# Investor Irrationality and the Nasdaq Bubble

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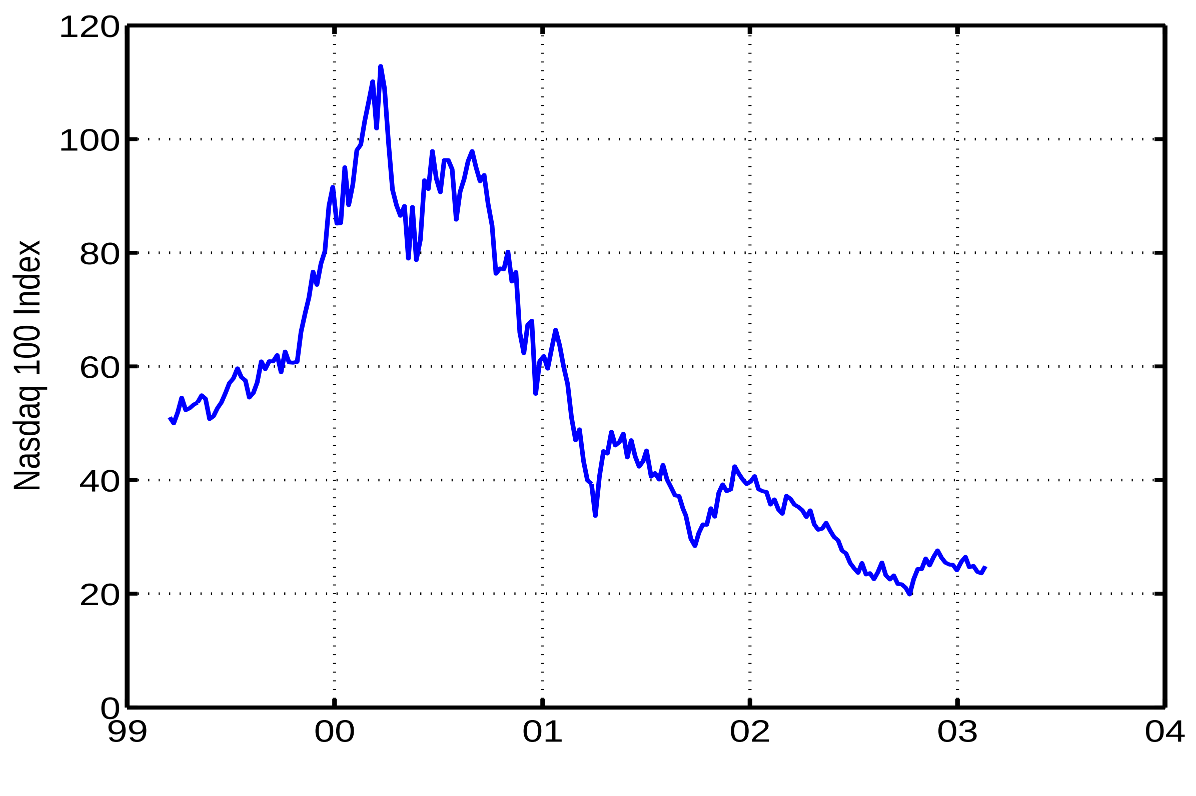
# The up and down of the Nasdaq




- Is this a bubble?
- How can we identify a bubble?

# What is a bubble?

A period of sharp price increase of an asset followed by its dramatic drop, *all the while without the corresponding identifiable movement in the underlying fundamentals or cash flows.*

- If future cash flows do not change, what can lead to price changes?
  - Investors' subjective probability deviates from the objective probability.
  - Investors' risk preference varies over time for no fundamental reason.
  - Other market frictions (short-sale constraints,...)
- In the formal language of asset pricing:

$$\text{Price}_t = \mathbb{E}_t^{\mathbb{P}}\left[\int_t^\infty \text{Pricing kernel}_{t,s}\text{Cashflow}_s\, ds\right]$$

$\mathbb{E}_t^{\mathbb{P}}$ — Expectation under the objective (true, physical) probability measure $\mathbb{P}$.

  - The pricing kernel is a reduced-form summary of investors' subjective probability (deviations) and risk preferences.
  - It defines the market prices on various sources of risks.

