ROBERT H. SMITH SCHOOL OF BUSINESS

Financial Management

Options

OPTIONS

- What are Options?
  - Options are the right or "option" for the holder to buy (Call Option) or sell (Put Option) at given contract terms.
    Common types of options: Calls, Puts, Warrants, Convertible portion of Convertible Debt. Option "like" features are found in many corporate securities.
  - In this note we will present the basics of option theory, show how options are valued *relative to* other securities and *use* option theory to value some corporate securities including warrants
- Options are part of larger group of securities called contingent claims or derivative securities
  - Value of the value of these securities is contingent on value of underlying security (usually equity)

Examples of Questions using option pricing:

- What is the value of convertible debt?
- What is the value of deciding to default on bonds?
- Convertible Debt usually has a lower interest rate for the firm. Does this mean that it is "cheaper?" Anw. NO.
- We want to be able to know how the risk and expected return of convertible debt securities differ from straight debt so we can include them in the WACC for the firm.

Basic insight of option theory

- Options are priced relative to other assets.
- The payoffs to options can be exactly replicated by a portfolio of risk free bonds and the underlying stocks.
- Thus, options can be priced by arbitrage methods.
- Investors attitudes about risk do not matter for pricing an option, as it is priced relative to bonds and stocks - however the risk of an option and thus its expected return can be substantial.

Where are options traded?

- Chicago Board Options Exchange
  - Individual stocks
  - General stock market indices
  - Treasury bonds
- AMEX
  - Individual stocks
  - General stock market indices
  - Oil and gas index
  - Transportation index
  - Treasury bills and notes
- Philadelphia Exchange
  - Individual stocks
  - Foreign currencies
  - Gold and silver indexes

1. Option Terminology

- **Call Option, C(K,T,σ,S):** The right to **buy** a financial security or commodity at a fixed price (strike price) K at time T in the future.
  - σ, S are underlying volatility and Price of the financial security (stock).
- **Put Option, P(K,T):** The right to **sell** a financial security or commodity at a strike price K at time T in the future.
  - Note 1: You can both buy and sell both types of options: call and put options.
- **Strike Price, K** : The amount you pay for the security or commodity when you exercise the option.

Option Terminology

- **European Options:** Options that cannot be exercised before fixed exercise date.
- **American Options:** Option holder can choose to exercise before expiration.
  - Note 2: Call and Put options can be either American or European.

Option Payoffs

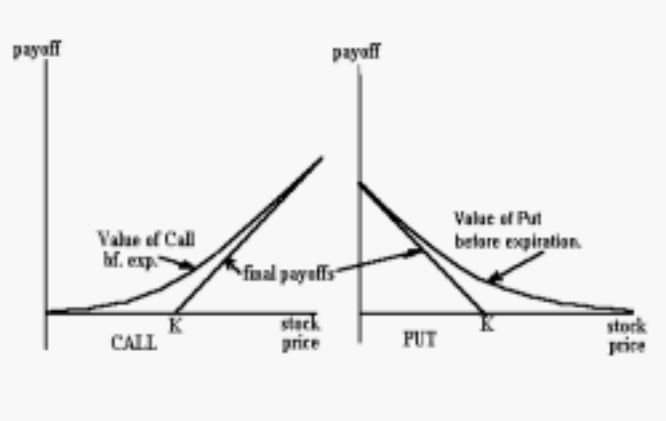
- **Call Option:** The buyer has the choice or option of whether or not to buy in the future at a predetermined exercise price or strike price, K, at a predetermined date, T, - the exercise date.
- Buyer pays the call price, Ct, today to the seller - receives the option to buy in the future.
- **Call Option Payoffs:** (S_T = Stock or asset price at exercise date T)

	At Contract Date	*At Exercise*
Seller	+ C_t	- Max{ 0 , S_T - K }
Buyer	- C_t	+ Max{ 0 , S_T - K }

- **Put Option:** The buyer has the choice or option of whether or not to sell in the future at a predetermined exercise price or strike price, K, at a predetermined date, T, - the exercise date.
- Buyer pays the put price, Pt, today to the seller - receives the option to sell in the future.
- **Put Option Payoffs:** (S_T = Stock or asset price at exercise date T)

	At Contract Date	*At Exercise*
Seller	+ P_t	- Max{ 0 , K - S_T }
Buyer	- P_t	+ Max{ 0 , K - S_T }

Payoff graphs for Call and Put Options:



* Remember, an investor writing a call or a put receives the exact opposite payoffs.

