Competitive Analysis of Online Booking: Dynamic Policies

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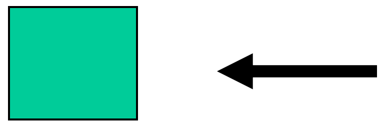
Competitive Analysis



$i_1, i_2, i_3, \ldots$

Algorithm Designer | ***algorithm*** | ***input stream*** | Evil Adversary

Competitive Ratio =

$$\textbf{Min}_{\text{input streams}} \{(\text{alg performance})/(\text{best performance})\}$$

“Traditional” revenue management analysis has assumed:

- **Demand can be forecast reasonably well**
- **Risk neutrality**

Are these valid??

Sample Result

- Flight has 95 available seats, three fare classes: $1,000, $750, $500
- Policy that guarantees at least 63% of the max possible revenue:
  - Protect 15 high fare seats
  - Protect 35 seats for two higher fare classes (i.e. sell at most 60 lowest fare seats)

Two-Fare Analysis

- n = number of seats
- f1 = higher fare; f2 = lower fare.
- r = f2/f1 = discount ratio.
- Key quantity: $b(r) = 1 / (2 – r)$

Policy intuition:

Always best to accept any high fare (H) that comes along

➔ Adversary will start off with low fare requests (L): Question – how many to accept before only H's are accepted??

Basic Trade Off

Note: must accept first order, o.w. adversary will stop after submitting one order with algorithm performance = 0.

L L L L | L L L L L L L L L L L L

available seats

Stop accepting L's

Accept too few L's ➔ adversary will only send additional L's

L L L L L L L L L L L L L | H H H | H H H H H H

Stop accepting L's

Accept too many L's ➔ adversary will only send additional H's

Best Two-Fare Policy

Proposal: protect (1 – b(r)) n = (1 – 1/(2 – r)) n
… assume for the moment that b(r) n is integer ..

