

# Volatility Risk Premium Embedded in Individual Equity Options: Some New Insights

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## Abstract

Previous research has indicated that index options price a negative volatility risk premium, thus providing a possible explanation of why Black-Scholes implied volatilities of index options exceed realized volatilities. In this paper, we examine the empirical implication of market volatility risk premium on 25 individual equity options. We first note that, although the Black-Scholes implied volatilities of individual equity options are also greater than historical return volatilities, the difference between these two volatilities is much smaller than for the market index. Next, consistent with index option market, the individual equity option prices also embed a negative market volatility risk premium. However, the magnitude of this risk premium is much smaller than for the index option and idiosyncratic volatility does not appear to be priced. Our empirical results provide a potential resolution of why buyers of individual equity options leave less money on the table as compared with buyers of index options.

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# I Introduction

In a recent paper, Bakshi and Kapadia (2003) have shown the existence of a negative volatility risk premium in index options. One crucial impact of a negative volatility risk premium is to make index options more expensive; Negative market volatility risk premium provides, at the least, a partial explanation for the stylized finding that *index* implied volatilities are typically greater than realized volatilities (Jackwerth and Rubinstein (1996)). In essence, adding options to a market portfolio will help hedge market risks as market volatility tends to increase when stock market falls and hence consistent with a negative volatility risk premium.

In this paper, we investigate the pricing of market volatility risk in individual equity options. There are several reasons why it is important to extend existing evidence on the index option market to individual equity options. First, given that stock returns have a significant market component, the existence of a market volatility risk premium also has implications on how individual equity options are priced. Potentially, this allows us to verify that market volatility risk is compensated and that existing results from the index option market are not driven by other factors (say, demand for index options for portfolio insurance purposes). Second, it is of economic importance to understand the magnitude of the volatility risk premium imbedded in individual stock options. Given that individual risk-neutral distributions are systematically different from the market index (see Bakshi, Kapadia and Madan (2003)), how volatility risk is priced in individual options can shed additional insights on the pricing structure of individual equity options.

Following the theoretical arguments in Bakshi and Kapadia (2003), we consider the gains on a delta-hedged option portfolio - a portfolio of a long call position hedged by a short position in the stock, with the net investment earning the risk-free interest rate. If volatility risk is not priced, average delta-hedged gains are zero even when volatility is stochastic. If volatility risk is priced, then the sign and magnitude of the average delta-hedged gains are determined by the volatility risk premium. In particular, a testable implication of a non-zero market volatility risk premium is that delta-hedged gains are correlated with the level of market volatility and not with idiosyncratic return volatility.

We apply our framework to empirically examine the pricing of market volatility risk in 25 individual equity options. Our first finding is that, on average, near-money Black-Scholes implied volatilities of individual equity options are greater than historical realized

volatilities. More importantly, the magnitude of this difference between implied and realized volatilities for the individual equity options is much smaller than that for the index.

Second, delta-hedged gains of individual equity options are more negative than positive. Almost twice as many firms have significant negative gains as compared with significantly positive gains. Over all firms, on average, the delta-hedging strategy loses a statistically significant 0.03% of underlying asset value. The same delta-hedging strategy for the index has a loss of 0.07% of the underlying index level.

Third, individual equity delta-hedged gains are significantly negatively correlated with the level of market volatility. Moreover, the market volatility subsumes the effect of the firm's own volatility and idiosyncratic volatility does not appear to be priced. Our results are consistent with the implication of a non-zero volatility risk premium, and in particular a negative market volatility risk premium. However, the volatility risk premium embedded in individual equity options is much smaller than for index options.

What are the economic implications of these results? The difference between Black-Scholes implied volatility and realized volatilities indicates that, like the buyers of index options, buyers of individual equity options also lose money and, in other words, "leave money on the table." The analysis of delta-hedged gains provides a reason why - individual equity options, like index options, also incorporate a negative market volatility risk premium. Our estimates of the volatility risk premium is much smaller for individual equities thus providing an explanation on why the implied and realized volatility are empirically closer in individual option markets.

## II The Basic Model

The theoretical framework is a variant of that adopted in Bakshi and Kapadia (2003). To fix convention, we denote the stock price and variance of firm  $i$  as  $S_i(t)$  and  $V_i(t)$  respectively, and the market index variance as  $V_m(t)$ . Under the physical probability measure assume that:

$$\frac{dS_i(t)}{S_i(t)} = \mu_i[S_i, V_i] dt + \sqrt{V_i(t)} dW^1(t), \quad (1)$$

$$V_i(t) = \beta_i V_m(t) + Z_i(t), \quad \beta_i > 0, \quad (2)$$

and

$$dV_m(t) = \theta[V_m] dt + \eta[V_m] dW^2(t). \quad (3)$$

To explain stock price dynamics assumptions outlined in (1)-(3), we first note that equation (2) implies a single-factor model of individual stock variance. Equation (2) stems from the assumption that stock returns have a market component and an idiosyncratic component, as in  $R_i(t) = \hat{\alpha}_i + \hat{\beta}_i R_m(t) + \hat{\varepsilon}_i(t)$ . Specifically, if the idiosyncratic stock variance  $Z_i(t)$  is uncorrelated with market index variance then  $\beta_i \equiv \hat{\beta}_i^2$  is the sensitivity of individual variance with respect to the variance of the market index. That is, we are essentially staying within the Black-Scholes framework, with a modification allowing for stochastic volatility in the individual stock price process (1). See Bakshi and Kapadia (2003) for a framework that relates the losses on delta-hedged portfolio to return-jumps. Given the low negative risk-neutral skewness found in individual equity options, Merton (1976) type return-jumps are omitted to maintain focus on the volatility risk premium.