Factors affecting value of call

- The value of a call is contingent on certain characteristics of the underlying security:

$C = f(S_t, \sigma^2, K_T, \tau, r_f)$

where

S = Stock price (+ related to call price as the payoff increases with the stock price)

σ^2 = Variance of stock price (+ related as increased chance of exercise)

K = Exercise price (- related as lower probability of being exercised)

τ = Time til maturity (+ related as greater chance of exceeding exercise price)

r_f = Risk free rate (+ related as present value of the delay of payment of exercise price becomes more valuable as interest rates rise)

2. Put-Call Parity

A central relationship that is useful for puts and calls:

(Also useful in pricing some corporate securities.)

$$S = C(K) + Ke^{-r\tau} - P(K)$$

(equals for European options, less than or equal to for American options)

- where S = current price of the stock,

K is the common exercise price of the options

τ is the time to maturity of the options,

r the riskless rate of interest,

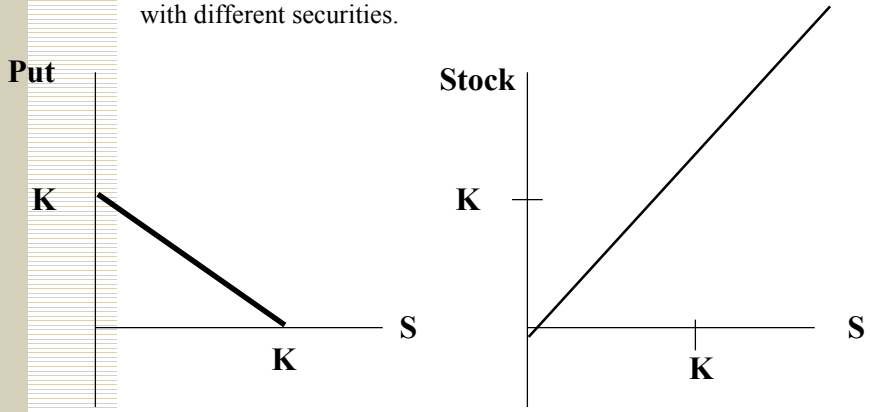
C(K), P(K) the current prices of the calls and puts.

Ke^{-rt} = present value of the exercise price.

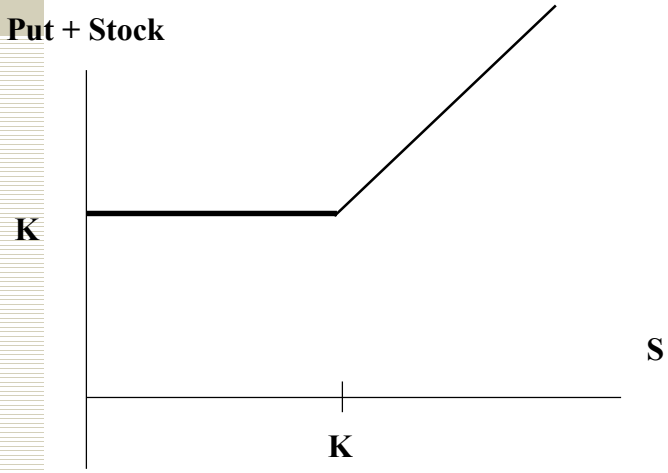
Graphical Illustration of Put-Call Parity

- Compare the following combinations:
  Buy a Put + share of stock (S) vs. Buying a Call + Bond

