having the manager consider projects of types  $\tilde{\phi} \in (\bar{\phi}^{\text{FB}}, \bar{\phi}']$  are not worth the cost of creating incentives for him to do so. This problem is alleviated when the same risk-averse manager is also overconfident. Because his overconfidence makes it possible for the firm to attain any  $\bar{\phi}$  with a flatter and cheaper compensation contract, the firm can afford to reach threshold levels that are closer to first-best. The dashed line in Figure 4.3 shows how this also increases the value of the firm.

As in the previous section, it is important to remember that the gain in firm value does not come at the expense of the manager, i.e., it is not the case that the manager's overconfidence effectively subsidizes the firm. Because the retention constraint is satisfied, the manager's expected utility is  $\bar{u}$ . Because of his overconfidence, of course, the manager's ex ante expected utility is more than  $\bar{u}$ , but this extra utility that he mistakenly perceives does not hurt him.

#### 4.4 Competitive Labor Markets

In our analysis so far, the manager is assumed to have exogenous opportunities outside the firm that are worth  $\bar{u}$  in expected utility. In this section, we endogenize these opportunities by considering a competitive labor market. In such a market, firms compete for the services of a skilled manager, who is then able to capture some firm surplus. Effectively, this also endogenizes the manager's reservation utility, which is then given by the utility he can expect from working for a competing firm.

Let us assume that two firms are competing to hire one manager, who chooses which firm to work for based on the utility he expects to receive from their compensation contract.<sup>10</sup> We assume that these two firms are identical to the one firm that we have modelled so far. Also, as before,

<sup>10</sup>We assume that he picks a firm randomly if his expected utility is the same for both.

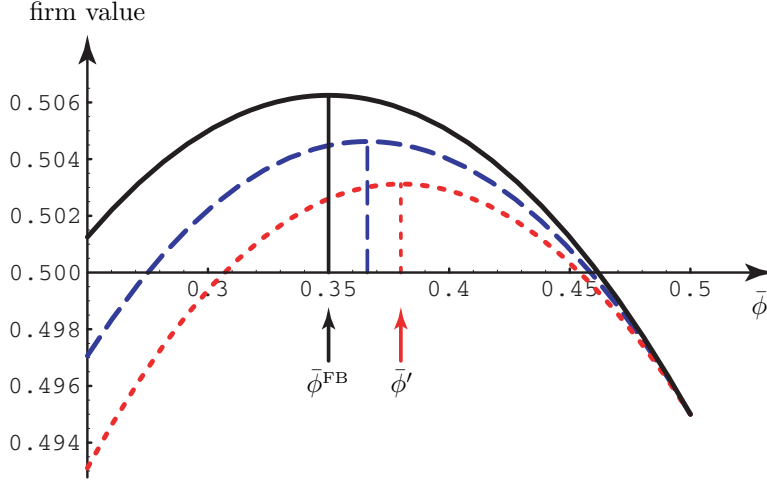


Figure 1: In this figure, the continuous line shows the maximization problem of the firm assuming that its manager is risk-neutral and rational. In that case, the  $\tilde{\phi}$ -threshold that is optimally chosen is the one that yields first-best, i.e.,  $\tilde{\phi} = \bar{\phi}^{\text{FB}}$ . When the rational manager is risk-averse, the value of the firm is maximized with a threshold  $\tilde{\phi}'$ , which is larger than  $\bar{\phi}^{\text{FB}}$ , as illustrated by the dotted line. The resulting value of the firm is lower. As this risk-averse manager becomes overconfident, however, the manager starts making decisions that are more naturally aligned with the value-maximizing objective of the firm. As shown by the dashed line, the resulting threshold for  $\tilde{\phi}$  is somewhere between  $\bar{\phi}^{\text{FB}}$  and  $\tilde{\phi}'$ , and the value of the firm increases. This figure was generated using the following parameter values:  $a = 0.3$ ,  $r = 0.9$ ,  $\delta = 0.3$ ,  $\bar{u} = 0.005$ .

the manager's ability, risk aversion and overconfidence are given by  $a$ ,  $r$  and  $\delta$ . For simplicity, we assume that the firm that fails to hire this one manager must operate with a skill-less manager ( $a = 0$ ) or without one altogether. Thus the firm without the skilled manager cannot generate any value from risky projects, and so is worth  $\frac{1}{2}$ . This assumption captures the idea that managerial skill is a scarce resource, and this is what allows the manager to capture the surplus that he creates for the firm.<sup>11</sup>

Because the two firms compete for the manager's services, it will be the case that, in equilibrium, they will both be worth  $\frac{1}{2}$  whether or not they are successful in hiring the manager. The manager's presence increases the firm's revenues from  $\frac{1}{2}$  to  $\frac{1}{2} + (p_H - p_L) \frac{1}{2}$ , as the firm gets an extra  $\frac{1}{2}$  in revenues in the high state but loses  $\frac{1}{2}$  in revenues in the low state. Thus, in equilibrium, the compensation contract  $\{\Delta_M, \Delta_H\}$  that the manager will end up accepting must satisfy

$$p_M \Delta_M + p_H (\Delta_M + \Delta_H) = (p_H - p_L) \frac{1}{2}, \quad (13)$$

<sup>11</sup> Assuming that firms pick managers from a pool of individuals with different skills would generate similar results as long as the highest-skill managers are fewer (or have a smaller measure, if one assumes an infinite number of firms) than firms.

where the left-hand side of this last equation shows the compensation that the firm expects to pay the manager. Indeed, any compensation that leaves a firm with some surplus will be improved upon by the other firm until all surplus disappears.

Of course, the probabilities  $p_L$ ,  $p_M$  and  $p_H$  in (13) are endogenous. They depend on the threshold  $\bar{\phi}$  that the manager will set for the firm, which in turn depends on the contract that the manager ultimately receives, as well as his personal characteristics,  $a$ ,  $r$  and  $\delta$ . Still, (13) restricts the set of contracts that will be received by the manager in equilibrium. Let us denote this set by  $\Pi_0$ . As the following lemma shows, the one contract that the manager will end up receiving out of this zero-profit set is the contract that maximizes his (biased) expected utility.

**Lemma 4.1** *The contract  $\{\Delta_M^C, \Delta_H^C\} \in \Pi_0$  that the manager receives in equilibrium is such that his (biased) expected utility cannot be improved with any other contract in  $\Pi_0$ .*

This result is intuitive. Suppose that the equilibrium contract is  $\{\Delta'_M, \Delta'_H\} \in \Pi_0$ . Suppose also that there is a contract  $\{\Delta^*_M, \Delta^*_H\} \in \Pi_0$  that gives the manager more expected utility, and let us denote by  $\bar{\phi}^*$  the  $\bar{\phi}$ -threshold chosen by the manager with this contract. Then one of the competing firms can offer a contract,  $\{\Delta''_M, \Delta''_H\}$ , that is arbitrarily close to  $\{\Delta^*_M, \Delta^*_H\}$ , but offers less compensation to the manager in both the medium and high states. If the reduction from  $\{\Delta^*_M, \Delta^*_H\}$  to  $\{\Delta''_M, \Delta''_H\}$  is small enough, the manager is better off with this new contract than with  $\{\Delta'_M, \Delta'_H\}$ . Also, if the reduction is done in such a way that it does not affect the manager's choice of  $\bar{\phi}^*$ , the firm's value rises above  $\frac{1}{2}$ , as the firm's revenues are unaffected but the manager's compensation becomes cheaper. Thus  $\{\Delta'_M, \Delta'_H\}$  is clearly not an equilibrium contract.

