The Principal Principle [1]

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Abstract

The Principal Principle

Lenders will often restructure a loan rather than foreclose on a property because it is less value-destroying. A loan modification primarily entails a change in the loan rate, principal balance and/or remaining time to maturity. We analyze optimal loan modification schemes in a stochastic home price and stochastic interest-rate environment. Lenders maximize their loan values by managing the value of the borrower's option to default on the loan and prepayment option. In the presence of negative equity, controlling for the borrower's ability to pay, loan modifications via rate reductions and maturity extensions result in a higher probability of re-default by homeowners even after modification of their loans. In contrast, loan write-downs (the Principal Principle), not a favored recipe, and sometimes prohibited by covenants, are value-maximizing for the lender. Once negative equity has been tackled by principal write-downs, rate reductions are optimal. A useful structuring device, the shared appreciation mortgage, enhances the ability to pay, mitigates adverse selection, and reduces the present value of expected deadweight foreclosure costs.

1 Introduction

The current housing crisis has resulted in a high level of delinquincies—as of May 2011, over 4 million mortgages in the U.S. were over 90 days delinquent or in foreclosure.[1] One in every 380 housing units received a foreclosure filing in August 2010.[2] This is at the same rate as the previous year—for Q3 2009, total filings (default notices, scheduled auctions, and bank repossessions) numbered 937,840 (approximately 1 of every 136 housing units), but up 23% from the previous year (i.e., 2008).[3] Lender Processing Services (LPS), in its September 2010 report, states that pre-sale foreclosure inventory stands at 2.038 million homes. About 4.9 million mortgages are 30 days overdue, of which 2.374 million are 90 days past due. Negative equity is a persistent problem, especially in the 2006–2007 loan cohorts, and about 11 million borrowers, i.e., 23% of households with a mortgage are underwater.[4] Hope Now reports that in February 2011, there were 87,083 permanent loan modifications, of which three-fourths were proprietary, and the remaining were under the government HAMP program.

This paper deals with the optimal modification of distressed home loans, and explains why lenders should forgive, not forsake mortgages. When dealing with failing borrowers, lenders need to decide between modifying the loan or foreclosing. Modification results in a lowering of the present value of the loan. On the other hand, foreclosing results in deadweight costs that reduce the recovery value of the home. This paper develops a framework for restructuring loans to maximize their value to the lender, accounting for the incentives of the borrower with negative equity to default. Despite best efforts, a loan modification often ends up in re-default, at rates ranging from 20% to 60% over a year, depending on the lender and type of loan.[5] Moodys reports that principal modified loans have the lowest re-default rates[6], and that for subprime loans, a 20% reduction in monthly payments reduces the one-year re-default rate by 10%.

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[1] Per Loan Processing Services: `http://www.dsnews.com/articles/lps-finds-serious-delinquencies-outnumber-foreclosure-sales-501-2011-06-29`.

[2] `www.realtytrac.com`

[3] RealtyTrac. RealtyTracs report incorporates documents filed in all three phases of foreclosure: default—Notice of Default (NOD) and Lis Pendens (LIS); Auction—Notice of Trustee Sale and Notice of Foreclosure Sale (NTS and NFS); and Real Estate Owned, or REO properties (that have been foreclosed on and repurchased by a bank). If more than one foreclosure document is filed against a property during the month or quarter, only the most recent filing is counted in the report.

[4] Based on CoreLogic's estimates, reported in the Wall Street Journal, September 4, 2010. This is down from 24% in the previous quarter, but this is not good news—the reduction in negative equity loans comes primarily from these loans moving to foreclosure status.

[5] `http://www.dsnews.com/articles/fitch-subpar-loan-mod-results-making-us-foreclosures-reality-2011-02-07`

[6] `http://www.dsnews.com/articles/moodys-takes-closer-look-at-dynamics-of-mortgage-re-defaults-2011-02-04`

An optimal loan modification must be cognizant of the borrower's ability to pay *and* willingness to pay. Foote, Gerardi, Goette and Willen (2009) state that the data suggests that loan modification to reduce foreclosure should focus on two important reasons for foreclosure—falling home prices (impacting willingness to pay) and adverse life events (affecting ability to pay). The optimal strategies of the lender (the bank/ servicer[7]) and the borrower (the homeowner) depend on the value of an American (more appropriately, Bermudan) put option, i.e., the option the borrower has to put the house back to the lender—see Kau and Keenan (1999); Deng, Quigley, and van Order (2000); and Ambrose, Capone and Deng (2001). This option influences the borrower's willingness to pay. It also interacts with the borrower's option to prepay the loan—interestingly, mitigating the likelihood of the borrower exercising the option to default increases the likelihood that he/she will prepay the loan, given that no default occurs.

There are various motives for borrowers to strategically default on their home loans. Cohen-Cole and Morse (2009) provide evidence that foreclosure in the presence of ability to pay may arise from a precautionary liquidity motive, i.e., faced with a choice between paying a home loan and a credit card, liquidity concerns drive borrowers to pay the credit card first. Guiso, Sapienza and Zingales (2009) suggest that 26% of defaults are strategic in nature.[8] Strategic default by homeowners has parallels in the corporate debt literature—see Anderson and Sundaresan (1996). Borrowers have become increasingly aware of the strategic option to default, and are showing an increasing willingness to exercise this option. A recent report by Pew Research[9] states that 19% of Americans believe that it is unconditionally acceptable to walk away from their homes, and a further 17% say that they would do so depending on personal circumstances.

In this paper, we focus on loan modification schemes that adjust loan terms to make the loan affordable on a monthly payment basis, but need tuning to manage strategic default.[10] Errors in managing strategic default often make loan modification suboptimal, leading to dissipation in loan value to the lender, and a high probability of re-default by homeowners even after modification of their loans—the OCC and OTS Mortgage Metrics Report (2009)

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[7] Whereas loan modifications are undertaken by the loan servicer in most cases, and sometimes by investors who buy distressed loans from banks, we will use the term "lender" throughout the paper as the nomenclature of the entity that is tasked with making loan modifications.

[8] "Strategic" default is defined as the decision by a borrower to stop making payments on a mortgage despite having the financial ability to make the payments. That is, an unwillingness to pay, despite the ability to do so. Guiso, Sapienza and Zingales (2009) employ a survey approach to arriving at their strategic default estimate.

[9] Pew Research Center Report, September 15, 2010, by Rich Morin, Senior Editor.

[10] Default is often triggered by a combination of income loss and negative equity. By giving up some of the present value of the loan, the lender is able to make the loan affordable and manages to stave off deadweight costs of foreclosure on default.

finds that two-thirds of modified loans re-default within a year of modification—more recent estimates by Moody's suggest this is one-third among major bank lenders. In this paper, we explore when principal modifications are useful in comparison to rate-reductions and maturity-extensions. Meadows (2009) and the FDIC (2009) Loan Modification Plan focus on these issues as well, but stop short of providing an option-based analysis. Given the severe deadweight costs of foreclosure—many articles, for example see Ambrose and Capone (1996) for an analysis of foreclosure and modification alternatives—suggest that loan modification to reset the ability and willingness to pay of the borrower is more often the socially better outcome. In contrast to prior work, this paper focuses specifically on the willingness to pay (strategic default) across various modifications, *after* equalizing them on the ability to pay. The paper also factors in ability-to-pay risk once willingness-to-pay is represented as a function of ability-to-pay.

A loan modification entails two considerations. First, it requires correctly determining the level of monthly service payments on the loan that are affordable to the distressed borrower. This calibrates the loan to the borrower's *ability* to pay. The second consideration involves the borrower's *willingness* to pay and foreclosure costs. Since there are many loan configurations that lead to the same monthly payment level, we develop guidelines to select the one that is most favorable in maximizing the value of the loan to the lender. Our analysis involves a value decomposition of the loan into various interacting components: (a) a risk-free component, (b) a default put option, (c) a call option to refinance the loan, and (d) a shared-equity component. The optimal loan configuration maximizes lender net value accounting for all these components.

We now describe the main results of the paper. In Section 2 we present the model framework in which we analyze the optimal loan modification problem in the presence of stochastic home prices and stochastic interest rates. The model is solved numerically using a quadrinomial lattice framework. The model comprises a Heath, Jarrow, and Morton (1990) (HJM) term structure model enhanced with a two-factor stochastic process for home equity. The use of the HJM model makes the model practitioner-friendly because these models are widely used and are easily calibrated. The quadrinomial tree framework makes it easy to accommodate all three options in the loan modification scheme: the borrower's options to prepay and default, and the lender's equity-sharing option that is implemented in loan modifications that give a shared-appreciation component to the lender.

Section 3 presents results of this analysis in terms of guidance for loan modification. First, negative equity loans result in rapid default, especially when volatility of house prices is low, and the borrower sees little upside in house values. Second, keeping service levels fixed on negative equity loans, rate reductions result in lower optimal loan values because they entail higher principal balances and greater levels of negative equity, increasing the probability of strategic default. Third, maturity extensions result in lower optimal loan values for the same reason that lower rates do, i.e., the default option is more in-the-money, and more valuable as its maturity is longer. Fourth, it is important to note that principal reductions in negative equity situations stave off strategic default and result in sharp increases in expected loan value to the lender, especially when the deadweight costs of borrower default are high. Finally, we show that once the negative equity problem has been staunched with an appropriate principal reduction, rate-reductions become optimal because they manage ability to pay risk most effectively.

Strategic default occurs in situations of negative equity, a proposition that has recent empirical backing. The negative equity problem is exacerbated by the phenomenon of cash-out refinancings, resulting in systemic increases in home owner leverage—see Mian and Sufi (2009); Khandani, Lo and Merton (2009). Section 3.2 also presents a brief empirical analysis of modified loans and finds that loan re-default rates decline significantly with principal modifications. Section 3 also analyzes shared appreciation mortgages (SAMs)—and shows that this feature improves the borrower's ability to pay, mitigates deadweight foreclosure costs, and tempers adverse selection in the loan modification market.[11]

In Section 4, we relate our theoretical prescriptions to what is being done in practice. Hedge funds and other investors that buy pools of distressed mortgages appear to be making optimal modifications by writing down loan balances. Section 5 offers discussion and concluding comments.

2 The Problem and Model Set Up

We adopt a two-state variable model in which both house value and interest rates are stochastic. The nominal value of the home is driven by a risk-neutral stochastic process $V(t)$ with two independent geometric Brownian motions, i.e.,

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[11] SAMs have been in place in the U.S. for some time already (since the 1980s), and usually take the form of the lender offering the borrower an interest-rate reduction on the loan in return for an appreciation share. The share of appreciated value is called "contingent interest" and Revenue Ruling 83-51 (1983) of the Internal Revenue Service specifies conditions under which the contingent interest in a shared appreciation mortgage may be considered tax-deductible mortgage interest. For loans with negative equity, SAMs reduce the likelihood of strategic default because they force a borrower who has the ability to pay to part with some of the upside of the home value.

$$\frac{dV(t)}{V(t)} = (r - \delta)\ dt + \sigma_1\ dZ_1(t) + \sigma_2\ dZ_2(t) \tag{1}$$

where r is the risk-free *spot* rate of interest, δ is the dividend or service flow from the housing asset, and the home's value variability is determined by the volatility coefficients σ_1 and σ_2. The randomness comes from independent standard Brownian motion increments dZ_1, dZ_2. Kelly (2006) points to considerable evidence showing that the level of homeowner's equity is a major determinant of default. Changes in equity, known as contemporaneous equity, are an even more critical determinant of foreclosure than initial equity. Therefore, the modeling of home equity movements is essential to the analysis of loan modifications. In this set up, the variance of returns on home value is $Var(dV/V) = (\sigma_1^2 + \sigma_2^2)\ dt$.

The loan balance on the home is denoted $L(t)$. Equity in the home is $E(t) = V(t) - L(t)$. We define troubled loans at a given time t as a home loan having negative equity, i.e., $E(t) < 0$. The loan-to-value (LTV) ratio is $L(t)/V(t)$, and LTV is greater than one when there is negative equity in the home.

