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A Refinement to Aït-Sahalia's (2002) "Maximum Likelihood Estimation of Discretely Sampled Diffusions: A Closed-Form Approximation Approach"*

This paper provides a closed-form density approximation when the underlying state variable is a one-dimensional diffusion. Building on Aït-Sahalia (2002), we show that our refinement is applicable under a wide class of drift and diffusion functions. In addition, it facilitates the maximum likelihood estimation of discretely sampled diffusion models of short interest-rate or stock volatility with unknown conditional densities. Our interest-rate examples demonstrate that the analytical approximation is sufficiently accurate.

I. Introduction and the Approach in Aït-Sahalia (2002)

In a recent contribution, Aït-Sahalia (2002) proposed a general method to derive the density function of a one-dimensional diffusion process. To describe his closed-form approximation approach, we follow his notation and consider a one-dimensional diffusion process for some state variable X_t:

$$dX_t = \mu(X_t;\theta)dt + \sigma(X_t;\theta)dW_t, \qquad (1)$$

where W_t is a standard Brownian motion. The functions $\mu(X_t;\theta)$ and $\sigma(X_t;\theta)$, respectively, are the coefficients of drift and diffusion, and θ represents the parameter vector in an open bounded set $\Theta \subset R^K$. Under the assumption that X_t is observed at dates $\{t = i\Delta | i = 0, \ldots, n\}$, Aït-Sahalia (2002) is interested in characterizing the conditional density

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