CONNECTING DATA WITH COMPETING OPINION MODELS

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OUTLINE

- Introduction and background
- Motivation and empirical findings
- Description of our model and agreement with data
- Analysis of the model
- Conclusions and future work
INTRODUCTION TO OPINION DYNAMICS

- Basic Idea: opinions or ideas propagate via social interactions between individuals

- Applications: adoption of languages, products, religions, etc.

- Why is important to model opinion dynamics?
  - Basic understanding: How do human interactions give rise to complex social phenomena?
  - Practical: how can we create the largest impact, e.g. vote shares, spread of social norms
INTRO TO OPINION DYNAMICS, CONT’D

- A statistical mechanical framework for opinion modeling:

  Atoms : properties of condensed matter → individuals : opinion dynamics

- How do we understand opinion formation?
  - Empirical data
  - Intuitive models (but are they well motivated?)
MODELING OPINION DYNAMICS

- Simplest model: adopt the opinion of a randomly chosen neighbor (Voter Model)
- Often seen (e.g. Voter Model Universality Class)
- Influential

Clifford and Sudbury, Biometrika (1973)
Redner MAPCON (2012)
TYPICAL MODEL ISSUES

- Inconsistent phenomenological models
- Motivation often based on results and not the dynamics themselves
- Unrealistic results (e.g., in equilibrium, everyone has a single opinion)
GOALS OF OUR MODEL

- Phenomenological: capture statistical patterns observed in data
- Realistic macro-dynamics: e.g., slow convergence to consensus/non-consensus in equilibrium
- Realistic micro-dynamics: Well-motivated mechanistic rules for the spread of ideas
MOTIVATING DATA

- **Top:** universal scaling for a candidate’s share of votes
  
  \[ Q: \text{# candidates (opinions)} \]
  
  \[ v: \text{# voters for a candidate (opinion holders)} \]
  
  \[ N: \text{# votes (nodes)} \]

- **Bottom:** vote correlation
  \[ \sim - \log(\text{distance}) \]

- Stubbornness (resistance to opinion change) seems to increases with time

UNIFYING OBSERVATIONS WITH A SINGLE MODEL

Our Model: Competitive Contagions with Individual Stubbornness (CCIS Model)

- Contagion-like spread of opinions (opinions are “caught” like a cold)
- The longer an individual has an opinion the less likely they are to change it (stubbornness)
- Individuals occasionally re-evaluate their opinions and “recover” to the unopinionated state

Well motivated micro-dynamics:

- Contagion-like spread, stubbornness, and recovery are all in qualitative agreement with empirical data
MODEL ASSUMPTIONS

- Fixed, undirected, network with $N$ nodes, and $Q$ opinion states
- A node $i$ is in state $\{0, 1, 2, \ldots, Q\}$ ($0$ is the un-opinionated state)
- At time $t = 0$, $n_0$ nodes are in state $0$, $n_1$ with opinion $1$, $\ldots$, $n_Q$ with opinion $Q$. 
MODELING DYNAMICS ON NETWORKS

- Dynamics modeled on a variety of networks
- Representative of real social networks

(a) Random network
(b) Scale-free network
(c) Lattice Network
(d) “Small-World” Network
ARE ASSUMPTIONS REASONABLE?

- Fixed number of opinions: e.g. jury verdict (“guilty” vs. “not guilty”)

- Assume a few (not necessarily all) individuals exhibit an innate opinion preference
  - Candidates not well known in the beginning
  - Some individuals seeded with preferences, e.g. candidate himself
MODEL OVERVIEW

- Convert un-opinionated neighbor with probability $\beta$

- Adoption probability decreases with time an individual $i$ has had an infection ($\tau_i$)

- After $\tau_i > 1/\mu$, an opinion “freezes”

- Opinionated (even frozen) individuals recover at a rate $\delta$
EXPLAINING VOTE DISTRIBUTIONS

- **Goal:**
  - Fit several voter distributions with consistent parameters

- **3 fitting parameters**
  - Persuasiveness ($\beta = 0.5$)
  - Initial fraction with a candidate preference (6%)
  - Network degree distribution ($p(k) \sim k^{-2.9}$)
  - All other parameters fixed

Vote probability distributions, shifted by decades (inset: original data collapse).