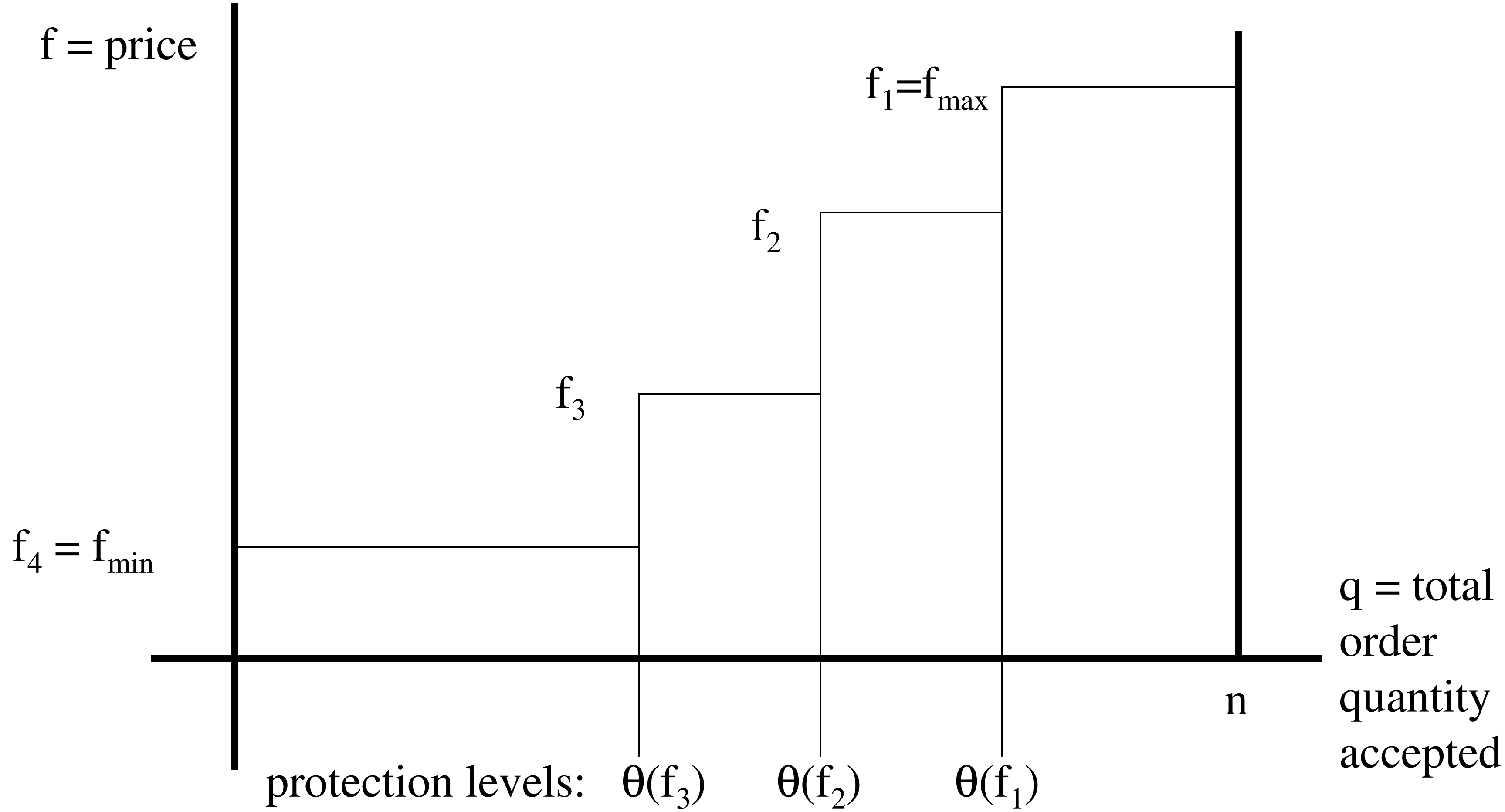
L L L L L L L L L L L L L | L L L L L L

All L's after stopping ➔
Performance = (f2 b(r) n) / (f2 n) = b(r)

L L L L L L L L L L L L L | H H H H H

All H's after stopping ➔
Performance = [f2 b(r) n + f1 (1 – b(r)) n] / (f1 n) = b(r)

