Procurement Auctions and Eroding Price Contracts in the Presence of Learning by Doing

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Abstract

In markets where suppliers experience learning by doing over time or, more generally, economies of scale in production, buyers are auctioning off longer-term contracts with an eroding price policy. Under an eroding price contract, the buyer initially competitively awards production to the lowest-bid supplier via an auction. Before the auction takes place, the buyer makes it clear to the suppliers that, if chosen, a sequence of price reductions will be mandatory in subsequent periods. In this paper, we study the design of the optimal eroding price contract in a two period setting where the magnitude of the cost reduction due to learning by doing is uncertain. We go on to compare the performance of the eroding price policy against sequential independent auctions under two cost frameworks : (1) Every supplier faces a new cost in each period, and (2) The supplier who wins the auction in the first period locks-in his cost for the future. Via analytical and numerical comparisons, we find that even in the presence of learning by doing/economies of scale in production, a buyer is often better off running sequential auctions with a reserve price, rather than limiting competition and contracting with a single supplier in the hopes of extracting a better future price.

1. Introduction

Since the concept of an "online procurement (reverse) auction" emerged in the mid-1990's, it has become an increasingly popular and powerful tool for outsourcing. Success stories at Hewlett-

Packard(Mohara et al.(2005)), Motorola(Arensman(2004), Minahan(2004), Metty et al.(2005)), and Volkswagon (Beall et al. (2003)), to name a few, have grabbed the attention of procurement managers across industries attempting to identify ways to reduce costs. One clear lesson that has emerged from the use of auctions in a variety of markets is that 'one size does not fit all'. Auction makers must take into account the characteristics of the market in designing their auctions.

For example, in the automotive industry, which has been an early adopter of B2B auctions, it is very common that each customer has his own design specifications even for pseudo-commodity products, such as leather car seats. Under such settings, a supplier frequently can not use the same production line for two different customers due to different design/engineering spec, e.g., colors, leather quality, number of wrinkles, etc. As a result, the supplier must incur a buyer-specific set-up cost to tailor the production line to meet the customer's specific demands. In addition, as the supplier (and often the buyer) learns more about the buyer's likes/dislikes, he is able to produce the good more efficiently and cost effectively. In markets such as this, where suppliers experience learning by doing over time and/or economies of scale in production, buyers are auctioning off longer-term contracts with an eroding price (a.k.a. cascading price) policy. Under an eroding price contract, a buyer awards a multi-period contract where the price paid to the supplier declines over time at a prespecified rate. The elements of a long-term contract, where a supplier is guaranteed a buyer's business, or is at least given the right of first refusal, combined with a pre-specified declining price are thought to be effective in identifying a win-win situation for both the buyer and winning supplier. What often complicates the design of these contracts is that the size of the cost reduction from learning by doing/economies of scale is not known with certainty at the contracting date.

While the benefits and advantage of such a procurement mechanism may at first seem intuitive, we are unaware of any work that has (*i*) studied the optimal design of the eroding price contract and/or (*ii*) demonstrated that an eroding price contract is more cost effective than alternative popular procurement methods. This paper aims to further our understanding of the appropriateness of eroding price contracts under various market settings. We assume that the buyer faces the same N potential suppliers in a two period setting, and compare the performance of eroding price contracts (EPC) against sequential independent auctions (SI) under two cost frameworks,

• No-Lock-In (NLI): Every supplier faces a new cost in each period. This assumption is reasonable when the duration between bidding periods is long or technology and industry environment changes fast (e.g., the electronics industry); under such a scenario a supplier may experience changes in its own subcontractors, the price of raw materials, financial situation, etc., and find itself with altered costs in the next bidding round.

• Lock-In (LI): The supplier who wins the auction in the first period ($T = 1$) 'locks-in' his (baseline) cost for the future; all other suppliers redraw their costs in the second period ($T = 2$). This setting can reflect a situation where the winning supplier is able to use his newly won contract to write a contract to lock-in his own input prices.[1]

We find that, while the form of the optimal eroding price contract is different depending under LI and NLI, sequential independent auctions perform better than the eroding price contract under many parameter settings for both cost frameworks. That is, the buyer is often better off running sequential auctions with a reserve price, rather than limiting competition and writing a long-term contract with a single supplier in the hopes of extracting a better future price.

Section 1.1 gives an overview of the relevant literature; we discuss our models and assumptions in sections 2 and 3. Using game-theoretic methods, we study the equilibrium behavior of suppliers under EPC and SI, and compare their performance under the NLI setting (section 4) and LI setting (section 5). We discuss the relevance of our results to procurement managers, extensions and future directions for research in sections 6 and 7.

1.1 Literature Review

Learning by doing can be more generally interpreted as a positive synergy (or complementarity) over a bundle of goods, where the cost of a bundle of goods is less than the sum of the costs of the goods individually. Krishna and Rosenthal (1996) study the bidding behavior of global and local bidders in a multi-unit 2^{nd} price auction, where a local bidder has positive value for only one (particular) object while a global bidder has positive synergies associated with winning more than one object. The global bidder's valuation from winning only one object is x, while his value from winning two objects is $2x + \alpha$, where x is i.i.d. and α is a *known* constant shared by all the global bidders. A key characteristic of this paper is that the global bidder faces a different set

[1]Under symmetric and strictly monotonically increasing bid functions at $T = 1$, the incumbent supplier would always be the least cost supplier at $T = 2$ if suppliers who lost at $T = 1$ kept the same costs across both periods; hence, we ignore this simple case.

of local bidders in each auction; since local bidders participate in only one of the auctions it is a (weakly) dominant strategy for a local to bid his true valuation. Elmaghraby(2005) examines a setting closer to our own, in that she assumes that the same set of local bidders can participate in both auctions, hence even local bidders will behave strategically and not bid truthfully in both auctions. This paper also studies sequential auctions of complementary goods; however, we consider a setting where (*i*) all of bidders are global bidders, implying that all of the bidders can participate and potentially win in both auctions, although only one will experience a cost reduction, and (*ii*) bidders who lose in the first auction redraw their costs at $T = 2$.

The models studied in Von der Fehr and Riis(2003), Grimm(2003), Jeitschko and Wolfstetter(2002), and Menezes and Monteiro(2003) are closer to our own. They each study a setting where all the bidders are global bidders and assume that the winner in the first auction has a cost advantage over the losers in the subsequent auction. However, all of the papers, with the exception of Von der Fehr and Riis(2003) assume a NLI cost setting, and hence assume that the costs in the first and the subsequent auction are independent for every bidder, including the winner in the first auction.[2] Our work deviates from their studies in three important ways: We (*i*) introduce and study the optimal design of an eroding price contract allowing for the size of the complementarity to be uncertain, and (*ii*) extend the analysis of sequential independent auctions to a LI setting; Von der Fehr and Riis(2003) also consider a LI cost setting, but focus only on establishing the equilibrium bidding behavior in sequential auctions.

There are a number of papers that address procurement in a dynamic setting (please see Elmaghraby(2000) for an overview of the literature). Many of the papers that have studied repeated procurements have focused on procurement strategies for the U.S. Department of Defense (DoD) procurement. DoD procurement is often comprised of multiple stages, encompassing concept design, R&D, initial production, and full production, and typically any cost reductions brought about by the incumbent are transferred to any new suppliers. Rob(1986), Anton and Yao(1987), Riordan and Sappington(1989), and Laffont and Tirole(1988) address the government's procurement strategy and discuss how the buyer can extract optimal effort from the developer (i.e., the selected

[2] Von der Fehr and Riis(2003) and Jeitschko and Wolfstetter(2002) study the effect of the future market opportunities on the bidding behavior and the auctioneer's selling price in the first and the subsequent auctions, while Grimm(2003) and Menezes and Monteiro(2002) compare the performance of a bundle auction with sequential independent auctions.

supplier) in the R&D stage under asymmetric information and experience (i.e., only the supplier knows his cost). In contrast, we focus on repeat procurement in a setting where the product(s) to be supplied are relatively well-defined (i.e., the buyer is not concerned with R&D) and where the learning advantage of the existing supplier is not transferable to other suppliers.

Under a static framework which does not consider possible shifts in supplier costs due to learning by doing/economies of scale, Bulow and Klemperer(1996) argue that a buyer is always better off maximizing the number of bidders (Corollary 1, pg. 189) and hence a buyer should not favor procurement mechanisms where she 'locks-in' to a single supplier in return for the supplier offering her a more competitive price. They consider a framework where a seller faces $N + 1$ buyers; the bidders' valuations can be either independent or affiliated (but are symmetric). Bulow and Klemperer compare the performance of two selling mechanisms; (*i*) an English auction with N bidders and a take-it-or-leave-it price offered to the last remaining bidder (where the take-it-or-leave-it price is determined after the seller has had the opportunity to gather information from the bidding process) and (*ii*) an English auction with one more bidder, $N + 1$, but without any additional negotiation stage. Under this setting, they prove that one additional bidder always yields a higher expected revenue to the seller than any negotiation process with one less bidder. This paper aims to answer if Bulow and Klemperer(1996)'s result carries over to a dynamic setting with synergies.

2. Model

A buyer wishes to procure Q units of a good in each of two time periods. The buyer wishes to sole source in each period, and incurs a switching cost of s if she switches suppliers at $T = 2$. A switching cost may occur due to the cost of running a new auction, training the new supplier about characteristics specific to that buyer and/or the cost of moving parts or changing tooling from the incumbent supplier to a new supplier. We assume that s is common knowledge.

The buyer faces $N \geq 3$ potential suppliers: We assume that all N suppliers have been prequalified to participate in the auction and have sufficient production capacity to fill the buyer's entire order. Thus, the buyer's sole criterion when selecting a supplier is price. Given the suppliers' adequate capacity, we can assume without loss of generality, that $Q = 1$.