Since home loan values are also determined by interest rates, we specify a model for the term-structure of interest rates. The model we choose is a one-factor Heath-Jarrow-Morton (HJM) model as described in Heath, Jarrow, and Morton (1990) and Heath, Jarrow, and Morton (1992). The forward-rate process we use in this model is given by the following system of equations:

$$df(t,T) = \alpha(t,T)\ dt + \beta(t,T)\ dZ_1(t), \quad \forall T = t \dots \tau. \tag{2}$$

where $f(t,T)$ is the forward rate that is seen at time t, for a forward period beginning at T. In the continuous-time representation above, there is a continuum of instantaneous forward rates at each T. Since the Wiener process dZ_1 is shared by the home value stochastic process in equation (1) and the term structure model in equation (2), correlation is induced between interest rates and real-estate values. The correlation is equal to $Corr(dV/V, df) = \frac{\sigma_1}{\sqrt{\sigma_1^2+\sigma_2^2}}$, which is easily derived by noting that he covariance between interest rates and home values is $Cov(dV/V, df) = \sigma_1 \beta\ dt$.

2.1 Discrete-time Model

Since a mortgage has embedded options (to prepay, default, and equity-sharing) that may be exercised at any time, we represent the continuous-time model in a discrete-time framework so that the American/Bermudan-style features of these options may be calculated by dynamic programming. Consider an N-period model with time-points numbered $\{t, T\} = 0, 1, 2, \ldots, N$. Each period is of length h years each, so the horizon τ of the model is equal to $N \cdot h$ years.[12] We extend the Cox, Ross and Rubinstein (1979) framework to build a "quadrinomial" (4–way) tree in home values, where the home value at time t, $V(t)$, moves to one of four values $V(t+1)$ at time $(t+1)$:

$$V(t+1) = \begin{cases} V(t)\exp\left(+\sigma_1\sqrt{h} + \sigma_2\sqrt{h}\right), & \text{w/prob } q/2 \\ V(t)\exp\left(+\sigma_1\sqrt{h} - \sigma_2\sqrt{h}\right), & \text{w/prob } (1-q)/2 \\ V(t)\exp\left(-\sigma_1\sqrt{h} + \sigma_2\sqrt{h}\right), & \text{w/prob } q/2 \\ V(t)\exp\left(-\sigma_1\sqrt{h} - \sigma_2\sqrt{h}\right), & \text{w/prob } (1-q)/2 \end{cases} \tag{3}$$

where $0 \leq q \leq 1$ is a parameter that determines the risk-neutral probability of transitions in V. We will shortly see how q is derived to ensure that the discrete stochastic model here is free from arbitrage. We need four branches from each node on this tree so that movements from both Brownian motions (dZ_1 and dZ_2) may be accommodated. On this quadrinomial lattice we superimpose the term structure of forward rates from the HJM model. We turn to this next.

The forward rate at time t for a *one-period* borrowing or investment at time T (where $T \geq t$ and $T \leq N-1$) is denoted $f(t, T)$. Note that $f(t, T)$ is quoted at time t but applicable to the period from T to $T+1$. All interest rates are quoted in continuously compounded and annualized terms. Since the time interval between T and $T+1$ is h years, this means that \$1 invested at time s at the rate $f(t, T)$ will grow by time $T+1$ to $\exp\{f(t, T) \cdot h\}$.

In this discrete-time setting, the initial forward curve at time 0 is composed of N forward rates: $f(0, 0), f(0, 1), f(0, 2), \ldots, f(0, N-1)$. At time 1, we are a step closer to the horizon of the model, so the forward curve has only $(N-1)$ forward rates $(f(1, 1), f(1, 2), \ldots f(1, N-1))$. Note that the rate $f(t, t)$ at any t denotes the spot rate $r(t)$ at that point. Spot rates will be superimposed on the quadrinomial tree of home values as they are necessary for period-to-period discounting.

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[12] We observe that t and T index the beginning of periods (points in time), but τ and h are in units of calendar time.

Let $P(t,T)$ denote the time-t price of a zero-coupon bond maturing at time T and with a face value of \$1. The usual spot-forward parity arguments result in the price of the zero-coupon bond being expressed in terms of forward rates, i.e.,

$$P(t,T) \;=\; \exp\left\{-\sum_{i=t}^{T-1} f(t,i)\cdot h\right\}. \tag{4}$$

In discrete-time the evolution of the forward rate term structure at time t, $f(t,T), \forall T)$ to time $(t+1)$, i.e., $f(t+1,T), \forall T$ is modeled as a binomial process, as in Heath, Jarrow, and Morton (1990):

$$\forall T, \quad f(t+1,T) = \begin{cases} f(t,T) + \alpha(t,T)\, h + \beta(t,T)\sqrt{h}, & \text{w/prob } \frac{1}{2} \\ f(t,T) + \alpha(t,T)\, h - \beta(t,T)\sqrt{h}, & \text{w/prob } \frac{1}{2} \end{cases} \tag{5}$$

where the risk-neutralizing drift terms $\alpha(t,T)$ are given by—see Heath, Jarrow, and Morton (1990) for a derivation approach:[13]

$$\sum_{t=1}^{T} \alpha(t,T) \;=\; \frac{1}{h^2} \ln\left[\cosh\left(\sum_{t=1}^{T} \beta(t,T)\, h^{\frac{3}{2}}\right)\right], \quad \forall T \tag{6}$$

Here the risk-neutral drifts $\alpha(t,T)$ are expressed as pure functions of the volatility term-structure $\beta(t,T)$. Equation (6) expresses these drifts recursively for all T.

The spot rates $r(t) = f(t,t)$ from the HJM binomial tree in forward-rate term structures based on equation (5) are embedded into the quadrinomial tree for home values given in equation (3) to obtain a combined quadrinomial tree. At each node of the combined tree we have a tuple $\{V, r\}$ of home value and spot rate. Each node bifurcates to one of four nodes one period ahead. Each of these nodes has four values of V given in equation (3). At the first and second of these nodes, we place the first value of r (the up shifted value). At the third and fourth of these nodes, we place the second value of r (the down shifted value). The probability structure is the same as that of equation (3), i.e., the four nodes have probability $\{q/2, (1-q)/2, q/2, (1-q)/2\}$. These probabilities are such that (a) they sum to 1, and (b) the sum of the first two and last two, both equal 1/2. Therefore, the probability of an up-move in interest rates and a down-move is 1/2, as stipulated in equation (5).

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[13] For a detailed derivation, see Chapter 30 of Sundaram and Das (2010).

Under the risk-neutral probability measure, expected future home value $E[V(t+1)]$ will be equal to the forward home value, i.e.,

$$\begin{aligned}
V(t)\, e^{(r(t)-\delta)h} &= E[V(t+1)] \\
&= V(t)\exp\left(+\sigma_1\sqrt{h}+\sigma_2\sqrt{h}\right) \times q/2 \\
&\quad +V(t)\exp\left(+\sigma_1\sqrt{h}-\sigma_2\sqrt{h}\right) \times (1-q)/2 \\
&\quad +V(t)\exp\left(-\sigma_1\sqrt{h}+\sigma_2\sqrt{h}\right) \times q/2 \\
&\quad +V(t)\exp\left(-\sigma_1\sqrt{h}-\sigma_2\sqrt{h}\right) \times (1-q)/2
\end{aligned}$$

Solving this equation for q gives:

$$q = \frac{2\, e^{(r(t)-\delta)h} - (u_2+u_4)}{u_1+u_3-u_2-u_4} \tag{7}$$

where

$$\begin{aligned}
u_1 &= \exp\left(+\sigma_1\sqrt{h}+\sigma_2\sqrt{h}\right) \\
u_2 &= \exp\left(+\sigma_1\sqrt{h}-\sigma_2\sqrt{h}\right) \\
u_3 &= \exp\left(-\sigma_1\sqrt{h}+\sigma_2\sqrt{h}\right) \\
u_4 &= \exp\left(-\sigma_1\sqrt{h}-\sigma_2\sqrt{h}\right)
\end{aligned}$$

With these probabilities, the quadrinomial tree set up is complete. Note that the service flow of the home (δ) has been incorporated into the tree probabilities in equation (7). As h, the time period on the tree, shortens, q remains within $(0,1)$ bounds. In all examples in the paper h is chosen such that the probabilities remain in this range. This tree model is computationally easier to implement than PDE approaches.

2.2 Modeling the mortgage

For a given loan balance L_0 with time T remaining on the loan, A is defined as the total flat payment per year including principal and interest. This is a function of the loan balance L_0 and the fixed interest rate on the loan, which we denote r_L per annum. Inferring A is easy because the annuity stream must equal the present value of the loan, i.e., L_0. The annuity equation resulting in the value of the loan in continuous time is

$$L_0 = A\int_0^T e^{-r_L t}\, dt = A\left(\frac{1-e^{-r_L T}}{r_L}\right) \tag{8}$$

Re-arranging this equation we get the flat payment rate per annum

$$A = \frac{r_L L_0}{1 - e^{-r_L T}} = \frac{r_L L_t}{1 - e^{-r_L (T-t)}} \tag{9}$$

As the equation notes, the flat payment A is the same at time 0 and at all subsequent times t.

In discrete time with (usually monthly) payments, we get the well known equation for periodic flat payments, analogous to that in equation (9):

$$A = m \times S = m \times \frac{i L_0}{1 - (1+i)^{-N}} \tag{10}$$

where A is the annual total of payments, S is the periodic payment, $i = r_L/m$ is the periodic interest rate, and the number of remaining periods is $N = mT$, where m is the number of periods in a year (for example $m = 12$ for monthly payments, which is standard in the U.S.).

The corresponding law of motion for the principal balance at any period t is

$$L(t+1) = L(t) \left[(1+i) - \frac{i}{1 - (1+i)^{-(N-t)}} \right] \tag{11}$$

Note that $0 < L(t+1) < L(t) < L(0)$ for all t. The quantity in brackets on the right-hand side of the preceding equation is the proportion of principal balance $L(t)$ remaining after period $(t+1)$.

The homeowner has negative equity, $E(t) < 0$, whenever $L(t)/V(t) > 1$. Note that if $E(t) > 0$, it is always better for the homeowner to sell the house (instead of foreclosure) and pay off the loan, thereby retaining the residual equity. The Wall Street Journal reported on August 5, 2009 that 24% of owner-occupied single-family homes had mortgage debt exceeding home value, i.e., negative equity. The situation was even more critical in states like Nevada (40% of homes with negative equity), Arizona (37%), and California (33%). CoreLogic reported that the negative equity situation at the end of 2010-Q3 is similar to that in 2009, i.e., 23%.

The homeowner has an American put option (denoted $P(t)$) on the home value $V(t)$ at a strike that is equal to the present value of future option-adjusted liabilities on the mortgage. The strike price changes each period because the loan balance changes as per the dynamics in equation (11), and also because interest rates and embedded options in the mortgage change in value. Because this put option has time value, it is not always optimal to exercise the put when it is just in-the-money, but it might be worth waiting to exercise when it is more valuable. Also, there are costs to exercising the put on one's home such as finding a new home, etc. (which we denote as "relocation costs" K_R), net of benefits such as the present value of rent in the "rent-free" period enjoyed during the foreclosure process. These features make this put option different from standard puts on stocks and bonds. The relocation costs include giving up the service flow δ from the property, and foregoing the benefit of remaining in the home rent-free. Hence, the homeowner will exercise his option to renege on the loan when it is sufficiently underwater so as to be optimal after relocation costs, present value of expected service flows, and other benefits of possessing the property.

In addition to an option to default, the borrower also has an option to refinance (i.e., call) the loan. This will occur when interest rates drop and the value of the loan exceeds unpaid principal $L(t)$. Therefore, the value of the loan to the lender (denoted $B^l(t)$) is the present value of future service payments, less the value of two options held by the borrower: a put option to default, and an option to call the loan. These two options are not independent of each other, because the exercise of one of them extinguishes the other, and hence, care needs to be taken in determining $B^l(t)$.

