MANAGEMENT SCIENCE
Vol. 52, No. 12, December 2006, pp. 1945–1956
ISSN 0025-1909 | EISSN 1526-5501 | 06 | 5212 | 1945

DOI 10.1287/mnsc.1060.0579

A Theory of Volatility Spreads

Gurdip Bakshi, Dilip Madan
Department of Finance, Robert H. Smith School of Business, University of Maryland, College Park, Maryland 20742
{gbakshi@rhsmith.umd.edu, dbm@rhsmith.umd.edu}

This study formalizes the departure between risk-neutral and physical index return volatilities, termed *volatility spreads.* Theoretically, the departure between risk-neutral and physical index volatility is connected to the higher-order physical return moments and the parameters of the pricing kernel process. This theory predicts positive volatility spreads when investors are risk averse, and when the physical index distribution is negatively skewed and leptokurtic. Our empirical evidence is supportive of the theoretical implications of risk aversion, exposure to tail events, and fatter left-tails of the physical index distribution in markets where volatility is traded.

Key words: risk aversion; physical return moments; pricing kernel; risk-neutral volatility; volatility spreads; spanning risk-neutral moments
History: Accepted by David Hsieh, finance; received June 8, 2005. This paper was with the authors 1 month for 3 revisions.

1. Introduction

Earlier empirical work from Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Bakshi et al. (2000), and Christoffersen et al. (2006) based on implied volatility, and now Bakshi and Kapadia (2003), Bollerslev et al. (2005), Britten-Jones and Neuberger (2000), Carr and Wu (2004), Jiang and Tian (2005), and Polimenis (2006) based on formal measures of risk-neutral volatility, suggests the finding that risk-neutral index volatility generally exceeds physical return volatility. Despite the advances in modeling volatility, there appears to be little theoretical work that connects the two entities within the same underlying economic equilibrium. The purpose of this paper is to formalize the departure between risk-neutral and physical index return volatilities, termed *volatility spreads*. What are the theoretical determinants of volatility spreads? What role does investor behavior play in explaining dynamic movements in volatility spreads? Our contribution lies in identifying the sufficient conditions on physical and risk-neutral densities that give rise to positive volatility spreads.

The compelling force behind the theory of volatility spreads is the notion that risk-neutral probabilities are physical probabilities revised by investor's risk preference as determined by the pricing kernel (Harrison and Kreps 1979). Rational risk-averse investors are sensitive to extreme loss states and are willing to counteract these exposures by buying protection. The desire to cover these losses typically drives up the risk-neutral probability relative to the actual probability of occurrence, and endogenously shifts probability mass to risk-neutral tails. This mechanism essentially argues that risk aversion introduces heterogeneity in the risk-neutral distributions and causes noninverted volatility spreads.

To finesse the above elementary intuition, we first present a four-state example in a two-date setting. Through a judicious design of physical probabilities and returns, this model distinctly parameterizes physical skewness and physical kurtosis and accommodates exposure to both up and down extremes. Using a naive pricing kernel representation that amplifies both the up and down risk-neutral tail probabilities, we analytically derive the parametric restrictions on physical skewness and physical kurtosis that guarantee the existence of positive volatility spreads. This analysis verifies the core intuition that volatility spreads get bigger when the investor dislike for unpleasant tail events gets progressively more severe. Theoretically, it is shown that the existence of positive volatility spreads warrant exposure to tail events that generate fat-tailed physical return distributions. Furthermore, the introduction of negative physical skewness, at the margin, tends to magnify the effect of physical kurtosis on volatility spreads.

The general modeling framework richens the state-space example and provides a deeper understanding of the issues by (i) relaxing the assumption that the parametric form of the physical density is known, and (ii) focusing on a family of theoretically appealing pricing kernels. Thus, it must be noted that our results hold for a wide class of physical return densities and are derived without imposing any structure on the dynamics of physical volatility and without assuming the functional form of the volatility risk premium. The main analytical characterization reveals an equation where volatility spreads are related to the pricing kernel and to the higher-order moments of the physical return distribution. For a broad set of pricing kernels that display risk aversion, we obtain approximate theoretical results that predict positive volatility