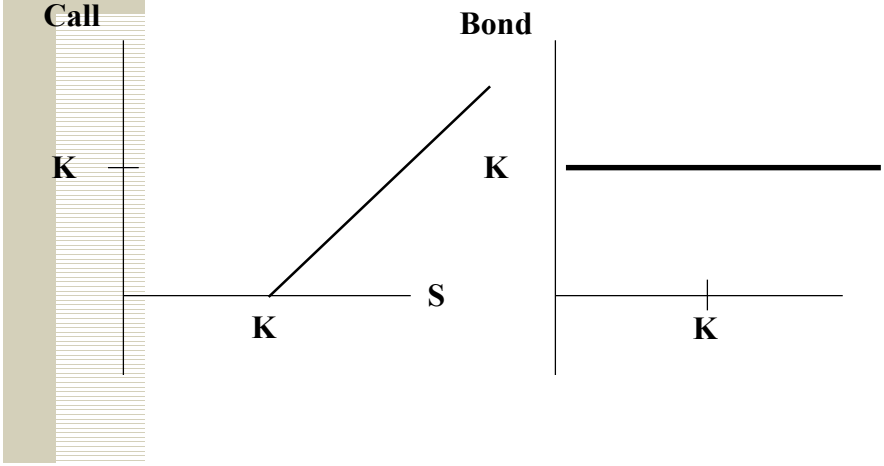
NOTE how the final graphs, slide 14 and slide 16, are the same with different securities.



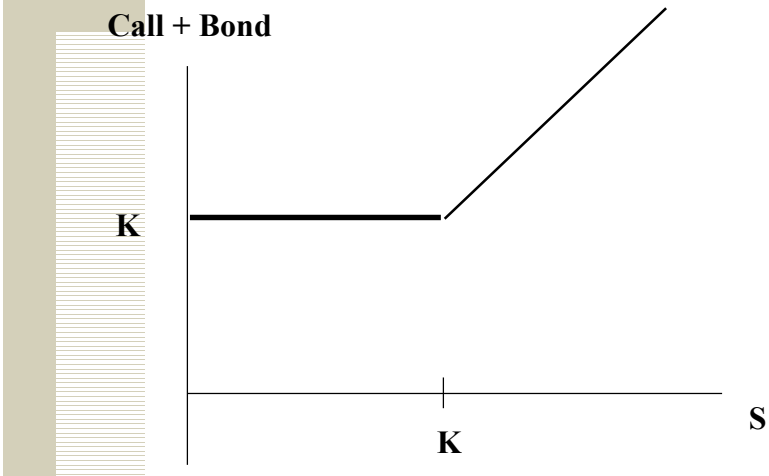
Combine put and stock



Buy call and bond that pays K at maturity



Combine call and bond



Replicating a Put Option

- Put-call parity theorem tells us that as long as you have 3 of the securities (Stock, Call, Put, Bond), you can always create the fourth.

Example:

- Suppose a European call option with a price of $4 has an exercise price of $90 with 6 months to maturity. The stock price is currently $75 and the riskless rate of interest for the six months is 4%.
- Assume that there is no put traded in the market. Could you reproduce the same cash flows as the Put Option? How? What Strategy?
- Part 2: Consider the scenario in which a put with an exercise price of $90 and 6 months maturity sells for $14. Is this an equilibrium price? How could you take advantage of this situation?

3. Black - Scholes Option Valuation:

- Going to continuous time we can derive the famous Black - Scholes option pricing formula:
- (for non-dividend paying stocks, for constant proportional dividend paying stocks a variant of this formula applies.):

$$C = S_t N(d_1) - Ke^{-r\tau} N(d_1 - \sigma\sqrt{\tau})$$

$$where\ d_1 = \frac{\ln(S_t / Ke^{-r\tau})}{\sigma\sqrt{\tau}} + \frac{1}{2}\sigma\sqrt{\tau}$$

N(x) is the standard normal distribution function. (a standard function in spreadsheets). σ = std. dev. of firms' stock return in continuous time, τ is the time to maturity of the options, S_t = current stock price.