More General Cases



m Fare Classes

Define: $\Delta = m - \sum_{\{i=2,m\}} f_i / f_{i-1}$

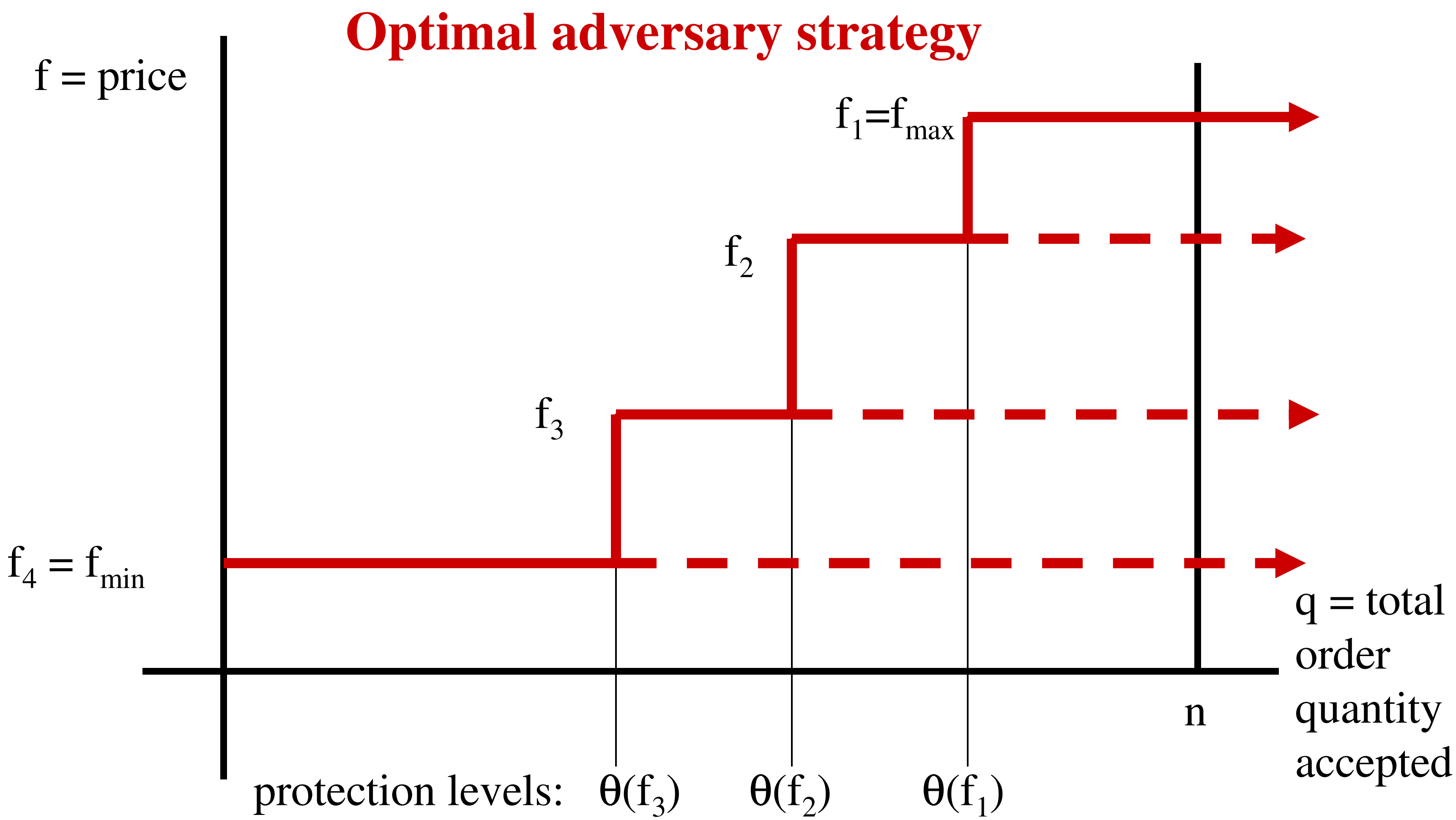
Theorem: For the continuous m-fare problem, no booking policy, deterministic or random, has a competitive ratio larger than $1 / \Delta$.