Let us fix the manager's ability ( $a$ ) and risk aversion ( $r$ ). Because the compensation contract that the manager receives in equilibrium will be affected by his overconfidence ( $\delta$ ), we denote this equilibrium contract by  $\{\Delta_M^C(\delta), \Delta_H^C(\delta)\}$  to emphasize its dependence on  $\delta$ . The following proposition, part of which could only be established numerically,<sup>12</sup> shows how this contract is affected by the manager's overconfidence, and how in turn this affects the manager's welfare.

**Proposition 4.4** *For any manager with risk aversion  $r \in (0, 1)$ , the equilibrium contract in the competitive labor market is such that  $\Delta_M^C(\delta)$  increases and  $\Delta_H^C(\delta)$  decreases as  $\delta$  increases from zero. The equilibrium expected utility of the manager, calculated from his biased perspective or from a rational perspective, increases as  $\delta$  increases from zero.*

<sup>12</sup>See proof for an outline of the procedure.

As in previous sections, the manager ends up receiving a flatter compensation contract when he is overconfident. The reason is different, however. In this case, it is the manager who effectively chooses his own compensation from a menu of contracts that leave the two firms with no net increase in value. His overconfidence, therefore, commits him to a compensation contract with a higher base salary and less performance-related compensation. The reason is that the manager thinks that he can get whatever extra compensation that the high state offers with a higher probability than he really can. As a result, the firms seek to attract him with a higher base salary which, because of the manager's risk aversion, he values more and is cheaper to the firm. Inadvertently, this increases the surplus that the firms can afford for his services. Not only does this increase the compensation that the manager can expect to receive but, because that compensation is more guaranteed (i.e., received in more states of the world), it makes him better off. Importantly, this improvement in manager welfare is not due to the fact that firms take advantage of his irrationality. Indeed, although the manager's bias makes him think that he is better off than he really is, it is also the case that the utility that he will experience on average will be higher than it would otherwise be if he were rational.

Of course, in a competitive labor market, the presence of overconfidence does not affect firm values, as any equilibrium contract leaves every firm's value unchanged at  $\frac{1}{2}$ . However, it will be the case, and it is easy to show, that firms with an overconfident manager will undertake more projects, generate more revenues and pay more in average compensation to the manager. Thus one could empirically detect top-management overconfidence from firm investments in real assets and average compensation paid to top managers.

## 5 Additional Considerations

### 5.1 Costly Effort

So far in this paper, we have assumed that the only agency cost present in the relationship between the firm's manager and its owners is due to the manager's risk aversion. In other words, compensation contracts only serve to realign the manager's incentives to take risks with those of shareholders. In reality, it is likely that other agency costs arise from the shareholder-manager relationship, and that compensation is used to reduce these as well. One agency cost that is likely to play an important role results from the moral hazard problem that is inherent in this principal-agent relationship.

In particular, although we have assumed so far that the information  $\tilde{s}$  that the manager receives

about  $\tilde{v}$  comes to him for free, it is likely that the acquisition of this information requires a costly effort on his part. Let us assume that the (additive) utility cost of this effort is  $e > 0$  for the manager. For a given compensation contract, this changes the manager's problem after he observes the project's type  $\tilde{\phi}$ . Indeed, the manager must now weight the benefit of acquiring a signal about  $\tilde{v}$  against the cost of doing so. For example, when  $\tilde{\phi}$  is slightly above the  $\bar{\phi}$  chosen by the manager in the absence of moral hazard, it will be the case that the manager will elect not to gather information if this requires a costly effort on his part. The firm then loses opportunities that are potentially valuable. We start our analysis of this problem with a lemma that characterizes the manager's information-gathering choices for a given compensation contract  $\{\Delta_M, \Delta_H\}$ .

**Lemma 5.1** *Suppose that the firm hires a manager with ability  $a$ , risk aversion  $r$ , overconfidence  $\delta$ , and effort cost  $e$ . As long as*

$$e < \frac{(a + \delta)(1 - r)\Delta_M\Delta_H}{\Delta_M + (1 - r)\Delta_H}, \quad (14)$$

*this manager gathers information if and only if the project's type  $\tilde{\phi}$  satisfies  $\bar{\phi}_e(\delta) < \tilde{\phi} < \underline{\phi}_e(\delta)$ , where*

$$\bar{\phi}_e(\delta) \equiv \frac{(1 - a - \delta)\Delta_M + 2e}{(1 + a + \delta)(1 - r)\Delta_H + (1 - a - \delta)\Delta_M}, \quad (15)$$

$$\underline{\phi}_e(\delta) \equiv \frac{(1 + a + \delta)\Delta_M - 2e}{(1 - a - \delta)(1 - r)\Delta_H + (1 + a + \delta)\Delta_M}. \quad (16)$$

*If (14) is not satisfied, then the manager never gathers any information.*

Clearly, since  $\bar{\phi}_e(\delta)$  is increasing in  $e$  and  $\underline{\phi}_e(\delta)$  is decreasing in  $e$ , the range for (and frequency with) which the manager gathers information shrinks as his effort cost goes up. Also,  $\bar{\phi}_e(\delta)$  is decreasing in  $\delta$  and  $\underline{\phi}_e(\delta)$  is increasing in  $\delta$ : the overconfident manager is more likely to exert the effort necessary to acquire information than his rational counterpart. In that sense, overconfidence commits the manager to a higher level of effort. This is because the overconfident manager overestimates the value of his information, and so is less reluctant to “invest some utility” into gathering it: for him, the effort cost appears small relative to the gain he can expect from the information he gathers.

As in Proposition 4.1, realigning the incentives of the manager with those of the shareholders requires finding the relationship between  $\Delta_M$  and  $\Delta_H$  that makes the manager choose  $\bar{\phi}_e(\delta) = \bar{\phi}^{\text{FB}}$ . Using (15) and the fact that  $\bar{\phi}^{\text{FB}} = \frac{1-a}{2}$ , it is easy to show that the resulting contract must satisfy

$$\Delta_H = \frac{4e + (1 + a)(1 - a - \delta)\Delta_M}{(1 - r)(1 - a)(1 + a + \delta)}. \quad (17)$$

Because the reward for effort must be increased as  $e$  increases, the resulting contract can inadvertently create undesirable incentives for the manager. More precisely, it is possible for  $\Delta_H$  to reach a level such that, for the manager, undertaking some projects without any effort or information becomes optimal. Indeed, when  $\Delta_H$  is so large relative to  $\Delta_M$ , the manager will sometimes prefer not to exert any effort and to undertake the risky project. This can be seen from Lemma 5.1: when  $\underline{\phi}_e(\delta)$  is lower than  $\frac{1}{2}$ , the manager does not exert any information-gathering effort after he observes  $\tilde{\phi} \in [\underline{\phi}_e(\delta), \frac{1}{2}]$ . Thus, when the relationship in (17) between  $\Delta_H$  and  $\Delta_M$  that is necessary for first-best realignment leaves  $\underline{\phi}_e(\delta)$  smaller than  $\frac{1}{2}$ , first-best realignment is simply not possible. As the following proposition shows, however, the range of effort costs for which first-best realignment is possible increases with overconfidence.

