RISK, DIVERSIFICATION, AND THE SECURITY MARKET LINE (SML)

In this session we will cover and examine:

1. The difference between expected and realized returns
2. Types of Risk: Systematic vs. Unsystematic
3. The ability for investors to reduce risk through diversification.
4. The Capital Market Line and Diversification
5. What a Stock's Beta measures.
6. The Security Market Line - SML
   - Beta and the Risk Premium
   - The Security Market Line
7. Conclusions

1. EXPECTED RETURNS

GOAL: find expected returns and risk given probabilities of future events.

A. Expected Return

Let S denote the total number of states of the world, r_{is} the return in state s, for stock i, and p_s the probability of state s. Then the expected return is given by:

$$E(r_i) = \sum_{s=1}^{S} p_s * r_{is}$$

Note the difference between historical returns. However, it is difficult to get these probabilities and returns by state.

Example:

State of Economy	(1) Probability of State	(2) Return in state	(3) Product
+1% change in GNP	.25	-.05	-.0125
+2% change in GNP	.50	.15	.0750
+3% change in GNP	.25	.35	.0875
	1.00		E(r) =.15

Projected or expected risk premium

= Expected return - Risk-free rate = E(r) - r_f

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2. DIVERSIFICATION AND RISK

A. Systematic and Unsystematic Risk

- Risk consists of surprises - realization of uncertain events. Surprises are of two kinds:
- **systematic risk** - a surprise that affects a large number of assets, each to a greater or lesser extent - sometimes called market risk.
- **unsystematic risk** - a risk or surprise that affects at most a small number of assets sometimes called unique risk.
- Examples:

market risk:

Unique risk:

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B. The Principle of Diversification

- Principle of diversification: variability of multiple assets held together less than the variability of typical stock.
- The portion of variability present in a typical single security that is not present in a large group of assets held together (**portfolio of assets)** is termed **diversifiable risk or unique risk**.
-
- Why does risk go down for a portfolio? Unique risks tend to cancel each other out.
- The level of variance that **is** present in collections of assets is termed **undiversifiable risk or systematic risk.**
- A typical single stock on NYSE σ(annual) = 49.24%,
- 100 or more stock portfolio - NYSE stocks - σ(annual) < 20%

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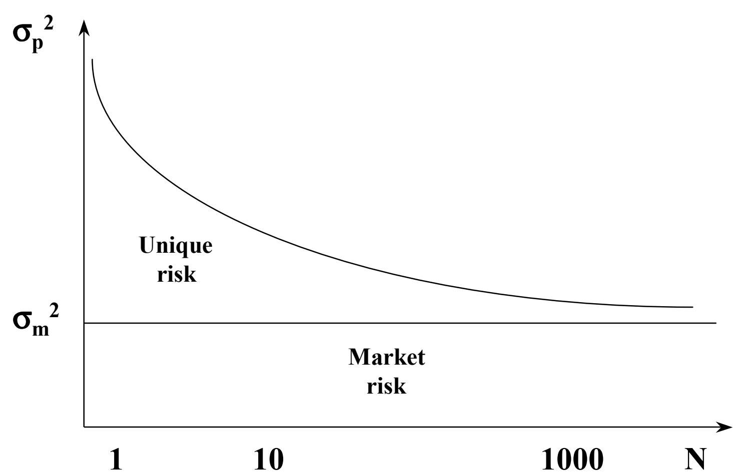
<u>Effects of diversification</u>

Total Portfolio Risk as you add stocks to your portfolio



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3. PORTFOLIOS

A portfolio is a collection of securities, such as stocks and bonds, held by an investor.

A. Portfolio Weights

- Portfolios can be described by the percentages of the portfolio's total value invested in each security, i.e., by the security's portfolio weights, α_i.