$\Rightarrow$ *Identifying a price bubble amounts to identifying structural shifts in the market prices of various sources of risks.*

# How do we identify the pricing kernel?

- Joint analysis of options and the underlying security returns.
  - Statistical distribution can be identified from the time series return.
  - Risk-adjusted (risk-neutral) distribution can be identified from option prices.
  - The difference defines the pricing kernel.
- We use the time-series returns and options on Nasdaq 100 to study the *variations in the risk level and the market prices of various sources of risks* around the bubble period .
- We investigate three hypotheses on the Nasdaq bubble:
  - A rational response to changes in return risk.
  - Investors are driven by irrational euphoria with systematic shifts in the market prices of risks (Greenspan, Shiller (2000), Shleifer (2000)).
  - A random event unaccompanied by any discernible changes in the risk levels and/or market price of risks.

# Existing studies on the Nasdaq bubble

- Pastor, Veronesi (05): Analyze the variations of the fundamentals (cashflows) around the bubble. Hypothesize a positive relation between prices, volatility, and uncertainty about profitability.
  - Risk increased with the soaring Nasdaq valuation.
  - *Not necessarily a bubble*: Price change can be traced to cashflow change.
- Ofek, Richardson (2003): Short-sale constraints limit the implementation of arbitrage trading that can bring down the Nasdaq. Put-call parity is violated.
- Battalio, Schultz (2005): Use intraday options data to show that short-sale constraints are not binding for options trading and hence not the reason for the bubble (high valuation).
- Brunnnermeir, Nagel (2004): Are hedging funds correcting or riding the bubble? $\Rightarrow$ riding.
- Griffin and Harris and Topaloglu (2004): Investor behavior over the rise and fall of Nasdaq
  - Both institutional ownership levels and volume on Nasdaq were high.
  - Institutions buy after market up-moves and sell following market dips.

  $\Rightarrow$ Institutions contributed more than individuals to the Nasdaq rise and fall.
- Josef Lakonishok, Inmoo Lee, Neil Pearson, Allen Poteshman: Study investor behavior in the option market

# Sources of risk in the Nasdaq market

$S_t$ — the Nasdaq 100 Index level:

$$\ln S_t/S_0 = \mu[\cdot] + W^s_{\mathcal{T}_t} + J^+_{\mathcal{T}_t} + J^-_{\mathcal{T}_t} - \xi \mathcal{T}_t, \quad \text{under } \mathbb{P}(\text{true measure})$$

- Four sources of risks:
  - $W^s_t$ — a standard Brownian motion capturing the diffusive return risk,
  - $J^+_t$ — a purely upside jump component, $\pi[x] = \lambda e^{-\beta_+ x} x^{-2}, x > 0$.
  - $J^-_t$ — a purely downside jump component, $\pi[x] = \lambda e^{-\beta_- |x|} |x|^{-2}, x < 0$.
  - $\mathcal{T}_t$ — a stochastic time change applied to return innovations to generate stochastic volatility (variation in the return risk level):

$$\mathcal{T}_t = \int_0^t V_s ds, \quad dV_t = \kappa(\theta - V_t)\, dt + \omega \sqrt{V_t} dW^v_t. \tag{1}$$

- Instantaneous return variance: $V_t\left(1 + \lambda\left(\beta_+^{-1} + \beta_-^{-1}\right)\right)$.

# Market prices of risk and risk premiums

The pricing kernel: *separate market prices for each of the 4 sources of risks:*

$$\mathcal{M}_t = \exp(-rt)\exp\left(-\gamma_s W^s_{\mathcal{T}_t} - \gamma_+ J^+_{\mathcal{T}_t} - \gamma_- J^-_{\mathcal{T}_t} - \gamma_v W^v_{\mathcal{T}_t} - \zeta \mathcal{T}_t\right),$$

- Theory: $\mathbb{P}$-dynamics + pricing kernel $\rightarrow$ $\mathbb{Q}$-dynamics $\rightarrow$ option pricing (tractable).
- Practice: Option prices $\rightarrow$ $\mathbb{Q}$-dynamics $\rightarrow$ pricing kernel (given $\mathbb{P}$-dynamics from time series).

