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Financial Management

Time Value of Money

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Time Value of Money (TVM)

- Present value and future value
  - how much is $1 now worth in the future?
  - how much is $1 in the future worth now?
- Business planning
  - compare current to future cash flows
    - is investment in a particular project worthwhile?
    - is one project a better investment than another?
- Personal financial planning
  - should you consume today or invest and consume later?
  - how will your money grow over time?
  - how much do you need to save today to retire later?

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$1 Today versus $1 Later

- Interest rates:
  - the supply and demand of money must be equal
  - the value of being patient; the cost of impatience
  - depends on horizon (maturity)
  - reflects expected inflation and a "real return"
- What about $ 1 for certain in five year's time versus $1 on average in five year's time?
  - Discount rates reflect both the "riskless" time value of money as well as the riskiness of the cash flows

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What techniques will we review?

- What is the value today of $\$C_T$ at time T?
- What is the value at time T of $\$C_0$ today?
- Compounding
  - Effective annual interest rate (EAIR) vs. Stated annual interest rate (SAIR)
  - Continuous vs. discrete compounding
- Multiple cash flow streams
  - perpetuity
  - growing perpetuity
  - annuity

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Time Value of Money: Notation

C_0	C_T
PV	FV
$ invested today	$ received in future

Define some terms

C_T	=	Future value at time T = FV; or cash flow at T
C_0	=	Current value = PV; or cash flow at O
r	=	Discount or interest rate
T	=	Number of future time periods

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Present Value and Discounting

Discounting: if we receive $1 in two years, what is it worth to us today (r = 9%)?

Year 0 1 2

C $0.842 ◄······ $1

Present value (C_0) = $\$1 / 1.09^2 = \$0.842$

The interest rate (9%) is also called the discount rate

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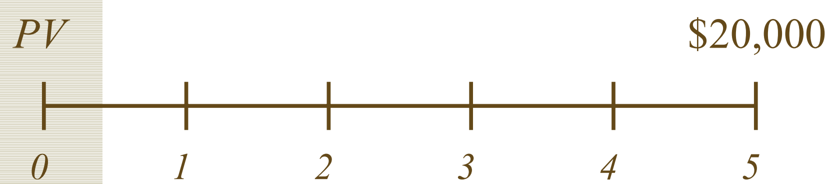
Present Value - another example

- How much would an investor have to set aside today in order to have $20,000 five years from now if the current rate is 15%?



$$\$9{,}943.53 = \frac{\$20{,}000}{(1.15)^5}$$

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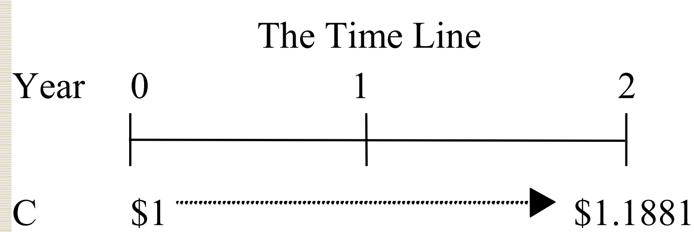
Future Value

Compounding: how much will $1 invested today at 9% be worth in two years?



Future value (C_2) = $1 x 1.09^2= $1.1881

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Compounding

- Compounding: interest on interest
  - Benjamin Franklin: "Money makes money, and the money that money makes makes more money"
- Which one would you prefer?
  1. simple interest: no compounding

     $FV_T = PV_0(1 + r*T)$
  2. annual compound interest

     $FV_T = PV_0(1 + r)^T$
  3. Semi-annual compound interest (bonds and banks)

     $FV_T = PV_0(1 + r/2)^{2T}$, where r is annualized,T in years
  4. continuous compound interest: compound "every second"

     $FV_T = PV_0*e^{rT}$ (to be shown later)

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"Money Makes Money…"

INTEREST RATE EXAMPLE

return	10%								
contribution	100								
			years						
	0	1	2	3	4	5	10	15	30
1. Simple Interest	100	110	120	130	140	150	200	250	400
2. Compound Interest	100	110	$121.00	$133.10	$146.41	$161.05	$259.37	$417.72	$1,744.94
3. Continuous Compound Interest	100	$110.52	$122.14	$134.99	$149.18	$164.87	$271.83	$448.17	$2,008.55