**Proposition 5.1** *There is a contract  $\{\Delta_M, \Delta_H\}$  that realigns the overconfident manager's incentives with the shareholders' as long as*

$$e \leq \frac{a(1-a^2) + \delta(2-2a^2-a\delta)}{2(2-a-a^2-a\delta)} \Delta_M. \quad (18)$$

It is straightforward to show that the right-hand side of (18) is increasing in  $\delta$ . So, once again, the presence of an overconfident manager is beneficial as it is possible for the firm to realign his incentives for larger effort costs. Thus, with moral hazard, there is one more reason why managerial overconfidence can be useful: even when the firm offers a compensation package that includes a large bonus in good states, the overconfident manager is less likely to succumb to the temptation of blindly undertaking risky projects without learning more about them (i.e., without exerting effort). In other words, because the overconfident manager not only overweighs positive information but also negative information, he is more likely to acquire information when the ex ante prospects of a risky project are good for him, given his compensation contract.

## 5.2 Optimism vs. Overconfidence

An overconfident manager puts too much weight on his information. As a result, he becomes positively biased about the prospects of the risky project after he receives positive information ( $\tilde{s} = 1$ ) about it, and negatively biased after he receives negative information ( $\tilde{s} = 0$ ). An *optimistic* manager, on the other hand, views all projects as better than they really are, even before receiving any information about them. As in Malmendier and Tate (2003) and Heaton (2002), we take optimism to describe the positive ex ante bias of the manager about his firm's projects. More precisely, we assume that the manager, upon learning the project's type  $\tilde{\phi}$ , thinks that the probability of a good

outcome for the risky project ( $\tilde{v} = 1$ ) is not  $\tilde{\phi}$ , but  $\tilde{\phi} + \gamma$ , where  $\gamma \in [0, \frac{1}{2}]$  measures the degree of optimism.

As before, let us assume that the manager is characterized by an ability of  $a$  and a risk aversion of  $r$ . Let us also assume that, instead of being overconfident, this manager is optimistic at a level of  $\gamma$ .<sup>13</sup> Let us also keep the assumption of section 5.1 that the manager incurs a utility cost of  $e$  when exerting effort to gather an informative signal  $\tilde{s}$  about  $\tilde{v}$ . The optimistic manager's use of information is rational in the sense that he correctly revises his beliefs upwards (downwards) after a positive (negative) signal. In fact, his posterior beliefs are given by (3) and (4) after  $\phi$  is replaced by  $\phi + \gamma$ . Because the manager starts with a positively biased view of the project, however, the resulting posteriors are always higher than they should be. Our first result about optimism is the analogue to Lemma 5.1.

**Lemma 5.2** *Suppose that the firm hires a manager with ability  $a$ , risk aversion  $r$ , optimism  $\gamma$ , and effort cost  $e$ . As long as*

$$e < \frac{a(1-r)\Delta_M\Delta_H}{\Delta_M + (1-r)\Delta_H}, \quad (19)$$

*this manager gathers information if and only if the project's type  $\tilde{\phi}$  satisfies  $\bar{\phi}_e(\gamma) < \tilde{\phi} < \underline{\phi}_e(\gamma)$ , where*

$$\bar{\phi}_e(\gamma) \equiv \frac{(1-a)\Delta_M + 2e}{(1+a)(1-r)\Delta_H + (1-a)\Delta_M} - \gamma, \quad (20)$$

$$\underline{\phi}_e(\gamma) \equiv \frac{(1+a)\Delta_M - 2e}{(1-a)(1-r)\Delta_H + (1+a)\Delta_M} - \gamma. \quad (21)$$

*If (19) is not satisfied, then the manager never gathers any information.*

The effects of optimism on the information-gathering incentives of the manager are not as clear as those of overconfidence. Although  $\bar{\phi}_e(\gamma)$  is decreasing in the manager's bias  $\gamma$ , it is also the case that  $\underline{\phi}_e(\gamma)$  is decreasing in  $\gamma$ . The first of these effects means that the manager's optimism does make him gather information for lower project types than he otherwise would if he were rational. Indeed, when the manager is risk-averse and his effort is costly, he tends to reject potentially valuable projects without gathering information about them (this can be seen from the fact that  $\bar{\phi}_e(\gamma)$  is increasing in  $r$  and  $e$ ). Optimism about risky projects can negate these effects by making the manager think twice before rejecting a project. However, this comes at a cost, captured by the

<sup>13</sup>In the vernacular, "overconfidence" implies both excessive certainty and excessive optimism. To facilitate analysis and emphasize the different roles that each trait has on corporate decision-making, we chose to model these aspects of overconfidence separately.

second effect. As his bias increases, the same manager no longer gathers information for project types close to  $\frac{1}{2}$ . Instead of increasing the range of project types for which the manager gathers information, optimism simply shifts that range.

As a result, perfectly realigning the manager's incentives with those of shareholders proves to be more difficult when the manager is optimistic than when he is overconfident. Indeed, setting the manager's compensation contract  $\{\Delta_M, \Delta_H\}$  in such a way that  $\bar{\phi}_e(\gamma) = \bar{\phi}^{\text{FB}}$  often results in him undertaking projects of type  $\tilde{\phi}$  close to  $\frac{1}{2}$  without gathering any information about them. That is, although optimism makes him work on projects of lower types, it also naturally creates an incentive for him to shirk on project of higher types. The following proposition shows that, as a result, perfect realignment of the manager's incentives becomes more difficult as his optimism increases. In fact, when  $\gamma$  is too large, perfect realignment is impossible, even if  $e$  is small.

**Proposition 5.2** *There is a contract  $\{\Delta_M, \Delta_H\}$  that realigns the optimistic manager's incentives with the shareholders' as long as*

$$e \leq \frac{a(1-a^2) + 4a\gamma(a-2\gamma)}{2(2-a-a^2+4\gamma)} \Delta_M. \quad (22)$$

Because the right-hand side of (22) decreases with  $\gamma$ , we see that optimism restricts the range of effort costs for which compensation contract can realign the incentives of the manager with those of shareholders. In fact, it is easy to show that the right-hand side of (22) becomes negative when

$$\gamma > \frac{1}{4} \left( a + \sqrt{2-a^2} \right). \quad (23)$$

In other words, when the manager's optimism is too large, no adjustment in his compensation contract yields first-best incentives. In fact, this is the case even in the absence of moral hazard problems, i.e., perfect realignment is then impossible even with  $e = 0$ .

## 6 Conclusion

Executive managers are likely to be overconfident. We examine how this bias may affect capital budgeting decisions, firm value, compensation, and manager welfare. To do so, we develop a simple capital budgeting problem in which a manager, using his information about the prospects of a risky project, must decide whether his firm should undertake this project or drop it in favor of a project with safe cash flows.

When decisions are made by a rational unbiased manager who is risk-averse (due to his inability to diversify firm specific human capital), some risky projects that are valuable to the firm may be

foregone by the manager unless the manager's compensation is properly adjusted by the firm's shareholders. More compensation must be offered to this manager in risky but positive states of the world for him to consider undertaking some risky projects with positive net present value. An overconfident manager, however, thinks that his information allows him to know the likely outcome of risky projects with better precision than is really the case. This manager is less reluctant to undertake risky projects. In effect, his overconfidence naturally realigns his incentives with those of shareholders and, as a result, less convexity is required in his compensation for him to make value-maximizing decisions.