# orem: Decomposition of Return Risk Premium

Suppose that (i) the asset dynamics is of the class (6), (ii) the jump structure is of the exponentially dampened power law class, (iii) the stochastic time change is of the class (1), and (iv) the pricing kernel specification is of the class (7). Then the instantaneous return risk premium is $\eta V_t$, where $V_t$ measures the time-$t$ risk level and $\eta$ measures the risk premium per unit risk, which can be decomposed into three components, reflecting the contribution from three risk sources $W_t^s, J_t^+, J_t^-$, respectively:

$$\eta \equiv \underbrace{\eta_{W^s}}_{\text{Market Price of Diffusion}} + \underbrace{\eta_{J^+}}_{\text{Market Price of Upside Jumps}} + \text{Market Pric}$$

with the market price on each source of risk determined by its respective cumulant exponent (Kuchler and Sorensen (1997)):

$$\begin{aligned} \eta_{W^s} &= \gamma_s + \gamma_v \rho, \\ \eta_{J^+} &= k_{J^+}[1] - k_{J^+}[1-\gamma_+] + k_{J^+}[-\gamma_+], \\ \eta_{J^-} &= k_{J^-}[1] - k_{J^-}[1-\gamma_-] + k_{J^-}[-\gamma_-], \end{aligned} \qquad (3)$$

where $k_{J^-}[u]$ and $k_{J^-}[u]$ are presented in the Appendix.

# Estimation: General idea and objectives

- Using index time-series returns and options to jointly identify the $\mathbb{P}$ and $\mathbb{Q}$ dynamics of the index return.
  - The dynamics of the return risk $V_t$:
    *How does it vary around the bubble period?*
  - General estimates of the market prices of risks:
    *How are the different sources of risks priced on average?*
  - Time-varying market prices:
    *Whether/How do the market prices of different sources of risks vary around the Nasdaq bubble period?*
- Data: Weekly data from March 17, 1999 to February 19, 2003.
  Options data are from OptionMetrics, 206 weeks, 20,160 options with maturities from one week to one year.

# Estimation procedure

I. Given initial guess on parameters $\Theta$, cast the model into a state-space form and extract the unobservable variance rates $\{V_t : t = 1, \ldots, T\}$ using an extended version of the Kalman filter (UKF).

- State propagation: an Euler approximation of the volatility dynamics:

$$V_t \quad = \quad A + \Phi\, V_{t-1} + \sqrt{\mathcal{G}_{t-1}}\, \varepsilon_t, \qquad V_t \in \mathbb{R}^+ \tag{4}$$

  where
  $\varepsilon_t \sim iid N(0,1), A = (1 - \exp(-\kappa \Delta t))\theta, \Phi = \exp(-\kappa \Delta t), \mathcal{G}_{t-1} = \omega^2\, V_{t-1} \Delta t$
  with $\Delta t = 1/52$.

- Measurement equation:

$$y_t \quad = \quad \mathcal{O}\left[V_t; \Theta\right] + e_t, \tag{5}$$

  $\mathcal{O}\left[V_t; \Theta\right]$–model-implied out-of-money option valued weighted by BS vega, as a function of $V_t$ and model parameters $\Theta$.

# Estimation procedures

II. Construct the log likelihood of options based on the UKF mean and variance forecasts, assuming normally distributed forecasting errors,

$$l^{O}_{t+1}[V_t;\Theta] = -\frac{1}{2}\ln\left|\overline{\Omega}_t\right| - \frac{1}{2}\left(\left(y_{t+1}-\overline{y}_{t+1}\right)^{\top}\left(\overline{\Omega}_{t+1}\right)^{-1}\left(y_{t+1}-\overline{y}_{t+1}\right)\right). \qquad (6)$$

III. Construct the likelihood of time-series returns conditioning on the extracted state $V_t$, $l^{s}_{t+1}[V_t;\Theta]$.

- Conditional on $V_t$, we can derive the characteristic function of the index return analytically, which is exponential affine in $V_t$.
- We then use FFT to obtain the density function.

IV. Choose model parameters to maximize the sum of conditional log likelihoods from both options and time-series returns,

$$\max_{\Theta}\mathcal{L}[\Theta,\{y_t\}_{t=1}^{T}] \equiv \sum_{t=0}^{T-1}\left(l^{O}_{t+1}[V_t;\Theta] + l^{s}_{t+1}[V_t;\Theta]\right). \qquad (7)$$

# Time-varying market prices of risk

- To study whether and how the market prices of various types of risks vary around the Nasdaq bubble period, we also re-estimate the model while allowing time-varying market prices.
- For identification, the returns are demeaned and $\gamma_d$ set to zero.
- Let $z_t \equiv (\ln \beta_+^{\mathbb{Q}}, \ln \beta_-^{\mathbb{Q}}, \gamma_v)$ denote the vector of market price of risks at time-$t$, we postulate the following dynamics for $z_t$:

$$z_t \quad = \quad \theta_z(1-\phi) + \phi\, z_{t-1} + \sigma_x\, x_t, \tag{8}$$

- The model can be estimated in similar fashion with expanded state vector: $V_t \equiv (V_t, z_t)$.