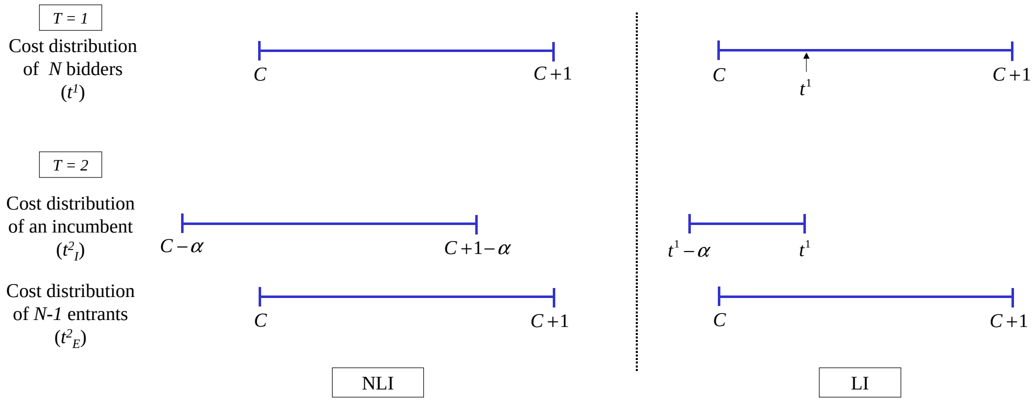

Figure 1: Costs at $T = 1$ and $T = 2$

A supplier t's cost at time T is given by t^T, for $T = 1, 2$. We assume that each bidder's cost at time T is private information and is drawn independently from a continuous and differentiable distribution. At $T = 1$, each bidder's cost is drawn from the *same* distribution $F(t^1) : F(C) = 0, F(C+1) = 1, C > 0$ with a corresponding density $f(t^1)$. We assume that $f(t^1)$ is strictly positive everywhere on $[C, C + 1]$.

A supplier who is not selected at $T = 1$ is referred to as an *entrant* at $T = 2$, and redraws his (privately known) cost at $T = 2$, t^2_E, from the same (original) common distribution function F (see figure 1). We refer to the supplier who fulfills the buyer's demand at $T = 1$ as the *incumbent* supplier. After supplying the buyer for one period, the incumbent supplier experiences a reduction in his production cost at $T = 2$ as a result of learning by doing and/or economies of scale in production (for the remainder of the paper, we will refer to this cost reduction as a learning by doing effect). We assume that the maximum possible amount of cost reduction is common knowledge and the same for all bidder types and is given by α (we consider relaxing this assumption in section 6). α is assumed to be small relative to C so that $C - \alpha \geq 0$ (without loss of generality, we assume that $C \geq 1$, and α is a fraction of 1). We define the incumbent's cost at $T = 2$ as t^2_I. In this paper, we study two different cost structures for the incumbent: (1) Under the No-Lock-In (NLI) setting, the incumbent's cost at $T = 2$, t^2_I, is drawn from the distribution $F_I(t^2_I) : F(C-\alpha) = 0, F(C+1-\alpha) = 1$, which shifts downward by α from F. (2) Under the Lock-In (LI) setting, the incumbent's cost at $T = 2$ is drawn from the distribution $F_I(t^2_I) : F(t^1 - \alpha) = 0, F(t^1) = 1$ (see figure 1).[3] All

[3]We may consider that the incumbent's cost at $T = 2$ is composed of two elements : his baseline production cost

distribution functions are common knowledge.

It is important to emphasize that this type of cost reduction occurs as a result of production (e.g., worker familiarity and experience with the production process) during $T = 1$ and is not the result of any cost-reducing measures that the supplier may undertake, such as upgrading equipment, educating workers, etc. The potential for learning by doing can be quite substantial in some industries: NASA hosts a learning curve calculator at http://www.jsc.nasa.gov/bu2/learn.html, and publishes statistics that estimate the cost savings arising from climbing the learning curve to be 15% in aerospace, up to 20% in shipbuilding and up to 25% in repetitive electrical operations.

We assume that all participants are risk-neutral and maximize(minimize) their expected profits(costs) and that N is common knowledge. Furthermore, we assume that suppliers behave rationally and they take into account expected future profit streams when determining their optimal bids at $T = 1$. The buyer (credibly) commits herself in advance to a procurement mechanism, i.e., a set of allocation policies and payment contracts. In this paper, we describe and compare two different types of procurement mechanisms that are currently being used in B2B auctions: (1) an Eroding price contract with conditional commitment (EPC), and (2) Sequential independent auctions (SI). Without loss of generality, we focus on direct mechanisms, whereby suppliers bid by reporting a cost(type), which may differ from their true cost. The winner is selected based on the reported cost and the buyer uses the suppliers' reported types in designing her optimal eroding price schedule.

3. Procurement Mechanisms

Mechanism 1: Eroding Price Contract (EPC) (see figure 2) When it is common knowledge that a supplier may experience some learning by doing and hence a reduction in production costs at $T = 2$, some buyers wish to use a long-term contract to exert some control over the price that they pay at $T = 2$ and ask their suppliers to adhere to an eroding price contract. Under an eroding price contract, the buyer holds an auction at $T = 1$ to select a supplier and each supplier simultaneously submits a bid indicating the amount he wishes to be paid per unit at $T = 1$. The lowest bidding

and learning effect cost reduction. Under NLI setting, the learning effect cost is deterministic and is given by α, while the baseline production cost at $T = 2$ is uncertain. In contrast, under LI setting, the size of the learning effect is uncertain, while the supplier's baseline production cost is known to be t^1.

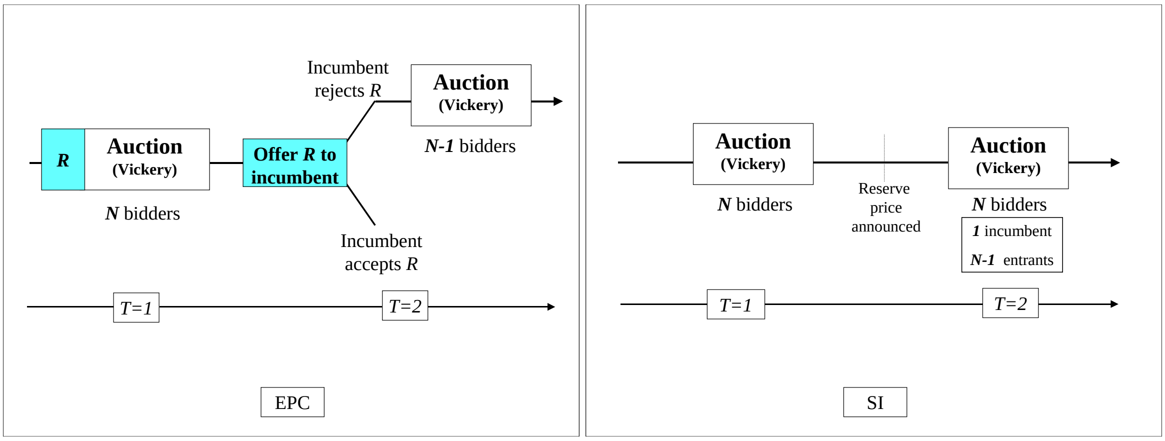

Figure 2: Procurement Mechanisms

supplier is awarded the buyers' business at $T = 1$ and is paid the lowest rejected price (Vickrey auction).[4] However, the buyer announces upfront a price schedule to which the winning supplier must adhere. That is, before the auction at $T = 1$, the buyer announces an eroding price schedule, $R(B^1_{EPC}(t^1))$, which states for each submitted bid at time $T = 1$, the price the supplier will be paid at $T = 2$. Under strictly monotonically increasing bid functions and a direct mechanism, we can model the eroding price schedule as a function of a supplier's true type, i.e., $R(B^1_{EPC}(t^1)) = R(t^1)$.

We assume that $R(\cdot)$ is continuous, differentiable everywhere and that $R(t^1)$ in the range of the incumbent's cost, i.e., $R(t^1) \in [C - \alpha, C + 1 - \alpha]\ \forall\ t^1$ under NLI, and $R(t^1) \in [t^1 - \alpha, t^1]\ \forall\ t$ under LI. This implies that the buyer does not consider prices that are guaranteed to make the supplier unprofitable at $T = 2$ or pay the supplier more than his highest possible cost.

Under EPC, the incumbent supplier is given the *option* to reject $R(t^1)$ *after* he has observed his cost at $T = 2$. If the incumbent decides to reject $R(t^1)$, then the buyer holds a second (Vickrey) auction with the remaining $N - 1$ suppliers at $T = 2$, where the incumbent is not invited to participate in the second auction. The rationale for excluding the incumbent is as follows: If the incumbent cannot participate in a second auction, then it is optimal for him to accept $R(t^1)$ if

[4]The most common auction format adopted by FreeMarkets is an open descending price auction. Under this format, the winning bidder is paid the price at which his last opponent drops out of the auction. It is well-known that a Vickrey auction and an open-auction yield the same outcome in the single unit case (Myerson(1981)). We use a Vickrey auction as a stylistic representation of the open ascending auction format, with the acknowledgment that the two formats *may* not lead to the same equilibrium bidding behavior under our modeling assumptions. The strategic and revenue equivalence of the two auction formats remains an open research question.

$R(t^1) > t_I^2$ and reject it otherwise. If the incumbent were not excluded from the second auction, then he would have the incentive to trade-off his profit under $R(t^1)$ with his expected profit from participating in the second auction, and may not accept $R(t^1)$ even if it is profitable. Under EPC, each supplier has two decision variables; his bid at $T = 1$ and his bid at $T = 2$ if he should lose at $T = 1$ and the incumbent should reject the contract at $T = 2$.

Mechanism 2 : Sequential Independent Auctions (SI) (see figure 2) Under SI auctions, the buyer prefers to maintain short-term contracts with her suppliers, i.e., she commits to a supplier for only one period, and selects the winning supplier in each period via a Vickrey auction. The buyer uses information gathered in the first auction to set the second period reserve price, r^2; if $B_{SI}^1(t^1)$ is the winning bid at $T = 1$, the buyer sets r^2 to be the incumbent's highest possible cost at $T = 2$, i.e., $r^2 = C + 1 - \alpha$ under NLI and $r^2 = t^1$ under LI.[5]

It is common in SI auctions for the buyer to announce upfront that she will not switch suppliers unless the cost savings from an alternative supplier are greater than the cost of switching. Under SI, the buyer adds on the switching cost s to each *submitted* bid by an entrant at $T = 2$ to derive an *effective* bid for each entrant (note that the *submitted* bid and *effective* bid are the same for the incumbent supplier). If, given the effective bids, the lowest cost bidder is an entrant, then the buyer will pay him the second lowest *effective* bid minus s; if an incumbent is selected then she pays him an amount equal to min[reserve price, the second lowest *effective* bid].[6] We define F_E to be the entrants' 'adjusted' cost distribution with support $[C + s, C + 1 + s]$, i.e., $F_E(C + s) = 0, F_E(C + 1_s) = 1$.