We use our quadrinomial lattice to compute the loan value at all nodes and implement the refinancing option simultaneously with the option to default in determining the loan value at all times. We keep track of the lender's expected loan value separately from that of the borrower (the difference between the two comes from the presence of deadweight foreclosure costs). Pricing on the tree is undertaken by implementing the following backward recursion steps on the tree

1. *At maturity*: The lender's value of the loan at final maturity T is denoted by asset $B^l(T)$, and to the borrower is a liability $B^b(T)$, both equal to the terminal periodic payment S. The borrower will make the final payment as long as it less than the value of the house $V(T)$, adjusted for relocation costs. Thus, the put option to default has value $P(T) = \max(0, S - V(T) - K_R)$. Unless the house is almost worthless, it is unlikely that the put option to default will be exercised at maturity. If the put option is exercised, set $B^l(T) = \phi \cdot V(T)$, where $\phi \in (0, 1)$ is the fraction of home value recovered by the lender on foreclosure. Set $B^b(T) = V(T)$, because the borrower's liability does not exceed the value of the house when he defaults, and he extinguishes the liability by delivering the house to the lender. The difference in value of the loan to the lender and the liability of the borrower arises from the deadweight cost of foreclosure, i.e., $(1 - \phi)\, V(T)$. The loan balance will be $L(T) = 0$ at maturity.

2. *Backward recursion*: At nodes prior to maturity, the borrower will continue to service the loan provided the current value of his liability $B^b(t)$ (including the current due

payment) less relocation costs is less than the value of the house $V(t)$. Therefore, we discount the expected value from succeeding nodes and add the current periodic payment, required to keep the loan current, i.e., compute $B^l(t) = e^{-r(t)h} \cdot E^q[B^l(t+1)] + S$ (lender's point of view) and $B^b(t) = e^{-r(t)h} \cdot E^q[B^b(t+1)] + S$ (borrower's liability). Here $E^q[\cdot]$ denotes expectation under the risk-neutral probability measure. Likewise, we compute the continuation value of the option to default: $P(t) = e^{-r(t)h} \cdot E^q[P(t+1)]$. Also compute the remaining loan balance $L(t)$ using equation (11)—this is required to check if the prepayment option is to be exercised or not.

3. *Refinancing*: After the preceding step, check if the loan may be refinanced, i.e., is the borrower's current liability $B^b(t) > L(t) + S$? If so, then set $B^b(t) = L(t) + S$, because the refinancing option does not permit the value of the loan to exceed its current balance. If refinanced, also set $B^l(t) = B^b(t) = L(t) + S$.[14]

4. *Exercising the default option*: Next, check if the borrower's option to default is worth exercising at time t, i.e., determine if the value of the un-exercised default put option $P(t)$ is less than the value of immediate exercise, i.e., $B^b(t) - V(t) - K_R$ (note that $B^b(t)$ already includes the current due payment). The default option is exercised early when the value of the house $V(t)$ is less than the present value of the borrower's liability $B^b(t)$ (which accounts for optimal exercise of options on succeeding paths) less net relocation costs/benefits. Set $P(t) = \max[P(t), B^b(t) - V(t) - K_R]$. If default option exercise occurs, then set $B^l(t) = \phi \cdot V(t)$, where ϕ is the foreclosure recovery fraction. Set $B^b(t) = V(t)$ because the loan liability to the borrower does not exceed the value of the house. Note that $B^b(t)$ captures the value of the borrower's liability less value from future optimal exercise of default and refinancing options. It is deadweight costs of foreclosure that drive a wedge between the lender's value of the loan $B^l(t)$ and that of the borrower's liability $B^b(t)$. We need to keep track of $B^b(t)$ to determine the optimal decisions by the borrower, and of $B^l(t)$ to track the loan value to the lender.

5. *Iterate*: This completes processing of period t. Now, proceed to period $(t-1)$ and repeat steps 2–5. And so on, until $t = 0$.

This procedure computes the current value of the mortgage $B^l(0)$ to the lender, properly accounting for the strategic behavior of the borrower vis-a-vis default and prepayment. The

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[14] An additional condition that may be applied here is one where the lender refuses to allow refinancing unless the LTV is lower than a given cut off, for example 80%. In a loan modification situation, restructuring via interest-rate reductions is economically similar to refinancing. We do not restrict refinancing in order to allow the call option to have its highest value. Costs related to refinancing are also ignored. Hence, our approach is most conservative from the lender's point of view. Random repayments on account of borrower mobility are not modeled, since the loans we are considering are distressed ones, not prematurely repaid ones.

approach taken here for boundary conditions and optimal exercise behavior is similar to the model of Kau, Keenan, Muller, and Epperson (1992); Kau and Keenan (1999); and Ambrose, Capone and Deng (2001), but rather than solve a partial differential equation numerically, we set up a quadrinomial tree. In addition, the HJM term structure model here is richer than the CIR model used in past work, and is easier to calibrate to the initial term structure of rates and volatilities because no risk premium needs to be estimated. Further, the approach correctly accounts for the interlocking nature of the default and refinancing options, as well as the difference in loan/liability value between the lender and borrower on account of deadweight foreclosure costs. In this way our pricing model complements and contributes to extant models for pricing fixed-rate mortgages.[15]

2.3 Loan Modification

In the event of the homeowner defaulting on the loan, the bank has the option to foreclose or to modify the loan. A loan modification at time t involves changing the loan maturity date T, the principal balance $L(t)$, or the loan rate r_L. These modifications translate into a lower annual flat payment A. It is necessary that $A \leq A_{max}$, the maximum annual payment the borrower can afford. This amount may be set by a regulatory agency such as the FDIC, expressed as a function of the borrower's income, i.e., the housing-to-income (HTI) ratio.[16] However, if there is no feasible level of monthly payment that will work, then modification is moot. For example, if the borrower has no ability to pay, or can only pay at a level where the present value of the optimally modified loan is less than what the lender will obtain from foreclosure, then foreclosure is the optimal strategy.

There may be many level sets of A that are obtained by various combinations of $\{T, L(t), r_L\}$. Among these level sets, the lender will want to choose the set that maximizes the value of $B^l(t)$, after accounting for optimal default and prepayment behavior by the borrower. The borrower of course, is not indifferent across these loan configurations, and aims to minimize the present value of loan liability by optimally choosing to default or prepay.[17]

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[15] There are some technical differences in the approaches that are worth highlighting. In determining the value on exercise of the option, the literature points out that the option to default is in-the-money when the current value of the house is less than the present value of future payments due (e.g., Kau and Keenan (1999)). Another approach is to assume that the option is in-the-money by the amount of the unpaid loan balance minus the home value (e.g., Ambrose, Capone and Deng (2001)). In the paper here, we develop a third approach where we treat the option as being in-the-money by the difference between the borrower's current liability and the value of the house, i.e., $B^b(t) - V(t)$. We note that $B^b(t)$ correctly accounts for all future options to default/prepay, and is different from the present value of future payments, assumed in the extant literature. However the qualitative nature of the results should be the same under both approaches.

[16] The FDIC (2009) HAMP plan stipulates an HTI of 31–38%.

[17] Probably more importantly, it is likely that the different approaches and borrower condtions will influence

We ignore agency costs between the lender and borrower in this model. Harding and Sirmans (2002) use a single-period model to show how loan modification may be used to mitigate agency costs of underinvestment in the property by the borrower, or excessive risk-taking with the property—by choosing appropriately between maturity extension and principal reduction. Our model may be used to analyze the agency problem in a multi-period setting in future work.[18] For now, the analysis here is assumed to be agency cost-neutral.

The three choice variables—maturity T (or number of periods N), loan balance L, and loan rate r_L—are the determinants of the flat periodic payment A/m. Hence, if we hold fixed a given level of periodic payment A, then the choice of any two of these three variables automatically determines the third. In the next section, we compare different modified loans where the periodic payments are the same. We will examine the impact on loan value $B^l(t)$ as we change the loan rate r_L, the maturity T, and the principal balance $L(t)$, holding annual payment A fixed. This will deliver insights into the direction in which the loan parameters should be changed in order to maximize loan value.

3 Analysis

The initial driver of the loan modification decision is the debt service level that the borrower can maintain per year. This amount is denoted A_{max} as stated earlier. We first analyze the case when A_{max} is known with certainty, and then extend the analysis to the case with uncertainty. The amount paid per payment period is A_{max}/m, where m is the frequency of payments per year. If there are N remaining payment periods, and we set a loan rate r_L, then given A_{max} we can solve for the implied loan balance by re-arranging equation (10):

$$L = \frac{A_{max}}{m}\left[\frac{1-(1+r_L/m)^{-N}}{r_L/m}\right] \qquad (12)$$

This equation gives the loan balance L that we reset the loan to with loan rate r_L and maturity T to provide exactly A_{max} in loan service per year. To fix ideas, we use a base case. Assume there is a home with a current loan balance of \$300,000 but that the value of the home has fallen to $V_0 = \$250,000$, i.e., there is substantial negative equity. The current remaining maturity on the loan is $T = 25$ years, the loan rate is $r_L = 6\%$. The annual service payments on this loan amount to \$23,194.80. Suppose the lender determines

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the borrower's behavior. Theory and previous research—see Harding, Miceli, and Sirmans (2000)—shows that current LTV ratio influences owner maintenance expenditures.

[18] Risk-taking by the homeowner may be analyzed in the same model by adjusting volatility upwards. Agency costs may also be built into K_R. A full-blown analysis of dynamic agency costs in the Harding and Sirmans (2002) model in our multi-period framework is non-trivial.

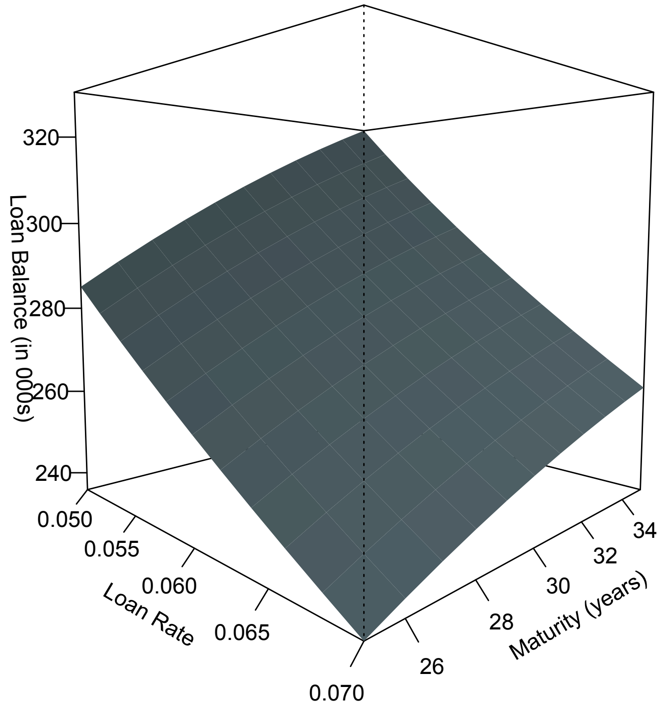

Figure 1: Loan balance for equal loan service. The plots are shown as the loan rate (r_L) and maturity (T) vary, keeping the debt service fixed at $A_{max} = \$20,000$ per year ($1,666.67 a month). The plot shows what the loan balance L_0 needs to be to constrain debt service at this level computed using equation(12). The number of periods per year is $m = 12$, and the maturity is $t = 25$ years.

that the borrower's ability to pay only supports an annual service amount of $20,000, and we decide to vary the maturity on the loan T from 25 to 35 years and the loan rate r_L from 5% to 6%. For each of these values, we compute the loan principal that delivers an annual service payment of $20,000. If we construct a 3D plot of L against $\{r_L, T\}$, we obtain an "iso-service" loan-level graph, i.e., a plot of L against r_L and T where the debt service requirements remain constant. For an example, see the plot in Figure 1. The lender's objective is to find the configuration of $\{r_L, T, L\}$ that gives the maximum loan value from amongst these iso-service loans.

