Structuring the Initial Offering: Who to Sell To and How to Do It

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This revision: March 2004

[1]We thank Sugato Bhattacharyya, Jos von Bommel, Zhoahui Chen, Robert Hauswald, Dilip Madan, Kristian Rydqvist, Alex Stomper, Meg Van De Weghe and participants at the University of Maryland, Indiana University, Carnegie Mellon and Tulane University finance seminars for their helpful comments and suggestions. This paper has evolved from an earlier paper titled Private versus Public Offerings: Optimal Selling Mechanisms with Adverse Selection, which has been presented at the 1999 Western Finance Association meetings. The authors may be contacted by electronic mail at vmaksimovic@mbs.umd.edu and pichler@mit.edu.

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Abstract

This paper shows how the owner of a nonpublic firm, who wishes to maximize revenue from the sale of shares, should structure the sale. We develop a unified model of adverse selection in the retail market and mechanism design for information gathering that enables us to determine the optimal amount of information gathering before setting the issue price. We show that the need to induce truthtelling does not, without allocation restrictions, lead to underpricing when bookbuilding is conducted prior to an initial public offering. We obtain a number of new empirical predictions relating the information gathering decision and underpricing to characteristics of the bookbuilding process and issue characteristics such as issue size and uncertainty about share value.

1 Introduction

Issuers of unseasoned securities must make a number of strategic decisions when determining how to place their offerings. Issuers must first decide whether to place securities privately or publicly. If they place shares publicly, then they will engage in an initial public offering (IPO). Issuers must decide whether to engage in potentially costly information gathering activities prior to setting the offer price, and if so, how much information to gather. This last decision is also directly linked to the decision of whether to stage an offering, that is break the offering into parts. The advantage of doing so in a public offering is that trading in securities sold in the first stage can enable the public release of information, thus facilitating the pricing of securities in the second stage.

If issuers do not gather information prior to pricing their shares, then they may face a wealth loss due to adverse selection risk. For example, if some potential investors have superior information about the value of the unseasoned securities, then these investors will participate in the offering only if their information is good relative to the price of the issue. If their information is bad, then they will refrain from participating. As a result, uninformed investors are faced with a winner's curse in that they expect to receive a disproportionate allocation if the issue is overpriced. As modeled by Rock (1986), in order to induce uninformed investors to participate in the presence of such adverse selection risk, issues must on average be underpriced.[1] This underpricing is costly to the issuer in that it represents money left on the table at the time of the issue. Alternatively, the adverse selection risk may be lessened by gathering information prior to setting the issue price. For example, underwriters acting as agents for issuers can elicit information directly from potential investors, who are then typically given priority when the securities are allocated. This process is often referred to as bookbuilding. Benveniste and Spindt (1989) modeled this process as a direct mechanism that uses underpricing in order to induce investors to truthfully reveal good information about the issue. The basic idea is that investors do not want to reveal positive information that can lead to a higher issue price, and so the underwriter implicitly commits to allocate underpriced shares to investors who provide such information. Thus, the process of eliciting information results in money left on the table by the issuer.

In this paper, in order to analyze the trade-offs facing the issuer, we develop a model

[1]If uninformed investors refuse to participate, then the issuer may be faced with an unacceptably high probability of failure. Thus, the issuer will underprice the shares so as to ensure uninformed participation.

that takes into account both adverse selection risk and the mechanism design problem for gathering information. In our model the mechanism for eliciting information determines the amount of adverse selection risk remaining among retail investors, *and* the adverse selection risk affects the mechanism design. The advantage of such a unified approach is that we are able to determine the optimal amount of information gathering, using a methodology that endogenously determines the marginal benefit and cost of gathering information prior to pricing the shares. We use this model to answer the following questions: Should a private firm engage in a private or public offering? Should an issuer/underwriter gather information from investors prior to setting the issue price? If so, what is the optimal amount of information gathering? Finally, under what conditions is it optimal for an issuer to stage an IPO? The unified modeling approach that we take enables us to obtain empirically testable relations between underpricing, issue method, and issue characteristics such as the amount of a priori (before gathering any information) and ex post (after gathering the optimal amount of information) uncertainty about share value.

A key decision faced by the issuer is whether to poll informed "regular" investors who are then given preferential access to the issue in exchange for information. That is, an issuer must decide whether to engage in bookbuilding-type information gathering. In contrast to the existing litertature which claims that inducing truth-telling in this process leads to underpricing, we show that this causes underpricing only if there are restrictions on how the shares may be allocated. The most extreme restriction occurs in a private placement, in which the issuer is unable to sell any of the issue to retail investors. In an initial public offering, if the issuer has no restrictions in allocating the issue between regular investors and retail investors, then we can design a uniform price mechanism that truthfully extracts information from regular investors, and does not require the issuer to cede any rents to these investors for their information. Thus, our mechanism leads to zero expected underpricing. This result is important in that it tells us that, if information gathering through bookbuilding leads to underpricing, then this must be due to some constraint that has been imposed on the process. Otherwise, such information gathering is costless.[2]

As we know from the empirical literature, zero underpricing in IPOs is not the norm. In addition, there is evidence that IPO offer prices are only partially adjusted in response to information learned during the bookbuilding process.[3] Such partial adjustment is consistent

[2]It is possible that regular investors will simply refuse to participate if they do not expect to receive positive rents. This possibility is also captured in the model.

[3]Revisions from preliminary price ranges to the offer price have been found to significantly predict underpricing, especially

with the notion that investors who participate in the bookbuilding process are rewarded, by receiving underpriced shares, when positive information is learned by the underwriter. Thus, our first result, together with the empirical literature, implies that additional contraints, apart from just the need to induce truth-telling on the part of investors, are likely present in the bookbuilding process. In conversations with a banker we have been told that any regular investor who participates actively in the pricing process, by providing information, is offered an allocation, regardless of the nature of the information. Underwriters may also follow a policy of allocating a minimum fraction of the issue to institutional investors in order to both provide validation for the issue and to encourage analyst coverage after the offering is completed.[4] These policies represent constraints on the allocation of shares and do require the issuer to underprice in order to induce truth-telling, thus making information gathering costly.

Recognizing what does, and does not, cause underpricing in bookbuilding is important in order to understand the empirical relation between underpricing and characteristics of the issuing firm. This is because the exact nature and extent of the constraints faced by issuers in the bookbuilding process affects the optimal amount of information gathering prior to pricing, and thus affects the expected underpricing. In general, more stringent constraints lead to less information gathering and greater underpricing. But, our analysis enables us to extend our understanding well beyond this. For example, due to adverse selection risk, information gathering becomes *more valuable* as uncertainty about share value increases. Thus, we would expect issuers to optimally gather more information when uncertainty is higher. However, if issuers are faced with allocation constraints in the bookbuilding process, then information gathering also becomes *more costly* as uncertainty increases. The net effect is that, in the presence of allocation constraints, the optimal amount of information gathering is independent of uncertainty, and therefore expected underpricing increases proportionally with uncertainty. If instead, bookbuilding causes underpricing due to a constraint other than allocational,[5] then the optimal amount of information gathering increases with uncertainty, and expected underpricing increases less than proportionally with uncertainty.

There exists a rather extensive literature that documents underpricing in IPOs. Ibbot-

for positive revisions. This has been documented by Hanley (1993) and Maksimovic and Unal (1993). More recent evidence of this phenomenon can be found in Bradley and Jordan (2002), Loughran and Ritter (2002a) and Lowry and Schwert (2002).

[4]Participation of institutional investors can reassure retail investors that the issue is priced fairly. Analyst coverage can be valuable after an offering for maintaining investor interest. Analysts are evaluated by institutional investors, and so prefer to cover firms that have institutional ownership.

[5]An alternative type of constraint is described in the paper.

son, Sindelar and Ritter(1988), Loughran and Ritter (2002b), and Ritter (1987) all provide evidence of significant underpricing, defined as positive returns from the offer price to the first day closing price. The evidence linking allocations to regular investors and information gathering is somewhat mixed. Cornelli and Goldreich (2001) examine bookbuilding by one European investment bank and find that investors who post more informative bids on average earn higher profits since they receive more favorable allocations of IPO shares. Ljungqvist and Wilhelm (2002) address the link between information gathering and allocations to institutional investors, using data from France, Germany, the United Kingdom and the United States. They find a linkage between these allocations, price revisions and underpricing that is consistent with the idea that regular investors are rewarded for providing information prior to an IPO. Jenkinson and Jones (2003), however, find somewhat different evidence. They examine data from order books of European IPOs and find that while institutional bidders are favored in the allocation of IPO shares, this favorable treatment is not necessarily a reward for information contained in their orders.

A number of papers have presented models of the pricing and selling of shares when there is asymmetric information among investors. Habib and Ljungqvist (2001) model promotion activities for selling IPO shares in the presence of adverse selection risk. Their approach has some similarities to ours, but they treat promotion costs as exogenous, and do not consider a mechanism for gathering information. Back and Zender (1993) solve for an optimal sale mechanism for Treasury auctions with two regular investors. The solution of our initial base-case mechanism follows closely from their solution, except that in their model the whole issue is, by assumption, sold to the regular investors. Sherman and Titman (2001) model the moral hazard problem of providing incentives for regular investors to collect information. They do not consider the role of retail investors in the bookbuilding process and so do not consider the tradeoff between information gathering costs and adverse selection risk. Benveniste and Wilhelm (1990) compare differential and uniform pricing policies in a model that considers adverse selection risk and mechanism design, but they do not address the question of optimal information gathering and their solution to the basic model with a fixed amount of information gathering also differs markedly from ours.[6] Biais and Faugeron–Crouzet (2002) expand on the Benveniste and Spindt (1989) model in order to compare bookbuilding with the French mise-en-vente. Sherman (2000) extends Benveniste and Spindt (1989) in order to examine long-term relationships between investors and investment banks.

[6]This difference may be due, at least in part, to some errors they make in information updating.

Neither of these papers considers adverse selection risk in the retail market, or addresses the question of optimal information gathering. Our work differs from all of the above papers in that we endogenize the cost of gathering information through a mechanism. As such we are able to present empirical predictions on the functional forms of the relations between uncertainty, IPO pricing strategies and underpricing that cannot be obtained from other papers.

The rest of the paper is organized as follows. In the following section we describe the basic model. In section 3 we derive the expected underpricing in two polar cases: when the issuer gathers no information and when the issuer is able to very easily gather all information. This work sets the foundation for the results on bookbuilding restrictions and optimal information gathering that are presented in Section 4. Section 5 presents results on wealth loss and addresses the question of staging an IPO. Section 6 concludes. The appendix includes all proofs and a list of notation.

2 The Model

We compare three alternative selling strategies that correspond to existing institutional practices. For each of the strategies we derive a selling mechanism that maximizes the expected value for an entrepreneur who wishes to issue securities.[7] The market for the securities consists of a potentially large number of investors, all of whom have a common knowledge prior belief about the value of the issuing firm. Some of the investors, who we call "informed", have additional relevant information about the value of the firm. The entrepreneur and the remaining investors do not possess this additional information.

The first selling strategy that we consider is the Retail strategy (R) in which the entrepreneur sets a price and sells directly to the financial market without attempting to pregather information possessed by informed investors. This corresponds to a best efforts offering and is the case analyzed by Rock (1986). Because no information is gathered prior to setting the issue price, there will be an adverse selection risk for uninformed investors.

The pool-only strategy (P) is a private offering in which the entrepreneur sells the entire issue to a limited number (pool) of informed investors. This and the following strategy differ from the retail-only strategy in that the issue price is set after the issuer gathers information

[7] While there may be an invesment banker facilitating the sale, we will assume that the incentives of the entrepreneur and the investment banker are perfectly aligned.

from this pool of investors. To examine this strategy, we transform the issuer's problem into a mechanism design problem of the type previously studied in the literature by Benveniste and Spindt (1989) and Laffont and Mortimort (1997).

The third selling strategy corresponds to the bookbuilding method used for pricing and distributing most IPOs in the United States. This method has also increased in use throughout the world.[8] In this strategy the issuer also forms a pool of investors from which he solicits information before setting the price of the issue. The issuer may grant these investors preferential access to shares in return for information, but the issuer retains the option to sell a fraction of the issue to the retail market. The proportion of the issue sold to the pool is a function of the information received from the pool. We call this the Pool-Retail strategy (PR).

In principle, the issuer could consider a fourth strategy: a private offering to the informed pool and to institutional investors outside the pool. However, the adverse selection risks facing investors not in the pool in a private offering are likely to be more severe than those facing investors in the retail market. In addition, there is a potential moral hazard problem in this fourth strategy. If the investment banker sells to nonpool members in a public offering these investors have a secondary market price to check that the banker has not cheated them. Any such cheating will result in costly damage to the banker's reputation. In a private offering there is no secondary market price to keep the banker honest. [9] Thus, such a fourth strategy will be dominated by the pool and retail strategy (PR) and so is not modeled here.