Table 1 summarizes the strategy space in each of the two mechanisms. We use $B_M^T(t^T)$ to denote the bidding strategy used by supplier t under procurement mechanism M in the T^{th} auction. Given the frequent use of these two mechanisms, we pose and answer the following two questions:

- What is the optimal eroding price schedule $R(t^1)$ under EPC?

[5]Setting r^2 in this manner is not necessarily optimal; we still find that SI outperforms EPC under a variety of market settings. This result is only further strengthened if the reserve prices in both periods are set optimally under SI (Oh and Elmaghraby(2005) study the optimal reserve price policy under NLI setting).

[6]For example, if $s = 2$, the incumbent's bid at $T = 2$ is equal to 10, and the first and second lowest submitted bids from entrants are 6 and 9 respectively, then an entrant wins and is paid 8 (=10 - 2). Note that this payment rule can result in the buyer paying more than the second lowest submitted bid to a winning incumbent at $T = 2$. For example, if $s = 2$, the reserve price at $T = 2$ is 10, the lowest submitted bid from entrants is 9 and the incumbent's bid is 7; then the incumbent will win and be paid 10 (although the second lowest submitted bid is 9, the effective bid is 11). We adopt this payment rule for the sake of consistency.

Table 1: Strategy space in each procurement mechanism.

Mechanism	Buyer	Suppliers
EPC	$R(t^1)$	$B^1_{EPC}(t^1),\ \ B^2_{EPC}(t^2)$
SI		$B^1_{SI}(t^1),\ \ B^2_{SI}(t^2)$

• Can an eroding price contract outperform standard auctions, by yielding the buyer a lower expected total procurement cost?

We first consider the NLI setting in section 4. We solve for the suppliers' equilibrium bids and then go on to compare the buyer's expected costs under EPC and SI, respectively. We then perform a similar analysis for the LI setting in section 5. Before we proceed to analyze the NLI and LI settings, we make the following observation that is valid under both settings: The 2^{nd} price nature of the auctions simplifies the search for the bidders' optimal bidding behaviors in the last auction: Vickrey(1961) demonstrated that it is a (weakly) dominant strategy for the suppliers to bid their true costs in the last auction.

$$B^2_{EPC}(t^2) = B^2_{SI}(t^2) = \begin{cases} t^2_I, & \text{if } t \text{ is an incumbent} \\ t^2_E & \text{if } t \text{ is an entrant} \end{cases} \tag{1}$$

The assumption of 2^{nd} price auctions allows us to focus our attention on the bidding behavior in the first auction, $B^1_M(t^1)$. Note that the 2^{nd} price nature of the auctions also implies that the bidders do not use any information gathered after the first auction under SI or EPC strategically when submitting a second bid; i.e., the information flow that occurs under SI and EPC does not affect bidding behavior at $T = 2$. In equilibrium, each supplier will adopt a strategy that maximizes his expected profit conditional on what strategies he believes his opponents are using. Given the symmetry in beliefs at $T = 1$, it is natural to posit that two suppliers of the same type will bid the same in equilibrium. Hence, we focus our attention on symmetric equilibrium bidding functions that are continuous and strictly monotonically increasing in type.

4. No Lock-In (NLI) Setting

We begin by first considering a model where the incumbent's baseline cost at $T = 2$ is independent of his cost at $T = 1$; his cost is redrawn from $[C - \alpha, C + 1 - \alpha]$. Under this setting, any supplier at $T = 1$ faces the same expected profit as an incumbent or an entrant at $T = 2$. As a result, we have our first observation.

Observation 1. *Under NLI, it is optimal for the buyer to offer all suppliers the same price, R, at $T = 2$, i.e., $R(t^1) = R \ \ \forall t^1$.*

To obtain the buyer's expected total costs, we must identify the supplier's equilibrium bidding strategies under each mechanism.

Proposition 1. *In combination with the second period bids outlined in equation (1), the following constitutes a subgame perfect equilibrium bidding strategy under EPC,*

$$B^1_{EPC}(t^1) = t^1 - \underbrace{\int_{C-\alpha}^{R} F_I(x)\,dx}_{E[\Pi^2_{EPC}(t^2_I);R]} + \underbrace{[1 - F_I(R)] \int_{C}^{C+1} \left[1 - F_{(1:N-2)}(x)\right] F(x)\,dx}_{E[\Pi^2_{EPC}(t^2_E);R]} \qquad \forall\ t^1 \tag{2}$$

Proof. see online appendix B.1. □

We use $F_{(i:j)}$ to denote the distribution of the ith lowest order statistics out of j suppliers. The ability of the incumbent supplier to reject R at $T = 2$ has two opposing effects on a supplier's bid at $T = 1$: (1) the supplier is guaranteed to have a non-negative profit as an incumbent at $T = 2$ and hence the supplier always *shades* his bid by this expected non-negative profit given R ($E[\Pi^2_{EPC}(t^2_I); R]$); and (2) if he loses at $T = 1$, the supplier has the opportunity to bid again with a new set of costs as an entrant (possibly higher or lower costs); this opportunity causes the supplier to *inflate* his bid ($E[\Pi^2_{EPC}(t^2_E); R]$).[7]The fact that all suppliers draw their expected costs at $T = 2$ from the same distributions F_I and F implies that they all shade/inflate their bids by the same amount.

When R or(and) α increases, the supplier's expected profit as an incumbent ($E[\Pi^2_{EPC}(t^2_I); R]$) increases, but his expected profit as an entrant ($E[\Pi^2_{EPC}(t^2_E); R]$) decreases as a result of it being

[7]Under the uniform distribution, the supplier t will always submit his bid below his cost since the positive gains as an incumbent at $T = 2$ dominates the gains as an entrant($E[\Pi^2_{EPC}(t^2_I); R] > E[\Pi^2_{EPC}(t^2_E); R]$, see online appendix B.3).

less likely that, should he lose at $T = 1$, an opponent would reject R at $T = 2$. The supplier's profit as an incumbent does not change as N increases, while his profit as an entrant decreases because of the decreasing difference between his bid and the second lowest entrant's bid. This, in turn, exerts a downward pressure on his bid. Note that s does not appear in $B^1_{EPC}(t^1)$; if an auction takes place at $T = 2$, it is only *after* the incumbent has rejected R and involves only entrants.[8]

Proposition 2. *In combination with the second period bids outlined in equations (1), the following constitutes a subgame perfect equilibrium bidding strategy under SI,*

$$B^1_{SI}(t^1) = \begin{cases} t^1 - [r^2 - E[t^2_I]], & \text{if } \alpha + s \geq 1 \\ t^1 - \underbrace{\left[\int_{C-\alpha}^{C+s} F_I(x)\,dx + \int_{C+s}^{C+1-\alpha} F_I(x)\left[1 - F_{E(1:N-1)}(x)\right][1 - F_E(x)]\,dx\right]}_{E[\Pi^2_{SI}(t^2_I)]} \\ \quad + \underbrace{\int_{C+s}^{C+1-\alpha} [1 - F_I(x)]\left[1 - F_{E(1:N-2)}(x)\right] F_E(x)dx}_{E[\Pi^2_{SI}(t^2_E)]}, & \text{if } \alpha + s < 1, \end{cases} \tag{3}$$

where $r^2 = C + 1 - \alpha$ *(reserve price)*

Proof. see online appendix C.1. □

The incentives for bid shading and inflating are similar under SI and EPC. The opportunity to participate in the second auction with a (possibly) reduced cost after experiencing learning by doing as an incumbent supplier incents a supplier to shade his bid ($E[\Pi^2_{SI}(t^2_I)]$). However, the opportunity to participate with a new cost at $T = 2$ incents a supplier to inflate his bid ($E[\Pi^2_{SI}(t^2_E)]$). The incentive to shade dominates the incentive to inflate, and hence the equilibrium bid is always below t^1 (please see online appendix C.3). If $\alpha + s \geq 1$ (i.e., the incumbent's cost at $T = 2$ is always less than or equal to the minimum possible entrant's cost), there is no chance for a supplier to lose at $T = 1$ and then win at $T = 2$, and hence there is no inflation effect on his bid at $T = 1$. As with EPC, all cost types shade their bids by the same amount.

[8] As we will see in the next section, the optimal R is increasing in s. Thus, s also indirectly affects the bidding behavior of bidder t.

4.1 Expected Total Cost and the Optimal Eroding Price R

Given the equilibrium bids derived in the propositions 1 and 2, we are one step away from comparing the performance of each procurement mechanism. While the expected costs associated with SI are completely determined by the suppliers' bids, EPC requires the buyer to select an eroding price R (see table 2 below). We use $x_{(i:j)}$ to denote the i^{th} lowest order statistic out of j random draws, and P_1 to denote the probability that the incumbent supplier accepts R at $T = 2$, i.e., $P_1 = F_I(R)$ ($P_0 = 1 - P_1$).

Table 2: Expected total cost in each procurement mechanism.

Mechanism	Expected Total Cost (ETC)
EPC	$ETC_{EPC}(R) \quad = E[B^1_{EPC}(t^1)_{(2:N)}] + P_1 \times R + P_0 \times E[B^2_{EPC}(t^2)_{(2:N-1)} + s]$
SI	$ETC_{SI} \quad = E[B^1_{SI}(t^1)_{(2:N)}] + E[\min(\tilde{B}^2_{SI}(t^2)^{\S}_{(2:N)},\, r^2)]$

$\S \tilde{B}^2_{SI}(t^2)_{(2:N)}$ is the effective bid, i.e., if an entrant defines the payment, $\tilde{B}^2_{SI}(t^2)_{(2:N)} = B^2_{SI}(t^2)_{(2:N)} + s$.