From Figure 1 we can see that as the interest rate (r_L) rises, keeping maturity T constant, to remain on the iso-service surface for annual payments totaling $A_{max} = \$20,000$, the loan balance L must be reduced, i.e., the lender will write down the value of the loan. On the other hand, keeping interest rates fixed, if the lender increases the maturity of the loan, the loan balance will increase in order to remain on the iso-service surface. The latter situation is what is normally done by a bank when modifying a home loan via principal forebearance, i.e., the maturity is extended and principal is increased, usually by the amount of unpaid loan installments.

Our next few examples use the quadrinomial tree model to elicit salient guidelines for an optimal loan modification. The initial result is that for loans with negative equity, the optimal modification will require principal reduction, no lowering of loan rates, and no extension of maturity. Intuitively, these steps minimize the borrower's option to default on the loan, thereby maximizing the value of the loan for the lender.

Figure 2 presents results for our base case analysis. We consider five levels of ability to pay, by varying the annual payment in the set $A = \{17000, 18000, 19000, 20000, 21000\}$. For each of the these payments, we vary the loan rate in the set $r_L = \{5.0\%, 5.5\%, 6.0\%\}$, though we also explore vastly higher rates up to 8%. The maturity across all loans is fixed at $T = 25$ years. The upper left graph of the figure shows the principal balance that delivers the required service level A. The upper right graph shows the loan value for each level of interest rate, and annual payment level A. Looking at the upper left graph, we see that for each level of annual payment A, as we increase the loan rate r_L, the principal declines—a loan with higher principal will need a lower loan rate in order to keep the payments the same. We also see that as we increase the annual payment level A, the principal balance increases, because, keeping interest rates and maturity fixed, to get a higher payment level, the principal must be increased. The LTV ratio of the loan may be inferred from the principal balance L, because the home value is fixed at $V = 250,000$. In the upper left graph of Figure 2, we see that when $A = \{17000, 18000\}$, the principal balance is such that the LTV is less than one (except when $r_L = 0.05$), but when $A = \{19000, 20000, 21000\}$, the LTV is greater than one, i.e., there is negative equity.

In the upper right graph in Figure 2, the loan values calculated on the quadrinomial lattice are shown. There are several implications to be drawn from this graph.

1. When loans have negative equity, the default put option is more likely to be exercised, leading to a drop in the value of the loan to the lender because of the deadweight costs of foreclosure. Therefore, even when the borrower has the ability to make annual payments at a higher level, if the principal is not lowered to remedy the negative equity situation, the high probability of strategic default makes the loan less valuable to the lender, and suggests lowering the principal below the level based on ability to pay. Since the default put captures the borrower's willingness to pay, the loan should be reset such that the annual payment is the lower of the borrower's ability and willingness to pay.

2. Lowering the interest rate keeping maturity and service payment fixed, requires raising loan principal, pushing the loan further into negative equity territory, thus resulting in greater expected deadweight costs from a higher probability of default, and accordingly,

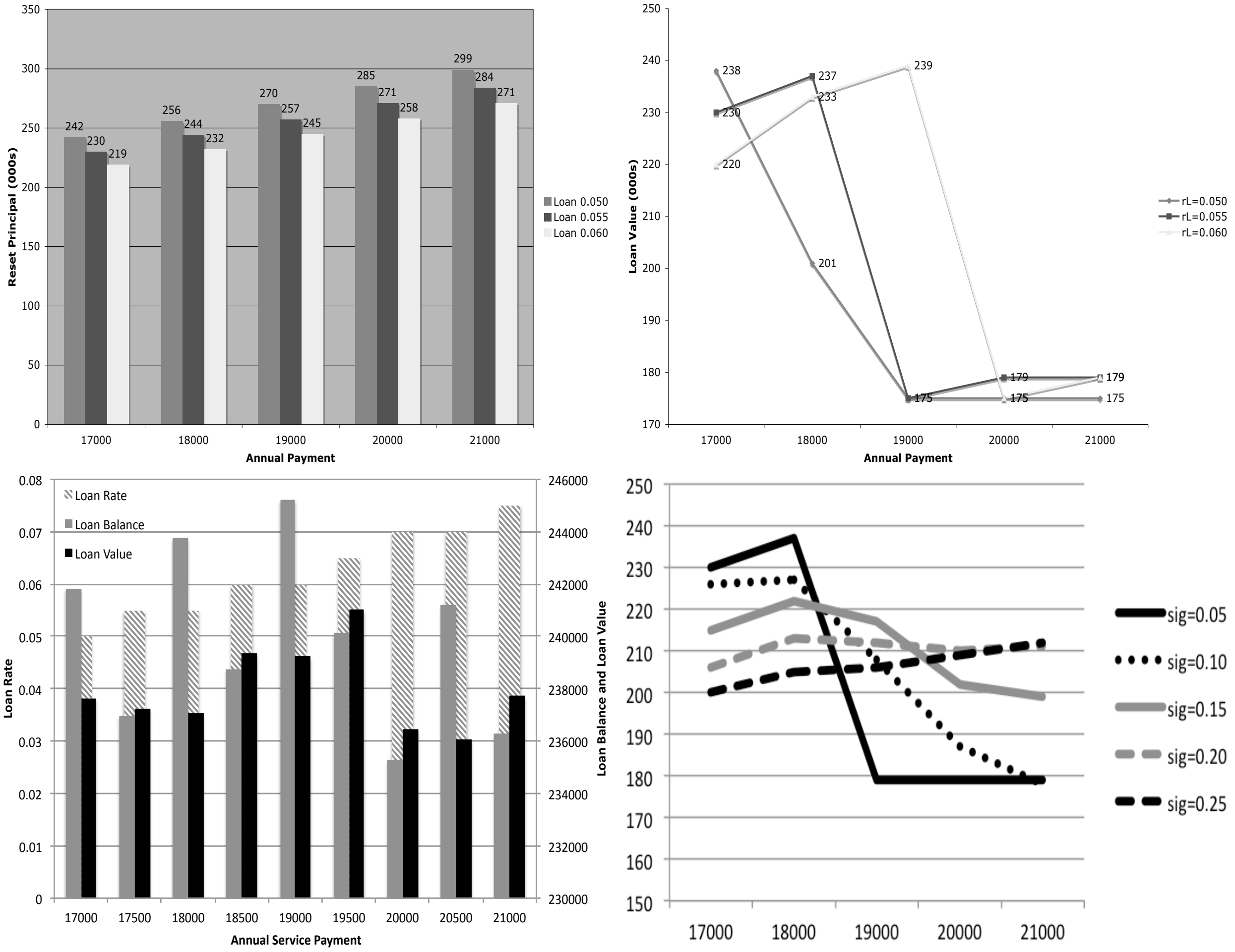

Figure 2: Loanvalues for iso-service loans. We consider five cases of ability to pay, by varying the annual payment in the set $A = \{17000, 18000, 19000, 20000, 21000\}$. For each of these payments, we vary the loan rate in the set $r_L = \{5.0\%, 5.5\%, 6.0\%\}$. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$ (increased in a later example), foreclosure recovery rate $\phi = 0.7$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%. The upper left panel of the figure shows the principal balance that delivers the required service level A. The upper right panel shows the loan value for each level of interest rate, and annual payment level A. The lower left panel shows the optimal principal balance and loan rate, along with optimized loan value, extending rates out to 8%, The lower right panel shows loan values for different total volatility $(\sigma_1 + \sigma_2)$, and loan rate $r_L = 0.055$. For robustness, we changed the time step from $h = 1/4$ to $h = 1/6$, with similar results, quantitatively and qualitatively.

lower loan value. We see from the upper right graph that initially, as annual payment A increases, the value of the loan increases, but when negative equity occurs, the loan begins to drop in value. The cut over point at which the loan value peaks and then begins to drop occurs at a higher annual payment for the 6% loan rate than for the 5% loan rate. See for example, that the 6% loan is worth much more at an annual service level of $19,000 than the two others loans at 5.5% and 5%. Therefore, rate reductions, keeping the service payment fixed, are not recommended, when the loan is at or close to a negative equity situation.[19]

3. The upper graphs also provide the intuition for an "optimal" LTV at which to reset the loan. Take the case of a loan at 6%. We see that the value of the loan is maximized when the annual payment is set to $A = 19000$. At lower levels of $A = 17000$, the payments on the loan are lower, and hence, the present value of loan payments is lower. Because the principal is also low, the probability of default is low. As we raise A from 17000 to 18000 to 19000, the present value of payments increases from the higher payment stream, but the principal has been set higher (because loan rate and maturity are fixed), so that there is an increase in the probability of default. Still, at $A = 18000$, the increase in payments offsets the higher probability of default, resulting in an increase in loan value. The same is true for $A = 19000$, where the loan value is maximized. When payments are raised to $A = 20000$, the iso-service loan balance is such that the loan enters negative equity territory and the expected deadweight costs of default outweigh the increased present value of payments, resulting in a steep drop in loan value. There is therefore a critical point at which the LTV must be set in order to maximize loan value. The experiments suggest that LTV cannot exceed unity by too much.

   The lower left graph summarizes the four variables of interest in one plot, i.e., annual service payment, optimal loan rate, optimal loan balance, and optimized loan value. As the annual service payment (ability to pay) rises, the optimal loan rate also rises, and the loan balance is adjusted to meet the annual payment. But after a point, the loan balance needs to be dropped to avoid negative equity, while continuing to raise the rate. Hence, when the borrower has the ability to pay, it is important to write down principal and raise rates to stave off strategic default. Conversely, when ability to pay is low, it is better to lower rates after mitigating negative equity.

4. The lower right graph in Figure 2 shows loan values when total volatility of home

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[19] Our analysis here examines changing one aspect of the loan only, but of course, one could imagine combination modifications, where rates and principal are both modified. Looking at these in isolation helps detect the driving dynamics of each loan feature. We look at rate modifications subsequently.

value ($\sigma_1 + \sigma_2$) is increased. Guerrieiri, Hartley, and Hurst (2010) noted that home value volatility can be quite different at the zip code level and can vary from aggregate Case-Shiller volatility. We notice that (a) When volatility is low, loan values are optimized by lowering the annual service payment substantially by cutting principal. (b) When volatility is high, loan value is optimized by keeping the annual service payment a bit higher (if ability to pay allows it). (c) As volatility increases, the value of the default put option also increases, and optimal modified loan values decline, but if the borrower has the ability to pay, then the principal write down is smaller than when volatility is low. This is because at higher volatility the propensity to delay exercising the option to default is higher, and this pushes up loan values in the presence of deadweight foreclosure costs. Intuitively also, using the notion of put-call parity, higher volatility implies a possibly greater upside to the homeowner in the value of the house, and incents a delay of exercise of the default option. To fix ideas, notice that when volatility is 5%, the optimal is at $A = 18000$, but when volatility is 25%, the optimal is $A = 21000$. However, the optimized value is lower at the higher volatility because the default put is worth more.

What is the role of rate reductions at different volatility levels? When ability to pay is low and we need to lower the annual payment substantially, loan value is optimized at lower interest rates. This pattern is the same irrespective of volatility level. Hence, there is no change in policy needed for different volatility zip codes when ability to pay is low. But when ability to pay is high, to stem off strategic default in low volatility zip codes, a greater principal write down is possible and a higher loan value is attained than in high volatility zip codes.