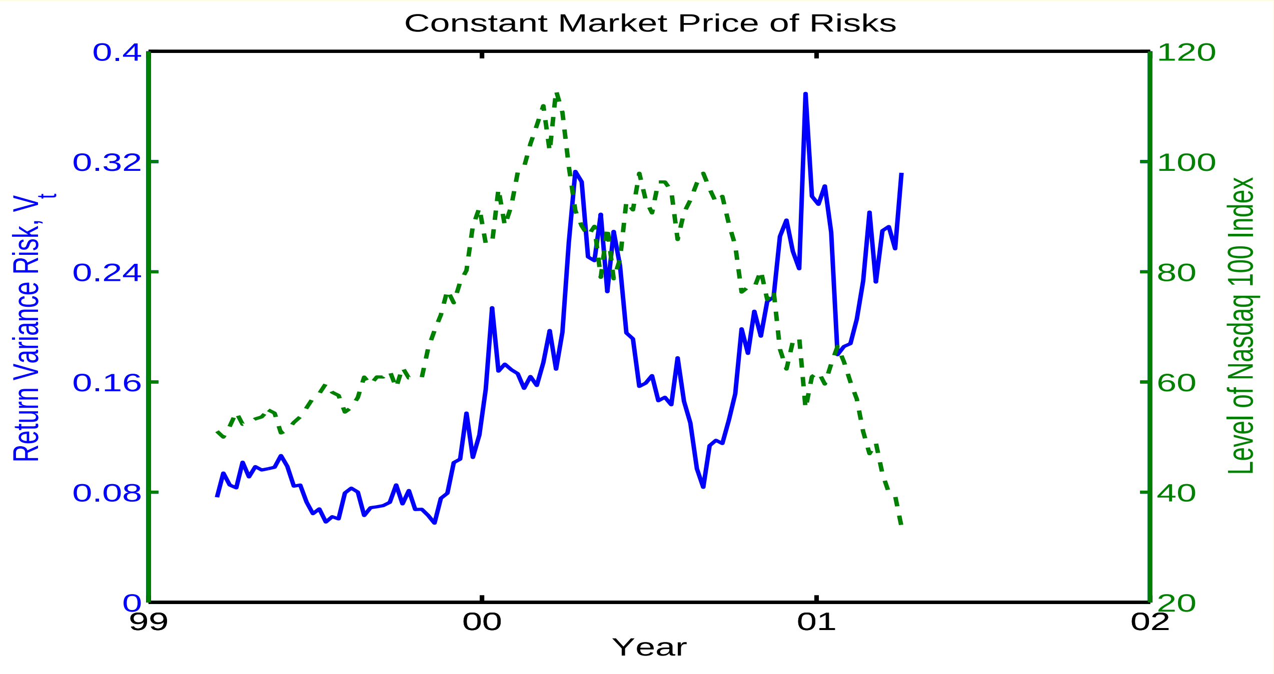
# Parameter estimates

(Absolute $t$-statistics in parentheses)

| Return Risk Dynamics, $V_t$: | | | Jump Risk Structure, $\pi[x]$: | | | Market Prices of Risks: | | |
|---|---|---|---|---|---|---|---|---|
| $\kappa$ | 6.2608 | ( 14.82 ) | $\lambda$ | 1.4198 | ( 34.35 ) | $\gamma_v$ | -3.0556 | ( 3.84 ) |
| $\theta$ | 0.1949 | ( 21.56 ) | $\beta_+$ | 25.4845 | ( 0.39 ) | $\beta_+^{\mathbb{Q}}$ | 4.3234 | ( 162.83 ) |
| $\omega$ | 0.5603 | ( 34.37 ) | $\beta_-$ | 16.5948 | ( 0.35 ) | $\beta_-^{\mathbb{Q}}$ | 1.8810 | ( 48.56 ) |
| $\rho$ | -0.8878 | ( 41.73 ) | | | | | | |