Proposition 3. *Under EPC, the buyer's optimal choice of R^* is as follows.*

$$R^* = \min[\hat{R},\ C + 1 - \alpha], \tag{4}$$

where

$$\hat{R} = \underbrace{\int_C^{C+1} \left[1 - F_{(1:N-2)}(x)\right] F(x)\, dx}_{E[\Pi^2_{EPC}(t^2_E)]} + \underbrace{(C + 1 + s) - \int_{C+s}^{C+1+s} F_{E(2:N-1)}(x)\, dx}_{E[t^2_{E(2:N-1)}+s]}{}^{9} \tag{5}$$

The buyer's expected total cost with R^* is given by.

$$ETC_{EPC}(R^*) = \begin{cases} E[t^1_{(2:N)}] + E[t^2_I], & \text{if } R^* = C + 1 - \alpha \\ E[t^1_{(2:N)}] + E[t^2_I] - \underbrace{\int_{\hat{R}}^{C+1-\alpha} [1 - F_I(x)]\, dx}_{\Psi}, & \text{if } R^* = \hat{R} \end{cases} \tag{6}$$

Proof. see online appendix B.2. □

[9] The reader should distinguish $E[\Pi^2_{EPC}(t^2_E)]$ from $E[\Pi^2_{EPC}(t^2_E); R]$. The former is the expected profit as an entrant, while the latter is the expected profit conditional on the incumbent rejecting R. Under the uniform distribution, $E[\Pi^2_{EPC}(t^2_E)] = \frac{1}{N(N-1)}$

R has two opposite effects on the buyer's expected total cost, a decrease in R decreases the buyer's payment at $T = 2$, but increases her payment at $T = 1$ (due to an increase in the bids at $T = 1$, as described in proposition 1). The optimal R balances these two effects.[10] It is interesting to note that $\hat{R}$ is *not* a function of α; this is because the factors that influence $\hat{R}$ are the potential profits as an entrant as well as the expected cost from an auction with only entrants, neither of which are impacted by α.

We can make the following observation,

Observation 2. *$R^* = \hat{R}$ is a sufficient condition for $\alpha + s < 1$.*

Proof. From equation (5), we can rewrite $\hat{R} = \Gamma + C + s$, where Γ is a positive term (this follows since both integrals in equation (5) are positive; furthermore, the second integral represents the expected second lowest order statistic in the range of $[C+s,\, C+s+1]$, which is greater than $C+s$). We have that $\hat{R} = \Gamma + C + s < C + 1 - \alpha \Rightarrow \alpha + s < 1 - \Gamma < 1$. □

Given the optimal R^*, the buyer's expected total cost is composed of her payment to the supplier for the first unit of the good ($E[t^1_{(2:N)}]$) and the cost for the second unit ($E[t^2_I] - \Psi$). The last term $-\Psi = \int_{R^*}^{C+1-\alpha} (R^* - x)\, f_I(x)dx$, is a negative term and can be interpreted as the expected reduction in procurement cost when the incumbent rejects the R^* at $T = 2$ and an entrant supplies the buyer. When the potential entrant pool is relatively uncompetitive (this adjective describes a market when N is small, and/or α and s are large), the buyer does not wish to expose herself to a second auction. Hence she sets $R^* = C + 1 - \alpha$ and keeps the same supplier for both periods, and $\Psi = 0$. However, as the entrant pool becomes competitive (N increases, and/or α and s decrease), the buyer will find it optimal to reduce R below $C + 1 - \alpha$ and reduce her costs by Ψ.

Proposition 4. *Given the bidding strategy as in proposition 2, the expected total cost of the buyer*

[10] For example, consider a setting where $R^* = \hat{R} < C+1-\alpha$; suppose that the buyer sets $R^* > \hat{R}$, i.e. $R^* = \hat{R}+\epsilon$. The expected cost at $T = 1$ decreases by the amount $\int_{\hat{R}}^{\hat{R}+\epsilon} F_I(x)dx + K\int_{\hat{R}}^{\hat{R}+\epsilon} f_I(x)dx$, where $K = E[t^2_{E(2:N-1)} + s]$, as a result of a lower bids at $T = 1$, but the expected cost at $T = 2$ increases by $\epsilon F_I(\hat{R}+\epsilon) + K\int_{\hat{R}}^{\hat{R}+\epsilon} f_I(x)dx$. Since $F_I(x)$ is a strictly increasing function, the increase in cost at $T = 2$ is larger than the decrease in cost at $T = 1$. Similarly, if the buyer sets $R^* = \hat{R} - \epsilon$, the increasing amount of aggressiveness of bid at $T = 1$ ($\int_{\hat{R}-\epsilon}^{\hat{R}} F_I(x)dx + K\int_{\hat{R}-\epsilon}^{\hat{R}} f_I(x)dx$) becomes larger than the decreasing amount of cost at $T = 2$ ($\epsilon F_I(\hat{R} - \epsilon) + K\int_{\hat{R}-\epsilon}^{\hat{R}} f_I(x)dx$).

under SI is as follows.

$$ETC_{SI} = \begin{cases} E[t^1_{(2:N)}] + E[t^2_I], & \text{if } \alpha + s \geq 1 \\ \\ E[t^1_{(2:N)}] + E[t^2_I] \\ \underbrace{-\int_{C+s}^{C+1-\alpha} [1 - F_I(x)]\, F_{E(2:N-1)}(x)dx}_{\Phi 1} & \text{if } \alpha + s < 1 \\ \underbrace{+\int_{C+s}^{C+1-\alpha} [1 - F_I(x)]\, \left[1 - F_{E(1:N-2)}(x)\right] F_E(x)dx}_{\Phi 2\,(=E[\Pi^2_{SI}(t^2_E)])} \end{cases} \tag{7}$$

Proof. see online appendix C.2. □

In contrast to EPC, the buyer always conducts a second auction with all N suppliers at $T = 2$ under SI. As described for EPC, the chance to participate in the auction at $T = 2$ incents a supplier to inflate his bid, and this exactly translates into the cost of the buyer ($\Phi 2 = E[\Pi^2_{SI}(t^2_E)]$). However, the second auction with N bidders offers the buyer a chance to procure from a more competitive entrant at $T = 2$ without sacrificing a reduction in the number of bidders, which is captured in $\Phi 1$.[11]

Given the buyer's expected total cost under the two mechanisms, we are now ready to compare the performances of the two mechanisms.

Proposition 5. *When $\alpha + s \geq 1$, the buyer's expected total costs are the same under EPC and SI.*

Proof. We prove that $\alpha + s \geq 1$ is a sufficient condition of $R^* = C + 1 - \alpha$. Recall that we can rewrite $\hat{R}$ as $\hat{R} = \Gamma + C + s$, where Γ is a positive term. If $\alpha + s \geq 1$, $\Gamma + C + s \geq \Gamma + C + 1 - \alpha > C + 1 - \alpha$. Thus, $R^* = C + 1 - \alpha$ from equation (4). □

When $\alpha + s > 1$, the cost ranges of an incumbent and the entrants do not overlap. Under EPC, the optimal $R^* = C + 1 - \alpha$, while the winning supplier at $T = 1$ will always win at $T = 2$ under SI with the reserve price $r^2 = C + 1 - \alpha$. Hence, the two mechanisms are cost equivalent. However, when the bidders' costs overlap (i.e., $\alpha + s < 1$), either EPC or SI can minimize the buyer's expected total cost. Propositions 6 and 7 and table 3 present the conditions under which each mechanism

[11] ETC_{SI} (if $\alpha + s < 1$) in equation (7) mirrors Grimm(2003)'s findings. She denotes $\Phi 1$ as the "value of competition" and $\Phi 2$ as the "looser's option value".

Table 3: Conditions for comparisons of the buyer's expected total costs under EPC and SI

R^*	$C+1-\alpha$	$C+1-\alpha$	$\hat{R}$
Market Conditions	α, s large ($\alpha+s \geq 1$)	N small, α, s large($\alpha+s<1$)	N large, α, s small ($\alpha+s<1$)
Comparisons (Conditions)	$ETC_{EPC} = ETC_{SI}$	$ETC_{EPC} < ETC_{SI}$ ($\Leftrightarrow 0 > \Phi1 - \Phi2$)	$ETC_{EPC} < ETC_{SI}$ ($\Leftrightarrow \Psi > \Phi1 - \Phi2$)
		$ETC_{EPC} > ETC_{SI}$ ($\Leftrightarrow 0 < \Phi1 - \Phi2$)	$ETC_{EPC} > ETC_{SI}$ ($\Leftrightarrow \Psi < \Phi1 - \Phi2$)

dominates, while table 1 in the online appendix D presents representative numerical instances that support each scenario.

Proposition 6. *When $R^* = C+1-\alpha$ and $\alpha+s<1$, the buyer's expected total cost under EPC is less than that under SI when $\Phi1 < \Phi2$ is satisfied. Conversely, SI outperforms EPC when $\Phi1 > \Phi2$ is satisfied.*

Proposition 7. *When $R^* = \hat{R}$, the buyer's expected total cost under EPC is less than that under SI when $\Psi > \Phi1 - \Phi2$ is satisfied. Conversely, SI outperforms EPC when $\Psi < \Phi1 - \Phi2$ is satisfied.*

The buyer will generally find it optimal to set $R^* = C+1-\alpha$ when the entrant pool is uncompetitive, i.e., N is small and/or when α or s are large. Whether or not SI will outperform EPC then depends on which factor exerts the larger force; the expected cost savings from holding an auction at $T=2$ with N bidders ($\Phi1$) versus the degree to which bidders inflate their bid at $T=1$ due to the presence of the second auction($\Phi2$). As the entrant pool becomes more competitive (N increases and/or α or s decrease), the buyer will find it optimal to set $R^* = \hat{R} < C+1-\alpha$ and expose herself to the incumbent rejecting the contract at $T=2$ and running a second auction with an associated cost savings of Ψ. Whether or not EPC dominates SI will depend on the value of having an addition bidder and having the price be competitively determined at $T=2$.

While the derivation of the equilibrium bids and optimal R^* proved to be an interesting theoretical exercise, we found that the actual cost difference between EPC and SI was very small when F follows a uniform distribution[12].

[12] To test the robustness of our result, we also ran numerical comparisons under several different beta distributions (e.g., $B(2,3)$, $B(2,2)$, $B(3,2)$). The results mirrored those of the uniform distribution. As a general rule, we also found that SI will dominate EPC if $R^* = \hat{R} < C+1-\alpha$; the only exception to the rule occurred when $N=3$.