Surprisingly, this failure to correctly maximize his own expected utility may not hurt the manager, even though it makes his firm more valuable. When the manager has opportunities outside the firm and the firm's owners seek to retain his services or attract him from other firms, the manager's overconfidence does not make him vulnerable to opportunistic behavior by the firm. Since risk-taking incentives can be created with less upside compensation for the manager, the firm can use its savings from the upside states to offer the manager more base compensation in its effort to keep or attract him. As we show, such a flatter compensation schedule can translate into more utility for the manager and at the same time more affordable compensation for the firm.

Our results on managerial overconfidence get even stronger when we add moral hazard to the incentive problems that the manager's compensation contract must resolve. Because the overconfident manager thinks that his effort to learn about projects yields larger reductions in risk than it actually does, little incentive compensation is required to make him exert that effort. In short, an overconfident manager naturally overcomes the obstacles that his risk aversion and costly effort bring about.

This is not necessarily the case for optimistic managers, who have positive ex ante biases about the prospects of their firm's projects. While their bias does make them more willing to take on risk and so helps realign their incentives with those of shareholders, exerting effort to gather information may be less attractive for these managers. This is because an optimistic manager is likely to think that the ex ante prospects of a project are so good that the benefits to learning more about the project are simply not worth the cost of the required effort. As a result, a firm with an optimistic manager may be limited in terms of the set of incentives it can create for its manager.

There are many reasons to expect CEOs and other top managers to be overconfident and optimistic. Not only are these traits prevalent in the population, but they are prevalent among experts.<sup>14</sup> Optimism is most severe among more intelligent individuals (Klaczynski and Fauth,

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<sup>14</sup>Clinical psychologists (Oskamp, 1965), physicians and nurses (Christensen-Szalanski and Bushyhead, 1981; Bau-

1996). Those who are overconfident and optimistic about their management abilities are more likely to pursue careers as managers. Risk-taking induced by overconfidence and optimism may, in a tournament type setting, result in promotions (Goel and Thakor, 2000). Moreover, successful CEOs may attribute too much of their success to their own abilities rather than other factors and thus become increasingly overconfident (Gervais and Odean, 2001).

While CEOs are likely to be overconfident and optimistic, the normative advice provided by academic finance to those who wish to align the decisions of managers with the desires of shareholders largely ignores these traits (e.g., Barnea, Haugen and Senbet, 1985; Smith and Stulz, 1985; Lambert, 1986; Hirshleifer and Suh, 1992; and Hemmer, Kim and Verrecchia, 2000). To mitigate agency concerns, corporate boards are advised to offer CEOs convex compensation packages (e.g., salary plus options) without any discussion of how CEO overconfidence and optimism may affect the need for convexity.

In our model, fewer options are needed to align the decisions of overconfident, optimistic managers with the interests of risk-neutral shareholders. When such managers are compensated as if they were rational, they are paid more than is optimal for the firm and, furthermore, the additional compensation may motivate them to take risks that are *not* in the best interests of shareholders. The inappropriateness of these costs and risks are exacerbated when shareholders are actually averse to firm specific risk. Average CEO, compensation as a multiple of average worker compensation rose from 45 in 1980, to 96 in 1990, and to 458 in 2000 (Sklar, Mykyta and Wefald, 2001), with 79 percent of the increase in median CEO compensation from 1992 through 2000 coming in the form of long-term incentives, primarily stock options.<sup>15</sup> Our analysis suggests that one reason for the dramatic growth in CEO compensation could be that corporate boards have heeded academic advice to compensate CEOs with options, but have not considered the implications of managerial optimism and overconfidence.

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mann, Deber and Thompson, 1991), investment bankers (Stäel von Holstein, 1972), engineers (Kidd, 1970), entrepreneurs (Cooper, Woo and Dunkelberg, 1988), lawyers (Wagenaar and Keren, 1986), negotiators (Neale and Bazerman, 1990), and managers (Russo and Schoemaker, 1992) have all been observed to exhibit overconfidence in their judgments.

<sup>15</sup>The Conference Board Commission on Public Trust and Private Enterprise, September 17, 2002.

## Appendix A

### Proof of Proposition 2.1

As explained in the paragraph preceding the proposition, the manager will only possibly choose to undertake the project when he receives a positive signal about it. When the manager does observe that  $\tilde{s} = 1$ , he must choose whether undertaking the risky project is preferable to holding onto cash, i.e., he must calculate whether the risky project has a positive net present value. Given (3), the project's net present value is  $\frac{(1+a)\phi}{1-a+2a\phi} - \frac{1}{2}$ , which is positive when  $\phi > \frac{1-a}{2} \equiv \bar{\phi}^{\text{FB}}$ . With this strategy and using the fact that  $\tilde{\phi}$  is uniformly distributed on  $[0, \frac{1}{2}]$ , the initial value of the firm is given by

$$F^{\text{FB}} = \frac{1}{2} + \int_{\bar{\phi}^{\text{FB}}}^{\frac{1}{2}} \Pr \left\{ \tilde{s} = 1 \mid \tilde{\phi} = \phi \right\} \left[ \frac{(1+a)\phi}{1-a+2a\phi} - \frac{1}{2} \right] 2 d\phi. \quad (24)$$

It is straightforward to show that  $\Pr \left\{ \tilde{s} = 1 \mid \tilde{\phi} = \phi \right\} = \frac{1-a+2a\phi}{2}$  and, as a result,  $F^{\text{FB}}$  can be shown to be equal to (5). ■

### Proof of Proposition 2.2

We can apply the exact same reasoning as in the proof of Proposition 2.1 with  $\bar{\phi}^{\text{RA}} = \frac{1-a}{2-r(1+a)}$  replacing  $\bar{\phi}^{\text{FB}}$ . In particular, the initial value of the firm is given by

$$\begin{aligned} F &= \frac{1}{2} + \int_{\bar{\phi}^{\text{RA}}}^{\frac{1}{2}} \Pr \left\{ \tilde{s} = 1 \mid \tilde{\phi} = \phi \right\} \left[ \frac{(1+a)\phi}{1-a+2a\phi} - \frac{1}{2} \right] 2 d\phi \\ &= F^{\text{FB}} - \int_{\bar{\phi}^{\text{FB}}}^{\bar{\phi}^{\text{RA}}} \Pr \left\{ \tilde{s} = 1 \mid \tilde{\phi} = \phi \right\} \left[ \frac{(1+a)\phi}{1-a+2a\phi} - \frac{1}{2} \right] 2 d\phi, \end{aligned}$$

and this last integral reduces to  $\frac{1}{8} \left[ \frac{(1-a^2)r}{2-r(1+a)} \right]^2$  after straightforward manipulations. ■

### Proof of Proposition 3.1

The  $\tilde{\phi}$ -threshold chosen by the overconfident manager,  $\bar{\phi}^{\text{OV}}$ , is found by replacing  $a$  by  $a + \delta$  in  $\bar{\phi}^{\text{RA}}$ , that is,  $\bar{\phi}^{\text{OV}} = \frac{1-a-\delta}{2-r(1+a+\delta)}$ . This threshold is equal to the first-best threshold,  $\bar{\phi}^{\text{OV}} = \frac{1-a}{2}$ , when  $\delta$  is given by  $\delta^{\text{FB}}$  in (7). Using the same reasoning as in the proof of Proposition 2.1, the value of the firm, when led by the overconfident manager, is given by

$$F = F^{\text{FB}} - \int_{\bar{\phi}^{\text{FB}}}^{\bar{\phi}^{\text{OV}}} \Pr \left\{ \tilde{s} = 1 \mid \tilde{\phi} = \phi \right\} \left[ \frac{(1+a)\phi}{1-a+2a\phi} - \frac{1}{2} \right] 2 d\phi. \quad (25)$$