B. Portfolio Expected Returns

- The expected return to a portfolio is the sum of the product of the individual security's expected returns and their portfolio weights. The portfolio expected return:

$$E(r_p) = \sum_{i=1}^{N} (\alpha_i \text{ x } E(r_i))$$

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C. Portfolio Variance

For a 2 stock portfolio:

$$\sigma_p^2 = \alpha_i^2 \sigma_i^2 + \alpha_j^2 \sigma_j^2 + 2 * \alpha_i \alpha_j Cov(r_i, r_j)$$

For an n-stock portfolio:

$$\sigma_p^2 = \sum_{i=1}^{N} \alpha_i^2 \sigma_i^2 + 2 * \sum_{i=1}^{N} \sum_{j=i+1}^{N} \alpha_i \alpha_j Cov(r_i, r_j)$$

Unlike expected return, the variance of a portfolio is not the weighted sum of the individual security variances.

Combining securities into portfolios can reduce the variability of returns.

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As N gets large

As N gets large, the average covariance of the securities with the portfolio dominates any individual security's measure of risk.

Left with COV(i,p)

- Measure of how much risk any one security contributes to portfolio

Proportion of risk any one asset contributes to overall portfolio risk is:

$$\frac{COV(i,p)}{\sigma_p^2}$$

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Definition of covariance

Covariance is also product of individual asset standard deviations and correlation between them

$$COV(r_A, r_B) = \rho_{AB}\,\sigma_A\,\sigma_B$$

where $-1 \le \rho_{AB} \le 1$

What determines the sign of the covariance?

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Two asset portfolio practice problem

Your broker calls about 2 stocks: Weber (W) and Unix (U) that she believes has good fit with your investment objectives.

The expected rates of return and variances are:

	Weber	Unix
Expected return	.05	.03
Standard deviation	.032	.051

The correlation between the two assets is -.70.

You tell your broker to invest 60% of your wealth in W and 40% of your wealth in U. What is the expected return and standard deviation of this portfolio?

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D. The Return and Risk for Portfolios

	Stock fund		Bond Fund	
Scenario	**Rate of Return**	**Squared Deviation**	**Rate of Return**	**Squared Deviation**
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

Note that stocks have a higher expected return than bonds and higher risk. Let us turn now to the risk-return tradeoff of a portfolio that is 50% invested in bonds and 50% invested in stocks.

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The Return and Risk for Portfolios

Scenario	Rate of Return Stock fund	 Bond fund	 Portfolio	 squared deviation
Recession	-7%	17%	5.0%	0.160%
Normal	12%	7%	9.5%	0.003%
Boom	28%	-3%	12.5%	0.123%
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

The rate of return on the portfolio is a weighted average of the returns on the stocks and bonds in the portfolio:

$$r_P = w_B r_B + w_S r_S$$

$$5\% = 50\% \times (-7\%) + 50\% \times (17\%)$$

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The Return and Risk for Portfolios

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Recession	-7%	17%	5.0%	0.160%
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$$9.5\% = 50\% \times (12\%) + 50\% \times (7\%)$$

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The Return and Risk for Portfolios

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$$12.5\% = 50\% \times (28\%) + 50\% \times (-3\%)$$

The Return and Risk for Portfolios

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The *expected* rate of return on the portfolio is a weighted average of the *expected* returns on the securities in the portfolio.

$$E(r_P) = w_B E(r_B) + w_S E(r_S)$$

$$9\% = 50\% \times (11\%) + 50\% \times (7\%)$$

The Return and Risk for Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
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The variance of the rate of return on the two risky assets portfolio is

$$\sigma_P^2 = (w_B\sigma_B)^2 + (w_S\sigma_S)^2 + 2(w_B\sigma_B)(w_S\sigma_S)\rho_{BS}$$

where ρ_{BS} is the correlation coefficient for the stock and bond funds. However in the above 3.08% we can use the portfolio returns directly and just use the simple variance formula from the last notes.