Define: $\theta_i = (n / \Delta)\ (i - \sum_{\{j=1,i\}} f_{j+1} / f_j)$

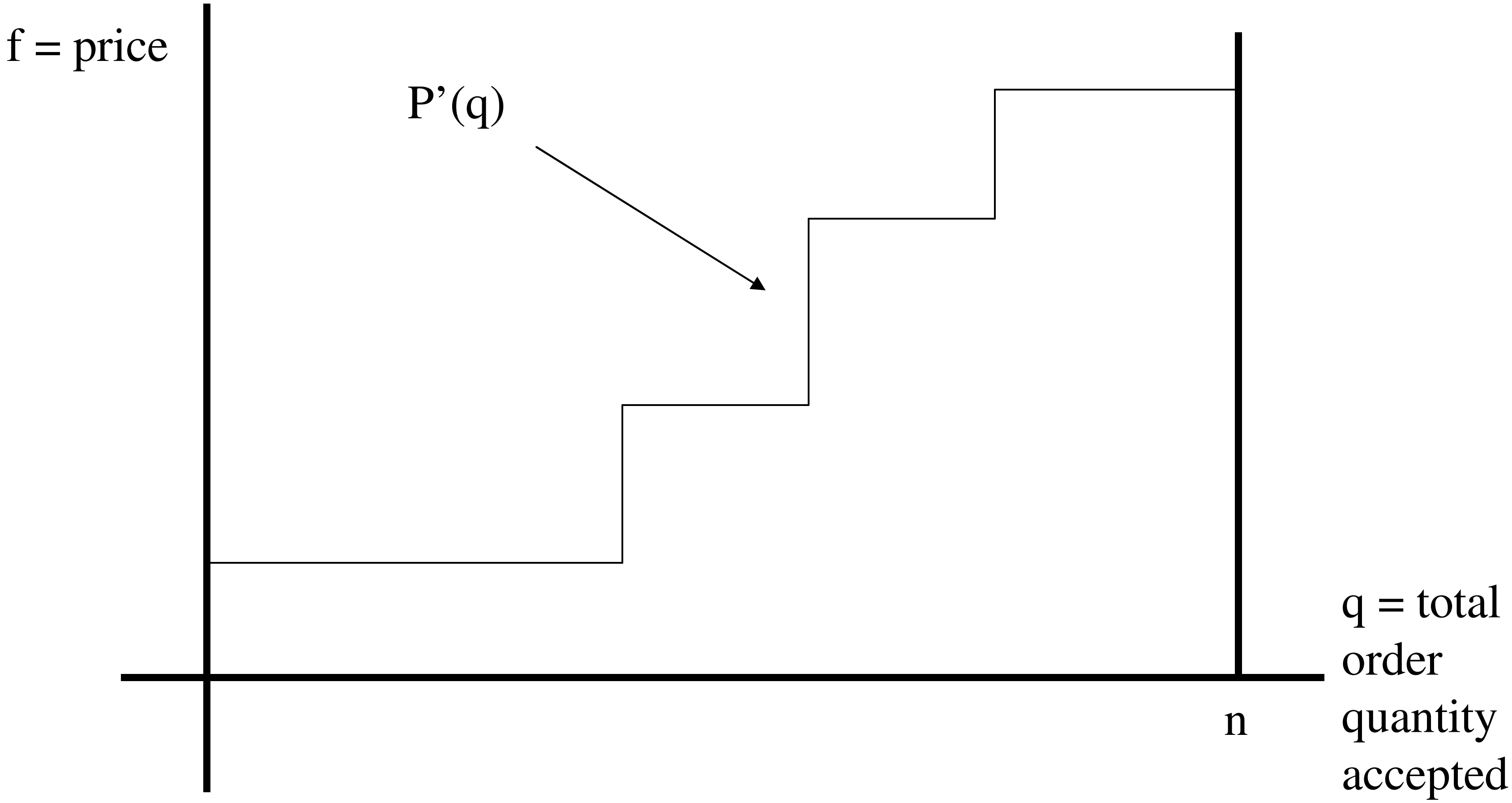
Theorem: For the continuous m-fare problem, the protection level policy using protection levels θ_i achieves a competitive ratio of at least $1 / \Delta$.