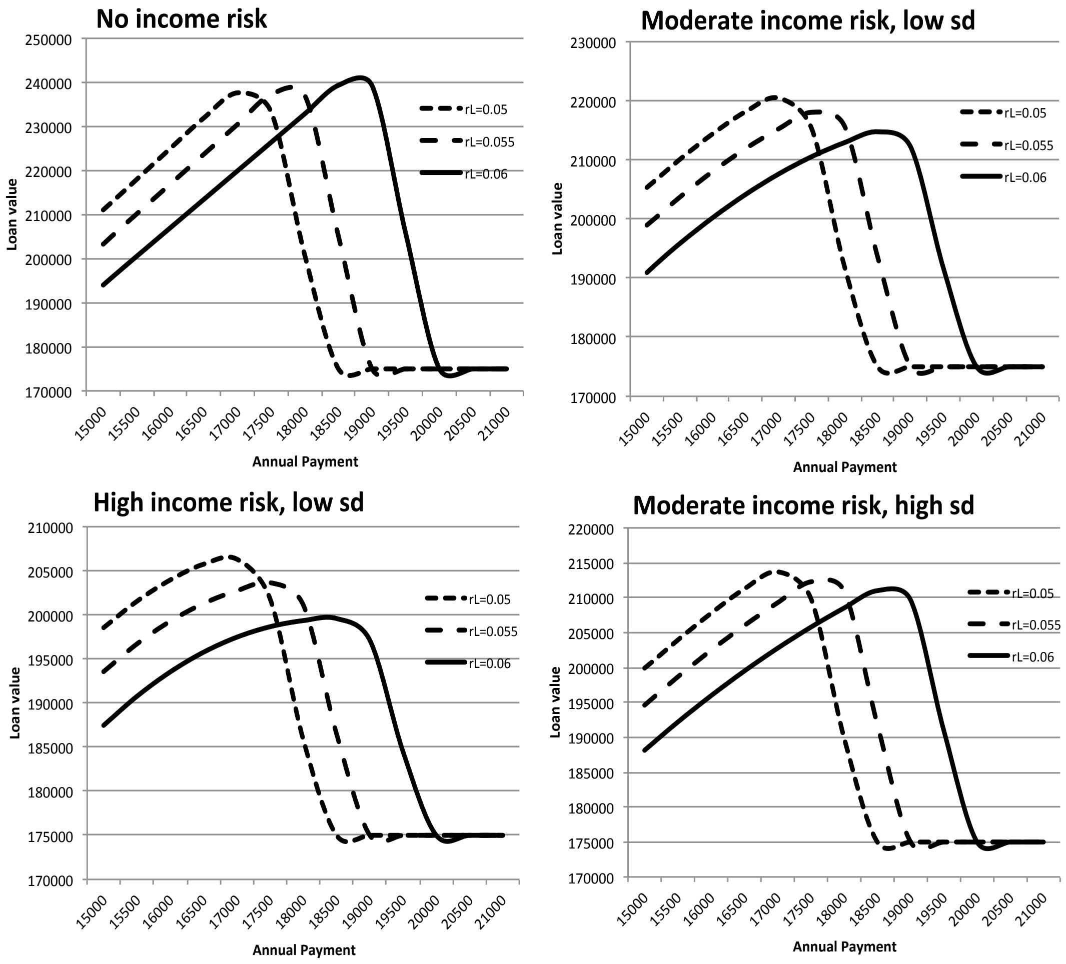
5. The analysis so far has focused on strategic default, i.e., the willingness to pay by the borrower. A larger question is whether the borrower can pay at all, i.e., the role of income risk, or ability to pay. Income risk is addressed in Adelino, Gerardi and Willen (2009) and the lender faces two types of risk. (a) Cure risk, i.e., the risk of unnecessarily providing loan relief, i.e., reducing the monthly payment, when the borrower would not have ultimately defaulted. (b) Redefault risk, i.e., a futile loan write down that eventually results in default occurring again. Assume an estimate of borrower income A available for housing service, with a mean of μ_I and a standard deviation of σ_I (the error of our estimate, normally distributed). The loan value, accounting for income risks is

$$\text{Loan Value} = \mathcal{B}(A) \cdot \left[1 - N\left(\frac{A - \mu_I}{\sigma_I}\right)\right] + \phi V_0 \cdot N\left(\frac{A - \mu_I}{\sigma_I}\right) \tag{13}$$

Here $\mathcal{B}(A)$ is the value of the loan accounting for willingness to pay as priced in Section 2 (see Figure 2), and this is multiplied by the probability that the borrower is able to pay. The second term in the equation above deals with redefault arising from lack of ability to pay, i.e., redefault risk after the loan is modified. All components of loan value depend critically on the modified service payment A. When considering both default from lack of ability to pay (income risk) and from willingness to pay (negative equity risk leading to strategic default), we see an interesting interplay between principal reduction and interest-rate reduction. If the loan is in a negative equity situation, and there is no income risk, principal reduction and no rate reduction is best, as seen in Figure 2. When ability to pay is uncertain, we expect to see that a further reduction in service payments is needed to offset default from income risk, and the optimal loan value is also lower given the additional risk of default. However, once the loan balance (and annual service payments) have been lowered to mitigate the negative equity in the loan, it is better to provide additional relief through rate reductions, *not* principal reductions.

These results are portrayed in Figure 3. The top left plot shows loan values when the ability to pay is known with certainty, i.e., $\mu_I = A, \sigma_I = 0$ (no income risk). This is the case that was previously considered in Figure 2. We see the familiar pattern where a reduction in service payments through principal reduction raises value. As we reduce annual payment levels, keeping the rate high gives the best loan value. Next, we inject some income risk: the top right plot has $\mu_I = 20,000, \sigma_I = 5,000$. This is a case of moderate income risk with some estimation error. As expected, the optimal loan values have been reduced from when there is no income risk. The optimal prescription now shows that a greater reduction in A gives the optimal loan value, accounting for uncertainty in ability to pay, and this is implemented partially through a rate reduction as well. In the bottom left plot, the mean ability to pay is further reduced to $\mu_I = 17,000$, and we see even lower loan values. The pattern of rate reductions for lower annual payments is also seen here. Finally, in the bottom right plot, where ability to pay is reset to $\mu_I = 20,000$ but estimation risk is increased to $\sigma_I = 10,000$, we see lower loan values than in the top right plot. This captures the effect of uncertainty of income or borrowing capacity as considered in Campbell and Cocco (2010). Overall, the presence of income risk calls for greater reduction in annual payments, to avoid the deadweight costs of foreclosure from lack of ability to pay. This compensates for the negative equity, removing willingness to pay risk, so that less principal write down is needed, and interest rates are reduced more for the same annual payment level.

Linked to the ability to pay question is an adverse selection problem. A lender facing a distribution of borrowers with varying ability to pay may propose a modified service payment A, but even borrowers for whom the payment A is affordable may still choose to default if that is more attractive at the current time, or becomes more attractive



No income risk

Moderate income risk, low sd

High income risk, low sd

Moderate income risk, high sd

Figure 3: Loan values for differing income risk. We vary the annual payment from \$15,000 to \$21,000. For each of the these payments, we vary the loan rate in the set $r_L = \{5.0\%, 5.5\%, 6.0\%\}$. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$ (increased in a later example), foreclosure recovery rate $\phi = 0.7$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%. The mean and standard deviation of the ability to pay (income risk) is varied across the plots: (a) top left plot, no risk; (b) top right, mean ability to pay \$20,000 per year, sd=\$5,000, moderate risk; (c) high income risk, mean=\$17000, sd=\$5000; (d) moderate income risk, with high range, mean=\$20,000, sd=\$10,000.

subsequently. The dynamic model in this paper accounts for this by optimizing loan value accounting for optimal exercise of the default option by the borrower. Nevertheless, the optimal loan modification does not stave off possible future default for sure, only sets the probability of such default to a level that maximizes the value of the loan to the lender.

6. It is important to note that as principal is written down and the annual payment is reduced, the role of loan rate changes. To the right side of the plots for levels of monthly payment that are not steeply reduced, keeping the interest rate high, i.e., writing down more principal holding A fixed, helps mitigate the negative equity problem and delivers greater value. But on the left side of the plots, when substantial annual payment reductions are called for and negative equity is no longer an issue, loan value is maximized by keeping the interest rate low, since it requires less principal write-down for the same annual payment level. In the case of ARM loans, a rate revision upwards at the end of a teaser period will increase income risk, and further relief will be needed. On the other hand, for an ARM, declining rates will make it easier to mitigate income risk.

7. The lower right graph in Figure 2 shows loan values when total volatility of home value ($\sigma_1 + \sigma_2$) is increased. Guerrieiri, Hartley, and Hurst (2010) noted that home value volatility can be quite different at the zip code level and can vary from aggregate Case-Shiller volatility. We notice that (a) When volatility is low, loan values are optimized by lowering the annual service payment substantially by cutting principal. (b) When volatility is high, loan value is optimized by keeping the annual service payment high (if ability to pay allows it). (c) As volatility increases, the value of the default put option also increases, and optimal modified loan values decline, but if the borrower has the ability to pay, then the principal write down is smaller than when volatility is low. This is because at higher volatility the propensity to delay exercising the option to default is higher, and this pushes up loan values in the presence of deadweight foreclosure costs. Intuitively also, using the notion of put-call parity, higher volatility implies a possibly greater upside to the homeowner in the value of the house, and incents a delay of exercise of the default option. To fix ideas, notice that when volatility is 5%, the optimal is at $A = 18000$, but when volatility is 25%, the optimal is $A = 21000$. However, the optimized value is lower at the higher volatility.

   What is the role of rate reductions at different volatility levels? When ability to pay is low and we need to lower the annual payment substantially, loan value is optimized at lower interest rates. This pattern is the same irrespective of volatility level. Hence, there is no change in policy needed for different volatility zip codes when ability to pay

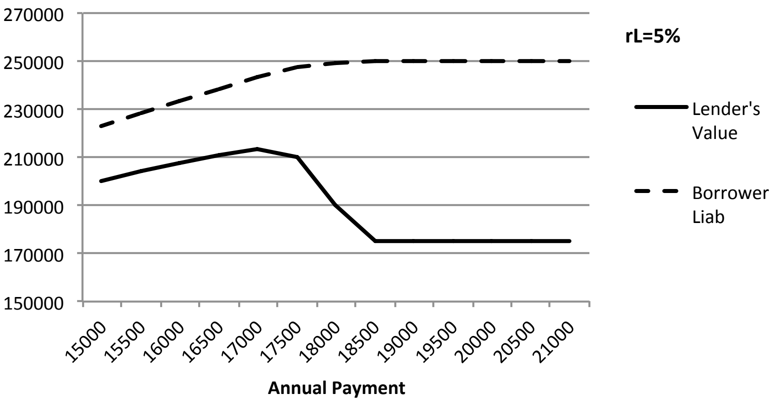


Figure 4: Loan values and borrower's liability with income risk. We vary the annual payment from \$15,000 to \$21,000. The loan rate is $r_L = 5.0\%$. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$ (increased in a later example), foreclosure recovery rate $\phi = 0.7$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%. The mean and standard deviation of the ability to pay (income risk) is $\mu_I = \$20,000$ per year, sd=\$5,000.

is low. But when ability to pay is high, to stem off strategic default in low volatility zip codes, a greater principal write down is possible and a higher loan value is attained than in high volatility zip codes.

8. One might well ask, what is the modification that maximizes the value to the lender among all that leave the borrower the same? More generally, since we are taking the viewpoint of the lender, is the borrower worse off? Figure 4 shows that both lender and borrower are better off, considering both ability- and willingness-to-pay effects. At high levels of annual payment with negative equity, the loan will default and the borrower's liability is capped at the home's value. As the annual payment is reduced, the present value of borrower's liability eventually falls, whereas the loan value to the lender increases as expected deadweight costs of foreclosure decline. Hence, reducing the service burden is pareto-improving to lender and borrower! In short, for underwater homes, the borrower is not worse off, even if the loan is optimized from the lender's viewpoint.

There are further insights on negative equity loans. The reason that loan value drops off sharply when the default option is in-the-money comes from two salient features of mortgages: (a) low return volatility of house prices, and (b) convex and declining (in time) strike prices for the default option. An examination of Case-Shiller indices reveals that house price volatility is low in comparison to equities. Using the U.S. composite of 20 regional Case-Shiller indexes, the SPCS20R-SA monthly series, from January 2000 to June 2010, we find that annualized volatility of returns on the index is just 3.5%. When volatility is low, the continuation value of a put option that is in-the-money is low, and there is little to be gained from the borrower postponing immediate exercise of the default option. Figure 5 shows a slice of the 3-D tree of home values and superimposes on this tree the region where the default put is optimally exercised. The panels of the figure from top to bottom show the default exercise region for loans with principal balances of \$200,000, \$225,000, and \$250,000 respectively, relative to a house value of \$250,000. The home values over 25 years demonstrate the shape of exponential growth that derives from the stochastic process in equation (1). The left-hand side of the panels shows the entire evolution of home values and the region in which the default option is exercised (lower part of the tree). The right-hand side of the panels zooms in on a region of home values between zero and \$600,000 so that the exercise region may be viewed in more detail.