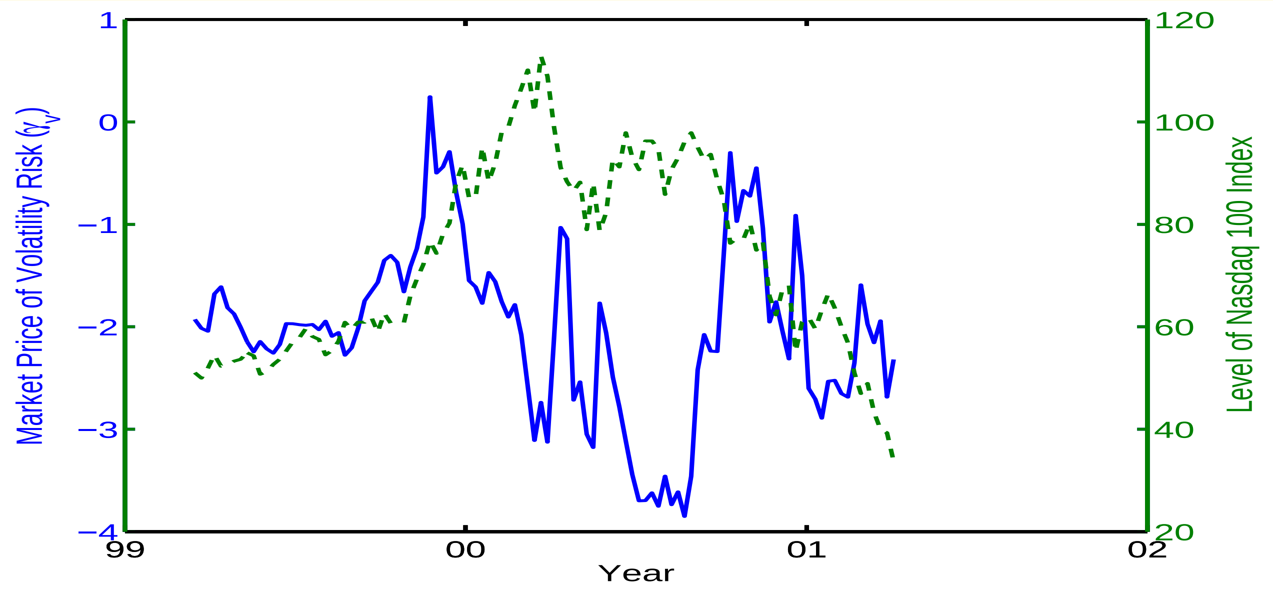
- General pictures:
  - Mean return volatility level: 47.16% — $\sqrt{\theta(1+\lambda(\beta_+^{-1}+\beta_-^{-1}))}$.
  - Stochastic return variance ($\omega$), leverage effect ($\rho$), strong mean reversion ($\kappa$).
  - Could not identify large jumps from the time series returns ($\beta_+, \beta_-$).
  - Identify significantly fatter tails (larger jumps) from options ($\mathbb{Q}$), especially the left tail (negative jumps) ($\beta_+^{\mathbb{Q}}, \beta_-^{\mathbb{Q}}$).
  - Negative market price (on average) on variance risk ($\gamma_v$).

# How did the risk level vary?




- The return variance rate $V_t$ started at a relatively low level in March 1999, but steadily increased as the Nasdaq 100 index climbed up. (Schwert (02)).
- Return volatility generally remained high even after the bursting of the bubble in March 2000.
- Our take:
  - *At normal times, risk moves reversely with the index level (leverage effect).*
  - *At bubble times, risk moves together with the index level (uncertainty, jittery).*