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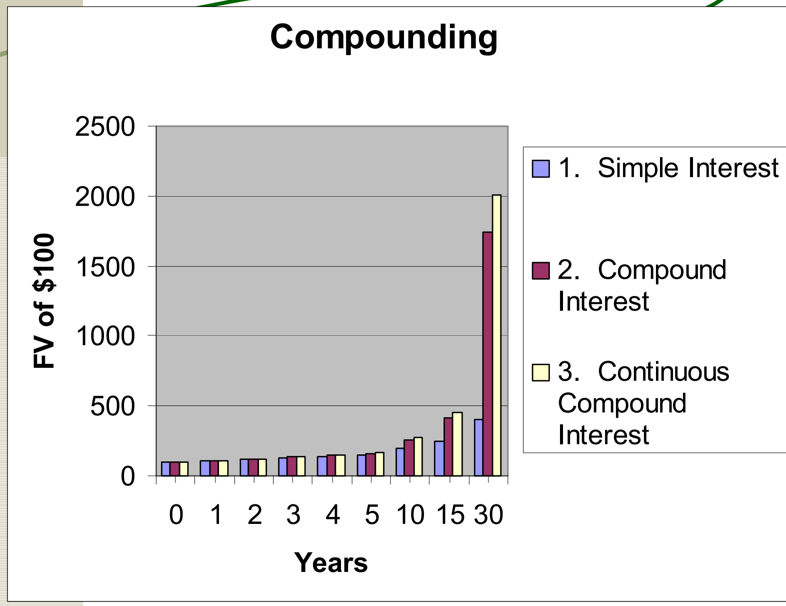

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Effective Annual Rates (EAIR)

- The formula for computing the return when there are multiple compounding periods is:

  $(1+SAIR/m)^m = (1+EAIR)$

  m is the number of times interest is paid annually

  SAIR represents the stated annual interest rate

  EAIR represents the effective annual interest rate

- If the SAIR is 12%, how much will I receive at the end of the year if interest is paid monthly vs. semi-annually vs. yearly?

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EAIR: Example

If you invest $50 for 1 year at 12% (the SAIR) compounded semi-annually - what is the EAIR on that investment?

$$FV = \$50 \times (1 + \frac{.12}{2})^2 = \$50 \times (1.06)^2 = \$56.18$$

The EAIR is the annual rate that would give us the same end-of-investment wealth after 1 year:

$$\$50 \times (1 + EAIR) = \$56.18$$

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EAIR: Solution

$$FV = \$50 \times (1 + EAIR) = \$56.18$$

$$(1 + EAIR) = \frac{\$56.18}{\$50}$$

$$EAIR = .1236$$

So, investing at12.36% compounded annually (i.e. the EAIR) is the same as investing at the SAIR of 12% compounded semiannually.

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Continuous Compounding

- The general formula for the future value of an investment compounded continuously over many periods can be written as:

$$FV = C_0 \times e^{rT}$$

Where

C_0 is cash flow at date 0,

r is the stated annual interest rate,

T is the number of periods over which the cash is invested, and

e is a special number (= 2.718...). e^x is a key on your calculator, and the inverse function is the natural logarithm (ln).

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Present Values with Continuous Compounding

- Two types of compounding - be careful!
  - Discrete Compounding
    - annual or semi-annual most common: $PV = \frac{FV}{(1+r)^T}$
  - Continuous Compounding: $PV = \frac{FV}{e^{r*T}}$
  - very close for small r but not for large r: watch out for those "small" discrepancies that make your bankers rich!
- While no bank offers continuously compounded rates of return, continuous compounding is often used in finance because it facilitates calculations.

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How long does it take to double your money?

If we deposit $5,000 today in an account paying 10% annually, how long does it take to grow to $10,000?

$$FV = C_0 \times (1+r)^T \qquad \$10{,}000 = \$5{,}000 \times (1.10)^T$$

$$(1.10)^T = \frac{\$10{,}000}{\$5{,}000} = 2$$

$$\ln(1.10)^T = \ln 2 \Rightarrow T \ln(1.10) = \ln 2$$

$$T = \frac{\ln 2}{\ln(1.10)} = \frac{0.6931}{0.0953} = 7.27 \text{ years}$$

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Double your money ... using continuous compounding

- Continuous compounding: FV/PV = e^{rT}
- Double your money FV/PV = 2
- $e^{rT} = 2 \Rightarrow rT = \ln(2)$ (if $y=e^x$, then $x = \ln(y)$)
- rT = .69
  - if r=.10, T = 6.9 years (compare to 7.3 on last slide)
  - if r=.20, T = 3.5 years
- The rate or time required to double your money can be quickly figured out using the “Rule of 72” (an approximate rule based on annual compounding).