Table 4: Comparison of EPC and SI$\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\%\right)$ under NLI

$s = 0.1$							$\alpha = 0.4$						
$\alpha \setminus N$	3	4	5	6	10	20	$s \setminus N$	3	4	5	6	10	20
0.1	-0.6	0.5	0.6	0.5	0.3	0.1	0.1	-0.4	0.0	0.4	0.5	0.3	0.1
0.2	-0.6	0.4	0.6	0.5	0.3	0.1	0.2	-0.2	-0.1	0.1	0.3	0.3	0.1
0.3	-0.5	0.2	0.5	0.5	0.3	0.1	0.3	-0.1	-0.1	0.0	0.1	0.2	0.1
0.4	-0.4	0.0	0.4	0.5	0.3	0.1	0.4	-0.0	-0.0	-0.0	0.0	0.1	0.1
0.5	-0.3	-0.1	0.1	0.3	0.3	0.1	0.5	-0.0	-0.0	0.0	0.0	0.0	0.0
0.7	0.0	0.0	0.0	0.0	0.1	0.1	0.7	0.0	0.0	0.0	0.0	0.0	0.0

Observation 3. *The expected cost difference between EPC and SI is less than* 1% *(table 4).*

5. Lock-In (LI) Setting

As suggested in the introduction, in some market settings, an incumbent's cost at $T = 2$ depends on his cost at $T = 1$. For example, the incumbent supplier may be able to write a long-term contract fixing his own input prices over both periods. Under such settings, the incumbent's cost at $T = 2$ is drawn from the range of values $[t^1 - \alpha,\, t^1]$.

Since the cost distribution of an incumbent depends on his type at $T = 1$, the dynamics of the bidder's behaviors as well as the choice of the optimal $R(t^1)$ under a LI setting become more complex when compared to those under NLI. For example, when the bidder submits his bid at $T = 1$, he must consider the winning opponent's cost $T = 1$ should he lose and its impact on his potential profits at $T = 2$. The same is not true under NLI.

Similarly, the buyer's selection of the optimal eroding price contract is complicated by the incumbent's temporal cost dependency; it is no longer optimal to offer all supplier types the same price at $T = 2$. Rather, the buyer should offer prices as a function of t^1, $R(t^1)$. In our discussions with companies that have employed EPC,[13] we learned that *non-discriminatory* eroding price contracts are the most commonly used format, i.e., *any* winning supplier is required to reduce its

[13]During the summer of 2003, Se-kyoung Oh had the benefit of interning at FreeMarkets.com, where she was able to observe their auction practices across multiple markets. We wish to extend our thanks to Jack Allamon, Bill Blair, Michael Bryson, Brenna Bulwinkle, and David Talbot for sharing with us their time and auction experiences.

price in the second period by the same fixed percent $(1-k)$ and is paid the price $R(t^1) = kt^1$ where $0 < k \leq 1$. Markets for which non-discriminatory eroding price contracts have been used in FreeMarkets auctions include PCA/PCBs, capacitors, transceivers, monitors, and wire harnesses (please see the online appendix A for additional examples).

Therefore, in the analysis below, we search for the optimal non-discriminatory eroding price contract. As discussed in section 3, we assume that $R(t^1) \in [t^1 - \alpha, t^1]\ \forall\ t^1$; this implies that the buyer does not consider prices that are guaranteed to make the supplier unprofitable at $T = 2$ or pay the supplier more than his possible highest cost.

5.1 Equilibrium Bidding Strategy and the Optimal Eroding Price $R(t^1)$ under EPC

We first solve for the equilibrium bidding strategies under EPC, given truthful bidding at $T = 2$.

Proposition 8. *In combination with the second period bids outlined in equation (1), the following constitutes a subgame perfect equilibrium bidding strategy under EPC,*

$$\begin{aligned} B^1_{EPC}(t^1) &= t^1 - \underbrace{\int_{t^1-\alpha}^{R(t^1)} F_I(x)\,dx}_{E[\Pi^2_{EPC}(t^2_I);R(t^1)]} + \underbrace{\left[1 - F_I(R(t^1))\right] \int_C^{C+1} \left[1 - F_{(1:N-2)}(x)\right] F(x)\,dx}_{E[\Pi^2_{EPC}(t^2_E);R(t^1)]} \\ &\quad + R'(t^1) F_I(R(t^1)) \left(\frac{1 - F(t^1)}{(N-1)f(t^1)} \right) \qquad \forall\ t^1, \end{aligned} \tag{8}$$

Proof. see online appendix E.1 □

In contrast with $B^1_{EPC}(t^1)$ under NLI (equation (2)), suppliers may inflate or shade their bids at $T = 1$ differentially (i.e., not by the same amount). In equilibrium, supplier t submits a bid that reflects his unit cost at $T = 1 (t^1)$, shaded by his expected profit at $T = 2$ as an incumbent $(E[\Pi^2_{EPC}(t^2_I); R(t^1)])$. However, this downward pressure on a supplier's bid is countered by (1) the likelihood that he is the lowest cost supplier in the auction at $T = 1$ $(\frac{1-F(t^1)}{(N-1)f(t^1)})$, and the attractiveness of misrepresenting his type at $T = 1$ in order to affect his payment at $T = 2$ $(R'(t^1) = \frac{\partial R(t^1)}{\partial t^1})$, and (2) the possibility of earning money in the second auction, if it is held $(E[\Pi^2_{EPC}(t^2_E); R(t^1)])$.

As we saw in proposition 1 in section 4, the ability of the incumbent supplier to reject $R(t^1)$ at $T = 2$ exerts both bid shading and bid inflating forces on his bid at $T = 1$.[14]

The eroding price contract influences a supplier's bid in two ways. The *absolute value* of $R(t^1)$ directly affects a supplier's expected profitability at $T = 2$. The higher a supplier's expected profit at $T = 2$, the more aggressively he will bid at $T = 1$. Conversely, if $R(t^1)$ is set so low as to make profitable production at $T = 2$ very unlikely, then a supplier will compensate for this future negative profit stream by inflating his bid at $T = 1$ proportionately. The *shape* of $R(t^1)$ affects how competitively a supplier bids at $T = 1$ with respect to his competitors. As $R'(t^1)$ increases, supplier t has a greater incentive to bid as a higher type and hence secure a higher payment at $T = 2$, if he should win at $T = 1$.

Given the increased complexity of the buyer's decision problem, i.e., determining an entire schedule of prices as opposed to a single price, we now limit our focus to a setting where F follows the uniform distribution. While this assumption was not needed to derive the equilibrium bidding strategies, it was necessary in order to derive the optimal eroding price contract (we were unable to solve the buyer's decision problem under the general distribution).[15]

Proposition 9. *Under the non-discriminatory eroding price schedule $R(t^1) = kt^1$, the buyer's expected total cost under EPC is minimized when $k = 1$. The buyer's expected total cost is as follows.*

$$ETC_{EPC}(R^*(t^1)) = E[t^1_{(2:N)}] + E[t^2_{I(2:N)}] \tag{9}$$

Proof. see online appendix E.2 and E.4 □

One might initially think that the buyer would find it optimal to set $k < 1$, due to the possibility of a high type supplier winning at $T = 1$. With $k < 1$, the buyer could then hopefully procure from a lower cost supplier in the auction at $T = 2$ if the incumbent does not experience a sufficient

[14]A bidder t's expected profit as an entrant is considered only if an incumbent $w(\neq t)$ rejects the buyer's offer $R(w^1)$ at $T = 2$. This is captured in $1 - F_I(R(t^1))$ in the last term in equation (8). At $T = 1$, the incumbent's cost w^1 should be no greater than t^1 to be the winner ($w^1 \sim [C, t^1]$, and hence $1 - F_I(R(t^1))$ is the probability that the incumbent with the marginal cost, t^1 rejects $R(t^1)$.

[15]To get an idea of the robustness of our result, we used numerical methods to solve for the optimal k under different beta distributions (e.g. beta functions $B(2, 3)$, $B(2, 2)$, $B(3, 2)$). We found that in all settings, the optimal solution was always $k^* = 1$. This leads us to conjecture that $k^* = 1$ under most general distribution settings.

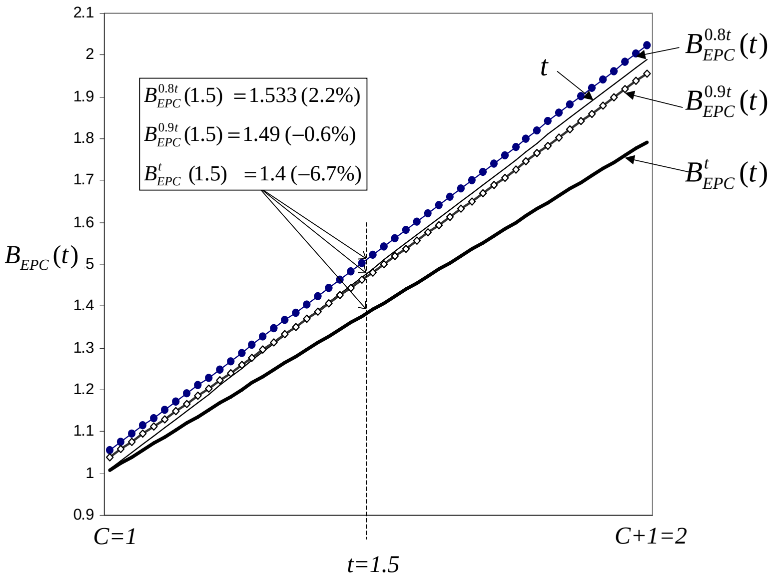

Figure 3: Equilibrium bidding strategies in non-discriminatory EPC, with N = 6, α = 0.4 and s = 0.1 (t used instead of t^1)

cost reduction. However, a non-discriminatory $R(t^1)$ does not offer the buyer the opportunity to differentially set k for each t^1. That, in combination with the fact that suppliers' bids at $T = 1$ become more aggressive as k increases and the incumbent's cost at $T = 2$ is bounded from above by t^1 (which was not the case under NLI) implies that it is optimal for the buyer to guarantee a non-negative profit stream to any incumbent supplier at $T = 2$, and hence keeps her incumbent supplier with probability equal to one.