Equation (3) generically specifies the market variance as a one-dimensional diffusion with drift and diffusion coefficients given by  $\theta[V_m]$  and  $\eta[V_m]$ . While certain choices for  $\theta[V_m]$  and  $\eta[V_m]$  can lead to empirically unappealing variance dynamics (i.e., arithmetic and non-mean-reverting), we nonetheless keep the functional form of  $\theta[V_m]$  and  $\eta[V_m]$  unspecified with the understanding that  $\theta[V_m]$  and  $\eta[V_m]$  imply a well-specified variance process. Suppose  $\theta[V_m] \equiv \theta - \kappa V_m$  and  $\eta[V_m] \equiv \eta\sqrt{V_m(t)}$ , then it admits mean-reversion in market return variance and a stationary distribution for both  $V_m(t)$  and  $V_i(t)$  (Heston (1993)). Let  $\rho_i$  be the correlation between the standard Brownian motions  $W^1(t)$  and  $W^2(t)$ .

For ease of analysis, assume that idiosyncratic return variance,  $Z_i(t)$ , is a constant for all  $t$  so that we can set  $dZ_i(t) = 0$  in  $dV_i(t) = \beta dV_m(t) + dZ_i(t)$ . With constant  $Z_i$ , individual return variance obeys a one-dimensional diffusion. However, as shown below, our key results will be unaffected with stochastic  $Z_i(t)$  provided  $Z_i(t)$  is unpriced (i.e., uncorrelated with the pricing kernel). In a later empirical exercise we investigate and reject that idiosyncratic return volatility is priced.

Denote  $C_i(t; K, \tau)$  as the call option on the individual stock with strike  $K$  and maturity  $\tau$ . Then by Ito's lemma,

$$C_i(t + \tau) = C_i(t) + \int_t^{t+\tau} \Delta_i(u) dS_i(u) + \int_t^{t+\tau} \frac{\partial C_i}{\partial V_i} dV_i(u) + \int_t^{t+\tau} b_i(u) du, \quad (4)$$

where

$$b_i(u) \equiv \frac{\partial C_i}{\partial u} + \frac{1}{2} V_i S_i^2 \frac{\partial^2 C_i}{\partial S_i^2} + \frac{1}{2} \beta_i^2 \eta[V_m]^2 \frac{\partial^2 C_i}{\partial V_i^2} + \rho \beta_i \eta[V_m] \sqrt{V_i} S_i \frac{\partial^2 C_i}{\partial S_i \partial V_m}, \quad (5)$$

and

$$\Delta_i \equiv \partial C_i / \partial S_i, \quad (6)$$

is the call option delta. In the empirical implementation we treat the Black-Scholes delta as a close enough approximation to the true delta that it can be used as a proxy.

The valuation equation that determines the price of the call option is:

$$\begin{aligned} & \frac{1}{2} V_i S_i^2 \frac{\partial^2 C_i}{\partial S_i^2} + \frac{1}{2} \eta[V_m]^2 \beta_i^2 \frac{\partial^2 C_i}{\partial V_i^2} + \rho \beta_i \eta[V_m] \sqrt{V_i} S_i \frac{\partial^2 C_i}{\partial S_i \partial V_i} + r S_i \frac{\partial C_i}{\partial S_i} \\ & + \beta_i (\theta[V_m] - \lambda[V_m]) \frac{\partial C_i}{\partial V_i} + \frac{\partial C_i}{\partial t} - r C_i = 0, \end{aligned} \quad (7)$$

where

$$\lambda[V_m] \equiv -\text{Cov}_t \left( \frac{dm(t)}{m(t)}, dV_m(t) \right) \quad (8)$$

represents the price of volatility risk for a pricing kernel process  $m_t$  and  $\text{Cov}_t(\cdot, \cdot)$  is a conditional covariance operator divided by  $dt$ . Note that if volatility is nonstochastic, as in the basic Black-Scholes, volatility risk is zero and it does not matter whether volatility is high or low as the resulting delta-hedge is riskless in theory. Allowing volatility to be stochastic exposes the investor to random variability in volatility through its covariance with changes in the pricing kernel as made precise in equation (8).

If the market volatility risk premium is non-zero, as empirically shown by Bakshi and Kapadia (2003) and  $\beta_i > 0$ , the individual volatility risk premium will share the same sign as the market volatility risk premium. This characterization holds if we had defined volatility risk as:  $-\text{Cov}_t \left( \frac{dm(t)}{m(t)}, d\sqrt{V_m(t)} \right)$  which, by Ito's Lemma, is the same as:  $-\frac{1}{2\sqrt{V_m}} \text{Cov}_t \left( \frac{dm(t)}{m(t)}, dV_m(t) \right)$ . In particular, if  $\lambda[V_m]$  is assumed proportional in  $V_m$  then  $\lambda[\sqrt{V_m}]$  is proportional to  $\sqrt{V_m}$ . Thus, in what follows, we adopt the convention that volatility risk is as specified in (8) and that volatility and variance are interchangeable.