It is easy to verify that  $\bar{\phi}^{\text{OV}}$  is decreasing in  $\delta$ . Since  $\frac{(1+a)\phi}{1-a+2a\phi} - \frac{1}{2} > 0$  for  $\phi > \bar{\phi}^{\text{FB}}$ , this implies that this last integral gets smaller, and thus  $F$  gets larger, as  $\delta$  increases towards  $\delta^{\text{FB}}$ . Similarly, because  $\frac{(1+a)\phi}{1-a+2a\phi} - \frac{1}{2} < 0$  and  $\bar{\phi}^{\text{FB}} > \bar{\phi}^{\text{OV}}$  for  $\phi < \bar{\phi}^{\text{FB}}$ , the integral in (25) increases, and thus  $F$  decreases, as  $\delta$  is increased past  $\delta^{\text{FB}}$ . ■

### Proof of Proposition 3.2

We know from Proposition 3.1 that a manager with ability  $a$  and overconfidence  $\delta^{\text{FB}}$  restores the first-best outcome, that is a firm value of  $F^{\text{FB}}$ , as derived in Proposition 2.1. So a firm value exceeding  $F$  can be obtained with a manager of ability  $a'$  as long as

$$\frac{1}{2} + \frac{(a')^2}{8} > F,$$

or equivalently, as long as

$$a' > 2\sqrt{2F - 1} \equiv \bar{a}.$$

This completes the proof. ■

### Proof of Proposition 4.1

Suppose that the manager has observed that the project's type is  $\tilde{\phi} = \phi$ , and that he has received positive information ( $\tilde{s} = 1$ ) about this project. If the manager chooses not to undertake the project and hold onto cash, he receives  $\Delta_{\text{M}}$ . Conditional on observing  $\tilde{s} = 1$ , the manager thinks that the probability that the project will be successful is

$$\phi_1^{\text{OV}} \equiv \frac{(1+a+\delta)\phi}{1-(a+\delta)(1-2\phi)}, \quad (26)$$

which is obtained by replacing  $a$  by  $a+\delta$  in (1). Thus the manager's (biased) expected utility from undertaking the risky project is

$$\phi_1^{\text{OV}} [\Delta_{\text{M}} + (1-r)\Delta_{\text{H}}] = \frac{(1+a+\delta)\phi}{1-(a+\delta)(1-2\phi)} [\Delta_{\text{M}} + (1-r)\Delta_{\text{H}}],$$

which is greater than  $\Delta_{\text{M}}$  if and only if

$$\phi > \frac{(1-a-\delta)\Delta_{\text{M}}}{(1-a-\delta)\Delta_{\text{M}} + (1+a+\delta)(1-r)\Delta_{\text{H}}}. \quad (27)$$

We know from Proposition 2.1 that first-best requires the manager to choose a  $\tilde{\phi}$ -threshold equal to  $\frac{1-a}{2}$ . The right-hand side of (27) is equal to this first-best threshold when (8) is satisfied. Because first-best also requires every negative-information project to be dropped, we need to ensure that

this compensation does not create an incentive for the manager to undertake the risky project after he observes  $\tilde{s} = 0$ . When the manager does receive  $\tilde{s} = 0$  for a project of type  $\tilde{\phi} = \phi$ , he thinks that the probability of success is

$$\phi_0^{\text{OV}} \equiv \frac{(1-a-\delta)\phi}{1+(a+\delta)(1-2\phi)}, \quad (28)$$

which is obtained by replacing  $a$  by  $a + \delta$  in (2). The manager's expected utility from undertaking this project is  $\phi_0^{\text{OV}} [\Delta_M + (1-r)\Delta_H]$ , which simplifies to

$$\frac{2\phi(1-a-\delta)[(1-a)(1+a)-a\delta]\Delta_M}{(1-a)(1+a+\delta)[1+(a+\delta)(1-2\phi)]}$$

after replacing  $\Delta_H$  and  $\phi_0^{\text{OV}}$  by their expressions in (8) and (28) respectively. Since the project of type  $\tilde{\phi} = \frac{1}{2}$  is the most appealing, even after  $\tilde{s} = 0$  is observed, it is sufficient to verify that this project is not undertaken by the manager following a negative signal, that is, it is sufficient to verify that

$$\frac{(1-a-\delta)[(1-a)(1+a)-a\delta]\Delta_M}{(1-a)(1+a+\delta)} < \Delta_M.$$

This follows immediately from the observation that  $(1-a) > (1-a-\delta)$  and  $(1+a+\delta) > (1-a^2-a\delta) = (1-a)(1+a) - a\delta$ . ■

## Proof of Proposition 4.2

As long as he sticks to riskfree cash, the manager is sure of receiving  $\Delta_M$ . When his compensation perfectly realigns his incentives with those of shareholders, we know from Proposition 2.1 that the manager then undertakes the risky project if and only if  $\tilde{\phi} > \frac{1-a}{2}$  and  $\tilde{s} = 1$ . In that event, the manager receives additional utility of  $(1-r)\Delta_H$  if the project is successful (i.e., with probability  $\phi_1$ ), but gives up  $\Delta_M$  if it fails (i.e., with probability  $1 - \phi_1$ ). Therefore, the manager's expected utility is given by

$$\begin{aligned} & \Delta_M + \int_{\frac{1-a}{2}}^{\frac{1}{2}} \Pr \left\{ \tilde{s} = 1 \mid \tilde{\phi} = \phi \right\} [\phi_1(1-r)\Delta_H - (1-\phi_1)\Delta_M] 2 d\phi \\ &= \Delta_M + \int_{\frac{1-a}{2}}^{\frac{1}{2}} \frac{1-a+2a\phi}{2} \left[ \frac{(1+a)\phi}{1-a+2a\phi} (1-r)\Delta_H - \frac{(1-a)(1-\phi)}{1-a+2a\phi} \Delta_M \right] 2 d\phi \\ &= \Delta_M + \int_{\frac{1-a}{2}}^{\frac{1}{2}} \left[ (1+a)\phi(1-r)\Delta_H - (1-a)(1-\phi)\Delta_M \right] d\phi, \end{aligned}$$

which, after some manipulations, reduces to

$$\Delta_M + \frac{a}{8}(1+a)(2-a)(1-r)\Delta_H - \frac{a}{8}(1-a)(2+a)\Delta_M.$$

From Proposition 4.1, we know that the relationship between  $\Delta_H$  and  $\Delta_M$  must satisfy (8), and so this last expression, after more manipulations, further reduces to

$$\frac{\Delta_M}{4(1-a)(1+a+\delta)} \left[ (1-a^2)(4+a^2) + 2\delta(2-3a) \right].$$

The retention constraint is satisfied when this last quantity is equal to  $\bar{u}$ , that is, when (9) is satisfied. Finally, (10) is obtained by applying (8) on (9).