The three strategies expose the issuer to different costs. An issuer who chooses a private placement to a prespecified pool of investors incurs the cost of motivating these investors to bid aggressively for shares of the issue, as well as any participation cost demanded by these investors. The optimal mechanism that we design results in all information that is held by pool members being elicited from them before setting the issue price. Thus, since the entire issue is placed with the pool, the issuer does not have to underprice the issue in order to compensate purchasers for adverse selection risk. By contrast, an issuer who sells an IPO directly to the retail market does not bear a participation cost or the cost of motivating informed investors to bid aggressively. The issuer also does not pregather information from

[8]See Ljungqvist, Jenkinson and Wilhelm (2001) and Sherman (2001).

[9]While we do not explicitly model these reputational issues, everything we do is consistent with this aspect of the investment banking industry.

Figure 1: Timeline for Security Issuance
If strategy R is chosen, then nothing happens at times t_1 and t_2.

investors and thus, the investors are subject to adverse selection risk. The IPO strategy in which the issuer grants preferential access to a pool of informed investors, while retaining an option to sell to retail investors, potentially incurs all these types of costs, but each may be incurred to a lesser degree.

The sequence of events is depicted in Figure 1. At time t_0 the issuer decides whether to conduct a public or private offering. If he has chosen a public offering, then he must decide whether to pregather information (PR strategy), or not (R strategy). At time t_1, if the issuer has chosen strategy P or PR then he decides how large the pool should be and he forms the pool. In strategies P and PR we will design a mechanism such that pool members agree to provide information to the issuer, on the understanding that they will receive an allocation of shares that depends on the information reported by themselves and other pool members. If a pool is formed, pool members report their information to the issuer at time t_2. At time t_3 the issuer sets the issue price. The shares are allocated to each pool member (strategies P and PR) and to the retail market (strategies R and PR) according to the allocation rules.

It is essential to understand that the decision of whether to do a public or private offering is made a priori – before information is elicited from pool members. We make this assumption in accordance with the fact that preparing for an initial public offering takes time and thus choosing the selling strategy is not a decision that can be made just before pricing an issue. The issuer chooses a public offering if, prior to gathering information, a public offering dominates a private offering. If the PR strategy is chosen, it is possible that ex post (after eliciting information), the issuer will optimally choose not to allocate shares to the retail market. Because the issuer chose a priori to obtain the option to sell to the retail market, this is still a public offering. A private offering occurs in our model only when the issuer decides a priori not to obtain the option to sell to the retail market.[10]

For most of the analysis we assume that a fixed number Q of shares will be sold. In order

[10]This means that in the absence of a cost to going public, a private offering will be optimal only when it is not strictly suboptimal.

to keep the algebra simple so that we can focus on information effects, we assume that the offering is a pure secondary offering.[11] These two assumptions imply that the objective of maximizing the issuer's value is equivalent to minimizing expected underpricing. In Section 5 we ease the assumption of a fixed number of shares. We then apply our predictions on wealth loss versus issue size to examine the value of staging an IPO.

We assume that an exogenously given fraction, z_I, of retail investors is informed, where z_I is common knowledge. Each informed investor observes exactly one of two pieces of available information. Each informed investor has a probability one-half of seeing each piece of information. The two signals are independent of the prior belief on firm value and of each other. We assume that the secondary market price will include both signals and that each signal can take on one of two possible values, so that the secondary market value of a share will be:

$$\hat{v} \in \{v_0 - w,\ \ v_0,\ \ v_0 + w\} \tag{1}$$

with the probability distribution $\left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right\}$. The prior expected per share value of the firm is v_0. The factor w is the most that the secondary market price can differ from the prior expected share value. This factor reflects the level of a priori uncertainty about the issue value.

The information structure captures in a simple way the economic fact that informed investors' signals are positively, but not perfectly, correlated. The effect of this is that, in terms of a priori expected value, the benefit to polling an additional informed investor is positive, but the expected added benefit is decreasing in the number of investors who have already been polled. As will be shown in later sections, a key effect of this is that if information gathering is costless, the expected issue price will be an increasing, concave function of the number of informed investors polled.

3 Underpricing in two polar cases

In this section we solve two polar cases: the case where no information is gathered and the case where all information can be gathered. The second case is presented in the simplest possible manner in order to ensure that the results are intuititive and accessible. We will

[11] The analysis can be extended to a combination primary and secondary offering so as to take into account the effect of dilution when an issue is underpriced. This extension is fairly straightforward, but greatly adds to the algebraic complexity without significantly altering the intuition.

show that even with very simplistic assumptions we obtain some fundamental results that will be important as we expand in later sections into a richer and more realistic setting.

3.1 The R strategy: No information gathering prior to pricing

We first determine the expected underpricing when the issuer sells directly to the retail market, without pregathering information from regular investors (Strategy R). It is assumed that investors arrive randomly in the retail market. Each investor who arrives receives e number of shares, where $\frac{e}{Q}$ is very small. Shares are distributed on a first-come first-served basis until all of the shares are sold. We do not consider the possibility of retail investors expending any significant cost to become informed. This is because individual expected allocations are very small and so a rational investor would not expect to recover any significant information cost. Instead, we assume that the fraction, z_I, of retail investors who are informed have become so serendipitously.

When all of the informed investors participate, on average a fraction z_I of the shares will go to informed investors. If an uninformed investor participates in an offering, then he has a higher probability of receiving an allocation when informed investors don't participate, that is when informed investors have negative information. This is the source of the adverse selection risk.

Each informed investor sees a signal S_i and determines that one share is worth v_i. Informed investors form their beliefs knowing there is one other signal that is independent of their own:

$$v_i = E[\hat{v}|S_i] \in \left\{ v_0 - \frac{w}{2}, v_0 + \frac{w}{2} \right\} . \tag{2}$$

In order to ensure the participation of uninformed investors, and thus ensure the success of the offering, the issuer must on average underprice the issue. That is, the shares must be priced below the a priori expected value of the shares. The expected per share underpricing when information is not gathered and the IPO shares are sold entirely to the retail market is:[12]

$$Eu_R = Eu_{Retail} = \frac{wz_I}{2(2 - z_I)} . \tag{3}$$

The analysis of the retail strategy presented here is similar to the Benveniste and Wilhelm (1990) version of the Rock (1986) adverse selection model. However, the underpricing predicted here is somewhat smaller than that predicted by Benveniste and Wilhelm because

[12] See the Appendix for the derivation.

we allow for two independent classes of informed investors. Earlier adverse selection models have assumed that all informed investors observe the same signal. As a result, when the secondary market price is equal to the prior, all of the informed investors participate. In the model developed here only half of the informed investors participate in this state, thus somewhat decreasing the adverse selection problem.

3.2 The P and PR strategies when all information is gathered prior to pricing

In this section we assume that there exist just two informed investors and that these investors have independent signals. The issuer thus forms a two-member pool that contains, with probability one, all the relevant available information. This assumption enables us to obtain, in the simplest possible setting, some fundamental results concerning the differences between mechanism design in a public versus private offering. The drawback of this assumption is that it makes the question of optimal information gathering moot. For this reason we will eliminate this assumption in the following section.

The problem faced by the issuer is to design a mechanism that minimizes the expected underpricing. Prior to communicating with the pool members, the issuer (implicitly) commits to a mechanism that specifies the issue price and the allocation that each pool member receives, contingent on the information that the pool members provide to the issuer. After the pool members report their information to the issuer, the issuer sets the issue price and allocates shares to pool members according to the agreed upon mechanism. Any shares that are not allocated to the pool are then sold to retail investors on a first-come first-served basis. In a private offering all of the shares must be allocated to the pool.[13]

In solving for the optimal mechanism, we proceed in two stages. First, we assume that, in a public offering, the issuer is not constrained in how he allocates the issue to retail investors and pool members, and that the issuer does not need to compensate pool members for any cost of becoming informed. Next, we generalize the model to take into account possible institutional constraints that can limit the issuer's flexibility.

The design of the mechanism is affected by two considerations. First, the issuer must motivate the pool members to report truthfully.[14] Second, the issuer must price the issue so that all participants at least break even in expected value, otherwise they will not wish to

[13] As discussed above, a private offering that includes allocations to investors outside the pool will be dominated by a public offering and so is not considered.

[14] We invoke the Revelation Principle which says that the issuer can do at least as well with a mechanism that induces truthtelling as with any mechanism that does not. See Myerson (1981) for a discussion of this principle.

participate. Because the pool has all available information, if pool members tell the truth, the share value given the information reported by the pool (v_p) is equal to the secondary market share value.

The unconstrained case.

The mechanism design problem faced by the issuer is to choose a pricing and allocation mechanism to satisfy the following set of equations:

$$\min \; Eu = \frac{1}{4}(u^{++} + 2u^{+-} + u^{--})$$

where + (−) signifies positive (negative) information and u^{ab} = underpricing when one pool member reports a and the other reports b. This function is minimized subject to four sets of constraints.

(i) Participation of pool members:

$$EV^{+} + EV^{-} \geq 0 \qquad (PC)$$

$$\begin{aligned} EV^{+} &= \frac{1}{2}\left(u^{++}q^{++} + u^{+-}q^{+-}\right) \geq 0 \\ EV^{-} &= \frac{1}{2}\left(u^{+-}q^{-+} + u^{--}q^{--}\right) \geq 0 \end{aligned}$$

where EV^{a} = expected excess return to an investor who sees and reports a;
q^{ab} = quantity received by a pool member who says a when the other says b.

(ii) Incentive Compatibility for pool members:[15]

$$EV^{+} \geq EV^{-} + \frac{w}{2}(q^{-+} + q^{--}) \qquad (IC^{+})$$

$$EV^{-} \geq EV^{+} - \frac{w}{2}(q^{++} + q^{+-}) \qquad (IC^{-})$$

(iii) Allocation constraints:

$$2q^{++},\; 2q^{--},\; q^{+-} + q^{-+} \leq Q \qquad \text{and} \qquad q^{++},\; q^{--},\; q^{+-},\; q^{-+} \geq 0.$$

In a private offering the first set of allocation constraints must be satisfied with equality.

(iv) Participation of the retail market:

$$u^{++}(Q - 2q^{++}) \geq 0 \qquad 2u^{+-}(Q - q^{+-} - q^{-+}) \geq 0 \qquad u^{--}(Q - 2q^{--}) \geq 0$$

[15] An investor who sees a positive signal and reports a negative signal lowers v_p (the expected share value given the pool's reported information) by an amount w and expects an allocation of $\frac{q^{-+}+q^{--}}{2}$. Similarly, an investor who sees a negative signal and reports a positive signal raises v_p by an amount w and expects an allocation of $\frac{q^{++}+q^{+-}}{2}$.

Because the IPO price will fully reveal the information learned from the pool, this implies that the issuer will not overprice in any state in which retail participation is desired.

We refer to this problem as the "unconstrained case" because the participation constraints require only that the informed investors expect a nonnegative return, and the allocation constraints require only that allocations be nonnegative and that, for a public offering, they not add up to more than the total issue size.

In the following proposition we present the expected underpricing for both private and public offerings. In the private offering the issuer is restricted to sell all of the shares to pool members, regardless of the information learned. In a public offering the issuer may allocate shares to the retail market after eliciting information from the pool. Allocations and price rules for each type of offering are given in the Appendix.

Proposition 1. Zero Expected Underpricing. *When the issuer forms a pool of informed investors who have all the available information, if there are no allocation restrictions or direct costs of information:*
i) Expected underpricing in a public offering is zero.
ii) Expected underpricing in a private offering is strictly positive: $Eu_P = \frac{w}{4}$.

In order to minimize the cost of ensuring truth-telling the issuer should employ a mechanism that allocates as few shares as possible to any pool member who says that the firm has a low value. In a private offering, the issuer is restricted to selling shares only within the pool. Hence, if a pool member deviates and reports a negative signal after observing a positive signal while the other pool member also reports a negative signal, then each receives half of the shares. Selling the offering only to pool members limits the issuer's ability to minimize the cost of eliciting information. As a result, the requirement that the whole offering is sold to the pool means that the issue has to be underpriced. In contrast, in the case of a public offering, a pool member who deviates and reports a negative signal will receive no shares, regardless of what the other pool member reports. As a result, if there are no allocation restrictions and if the informed investors do not require strictly positive compensation for participating, then there is zero expected underpricing in a public offering.

It is important to understand that the zero underpricing result given here is the *a priori* (before gathering information) expected underpricing. Even in a private offering, in our model, there are possible *ex post* outcomes that will result in zero underpricing. However, due to the allocation restrictions that are implicit in a private offering, the a priori expected underpricing is strictly positive. Our result of a priori expected zero underpricing in a public offering, with no allocation restrictions or strictly positive participation cost, is much stronger than previous results of ex post realized zero underpricing.[16]

The results of Proposition 1 were obtained with some very strong assumptions regarding the informed investors: i.e., that there exist exactly two informed regular investors who have seen independent signals. One could argue that, if there really were only two different signals, if the issuer could identify which investors have observed which signals and if there were more than two such investors, then the issuer could eliminate underpricing, even in a private offering, by polling four investors, two who have observed each signal.[17] While this is technically correct, it is not a robust result. A small perturbation of the information assumptions would eliminate such a result. The result presented here is robust: *underpricing is not required to induce truthtelling in a public offering without allocation constraints.* In a later section we will adopt the more realistic assumptions that there are more than two informed investors, but that all of the information cannot be obtained for certain from only a few informed investors. Under this assumption some underpricing may be needed in a public offering due to residual adverse selection risk, but with no allocation constraints underpricing is not needed to induce truthtelling. In a private offering underpricing is needed in order to induce truthtelling.