Assumptions of Black-Scholes

- No restrictions on short selling
- Transactions costs and taxes are zero
- European option
- No dividends are paid
- Process describing stock price return is continuous
- Market has continuous trading
- Short-term interest rate is known and constant
- Stock returns are lognormally distributed

Example using Black-Scholes

- Private Equipment Company (PEC), on October 4, 1994 has an April 49 call option with a closing value of $4. The stock itself is selling at $50. On October 4, the option had 199 days to expiration. The annual risk free rate is 7%.

- We can easily get four of the necessary components:
  - Stock price (S) is $50
  - Exercise price (K) is $49
  - Risk free rate (r_f) is 0.07
  - Time to maturity (τ) = 199/365 = .545

- You would have to calculate σ, the standard dev. of the firm's stock price. How? Calculate standard deviation of stock's continuous return using daily data and annualize - continuous return = $\ln(S_t/S_{t-1})$

Black-Scholes with Dividends

- Dividends are a form of "asset leakage". If dividend are paid repeatedly we adjust Black Scholes to allow constant proportional dividends:

$$C = S_{\delta} N(d_3) - Ke^{-rt} N(d_3 - \sigma\sqrt{t})$$

$$\text{where } d_3 = \frac{\ln(S_{\delta} / K) + [r + \sigma^2 / 2]t}{\sigma\sqrt{t}}$$

$$\text{and } S_{\delta} = Se^{-\delta t} \text{ and } \delta \text{ is a constant dividend yield.}$$

4. VALUING WARRANTS

- Warrants are a direct application of option theory. Warrants are rights to buy shares of stock at a set exercise price.
- They are usually issued in conjunction with underwritings and private placement bonds.
- In addition warrants are commonly given to underwriters as partial payment for services.
- A warrant on a stock with no dividends can be valued with the standard Black - Scholes formula. Its payoffs are similar to a Call Option.
- To Value Warrant: Use Black - Scholes for a Call options:

Valuing Warrants (Continued)

- However, two complications arise in valuing warrants.

  1. Dilution: upon exercise more shares will be outstanding.

  2. Change in the risk of equity.

- The 2nd complication is that the standard dev. of this "new" firm's equity without warrants is different than the real firm whose warrants we are trying to price.

Dilution

•Note share price after exercise = S' = $\frac{V + NqK}{N + Nq}$

where
V = value of equity.
N = Number of shares outstanding.
q = % of warrants outstanding per share of stock.
K = exercise price on the warrant.

- Similar to a call option, but with the new share price:
- Warrant value = max(S'- K, 0)

  = 1 / (1+q) * max(V/N - K , 0)

- We can now value this using Black - Scholes, on a firm without any warrants outstanding.

Change in Risk of Equity

- To find the new standard deviation of equity, use the following formula:

• s.d. of assets = (assume constant)	proportion of equity after issue	X	new s.d. of equity ("new firm")

(s.d. = standard deviation.)

- The first item (s.d. of assets) we get by unlevering the standard deviation of the leveraged firm.
- s.d. of assets = $\frac{E^o}{V^o} * old$ s.d. of common stk .

$\frac{E^o}{V^o}$ = pre-warrant equity value / pre-warrant firm value.

•The second item in the formula is the equity to value ratio after the warrant has been exercised.

Final Complication:

- The warrant may be on a dividend paying stock:
- This can be valued using adjustments to the Black-Scholes for constant proportional dividends.
- Generally, you have to compare the value of exercising at each point in time and getting the dividends with the value of holding the option or warrant.

- (Numerical techniques exist to value the dividend paying feature. Exact formulas can be derived for special cases.)
- Once the value of the warrant is calculated - you may want to compute its required return and Beta so that you can include it in the cost of capital for the firm. (SEE: Cox - Rubinstein Options Markets (1985))

CONCLUSIONS

You should understand the following principles from option theory:

* It is valuable to wait to exercise an option. Remember you only exercise in good states, you save your exercise money if you wait and the bad (low price) state results.
* The longer the time to expiration, the higher the value of the option.
* The cost of waiting on a stock option is the lost profits (dividends) that would be yours from owning the stock.
* These results hold no matter what the distribution of the asset is (continuous, binomial, jump process). You do have to assume a process that describes how the asset's or investment's returns change over time in order to get an explicit value for this option.