Under a first-best investment policy, the probability that the risky project is undertaken and is successful is given by

$$p_H^{\text{FB}} = \int_{\bar{\phi}^{\text{FB}}}^{\frac{1}{2}} \Pr \left\{ \tilde{s} = 1 \mid \tilde{\phi} = \phi \right\} \phi_1 2 d\phi = \int_{\frac{1-a}{2}}^{\frac{1}{2}} (1+a)\phi d\phi = \frac{1}{8}a(1+a)(2-a), \quad (29)$$

the probability that the risky project is undertaken and fails is given by

$$p_L^{\text{FB}} = \int_{\bar{\phi}^{\text{FB}}}^{\frac{1}{2}} \Pr \left\{ \tilde{s} = 1 \mid \tilde{\phi} = \phi \right\} (1-\phi_1) 2 d\phi = \int_{\frac{1-a}{2}}^{\frac{1}{2}} (1-a)(1-\phi) d\phi = \frac{1}{8}a(1-a)(2+a), \quad (30)$$

and the probability that the risky project is not undertaken at all is  $p_M^{\text{FB}} = 1 - p_H^{\text{FB}} - p_L^{\text{FB}} = 1 - \frac{a(2-a^2)}{4}$ .

Thus the compensation that the firm expects to pay its manager is

$$\bar{C} = p_M^{\text{FB}} \bar{\Delta}_M + p_H^{\text{FB}} (\bar{\Delta}_M + \bar{\Delta}_H) = (1 - p_L^{\text{FB}}) \bar{\Delta}_M + p_H^{\text{FB}} \bar{\Delta}_H. \quad (31)$$

Given that the first-best revenues are always equal to  $\frac{1}{2} + \frac{a^2}{8}$ , as calculated in Proposition 2.1, the value of the firm is decreasing in  $r$  and increasing in  $\delta$  if the expected compensation to the manager is increasing in  $r$  and decreasing in  $\delta$ . Notice that only  $\bar{\Delta}_H$  depends on  $r$  in (31) and, from (10),  $\bar{\Delta}_H$  is clearly increasing in  $r$ . Thus the manager's expected compensation is increasing in  $r$ . Similarly, only  $\bar{\Delta}_M$  and  $\bar{\Delta}_H$  depend on  $\delta$  in (31). From (9) and (10), it is straightforward to show that

$$\frac{\partial \bar{\Delta}_M}{\partial \delta} = \frac{4a(1-a^2)(1+a)(2-a)\bar{u}}{[(1-a^2)(4+a^2) + 2\delta(2-3a)]^2}$$

and

$$\frac{\partial \bar{\Delta}_H}{\partial \delta} = \frac{-4(1-a^2)(8-2a+a^2+a^3)\bar{u}}{[(1-a^2)(4+a^2) + 2\delta(2-3a)]^2(1-r)}.$$

Using these two expressions along with (29) and (30) in (31), we have

$$\begin{aligned} \frac{\partial \bar{C}}{\partial \delta} &= \frac{4a(1-a^2)(1+a)(2-a)\bar{u}}{[(1-a^2)(4+a^2) + 2\delta(2-3a)]^2} \left[ 1 - \frac{1}{8}a(1-a)(2+a) - \frac{8-2a+a^2+a^3}{8(1-r)} \right] \\ &< \frac{4a(1-a^2)(1+a)(2-a)\bar{u}}{[(1-a^2)(4+a^2) + 2\delta(2-3a)]^2} \left[ 1 - \frac{1}{8}a(1-a)(2+a) - \frac{8-2a+a^2+a^3}{8} \right] = 0. \end{aligned}$$

This completes the proof. ■

### Proof of Proposition 4.3

Let us denote by  $p_L(\bar{\phi})$ ,  $p_M(\bar{\phi})$  and  $p_H(\bar{\phi})$  the probabilities of the low, medium and high states when the manager uses a given  $\bar{\phi}$  as his  $\tilde{\phi}$ -threshold.<sup>16</sup> To make the manager choose  $\bar{\phi}$  as his  $\tilde{\phi}$ -threshold, the contract  $\{\Delta_M, \Delta_H\}$  must make the right-hand side of (27) equal to  $\bar{\phi}$  which, after simple manipulations, reduces to

$$\Delta_H = \frac{1 - \bar{\phi}}{\bar{\phi}} \frac{1 - a - \delta}{1 + a + \delta} \frac{\Delta_M}{1 - r}. \quad (32)$$

The solution contract  $\{\Delta_M, \Delta_H\}$  must also satisfy the retention constraint (and will do so exactly as the firm need not give the manager more surplus), that is, we must have

$$\Delta_M p_M(\bar{\phi}) + [\Delta_M + (1 - r)\Delta_H] p_H(\bar{\phi}) = \bar{u}. \quad (33)$$

Let us denote by  $\{\Delta_M(\bar{\phi}), \Delta_H(\bar{\phi})\}$  the contract that satisfies (32) and (33). The firm's maximization problem then simply becomes one of choosing  $\bar{\phi}$  to maximize firm value, that is,

$$\max_{\bar{\phi}} \left[ \frac{1}{2} - \Delta_M(\bar{\phi}) \right] p_M(\bar{\phi}) + [1 - \Delta_M(\bar{\phi}) - \Delta_H(\bar{\phi})] p_H(\bar{\phi})$$

which, after using (33), reduces to

$$\max_{\bar{\phi}} \frac{1}{2} p_M(\bar{\phi}) + [1 - r\Delta_H(\bar{\phi})] p_H(\bar{\phi}) - \bar{u}. \quad (34)$$

Let us denote the solution to this problem by  $\bar{\phi}^*$ . Also, let us denote  $p_L(\bar{\phi}^*)$ ,  $p_M(\bar{\phi}^*)$ ,  $p_H(\bar{\phi}^*)$ ,  $\Delta_M(\bar{\phi}^*)$  and  $\Delta_H(\bar{\phi}^*)$  by  $p_L^*$ ,  $p_M^*$ ,  $p_H^*$ ,  $\Delta_M^*$  and  $\Delta_H^*$  respectively. Clearly, the optimized value of the firm is  $\frac{1}{2} p_M^* + (1 - r\Delta_H^*) p_H^* - \bar{u}$ , which we denote by  $\bar{F}^*$ .

Suppose now that we start increasing  $\delta$  (to  $\delta'$ , say) but that, at the same time, we change the compensation contract in such a way that the manager still chooses  $\bar{\phi}^*$  as his  $\tilde{\phi}$ -threshold and that the retention constraint is still satisfied. That is, instead of (32) and (33), this new contract  $\{\Delta'_M, \Delta'_H\}$  must satisfy

$$\Delta'_H = \frac{1 - \bar{\phi}^*}{\bar{\phi}^*} \frac{1 - a - \delta'}{1 + a + \delta'} \frac{\Delta'_M}{1 - r} \quad (35)$$

and

$$\Delta'_M p_M^* + [\Delta'_M + (1 - r)\Delta'_H] p_H^* = \bar{u}. \quad (36)$$

<sup>16</sup>Calculations similar to the ones made in (29) and (30), with  $\bar{\phi}^{\text{FB}}$  replaced by  $\bar{\phi}$  in both integrals, yield  $p_L(\bar{\phi}) = \frac{1}{2}(1 - a) \left(\frac{1}{2} - \bar{\phi}\right) \left(\frac{3}{2} - \bar{\phi}\right)$ ,  $p_M(\bar{\phi}) = \frac{1}{2} + \bar{\phi} + a \left(\frac{1}{2} - \bar{\phi}\right)^2$ , and  $p_H(\bar{\phi}) = \frac{1}{2}(1 + a) \left(\frac{1}{2} - \bar{\phi}\right) \left(\frac{1}{2} + \bar{\phi}\right)$ .

