ROBERT H. SMITH SCHOOL OF BUSINESS

BMGT - Corporate Finance

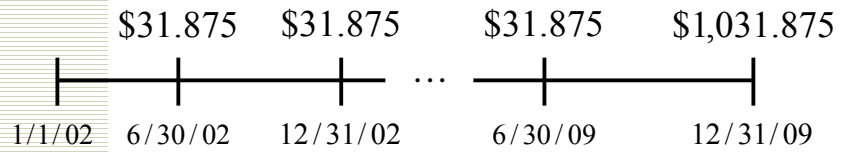
Bond Valuation

Outline

- Bond characteristics/terminology
- Valuing a "Discount" Bond (given a yield)
- Backing out a Yield from a Price
- Valuing a Coupon Bond
- Bond risk relationship to duration
- Term structure of interest rates (the yield curve)
- Valuing risky bonds: Default risk

A Standard U.S. Treasury Bond

- Consider a U.S. government bond issued on Jan. 1, 2002 with a 6 3/8% coupon, maturing in Dec. 2009.
  - The *Par Value* of the bond is $1,000.
  - *Coupon payments* are made semi-annually (June 30 and December 31 for this particular bond). Since the *coupon rate* is 6 3/8 the semi-annual payment is $31.875.
  - On January 1, 2002 the size and timing of cash flows are:



Bond Market Lingo

Some terminology:

- **Principal:** The amount borrowed
- **Par (Face) Value:** The amount repaid at end of loan
  - Usually face value is equal to principal, but for bonds the original issue price (principal) may be slightly higher or lower
- **Maturity:** Years Until Face Value Repaid
- **Coupon:** Interest Payment
  - Bonds usually have a fixed interest payment
  - Loans often have floating coupon rates, tied to an index
  - Bonds/loans that have no interest are called zero-coupon or discount bonds/loans
  - Typically paid out twice a year (semi-annually) for bonds
  - Coupon typically quoted as coupon rate = coupon/face value
  - (Current yield = coupon/current price)
- **Bonds** are standardized instruments that are generally tradable
- **Debentures** are unsecured bonds with maturities of at least 15 years
- **Bills** are bonds with less than 1 year maturity (no interest payments)
- **Amortized loan** (like a mortgage) – equal repayment each period (part principal, part interest)

Bond or Loan Structure

- *Covenants*: legal provisions that when violated give bondholder right to specific action such as forcing bankruptcy and demanding repayment.
- *Security*: pledged assets committed to paying off debt
- *Seniority*: order of payment in bankruptcy
  - Secured debt, then Senior Debt, then Subordinated Debentures.
- *Call provision*: enables corporation to repay and retire the debt at will.
- *Sinking funds*:
  - fund that firm contributes cash to for repayment.
- *Rating:*
  - Investment Grade Bonds: Bonds rated Baa or higher by Moody's or BBB or higher by Standard and Poors.
  - High-yield or "junk" bonds: not investment grade

Bond Prices, Values and Yields

- Three key "numbers" we look at with bonds:
  - Price - what it is trading at in the market
  - Value - what we believe its true value is (discounted future cash flows)
  - Yield-to-Maturity (YTM)
    - the rate of return on the bond, using today's bond price and assuming that all promised payments will in fact be made
    - equivalently, the required discount rate that sets the discounted cash flows equal to the market price of the bond

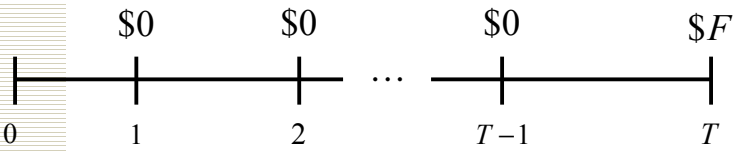
Pure Discount Bonds

Information needed for valuing pure discount bonds:

Time to maturity (T) = Maturity date - today's date

Face value (F)

