

Introduction

Share-of-choice is a combinatorial optimization problem which aims to design the product profile that will return the largest market share using part-worth data obtained after a conjoint analysis.

Share-of-choice problem

$$\begin{aligned} & \text{Max} \sum_{s=1}^{|V|} y_s, && \text{Market share} && (1) \\ \text{subject to} & \sum_{k=1}^{|K|} \sum_{l=1}^{|L_k|} u_{kl}^s x_{kl} \geq h_s y_s, && \text{Product utilities vs. hurdles} && (2) \\ & \sum_{l=1}^{|L_k|} x_{kl} = 1 && \text{One level for each attribute} && (3) \\ & y_s, x_{kl} = 0 \text{ or } 1 && \forall s \in V, \forall k \in K, \forall l \in L_k && (4) \end{aligned}$$

Kohli and Krishnamurti 1989 showed that it is NP-Hard.
Solution approaches: Green and Krieger 1989, Balakrishnan and Jacob 1996, Shi and Olafsson 2001, Camm et al. 2006, Wang et al. 2009

Can we isolate our purchase or usage decisions from our societal networks? In this paper, we consider the setting where peer influence plays a significant role in a consumer's choice or there is a tangible benefit from using the same product as the rest of one's social network.

Consistent with social contagion research, we follow a multi-attribute linear utility-maximization approach. An individual can only observe the outcome of a consumer's purchase decision, but not the relative preferences among each attribute level.

Share-of-choice model incorporating social network effects

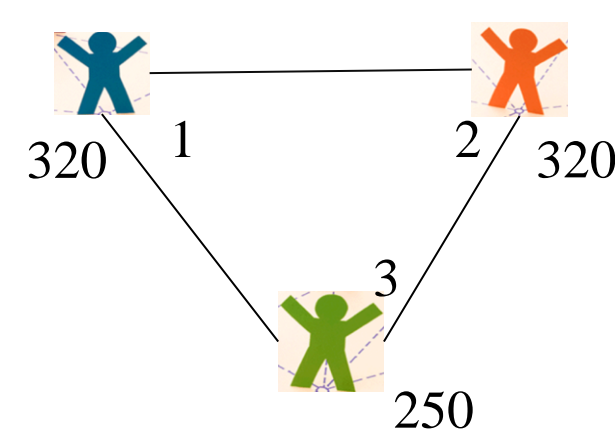
$$\text{subject to} \quad \sum_{k=1}^{|K|} \sum_{l=1}^{|L_k|} u_{kl}^s x_{kl} \geq h_s^H y_s - \Delta_s \sum_{j \in V} a_{sj} y_j \quad s = 1, 2, \dots, |V|$$

Parameters: $a_{sj} = 1$ if $(s,j) \in E$ $\Delta_s = \frac{h_s^H - h_s^L}{deg(s)}$

Why are social network effects important?

Single product, single attribute with 2 levels. $\Delta_s = 20$ for each person.

| | Utilities | |
|---|-----------|---------|
| | Level 1 | Level 2 |
| 1 | 200 | 280 |
| 2 | 320 | 280 |
| 3 | 205 | 240 |



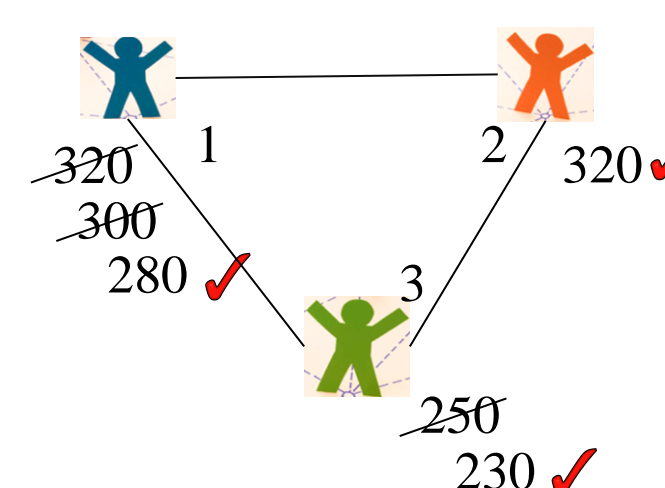
The solution with the original SOC model:
Choose level 1 → Market share = 1 out of 3.