It is easy to show that the contract  $\{\Delta'_M, \Delta'_H\}$  that satisfies (35) and (36) is such that  $\Delta'_M$  ( $\Delta'_H$ ) increases (decreases) as  $\delta'$  increases. Using similar arguments as the ones we used to derive (34), the firm's value under this contract is equal to

$$\frac{1}{2}p_M^* + (1 - r\Delta'_H)p_H^* - \bar{u}.$$

Since  $\Delta'_H$  is the only quantity in this expression affected by the change in  $\delta'$  and because it is decreasing in  $\delta'$ , the value of the firm is increasing in  $\delta'$ . Because  $\{\Delta'_M, \Delta'_H\}$  satisfies the retention constraint by construction, it is a candidate for the firm's maximization problem with the higher  $\delta'$ . Thus the firm's optimized value (i.e., with the optimal  $\tilde{\phi}$ -threshold, which may not be  $\bar{\phi}^*$ ) has to be larger than  $\bar{F}^*$ . ■

### Proof of Lemma 4.1

Suppose that the equilibrium contract  $\{\Delta'_M, \Delta'_H\}$ , which must satisfy (13), does not make the manager as well-off as another contract,  $\{\Delta_M^*, \Delta_H^*\}$ , also satisfying (13). Then the competing firm can always offer the manager a contract that makes him better off and increases the value of the firm above  $\frac{1}{2}$ . To achieve this, the firm takes  $\{\Delta_M^*, \Delta_H^*\}$  and modifies it as follows: decrease  $\Delta_M^*$  and  $\Delta_H^*$  in proportions such that the  $\tilde{\phi}$ -threshold chosen by the manager remains the same as with contract  $\{\Delta_M^*, \Delta_H^*\}$ . As (32) in the proof of Proposition 4.3 shows, this means that, as  $\Delta_M$  is decreased by  $\varepsilon$ ,  $\Delta_H$  must be decreased by  $\left(\frac{1-\bar{\phi}^*}{\bar{\phi}^*} \frac{1-a-\delta}{1+a+\delta} \frac{1}{1-r}\right) \varepsilon$ , where  $\bar{\phi}^*$  is the  $\tilde{\phi}$ -threshold chosen by the manager with contract  $\{\Delta_M^*, \Delta_H^*\}$ . The resulting contract,  $\{\Delta''_M, \Delta''_H\}$ , makes the manager better off than with  $\{\Delta'_M, \Delta'_H\}$  as long as  $\varepsilon$  is small enough. Because the manager chooses the same threshold for  $\tilde{\phi}$  as the contract changes, the firm's revenues do not change. Since  $\Delta_M^*$  and  $\Delta_H^*$  are reduced to  $\Delta''_M$  and  $\Delta''_H$  respectively, the contract is cheaper for the firm, and the reduced expected compensation translates into a firm value that rises above  $\frac{1}{2}$ . Clearly, therefore,  $\{\Delta'_M, \Delta'_H\}$  is not an equilibrium contract. ■

### Proof of Proposition 4.4

Establishing that the manager's biased expected utility increases as  $\delta$  increases can be done in essentially the same way as in the proof of Proposition 4.3. The effect that  $\delta$  has on  $\Delta_M^C(\delta)$ ,  $\Delta_H^C(\delta)$  and the manager's expected utility, calculated from a rational perspective, requires that we solve for the equilibrium contract. As in the proof of Proposition 4.3, this contract can be expressed as the solution to an unconstrained maximization problem over  $\bar{\phi}$ , the threshold for  $\tilde{\phi}$ . The solution to this maximization problem is the one root of a polynomial of degree six that is included in  $(0, \frac{1}{2})$ ,

and requires numerical solutions. Because we are interested in the behavior of the equilibrium at  $\delta = 0$ , only two parameters,  $a$  and  $r$ , move freely. Both of these are in the  $(0, 1)$  interval, and so can be easily spanned numerically. The details of this numerical procedure are available from the authors upon request. ■

### Proof of Lemma 5.1

Suppose that the manager observes that  $\tilde{\phi} = \phi$ . Without any information, the manager's expected utility is  $\Delta_M$  if he holds onto cash, or  $\phi[\Delta_M + (1 - r)\Delta_H] + (1 - \phi)(0)$  if he undertakes the risky project. Holding onto cash is better for the manager when

$$\phi < \frac{\Delta_M}{\Delta_M + (1 - r)\Delta_H} \equiv \hat{\phi}. \quad (37)$$

Suppose that this condition is satisfied. The manager then gets  $\Delta_M$  if he chooses not to acquire any information. If he does choose to acquire information, he will undertake the risky project if and only if  $\tilde{s} = 1$ . His expected utility is then equal to

$$\Pr_b \left\{ \tilde{s} = 1 \mid \tilde{\phi} = \phi \right\} \left\{ \phi_1^{\text{OV}} [\Delta_M + (1 - r)\Delta_H] + (1 - \phi_1^{\text{OV}})(0) \right\} + \Pr_b \left\{ \tilde{s} = 0 \mid \tilde{\phi} = \phi \right\} \Delta_M - e,$$

where the “b” subscript on “Pr” denotes the fact that the probability is calculated by a biased manager, and where  $\phi_1^{\text{OV}}$  is given by (26). Using  $\Pr_b \left\{ \tilde{s} = 1 \mid \tilde{\phi} = \phi \right\} = \frac{1 - (a + \delta)(1 - 2\phi)}{2}$  and (26), the manager's expected utility from exerting an effort to gather information reduces to

$$\frac{(1 + a + \delta)\phi}{2} [\Delta_M + (1 - r)\Delta_H] + \left[ 1 - \frac{1 - (a + \delta)(1 - 2\phi)}{2} \right] \Delta_M - e. \quad (38)$$

This quantity exceeds  $\Delta_M$  if and only if  $\phi > \bar{\phi}_e(\delta)$ , where  $\bar{\phi}_e(\delta)$  is given by (15).