Combining (7) and substituting out  $b_i(u)$  in the stochastic differential equation (4), we

obtain

$$\begin{aligned}
C_i(t + \tau) - C_i(t) &= \int_t^{t+\tau} \frac{\partial C_i}{\partial S_i} dS_i + \int_t^{t+\tau} r \left( C_i - \frac{\partial C_i}{\partial S_i} S_i \right) du + \int_t^{t+\tau} \beta_i \lambda[V_m] \frac{\partial C_i}{\partial V_i} du \\
&+ \int_t^{t+\tau} \beta_i \eta[V_m] \frac{\partial C_i}{\partial V_i} dW^2.
\end{aligned} \tag{9}$$

Define the delta-hedged gains,  $\Pi_i(t, t + \tau)$ , as the gain or loss on a delta-hedged option position (where the net investment earns the risk-free rate):

$$\Pi_i(t, t + \tau) \equiv C_i(t + \tau) - C_i(t) - \int_t^{t+\tau} \Delta_i dS_i - \int_t^{t+\tau} r (C_i - \Delta_i S_i) du. \tag{10}$$

Then, from (9) and (10), we can write the delta-hedged gains as:

$$\Pi_i(t, t + \tau) = \int_t^{t+\tau} \beta_i \lambda[V_m] \frac{\partial C_i}{\partial V_i} du + \int_t^{t+\tau} \beta_i \eta[V_m] \frac{\partial C_i}{\partial V_i} dW^2, \tag{11}$$

and from the martingale property of the Ito integral

$$\mathbb{E}_t(\Pi_i(t, t + \tau)) = \int_t^{t+\tau} \mathbb{E}_t \left( \beta_i \times \lambda[V_m] \times \frac{\partial C_i}{\partial V_i} \right) du, \tag{12}$$

where  $\mathbb{E}_t(\cdot)$  is the expectation operator under the physical probability measure.

The testable implication of (12) is that the magnitude and the sign of the individual equity delta-hedged gains are related to the sign and magnitude of  $\lambda[V_m]$ . In particular, as  $\lambda[\cdot]$  is some function of market variance,  $V_m$ , it follows that, if market volatility risk is priced, then the delta-hedged gains will be related to the level of market variance. In other words, we can infer the impact of the market volatility risk premium by the relation between individual equity delta-hedged gains and market variance.

Equation (12) also holds if we relax the assumption that idiosyncratic return volatility is constant. To allow for stochastic idiosyncratic volatility, let

$$dZ_i(t) = \theta_i^z [Z_i] dt + \eta_i^z [Z_i] dW^3(t), \tag{13}$$

then under the assumption that standard Brownian motion  $W^3(t)$  is independent of all

sources of stochastic variation, we can derive, as before, the delta-hedged gains as:

$$\Pi_i(t, t + \tau) = \int_t^{t+\tau} \beta_i \lambda[V_m] \frac{\partial C_i}{\partial V_i} du + \int_t^{t+\tau} \beta_i \eta[V_m] \frac{\partial C_i}{\partial V_i} dW^2 + \int_t^{t+\tau} \eta_i^z[Z_i] \frac{\partial C_i}{\partial V_i} dW^3. \quad (14)$$

Because idiosyncratic volatility is not priced, equation (12) still applies; stochastic idiosyncratic volatility simply adds noise to the delta-hedged gains. Moreover, this fact provides an additional test of whether market volatility risk is priced: Mean delta-hedged gains must be correlated with market volatility and *not* with firm-specific idiosyncratic volatility. The analysis of this paper can be extended in a straightforward manner to allow for multi-factor models of individual return volatility.

In general, the relation between the delta-hedged gains and market volatility in (12) may be of any functional form. However, for the square root process of Heston (1993), this relation is strikingly simple. As before, to fix ideas, let  $\theta[V_m] \equiv \theta - \kappa V_m$ ,  $\eta[V_m] \equiv \eta \sqrt{V_m(t)}$  and  $\lambda[V_m] \equiv \lambda V_m(t)$ , then Bakshi and Kapadia (2003) show, for at-the-money options, the *scaled* delta-hedged gains, defined as  $\Pi_i(t, t + \tau)/S_i(t)$ , is linearly related to the level of the market volatility (see their proposition 2). The scaling of the delta-hedged gains by the price of the underlying asset ensures that we can compare delta-hedged gains over time.

The linearity of the scaled delta-hedged implies that a linear regression can be used to test the implications of the market volatility risk premium. Thus, there exists a relatively straightforward test that allows us to answer the two questions of importance: is market volatility risk priced, and, if so, what is the sign and strength of the risk premium? To obtain additional insights, we now turn to our sample of individual equity options.

### III Description of Individual Equity Options

Obtained from Berkeley Options Database, our empirical tests use bid-ask call option quotes on 25 individual stocks and the S&P 500 index. These options are traded on the Chicago Board Options Exchange and have American-style expirations. For each of the 1258 days in the sample period of 1/1/91 through 12/31/95, we retain the last quote prior to 3:00 PM (CST). The tickers and names of the individual stock options are displayed in the first two columns of Table 1. This sample includes the largest stocks as their options are likely to be more liquid.

Several filters are employed to construct the call sample. First, we screen the data to

eliminate bid-ask option pairs with missing quotes or zero bids. Second, we remove option prices violating arbitrage restrictions  $C(t, \tau; K) < S(t)$  or  $C(t, \tau; K) > S(t) - \text{PVD}[D] - \text{PVD}[K]$ , for present value function  $\text{PVD}[\cdot]$  and dividends  $D$ . Third, options with remaining days to expiration less than 14 days and greater than 30 days are not considered. Finally, we use close to the money options within a moneyness range of  $-2.50\% \leq y(t) - 1 \leq 2.50\%$  where  $y(t) \equiv S(t)e^{r\tau}/K$  is option moneyness.

Firm specific dividends are obtained from CRSP and assumed known. The source of S&P 500 dividends is the S&P Information Bulletin. Following convention, the current stock price is adjusted by subtracting the present value of dividends. As in Bakshi and Kapadia (2003), the interest rate is interpolated to match option maturity using the overnight and 3-month Eurodollar interest rates.