e.g. $f_1 = 1000$, $f_2 = 750$, $f_3 = 500$, $1 / \Delta \approx 63\ \%$

Approach to alternate type of policy



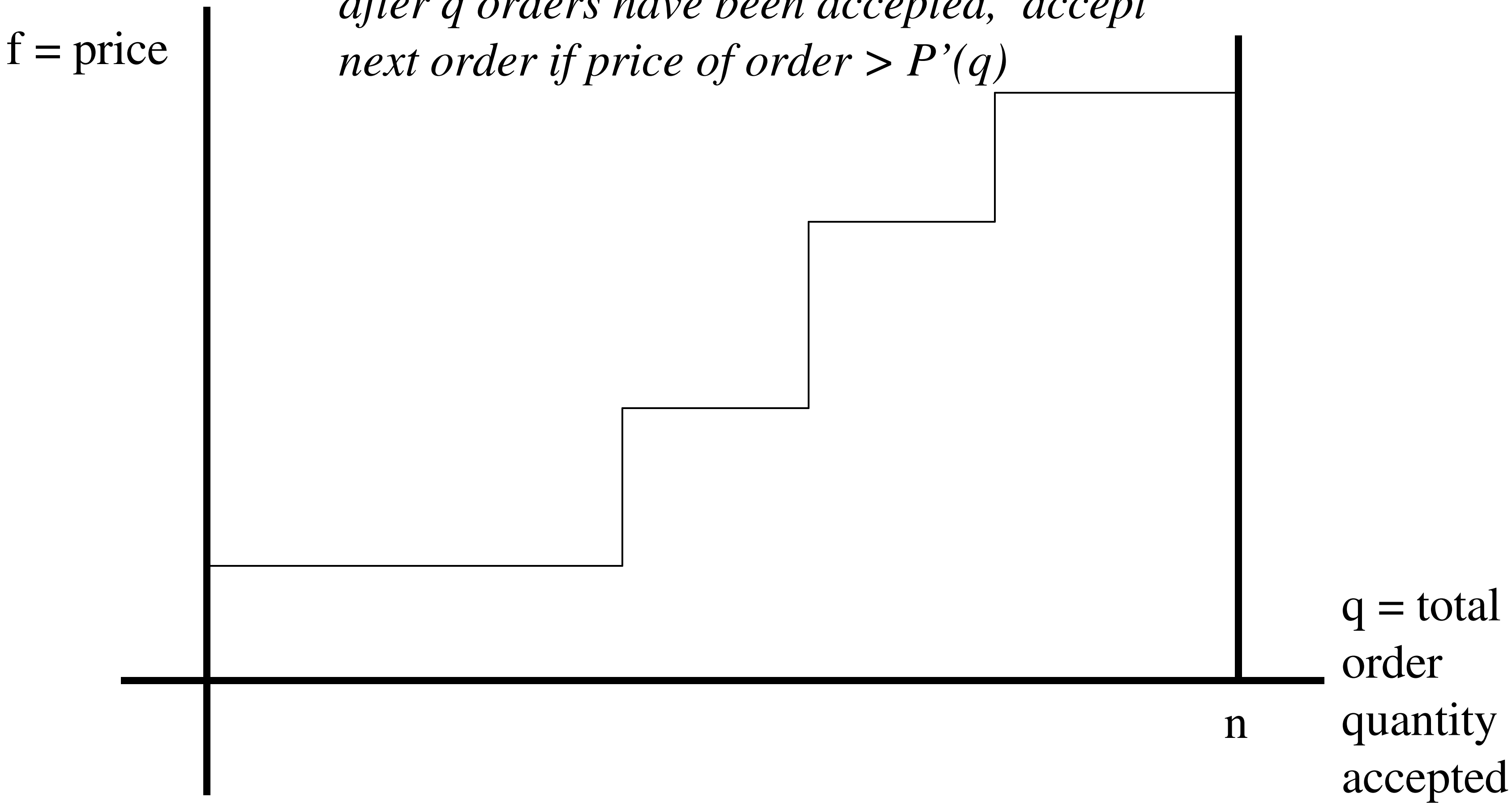
Order Quantity Control Fcn



Order Quantity Control Policy

Order quantity control policy:

after q orders have been accepted, accept next order if price of order > P'(q)