Discount rate (r): YTM



Present value of a pure discount bond at time 0:

$$PV = \frac{F}{(1+r)^T}$$

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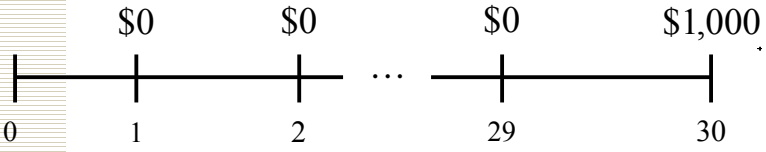
Pure Discount Bond Example

Find the value of a 30-year zero-coupon bond with a $1,000 par value and a YTM of 6% (assuming annual compounding).



$$PV = \frac{F}{(1+r)^T} = \frac{\$1{,}000}{(1.06)^{30}} = \$174.11$$

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How risky are discount bonds?

- What if the discount rate were 5% instead of 6%?
  - The bond's value would be 231.38 (=1000/$(1.05)^{30}$)
  - So, when the discount rate decreases by 1%, the bond's value appreciates from 174.11 to 231.38, a 33% increase!
- Consider a 1-year pure discount (zero-coupon) bond with a face value of $1000.
  - If the discount rate is 6%, the bond's value would be $943.40.
  - If the rate were instead 5%, the price would be $952.38.
  - Thus, a 1% *decrease* in the discount rate, leads to a 1% *increase* in the value of the one-year discount bond.
- **The 30 year discount bond is approximately 30 times as risky (as sensitive to rate changes) as the 1 year discount bond. Long maturity bonds can be very risky!**

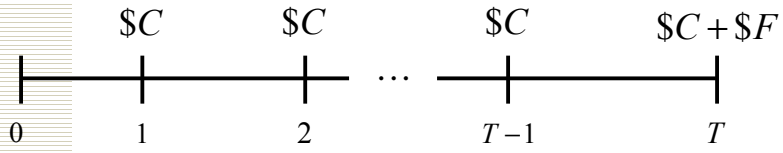
YTM Calculation Example

- We have seen how you can compute the bond value if you know the YTM (discount rate).
- Now, find the YTM based on the known price of the discount bond:
  - If the quoted price of a 5-year discount bond is 71.30, what is the YTM (assuming annual compounding)?
  - 71.30 = 100 / $(1.07)^5$ (.07 = $(100/71.30)^{1/5}$ - 1)

Valuing Bonds with Coupons

Information needed to value level-coupon bonds:

- Coupon payment dates and time to maturity (T)
- Coupon payment (C) per period and Face value (F)
- Discount rate: YTM = *r*



Value of a Level-coupon bond
= PV of coupon payment annuity + PV of face value

$$PV = \frac{C}{r}\left[1-(1+r)^{-T}\right]+\frac{F}{(1+r)^{T}}$$

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Example 1: Bond Trading at Par Value

- Suppose Harley Davidson issued $1,000 face value bonds
  - 20 years to maturity.
  - Annual coupon is $110 (11% Coupon Rate)
- What is the bond's fair value if similar bonds have a yield to maturity of 11% (assuming annual compounding)?
  - Present value of face value = $1,000/$(1.11)^{20}$ = $124.03
  - Present value of coupons = $110 x (1-1/$(1.11)^{20}$)/.11
    = $110 x **7.9633** = $875.96
- Adding the discounted face value and coupons together:
  - Bond Value = $124.03 + $875.96 = $1000
- If the YTM and coupon rate are the same, the bond must be trading at its Par Value!

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Example 2: Bond Trading at Discount

- Assume now that the yield on comparable bonds is 13% (rather than 11% in the previous example).
- Value of Bond:
  - Present value of Principal = $1,000/(1.13)^{20} = $86.78
  - Annuity present value of coupons
    = $110 x (1-1/(1.13) ^{20})/.13 = $110 * **7.0248** = $772.72
  - Total value = $86.78 + $772.72 = $859.50
- The bond should thus be trading at a discount of $140.50 (= 1,000 - 859.50) to its face (par) value.