For our calculations involving realized volatility, we use a measure of sample standard deviation which is computed as:

$$\text{VOL}_{t-\tau, t} = \sqrt{\frac{252}{\tau} \sum_{n=t-\tau}^t R_{n-1, n}^2}, \quad (15)$$

where  $\tau$  is set to 30/360 for monthly volatility. We do not subtract the sample mean return as this estimate of expected return can be unreasonable. As empirical results using GARCH estimation are essentially similar (see the robustness exercises in Bakshi and Kapadia (2003)), the resulting duplication is avoided to save on space. The measure of standard deviation in (15) is convenient as the rolling procedure produces estimates where the estimation error is serially uncorrelated through time for non-overlapping periods.

## IV Implied Volatilities and Realized Volatilities for Individual Equities

To examine whether volatility risk is priced in individual equities we first investigate the relation between implied and realized volatilities. Note that, in the context of index options, Jackwerth and Rubinstein (1996) document that Black-Scholes implied volatilities for at-the-money options are, on average, greater than realized volatilities. This empirical finding has proven difficult to reconcile by just relaxing the constant volatility assumption as, for example, allowing for stochastic volatility or jumps in the stock return dynamics (e.g.,

Bakshi, Cao and Chen (1997) and Bates (2000)). Recent research in Bakshi and Kapadia (2003) and Buraschi and Jackwerth (2001) suggests one possible answer - that market volatility risk is priced in equity options.

Specifically, Bakshi and Kapadia (2003) reason that option prices incorporate a negative volatility risk premium. Their empirical tests are motivated by the finding that shocks to market volatility are negatively correlated with market returns (e.g., French, Schwert and Stambaugh (1987)). The negative correlation implies that market volatility increases when the market return is negative. In this case, including options to a portfolio will help hedge market risk as the option vega is positive. Due to this hedging motive, investors are willing to pay a risk premium for a long option position implying a negative volatility risk premium. The effect of a negative volatility risk premium is to increase the option price, resulting in implied volatilities that are higher than realized volatility. More precisely, the drift of the risk-neutral volatility process will exceed the counterpart under the physical probability measure. Because individual volatilities are generally positively correlated with market volatility (see, equation (2)), we might expect the individual implied volatilities to similarly exceed realized volatilities.

Panel A of Table 1 reports the Black-Scholes implied volatilities and the realized volatilities for each ticker. Consistent with our prior of priced volatility risk, it can be observed that the implied volatilities of individual equity options tend to be higher than the realized counterparts. For example, the average annualized implied volatility for GE is 18.8% as compared with an annualized historical volatility of 17.4% realized over the remaining lifetime of the option. This conclusion is robust even if return volatility is measured using returns over the past 30 calendar days.

However, for every option in our sample, this difference between implied and historical volatilities is *less* than that for S&P 500 index options. For instance, the average difference between implied and realized volatility for SPX calls is 3.3% (on an annualized basis), while the average across the 25 stocks in our sample is only 1.5%. Given that the options on individual stocks are priced at a higher level of volatility, this difference between the implied and the realized has a price impact that is considerably smaller for individual equity options (compared to the market index).

One possible concern with our results is that individual equity options are American-style while the SPX option is European. To assess the impact of early exercise on our results,

we make use of the fact that if there are no dividends paid in the remaining maturity of the call, the early exercise of American call will not be optimal. Guided by this result, we also compare implied volatilities of options that have a dividend in the remaining maturity to the implied volatility when there is no dividend during the remaining maturity. This exercise shows that the early exercise premium is equivalent to about 2% volatility points for our sample of short-term near money calls.

To eliminate the impact of early exercise on our results in the empirical work that follows, we eliminate all call observations where the stock pays a cash dividend over the remaining lifetime of the contract. Panel B of Table 1 reports the results of the option sample omitting dividends. Because of quarterly dividend payouts about 25% of the original individual equity option sample is eliminated. The average difference between the Black-Scholes implied and realized volatility across the 25 firms is now 1.07%, and lower than the corresponding estimate in Panel A of Table 1. However, we note that for 23 out of 25 firms, the average implied volatility still exceeds realized volatility. Moreover, this finding is also robust when the return volatility is estimated using the prior 30 days stock returns.

Overall, two chief conclusions can be drawn based on Table 1. First, the pricing of individual equity options and the index options are consistent with the notion that implied volatilities are, on average, higher than realized volatilities. This suggests that volatility risk is negatively priced in both individual and index option markets. Second, the difference between implied and realized volatilities is far more pronounced for index options than for individual equity options. It appears that individual equity option buyers, in general, leave money on the table but do so more for index options. Below, we examine whether the hypothesis of a market volatility risk premium can help explain these findings.

## V Insights from Individual Equity Options

To examine the implications of a priced market volatility risk factor, we first construct delta-hedged gains for every option in our sample. The option position consists of a long call bought at time  $t$  and hedged discretely until expiration,  $t + \tau$ . Total delta-hedged gains for each option up to the maturity date is calculated as:

$$\Pi(t, t + \tau) = \max(S(t + \tau) - K, 0) - C(t) - \sum_{n=0}^{N-1} \Delta(t_n)(S(t_{n+1}) - S(t_n))$$

$$- \sum_{n=0}^{N-1} r (C(t) - \Delta(t_n)S(t_n)) \frac{\tau}{N} \quad (16)$$

where  $t_0 = t$ ,  $t_N = t + \tau$  is the maturity date, and  $\Delta(t_n)$  is the hedge ratio at  $t_n$  (re-computed daily). For tractability,  $\Delta_{t_n}$  is computed as the Black-Scholes hedge ratio,  $\Delta_{t_n} = \mathcal{N}[d_1(S_{t_n}, t_n)]$ , where  $\mathcal{N}[\cdot]$  is the cumulative normal distribution, and  $d_1 \equiv \frac{1}{\text{VOL}_{t,t+\tau} \sqrt{\tau_n}} \log(y_n) + \frac{1}{2} \text{VOL}_{t,t+\tau} \sqrt{\tau_n}$ .