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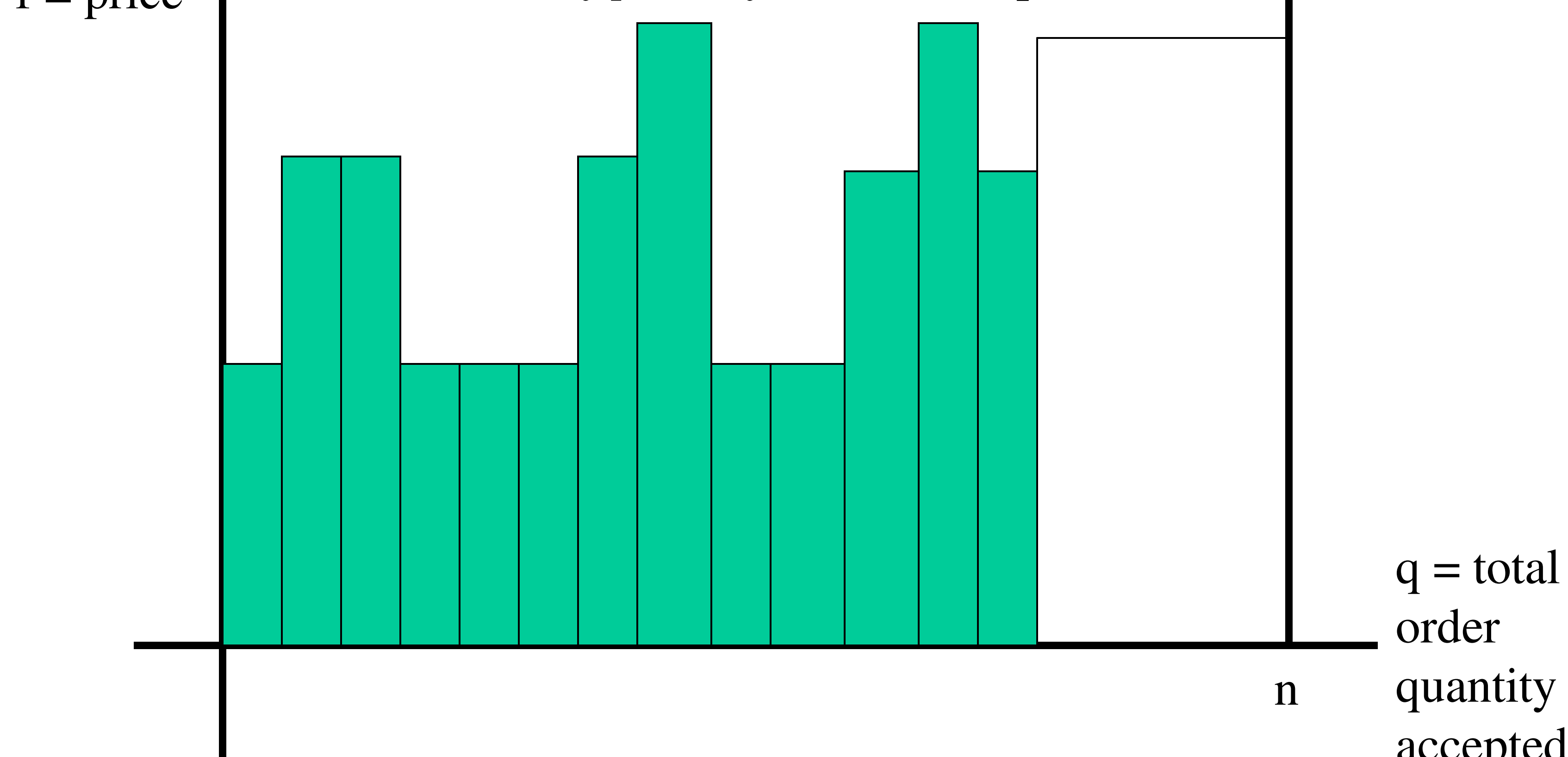
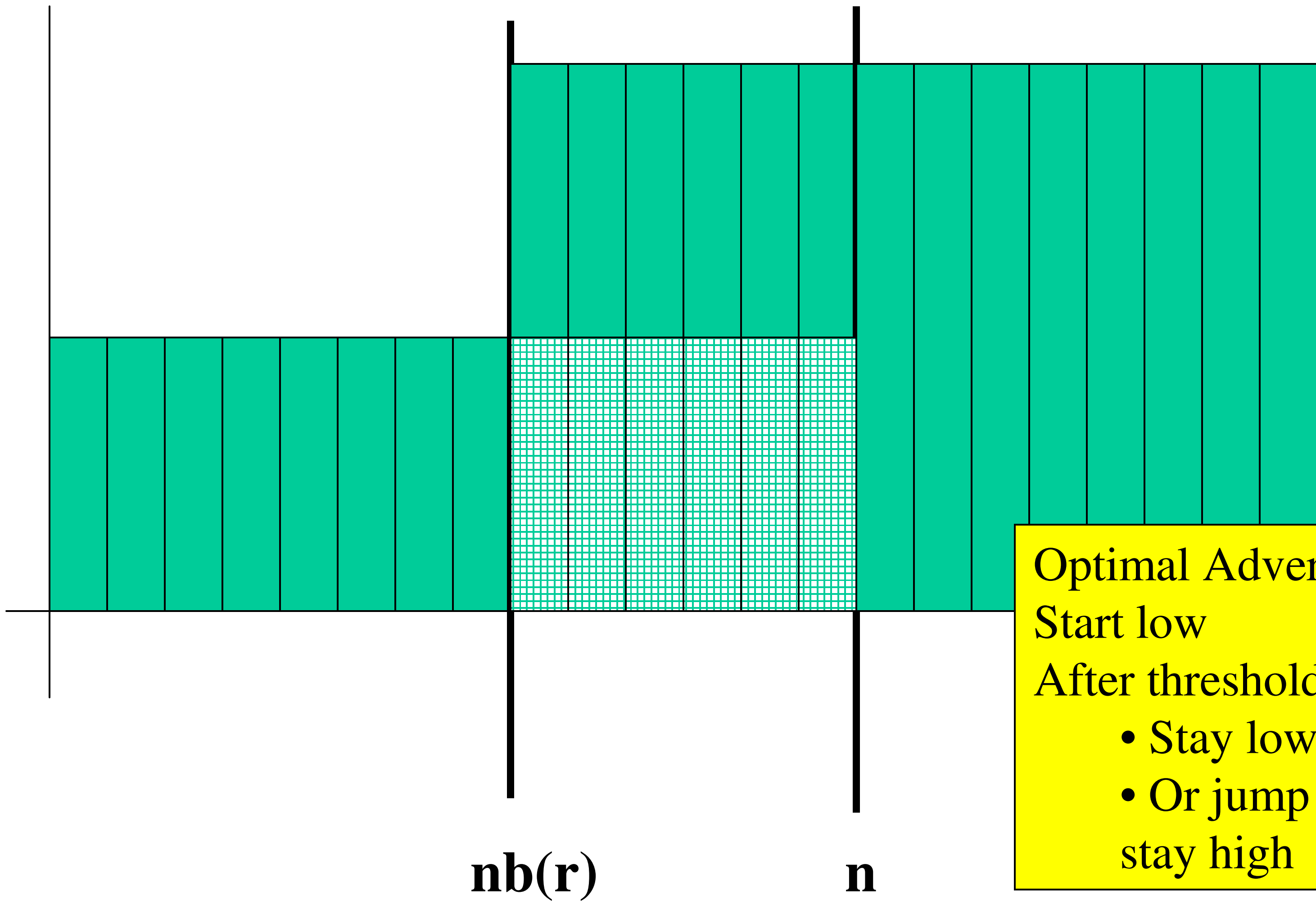


Accept!! or *Reject!!*

Dynamic Policy Motivation:
Taking advantage of an inferior adversary



Optimal Adversary Policies:
Start low
After threshold:
- Stay low
- Or jump to high and stay high