The constrained case.

Proposition 1, together with the empirical literature cited in the introduction, suggests that allocation and/or participation constraints are likely to be important factors in underpricing of public offerings. (The inability to gather all information for certain from a small number of investors also results in underpricing, but as we will see later, in the absence of significant allocation or participation constraints, this source of underpricing will be largely eliminated.) We consider several possible classes of constraints. First, we allow for the possibility that each pool member requires a minimum nonnegative expected excess return, $\gamma/2$, to be induced to participate. While we allow the value of γ to equal zero, we expect that if potential pool members have a cost to obtaining information, then they will require strictly positive rents.[18]

[16] A priori expected zero underpricing may be obtained by extending the Benveniste and Spindt (1989) results. It is not, however, presented in that paper and we present it here as an important base case to what follows.

[17] The issuer does this by punishing investors who have seen identical signals, but give different reports.

[18] We model the per investor cost as $\gamma/2$, rather than γ purely for ease of notation: in most constraints this quantity is multiplied by 2.

Second, we allow for the possibility that the issuer is constrained to offer each pool member a minimum number, q_L, of shares of the issue. The intuition for q_L comes from discussions with an investment banker. Large investors who are involved in discussions concerning the value of an IPO typically are included when the IPO is sold. While there may be no formal constraint, in practice the banker may be unwilling to assign a zero allocation to a regular investor who has placed a positive order at the issue price, even if the optimal mechanism calls for only the most enthusiastic pool members to receive a positive allocation.[19] We capture this loss of flexibility by assuming that each pool member's allocation must be at least q_L. While we allow for the possibility that this minimum number of shares is zero, we expect that when pool members are large institutional investors, q_L will be strictly positive.

These two constraints, a strictly positive minimum allocation requirement and a strictly positive participation constraint, are similar in that both are means by which informed investors can demand rents from the issuer. However, these two means of demanding rents are qualitatively quite different in that an allocation restriction will directly affect the cost of inducing truth-telling, while the participation constraint does not. For this reason, the form that a demand for rents takes, minimum allocation requirement or participation cost, will affect the empirical predictions regarding the optimal amount of information gathering and the resulting expected underpricing. In developing the model, we considered allowing for a somewhat different minimum allocation restriction. As discussed in the introduction, an issuer may wish to allocate a minimum fraction of the issue to institutional investors as a group. This is a somewhat less restrictive allocation constraint than the one described above, but is otherwise quite similar in that it would affect the cost of inducing truth-telling in the same way. Rather than over-complicate the analysis with too many similar constraints, we choose to include only the more restrictive of the minimum allocation constraints. This choice will not affect the qualitative results presented below.

Third, we assume that in a public offering with gathering of information (PR) the issuer may be constrained to sell a minimum number, q_R, of shares to the retail market. In some countries a constraint requiring allocations to the retail market may be legally mandated. This is especially true for privatizations. Even if such allocations are not legally mandated, investment banks that specialize in distributing IPO shares to retail investors may negotiate

[19] In our model we abstract away from the demand schedules typically submitted by institutional investors. Such investors may submit (somewhat small) orders at prices that are consistent with other investors being more enthusiastic than themselves about the issue. For a regular customer an investment bank may feel obligated to fill at least part of this order.

up front to receive a certain minimum number of shares. In a private offering q_R is automatically set to zero. Both q_R and q_L act as restrictions on the issuer's ability to freely allocate shares between informed institutional investors and the retail market. While we make no claims in our work regarding the size of these parameters (or even if they are strictly positive) we do show that these parameters play significant roles in determining expected underpricing and optimal issue strategies.

We also considered, as a fourth constraint, the possibility that individual pool members have a maximum limit on the shares they can purchase. We have chosen not to present this constraint in the paper because by itself it does not cause underpricing. If each of the other constraint parameters, γ, q_L and q_R, is set to zero, and if the issuer may enlarge the pool size as permitted in the following section, then expected underpricing is zero, even with an upper limit on allocations. Such an upper limit can increase expected underpricing in the presence of other binding constraints, but in the interest of minimizing the complexity of the analysis we choose to focus on the three types of constraints that by themselves can cause underpricing.[20]

The problem is unchanged except for the participation and allocation constraints:

$$EV^+ + EV^- \geq \gamma \tag{PC}$$

where γ is an exogenously given constant, $\quad 0 \leq \gamma < \frac{w}{2}(Q - q_R)$;

$$q_L \leq q^{++},\ \ q^{--} \leq \frac{Q - q_R}{2} \qquad\qquad q^{+-} + q^{-+} \leq Q - q_R \qquad\qquad q^{+-},\ \ q^{-+} \geq q_L$$

where q_L and q_R are exogenously given constants, $\quad q_L \in \left[0, \frac{Q-q_R}{2}\right] \quad$ and $\quad q_R \in [0, Q - 2q_L]$.

Proposition 2. Expected Underpricing, with constraints. *When the issuer forms a pool of two informed investors who have all the available information:*
i) The expected underpricing in a public offering is

$$Eu_{PR} = \max\left[\frac{wq_L}{Q - q_R}, \frac{\gamma}{Q - q_R}\right]. \tag{4}$$

ii) The expected underpricing in a private offering is

$$Eu_P = \max\left[\frac{w}{2}\left(\frac{q_L}{Q} + \frac{1}{2}\right), \frac{\gamma}{Q}\right]. \tag{5}$$

[20] In this section a positive q_R also does not by itself cause underpricing. In the following section, with a more realistic information structure, a positive q_R does by itself cause underpricing.

The expected underpricing in a private offering is now driven either by the cost of inducing turth-telling or by the cost of ensuring the participation of the pool members, whichever is greater. The first term, $\frac{w}{2}\left(\frac{q_L}{Q}+\frac{1}{2}\right)$, is the per share cost of inducing truth-telling, where $\frac{1}{2}\left(\frac{q_L}{Q}+\frac{1}{2}\right)$ is the expected share allocation to any pool member who reports a low value for the issue. We can see that the per share cost of inducing truth-telling is decreasing in the issue size (Q), and increasing linearly in the a priori uncertainty, as represented by w. The last term in Eu_P, $\frac{\gamma}{Q}$, is the per share cost of ensuring participation. This cost, which we expect will increase as the cost of obtaining information increases, decreases as issue size increases.

In both a public and private offering we see that the size of the expected underpricing is increasing in q_L and γ. Which of these is the dominant factor is an open empirical question. The two costs affect underpricing in similar ways. For example, expected relative underpricing is decreasing in issue size, regardless of whether q_L or γ is the dominant factor. However, if the minimum allocation requirement, q_L, is the dominant factor, then underpricing is linearly related to a priori uncertainty (w). This direct relationship between expected underpricing and a priori uncertainty does not occur if the participation cost is the dominant factor.

As long as the first term in the maximand of equation (4) is larger than the second, a public offering will lead to lower expected underpricing than a private offering. If the second terms of the maximands in both equations (4) and (5) are larger than the first (γ is the dominant factor in determining expected underpricing), then there is no advantage in doing a public offering instead of a private offering. In fact, there is a disadvantage if the underwriter is required to allocate shares to the retail market, $q_R > 0$. The following proposition states this result in a qualitative manner. The precise mathematical conditions for the result are given in the Appendix.

Proposition 3. Choice of issue method. *A public offering with gathering of information (PR) will dominate a private offering (P), unless in a public offering such a large portion of the offering must be reserved for retail investors that it substantially interferes with the issuer's ability to extract information from regular investors.*

A private offering strictly dominates a public offering with information gathering only in the case that γ is very large *and* q_R is nonzero, or in the case that q_R is very large relative to the total number of shares sold, Q. In this case, however, doing no pregathering of information (R) will dominate a private offering unless the adverse selection risk in the retail market is very high.[21] In most offerings, allowing retail investors to participate leads to less underpricing. Thus, we predict that an initial public offering will typically have less underpricing than will a private issue of shares in a private firm. The exception occurs when information is both widely distributed (z_I large) and expensive to gather, two phenomena that we do not expect to occur together.

Ultimately the entrepreneur should care about total wealth loss rather than per share underpricing, where total wealth loss is defined as the number of shares sold times per share underpricing. As long as we assume that the number of shares to be sold is fixed, then minimizing wealth loss and minimizing underpricing are equivalent. We will later ease this assumption so it is useful at this point to note that wealth loss due to underpricing caused by adverse selection risk is a variable cost, in that it increases linearly in the number of shares sold. This is seen by the fact that Eu_R is independant of Q. Wealth loss due to underpricing caused by bookbuilding is either a fixed cost, or is decreasing in issue size if $q_R > 0$. A direct implication of this is that we should expect gathering of information through pool formation (bookbuilding) to be more common for larger IPOs, as compared to smaller IPOs.

4 Optimal selling mechanism and underpricing: the general case

In this section we extend the earlier model by adding the more realistic assumption that there are more than two informed regular investors and the issuer cannot be certain, after polling a limited number of regular investors, of having obtained all available information. For simplicity sake we will continue to assume that there exist two independent pieces of information and that each informed investor sees exactly one of these. But, we will now assume that, while the issuer can identify informed investors, the issuer cannot determine a priori which investor has seen which signal. Thus, with a limited pool size the issuer cannot be certain that both signals have been captured. This changes the issuer's problem in a number of ways. First, in a public offering uninformed retail investors face a "residual" adverse selection risk, because of the possibility that only some of the available information has been learned before setting the issue price. The issuer must underprice the issue sufficiently in states in which he sells to the retail market in order to compensate uninformed investors

[21] The exact condition for R to dominate is $\min\left[\frac{q_L}{Q-q_R}, \frac{q_L}{2Q}+\frac{1}{2}\right] > \frac{z_I}{4-2z_I}$ or $\frac{\gamma}{Q} > \frac{wz_I}{4-2z_I}$.

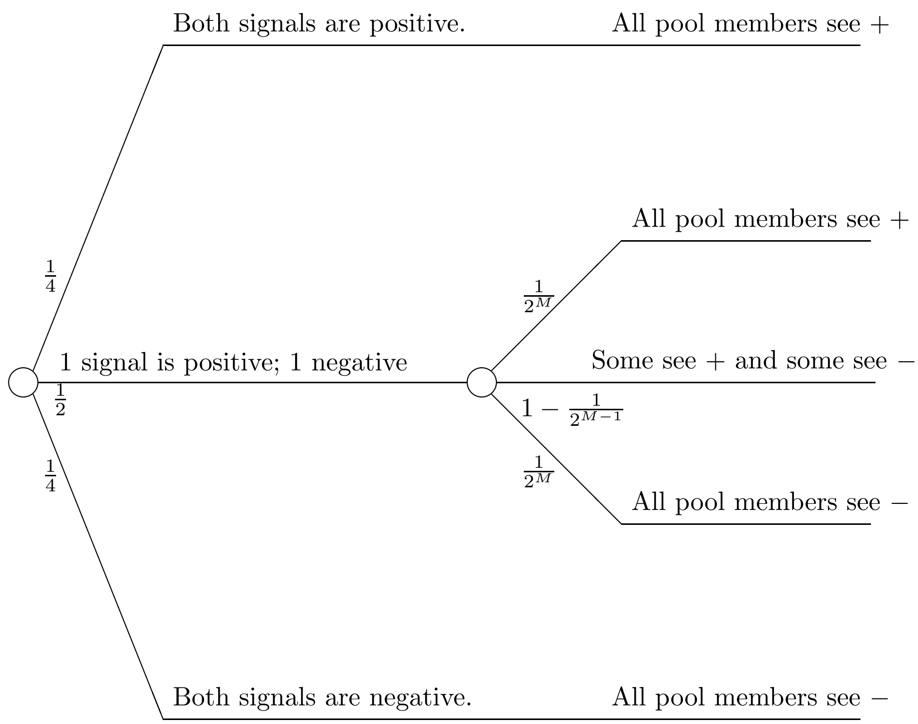


Figure 2: Probability Tree when Pool may not have all information
M = number of pool members

for this residual risk. Second, pool members' information is now positively correlated. This helps the issuer to detect false reports and thus affects the design of the optimal mechanism to elicit information. Third, the issuer may wish to increase the pool size so as to increase the probability that the pool contains all available information. As the pool size increases, the probability of obtaining both pieces of information increases. The probability tree for a pool of size M is illustrated in Figure 2.

We can define the following variables, where their values are derived from Figure 2:

$$v_p^+ \equiv E[\hat{v}|\text{all } M \text{ pool members report } +] = v_0 + \frac{2^{M-1}w}{1+2^{M-1}} < v_0 + w \tag{6}$$

$$v_p^- \equiv E[\hat{v}|\text{all } M \text{ pool members report } -] = v_0 - \frac{2^{M-1}w}{1+2^{M-1}} > v_0 - w \tag{7}$$

There will be residual adverse selection risk, because some investors in the retail market may believe something different from the expected value, given the pool's aggregate information. If, however, some pool members report positive information and some report negative, and there is truthful reporting, then the issuer knows that all of the information has been captured in the pool. In this case $v_p = \hat{v} = v_0$ and there is no residual adverse selection risk.