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What Rate Is Enough?

Assume the total cost of a college education will be \$50,000 when your child enters college in 12 years. You have \$5,000 to invest today. What annual rate of interest must you earn on your investment to cover the cost of your child's education? About 21.15%.

$$FV = C_0 \times (1+r)^T \qquad \$50{,}000 = \$5{,}000 \times (1+r)^{12}$$

$$(1+r)^{12} = \frac{\$50{,}000}{\$5{,}000} = 10 \qquad (1+r) = 10^{1/12}$$

$$r = 10^{1/12} - 1 = 1.2115 - 1 = .2115$$

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Multiple Cash Flows

- Now, a more realistic and complex setting
  - Most investments generate multiple cash flows over time
  - Special cases:
    - Annuities: same cash flow repeated for a fixed number of periods
    - Perpetuities: same cash flow repeated periodically, forever
    - Growing annuities and perpetuities
- Objective: understand Discounted Cash Flow (DCF) and Net Present Value (NPV) so as to:
  - value and price securities and companies
  - evaluate projects: "capital budgeting"

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Present and Future Value: Multiple Cash Flows

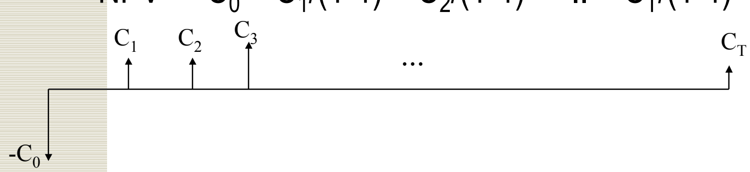
- Net Present Value and Investment Projects
  - **Net Present Value** is a measure of the difference between the market value of an investment and its cost. If the NPV is positive, this indicates a profitable investment (if the NPV < 0, the investment should not be made.
- Estimating Net Present Value

  Discounted cash flow (DCF) valuation consists of finding the market value of assets or their benefits by taking the present value of future cash flows, i.e., by estimating what the future cash flows would trade for in today's dollars.

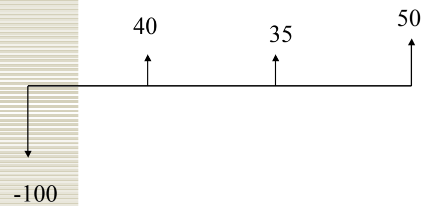
Net Present Value (NPV)

- Some rules for computing NPV:
  - use a simple time line to lay out all the cash flows
  - only do simple addition or subtraction of cash flows if they are from the same time period
  - use an appropriate discount rate: later in the course
- The general formula for calculating NPV:

$$NPV = -C_0 + C_1/(1+r) + C_2/(1+r)^2 + .. + C_T/(1+r)^T$$



Simple NPV Example



Assume r = 10%

$$NPV = -100 + 40/(1+.10) + 35/(1+.10)^2 + 50/(1+.10)^3 = 2.85$$

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Annuities

Series of equal period payments

0 1 2 3 4 .... T

|____|_____|______|______|____________|

$C_1 = C_2 = C_3 = C_4 = C_T$

- For simplicity, we will assume a single rate of interest which allows us to easily use the interest tables.