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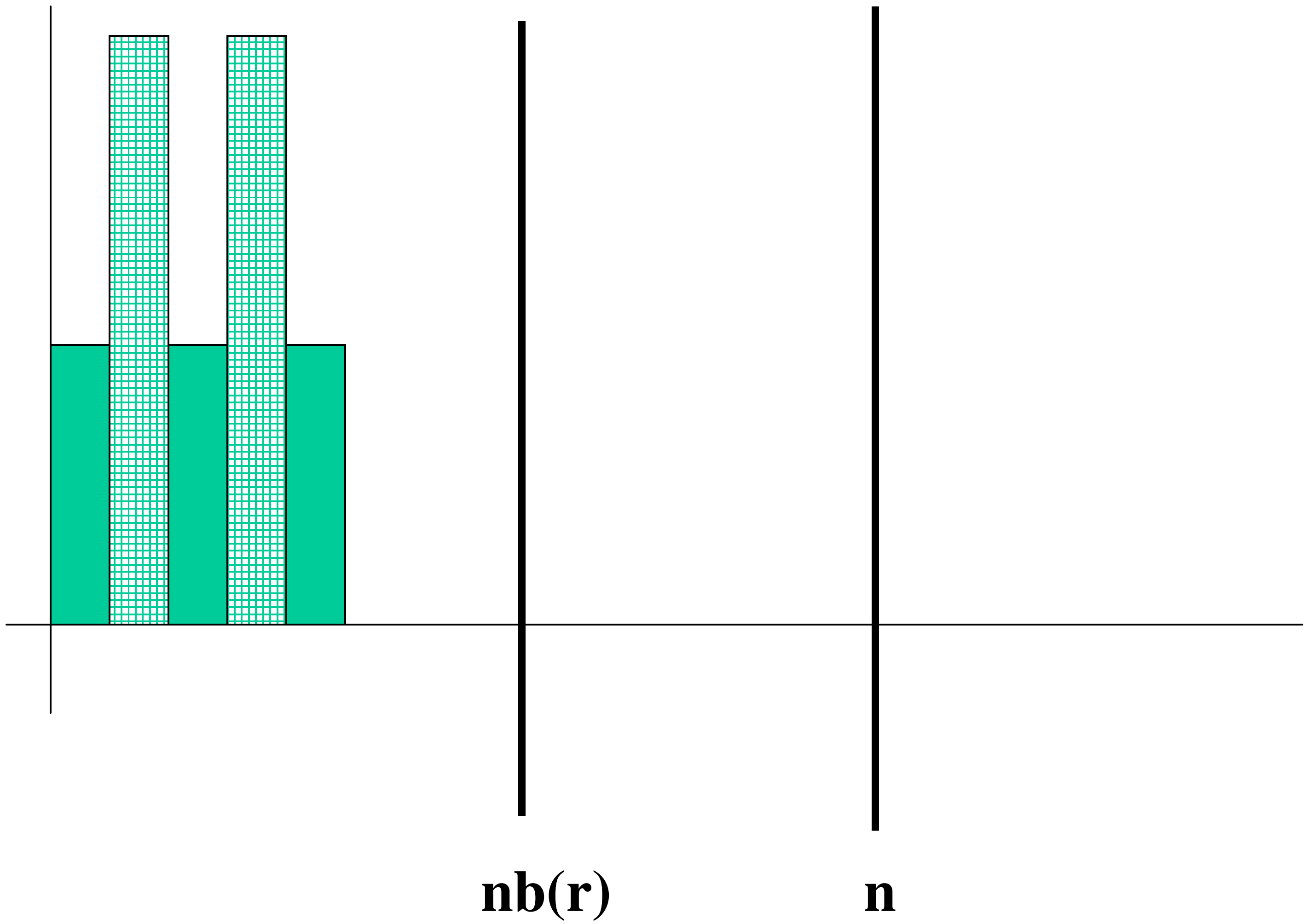