It is illuminating to observe the shifts in $B^{kt^1}_{EPC}(t^1)$ as k increases. Figure 3 illustrates that suppliers bid more aggressively as k increases: This reflects the increase in expected profit at $T = 2$ if the supplier should win at $T = 1$. For example, under $k = 0.8$, the supplier with cost $t^1 = 1.5$ rejects $R(t^1)$ with a probability of 75%; he takes this into account when submitting his bid at $T = 1$ and inflates his bid above cost (2.2% inflation) to reflect the opportunity cost of winning at $T = 1$ and not being able to participate in an auction at $T = 2$ with a new and potentially lower cost. As k increases, the probability that the incumbent rejects the contract decreases significantly (Pr(reject) = 37.5% for $k = 0.9$, is equal to 0 for $k = 1$). When $k = 1$, supplier $t^1 = 1.5$ *shades* his bid below his cost by 6.7%. Again, the differential bid shading reflects a supplier's comparative advantage at $T = 1$ as well as his expected profit at $T = 2$ as an incumbent and an entrant.

The rate of decrease in bids in combination with the reduced likelihood of incurring s dominates the rate of the increase in $R(t^1)$ (the buyer's expected total cost is decreasing convex function of k, and hence $k = 1$ minimizes the buyer's total expected costs - online appendix E.2).[16]

5.2 Equilibrium Bidding Strategy under SI and Comparisons between EPC and SI.

While we were able to derive a closed form solution for the bids under EPC, we were unable to do so under SI.[17]

Table 5: Parameters for numerical examples

C	1 , $t \sim U[1, 2]$
N	3, 4, 5, . . ., 10, 20, 30
α	0.05, 0.1, 0.2, . . ., 0.9
s	0.05, 0.1, 0.2, . . ., 0.9

We ran our algorithm over the experimental settings in table 5 and varied the values of α, s, and N to capture a wide spectrum of market settings (based on this, we obtained the buyer's expected total cost under SI and compared it with the buyer's cost under EPC. We ran a total of 1210 market instances.). We approximate the uniform distribution of supplier types by discretizing the uniform distribution over the interval $[C, C + 1]$ by increments of $\frac{1}{n}$, i.e., $(t_1, t_2, \ldots, t_n) = (C, C + \frac{1}{n} \ldots, C + \frac{n-1}{n})$. Under these market settings, we first discuss the equilibrium bidding

[16] In online appendix E.3, we show the sensitivity of the choice of $R^*(t^1) = kt^1$. The buyer's cost are more sensitive to a chance of k (i.e., his cost would rapidly increase), as α and N decrease and s increases.

[17] The bid monotonicity assumption was violated under a few market settings; this implies that strictly monotonically increasing bids may fail to exist for SI when F is continuous. Our next step was to assume a discrete distribution for the supplier type space, and use numerical methods to solve for the equilibrium bids. Under this altered setting, we were able to find equilibrium bids that satisfy strict monotonicity. For each supplier type, we iteratively search for the optimal bid which maximizes his profit given other suppliers' bids, $\Pi_i(B_1, ..., B_{i-1}, B_{i+1}, ..., B_n)$ (see online appendix F for the payoff function of bidder t). MATLAB was used for this experiments (interested readers may contact the authors for a copy of the algorithm).

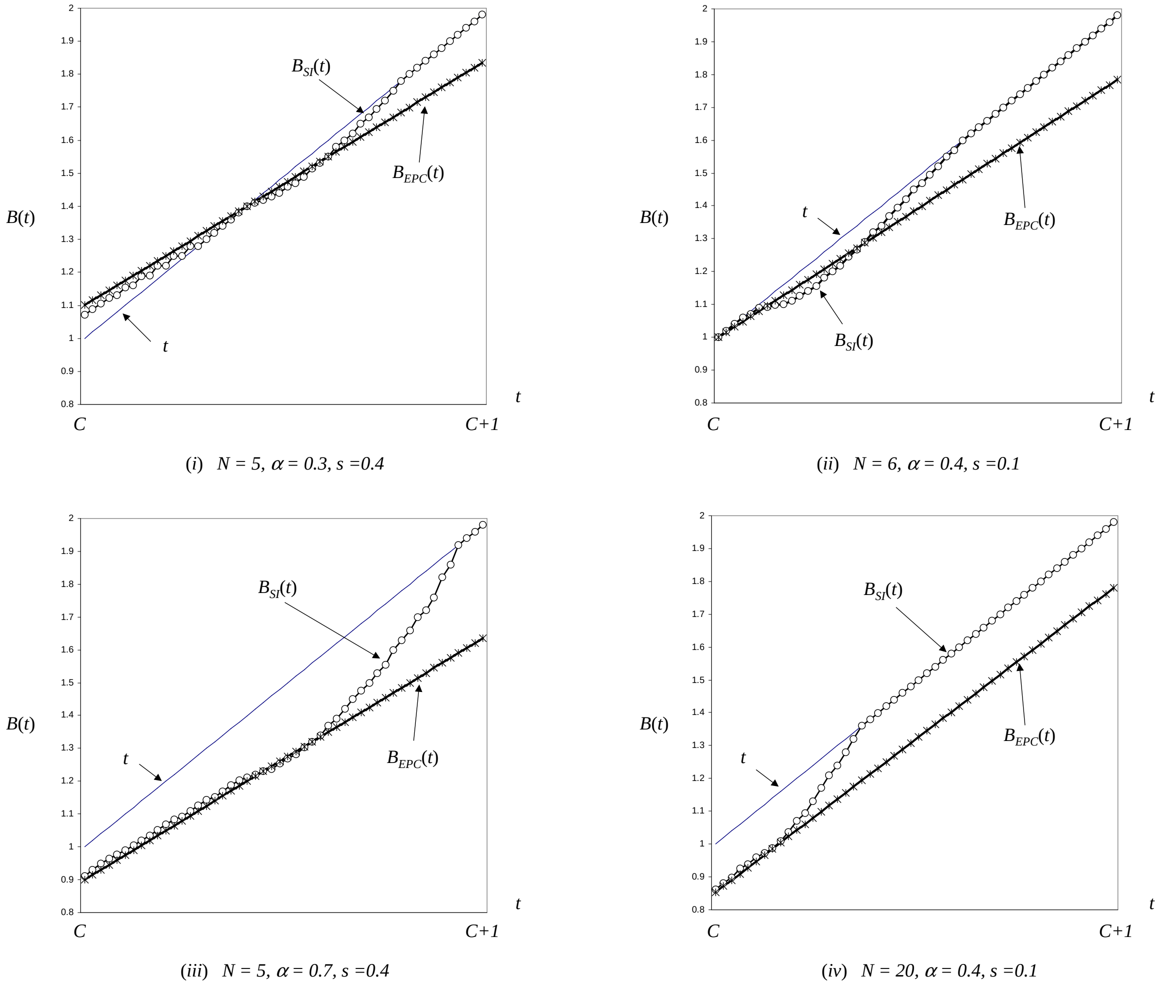

Figure 4: Example of Bids under EPC and SI (t used instead of t^1)

behavior under SI and proceed to compare the buyer's expected total cost under SI and EPC.[18]

Figure 4 plots the equilibrium bids under SI and EPC in four different market settings. As we can see, the bids under SI and EPC are similar for low supplier types, while the difference become larger for high types.

Under EPC, the supplier who wins at $T = 1$ will always supply the buyer at $T = 2$ (and there will be no second auction if he should lose at $T = 1$ and hence there is no opportunity cost associated with winning at $T = 1$). As a result, the bidder submits an aggressive bid to win at $T = 1$; for example, the highest type ($= C + 1$) shades his bid by amount of his expected profit

[18]Results illustrated in this section are based on experiments with $n = 50$.

at $T = 2$, $E[\Pi^2_{EPC}(t^2_I)]$. Under SI, the supplier who wins at $T = 1$ must compete again for the buyer's business at $T = 2$. For a high type, his cost at $T = 2$ (after experiencing a cost reduction) may not be competitive vis-a-vis the entrants' costs, leaving open the possibility of losing at $T = 2$. Conversely, should he lose at $T = 1$, he can reenter the auction at $T = 2$ with a new and potentially more competitive cost, hence his bid at $T = 1$ is conservative. For lower types, it is more likely that a supplier who wins at $T = 1$ will also win in the auction at $T = 2$, and his bid at $T = 1$ reflects this in the form of greater bid shading. For a very low type, they will win again at $T = 2$ with almost certainty and be paid the reserve price (equal to his cost t^1), as would be the case under EPC.

Given the equilibrium bidding strategies, we next compare the buyer's expected total cost under EPC and SI.

Observation 4. *The performance of EPC when compared to SI improves as we increase s, α or N. The buyer's expected total cost under EPC is less than or (at least) same as that under SI when at least two of the market parameters (N, α, and s) are large. Conversely, her cost under SI is less than that under EPC when at least two of the parameters are small.*

Table 6 illustrates this observation as we vary N, α, and s. These comparisons illustrate that EPC's performance will improve as α, s and N increase. The reason for this is as follows: (1) Under EPC, the incumbent is guaranteed the buyer's business at $T = 2$ ($R^*(t^1) = t^1$), while the same is not true under SI. Hence, as α increases, the bidders pass along more of this cost advantage into their bids at $T = 1$ under EPC than SI. (2) While an increase in s does not affect the bids and the buyer's expected total cost under EPC (see equation (9) in proposition 9), the buyer's cost at $T = 1$ and 2 increase as a function of s under SI, and hence EPC become attractive to the buyer. Note that with a large s, the incumbent would always be the winner at $T = 2$ and be paid the reserve price, which implies that the cost at $T = 2$ would not change under SI. Therefore, after some threshold level of s, the difference between EPC and SI would remain a constant. (3) When N increases, the expected cost difference at $T = 2$ between the incumbent and the lowest entrant decreases, which affects the supplier's expected profit at $T = 1$ under SI, while it does not under EPC. As a result, suppliers will bid (relatively) less competitively under SI than EPC for a given change in N. Note that the effect of an increase in α is more significant than that of s. For example, when $N = 6$, $\alpha = 0.1$ and $s = 0.1$, the difference is 3.1% (SI outperforms EPC). If s increases to

Table 6: Comparison of non-discrim. EPC ($k = 1$) and SI$\left(\frac{ETC^t_{EPC} - ETC_{SI}}{ETC_{SI}} \%\right)$ under LI