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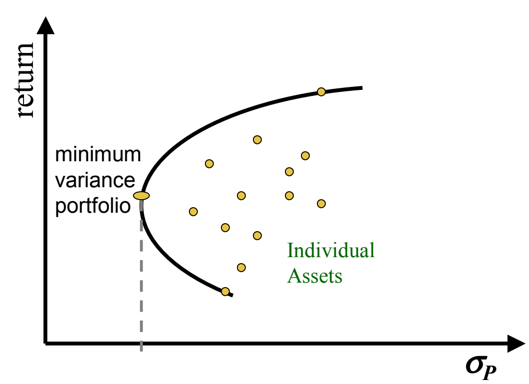
E. The Efficient Set (FRONTIER) for Many Securities



Given the *opportunity set* we can identify the **minimum variance portfolio**.

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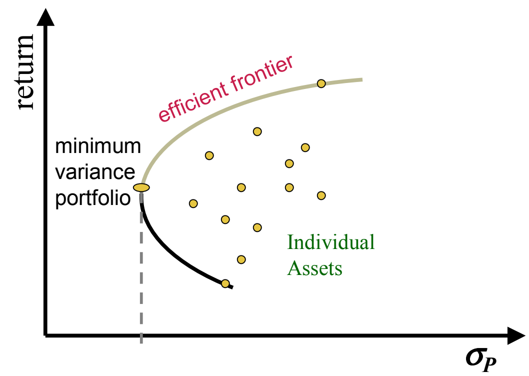
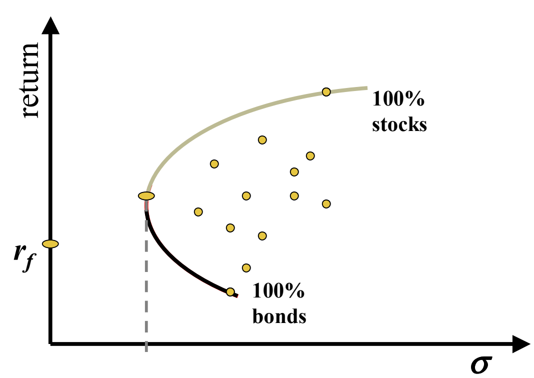
The Efficient Set for Many Securities



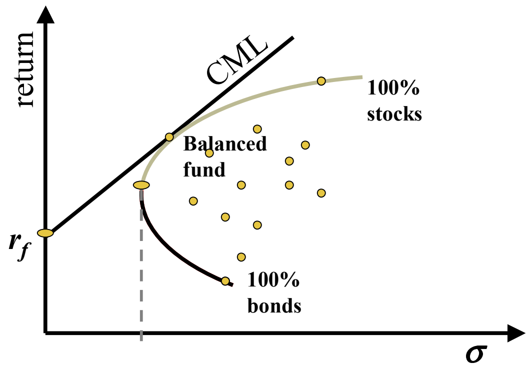
The section of the opportunity set above the minimum variance portfolio is the efficient frontier.

Optimal Risky Portfolio with a Risk-Free Asset



In addition to stocks and bonds, consider a world that also has risk-free securities like T-bills and bonds.

4. Capital Market Line: Riskless Borrowing and Lending



Now investors can allocate their money across the T-bills and a balanced mutual fund

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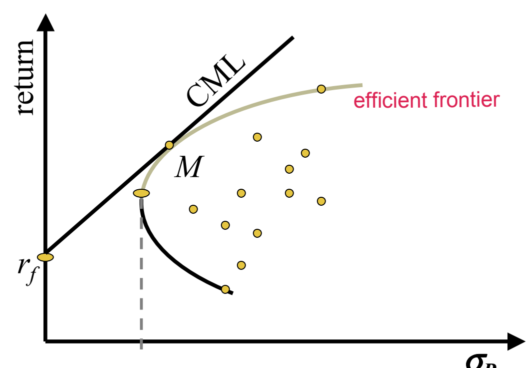
Market Equilibrium



With the capital allocation line identified, all investors choose a point along the line—some combination of the risk-free asset and the market portfolio *M*. In a world with homogeneous expectations, *M* is the same for all investors.