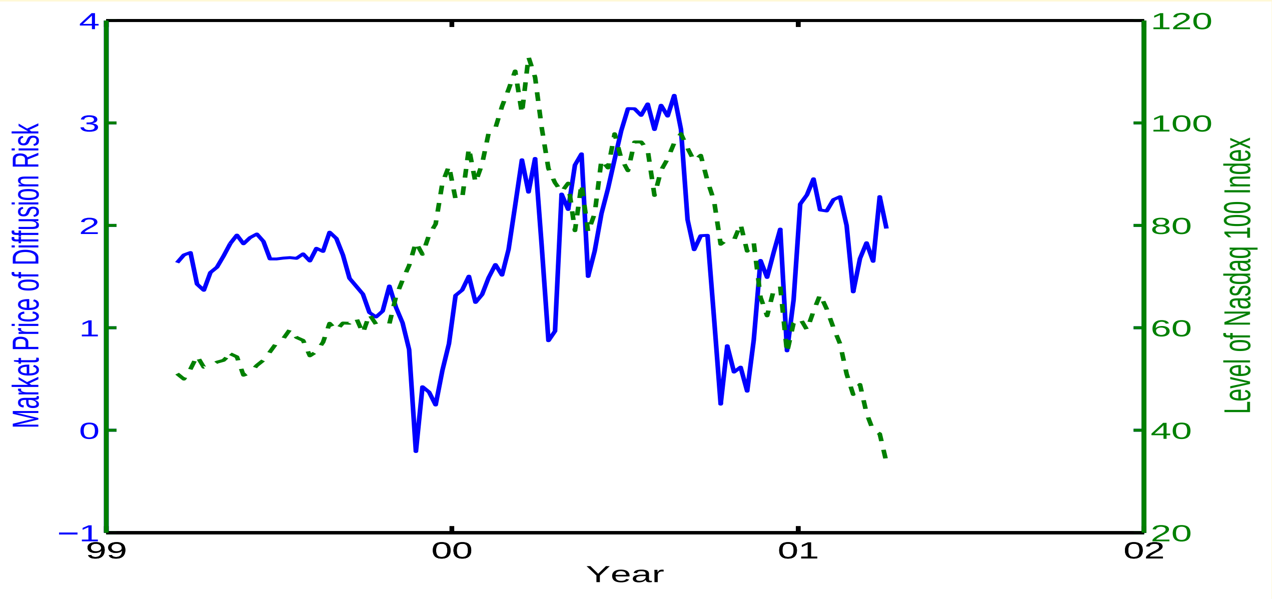
# Time varying market price of volatility risk $\gamma_v$




- $\gamma_v$ started negative and stayed negative on average, suggesting that investors do not like exposures to volatility risk — *standard result*.
- In late 1999, $\gamma_v$ spiked up and even became positive momentarily, following the soaring Nasdaq valuation — *risk (volatility) loving?*
- With the burst of the bubble, investors' dislike for volatility risk reached historical high level – resumed risk aversion.

$\Rightarrow$ *The market price of volatility risk becomes "irrationally" low as the Nasdaq bubble builds.*