Notice that as we proceed from the top panel down, the loan moves from positive equity to zero equity. Correspondingly, the earliest possible time of exercise moves from about two years for the top panel to one year for the middle panel, to almost immediate in the lower panel. Second, the upper envelope of the exercise boundary is declining and concave, because the strike level of the default option declines as time moves on, since the principal balance is being paid off periodically. This early-exercise boundary for mortgages is different from that of standard American-style options, where the boundary is increasing and convex (exactly the opposite of what we have here). Given the shape of the exercise boundary shown in Figure 5, it becomes clear that the option to default on a mortgage is best exercised almost as soon as negative equity occurs, because a declining exercise boundary results in a low continuation value of the put option, especially since volatility underlying the option is also very low.

We move on to a consideration of relocation costs. A borrower is less likely to entertain foreclosure if there are relocation costs. We imposed a fixed relocation cost of \$20,000 and calculated loan values, shown in Figure 6. Comparing the lower graph of this figure to Figure 2 we see that the optimal point that maximizes loan value moves to the right, i.e., a higher annual payment may be set by the lender on loan modification given that the relocation costs require the default option to be more in the money than previously, before the borrower is willing to default.

Finally, we examine the implication of increasing loan maturity while remaining on the iso-service loan surface. Note that as maturity is increases, keeping the loan rate fixed, the principal balance must be increased to remain at the same annual service payment. This makes it more likely that the loan will be restructured with negative equity, making default

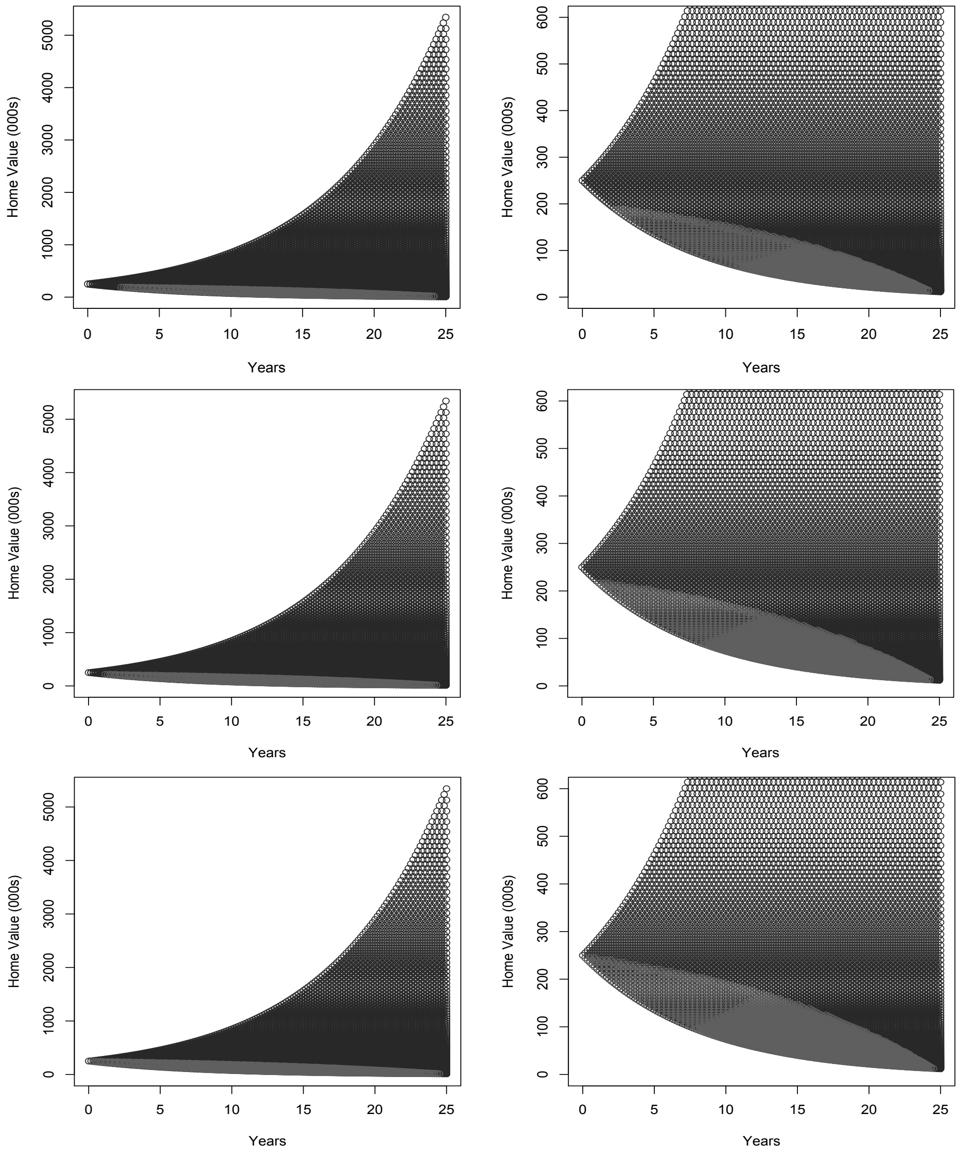


Figure 5: Default put option exercise region. The evolution of house price is shown with the lower section depicting the exercise regions for three loan principal balances: \$200,000, \$225,000, and\$250,000, given a current home value of \$250,000. The upper panel shows the exercise region when the loan principal is \$200,000, the middle panel for principal equal to \$225,000, and the lower panel is for principal equal to \$250,000. The left-hand side of the panels shows the entire evolution of home values and the region in which the default option is exercised. The right-hand side of the panels zooms in on a region of home values between zero and \$600,000 so that the exercise region may be viewed in more detail. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$, foreclosure recovery rate $\phi = 0.7$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%.

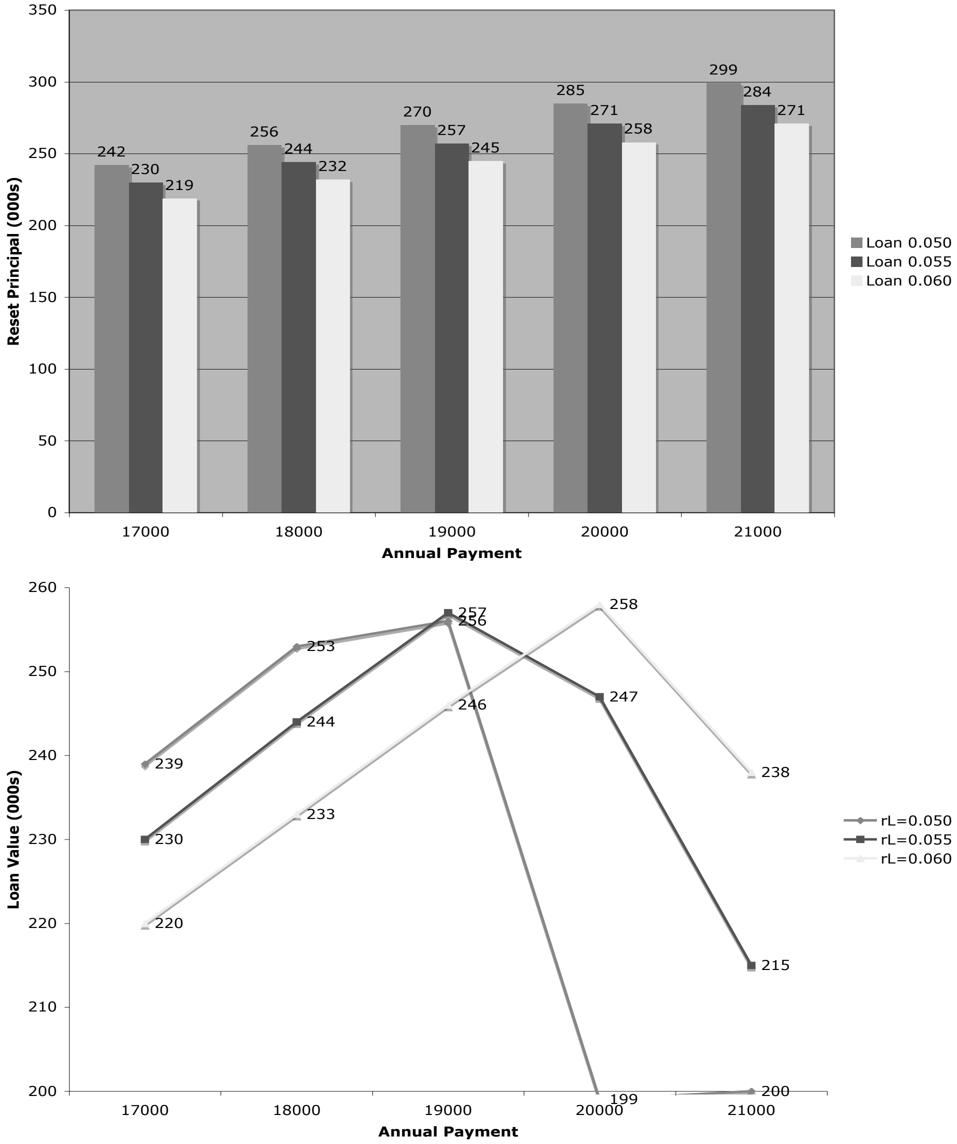


Figure 6: Loanvalues for iso-service loans with nonzero relocation costs. We consider five cases of ability to pay, by varying the annual payment in the set $A = \{17000, 18000, 19000, 20000, 21000\}$. For each of the these payments, we vary the loan rate in the set $r_L = \{5.0\%, 5.5\%, 6.0\%\}$. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, relocation costs $K_R = \$20,000$, foreclosure recovery rate $\phi = 0.7$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%. The upper panel of the figure shows the principal balance that delivers the required service level A. The lower panel shows the loan value for each level of interest rate, and annual payment level A.

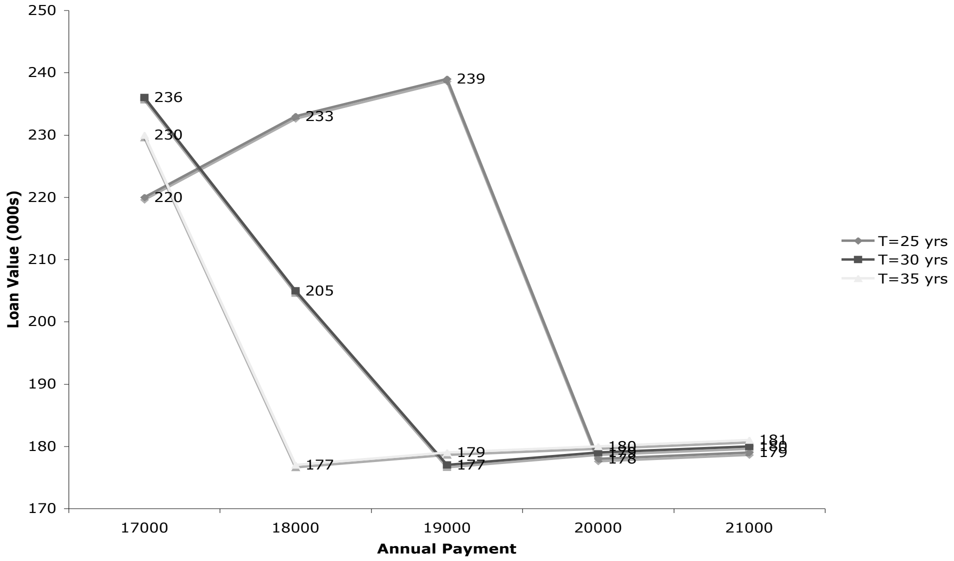


Figure 7: Loanvalues for iso-service loans with varying maturity. We consider five cases of ability to pay, by varying the annual payment in the set $A = \{17000, 18000, 19000, 20000, 21000\}$. For each of the these payments, we vary the loan maturity in the set $T = \{25, 30, 35\}$ years. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$, foreclosure recovery rate $\phi = 0.7$, and a flat forward rate curve at 5%.

more likely and resulting in a drop in loan value as maturity is increased. This outcome is clearly seen in Figure 7. When maturity is set to 25 years the annual payment at which the loan value is maximized is greater than when loan maturity is set to 30 or 35 years. The maximized loan value is also higher for the shorter maturity loan than the ones of longer maturity.