The probability of zero residual adverse selection risk is $\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{M-1}\right)$. This probability is clearly increasing in the number of pool members, M. If each pool member reports good information, then the underpricing due to residual adverse selection risk is:

$$u^{+}_{AS} = \frac{2^{M-2}wz_I}{(1+2^{M-1})(1+2^{M-1}-(\frac{1}{2}+2^{M-1})z_I)} \ .$$

If each pool member reports good information, then the underpricing due to residual adverse selection risk is:

$$u^{-}_{AS} = \frac{2^{M-2}wz_I}{(1+2^{M-1})(1+2^{M-1}-z_I/2)} \ .$$

It can easily be shown that u^{+}_{AS} and u^{-}_{AS} are both decreasing in M. Thus, the cost of adverse selection risk is decreasing in pool size.

In addition, the impact of a single lie on $v_p = E[\hat{v}|\text{all reports of pool members}]$ is a decreasing function of M. In a pool of size M, ($M \geq 2$), the absolute value of the per share impact of a lie is:

$$w_M = \frac{2^{M-1}w}{1+2^{M-1}}\text{Prob}\{M-1 \text{ others agree with each other}\} = \frac{(1+2^{M-2})w}{1+2^{M-1}} \ .$$

For $M \geq 2$, w_M is in the interval $(\frac{w}{2}, \frac{2w}{3}]$. Because the per share impact of a lie is strictly decreasing with pool size, we would expect that with a larger pool size the issuer could design an optimal selling mechanism that results in less underpricing. This, however, may not hold true, because as the pool size increases the issuer loses allocational flexibility. This latter effect works to increase expected underpricing. In addition, the second derivative of w_M with respect to M is positive, meaning that the incremental informational benefit obtained from adding a pool member is a decreasing function of pool size. In other words, there are decreasing informational returns to pool size. The result of these effects is that, for a wide range of parameter values there will be an optimal value of M that is less than the maximum possible $(\frac{Q}{q_L})$ and greater than 2. That is, there is an interior optimum.

In order to determine the optimal value of M, we must first determine the expected underpricing as a function of M, for the different selling methods. The following proposition gives the expected underpricing when a pool of size M is formed to obtain information. The revised mechanism design problem is specified in the proof of Proposition 4.

Proposition 4. Expected Underpricing, M-member pool may not have all information *Assume that the issuer forms a pool of size M ($M \geq 2$) that may not contain all available information.*

i) If there are no allocation restrictions or cost of information, expected underpricing in a public offering is positive, but significantly less than in a private offering.
ii) The expected underpricing in the P strategy is:

$$Eu_P = \max\left[\frac{(1+2^{M-2})w}{2^{M+1}(1+2^{M-1})}\left(\frac{(2^M-1)Mq_L}{Q}+1\right), \frac{M\gamma}{2Q}\right] \tag{8}$$

iii) The expected underpricing in the PR strategy is:

$$\begin{aligned} Eu_{PR} = \frac{wz^-}{4} + \max\Bigg[& \frac{wMq_L(1+2^{M-2}+\frac{z^-}{2})}{2(1+2^{M-1})(Q-q_R)}, \quad \frac{Jwz^+}{4}, \\ & \frac{(1+2^{M-2}+2^{M-1}z^-)M\gamma}{2(1+2^{M-2})(1+2^M z^-)(Q-q_R)} + \frac{(2^M-1)Mwz^-q_L}{4(1+2^M z^-)(Q-q_R)}, \frac{M\gamma}{2(Q-q_R)} - \frac{wz^-}{4}\Bigg] \end{aligned} \tag{9}$$

$$\textit{where} \quad z^- = \frac{z_I}{2+2^M-z_I} \quad \textit{and} \quad z^+ = \frac{z_I}{2+2^M-(1+2^M)z_I},$$
$$J = 1 \textit{ if } q_R > 0 \quad \textit{and} \quad J = 0 \textit{ otherwise.}$$

The results in Proposition 4 are much more complex than those of Proposition 2, but they are really quite similar. The most significant difference is that underpricing in a public offering may now be caused by residual adverse selction risk. This part of the underpricing is found in the terms of equation (9) that contain either z^- or z^+. Looking at the equations for z^- and z^+, we see that underpricing due to residual adverse risk decreases as the pool size, M, increases. A second difference, relative to the earlier results, is caused by the correlation of the pool members' signals. As in Proposition 2, the first term of the expected underpricing with the P strategy is the per share cost of inducing truth-telling. Suppose, for comparison with Proposition 2, that $M = 2$. The first term of equation (8) is then $\frac{w}{2}\left(\frac{q_L}{Q}+\frac{1}{6}\right)$. This first term is smaller than the corresponding term in Proposition 2, because of the benefit of using correlation to help induce truthtelling. The first term of the maximand in equation (9) is: $\frac{w}{2}\left(\frac{(4+z^-)q_L}{3(Q-q_R)}\right)$. If the residual adverse selection risk is not large, then this is also smaller than in Proposition 2. Thus, the results of Proposition 3 continue to hold, even in this richer environment. A public offering will generally dominate a private offering, unless the allocation restrictions in a public offering are too onerous.

The richer information structure that we model here enables us to address an additional tradeoff faced by the issuer. By increasing pool size, and thus collecting more information, the issuer lowers the residual adverse selection risk. However, if there are either allocation

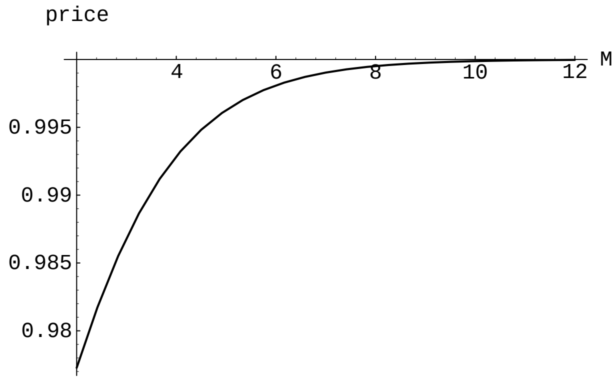


Figure 3: Relative Issue Price versus Pool Size
PR strategy. $q_L = \gamma = 0$ $(w = 1,\, z_I = .5)$

restrictions or a participation cost, then an increase in pool size will also increase the cost of information gathering. Figure 3 presents the expected issue price as a function of M for the case such that there are no allocation restrictions or participation cost, $\gamma = q_L = 0$. From the equations in Proposition 4, it is clear that if $\gamma = q_L = 0$, then the optimal solution calls for a public offering in which pool size (M) goes to infinity and expected underpricing goes to zero. We do not suggest this solution as an economically realistic solution, but rather as an important base case that helps us to understand the source(s) of underpricing.[22] Because of the concavity of the production function in Figure 3, if either γ or q_L is strictly positive, then the optimal pool size will be finite.

A strictly positive value for either γ or q_L represents a constraint that enables regular investors to capture rents in the bookbuilding process. The optimal amount of information gathering (pool size) depends both the nature of such constraints and on issue characteristics, such as issue size and uncertainty. The following proposition summarizes these results.

Proposition 5. Information gathering. *For both public and private offerings:*
a) The optimal amount of information gathering is increasing in issue size, and decreasing in the relative strength of allocation constraints ($\frac{q_L}{Q}$) and the relative strength of participation constraints ($\frac{\gamma}{Q}$).
For public offerings:
b) If the cost of gathering information is driven by a minimum allocation restriction (q_L), then the optimal amount of information gathering is independent of the a priori uncertainty about issue value (w).

[22]It is also true that if the investment bank has its own administrative costs that are increasing in pool size, then the bank would wish to limit the pool size. The analysis can be generalized to address such costs.

c) If the cost of gathering information is driven by a per investor participation cost (γ), then the optimal amount of information gathering is increasing in the a priori uncertainty about issue value (w).

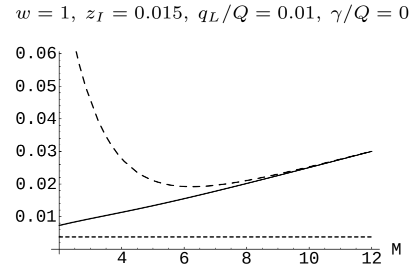
As the amount of information gathering (pool size) increases, the underpricing that is necessary to eliminate the adverse selection risk decreases, while the underpricing that is necessary to cover information gathering costs increases. Underpricing in a public offering with information gathering is the maximum of these two.[23] Thus, the optimal amount of information gathering prior to a public offering is the amount which equates these two levels of underpricing.

As discussed at the end of the last section, wealth loss that is due to underpricing caused by adverse selection risk is a variable cost, in that it increases proportionally with issue size. In contrast, for any given amount of information gathering, wealth loss due to information gathering is a fixed cost; it is independent of issue size. Thus, the optimal amount of information gathering is increasing in issue size.

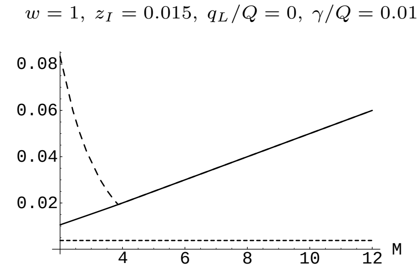
The relation between the level of a priori uncertainty about issue value (w), the amount of information gathering and the level of ex post (at the time of pricing) uncertainty depends on the nature of the constraints that enable investors to capture rents for disclosing information. If the issuer is faced with allocation restrictions ($q_L > 0$), then inducing truth-telling on the part of regular investors is the source of the information gathering cost. This cost is increasing in a priori uncertainty. That is, the more valuable the investors' information, the more the issuer must pay for the information. Because both the cost and benefit of information gathering increase with a priori uncertainty, the optimal amount of information gathering does not increase. That is, the optimal amount of information gathering is independent of the a priori level of uncertainty, and as a result, the residual uncertainty (at the time of pricing) will be proportionally related to the level of a priori uncertainty.

Suppose instead that there are no allocation restrictions, but that the regular investors simply require a fixed expected level of remuneration for collecting information. Information gathering costs from this source are not directly linked to the value of the information. Thus, more information gathering will be done as the level of a priori uncertainty increases, and as a result, the remaining uncertainty will be *less than* proportionally related to the level of a priori uncertainty.

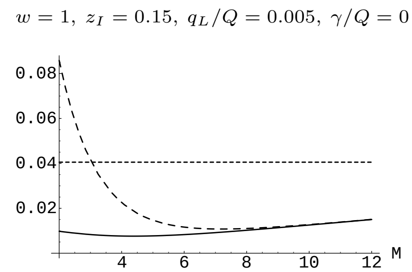
[23]Residual adverse selection risk is not a concern in a private offering.



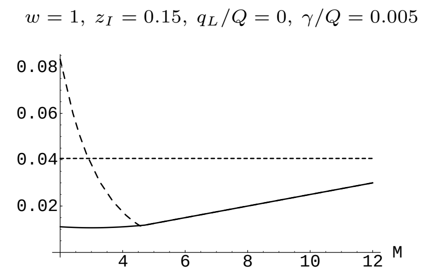
4a: R is Optimal Method



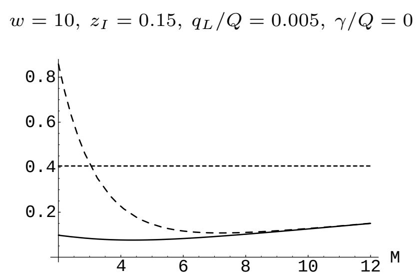
4b: R is Optimal Method



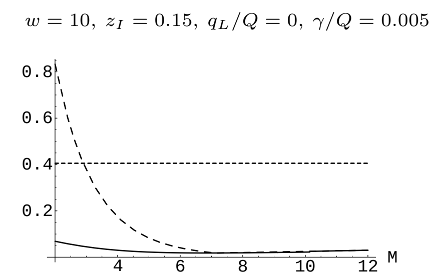
4c: PR is Optimal Method



4d: PR is Optimal Method



4e: PR is Optimal Method



4f: PR is Optimal Method

Figure 4: Expected Underpricing versus Extent of Information Gathering
The convex functions are expected underpricing, given that information is gathered:
— Eu_{PR} - - - Eu_P The horizontal dashed line is Eu_R

In Figure 4 we present a number of numerical examples that illustrate the results of Proposition 5 and also shed light on the choice of issue method. Each graph in Figure 4 shows expected underpricing versus pool size for the three issue strategies: pool and retail (PR), pool only (P) and retail only (R). The expected underpricing, given PR or P is the expected underpricing determined in Proposition 4. In Figures 4a, 4c and 4e, $q_L/Q > 0$ and $\gamma/Q = 0$. In Figures 4b, 4d and 4f, $q_L/Q = 0$ and $\gamma/Q > 0$. The graphs differ further according to the level of a priori uncertainty about issue value, as measured by w, and the fraction of informed investors, z_I. In graphs 4a, 4b and 4d w and z_I are low. In graph 4c w is low and z_I is high. In graphs 4e and 4f both w and z_I are high. In all of the graphs q_R is set equal to zero.