Closed markers represent empirical data. Open markers represent model results.
EXPLAINING OBSERVATIONS: LONG-RANGE CORRELATIONS

- Assume if $\delta$ (recovery) $\to 0$, model behaves diffusively
  - $C(r) \sim -\log(r)/\log(t)$
    - (2 dimensions)
- Slow consensus!
- Simulations corroborate this behavior
AGREEMENT WITH SIMULATION

- 2D grid with rewired connections (with no recovery or stubbornness)
  - 2D: Correlation falls as $\sim \log(r)$,
  - Real network has small geodesic distance ("small world")
  - Rewire: small world, with long-range correlations

A Lattice and "Small World"-type Graph
ANALYTIC TREATMENT: MEAN FIELD APPROXIMATION

Density Of Individuals With Opinion $A$ $\rho_A(t, \tau)$

Time $(t)$

Length Of Time Since Adopting $A$ $(\tau)$
**MEAN FIELD ANALYSIS**

- Mean field PDE:
  \[
  [\partial_t + \partial_\tau] \rho_A(t, \tau) = -\delta \rho_A(t, \tau) - \sum_{B \neq A} \beta k \theta(1 - \tau \mu)[1 - \tau \mu] \rho_A(t, \tau) P_B(t)
  \]

  \[P_A(t) \equiv \int_0^{\infty} \rho_A(t, \tau) d\tau\]

  - Cyan: losses due to “recovery”
  - Blue: losses due to competing opinions

- Close agreement with simulation:
  
  **Right:** Difference in \(Q = 2\) opinion densities versus persuasiveness.

\[\Delta P(t) = P_B(t) - P_A(t)\]

\[\Delta P(0) = 0.02, 0.05, 0.1, 0.2\]

\[N = 10^5, K\text{-regular random graph with } K = 10\]
A CLOSER LOOK AT THE MEAN FIELD EQUATION

\[
[\partial_t + \partial_\tau] \rho_A(t, \tau) = -\delta \rho_A(t, \tau) - \sum_{B \neq A} \beta k \theta(1 - \tau \mu)[1 - \tau \mu] \rho_A(t, \tau) P_B(t)
\]

\[
\rho_A(t, 0^+) = \delta(0^+) \beta k \left[ \sum_{B \neq A} P_B(t) \right]
\]

\[
[1 - P_A(t) + \sum_{B \neq A} -P_B(t) + \int_0^{\mu^{-1}} \rho_B(t', \tau')[1 - \tau' \mu] d\tau']
\]

Cyan/Blue: Loss/gain due to conversion to/from
Blue: contrary opinions
Cyan: the unopinionated state

Red: Total loss
Green: Total gain
MOTIVATION FOR A SLOW CONSENSUS MODEL

- Relatively little consensus in day-to-day ideas, even after many years:
  - GM vs. Toyota?
  - Obama vs. Romney?
- Long-range vote-correlations often = model with consensus
- Realistic models should have either slow consensus or non-consensus.

Example of Consensus

Friday, October 31, 14
SLOW CONSENSUS SIMULATIONS

- Consensus time minimized at non-trivial parameter values
- Analytic treatment: assume Voter Model-like dynamics
- Theory: $T_{\text{cons}} \sim \mu^2$, where $\mu$ is individual “stubbornness”
DEPENDENCE ON NETWORK SIZE

- In the mean field limit, consensus time scales as:
  \[ T_{cons} \sim \frac{N}{\langle k^2 \rangle (\beta \gamma / \mu)^2} \]

- For scale-free networks:
  \[ T_{cons} \sim N^{2(\alpha-2)/(\alpha-1)} \]

- Complete graph:
  \[ T_{cons} \sim 1/N \]
IMPLICATIONS OF OUR MODEL

- Checking model assumption: does stubbornness increase in time?

- Checking model prediction: if stubbornness = brand loyalty:

  Brand share probability should follow a distribution alike to the right figure

Rescaled Brand-share Probability Distribution

- \( P(v, Q, N) \)
- \( v: \text{Number of items sold} \)
- \( Q: \text{Number of brands} \)
- \( N: \text{total number of items} \)
SUMMARY OF RESULTS

- We created a minimal model with:
  - Strong agreement to empirical data
  - Realistic macrodynamics
  - Well motivated microdynamics
- Found analytically tractable results on dynamics and consensus
- Presented ways to further corroborate our model
ADDING GREATER REALISM

- Random opinion flipping
  - 3(!) critical points
  - Potentially in the Ising Model Universality Class

![Graph showing opinion difference versus random flip rate]
OPEN QUESTIONS

- What is the relation between model time-steps and the real world?
- How can we determine the microdynamics, versus aggregate macrodynamic data?
- How does stubbornness evolve?
Thank You

Questions?