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Example 3: Bond Trading at Premium

- Now, assume that discount rates in the market are lower than in the previous two examples: the appropriate yield is 9%
- Value of Bond:
  - Present value of principal = $1,000/(1.09)^{20} = $178.43
  - Annuity present value of coupons:
    = $110 x (1 -1/(1.09)^{20})/.09 = $110 x **9.1285** = $1,004.14
  - Total Value = $178.43 + $1,004.14 = $1,182.57
- The bond should be trading at a premium of $182.57 to its par value.

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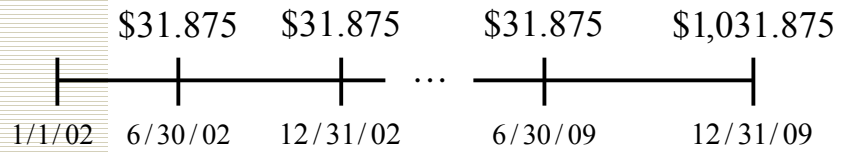
U.S. Treasury Bond (revisited)

- Recall the U.S. government bond issued on Jan. 1, 2002 with a 6 3/8% coupon, maturing in Dec. 2009.
  - The *Par Value* of the bond is $1,000.
  - *Coupon payments* are made semi-annually (June 30 and December 31 for this particular bond). Since the *coupon rate* is 6 3/8 the semi-annual payment is $31.875.
  - On January 1, 2002 the size and timing of cash flows are:



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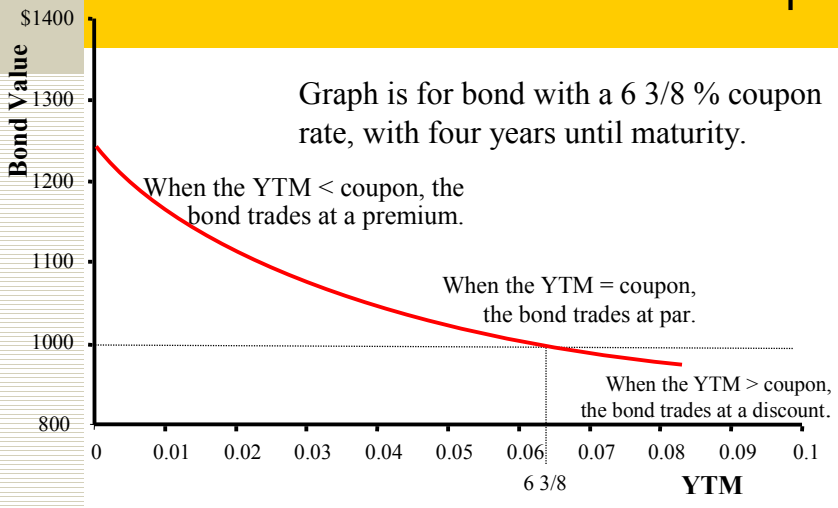
Yield - Bond Value Relationship



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Finding the Yield to Maturity

- Finding the YTM for a coupon-paying bond requires solving a "non-linear equation" - i.e. it is not easy to do quickly.
- Usually, we use a financial calculator, a spreadsheet program or annuity tables.
- We will revisit this when we discuss (in a couple of weeks) calculating the Internal Rate of Return (IRR) of an investment, which is exactly what the YTM is.

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Bond-Equivalent Yields

- Most bonds pay out coupons semi-annually - i.e. a 10% coupon bond pays out $50 twice a year on a $1000 face value.
- The bond markets have also adopted the convention of quoting yields assuming semi-annual compounding, called a "bond-equivalent basis." For example, if the bond-equivalent yield is 10%, that means that the equivalent annual rate (EAIR) is actually 10.25%.
- This convention ensures that a bond with a 10% coupon rate that yields 10% on a bond-equivalent basis trades at par:
  - To see this, take a one-year example (two coupons):
  - Value = $5/1.05 + 5/(1.05)^2 + 100/(1.05)^2 = 100$

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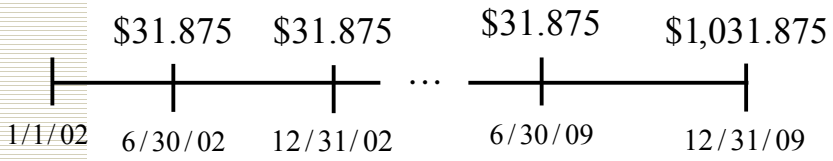
An example