As is illustrated in Figure 4, Eu_{PR} and Eu_P are convex functions of pool size, M. The optimal pool size is the value of M at which the expected underpricing is lowest. For a wide range of parameter values the optimal pool size (number of investors from whom to gather information) is greater than two and less than $\frac{Q}{q_L}$ (the maximum allowed by the allocation constraints). By comparing Figures 4c and 4e we see that if $q_L/Q > 0$ and $\gamma/Q = 0$, then expected underpricing increases proportionally with the increase in uncertainty, w. (The scale on the vertical axis differs across graphs.) Also, the optimal pool size is unaffected by a change in w. In contrast, Figures 4d and 4f show that if $q_L/Q = 0$ and $\gamma/Q > 0$, then while an increase in w does cause expected underpricing to increase, the increase in underpricing is less than proportional. In addition, the optimal pool size is larger with a larger value for w.

When choosing the optimal selling strategy the issuer faces the same tradeoffs that we saw in the previous section. The difference is that now for each strategy the expected underpricing is evaluated at the optimal level of information gathering for that strategy. All else equal, smaller allocation restrictions (smaller $\frac{q_L}{Q}$) make a public offering with information gathering (PR) more attractive as compared to both a private offering (P) and a public offering without information gathering (R). As demonstrated in Figure 4, for a wide range of parameter values, the optimal selling method will be a public offering, either with information gathering (PR) or without information gathering (R). It is only for rather extreme parameter values that a private offering (P) was found to be optimal. Thus, in the presence of asymmetric information across investors, a public offering will for most cases dominate a private offering.

The results of this section provide some additional empirical predictions. Total wealth loss at the optimal level of information gathering will be a combination of a fixed and a variable cost. This is because some, but not all, adverse selection risk will be eliminated through information gathering. Thus, wealth loss will be increasing in the number of shares sold, but less than proportionally. That is, our model predicts that total expected wealth loss is an increasing, but concave function of issue size.

The model also provides insight into the relation between underpricing and the number of institutions receiving significant allocations in an IPO. As our model points out, the number of regular investors to involve in information gathering is endogenous to the IPO pricing process. For a given IPO, expected underpricing is first decreasing, then increasing with the number of institutions involved in the process. However, when looking at a cross-section of issuers, if the issuers have all involved the optimal number of institutional investors then, after controlling for issue size, underpricing will appear to be an increasing function of the number of instutional investors. We should also not confuse the number of institutions that purchase a significant number of shares with the number of institutions that participate in the process. The former may be strictly less than the latter. However, due to the nature of the mechanism for gathering information, we also expect to see greater underpricing associated with a larger fraction of those institutions that are involved in the process also purchasing a significant number of shares.

5 Wealth Loss and Staging of IPOs

Up to this point we have taken issue size as given. In this section we extend the discussion of the previous section on wealth loss versus issue size and consider whether a firm should stage its IPO. If the issuer stages an IPO, then some of the Q shares are sold at the IPO and the remainder are sold in a follow-up offering. The advantage of staging the offering is that a secondary market for the shares is created between the two offerings, thus eliminating the need to underprice in the second offering.[24] We do not consider staging for a private placement, because there is no secondary market advantage.

Because wealth loss due to underpricing caused by adverse selection risk is increasing in issue size, staging the IPO is most advantageous when either no, or very little, information has been pregathered. In fact, the issuer may ideally wish to do no gathering of information and then sell very few shares at the IPO. The problem with this strategy is that the existence of a well-functioning secondary market requires that a sufficient number of shares be sold at the IPO.

We model the staging decision by assuming that the issuer may either sell all Q shares at the IPO, or Q_1 at the IPO and $Q - Q_1$ shares in a second offering. Of course, if $Q \leq Q_1$, then there can be no second offering. We also assume that there is an additional fixed cost F to going to the market to sell shares. This cost may be related to accountants' fees, investment bankers' fees and/or the announcement effect as modeled by Myers and Majluf (1984). While we assume that this cost is realized each time the firm goes to the capital market, we will only concern ourselves with the cost for the second part of a staged offering.

[24] We abstract away from the Myers and Majluf (1984) announcement problem where share price typically drops at the announcement of an equity offering. We do include a cost of going to the market which may proxy for this cost.

This is because the cost must be paid the first time for all selling methods and so is irrelevant to the decision at hand.

If the issuer does no gathering of information prior to pricing, then the amount by which the wealth loss is decreased by staging is:

$$Eu_R \times (Q - Q_1) \quad - \; F \, . \tag{10}$$

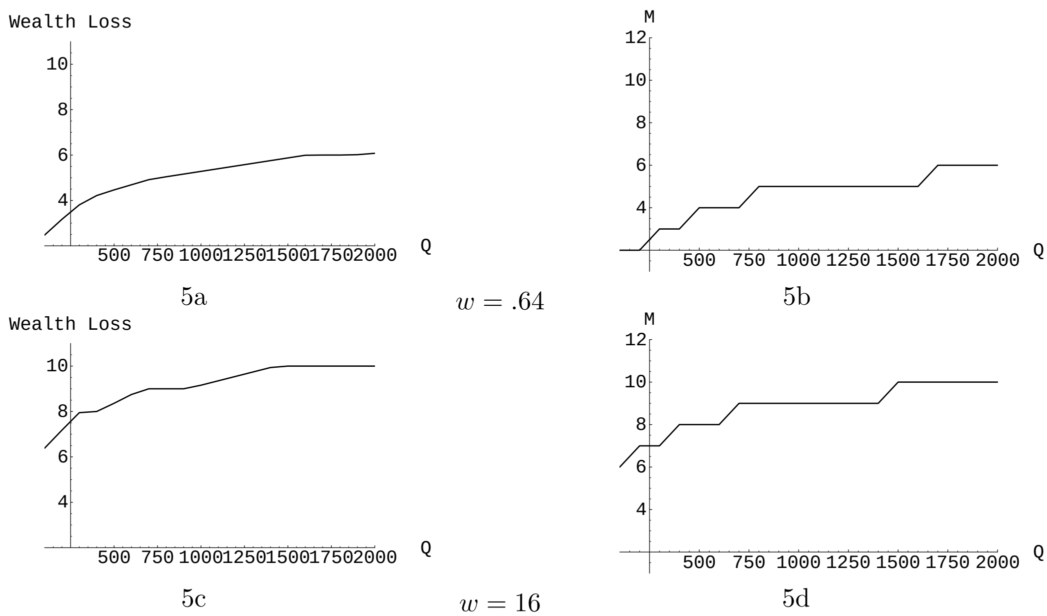
It is clear that this savings is increasing in total issue size, Q. If the issuer does gather information prior to pricing, then the savings is less straightforward. This is because the optimal amount of information gathering is a function of the number of shares issued at the IPO. Thus, the equation for the savings that can be achieved by staging an IPO with information gathering must be written more generally as:

$$Eu_{PR}(Q) \times Q \;\; - \; Eu_{PR}(Q_1) \times Q_1 \quad - \; F \tag{11}$$

This equation cannot be analytically solved, but the algorithm used to numerically solve it is well defined. As shown analytically, our model predicts that expected total wealth loss is generally increasing in issue size, but the relationship is concave. The fact that the extent of the concavity depends on the parameters that we consider is illustrated in Figures 5a, 5c, 6a and 6c. In Figures 5 and 6, the left-hand graphs illustrate expected total wealth loss versus issue size and the right-hand graphs illustrate the optimal amount of information gathering (pool size) versus issue size. As already demonstrated, the optimal amount of information gathering is increasing in issue size. It is this fact that causes the relationship between expected total wealth loss and issue size to be concave. Figures 5 and 6 illustrate the way in which this relationship is affected by two additional factors: the mechanism through which regular investors are remunerated (a minimum allocation, $q_L > 0$, or a direct participation reward, $\gamma/2 > 0$), and the level of a priori uncertainty about issue value, as measured by w.

In Figure 5 the cost of gathering information is driven by a fixed participation cost, $\gamma/2$. The only difference between the top two graphs and the bottom graphs is that the level of a priori uncertainty about firm value, as represented by w, is 25 times higher in the bottom graphs.[25] It is clear from Figures 5b and 5d that, when the cost of gathering information is driven by a fixed participation cost, the issuer optimally gathers more information when there is greater a priori uncertainty. As a result, while expected total wealth loss increases with uncertainty, it increases less than proportionally (by a factor of less than 25).

[25]For all examples in this section, the fraction of informed investors, z_I, is set to 0.25 and $q_R = 0$.



Expected Total Wealth Loss versus Issue Size

Optimal Pool Size versus Issue Size

Figure 5: Wealth Loss and Information Gathering versus Issue Size
$q_L = 0, \ \ \gamma > 0$

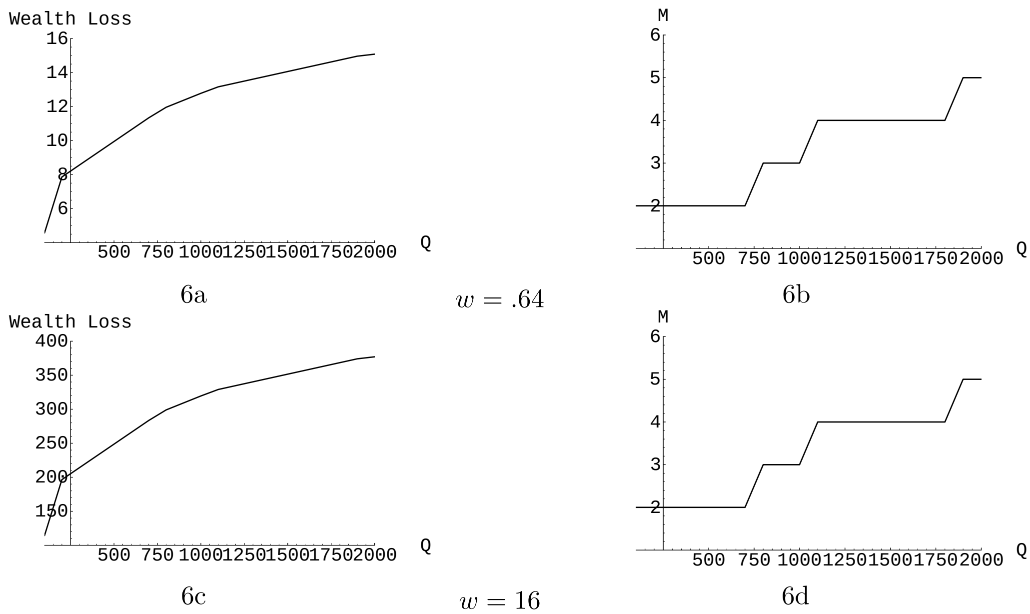
In Figure 6 the cost of gathering information is driven by minimum allocation restrictions, q_L. Again, the only difference between the top two graphs and the bottom graphs is that the level of a priori uncertainty about firm value (w) is 25 times higher in the bottom graphs. In Figures 6b and 6d we see that, when the cost of gathering information is driven by allocation restrictions, the issuer's optimal amount of information gathering is independent of a priori uncertainty. As a result, expected total wealth loss increases proportionally with uncertainty.[26]

To determine the potential value from staging we look at the change in expected total wealth loss between some minimum issue size, Q_1, and the total issue size, Q. Proposition 5, together with the numerical work, enables us to present a number of results. First, as is demonstrated in both Figures 5 and 6, if issue size increases by X%, then the value to staging will increase, but by less than X%.

Staging result 1: *The value of staging is increasing in issue size, but the increase is less than proportional.*

In Figure 5 we can see that even though the total expected wealth loss increases with

[26]Figures 5a and 5c were produced with identical scales. In Figures 6a and 6c the scales change proportionally with w. This was done to aid in comparisons of the graphs.



Expected Total Wealth Loss versus Issue Size

Optimal Pool Size versus Issue Size

Figure 6: Wealth Loss and Information Gathering versus Issue Size
$q_L > 0, \ \ \gamma = 0$

a priori uncertainty, the incremental wealth loss due to increases in issue size (above some rather small minimum size) does not. This illustrates the following result.

Staging result 2: *When the cost of gathering information from investors is driven by a fixed participation cost, then for any nontrivial Q_1 (minimum IPO size) and any fixed Q (total issue size), the value to staging is relatively independent of the level of a priori uncertainty.*

Figure 6 provides a contrast to this. The curves in Figures 6a and 6c are identical, but the scale in Figure 6c is 25 times larger, as is the level of uncertainty. As discussed in the previous section, when allocation restrictions are the source of information gathering costs, then the optimal amount of information gathering is independent of the amount of a priori uncertainty, and the underpricing is thus increasing proportionally with uncertainty. This leads to the third result.

Staging result 3: *When the cost of gathering information is driven by allocation restrictions, then the value of staging increases proportionally with a priori uncertainty.*

In Figure 6c wealth loss is driven largely by adverse selection risk, even at large issue sizes.[27] It is for this reason that the slope of the wealth loss function versus issue size is steeper than in other graphs. In this example a priori uncertainty is high and the cost of gathering information is driven by allocation restrictions. It is here that we find the greatest benefit to staging an IPO.