Bakshi and Kapadia (2003) note that the use of Black-Scholes hedge ratio as an approximation to the true hedge ratio does not significantly alter the conclusions and, furthermore, the results are robust to alternative estimates of return volatility in  $\Delta(t_n)$ .

Table 2 reports the magnitude of delta-hedged gains for 25 individual stocks and the SPX. As discussed earlier, we exclude equity option observations on dates where the firm paid a dividend during the remaining lifetime of the option. To make the delta-hedged gains comparable across the time-series and the cross-section, we prefer the normalized delta-hedged gains as  $\Pi(t, \tau)/S_t$  and  $\Pi(t, \tau)/C_t$ .

Consider the results on delta-hedged gains for index options. The delta-hedging strategy for the SPX loses money. On average, SPX calls lose 0.07% of the value of the index. In terms of the value of the option, the average loss is -3.31%. This loss is both statistically and economically significant; given the traded volume of index options, the dollar loss amounts to several 100 million dollars over the time period.

Why should buyers of options be willing to leave money on the table? As emphasized in the previous section, negative delta-hedged gains are consistent with a negative market volatility risk premium  $\lambda[V_m] < 0$ . A negative volatility risk premium increases the option price in comparison with its price when  $\lambda[V_m] \equiv 0$ . Because of the negative correlation between market index returns and market index volatility, buyers of options may be willing to pay a premium because a long position in volatility helps hedge market-wide risk.

Before we present the empirical results for the individual equity options, it is important to realize that the negative delta-hedged gains for the index option do not necessarily translate into negative delta-hedged gains for the individual equity option. Although we would expect a negative market volatility risk premium to bias the delta-hedged gains for individual equity options to be negative, the market volatility is merely one component of the firm's total volatility. The impact of the market volatility risk premium on the individual firm would depend on both the relation between the firm's total volatility and

the market volatility, as well as on whether non-market components of volatility are priced. The importance of the market volatility in determining the distribution of delta-hedged gains for individual equity options can only be determined empirically, and is the focus of interest of the discussion that follows.

From Table 2, it can be observed that the majority of the stocks have negative delta-hedged gains. For example, the average delta-hedged gain is negative for 14 of the 25 individual equity options, and 7 of the firms have their gains significantly less than zero at the 99% level (standard errors are not reported to conserve space but are available upon request). In contrast, only 4 firms have gains that are significantly positive. Our results remain robust when the median is used as a measure of central tendency.

To eliminate biases that may be caused by a few outliers, we also examine the relative outcomes of negative and positive gains across the firms. The last column of Table 2 displays the  $1_{\Pi < 0}$  statistic, which measures the frequency of negative outcomes. Of the 25 individual stocks, 20 have negative delta-hedged gains more than 50% of the times. On average, over the 25 stocks in our sample, the delta-hedging strategy has a loss of -0.03% of the underlying asset value. Although the delta-hedging trading strategy loses money for both the index and individual equity options, the loss, on average, for individual equity options is far less than that for the index. Overall, the evidence from individual firms is consistent with that observed for the index option in that, on average, the delta-hedged gains are negative.

Are the observed delta-hedged gains for individual equity options consistent with a market volatility risk premium? As discussed previously, for the negative delta-hedged gains to be consistent with a negative market volatility risk premium, they must be negatively correlated to the level of market volatility and unrelated to idiosyncratic return volatility. We empirically investigate these relations next.

## VI Delta-Hedged Gains, Market Volatility, and Idiosyncratic Volatility

An important theoretical implication of equation (12) is that the magnitude of the delta-hedged gains depend on the level of market volatility. Bakshi and Kapadia (2003) show in their Proposition 2 that, under mild assumptions, the relation between the scaled delta-hedged gains,  $\Pi_i(t, t + \tau)/S_i(t)$ , for near-money options is linear in  $\sqrt{V_m(t)}$ . Thus, we

can test for the relation between market volatility and delta-hedged gains through a linear regression framework.

For this test we choose the closest to the money option (within a  $\pm 2.5\%$  moneyness range) of maturity of exactly 30 days. Once again, options that have had a dividend paid over its maturity are eliminated from the sample. The number of observations for each ticker ranges from 19 to 40 with a total of 610 observations. The market volatility is estimated based on returns realized over the previous 30 calendar days.

We investigate the implication of a market volatility risk premium via two tests. First, we average the delta-hedged gains for every month over the 25 stocks in our sample, and then regress it on market volatility (letting  $\text{VOL}_m(t) \equiv \text{VOL}_{t-30,t}$ ).

$$\overline{\text{GAINS}}(t) = \Omega_0 + \Omega_1 \text{VOL}_m(t) + \epsilon_t \quad (17)$$

where  $\overline{\text{GAINS}}(t) = \frac{1}{25} \sum_{i=1}^{25} \text{GAINS}_i(t)$ . As firms have different dividend payment cycles, averaging across all firms in any period allows us to construct a time-series of observations that is complete over the 60 months of our data period. Although this test does not allow us to discern firm-specific components of the delta-hedged gains, it does allow us to test whether, on average, market volatility risk is priced in individual equity options.

Tables 3 reports the regression results for the specification in (17). We observe that the average delta-hedged gain is, in fact, negatively correlated with the market volatility. The estimate of  $\Omega_1$  is  $-0.018$  and is statistically significant. For comparison, we also report the regression results when the dependent variable is the delta-hedged gain for the SPX option,

$$\text{GAINS}_m(t) = \Omega_0 + \Omega_1 \text{VOL}_m(t) + \epsilon_t^* \quad (18)$$

The estimate of the slope coefficient is  $-0.03$ , and again statistically significant. The negative sign of the slope coefficient in both regressions is consistent with the existence of a negative market volatility risk premium.