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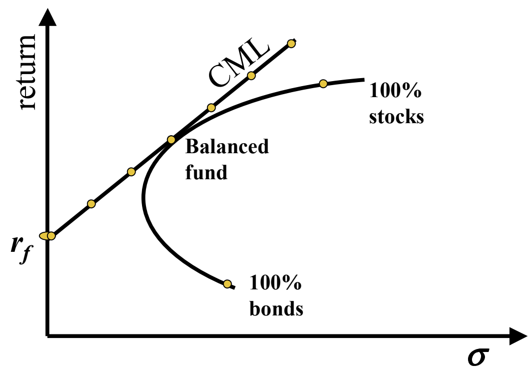
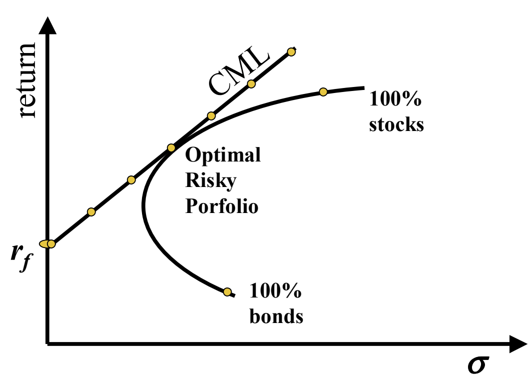
Market Equilibrium



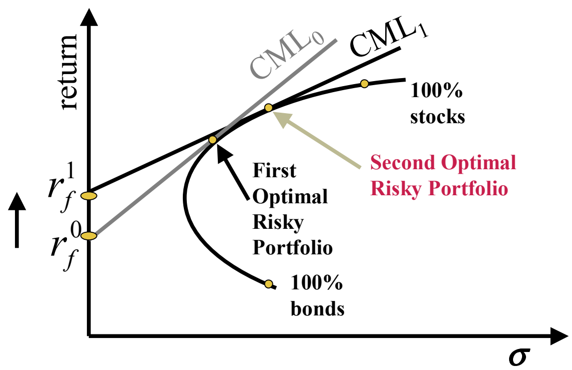
Just where the investor chooses along the Capital Asset Line depends on his risk tolerance. The big point though is that all investors have access to the same CML.

The Separation Property



The separation property implies that portfolio choice can be separated into two tasks: (1) determine the optimal risky portfolio, and (2) selecting a point on the CML.

Optimal Risky Portfolio with a Risk-Free Asset



By the way, the optimal risky portfolio depends on the risk-free rate as well as the risky assets.

5. SYSTEMATIC RISK AND BETA

A. The Systematic Risk Principle

- The principle:
- The reward for bearing risk depends only upon the systematic or undiversifiable risk of an investment
- What about unsystematic or diversifiable risk?

B. Measuring Systematic Risk:

- Beta coefficient, β: A measure of how much systematic risk an asset has relative to an average risk asset WHEN investors hold large portfolios.

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)}$$

where r_m = the return on the market portfolio, (typically we use S&P 500). Beta thus measures the responsiveness of a security to movements in the market portfolio.

Relation of β and variance of portfolio

Variance of a portfolio is composed of two parts:

σ_p^2 = Market risk + Unique risk

$$\beta_p^2 \sigma_m^2 \quad + \quad \frac{1}{N}\sum_{i=1}^{N} \sigma_{\varepsilon i}^2$$

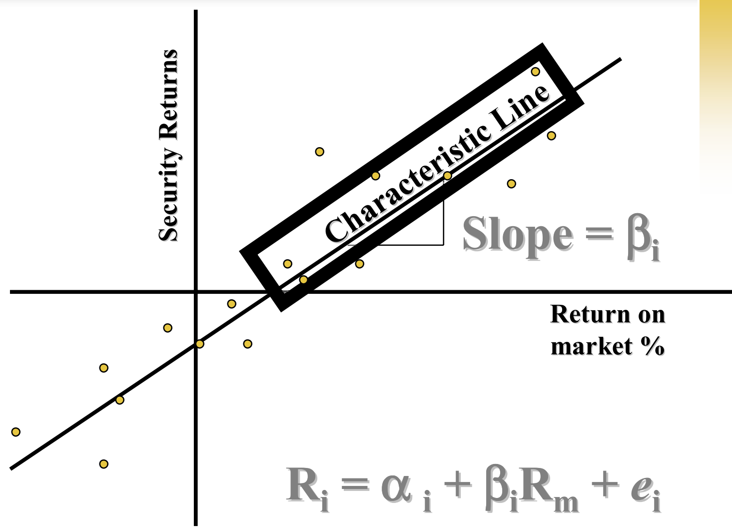
As N becomes large $\sigma_{\varepsilon i}^2$ --> 0 only market risk of a security remains