Staging result 4: *If the cost of information gathering is driven by allocation restrictions, then there is generally a greater benefit to staging an IPO than if the cost of information gathering is driven by a fixed participation cost.*

While the benefit to staging an issue always increases with total issue size, this is not true of the amount of information that will optimally be gathered prior to pricing. All else equal, small firms will generally gather small amounts of information and not stage their offerings. Medium-sized firms will gather more information, and also not stage. For large firms, however, it will be optimal to gather less information prior to pricing, and then stage the initial offering. This nonmonotonicity occurs because of the fixed cost of going to the market.

6 Conclusion

We show that although the IPO paradigm whereby an underwriter elicits information from a limited number of informed investors to whom he gives special access (bookbuilding) is optimal for a wide variety of security offerings, the amount of information that is optimally gathered prior to setting the issue price will vary across firms. We first solve for the optimal mechanism for extracting information from investors, and then apply this result to determine the optimal amount of information gathering as a function of characteristics of the firm and the bookbuilding process. We also determine conditions under which it is optimal to stage an IPO.

In designing an optimal mechanism, the issuer would ideally like to have complete freedom to discriminate among investors, both on share allocations and price. If, however, the issuer has complete freedom to discriminate on allocations, then for the most part price discrimination is unnecessary. In fact, if there are neither allocation restrictions nor an explicit cost of participation for informed investors, then bookbuilding in an IPO results in zero

[27]In Figure 6 underpricing is driven mostly by the adverse selection risk. For an issue size of 200 shares or less, the optimal issue method is R, no pregathering of information. For this reason, wealth loss is increasing with issue size at a fairly steep rate below 200 shares. Above 200 shares, it is optimal to do some gathering of information, but there still remains significant residual adverse selection risk.

expected underpricing. Thus, in order to understand what causes underpricing in an IPO with bookbuilding, we develop a model that allows for a number of restrictions that may be imposed in the bookbuilding process. We develop empirical predictions that relate underpricing to these different types of restrictions and to issue characteristics. We show that in bookbuilt IPOs, restrictions requiring that a minimum number of shares be allocated to institutional investors have a greater impact on underpricing than do restrictions requiring that a minimum number of shares be allocated to retail investors. This is because the former restriction increases the cost of eliciting information in bookbuilding, and therefore affects the optimal amount of information gathering.

Our model provides a structure for examining a number of related issues. If information learned either during the issue process itself, or in secondary market trading after an IPO, can be valuable for resource allocation and if the information is costly, then the method of selling shares can directly affect the value of a firm. Costly and valuable information increases the attractiveness of the strategy in which an issuer elicits information from a limited number of informed investors, but also maintains the flexibility to sell to the retail market. The main reason for this is that the process of preferential allocation enables an issuer to compensate investors who gather information. And, the ability to sell to retail investors makes it unnecesary to over-compensate them.

Our results have implications for both the sale of corporate securities and of Treasury bonds. Primary dealers in U.S. Treasury securities may have proprietary information about the demand for a new issue. Our analysis implies that a better price may be obtained for the bonds if the Treasury reserves the right to sell part of the issue directly to retail investors, with this quantity being determined *after* bids are submitted by these dealers.

7 Appendix

Notation:

Strategies: R $\equiv$ retail–only, no pregathering of information, public offering
P $\equiv$ pool–only, pregathering of information, private offering
PR $\equiv$ pool and retail, pregathering of information, public offering

Q = number of shares to be sold
v_0 = prior expected per share value
S_i = signal observed by informed investor i
z_I = fraction of retail investors who are informed
$\hat{v}$ = secondary market per share value
v_p = expected value of $\hat{v}$, given all the information announced by pool members
v_p^k = expected value of $\hat{v}$, given that all of the pool members report k, $k \in \{+, -\}$
w = possible revision in share value, due to private information, $\hat{v} - v_0 \in \{-w, 0, w\}$
u = per share underpricing
Eu_X = expected per share underpricing when following strategy X, $X \in \{R, P, PR\}$
q_L = lower bound on number of shares that pool member will accept
q_R = minimum number of shares that must be sold to retail investors in a public offering
$\frac{\gamma}{2}$ = cost of gathering information (per pool member), $0 \leq \gamma < \frac{w}{2}(Q - q_R)$
EV^k = expected excess return to investor who sees and reports k, $k \in \{+, -\}$
EV^R = expected excess return to retail investors
M = number of pool members
u_{AS}^k = underpricing due to residual adverse selection risk when all M pool members report k, $k \in \{+, -\}$
w_M = per share impact of a lie by a single pool member, given M pool members

Derivation of v_i. Each informed investor sees one of two independent signals. The beliefs of an individual informed investor are $v_i = E[\hat{v}|S_i]$.
If S_i is positive, then $v_i = \frac{1}{2}(v_0 + w) + \frac{1}{2}(v_0) = v_0 + \frac{w}{2}$.
If S_i is negative, then $v_i = \frac{1}{2}(v_0 - w) + \frac{1}{2}(v_0) = v_0 - \frac{w}{2}$.

Derivation of Eu_R. The following table describes the participation in a sale to the retail market. To simplify the math we assume that if a particular class of informed agents participates, that class will receive a fraction $z_I/2$ regardles of whether the other class participates.

Secondary market value	$v_0 - w$	v_0	$v_0 + w$
probability	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
informed participation	0	$\frac{z_I}{2}$	z_I
fraction for uninformed	1	$1 - \frac{z_I}{2}$	$1 - z_I$

Let s = IPO price. The expected return to uninformed investors must be nonnegative. When underpricing is minimized, this expected return will be zero:

$$EV_U = 0 = \frac{1}{4}(v_0 - w - s) + \frac{1}{2}(v_0 - s)\left(1 - \frac{z_I}{2}\right) + \frac{1}{4}(v_0 + w - s)(1 - z_I)$$

The expected underpricing predicted by this model is: $Eu_R \equiv v_0 - s = \frac{wz_I}{4-2z_I}$.

Proof of Propositions 1 and 2. i) Private Offering: Expected underpricing can also be written as:

$$Eu = \left(\frac{1}{4}\right)\frac{2EV^+}{Q} + \left(\frac{1}{2}\right)\frac{(EV^+ + EV^-)}{Q} + \left(\frac{1}{4}\right)\frac{2EV^-}{Q} = \frac{EV^+ + EV^-}{Q}$$

The expected return to pool members of each type is:

$$EV^+ = \frac{1}{2}\left[u^{++}q^{++} + u^{+-}q^{+-}\right] \geq 0 \qquad\qquad EV^- = \frac{1}{2}\left[u^{+-}q^{-+} + u^{--}q^{--}\right] \geq 0$$

In the P strategy, $q^{++} = q^{--} = Q/2$ and $q^{+-} = Q - q^{-+}$, thus:

$$Eu = \frac{1}{Q}\left[EV^+ + EV^-\right] \geq \frac{\gamma}{Q}$$

Incentive Compatibility for those who see + requires:

$$2EV^+ - 2EV^- \geq w(q^{-+} + q^{--}) \geq w(q_L + \frac{Q}{2}) \Longrightarrow EV^+ \geq \frac{w}{2}(q_L + \frac{Q}{2}) \Longrightarrow Eu \geq \frac{w(q_L + \frac{Q}{2})}{2Q}$$

This is the lower bound on expected underpricing. This lower bound is feasibly attained with the solution given in the proposition, if the participation constraint is nonbinding. If the participation is binding, $Eu_P = \frac{\gamma}{Q}$. Thus, $Eu_P = \max\left[\frac{w}{2}\left(\frac{q_L}{Q} + \frac{1}{2}\right), \frac{\gamma}{Q}\right]$.

One optimal mechanism that achieves this expected underpricing is:

$q^{++} = q^{--} = \frac{Q}{2}, \qquad q^{+-} = Q - q_L, \qquad q^{-+} = q_L, \qquad u^{--} = u^{+-} = 0, \qquad u^{++} = 4Eu_P$

ii) Public Offering: Letting EV^R = expected excess return to retail investors, the problem becomes:

$$\min\ Eu = \frac{1}{Q}\left[EV^R + EV^+ + EV^-\right]$$

subject to:

$$EV^+ - EV^- \geq \gamma - 2EV^- \qquad (PC)$$

$$EV^+ - EV^- \geq \frac{w}{2}(q^{-+} + q^{--}) \qquad (IC^+)$$

$$EV^+ - EV^- \leq \frac{w}{2}(q^{++} + Q - q^{-+}) \qquad (IC^-)$$

In addition, the participation constraints for the retail market and the allocation constraints must be satisfied. If IC^+ is binding, then the objective function becomes:

$$\min \quad \frac{w}{2}(q^{-+} + q^{--}) + u^{+-}q^{-+} + u^{--}q^{--} + \frac{u^{++}}{4}(Q - 2q^{++}) + \frac{u^{+-}}{2}(Q - q^{+-} - q^{-+}) + \frac{u^{--}}{4}(Q - 2q^{--})$$

It is optimal to have $q^{++} = \frac{Q-q_R}{2}$ and $q^{+-} + q^{-+} = Q - q_R$:

$$\min \quad \frac{w}{2}(q^{-+} + q^{--}) + u^{+-}q^{-+} + \frac{u^{++}q_R}{4} + \frac{u^{+-}q_R}{2} + \frac{u^{--}}{4}(Q + 2q^{--})$$

$$\text{subject to:} \qquad \frac{w}{2}(q^{-+} + q^{--}) + u^{+-}q^{-+} + u^{--}q^{--} \geq \gamma \qquad (PC)$$

$$u^{++}q^{++} + u^{+-}(q^{+-} - q^{-+}) - u^{--}q^{--} = w(q^{-+} + q^{--}) \qquad (IC^{+})$$

$$q^{-+} + q^{--} \leq \frac{Q - q_R}{2} + q^{+-} + q_R \qquad (IC^{-})$$

IC^{-} is clearly nonbinding as long as $q^{-+} \leq q^{+-}$.
If PC is nonbinding, the solution is: $u^{--} = u^{+-} = 0$ and $q^{-+} = q^{--} = q_L$, resulting in:

$$u^{++} = \frac{4wq_L}{Q - q_R} \qquad Eu_{PR} = \frac{wq_L}{Q - q_R} \qquad EV^{-} = 0 \qquad EV^{+} = wq_L \qquad EV^{R} = \frac{wq_Lq_R}{Q - q_R}$$

PC is nonbinding iff $wq_L \geq \gamma$. If PC is binding, then $EV^{+} + EV^{-} = \gamma$ and $EV^{R} = q_REu$, so that $Eu = \frac{\gamma}{Q-q_R}$. Thus, the full solution is: $Eu_{PR} = \max\left[\frac{wq_L}{Q-q_R}, \frac{\gamma}{Q-q_R}\right]$.
One optimal mechanism that achieves this expected underpricing is: $q^{++} = \frac{Q-q_R}{2}$, $q^{+-} = Q - q_L - q_R$, $q^{-+} = q^{--} = q_L$, $u^{--} = u^{+-} = 0$, $u^{++} = 4Eu_{PR}$ ∎

Proposition 3. Precise conditions. A sufficient, but not a necessary, condition for PR to dominate P is that $q_R = 0$. If $q_R > 0$, PR still dominates P unless a) γ is so large that it determines underpricing or b) the minimum allocation restrictions are so severe that the issuer cannot reallocate much of the offering away from a pool member who reports bad news. That is, if $\frac{q_L}{Q-q_R}\left(\frac{Q+q_R}{Q}\right) > \frac{1}{2}$, which occurs only if q_L is close to its upper limit and $q_R > 0$.

Derivation of u^{+}_{AS} and u^{-}_{AS}. Suppose all pool members report positive information: An informed retail investor who sees an IPO price of $v^{+}_{p} - u^{++}$ and sees a positive signal will believe each share is worth $v^{++}_{p} = v_0 + \frac{2^M w}{1+2^M} > v^{+}_{p}$. This investor will participate in the retail market. An informed retail investor who sees an IPO price of $v^{+}_{p} - u^{++}$ and sees a negative signal will believe each share is worth v_0. As long as $u^{++} < v^{+}_{p} - v_0$, this investor will not participate. (It is shown below that underpricing due to the residual adverse selection risk satisfies this inequality.) We may form the following table for when the pool members all report positive information:

Secondary market value	v_0	$v_0 + w$
probability	$\frac{1}{1+2^{M-1}}$	$\frac{2^{M-1}}{1+2^{M-1}}$
informed participation	$\frac{z_I}{2}$	z_I
fraction for uninformed	$1 - \frac{z_I}{2}$	$1 - z_I$

Using this table and following the derivation of Eu_R, it is seen that the underpricing due to residual adverse selection risk is: $u^{+}_{AS} = \frac{2^{M-2}wz_I}{(1+2^{M-1})(1+2^{M-1}-(\frac{1}{2}+2^{M-1})z_I)}$.