Continuing, an alternative econometric specification could allow for both a firm-specific component and a market component in the delta-hedged gains. Recall from the theoretical discussion that the idiosyncratic component of the firm's total volatility will add noise to the delta-hedged gains. As this noise is firm-specific, the appropriate econometric specification

is a *random effects* panel regression. Consider the following regression framework,

$$\text{GAINS}_i(t) = \Omega_0 + \Omega_1 \text{VOL}_m(t) + \epsilon_{i,t} \quad (19)$$

$$\epsilon_{i,t} = e_{i,t} + u_i \quad (20)$$

The noise term  $u_i$  now captures the contribution of the idiosyncratic component of the volatility for firm  $i$ . The regression model is estimated by feasible generalized least squares, and results are presented in Table 4. The estimate of  $\Omega_1$  is  $-0.0263$  and it is statistically significant with a p-value of 0.01 (z-statistic of -2.55). These results confirm that individual equity options price a negative market volatility risk premium.

As discussed earlier, a stronger implication of a market volatility risk premium is that the idiosyncratic component of the firm's volatility, if not priced, must not be related to delta-hedged gains. To examine this hypothesis, we extend the specification of equation (19)-(20) by including the firm's total volatility as an explanatory variable,

$$\text{GAINS}_i(t) = \Omega_0 + \Omega_1 \text{VOL}_m(t) + \Omega_2 \text{VOL}_i(t) + \epsilon_{i,t}, \quad (21)$$

$$\epsilon_{i,t} = e_{i,t} + u_i \quad (22)$$

where  $\text{VOL}_i(t)$  is the  $i$ 'th firm's return volatility measured over the previous 30 calendar days. If both market and idiosyncratic volatility are priced, then in the augmented regression,  $\Omega_1$  should be insignificant and  $\Omega_2$  significant. On the other hand, if only market volatility is priced, then we should expect contrary results. Estimating the regression, we find that  $\Omega_1 = -0.0233$  with a p-value of 0.03 (z-statistic of -2.15), and  $\Omega_2 = -0.0042$  with a p-value of 0.39 (z-statistic of -0.86). Market volatility is significantly correlated with the firm's delta-hedged gains but not the firm's total volatility. The firm's total volatility is not significant even when the market volatility is omitted from the regression. Thus, the results are consistent with a volatility risk premium where only market volatility risk is priced and idiosyncratic return volatility is unpriced.

What is the economic implication of the estimated coefficient  $\Omega_1$ ? Consider, as an illustration, the call option on AIG. On 4/21/1993, the S&P 500 historical return was 10%. Given the estimate of  $\Omega_1 = -0.0263$ , the impact of the market volatility risk premium is -0.263% of the stock price level. With the AIG stock closing at 125.625, the impact of the market volatility risk premium on the AIG near-money call is \$0.33. In comparison, the

125 strike call was priced at \$3.82, implying a volatility risk premium of 8.65% as a fraction of the call price. Now consider SPX options. The estimate of  $\Omega_1$  is -0.03 and the index level on 4/21/1993 was 444.69. The impact of the volatility risk premium is \$1.33; With 440 strike call priced at \$8.25, the magnitude of the estimate risk premium is about 16% of the call price. Clearly, the impact of the market volatility risk premium is much higher for index options than for the average stock.

In summary, the results indicate that market volatility is negatively priced in individual equity options. However, the additional insight is that the effect of the market volatility risk premium is of a smaller magnitude and idiosyncratic return volatility is unpriced. These results have an important economic implication. If, as has been argued, that a negative market volatility risk premium results in implied volatilities being greater than return volatilities, then these results provide an explanation of why (as observed in our Table 1) this bias is greater for the index options as compared with individual equity options. Buyers of individual equity options lose less money because the impact of market volatility risk premium is smaller.

## VII Discussion and Conclusions

In this paper, we examine whether, and to what extent, market volatility risk is priced in individual equity options. If market volatility risk is priced, then gains on a delta-hedged option portfolio (long call and short stock) should be negatively correlated with market volatility. A negative volatility risk premium increases the price of the option relative to its value absent the volatility risk premium. The negative volatility risk premium arises because options serve as a hedge, and is consistent with an inverse correlation between volatility and stock returns.

Our work provides the following empirical insights. First, implied volatilities of individual equity options are higher than realized volatilities. However, the magnitude of this bias is substantially smaller compared with that of the market index. Second, we investigate the behavior of delta-hedged gains and show that average delta-hedged gains of individual stocks tend to be negative and are negatively correlated with market volatility. Our results support the hypothesis that the risk premium for market volatility is negative in individual equity markets. Third, the magnitude of the volatility risk premium indicates

a much smaller role for the risk premium for individual stocks as opposed to the market index. The impact of market volatility risk is smaller on the losses on delta-hedged gains and idiosyncratic volatility risk is not priced. Our empirical results provide a perspective on why the bias between implied and realized volatilities is smaller for individual equity options. Overall, our findings are consistent with investor risk aversion with respect to a market component of risk.

Why is the impact of the market volatility risk premium smaller on average on individual equity options than on the index option? First, due to the presence of an idiosyncratic volatility component, changes in market volatility may result in a smaller impact on the individual firm's total volatility. In our dataset, we find some evidence to support this hypothesis. For instance, when changes in individual volatility are regressed on changes in market volatility, we find that the sensitivity coefficient is generally smaller than unity. For this reason, the effect of the volatility risk premium is commensurately smaller on individual equity options. Second, the index option may be more sensitive to market volatility, because market volatility may also affect the pricing of other risks. In particular, Pan (2002) shows that market volatility affects the intensity of jumps and thus the pricing of jump risk. Given the low negative risk-neutral skewness in individual equities (Bakshi, Kapadia and Madan (2003)), jump risk appears to be a less important consideration for individual equities.