$s = 0.1$							$\alpha = 0.1$						
$\alpha \setminus N$	3	4	5	6	10	20	$s \setminus N$	3	4	5	6	10	20
0.1	2.8	3.4	3.3	3.1	1.7	0.1	0.1	2.8	3.4	3.3	3.1	1.7	0.1
0.2	2.5	2.9	2.8	2.1	0.6	-0.4	0.2	2.3	2.6	2.4	2.2	1.3	-0.1
0.3	2.3	2.3	1.7	1.2	0.0	-0.5	0.3	1.9	2.0	1.8	1.5	1.1	-0.1
0.4	1.9	1.5	1.1	0.6	-0.1	-0.6	0.4	1.5	1.5	1.4	1.2	1.1	-0.1
0.5	1.3	1.0	0.5	0.2	-0.3	-0.8	0.5	1.3	1.3	1.3	1.1	1.1	-0.1
0.7	0.5	0.1	-0.1	-0.2	-0.6	-1.1	0.7	1.2	1.2	1.2	1.1	1.1	-0.1
$s = 0.4$							$\alpha = 0.4$						
$\alpha \setminus N$	3	4	5	6	10	20	$s \setminus N$	3	4	5	6	10	20
0.1	1.5	1.5	1.4	1.2	1.1	-0.1	0.1	1.9	1.5	1.1	0.6	-0.1	-0.6
0.2	1.5	1.3	1.0	0.9	-0.1	-0.5	0.2	1.4	1.1	0.5	0.0	-0.4	-0.6
0.3	1.1	0.8	0.6	0.2	-0.5	-0.5	0.3	1.0	0.7	0.1	-0.3	-0.5	-0.6
0.4	0.8	0.5	0.0	-0.4	-0.6	-0.6	0.4	0.8	0.5	0.0	-0.4	-0.6	-0.6
0.5	0.4	0.0	-0.4	-0.5	-0.7	-0.8	0.5	0.6	0.4	-0.1	-0.4	-0.6	-0.6
0.7	-0.2	-0.5	-0.6	-0.7	-0.9	-1.1	0.7	0.6	0.3	-0.1	-0.5	-0.6	-0.6

0.4, it decreases to 1.2%. However, if α is increased to 0.4 (with $s = 0.1$), the difference is reduced to 0.6%.

As opposed to the NLI setting, the cost differences can be significant under LI, ranging anywhere from -1.3% to 5%. This implies that it would be worthwhile for the buyer to educate herself about her supplier pools' cost characteristics (i.e., s and α) as well as the size of the potential bidder pool N before determining whether or not to offer a longer-term relationship to the winner in the first auction. It is worth reiterating that under the optimal EPC, the buyer does not actually ask to share in any of the cost reductions at $T = 2$ ($k = 1$); therefore the superior performance under EPC is arising from the long-term stability of the relationship.

6. Extensions and Managerial Insights

6.1 Discriminatory $R(t^1)$ under LI

Under the NLI setting, the potentially complicated eroding price schedule $R(t^1)$ reduces its form to a simple R due to the independence of the incumbent's cost over time. However, under the

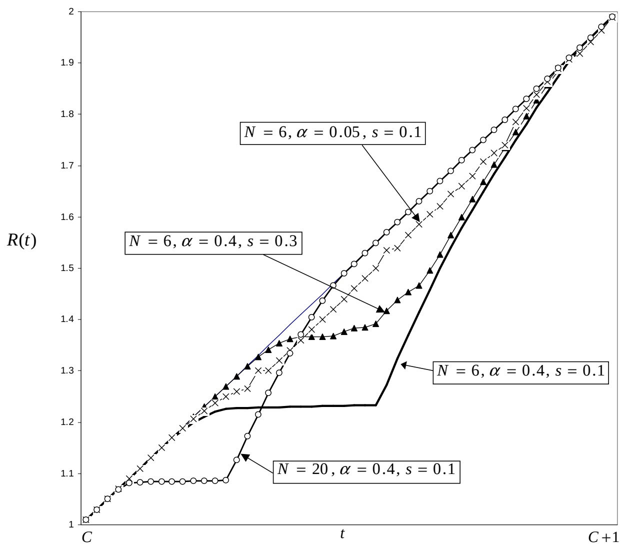

Figure 5: Optimal discriminatory $R(t^1)$ for different market settings $(N, \alpha,\ s)$ under LI

LI setting, the buyer should set a different price for each supplier type t^1 at $T = 1$. In practice, the buyers simplify a potentially complex price schedule by offering mainly nondiscriminatory contracts where all suppliers are asked for the same fractional price reduction, $R(t^1) = kt^1$. While using non-discriminatory price schedule may simplify the buyer's problem, it may also leave some 'money on the table' for the suppliers. If the buyer were to offer a discriminatory price schedule, $R(t^1) = \kappa(t^1)t^1$, it is clear that the buyer would be better off (or at least no worse off). What is not clear is how much better off would she be, what would be the form of the optimal contract and if the cost savings would warranty the added complexity of the contract design.

We assume that $R(t^1)$ is differentiable everywhere and is strictly monotonically increasing in t^1. Thus a buyer would never require a lower price from a higher type t; this assumption is particularly reasonable since the suppliers are assumed to have no control on their cost reduction, i.e., the cost reduction arises merely out of having supplied the buyer once before and is not a direct result of any cost-saving measures. By solving a nonlinear program, we were able to find the buyer's optimal $R(t^1)$ (or $\kappa(t^1)$) for a bidder t.

Observation 5. *Under the LI setting, the buyer finds it optimal to offer a contract* $R(t^1) = \kappa(t^1)t^1$ *that has the following properties: (1) guarantee for both 'low' and 'high' cost suppliers that they will not be unprofitable at* $T = 2$ *(i.e.,* $\kappa(t^1) \approx 1$*) and (2) extract some potential cost reductions for*

Table 7: Comparison of discrim. EPC and SI$\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\%\right)$ under LI

$s = 0.1$							$\alpha = 0.4$						
$\alpha \setminus N$	3	4	5	6	10	20	$s \setminus N$	3	4	5	6	10	20
0.1	2.8	2.7	2.4	2.0	0.7	-0.3	0.1	1.9	1.1	0.4	-0.1	-0.6	-0.8
0.2	2.5	2.1	1.9	0.9	-0.1	-0.7	0.2	1.4	0.9	0.2	-0.4	-0.7	-0.8
0.3	2.3	1.6	0.9	0.4	-0.6	-0.7	0.3	1.0	0.6	0.0	-0.5	-0.7	-0.7
0.4	1.9	1.1	0.4	-0.1	-0.6	-0.8	0.4	0.8	0.5	-0.1	-0.5	-0.6	-0.7
0.5	1.3	0.7	0.0	-0.4	-0.7	-1.1	0.5	0.6	0.4	-0.2	-0.5	-0.6	-0.7
0.7	0.5	-0.1	-0.5	-0.6	-1.0	-1.3	0.7	0.6	0.3	-0.1	-0.5	-0.6	-0.6

'moderate' cost suppliers(i.e., $\kappa(t^1) < 1$).

The buyer finds it optimal to set $\kappa(t^1) = 1$ for high cost types, thereby eliciting the most aggressive bidding behavior from them at $T = 1$. That, combined with the probability of a 'high' cost supplier winning at $T = 1$ being low, renders $\kappa(t^1) = 1$ optimal. The aggressive bidding behavior from this 'high' cost group cascades down and exerts a downward pressure on 'low' cost suppliers' bids. The reason for $\kappa(t^1) = 1$ for low cost suppliers is the incumbent supplier's ability to reject $R(t^1)$ at $T = 2$. Since (i) a low cost supplier will reject any unprofitable contract, and (ii) the probability that the expected payment at $T = 2$ from the auction plus switching cost is less than the incumbent's unit cost is low (the incumbent's cost at $T = 1$ is the lowest order statistic of N random draws, while the winner at $T = 2$ would be the lowest order statistic of $N - 1$ random draws), the buyer finds it optimal to set $\kappa(t^1) = 1$ for low cost suppliers. For the 'moderate' cost suppliers, the buyer finds it optimal to set $\kappa(t^1) < 1$, in the hopes of securing a lower price at $T = 2$ if the incumbent supplier should win and accept $R(t^1)$ and, if not, of securing a price from the second auction that is comparable to $R(t^1)$. The degree of $\kappa(t^1) < 1$ and the range of the types differ by the market settings (N, α, s) (see figure 5).[19]

This flexibility of the price schedule at $T = 2$ makes the buyer better off compare to the

[19] As we see in figure 5, The definition of the 'moderate' group changes across markets; this group shifts down (towards C) as N increases (N increases from 6 to 20), and shifts up (towards $C + 1$) as s increases (s increases from 0.1 to 0.3). For small s, $R(t^1) \to t^1 - \alpha$ for 'moderate' types when α is small, while $R(t^1) >> t^1 - \alpha$ when α is large. When α is small (e.g., $\alpha = 0.05$), the incumbent does not have a large cost advantage over entrants at $T = 2$; hence the buyer finds it optimal to set $R(t^1) = t^1 - \alpha$ for the moderate type suppliers in order to secure a price comparable to the cost of the lowest cost entrant at $T = 2$. When s is large ($s > 0.5$), the buyer wants to keep the incumbent for

Table 8: Comparison of non-discrim. EPC and SI with proportional $\alpha\left(\frac{ETC_{EPC}-ETC_{SI}}{ETC_{SI}}\%\right)$ under LI (when $R^*(t^1) = t^1$)

$s = 0.1$							$\alpha = 0.4$						
$\alpha \setminus N$	3	4	5	6	10	20	$s \setminus N$	3	4	5	6	10	20
0.1	2.6	3.1	3.1	2.9	1.5	0.0	0.1	0.9	0.7	0.4	0.0	-0.6	-0.7
0.2	2.1	2.4	2.0	1.5	0.3	-0.5	0.2	0.6	0.3	-0.1	-0.4	-0.7	-0.8
0.3	1.5	1.3	1.0	0.7	-0.2	-0.6	0.3	0.4	0.0	-0.3	-0.6	-0.7	-0.8
0.4	0.9	0.7	0.4	0.0	-0.6	-0.7	0.4	0.3	-0.1	-0.4	-0.6	-0.7	-0.8
0.5	0.5	0.2	-0.3	-0.4	-0.7	-0.8	0.5	0.0	-0.2	-0.4	-0.6	-0.6	-0.8
0.7	-0.5	-0.8	-0.7	-0.8	-1.1	-1.1	0.7	-0.1	-0.1	-0.4	-0.6	-0.7	-0.8

nondiscriminatory schedule. For example, when $N = 6$ $\alpha = 0.4$, $s = 0.1$, SI outperforms EPC with the nondiscriminatory price schedule $R(t^1) = kt^1$(the difference between the buyer's expected total cost under EPC and SI is 0.6%) but EPC outperforms SI with the discriminatory schedule $R(t^1) = \kappa(t^1)t^1$ (the difference is -0.1%) (see table 7).