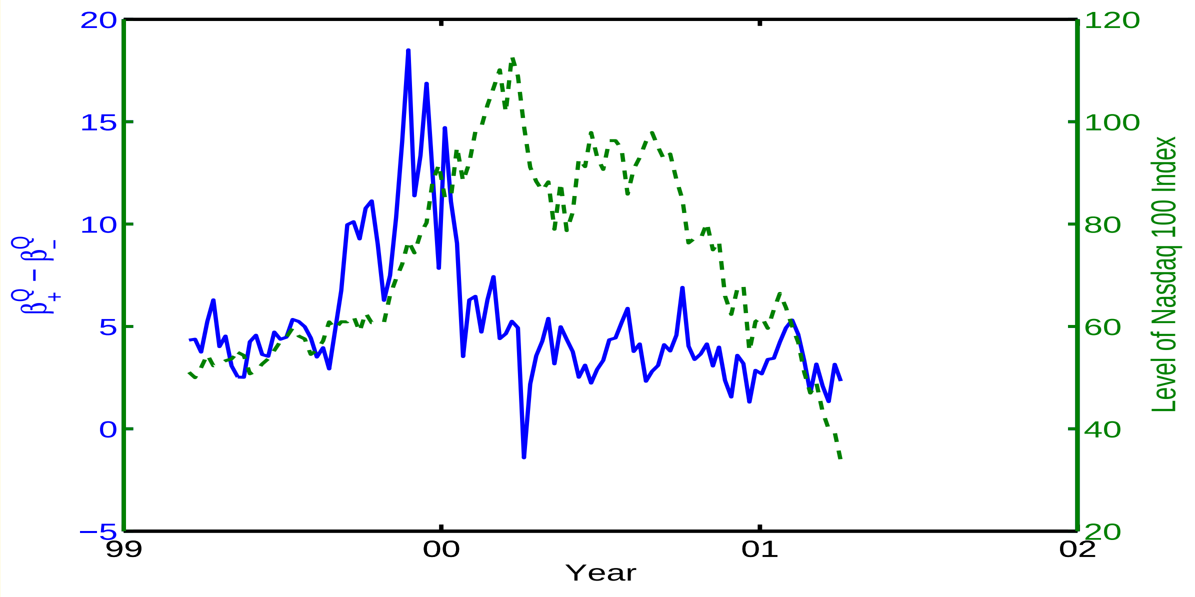
# Time varying diffusion risk premium




- Given $\rho < 0$, a negative market price of the volatility risk generates a positive risk premium on the index return ($\eta_{W^s} = \gamma_d + \rho\gamma_v$).
- The diffusion return risk premium started positive at slightly below two but became *negative* in late 1999, reflecting the market's strong buying pressure on Nasdaq stocks and a shift to "risk-loving" behavior.
- After the bubble burst, the diffusion return risk premium per unit risk became strongly positive, suggesting a reversion in risk attitudes.

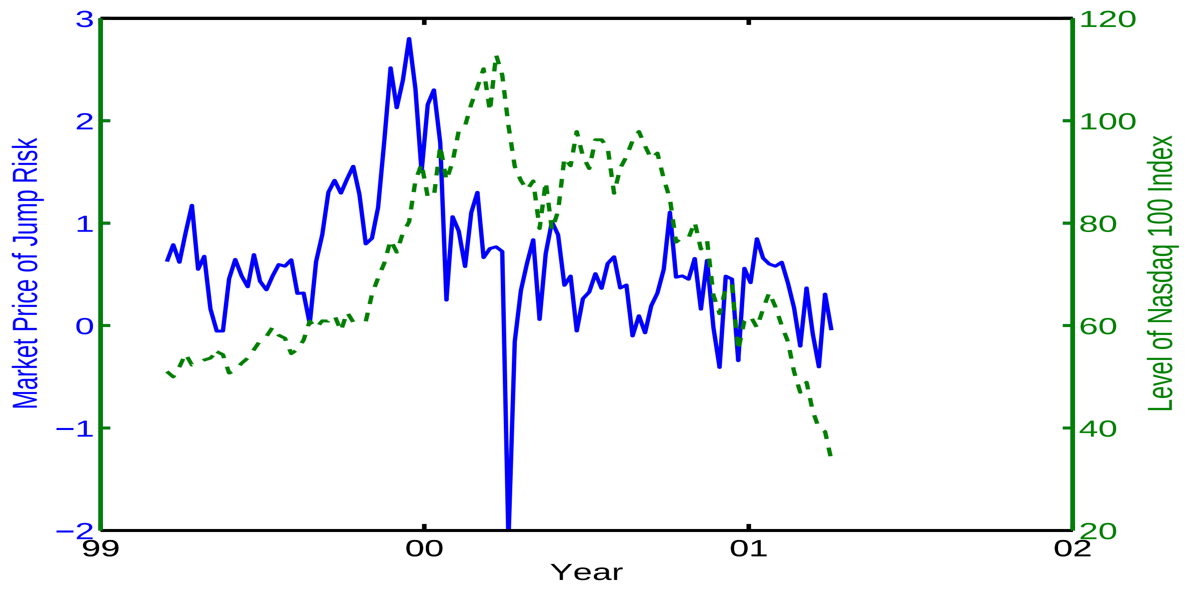
$\Rightarrow$ *Evidence of irrational exuberance.*

# Time varying market price of jump risk




- The demand for hedging against downward market jumps force $\beta_+^{\mathbb{Q}} > \beta_-^{\mathbb{Q}}$ and hence a negatively skewed index return distribution under $\mathbb{Q}$.
- The difference $\beta_+^{\mathbb{Q}} - \beta_-^{\mathbb{Q}}$ can be regarded as a measure of excess demand for out-of-the-money put options and a gauge of market concerns for downside jump risk.
- The market concern gauge moved in tandem with the rising Nasdaq valuation and reached its peak in the late 1999.
- This estimate went down after the burst and stayed close to the starting levels for the rest of the sample.