The solution with our model:
Choose level 2 → Market share = 3 out of 3.

Comparing utilities with hurdles, nobody buys. → We need an initial buyer.

If we persuade person 2 to buy by providing him with incentives;

Now, Person 3 buys, and further decreases Person 1's hurdle, and Person 1 buys.



Least Cost Influence Problem

$$\begin{aligned} & \text{Minimize} \quad \sum_{s \in V'} z_s, && \text{Incentives given out} \\ \text{subject to} & U_s \geq \text{Utility vs. hurdle} - \text{incentive} - \text{network effects} \\ & U_s \geq h_s^H y_{st} - z_s - \Delta_s \sum_{j \in V'} a'_{js} y_{j,t-1} \quad \forall s \in V', \forall t \geq 1, \\ & y_{st} \geq y_{s,t-1} && \text{Buyer in the successor periods} \\ & y_{sT} = 1 && \text{Each person buys by the end} \\ & y_{st} \in \{0, 1\} && \forall s \in V', \forall t \geq 0, \\ & z_s \geq 0 && \forall s \in V'. \end{aligned}$$

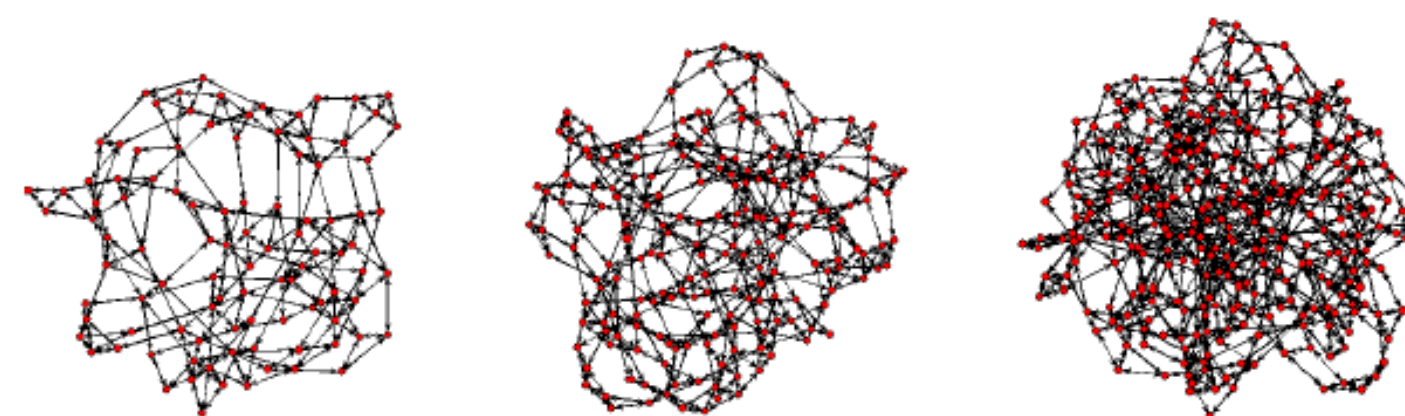


Figure 2.2: Social networks of 100, 200 and 300 people for $p = 0.2$, respectively.

Data generation for each person includes; Utilities for levels of attributes, high and low hurdle.

Genetic Algorithm (GA)

- Binary representation
Color: black, white
Size: small, medium, large
A small white product: (01 100)
- Evaluate using CPLEX
- Roulette-wheel selection
Single-point crossover
- Mutation rate
Random attribute, random level
- Choose the fittest 100 individuals of the current population.
- GO back to Step 3.
This is repeated until stopping condition is reached.

Step 1 [GENERATE]
Step 2 [EVALUATION]
Step 3 [CROSSOVER]
Step 4 [MUTATION]
Step 5 [REDUCTION]

Performance of the GA

The complete combination of product profiles;
(3,4,5,6) attributes, (2,3,4,5) levels, (0.1,0.2, 0.3) rewiring probabilities and (100,200,300) number of people.
Optimal hits: 122 out of 134 problems.
Average running time: 106 sec.