We summarize the results of this analysis in terms of guidance for loan modification. First, negative equity loans result in rapid default, especially when volatility is low. Second, keeping service levels fixed, rate reductions in the presence of negative equity result in lower optimal loan values because they entail higher principal balances and greater expected deadweight foreclosure costs. Third, maturity extensions result in lower optimal loan values for the same reason that lower rates do. Fourth, it is important to note that principal reductions in negative equity situations result in sharp increases in loan value to the lender, especially when the deadweight costs of borrower default are high. Finally, once principal reduction has mitigated the negative equity problem, income risk is optimally managed via rate reductions.

3.1 Mean-reversion in House Prices

The standard geometric Brownian motion with constant drift does not account for possible mean-reversion in home prices. For instance, it is likely that house prices will experience positive returns for some years followed by a period of negative drift. In our framework, we can adjust the drift of the process by making the flow parameter δ of the house price process a function of time. The parameter $\delta(t)$ is set to the net value of the growth/decline (according to mean reversion) in house prices and service flow.

To illustrate, using the same base case as before, we set normal service flow to be $\delta = 1\%$ for last 20 years of the entire 25-year life of the loan. But we allowed δ to take three different possible values for the first five years of the loan, i.e., $\delta = \{-0.01, 0.01, 0.07\}$ and then revert to 0.01 for the remaining 20 years. When $\delta = -0.01$, it means that house prices are drifting upwards at 2% per year, hence, net of standard service flow of 1%, the "dividend" rate becomes -1%. When $\delta = 0.01$, it means that house prices are assumed to grow at 0%, etc. As δ moves from negative to positive territory we expect that house price growth rates fall and loan values will drop. We show these results in Table 8. We also varied volatility between two levels by letting $\sigma_2 = \{0.03, 0.10\}$. As expected, loan values are lower when volatility is higher as the default put becomes more valuable. This approach to embedding mean-reversion is quite flexible and we may choose a different δ each period if so desired, without impacting the recombination property of the quadrinomial tree. Of course, risk-neutral probabilities on the tree are adjusted depending on δ as may be noticed from equation (7).

3.2 An Empirical Illustration

It is useful to ask whether these optimal modifications have the potential to stem re-default rates on modified loans. Using an illustrative sample of modified loans from a large financial institution, we assessed this question using logit regressions of re-default on the amount of changes in the loan rate, principal balance, and maturity. As controls, we used the LTV of the loan, the debt ratio of the household after the loan modification, and the percentage reduction in the monthly payment on the loan. The loan-to-value ratio of the home (LTV) is a proxy that captures the moneyness of the default option, and captures the willingness to default. The debt ratio of the household is a proxy for the borrower's characteristics. The percentage payment change (PPC) is the percentage amount by which the monthly payment on the loan was reduced. This is a proxy variable for the improvement in the ability to pay of the borrower. The sample covers two years, 2007 and 2008, and results are reported

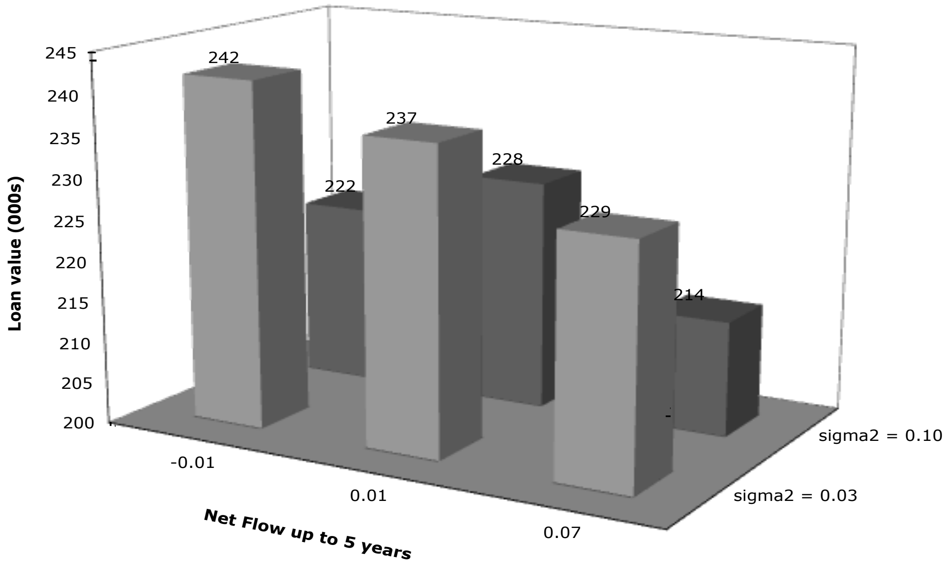


Figure 8: Loanvalues for with varying drift (net service flow incorporating mean-reversion). We set loan balance to \$245,000. Loan maturity is 25 years. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = \{0.03, 0.10\}$, service flow level $\delta = \{-0.01, 0.01, 0.07\}$ for the first five years and $\delta = 0.01$ thereafter, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/6 years$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$, foreclosure recovery rate $\phi = 0.7$, and a flat forward rate curve at 5%. The figure shows the loan value for three levels of δ in the first five years of the loan, and for two levels of volatility σ_2.

separately by year. The columns in the table stratify the analysis into five monthly payment ranges, thereby looking at the data in terms of equal ability to pay after loan modification.

We see from Table 1 that term extensions do not reduce subsequent re-default rates,[20] but that rate reductions and principal write-downs are effective in doing so. The magnitudes of the chi-square statistics show that in 2008, principal reductions are especially effective. Clearly the LTV of the loan also matters—the greater the LTV at modification, the higher the subsequent re-default rate. This suggests that making the default option held by the borrower less in-the-money is an effective way to contain re-default.[21]

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[20] Potential unobserved effects can affect the results. For example, certain mortgages, more likely to re-default after a modification, might be more likely to be securitized (due to agency problems) and in turn are then prevented from principal writedowns. These mortgages, if they obtain a modification, will necessarily have one of the other modifications, and at the same time are the ones more likely to re-default, and as a consequence this may explain the inefficacy of term modifications. We thank the referee for pointing this out.

[21] The goal here is *not* to provide a full-blown empirical analysis of the re-default of modified loans. Our intent in this paper is to provide an analysis of why principal reductions are likely to be better than rate reductions or maturity extensions. The logit results here are only indicative of the fact that principal reductions may be the most effective form of loan modification. We were not given access to the data—the bank that provided the data retained it and ran the regressions as per our request. Permission was only given to present just this one table of results from among all the empirical results obtained.

Table 1: Logit regressions of re-default. The table presents logit regressions to explain the re-default rates of modified loans based on the modifications undertaken. A re-defaulted loan is one that is 90 days delinquent in the six-month period following the loan modification. The three variables that characterize the loan modification are the changes (old value minus the new value) in interest rate on the loan, the principal balance, and the maturity (term) of the loan. The other three variables in the logit regressions are proxy variables. The loan-to-value ratio of the home (LTV) is an proxy that captures the moneyness of the default option, and captures the willingness to default. The debt ratio of the household is a proxy for the borrower's characteristics. The percentage payment change (PPC) is the percentage amount by which the monthly payment on the loan was reduced. This is a proxy for the improvement in the ability to pay of the borrower. The sample covers two years, 2007 and 2008, and results are reported separately by year. The columns in the table stratify the analysis into five monthly payment ranges, thereby looking at the data in terms of equal ability to pay after loan modification. The value to the right of the Wald statistic for the regression is the p-value. For parameter estimates, a chi-square statistic of greater than 3.84 is significant at the 5% level.

	Monthly Payment Amount after Modification									
2007	< 450		450-620		620-820		820-1120		> 1120	
	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2
Intercept	-2.91090	9.935	-3.23960	14.896	-1.75030	5.092	-2.71130	9.844	-1.47930	5.899
ΔRate	-0.11310	0.796	-0.28570	4.212	-0.31070	3.931	-0.33240	5.760	-0.36810	8.224
ΔTerm	-0.00640	5.427	-0.00133	0.220	0.00114	0.079	0.00016	0.002	-0.00038	0.008
ΔPrincipal	-0.00017	5.323	-0.00007	2.303	-0.00007	5.633	-0.00004	3.567	-0.00004	7.157
LTV	0.02270	6.143	0.02530	8.212	0.00163	0.040	0.02200	6.358	0.00141	0.043
Debt Ratio	-0.02110	2.021	-0.01060	0.802	-0.00291	0.056	-0.01790	3.182	-0.01150	1.357
PPC	-0.01000	0.421	-0.01340	1.888	-0.00324	0.033	-0.00463	0.106	0.00077	0.004
Re-Default	43		59		66		89		93	
No Default	230		330		382		459		545	
Wald Stat	18.9061	0.0043	18.8779	0.0044	15.1021	0.0195	23.4206	0.0007	20.8749	0.0019

	Monthly Payment Amount after Modification									
2008	< 450		450-620		620-820		820-1120		> 1120	
	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2
Intercept	-0.66310	3.595	-1.77870	18.532	-2.47180	39.303	-1.77290	22.438	-1.89300	39.145
ΔRate	0.13490	7.877	-0.16530	4.650	-0.01360	0.037	-0.30900	14.547	-0.14070	4.826
ΔTerm	0.00077	0.408	0.00028	0.042	0.00024	0.038	-0.00138	1.399	0.00035	0.143
ΔPrincipal	-0.00005	2.328	-0.00008	18.808	-0.00006	21.597	-0.00007	47.587	-0.00004	41.344
LTV	-0.00267	0.675	0.00806	4.207	0.01370	13.809	0.00653	3.627	0.00859	12.609
Debt Ratio	-0.01710	5.000	-0.00693	1.027	-0.00356	0.344	-0.00025	0.002	-0.00113	0.068
PPC	-0.02540	12.026	-0.00572	0.453	-0.01130	1.826	0.00540	0.417	-0.00754	1.286
Re-Default	177		205		286		375		522	
No Default	747		819		1000		1071		1564	
Wald Stat	32.3832	< 0.0001	42.1224	< 0.0001	46.1375	< 0.0001	83.5345	< 0.0001	81.2047	< 0.0001

3.3 The Shared Appreciation Mortgage (SAM)

In a standard variation in mortgage restructuring, a lender may agree to reduce the monthly payment on a loan to bring it down to a level that is affordable to the distressed borrower, and in return, the lender takes an equity share $\theta \in (0, 1)$ in the home, conditional on the home value increasing above a predetermined strike level K (which may be set to the current home value to strike the appreciation share option at-the-money). This is known as a "shared-appreciation mortgage" (SAM) or "home equity fractional interest" (HEFI) loan.[22]

Is the borrower more or less likely to strategically default if his loan is restructured as a SAM? Recall that a loan modification impacts both, the borrower's ability and willingness to make loan payments. SAMs enhance the borrower's ability to pay as they enable the lender to afford greater loan write-downs, but it may increase the willingness to re-default because the borrower has a smaller stake in the upside value of the home. However, a SAM delays immediate foreclosure, and postpones incurring resultant deadweight costs. An ex-post recovery of the housing market, rather than a decline, makes a SAM efficacious.