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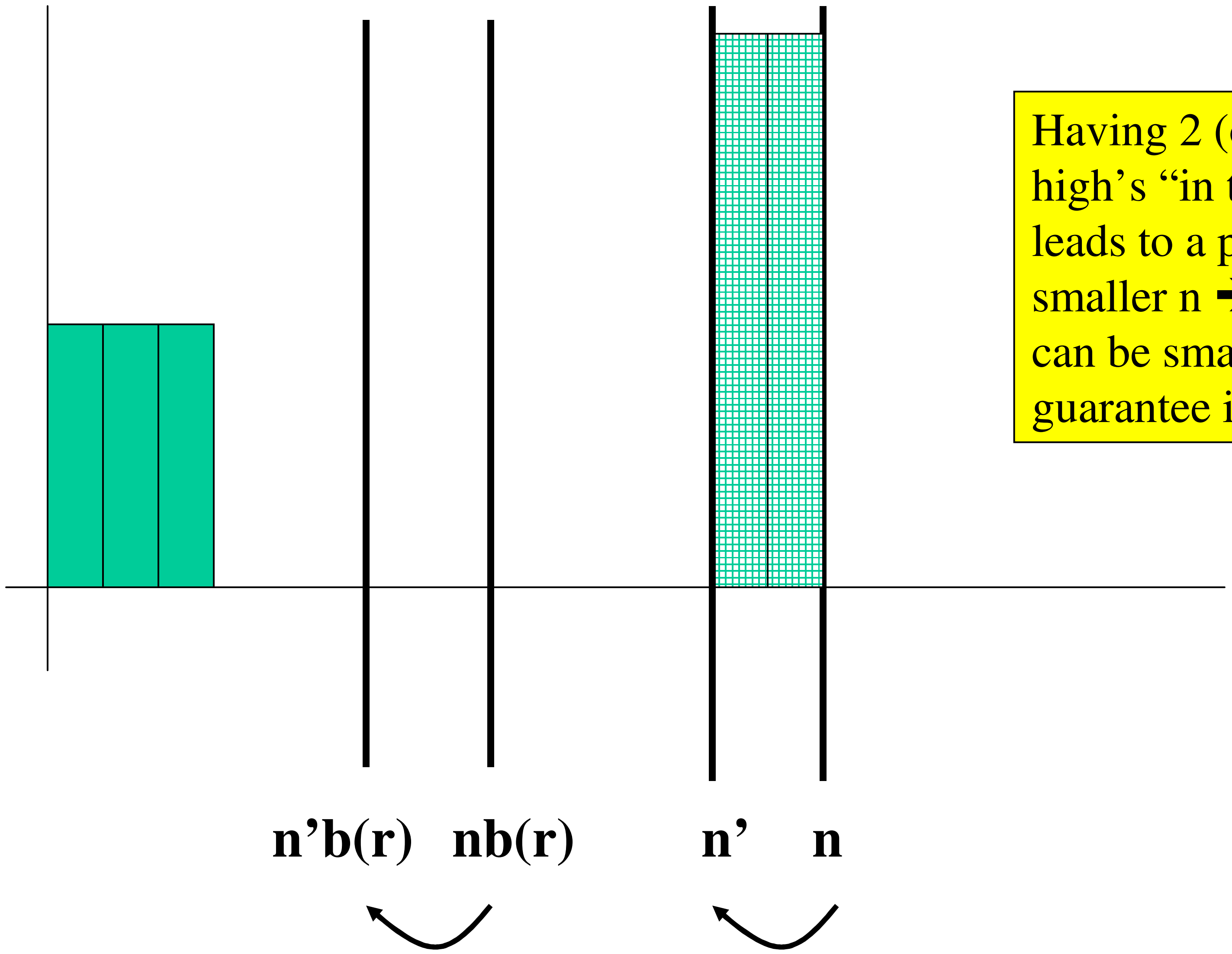

Having 2 (or more) high's "in the bank" leads to a problem on a smaller n ➔ threshold can be smaller & overall guarantee is improved.

Dynamically Revising Threshold

orig threshold: $n\, b(r) = n / (2-r)$

let: h' = # high fare request accepted so far

$\gamma = h'/n \qquad \alpha = (\gamma f_1 + (1 - \gamma) f_2) / f_2$

revised threshold:

$$n\,(1 + (1 - r)\,\gamma/r\,) / (1 + \alpha\,(1 - r)\,)$$

Dynamic Policy: numerical test

n = 100; r = .5

h’	ℓ”	dynamic guarantee	ℓ’	static guarantee
1	65.78	.67	66.67	.67
5	62.30	.69	66.67	.67
10	58.60	.71	66.67	.67
20	50.00	.75	66.67	.67
30	42.42	.79	66.67	.67
40	35.29	.82	66.67	.67
50	28.57	.86	66.67	.67

Final Thoughts

- No risk neutrality assumption
- No distribution information required
- Practical policies
- Applies to broad class of policies (not just protection level controls)
- Able to dynamically adjust to order history (promising area for further research)