Suppose that all pool members report negative information. An informed retail investor who sees an IPO price of $v_p^- - u^{--}$ and sees a positive signal will believe each share is worth v_0. This investor will participate. An informed retail investor who sees an IPO price of $v_p^- - u^{--}$ and sees a negative signal will believe each share is worth $v_p^{--} = v_0 - \frac{2^M w}{1+2^M} < v_p^-$. As long as $u^{--} < v_p^- - v_p^{--}$, this investor will not participate. (Again, the underpricing due to adverse selection in the retail market will be strictly less than this.) We may form the following table for when the pool members all report negative information:

Secondary market value	v_0	$v_0 - w$
probability	$\frac{1}{1+2^{M-1}}$	$\frac{2^{M-1}}{1+2^{M-1}}$
informed participation	$\frac{z_I}{2}$	0
fraction for uninformed	$1 - \frac{z_I}{2}$	1

Applying the results in the table, the underpricing due to residual adverse selection risk is: $u_{AS}^- = \frac{2^{M-2} w z_I}{(1+2^{M-1})(1+2^{M-1}-z_I/2)}$.

Proof of Proposition 4. Define the following variables:

$$
\begin{aligned}
u^{aa} &= \text{underpricing when all pool mbrs see a} \\
u_x^{+-} &= \text{underpricing when } 0 < x < M \text{ see } + \\
q^{aa} &= \text{qty to each pool mbr when all see a} \\
q_x^{+-} &= \text{qty to pool mbr who sees + when } 0 < x < M \text{ see } + \\
q_x^{-+} &= \text{qty to pool mbr who sees – when } 0 < x < M \text{ see } +
\end{aligned}
$$

Prob{the two signals are identical} $= \frac{1}{2}$
Prob{the two signals are different} Prob{x see + and $(M-x)$ see – | different signals} =
Prob{the two signals are different} Prob{$(M-x)$ see + and x see – | different signals} =
$\frac{1}{2^{M+1}} \frac{M!}{x!\,(M-x)!}$

$$
\begin{aligned}
\min\ Eu &= \left(\frac{1}{4} + \frac{1}{2^{M+1}}\right) u^{++} + \left(\frac{1}{2} - \frac{1}{2^M}\right) u^{+-} + \left(\frac{1}{4} + \frac{1}{2^{M+1}}\right) u^{--} \\
&= \frac{1}{Q}\left[EV^R + \frac{M}{2} EV^+ + \frac{M}{2} EV^-\right]
\end{aligned}
$$

$$
\text{where} \qquad u^{+-} = \frac{1}{2(2^{M-1}-1)} \sum_{x=1}^{M-1} \frac{M!\, u_x^{+-}}{x!\,(M-x)!}
$$

$$
\begin{aligned}
EV^R &= \left(\frac{1}{4} + \frac{1}{2^{M+1}}\right) u^{++}(Q - Mq^{++}) + \left(\frac{1}{4} + \frac{1}{2^{M+1}}\right) u^{--}(Q - Mq^{--}) \\
&\quad + \frac{1}{2^{M+1}} \sum_{x=1}^{M-1} \frac{M!\, u_x^{+-}(Q - xq_x^{+-} - (M-x)q_x^{-+})}{x!\,(M-x)!} \\
EV^+ &= \left(\frac{1}{2} + \frac{1}{2^M}\right) u^{++} q^{++} + \frac{1}{2^M} \sum_{x=1}^{M-1} \frac{(M-1)!\, u_x^{+-} q_x^{+-}}{(x-1)!\,(M-x)!}
\end{aligned}
$$

$$EV^- = \frac{1}{2^M}\sum_{x=1}^{M-1}\frac{(M-1)!\,u_x^{+-}q_x^{-+}}{(x)!\,(M-x-1)!} + \left(\frac{1}{2}+\frac{1}{2^M}\right)u^{--}q^{--}$$

The participation constraints are:

$$EV^+ + EV^- \geq \gamma \qquad (PC)$$

$$u^{++}(Q-Mq^{++}) \geq u_{AS}^{+}(Q-Mq^{++}) \qquad u^{--}(Q-Mq^{--}) \geq u_{AS}^{-}(Q-Mq^{--})$$

Incentive compatibility:

$$EV^+ \geq EV^- + \frac{u_{M-1}^{+-}q_{M-1}^{-+} - u^{--}q^{--}}{2} + \frac{w_M}{2^M}\left(2^{M-1}q_{M-1}^{-+} + \sum_{x=0}^{M-1}\frac{(M-1)!\,q_x^{-+}}{(x)!\,(M-x-1)!}\right) \qquad (IC^+)$$

$$EV^- \geq EV^+ + \frac{u_1^{+-}q_1^{+-} - u^{++}q^{++}}{2} - \frac{w_M}{2^M}\left(2^{M-1}q_1^{+-} + \sum_{x=1}^{M}\frac{(M-1)!\,q_x^{+-}}{(x-1)!\,(M-x)!}\right) \qquad (IC^-)$$

$$w_M = \text{expected impact of lie } = \frac{(1+2^{M-2})w}{1+2^{M-1}}$$

Allocation constraints:

$$q_L \leq q^{++} \leq \frac{Q-q_R}{M} \qquad q_L \leq q^{--} \leq \frac{Q-q_R}{M} \qquad xq_x^{+-} + (M-x)q_x^{-+} \leq Q - q_R$$

$$q_x^{+-}, q_x^{-+} \geq q_L \qquad q_L = \text{exogenously given, nonnegative number} \leq \frac{Q-q_R}{M}$$

Assumption: $\frac{\gamma}{Q} < \frac{w}{2}$ If this doesn't hold then (R) strategy will dominate all pool strategies, regardless of the values of the other parameters.

P Strategy:

$$\min \; EV^+ + EV^- = 2EV^- + (EV^+ - EV^-) \qquad \text{subject to:}$$

$$\frac{1}{2}\left(u_{M-1}^{+-}q_{M-1}^{-+} - u^{--}q^{--}\right) + \frac{w_M}{2^M}\left(2^{M-1}q_{M-1}^{-+} + \sum_{x=0}^{M-1}\frac{(M-1)!\,q_x^{-+}}{(x)!\,(M-x-1)!}\right) \leq$$

$$EV^+ - EV^- \leq \frac{1}{2}\left(u^{++}q^{++} - u_1^{+-}q_1^{+-}\right) + \frac{w_M}{2^M}\left(2^{M-1}q_1^{+-} + \sum_{x=1}^{M}\frac{(M-1)!\,q_x^{+-}}{(x-1)!\,(M-x)!}\right)$$

In the (P) strategy, $q^{++} = q^{--} = Q/M$, $xq_x^{+-} + (M-x)q_x^{-+} = Q$.
If PC is nonbinding, then: $u_x^{+-} = u^{--} = 0$, $q_x^{-+} = q_L$. The IC constraints become:

$$\frac{w_M}{2^M}\left(2^{M-1}q_L + \sum_{x=1}^{M-1}\frac{(M-1)!\,q_L}{(x)!\,(M-x-1)!} + \frac{Q}{M}\right) \leq EV^+ \leq$$

$$\frac{u^{++}Q}{2M} + \frac{w_M}{2^M}\left(2^{M-1}(Q-(M-1)q_L) + \sum_{x=1}^{M-1}\frac{(M-1)!\,\frac{1}{x}(Q-(M-x)q_L)}{(x-1)!\,(M-x)!} + \frac{Q}{M}\right)$$

Solution:

$$\left(1+\frac{1}{2^{M-1}}\right)\frac{u^{++}}{2} = EV^+\left(\frac{M}{Q}\right)$$

$$= \max\left[\frac{w_M}{2^M}\left(\frac{2^{M-1}Mq_L}{Q} + \sum_{x=1}^{M-1}\frac{(M-1)!\,Mq_L}{(x)!\,(M-x-1)!Q} + 1\right), \frac{M\gamma}{Q}\right]$$
$$= \max\left[\frac{w_M}{2^M}\left(\frac{(2^M-1)Mq_L}{Q} + 1\right), \frac{M\gamma}{Q}\right]$$

This solution is valid iff IC^- is satisfied. IC^- is clearly satisfied if PC is nonbinding (u^{++} is determined by the first term in the maximum above.) If PC is binding (u^{++} is determined by γ.), then we need:

$$\frac{M\gamma}{(1+2^{M-1})Q} \leq w_M\left(2^{M-1}(Q-(M-1)q_L) + \sum_{x=1}^{M-1}\frac{(M-1)!\,\frac{1}{x}(Q-(M-x)q_L)}{(x-1)!\,(M-x)!} + \frac{Q}{M}\right)\left(\frac{M}{Q}\right)$$

Sufficient for this is:

$$\frac{M\gamma}{Q} \leq (1+2^{M-2})w\left(\frac{(2^M-1)Mq_L}{Q} + 1\right)$$

Sufficient for this is $\frac{\gamma}{Q} \leq \frac{(1+2^{M-2})w}{M}$ and sufficient for this is $\frac{\gamma}{Q} \leq w$, which is true by assumption. Thus:

$$Eu_P = EV^+\left(\frac{M}{2Q}\right) = \max\left[\frac{w_M}{2^{M+1}}\left(\frac{(2^M-1)Mq_L}{Q} + 1\right), \frac{M\gamma}{2Q}\right]$$

PR Strategy: If IC$^+$ is binding:

$$\min\ \frac{M}{2}(EV^+ + EV^-) + EV^R = \frac{M}{2}(EV^+ - EV^-) + M\,EV^- + EV^R =$$
$$\frac{M}{4}\left(u^{+-}_{M-1}q^{-+}_{M-1} - u^{--}q^{--}\right) + \frac{Mw_M}{2^{M+1}}\left(2^{M-1}q^{-+}_{M-1} + \sum_{x=0}^{M-1}\frac{(M-1)!\,q^{-+}_x}{(x)!\,(M-x-1)!}\right) +$$
$$\frac{M}{2^M}\sum_{x=1}^{M-1}\frac{(M-1)!\,u^{+-}_x q^{-+}_x}{(x)!\,(M-x-1)!} + \left(1+\frac{1}{2^{M-1}}\right)\frac{Mu^{--}q^{--}}{2} +$$
$$\left(\frac{1}{4}+\frac{1}{2^{M+1}}\right)u^{++}(Q-Mq^{++}) + \left(\frac{1}{4}+\frac{1}{2^{M+1}}\right)u^{--}(Q-Mq^{--})$$
$$+ \frac{1}{2^{M+1}}\sum_{x=1}^{M-1}\frac{M!\,u^{+-}_x(Q-xq^{+-}_x-(M-x)q^{-+}_x)}{x!\,(M-x)!}$$

subject to: $$EV^+ + EV^- \geq \gamma \Longrightarrow EV^+ - EV^- \geq \gamma - 2EV^- \quad (PC)$$

$$EV^+ - EV^- \leq \frac{1}{2}\left(u^{++}q^{++} - u^{+-}_1 q^{+-}_1\right) + \frac{w_M}{2^M}\left(2^{M-1}q^{+-}_1 + \sum_{x=1}^{M}\frac{(M-1)!\,q^{+-}_x}{(x-1)!\,(M-x)!}\right) \quad (IC^-)$$

$$\text{If}\quad q^{--} < \frac{Q}{M} \text{ then } u^{--} \geq u^-_{AS} = \frac{2^{M-1}wz^-}{1+2^{M-1}} \quad \text{where} \quad z^- = \frac{z_I}{2+2^M-z_I}.$$

$$\text{If}\quad q^{++} < \frac{Q}{M} \text{ then } u^{++} \geq u^+_{AS} = \frac{2^{M-1}wz^+}{1+2^{M-1}} \quad \text{where} \quad z^+ = \frac{z_I}{2+2^M-(1+2^M)z_I}.$$

We can see that the issuer should set $q^{++} = \frac{Q-q_R}{M}$. As long as IC^- is satisfied then he should also set $xq^{+-} = Q - q_R - (M-x)q^{-+}$. It will be shown that IC^- is satisfied, so the only state in which the issuer may benefit from selling more than q_R to the retail market is $^{--}$. In order to make EV^- as small as possible the issuer will set $u^{--} = u^-_{AS}$, $u^{+-} = 0$ and $q_x^{-+} = q^{--} = q_L$ $(\forall x < M)$. IC^+ binding and the definitions of EV^- and EV^+ give us:

$$\begin{aligned} EV^+ - EV^- &= -\frac{2^{M-2}wz^-q_L}{1+2^{M-1}} + \frac{w_M q_L}{2^M}\left(2^{M-1} + \sum_{x=0}^{M-1}\frac{(M-1)!}{(x)!\,(M-x-1)!}\right) \\ &= -\frac{2^{M-2}wz^-q_L}{1+2^{M-1}} + w_M q_L = \frac{wq_L(1+2^{M-2}(1-z^-))}{1+2^{M-1}} \\ EV^- &= \left(\frac{1+2^{M-1}}{2^M}\right)\frac{2^{M-1}wz^-q_L}{1+2^{M-1}} = \frac{wz^-q_L}{2} \end{aligned}$$

$$EV^+ = \frac{wq_L(1+2^{M-2}(1-z^-))}{1+2^{M-1}} + \frac{wz^-q_L}{2} = \frac{wq_L(1+2^{M-2}+\frac{z^-}{2})}{1+2^{M-1}} = \left(\frac{1+2^{M-1}}{2^M}\right)\frac{u^{++}(Q-q_R)}{M}$$

$$\begin{aligned} \frac{u^{++}(Q-q_R)}{2M} &= \left(\frac{2^{M-1}}{1+2^{M-1}}\right)\frac{wq_L(1+2^{M-2}+\frac{z^-}{2})}{1+2^{M-1}} \\ EV^R &= \left(\frac{1+2^{M-1}}{2^{M+1}}\right)\frac{2^{M-1}wz^-}{1+2^{M-1}}(Q-Mq_L) + \left(\frac{1+2^{M-1}}{2^{M+1}}\right)u^{++}q_R \\ &= \frac{wz^-}{4}(Q-Mq_L) + \left(\frac{1+2^{M-1}}{2^{M+1}}\right)u^{++}q_R \end{aligned}$$