Another desirable property of a SAM is that it discourages adverse selection against lenders. Strategic borrowers with negative equity in their homes may be unwilling to make loan payments even though they have the ability to do so. Such borrowers have every incentive to approach the lender for a principal reduction by pretending that they are distressed. However, if these borrowers are offered a SAM, they may be less willing to behave strategically, as they would have to part with some of the upside in their home equity in return for the principal write-down. Since the SAM feature affects the default option, we account for adverse selection in our analysis.[23]

We enhance our pricing algorithm to account for appreciation sharing as follows. At each node of the pricing tree we carry the value of the upside in home value, i.e., an American call option $C(V(t), K, t)$ with strike K. Beginning with the terminal nodes on the tree, we use backward recursion to determined the price of this option at earlier nodes. At each node, the borrower accounts for the fact that he owns less of the home, i.e., $V(t) - \theta \cdot C(V(t), K, t)$, and that his exercise value of the default option is now given by $B^b(t) - [V(t) - \theta \cdot C(V(t), K, t)] - K_R$. Thus, the default option is more likely to be exercised as θ increases. We must also

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[22] The original idea for HEFI loans is described in O'Brien (2008).

[23] SAMs may support a separating equilibrium between borrowers who have low ability to pay (hopeless defaulters) versus those with low willingness to pay (strategic defaulters). Without a shared-appreciation structure, it is harder to separate the two types of borrowers. However, a strategic defaulter is less willing to take a modification at payment level A that has a greater rate reduction than principal reduction. A hopeless defaulter is only interested in relief so is more willing to take a rate reduction offer at the same payment level A, even if no principal reduction is offered.

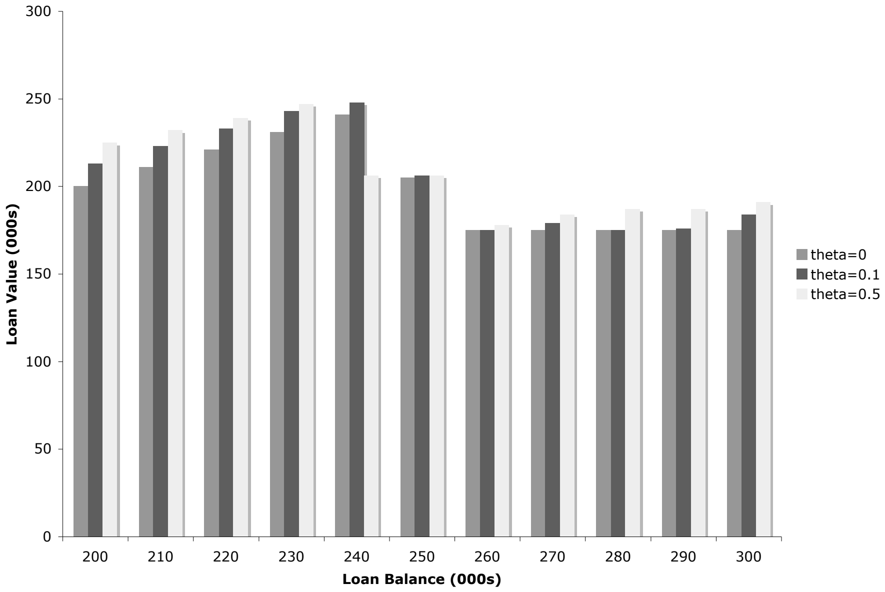

Figure 9: Loan values with appreciation sharing. The annual payment is $A = 19,000$. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$, foreclosure recovery rate $\phi = 0.7$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%. Appreciation share θ takes values in the set $\{0, 0.1, 0.5\}$, and the strike of the appreciation sharing agreement is \$250,000.

be careful in the backward recursion step to make sure that we set $C(V(t), K, t) = 0$ at each node where default occurs, because foreclosure eliminates the value of the appreciation share. And finally, the lender's loan value $B^l(t)$ is increased by $\theta \cdot C(V(t), K, t)$. These simple enhancements to the model enable pricing mortgages with the appreciation share feature.

Figure 9 shows the impact of appreciation sharing on loan values, adjusted for default and prepayment options. The familiar pattern of loan values seen in earlier plots is also evident here. Loan values are higher when there is positive equity and are lower in regions of negative equity in the presence of deadweight costs of foreclosure. When there is positive equity, as the lender's share θ increases, the loan value increases as well because the borrower will not default, yet a share is taken by the lender. Therefore, as shown in Figure 9 up to a loan balance of 230K, an increase in appreciation share θ results in increases in lender loan value. But when the loan balance is 240K, raising θ from zero to 0.10 results in an increase in loan value, but a further increase in θ to 0.50 triggers default, because the borrower does not wish to stay in the mortgage if he keeps only half the upside of home value. However, once the home value is in the negative equity region and default is imminent, increasing θ is advantageous again because on the small chance that the borrower does not default, the upside share is greater.

The loan modification with appreciation share is therefore quite simple to structure. After a principal reduction that brings the loan into positive equity territory, the appreciation share should not be too large. A borrower with a reasonable upside is less likely to default. This reduces expected deadweight costs and increases loan value through the upside-sharing component.

4 Relation to the extant environment

The theoretical analysis so far has shown that, across schemes that are matched on ability to pay, principal write-down modifications maximize the willingness to pay, thereby maximizing the loan's economic value to the lender. And once the negative equity problem has been staunched, rate reductions mitigate default from income risk. The OCC and OTS Mortgage Metrics Report (2009) covers two-thirds of all mortgages outstanding in the US, including subprime mortgages. More than half of the reported modifications entailed reduced monthly payments. Two-thirds of the modifications were combination modifications, entailing changes in two or more of loan rates, maturity, or adjustments to principal. Only 1.8% were write-downs of the principal balance. As a consequence the "re-default rate" evidenced is extremely high, about two-thirds of modified loans in 2008.

Managing negative equity (or loan-to-value ratios) is very important in manipulating the borrower's willingness to pay. Foote, Gerardi, Goette and Willen (2009) find that a 10% fall in house prices raises the probability of delinquency by more than 50%. Liebowitz (2009) analyzes a database from McDash Analytics of 30 million loans and shows that it is not the features of the loan (subprime, Alt-A, etc.) that explain foreclosure, but the extent of negative equity in the home matters more. While only 12% of the loans had negative equity, they accounted for 47% of all foreclosures. More than half of foreclosed homes are funded by prime loans, and the foreclosure rate for prime-loan homes grew at twice the rate of subprime homes. Liebowitz also reports that the level of induced foreclosures from raising rates is small—rate increases did not lead to greater foreclosures unless they were greater than 4%.

Loan modifications have shied away from writing down principal, because the accounting impact would be much more negative than with other types of iso-service loan modifications. Adelino, Gerardi and Willen (2009) report that the incidence of principal reductions in the data is low, and this may be on account of adverse incentive effects, where borrowers who are not in danger of default may be induced to seek foreclosure in the hope of having their loan balances written down. Lenders do have incentives to play hardball, as evidenced in residential mortgage renegotiation in the Great Depression (Ghent (2010)). Still, given that foreclosures have contagion effects on other homes in the neighborhood—see Harding, Rosenblatt and Yao (2009)—it may be more important for the tide of foreclosures to be stemmed by optimal loan write-downs.

The key insight in why writing down principal works is that it results in lower foreclosure rates, mitigation of the deadweight costs of foreclosure,[24] and an overall higher economic value of the loan to the lender, after accounting for the borrower's option to default—see Goodman (2010) for an excellent analysis of why the negative equity problem must be tackled head on with principal modifications. The recent introduction of the HAMP-PRA (Principal Reduction Alternative, effective October 2010) scheme by the Federal government acknowledged the tole of principal modification in the arsenal aimed at stemming foreclosure. Borrowers may be further incentivized to remain current on loans in a principal-reduction modification by deferring their rewards, as in the Responsible Homeowner Reward (RHR) scheme in Edmans (2010).

Hedge funds that invest in distressed home loans have recognized the optimality of principal write-downs. They buy loans from banks at deep discounts and are better able to manage the loan modification by writing down principal. Simon (2009) reports that—"Some mortgage investors have made principal reduction a part of their strategy, in part because it gives borrowers who owe more than their houses are worth an incentive to keep making payments. It is also easier to ultimately refinance or sell the mortgage if the borrower has equity."

5 Conclusions and Discussion

The housing crisis is deepening with about 3 million loans being foreclosed in the last three years, and forecasts of 5–7 million more by the end of 2012.[25] Levitin (2009) provides forceful arguments for the efficacy of mortgage modification in the current crisis. The analysis in this paper has shown that effective modifications require setting the loan rate on the mortgage equal to the current mortgage rate for the given borrower class, no extensions of maturity, and a writing down of the principal balance. These guidelines become more costly to violate when the lender's deadweight costs of foreclosure are high. White (2009a) argues that the failure of the 2008 voluntary mortgage contract modifications is attributable to the reluctance

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[24] This deadweight cost, also known as the "foreclosure discount" comprises damage repairs to restore the house to a sale-able condition, a distress sale discount, brokerage commissions and direct selling costs, taxes, insurance, and property management, and interest on capital for the holding period.

[25] See dsnews.com: July 22, 2011, by Krista Franks. An accompanying poll reports that only 49% of homeowners believe they are not underwater on their mortgages.

to write down principal.[26]

A suitable loan modification scheme must be cognizant of both, the borrower's ability to pay *and* willingness to pay. The latter criterion, dependent on the incentive effects of the modification, accounts for large differences across loan modifications that are otherwise neutral to the borrower's ability to pay. Therefore, in this paper, we optimize willingness to pay, after fixing the ability to pay. Further, once optimal principal write-downs mitigate willingness to pay risk, rate reductions may be used optimally to mitigate default from income risk.

The prescription to write down loan balances in a modification is bitter medicine for lenders, made especially so on account of accounting conventions that incentivize lenders to push losses into the future. The shared appreciation mortgage (SAM) is an important innovation that deserves more detailed analysis in future work. We showed that SAMs improve the borrowers ability to pay, reduce the deadweight costs of foreclosure and mitigate adverse selection in loan modification programs.

We did not consider the impact of securitization on loan modification—our analysis begins when the decision to modify the loan has been taken, assuming it is permissible. However, servicer restrictions and incentives might vary for securitized loans, and the empirical evidence on whether securitization affects loan modification is quite mixed. Piskorski, Seru and Vig (2008) show that securitization induces a foreclosure bias on the part of servicers. Elul (2009) points out that securitized loans perform worse than non-securitized ones and therefore, we should observe more modifications of securitized loans. Adelino, Gerardi and Willen (2009) show that the empirical reluctance to renegotiate loans is not driven by securitization.

There is no doubt, however, that pooling and servicing agreements (PSAs) for securitized loans often prohibit changes to principal, which rules out the channel for modification theoretically supported in this paper. Modification of securitized loans and the legal impediments to it are part of a larger game played between the homeowners, servicers and tranche holders in the securitization. Typically the AAA tranche, often comprising 80% of the collateral in the securitized pool will not agree to a loan modification that entails principal write-downs, even though it is optimal for them to do so as shown here. A possible approach would be to offer a government guarantee to these tranche holders if they would allow a modification, as from a societal point of view, this would stem the rate of foreclosures as well as eliminate the dead-weight costs of foreclosure.[27] Another approach to reducing foreclosure rates is to

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[26] See also White (2009b). As of July 2011, the pace of principal reductions has increased under the HAMP-PRA program, with a median principal reduction of 32.2%, or a median reduction of $69,500 (dsnews.com: July 01, 2011 by Carrie Bay).

[27] Suggested by Paul Kupiec of the FDIC.

promise a principal write-down in a year or two, provided the homeowner makes monthly payments till then. The incentive effects of all these modifications are also an issue that we leave for further modeling in subsequent work.

While this paper has focused on mitigating adverse incentive effects on willingness to pay in loan restructuring, Foote, Gerardi, Goette and Willen (2009) point out that it is also important to institute reforms that will enhance the ability to pay, i.e., economic stimulus steps to raise employment and stem the fall in house prices. This long-term solution to the housing crisis may be complemented by short-term tactical loan modification based on the guidelines developed in this paper.

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