- A $1,000 bond with an 8% coupon rate maturing in 10 years will have what price if the market quoted YTM is 10%?
  - The semi-annual discounting rate is 5% if the bond-equivalent yield is 10%. The coupon amount is $40 paid out every half-year.
  - Present value of face value
    = $1,000/$(1.05)^{20}$ = $376.89
  - Annuity present value of coupons
    = $40 x $(1-1/(1.05)^{20})/.05$
    = $40 x 12.4622 = $498.49
  - Adding the discounted face value and coupons together:
    = 376.89 + $498.49 = $875.38

Another example/exercise

Find the present value (as of January 1, 2002), of a 6-3/8 coupon T-bond with semi-annual payments, and a maturity date of December 2009 if the YTM is 5-percent (per annum) quoted on a bond-equivalent basis.

- The size and timing of cash flows are:



$$PV = \frac{\$31.875}{.025}\left[1-\frac{1}{(1.025)^{16}}\right]+\frac{\$1,000}{(1.025)^{16}} = \$1,049.30$$

Bond sensitivity to interest rate risk

- We have seen that as market interest rates, or yields, change, the value of a bond changes as well - in the opposite direction.
- We also saw earlier that a 30-year pure discount bond changed in price (on a percentage basis) approximately 30 times (33 times to be precise) as much as a 1-year pure discount bond. The maturity of the bond is thus a reasonably good measure of the risk of the bond, relative to the one-year bill.
- What about a 30-year bond with a 10% coupon rate?

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Duration

- We can think of the 30-year 10% coupon bond as a portfolio of 60 zero-coupon bonds:
  - a 1/2 year pure discount bond with face value of $5
  - a 1-year pure discount bond with face value of $5
  - ...
  - a 30-year pure discount bond with a face value of $105
- The interest rate risk of the coupon bond will be lower than the risk of a pure discount 30-year bond since it mixes this pure discount bond with shorter maturity bonds.
- If we take the weighted average maturities of all of the 60 pure discount bonds that make up the 10% coupon bond, this gives us what we call the "duration" of the bond - an important measure of the interest rate risk of a bond. The "weights" are the present values of each of the 60 bonds as a percentage of the total value of the coupon bond.

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The Yield Curve

- The pattern of interest rates (yields) from different maturity bonds is called the yield curve, or the term structure of interest rates.
- Since bonds with longer durations are riskier, one would expect that they should have higher yields. This implies a "positive yield curve" (i.e. yields increase consistently with maturity). This is typical, but is not always the case.
- To see why, note that bond yields should also reflect investors' expectations of future interest rate changes.
  - If the one-year interest rate is 4%, and investors expect that in one year's time the one-year interest rate will have increased to 6%, the two-year yield will likely be close to 5% (perhaps a little higher to reflect the risk in holding a two-year bond).
  - If investors believe instead that the interest rate will decrease to 2%, the two-year yield should be close to 3%, which is less than the one-year rate (i.e. the yield curve will have a negative slope).

Default Risk

- The value of a non-Treasury bond will reflect the default risk of the bond. The probability of default, as well as the loss in the event of default, will decrease the value (or price) relative to an "equivalent" Treasury bond. Stated in another way, the bond must offer a higher promised YTM than a default-free Treasury bond with the same duration.
- Comparing yield curves for bonds of different ratings with the Treasury-Bond yield curve gives us an indication of the premium (also called "spreads") that investors require for bearing credit risk (at different horizons).
- Corporate bonds move with the underlying stock of the company, but much less dramatically, since the bond value is highly sensitive to changes in company value only when default appears likely.

SUMMARY: BONDS

- AGAIN: Focus on the Cash Flows
- BONDS: Split Problem into 2 parts
  - What's the PV of the Coupon Payments
  - What's the PV of the Final Payment

==> Market Price of Bonds = Sum of these 2 amounts

- Remember: While cash flows are fixed, Market Value Changes