C. Portfolio Betas

- While portfolio variance is not equal to a simple weighted sum of individual security variances, portfolio betas are equal to the weighted sum of individual security betas.
- Example:

(1) Stock	(2) Amount Invested	(3) Portfolio Weight	(4) Beta Coefficient	Product (3) x (4)
IBM	$6000	50%	.75	.375
General Motors	$4000	33%	1.01	.336
Dow Chemical	$2000	17%	1.16	.197
Portfolio	100%			.91

Estimating β with regression



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Estimates of β for Selected Stocks

Stock	Beta
Bank of America	1.55
Borland International	2.35
Travelers, Inc.	1.65
Du Pont	1.00
Kimberly-Clark Corp.	0.90
Microsoft	1.05
Green Mountain Power	0.55
Homestake Mining	0.20
Oracle, Inc.	0.49

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Sharpe Ratio

Compares portfolios or individual assets based on standard deviation - allows comparison of undiversified portfolios.

$E(r_p)$ = expected return of portfolio

r_f = risk free rate

σ_p = standard dev. of portfolio

$$Sharpe\ \ Ratio = \frac{E(r_p) - r_f}{\sigma_p}$$

6. The Security Market Line

A. Beta and the Risk Premium

- A riskless asset has a beta of 0. Beta of portfolio is a weighted average of Betas of individual assets.
- **Example**:
- Let a portfolio be comprised of an investment in Portfolio A with a beta of 1.2 and expected return = 18%, and T-bills with 7% return.

Proportion invested in Portfolio A	Proportion Invested in R_f	Portfolio Expected return	Portfolio beta
0%	100%	7%	0
25%	75%	9.75%	.30
50%	50%	12.50%	.60
75%	25%	15.25 %	.90
100%	0%	18%	1.20
125%	-25%	20.75 %	1.5

Reward-to-Risk Ratio

- The combinations of portfolio expected return, beta in the previous example, if plotted, lie on a straight line with slope:
- Rise = $\frac{E(R_A) - R_f}{\beta_A} = \frac{(.18 - .07)}{1.2}$ = .092 = 9.2%
- Run
- This slope is sometimes called the "Reward-to-Risk" ratio. It is the expected return per "unit" of systematic risk.
- 2. The Fundamental Result: Reward-to-risk ratio must be the same for all assets in the market. That is,

$$\frac{E(r_A) - r_f}{\beta_A} = \frac{E(r_B) - r_f}{\beta_B}$$

If it were not- What would happen?

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B. The Security Market Line

- The line which gives the expected return - systematic risk combinations
- 1. Market Portfolios: "average" systematic risk, i.e., it has a beta
- of 1.
- Since all assets must lie on the security market line when appropriately priced, so must the market portfolio.
- Denote the expected return on the market portfolio $E(r_m)$. Then,

$$\frac{E(r_A) - r_f}{\beta_A} = \frac{E(r_m) - r_f}{1}$$

= SLOPE OF SML

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3. Relationship between Risk and Expected Return (CAPM)

Expected Return on the Market:

$$\overline{R}_M = R_F + \text{Market Risk Premium}$$

- Expected return on an individual security:

$$\overline{R}_i = R_F + \beta_i \times \underbrace{(\overline{R}_M - R_F)}_{\text{Market Risk Premium}}$$

This applies to individual securities held within well-diversified portfolios.