Suppose now that (37) is not satisfied, so that the manager's expected utility from not gathering information is  $\phi[\Delta_M + (1 - r)\Delta_H]$ . The manager then chooses to gather information if and only if this quantity is smaller than (38), which occurs when  $\phi < \underline{\phi}_e(\delta)$ , where  $\underline{\phi}_e(\delta)$  is given by (16). The range  $[\bar{\phi}_e(\delta), \underline{\phi}_e(\delta)]$  exists as long as  $\bar{\phi}_e(\delta) < \underline{\phi}_e(\delta)$ , which is equivalent to (14). ■

### Proof of Proposition 5.1

We know that (17) is necessary for first-best realignment. As argued in the paragraph preceding the proposition, first-best realignment will be achieved if  $\underline{\phi}_e(\delta) \geq \frac{1}{2}$  when (17) holds. Using (16), it is straightforward to show that  $\underline{\phi}_e(\delta) \geq \frac{1}{2}$  is equivalent to

$$(1 + a + \delta)\Delta_M \geq 4e + (1 - a - \delta)(1 - r)\Delta_H.$$

We can now replace  $\Delta_H$  by (17) in this last inequality, which then reduces to (18). ■

### Proof of Lemma 5.2

This result can be derived by setting  $\delta$  equal to zero and by replacing  $\phi$  by  $\phi + \gamma$  in the proof of Lemma 5.1. ■

### Proof of Proposition 5.2

For the manager to make the same decisions as first-best requires, it has to be the case that  $\bar{\phi}_e(\gamma) = \bar{\phi}^{\text{FB}} = \frac{1-a}{2}$  and that  $\underline{\phi}_e(\gamma) \geq \frac{1}{2}$ . Simple manipulations show that this equality and inequality are respectively equivalent to

$$\Delta_H = \frac{4e + (1-a)(1+a-2\gamma)\Delta_M}{(1-r)(1+a)(1-a+2\gamma)} \quad (39)$$

and

$$(1+a)(1-2\gamma)\Delta_M - 4e \geq (1-a)(1+2\gamma)(1-r)\Delta_H.$$

We can now replace  $\Delta_H$  by (39) in this last inequality, which then reduces to (22). ■

## References

- Adrian, T., and M. M. Westerfield, 2005, "Heterogeneous Beliefs and the Principal-Agent Problem," Working Paper, Reserve Bank of New York.
- Barnea A., R. Haugen, and L. Senbet, 1985, *Agency Problems and Financial Contracting*, Englewood Cliffs: Prentice-Hall.
- Baumann, A. O., R. B. Deber, and G. G. Thompson, 1991, "Overconfidence Among Physicians and Nurses: The 'Micro-Certainty, Macro-Uncertainty' Phenomenon," *Social Science and Medicine*, 32, 167-174.
- Bénabou, R., and J. Tirole, 2002, "Self-Confidence and Personal Motivation," *Quarterly Journal of Economics*, 117, 871-915.
- Benos, A., 1998, "Aggressiveness and Survival of Overconfident Traders," *Journal of Financial Markets*, 1, 353-383.
- Bernardo, A., and I. Welch, 2001, "On the Evolution of Overconfidence and Entrepreneurs," *Journal of Economics and Management Strategy*, 10, 301-330.
- Brunnermeier, M. K., and J. Parker, 2005, "Optimal Expectations," *American Economic Review*, 95, 1092-1118.
- Christensen-Szalanski, J. J., and J. B. Bushyhead, 1981, "Physicians' Use of Probabilistic Information in a Real Clinical Setting," *Journal of Experimental Psychology: Human Perception and Performance*, 7, 928-935.
- Cooper, A. C., C. Y. Woo, and W. C. Dunkelberg, 1988, "Entrepreneurs' Perceived Chances for Success," *Journal of Business Venturing*, 3, 97-108.
- Daniel, K., D. Hirshleifer, and A. Subrahmanyam, 1998, "A Theory of Overconfidence, Self-Attribution, and Security Market Under- and Over-reactions," *Journal of Finance*, 53, 1839-1885.
- De Long, J., A. Shleifer, L. Summers, and R. Waldmann, 1991, "The Survival of Noise Traders in Financial Markets," *Journal of Business*, 64, 1-19.

- Gervais, S., and I. Goldstein, 2005, "The Effects of Biased Self-Perceptions in Teams," Working Paper, Duke University.
- Gervais, S., and T. Odean, 2001, "Learning to Be Overconfident," *Review of Financial Studies*, 14, 1-27.
- Goel, A., and A. Thakor, 2000, "Rationality, Overconfidence, and Leadership," Working Paper, Michigan Business School.
- Hackbarth, D., 2003a, "Managerial Optimism, Overconfidence, and Capital Structure Decisions," Working Paper, University of California, Berkeley.
- Hackbarth, D., 2003b, "Determinants of Corporate Borrowing: A Behavioral Perspective," Working Paper, University of California, Berkeley.
- Heaton, J. B., 2002, "Managerial Optimism and Corporate Finance," *Financial Management*, 31, 33-45.
- Hemmer, T., O. Kim, and R. Verrechia, 2000, "Introducing Convexity into Optimal Compensation Contracts," *Journal of Accounting and Economics*, 28, 307-327.
- Hirshleifer, D., A. Subrahmanyam, and S. Titman, 1994, "Security Analysis and Trading Patterns When Some Investors Receive Information Before Others," *Journal of Finance*, 49, 1665-1698.
- Hirshleifer, D., and Y. Suh, 1992, "Risk, Managerial Effort, and Project Choice," *Journal of Financial Intermediation*, 2, 308-345.
- Jensen, M. C., and W. H. Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics*, 3, 305-360.
- Kahneman, D. and D. Lovallo, 1993, "Timid Choices and Bold Forecasts: A Cognitive Perspective on Risk Taking," *Management Science*, 39, 17-31.
- Kidd, J. B., 1970, "The Utilization of Subjective Probabilities in Production Planning," *Acta Psychologica*, 34, 338-347.
- Klaczynski, P. A., and J. M. Fauth, 1996, "Intellectual Ability, Rationality, and Intuitiveness as Predictors of Warranted and Unwarranted Optimism for Future Life Events," *Journal of Youth and Adolescence*, 25, 755-773.

- Kyle, A., and F. A. Wang, 1997, "Speculation Duopoly With Agreement to Disagree: Can Overconfidence Survive the Market Test?," *Journal of Finance*, 52, 2073-2090.
- Lambert, R. A., 1986, "Executive Effort and Selection of Risky Projects," *RAND Journal of Economics*, 17, 77-88.
- Malmendier, U., and G. Tate, 2003, "CEO Overconfidence and Corporate Investment," Working Paper, Harvard University.
- Neale, M. A., and M. H. Bazerman, 1990, *Cognition and Rationality in Negotiation*, New York: The Free Press.
- Odean, T., 1998, "Volume, Volatility, Price, and Profit When All Traders Are Above Average," *Journal of Finance*, 53, 1887-1934.
- Oskamp, S., 1965, "Overconfidence in Case-Study Judgments," *Journal of Consulting Psychology*, 29, 261-265.
- Roll, R., 1986, "The Hubris Hypothesis of Corporate Takeovers," *Journal of Business*, 59, 197-216.
- Russo, J. E., and P. J. H. Schoemaker, 1992, "Managing Overconfidence," *Sloan Management Review*, 33, 7-17.
- Sklar, H., L. Mykyta, and S. Wefald, 2001, *Raise the Floor: Wages and Policies that Work for All of Us*, Cambridge: South End Press.
- Smith, C., and R. Stulz, 1985, "The Determinants of Firms' Hedging Policies," *Journal of Financial and Quantitative Analysis*, 20, 391-405.
- Staël von Holstein, C.-A. S., 1972, "Probabilistic Forecasting: An Experiment Related to the Stock Market," *Organizational Behavior and Human Performance*, 8, 139-158.
- Treynor, J. L., and F. Black, 1976, "Corporate Investment Decisions" in *Modern Developments in Financial Management*, ed. Stewart C. Myers, New York: Praeger, 310-327.
- Wagenaar, W., and G. B. Keren, 1986, "Does the Expert Know? The Reliability of Predictions and Confidence Ratings of Experts," eds. E. Hollnagel, G. Maneini, and D. Woods, *Intelligent Decision Support in Process Environments*, Berlin: Springer, 87-107.