Checking that the IC^- constraint is satisfied:

$$\begin{aligned} \frac{wq_L(1+2^{M-2}(1-z^-))}{1+2^{M-1}} &\leq \frac{u^{++}(Q-q_R)}{2M} + \frac{w_M}{2^M}\left(2^{M-1}(Q-q_R-q_L) + \sum_{x=1}^{M}\frac{(M-1)!\,q_x^{+-}}{(x-1)!\,(M-x)!}\right) \\ &= \frac{wq_L(1+2^{M-2}+\frac{z^-}{2})}{1+2^{M-1}}\left(1-\frac{1}{1+2^{M-1}}\right) + \\ &\quad \frac{(1+2^{M-2})w(Q-q_R-q_L)}{2(1+2^{M-1})} + \frac{(1+2^{M-2})w}{2^M(1+2^{M-1})}\left(\sum_{x=1}^{M}\frac{(M-1)!\,q_x^{+-}}{(x-1)!\,(M-x)!}\right) \\ 0 &\leq \left(\frac{1+2^{M-1}}{2}\right)q_Lz^- - \frac{q_L(1+2^{M-2}+\frac{z^-}{2})}{1+2^{M-1}} + \\ &\quad \frac{(1+2^{M-2})(Q-q_R-q_L)}{2} + \frac{(1+2^{M-2})}{2^M}\left(\sum_{x=1}^{M}\frac{(M-1)!\,q_x^{+-}}{(x-1)!\,(M-x)!}\right) \end{aligned}$$

This constraint is clearly nonbinding, so if the PC constraint is satisfied then the solution is:

$$Eu_{PR} = \frac{1}{Q}\left[EV^R + \frac{M}{2}(EV^+ - EV^-) + M(EV^-)\right] = \frac{wz^-}{4} + \frac{wMq_L(1+2^{M-2}+\frac{z^-}{2})}{2(1+2^{M-1})(Q-q_R)}$$

Th PC constraint is satisfied iff: $EV^+ - EV^- \geq \gamma - 2EV^- \Longrightarrow \frac{\gamma}{Q-q_R} \leq \frac{wq_L(1+2^{M-2})(1+z^-)}{(1+2^{M-1})(Q-q_R)}$. If PC is not satisfied at $q^{--} = q_L$, then IC^+ is not binding at $q^{--} = q_L$, but IC^+ may be binding for larger q^{--}. (PC is binding if IC^+ is not binding at $q^{--} = q_L$. Thus, for a range of parameter values both PC and IC^+ are binding in the PR strategy.) If IC^+ is not binding at $q^{--} = \frac{Q}{M}$, then PC is binding for the P strategy, as well as the PR strategy, and the P strategy dominates the PR strategy. (Remember, PC often becomes binding at a lower value of γ for the PR strategy than for the P strategy.) If $\frac{\gamma}{Q-q_R} > \frac{wq_L(1+2^{M-2})(1+z^-)}{(1+2^{M-1})(Q-q_R)}$, then PC is binding and the problem for the PR strategy is:

$$\min \ \frac{M}{2}(EV^+ + EV^-) + EV^R = \frac{M\gamma}{2} + EV^R = \frac{M\gamma}{2} + \left(\frac{1}{4} + \frac{1}{2^{M+1}}\right) u^{--}(Q - Mq^{--})$$

subject to: (IC^+)

$$\gamma - 2EV^- \geq \frac{1}{2}\left(u^{+-}_{M-1}q^{-+}_{M-1} - u^{--}q^{--}\right) + \frac{w_M}{2^M}\left(2^{M-1}q^{-+}_{M-1} + \sum_{x=0}^{M-1}\frac{(M-1)!\,q^{-+}_x}{(x)!\,(M-x-1)!}\right)$$

(IC^-)

$$\gamma - 2EV^- \leq \frac{1}{2}\left(u^{++}q^{++} - u^{+-}_1 q^{+-}_1\right) + \frac{w_M}{2^M}\left(2^{M-1}q^{+-}_1 + \sum_{x=1}^{M}\frac{(M-1)!\,q^{+-}_x}{(x-1)!\,(M-x)!}\right)$$

The issuer wants u^{--} as small as possible and q^{--} as large as possible such that IC^+ is satisfied: $u^{--} = u^-_{AS} = \frac{2^{M-1}wz^-}{1+2^{M-1}}$, $u^{+-}_x = 0$, $q^{-+}_x = q_L$.

$$\begin{aligned}
EV^- &= \frac{(1+2^{M-1})u^-_{AS}q^{--}}{2^M} = \frac{wz^-q^{--}}{2} \\
EV^+ &= \gamma - EV^- = \gamma - \frac{wz^-q^{--}}{2} = \frac{(1+2^{M-1})u^{++}(Q-q_R)}{2^M\,M} \\
\frac{u^{++}}{2M} &= \frac{2^{M-1}}{1+2^{M-1}}\left(\frac{\gamma}{Q-q_R} - \frac{wz^-q^{--}}{2(Q-q_R)}\right)
\end{aligned}$$

$$(IC^+): \quad \gamma \geq wz^-q^{--} - \frac{2^{M-2}wz^-q^{--}}{1+2^{M-1}} + \frac{w_M}{2^M}\left(2^{M-1}q_L + \sum_{x=0}^{M-1}\frac{(M-1)!\,q^{-+}_x}{(x)!\,(M-x-1)!}\right)$$

$$\Longrightarrow \gamma \geq wq^{--}\left(z^-\frac{2^{M-2}z^-}{1+2^{M-1}} + \frac{1+2^{M-2}}{2^M(1+2^{M-1})}\right) + \frac{(1+2^{M-2})wq_L(2^M-1)}{2^M(1+2^{M-1})}$$

$$\Longrightarrow q^{--} \leq \frac{\left(\frac{1+2^{M-1}}{1+2^{M-2}}\right)2^M\gamma - (2^M-1)wq_L}{(1+2^Mz^-)w}$$

The IC^- constraint is:

$$\frac{\gamma}{Q-q_R} \leq$$

$$\frac{wz^-q^{--}}{Q-q_R} + \frac{2^{M-1}}{1+2^{M-1}}\left(\frac{\gamma}{Q-q_R} - \frac{wz^-q^{--}}{2(Q-q_R)}\right) + \frac{w_M}{2^M(Q-q_R)}\left(2^{M-1}q^{+-}_1 + \sum_{x=1}^{M}\frac{(M-1)!\,q^{+-}_x}{(x-1)!\,(M-x)!}\right)$$

$$\Longrightarrow \frac{\gamma}{Q-q_R} \leq \frac{(1+2^{M-2})wz^-q^{--}}{Q-q_R} + \frac{(1+2^{M-2})w}{2^M(Q-q_R)}\left(2^{M-1}q^{+-}_1 + \sum_{x=1}^{M}\frac{(M-1)!\,q^{+-}_x}{(x-1)!\,(M-x)!}\right)$$

We know that $q_x^{+-} \geq \frac{Q-q_R}{M}$, $\forall x \geq 1$, so the last term is $\geq$:

$$\frac{(1+2^{M-2})w}{2^M\, M}\left(2^{M-1}+\sum_{x=1}^{M}\frac{(M-1)!}{(x-1)!\,(M-x)!}\right)=\frac{(1+2^{M-2})w}{2^M\, M}\left(2^{M-1}+2^{M-1}\right)=\frac{(1+2^{M-2})w}{M}$$

Thus, sufficient for satisfying the IC^- constraint is: $\frac{\gamma}{Q-q_R} \leq \frac{(1+2^{M-2})wz^-q^{--}}{Q-q_R}+\frac{(1+2^{M-2})w}{M}$. Sufficient for this is $\frac{\gamma}{Q-q_R} \leq w$, which is true by assumption, so IC^- is nonbinding and we set:

$$q^{--}=\min\left[\frac{Q-q_R}{M},\frac{\left(\frac{1+2^{M-1}}{1+2^{M-2}}\right)2^M\gamma-(2^M-1)wq_L}{(1+2^Mz^-)w}\right]$$

Thus, if $\frac{\gamma}{Q-q_R} > \frac{wq_L(1+2^{M-2})(1+z^-)}{(1+2^{M-1})(Q-q_R)}$:

$$\begin{aligned} Eu_{PR} &= \frac{M\gamma}{2(Q-q_R)}+\frac{wz^-}{4}\left(1-\frac{Mq^{--}}{Q-q_R}\right) \\ &= \frac{M\gamma}{2(Q-q_R)}+\max\left[0,\frac{wz^-}{4}+\frac{z^-}{4(1+2^Mz^-)}\left(\frac{(2^M-1)wMq_L}{Q-q_R}-\left(\frac{1+2^{M-1}}{1+2^{M-2}}\right)\frac{2^MM\gamma}{Q-q_R}\right)\right] \end{aligned}$$

Note, if $\frac{\gamma}{Q-q_R} \geq \frac{(1+2^{M-2})w}{2^M(1+2^{M-1})}\left(\frac{(2^M-1)q_L}{Q-q_R}+\frac{(1+2^Mz^-)}{M}\right)$, then the issuer follows the (P) strategy with $Eu_P=\frac{M\gamma}{2Q}$.

Finally, we need to ensure that if $q_R>0$, then $u^{++} \geq u^+_{AS}$. This adds a fourth term to the maximand in equation (9): J$\times prob\{++\}\times$ $u^+_{AS} = \frac{Jwz^+}{4}$ where $z^+=\frac{z_I}{2+2^M-(1+2^M)z_I}$ and $J=1$ if $q_R>0$ and $J=0$ otherwise. ∎

Proof of Proposition 5.

a) Let $$y_1 \equiv \frac{(1+2^{M-2})(2^M-1)M}{2^{M+1}(1+2^{M-1})} \quad \text{and} \quad y_2 \equiv \frac{(1+2^{M-2})}{2^{M+1}(1+2^{M-1})}$$

Then $$Eu_P=\max\left[\frac{y_1wq_L}{Q}+y_2w,\frac{M\gamma}{2Q}\right]$$

$\frac{\partial y_2}{\partial M}<0$ and $\frac{\partial y_1}{\partial M}>0$. Thus, the optimal pool size in the P strategy is decreasing with $\frac{q_L}{Q}$ and $\frac{\gamma}{Q}$, and increasing with Q.

Let $$y_3 \equiv \frac{(1+2^{M-2}+\frac{z^-}{2})M}{(1+2^{M-1})}, \quad y_4 \equiv \frac{(1+2^{M-2}+2^{M-1}z^-)M}{(1+2^{M-2})(1+2^Mz^-)} \quad \text{and} \quad y_5 \equiv \frac{(2^M-1)Mz^-}{(1+2^Mz^-)}$$

$$Eu_{PR}=\frac{wz^-}{4}+\max\left[\frac{y_3wq_L}{2(Q-q_R)}\,,\,\frac{Jwz^+}{4}\,,\,\frac{y_4\gamma}{2(Q-q_R)}+\frac{y_5wq_L}{4(Q-q_R)}\,,\,\frac{M\gamma}{2(Q-q_R)}-\frac{wz^-}{4}\right]$$

$$\frac{\partial z^-}{\partial M}<0\,, \qquad \frac{\partial z^+}{\partial M}<0\,, \qquad \frac{\partial y_3}{\partial M}>0\,, \qquad \frac{\partial y_4}{\partial M}>0 \qquad \text{and} \qquad \frac{\partial y_5}{\partial M}>0$$

Thus, the optimal pool size in the PR strategy is decreasing with $\frac{q_L}{Q}$ and $\frac{\gamma}{Q}$, and increasing with $Q - q_R$.
b) Let $q_R = \gamma = 0$:

$$Eu_{PR} = w \left(\frac{z^-}{4} + \max \left[\frac{y_3 q_L}{2(Q - q_R)} , \frac{y_5 q_L}{4(Q - q_R)} \right] \right)$$

Eu_{PR} clearly increaes with w, but the optimal value of M is independent of w.
c) Let $q_R = q_L = 0$:

$$Eu_{PR} = \max \left[\frac{y_4 \gamma}{2(Q - q_R)} + \frac{w z^-}{4} , \frac{M \gamma}{2(Q - q_R)} \right]$$

In this case a tradeoff exists, such that the optimal value of M is increasing in w. ∎

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