Although, in this paper, we focus exclusively on the effect of a market volatility risk premium in the time-series, much remains to be learned of its effect across a cross-section of individual stock options. In particular, our conclusions regarding the existence of a market volatility risk premium suggests that some of the cross-sectional variation in the pricing of individual equity options can be explained by the sensitivity of the individual firm's volatility to market volatility. The question whether the market price of volatility risk is the same across stocks requires estimating individual volatility sensitivities is quite involved so we leave it to a follow up project. If the ongoing research on cross-sectional differences across stock prices and returns is a guide, the understanding of cross-sectional differences across stock options will likely enhance our understanding of how risk-averse investors price derivative assets.

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**Table 1: Implied Volatility versus Realized Volatility**

The table reports (i) Black-Scholes implied volatility for near-money individual equity calls (denoted IVOL), (ii) realized volatility (denoted  $VOL_{t,t+\tau}$ ) and (iii) prior 30-day realized volatility (denoted  $VOL_{t-30,t}$ ). The implied volatility is computed by equating the market option price to the Black-Scholes model price. The sample period is January 1991 to December 1995. In the implied volatility calculation, dividends are assumed known, discounted and subtracted from the stock price. All options have remaining days to maturity of 15-30 days. Near-money calls are defined to have a moneyness between  $-2.50\% \leq y_i(t) - 1 \leq 2.50\%$  where  $y_i(t) \equiv \hat{S}_i(t)e^{r\tau}/K_i$  and  $K_i$  is the strike of the option. Panel A reported the results using all call options while Panel B reports the results omitting options if the stock pays a cash dividend. OBS is the number of observations.

Ticker	Firm	Panel A: All Options				Panel B: Omitting Dividends			
		OBS	IVOL	$VOL_{t,t+\tau}$	$VOL_{t-30,t}$	OBS	IVOL	$VOL_{t,t+\tau}$	$VOL_{t-30,t}$
AIG	American Int'l	668	21.5	18.6	19.6	513	21.4	18.9	19.2
AIT	Ameritech	431	18.4	18.3	18.3	379	18.2	18.0	18.5
AN	Amoco	354	17.8	17.8	18.2	231	17.2	18.0	17.8
AXP	American Exp.	323	27.8	25.8	26.2	237	27.8	27.0	26.8
BA	Boeing	353	23.8	21.5	21.7	238	23.0	21.6	21.3
BAC	BankAmerica	307	26.9	24.1	24.9	224	26.1	24.1	23.8
BMY	Bristol Myers	452	19.7	17.8	19.0	349	19.6	17.6	19.2
CCI	Citigroup	244	29.3	28.8	30.9	215	29.2	29.4	31.0
DD	Du Pont	342	22.7	21.3	20.6	215	21.7	20.0	19.5
DIS	Walt Disney	499	26.2	23.1	24.1	327	26.5	22.8	25.4
F	Ford Motor	284	28.2	27.7	28.3	261	27.9	27.9	28.2
GE	General Electric	514	18.8	17.4	17.3	420	18.5	17.4	17.1
GM	General Motors	279	28.7	28.3	27.9	170	28.1	27.6	27.7
IBM	Int. Bus. Mach.	529	25.9	23.4	24.0	365	25.4	24.1	23.4
JNJ	J & J	476	22.9	22.1	22.6	315	22.6	21.7	21.9
KO	Coca Cola	374	22.0	20.4	20.4	279	22.3	20.9	20.5
MCD	MacDonald's	329	21.5	20.7	21.6	278	21.4	21.0	21.3
MMM	Minn. Mining	603	19.4	17.2	17.4	451	18.5	17.3	17.1
MOB	Mobil	546	17.5	16.7	16.7	407	17.1	16.8	17.0
MRK	Merck	483	23.9	21.5	22.1	365	23.9	22.5	21.9
PEP	Pepsico	261	21.5	20.6	21.7	172	21.6	21.5	21.1
SLB	Schlumberger	396	23.7	23.3	23.9	317	23.6	23.4	23.8
T	AT&T	298	18.5	17.9	18.5	255	18.3	17.7	18.4
WMT	Walmart	276	25.3	22.9	23.3	208	25.5	23.6	23.1
XRX	Xerox	572	24.3	21.3	22.3	440	24.2	22.0	21.6
SPX	S&P 500	2990	12.8	9.5	9.5	-	-	-	-

**Table 2: Magnitude of Delta-Hedged Gains**

We compute the gains on a portfolio of a long call position, hedged by a short position in the underlying stock, with the net investment earning the riskfree rate:

$$\Pi(t, t+\tau) = \max(S(t+\tau) - K, 0) - C(t) - \sum_{n=0}^{N-1} \Delta(t_n)(S(t_{n+1}) - S(t_n)) - \sum_{n=0}^{N-1} r(C(t) - \Delta(t_n)S(t_n)) \frac{\tau}{N}.$$

The hedge is re-balanced daily ( $\tau/N = 1$ ), and the option delta,  $\Delta_{t_n}$ , is computed as the Black-Scholes hedge ratio, evaluated at the volatility realized over the lifetime of the option. We report the delta-hedged gains normalized by the stock (i.e.,  $\Pi(t, t + \tau)/S_t$ ) and the option price (i.e.,  $\Pi(t, t + \tau)/C_t$ ), respectively.  $1_{\Pi < 0}$  is the proportion of  $\Pi$  that is less than zero. The sample consists of calls of maturity 15-30 days over the period January 1991 to December 1995. All calls are near-money,  $-2.50\% \leq y_i(t) - 1 \leq 2.50\%$  where  $y_i(t) = S_i(t)e^{r\tau}/K_i$  and  $K_i$  is the strike prices of stock  $i$  at  $t$ . Individual equity calls are excluded if the underlying stock pays a dividend within the remaining maturity of the option.