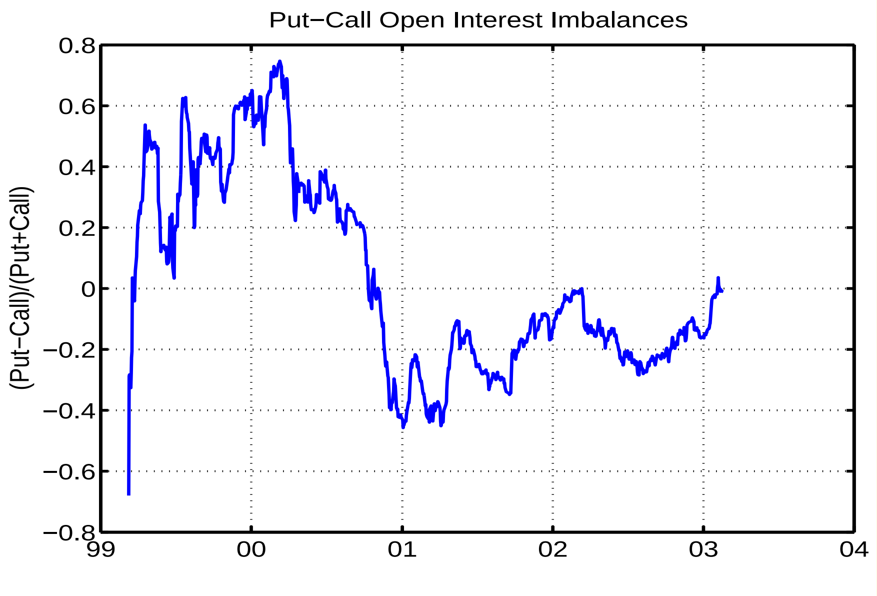
# Time varying market price of jump risk

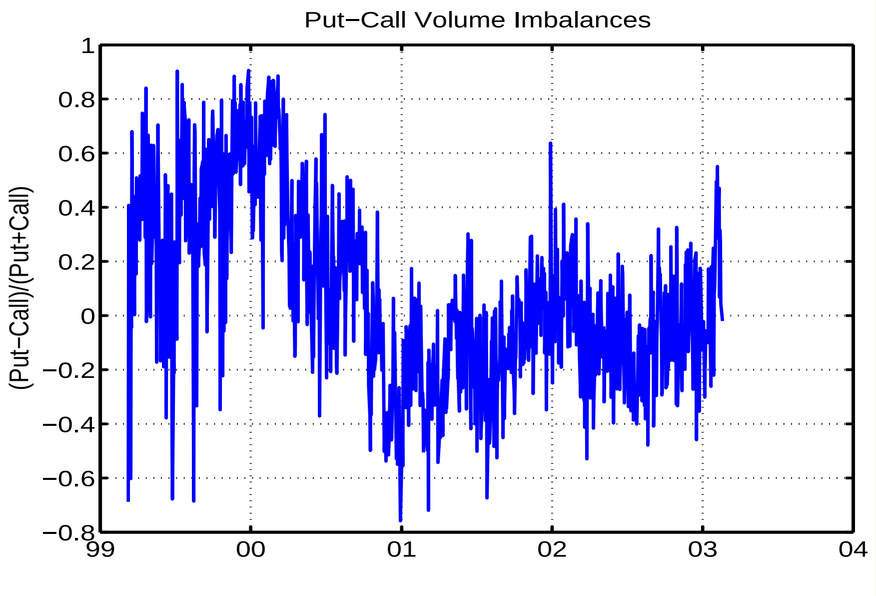



- Converting $(\beta_+^{\mathbb{Q}}, \beta_-^{\mathbb{Q}})$ into risk premiums (per unit risk), $\eta_{J+} + \eta_{J-}$, reveals the same pattern.
- $\Rightarrow$*With the Nasdaq soaring, players in the option markets grew increasingly apprehensive about the potentially significant market correction.*

# Further evidence from options trading

open interest and volume



# Conclusions

- Options market provides useful information about the risk level and market price variations around the bubble period.

- Using time series returns and options on QQQ, and a structural model, we find that the Nasdaq bubble period is associated with the following phenomena:

  - Risk (return volatility) increases with the rising valuation.
    (At normal times, they tend to move in opposite directions.)

  - Diffusion risk premium became close to zero and even became temporarily negative.
    (At normal times, it tends to be positive.)

  - Out-of-money put options became much more expensive than out-of-money call options.
    (At normal times, it is less so.)

  - Put open interest and volume were more than the call counterparts.
    (At normal times, they are more or less balanced.)

$\Rightarrow$ *Both the risk and the market prices of risk endured intriguing shifts during the bubble period.*