6.2 α under LI

Cost savings arising from learning by doing, economies of scale in production or other manifestations of synergies can occur in many forms. In section 5 we assumed that an incumbent's cost at $T = 2$ was drawn from the interval $[t^1 - \alpha, t^1]$, i.e., all suppliers faced the same absolute potential reduction in costs. An alternative cost framework is one where high cost suppliers have more potential for cost reduction (e.g., more 'low-hanging fruit'). We next consider the case where the maximum cost reduction is proportional to the supplier's type t^1, and as such, the cost range of an incumbent supplier is $[(1-\alpha)t^1,\ t^1]$, where α is the same for all suppliers.

Under the same market setting as in table 5, we found that (1) it is still optimal for the buyer to set $k = 1$ under a non-discriminatory EPC (see online appendix E.2.1) and (2) observation 4 continues to hold (see table 8). Note that the performance of EPC is improved under this proportional cost reduction setting. Since high cost types face a larger potential decrease in cost at $T = 2$, this larger cost reduction coupled with the optimal schedule $R(t^1) = t^1$, implies that they bid more aggressively at $T = 1$ and this aggression cascades down to lower types, while under

any α, and sets the optimal $R(t^1) = t^1 \ \ \forall t^1$, i.e., the set of 'moderate' suppliers is the null set.

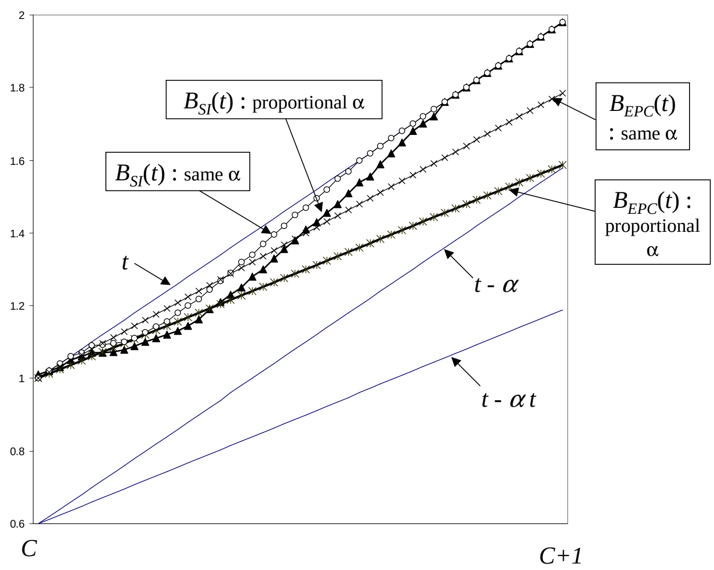

Figure 6: Bids under SI and EPC when the maximum possible cost reduction α is (1) the same for all types and (2) proportional to the type ($N = 6$, $\alpha = 0.4$, $s = 0.1$: t used instead of t^1)

SI, the larger cost reduction for higher types can not be fully captured in bid at $T = 1$ due to the positive probability of not winning at $T = 2$ (see figure 6). And hence, EPC's performance improves when the cost reduction is proportional to the bidder's type.

6.3 Cost distribution shift at $T = 2$

A final extension on our original model that we consider is a shift in the cost distribution at $T = 2$. In previous sections, we assumed that the distribution of costs for entrants at $T = 2$ stays same as in $T = 1$, i.e., $[C, C + 1]$, and only the incumbent redraws his cost from a new (lower) cost range. This is a valid assumption when industry costs are relatively stable over time, and/or there is only a single buyer in the market. However, the suppliers may be operating in a market where the cost of raw materials or technology increases or decreases over time. Alternatively, suppliers may be able to supply multiple buyers simultaneously and experience learning by doing through the production process for other buyers; however, the incumbent may still have an comparative cost advantage over the entrants due to buyer-specific learning-by-doing (e.g., leather seat example in the introduction). Both of these market effects can be modeled as a shift in the entrant's cost

Table 9: Comparisons of EPC and SI with entrant's cost shifts under LI : downward($\xi = -0.5$) and upward($\xi = 0.5$) $\left(\Delta_{ETC} \equiv \frac{ETC_{EPC} - ETC_{SI}}{ETC_{SI}} \%,\ R^*(t^1) = kt^1\right)$

			$\xi = -0.5$		$\xi = 0$		$\xi = 0.5$	
N	α	s	Δ_{ETC}	k	Δ_{ETC}	k	Δ_{ETC}	k
3	0.3	0.3	5.5	1.00	1.5	1.00	0.8	1.00
6	0.1	0.4	3.4	0.96	1.2	1.00	1.1	1.00
6	0.4	0.1	3.6	0.80	0.6	1.00	-0.4	1.00
10	0.2	0.3	3.1	0.90	-0.1	1.00	-0.2	1.00
10	0.3	0.2	3.0	0.85	-0.3	1.00	-0.5	1.00
20	0.1	0.5	1.0	1.00	-0.1	1.00	-0.1	1.00
20	0.5	0.1	1.7	0.76	-0.8	1.00	-0.8	1.00

range at $T = 2$, i.e., $[C + \xi,\ C + 1 + \xi]$ ($\xi > (<)0$).

Under the NLI setting, the incumbent does not lock in his previous cost at $T = 2$ and, like entrants, must redraw his 'baseline cost' at $T = 2$. If the industry experiences a shift in costs, it effects both the incumbent and the entrants, and their cost distributions at $T = 2$ become $[C - \alpha + \xi,\ C + 1 - \alpha + \xi]$ and $[C + \xi,\ C + 1 + \xi]$, respectively. Since both the incumbent's and the entrants' costs shifts upward/downward by ξ, these shifts canceled out each other and hence our analysis and results in section 4.1 carry over.

An industry shift in costs would manifest itself differently under the LI setting. Under this setting, the incumbent's cost at $T = 2$ would continue to be drawn from $[t^1 - \alpha, t^1]$, while the entrants' costs are drawn from $[C + \xi,\ C + 1 + \xi]$. Recall that the buyer uses effective bid rather than submitted bid for selecting the winning supplier at $T = 2$, due to the presence of the switching cost s; the shift in the entrants' cost distribution ξ plays a similar role and offers us some intuition as to the robustness of our results.

When the entrants' cost distribution shifts upward (i.e., $\xi > 0$), it is the same as an increase in s. The buyer would still find it optimal to set $k = 1$ and offer the contract $R(t^1) = t^1$, for now the incumbent is even more attractive to the buyer when compared to the case of $\xi = 0$. Similarly, when $\xi > 0$, EPC becomes more attractive than SI to the buyer. When the entrants' cost distribution shifts downward (i.e., $\xi < 0$), $R(t^1) = t^1$ may no longer be optimal. For example, when $N = 6$, $\alpha = 0.4$, $s = 0.1$, the % cost difference of between EPC and SI is 0.6% when $\xi = 0$.

It decreases to -0.4% when $\xi = 0.5$ (cost shift upward), and increases to 3.6% when $\xi = -0.5$ (cost shift downward) - see table 9 for more examples.

7. Conclusion

The aim of this paper was two-fold: (*i*) to study the optimal design of an eroding price contract when suppliers experience learning by doing and (*ii*) to establish when/if a buyer is better off committing to a single supplier in return for the supplier offering her a more competitive price. We asked these questions in two different cost frameworks, NLI and LI. We found that,

- Under LI where F follows a uniform distribution, it is optimal for the buyer to always guarantee her incumbent supplier a non-negative profit at $T = 2$ when using a non-discriminatory price schedule, i.e., $R(t^1) = t^1 \quad \forall t^1$. The buyer can further reduce her expected costs by using a discriminatory price mechanism. However, we found the additional cost savings to be very small, suggesting that the added complexity of designing of discriminatory EPC is not warranted.
- In contrast, the buyer may not find it optimal to guarantee the incumbent supplier her business under NLI (derived for general cost distribution F).
- Even in the presence of learning by doing, a buyer is often better off running sequential auctions with a reserve price, rather than limiting competition and contracting with a single supplier in the hopes of extracting a better future price. Our numerical examples show that the cost difference between the two mechanisms is very small under NLI, but can be substantial under LI.

As we suspected, EPC performs well when the incumbent has a strong competitive advantage over entrants, i.e., when α and s are large. When α and s are small, a buyer is better off not locking in a supplier for two periods and should instead run SI where the competition in each period is maximized. However, one result that ran counter to our intuition was the effect of N on the performance of EPC versus SI. We were surprised to find that the number of bidders must be moderate-to-large in order to EPC to outperform SI; if N is small, then SI outperforms EPC.

It is important to note that we have labeled the cost reduction that occurs at $T = 2$ as a learning by doing effect. However, our analysis and results apply equally for similar manifestations of economies of scale. We also considered a setting where the buyer has a known demand of Q in each period. It is possible that the buyer has only an estimate of her demand in each period, $E[Q]$. However, as long as both the buyer and seller's are risk-neutral, our analysis and results carry over.

There are a number of ways to expand our models and test the robustness of our results. The one that we feel is most pressing is to extend our model to include settings where the suppliers can reduce their costs over time by exerting costly effort, i.e., the cost reduction no longer occurs 'naturally' but rather is the result of a supplier's conscious effort to reduce production costs. Under such a framework, the buyer may use an EPC not only to try to 'capture' some of the cost savings that a supplier accrues, but also as a *catalyst* for the supplier to identify and undertake cost-saving effort/action and as a result, we conjecture that its performance will improve, relative to SI.

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