Ticker	Firm	OBS	Magnitude of $\Pi(t, t + \tau)/S(t)$		Magnitude of $\Pi(t, t + \tau)/C(t)$		$1_{\Pi < 0}$ (%)
			Average	Median	Average	Median	
AIG	American Int'l	565	-0.20	-0.30	-9.29	-14.99	67.08
AIT	Ameritech	409	0.04	-0.11	3.21	-6.33	57.70
AN	Amoco	256	0.12	0.06	8.65	3.71	44.14
AXP	American Exp.	255	0.05	0.01	0.78	0.41	48.63
BA	Boeing	255	-0.04	-0.16	1.66	-7.97	57.65
BAC	BankAmerica	242	-0.17	-0.17	-11.01	-9.21	64.46
BMJ	Bristol Myers	383	-0.12	-0.23	-5.50	-12.23	67.10
CCI	Citigroup	235	0.03	-0.06	2.27	-2.65	51.49
DD	Du Pont	228	-0.14	-0.21	-5.17	-10.06	63.16
DIS	Walt Disney	358	-0.26	-0.36	-9.48	-13.03	68.99
F	Ford Motor	286	0.22	0.07	10.44	2.49	46.85
GE	General Electric	460	-0.05	-0.08	-1.20	-3.68	54.57
GM	General Motors	185	-0.08	-0.15	-1.30	-5.36	56.22
IBM	Int. Bus. Mach.	396	-0.05	-0.19	-0.59	-8.92	63.64
JNJ	J & J	342	0.06	-0.09	1.27	-4.05	55.26
KO	Coca Cola	304	-0.11	-0.11	-1.31	-4.46	59.87
MCD	MacDonald's	303	0.10	0.06	4.58	3.87	44.88
MMM	Minn. Mining	488	0.05	-0.11	4.65	-7.28	60.25
MOB	Mobil	446	0.01	-0.00	0.44	-0.15	50.45
MRK	Merck	397	-0.13	-0.22	-4.26	-9.15	66.50
PEP	Pepsico	187	0.11	-0.02	5.62	-0.78	50.80
SLB	Schlumberger	345	0.11	0.00	11.76	0.12	49.57
T	AT&T	281	-0.01	-0.06	-2.16	-4.20	55.52
WMT	Walmart	224	-0.05	-0.21	-0.16	-9.90	59.38
XRX	Xerox	490	-0.05	-0.21	-2.68	-10.96	61.02
SPX	S&P 500	2990	-0.07	-0.10	-3.31	-5.98	65.79

**Table 3: Time Series Relationship between Delta-Hedged Gains and Volatility**

The table reports the results of the OLS regressions,

$$\overline{\text{GAINS}}(t) = \Omega_0 + \Omega_1 \text{VOL}_m(t) + \epsilon_t$$

and

$$\text{GAINS}_m(t) = \Omega_0 + \Omega_1 \text{VOL}_m(t) + \epsilon_t^*$$

where  $\overline{\text{GAINS}}(t) \equiv \frac{1}{25} \sum_{i=1}^{25} [\Pi_i(t, t + \tau) / S_i(t)]$  is the average delta-hedged gains for month  $t$  over the 25 individual stocks in the sample;  $\text{GAINS}_m(t)$  is the month  $t$  delta-hedged gains for the SPX; and  $\text{VOL}_m(t)$  represents the prior month return volatility for the S&P 500. We report the coefficient estimates, the t-statistics (in square brackets), and the adjusted- $R^2$  statistics. Standard errors are adjusted for heteroskedasticity using White's estimator.  $T$  is the number of observations used in the time-series regression. The sample employed in the test consists of closest to the money calls with moneyness in the range of  $\pm 2.50\%$ . Calls have maturity of 30 days, and are sampled monthly over the period of January 1991 to December 1995.

Dependent Variable	$T$	$\Omega_0$ $\times 10^{-2}$	$\Omega_1$	$R^2$ (in %)
$\overline{\text{GAINS}}$	60	0.11 [1.49]	-0.018 [-2.18]	3.51
$\text{GAINS}_m$	59	0.14 [1.32]	-0.030 [-2.92]	8.43

**Table 4: Cross-sectional and Time-Series (Panel) Regression**

The table reports the results of the feasible generalized least squares estimation of a random effects panel regression,

$$\begin{aligned} \text{GAINS}_i(t) &= \Omega_0 + \Omega_1 \text{VOL}_m(t) + \Omega_2 \text{VOL}_i(t) + \epsilon_{i,t}, & \text{and,} \\ \epsilon_{i,t} &= e_{i,t} + u_i, \end{aligned}$$

where  $\text{GAINS}_i(t) \equiv \Pi_i(t, t + \tau)/S_i(t)$ .  $\text{VOL}_m(t)$  represents the prior month return volatility for the S&P 500 index, and  $\text{VOL}_i(t)$  is the prior month return volatility for stock  $i$ . The table reports the coefficient estimates, and the z-statistics (in square brackets). The sample used in the test consists of the closest to the money call with moneyness in the range of  $\pm 2.50\%$ . Calls have maturity of 30 days, and are sampled monthly over the period of January 1991 to December 1995. The estimate of  $\Omega_0$  is suppressed. OBS refers to the size of the panel.

<b>OBS</b>	$\Omega_1$	$\Omega_2$
610	-0.0263 [-2.55]	-
610	-	-0.008 [-1.64]
610	-0.0233 [-2.15]	-0.0042 [-0.86]