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PV of an Annuity

- PV of annuity: $$PV = \sum_{t=1}^{T} \frac{C}{(1+r)^t} = C \cdot \frac{1 - \frac{1}{(1+r)^T}}{r}$$
  - NOTE: first payment is received in one year's time
  - Table A.2 in RWJ Appendix shows annuity factor values
  - generalization: RWJ give the formula for a growing annuity (equation 4.15 on page 92)
- How much are 5 equal annual payments of $8,190 (starting one year from today) worth today if the annual interest rate is 10%?
  - Answer = $31,046.54

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Annuity Practice Problem

- Installment purchases:
  - you are considering purchasing a house that costs $70,000
  - you intend to put 20% down and finance the remainder
  - if the 30 year mortgage rate is 7%, what will be your *monthly* mortgage payment?
- Note: we implicitly assume monthly compounding with end of period payments

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Amortization Table

Initial Data

LOAN DATA

House Price	**$700,000.00**
Loan amount:	**$560,000.00**
Annual interest rate:	**7.00%**
Term in years:	**30**
Payments per year:	**12**
First payment due:	**1/1/03**

TABLE DATA

Table starts at date:	1/1/03
or at payment number:	**1**
PVIFA	**150.3075679**

PERIODIC PAYMENT

Entered payment:	
Calculated payment:	**$3,725.69**
After-Tax payment	**$2,680.07**

The table uses the calculated periodic payment amount, unless you enter a value for "Entered payment."

CALCULATIONS

Use payment of:	**$3,725.69**	Beginning balance at payment 1:	**$560,000.00**
1st payment in table:	**1**	Cumulative interest prior to payment 1:	**$0.00**

Table

No.	Payment Date	Beginning Balance	Interest	Principal	Ending Balance	Cumulative Interest
1	1/1/03	560,000.00	3,266.67	459.03	559,540.97	3,266.67
2	2/1/03	559,540.97	3,263.99	461.70	559,079.27	6,530.66
3	3/1/03	559,079.27	3,261.30	464.40	558,614.87	9,791.95
4	4/1/03	558,614.87	3,258.59	467.11	558,147.76	13,050.54
5	5/1/03	558,147.76	3,255.86	469.83	557,677.93	16,306.40
6	6/1/03	557,677.93	3,253.12	472.57	557,205.36	19,559.52
7	7/1/03	557,205.36	3,250.36	475.33	556,730.03	22,809.89
8	8/1/03	556,730.03	3,247.59	478.10	556,251.93	26,057.48
9	9/1/03	556,251.93	3,244.80	480.89	555,771.03	29,302.28
10	10/1/03	555,771.03	3,242.00	483.70	555,287.34	32,544.28
11	11/1/03	555,287.34	3,239.18	486.52	554,800.82	35,783.45
12	12/1/03	554,800.82	3,236.34	489.36	554,311.46	39,019.79

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FV of an Ordinary Annuity

- Future Value

$$FV = C \cdot \frac{(1+r)^T - 1}{r}$$

  - FV annuity factors in Table A4 of RWJ
  - **Note that you can just calculate the PV of the annuity, and then find the FV by compounding the PV forwards (in other words, you don't need to learn yet another formula)**
- You wish to take a year off work in 5 years
  - If you save $8,190 each year for 5 years, how much will you have saved by the end of the time period if the interest rate is 10%? (assume you begin saving at the end of the year).
  - Hint: see page 30 (take FV of the PV calculated – you get $50K)

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Perpetuities

- A perpetuity is a stream of infinite cash flows with the first cash flow beginning one period from now

0 1 2 3 4 ………..
C C C etc
|_____|_____|_____|_____|__________|

- PV of a perpetuity: $PV = \frac{C}{r}$

The value of $10 every year forever if the interest rate is 5% is $200 (= $10 / .05)… quite simple!

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Valuing Growing Perpetuities

- A cash flow stream growing at a steady rate
  - cash flow each year; the first one (in one year's time) is $60, but cash flow grows at 2% a year
  - if the discount rate is 7%, how much would you be willing to pay? (Answer = $1200)
- Present value of growing perpetuity:

$$PV = \frac{C}{r-g}, \text{ for } r > g$$

- Useful when valuing companies (or equity).

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Summary

- Present values and Future values allow us to easily switch back and forth between cash flows separated in time
  - calculations are mechanical: look for shortcuts, use formulas / tables
- Simplifications: if possible reduce PV problems to
  - Perpetuity: an everlasting stream of constant cash flows
  - Growing perpetuity: stream of cash flows that grows at a constant rate forever
  - Annuity: a stream of constant cash flows that lasts for a fixed number of periods
  - Growing annuity: a stream of cash flows that grows at a constant rate for a fixed number of periods
- Important question for the future:
  - where do the cash flows come from?
  - what are their risks?

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