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Expected Return on an Individual Security

This formula is called the Capital Asset Pricing Model (CAPM)

$$\overline{R}_i = R_F + \beta_i \times (\overline{R}_M - R_F)$$

Expected return on a security	=	Risk-free rate	+	Beta of the security	×	Market risk premium

- Assume $\beta_i = 0$, then the expected return is R_F.
- Assume $\beta_i = 1$, then $\overline{R}_i = \overline{R}_M$

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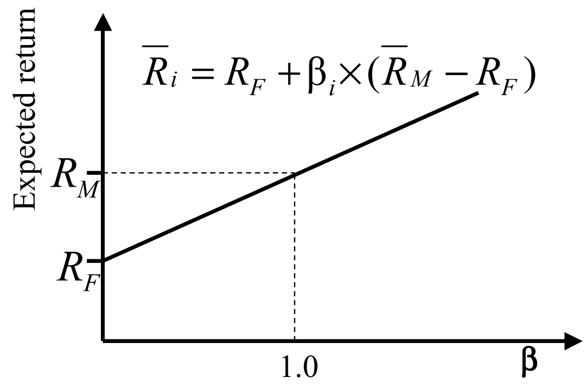
Relationship Between Risk & Expected Return



$$\overline{R}_i = R_F + \beta_i \times (\overline{R}_M - R_F)$$

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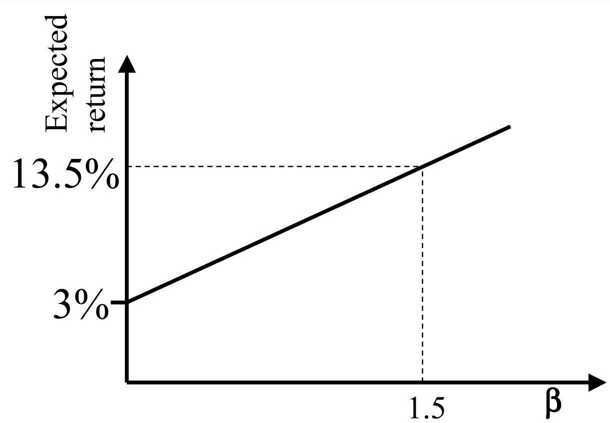
Relationship Between Risk & Expected Return



$\beta_i = 1.5 \qquad R_F = 3\% \qquad \overline{R}_M = 10\%$

$$\overline{R}_i = 3\% + 1.5 \times (10\% - 3\%) = 13.5\%$$

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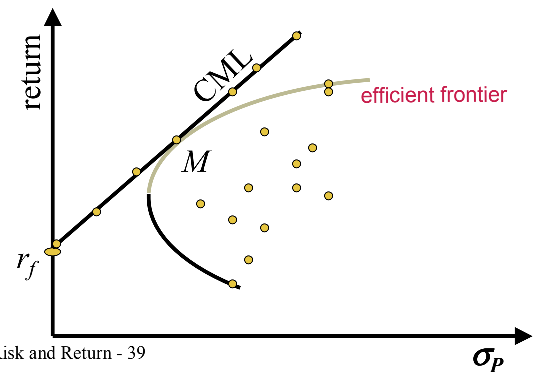
7. Summary and Conclusions

The efficient set of risky assets can be combined with riskless borrowing and lending. In this case, a rational investor will always choose to hold the portfolio of risky securities represented by the market portfolio.

- Then with borrowing or lending, the investor selects a point along the CML.



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Summary and Conclusions

- Unlike expected return, the variance of a portfolio is not the weighted sum of the individual security variances.
- Combining securities into portfolios can reduce the variability of returns - by reducing unsystematic (unique) risk.

The contribution of a security to the risk of a well-diversified portfolio is proportional to the covariance of the security's return with the market's return. This contribution is called the beta.

$$\beta_i = \frac{Cov(R_i, R_M)}{\sigma^2(R_M)}$$

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Summary and Conclusions

- The CAPM states that the expected return on asset depends upon:
- 1. The time value of money, as measured by r_f.
- 2. The reward per unit of systematic risk, $E(r_m) - r_f$.
- 3. The asset's systematic risk as measured by Beta, β.

$$\overline{R}_i = R_F + \beta_i \times (\overline{R}_M - R_F)$$

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