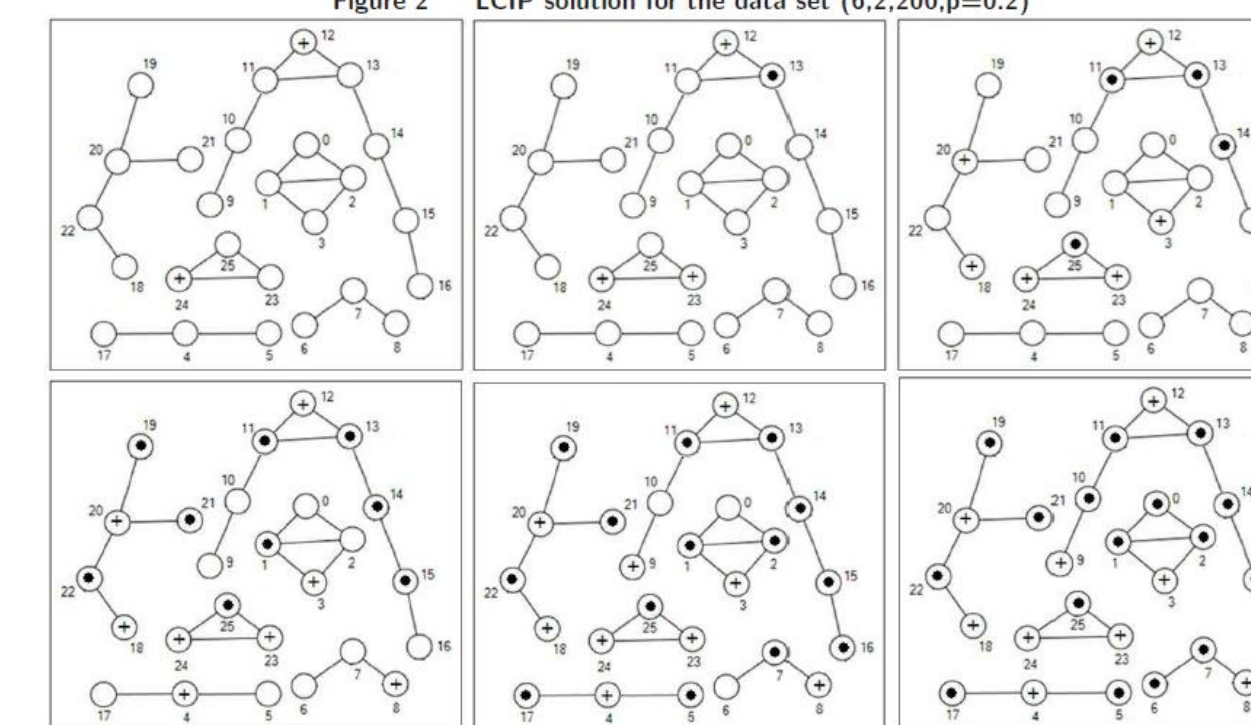
For large size data

10 attributes, 5 levels, Total number of possible product profiles = 9,765,625.
GA evaluates 2000 profiles. (0.0205 % of the total)

Network effects on market share

| Num. of attributes | Num. of levels | 200 people | | |
|--------------------|----------------|------------|--------|-------------|
| | | SOC | SOC+NE | GA+NE GA+IN |
| 6 | 2 | 59 | 91 | 114 140 |

Figure 2 LCIP solution for the data set (6,2,200,p=0.2)



Conclusion

To our knowledge, this is the first paper to model social network effects in the share-of-choice problem. We develop a robust matheuristic for the problem. We also model and solve a problem of maximizing/strengthening diffusion over the social network in the least expensive way.

Future work

We are currently extending our work to product-line design problem. Our future work consists of problems similar to pricing of the levels of attributes for a product as part of the design process, consideration of alternate assumptions of product selection amongst consumers and extension of the problem to stochastic network settings.