Online versus Offline Price Competition: Market Outcomes and the Effect of Internet Shopping Penetration

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Abstract

We analyze competition between an online firm and an offline firm, both selling a single physical product. We build on a generic two parameter product differentiation model, postulating reservation prices for every potential purchase and allowing for idiosyncratic differences in the valuations of the complete product which includes the physical good, service and delivery. We add an *Internet shopping penetration* parameter that represents the deterministic fraction of customers who comparison shop online. We present the unique Nash equilibria to the resulting dual market game and characterize them in terms of three parameters: the penetration parameter, and the Internet product's *niche appeal* and *degree of innovation*. The last parameter compares the innovativeness of the online product with that of the offline firm in terms of service execution and cost structure. The equilibria provide the basis for studying how the competitive consequences, such as prices, profits, and quantities sold, are affected by these three parameters. We identify conditions under which unwarranted enthusiasm for the online firm's prospects may appear. The degree of innovation plays a pervasive, pivotal role.

1 Introduction

There is substantial evidence that there is something fundamentally different for most consumers about purchasing on the Internet as compared to a physical store. Wolfinbarger and Gilly (2001) identify some benefits of shopping online such as: convenience, time and effort savings, accessibility, better selection and availability, search capabilities, increasingly accurate information, lack of lines, salespeople and crowds, and anonymity. Despite these many benefits, there are costs, or negatives, as well to shopping online as compared to offline. For example, Wolfinbarger and Gilly (2001) identify harder visual inspection, lack of touch, and delayed gratification. Tan (1999) adds that shopping online is often perceived as riskier than shopping at a store, while Morganosky and Cude (2000) mention that consumers are concerned about credit card security when shopping online. Consumers' needs and circumstances may also affect their individual valuations. For example, consumers who want the product immediately would place a low value on buying from an Internet firm that could not deliver immediately. The shipping charges for a consumer buying on the Internet channel may depend on where the consumer lives. Some consumers simply hate to grocery shop, and therefore prefer the electronic channel (Morganosky and Cude 2000). Furthermore, some prefer to learn about product features online while others prefer to learn from salespeople.

We adopt the approach taken by the literature on services, where the entire service, including both tangible and intangible components, is priced and valued (Grönroos 1990, Easton and Pullman 2001). Thus, because of the differences in the two channels, a consumer's assessment of the most that she would be willing to pay (her reservation price) differs for the two channels. Some of these differences, such as unit costs and application of sales taxes, may affect almost all potential customers in the same way. However, many of these differences are idiosyncratic, and we incorporate this fact into our model. In particular, we extend the linear reservation price market model developed by Schmidt and Porteus (2000), hereafter S/P, who postulate reservation prices for each product as functions of the customer type. Their model boils down to two parameters, the *degree of innovation* and the *niche appeal*, of the online (Internet) product compared to the offline (bricks and mortar) product, when applied in our setting. (The higher the degree of innovation, the more appealing the online product is to all customer types. The higher the niche appeal, the faster the appeal of the online product drops off as the customer type increases, moving toward potential customers who value the product less.)

There is also evidence that not all potential customers consider buying on the Internet. For example, in 2001, 57% of U.S. households had web access, but only 49% of them purchased anything online. By July 2003, access had increased to 64% and 51% of those had shopped online (McQuivey 2003). According to Jupiter Research, 34% of those who do not shop online, never plan to (Crenshaw 2002). To account for this phenomenon, our model incorporates an *Internet shopping penetration parameter* (for brevity, *penetration*) that represents the fraction of potential customers who comparison shop both online and offline. This leads to a dual market competition model. In the first market, the firms compete for the customers who compare prices on both channels. In the second market, there is effectively no competition as customers only shop offline. We assume the offline firm must offer the same price to all customers, whether or not they compare prices at the online firm. Our first main result is the specification of the unique Nash equilibrium for each static game of simultaneous price setting as a function of the degree of innovation, the niche appeal, and the penetration parameter. This result provides the basis for studying how the competitive consequences, such as prices, profits, and quantities sold, are affected by these three parameters.

We anticipate that, for some products, the price competition will be repeated over time, under continually changing conditions. For example, more customers will have access to the Internet; more will become aware of the viability of buying online; security issues will be resolved; consumers will become accustomed to searching online for products; bandwidth, hardware and software costs will continue to fall; and technology will improve the speed of transactions and the ability of consumers to view the product. These changes may affect all three parameters of the model. However, it is convenient to study the competitive consequences of changing one of these parameters at a time.

For example, Figure 1 shows, for a given niche appeal, what the market structure will be as a function of the degree of innovation and the penetration parameter. The functions L, M and N (defined in the Appendix and to be discussed in detail later) plotted on the figure represent the boundaries between the market structure regions, while S (also defined in the Appendix and discussed later) gives a strategic value of the degree of innovation for the Internet firm, where it is appropriate.

Holding the niche appeal and the penetration constant, moving up vertically in Figure 1 reveals the consequences of increasing the degree of innovation. If it is low, then the bricks and mortar firm will have a monopoly. As it increases (all else constant), the market structure changes to the bricks and mortar firm having a *constrained monopoly*, as defined by Greenstein and Ramey (1998), in which the Internet firm prices at cost, receives no sales, but limits the price that can be charged by the bricks and mortar firm. As the degree of innovation continues to increase, the market moves to a pure strategy duopoly, then to a mixed strategy duopoly, and, finally, to a double monopoly. Here the Internet firm has a monopoly in the *competitive market* (where consumers choose between the two firms), and the bricks and mortar firm has one in the *captive market* (where consumers only shop offline). An increase in the degree of innovation always increases profits for the Internet firm and decreases them for the bricks and mortar firm. Thus, increases in the degree of innovation should be highly sought by online firms.

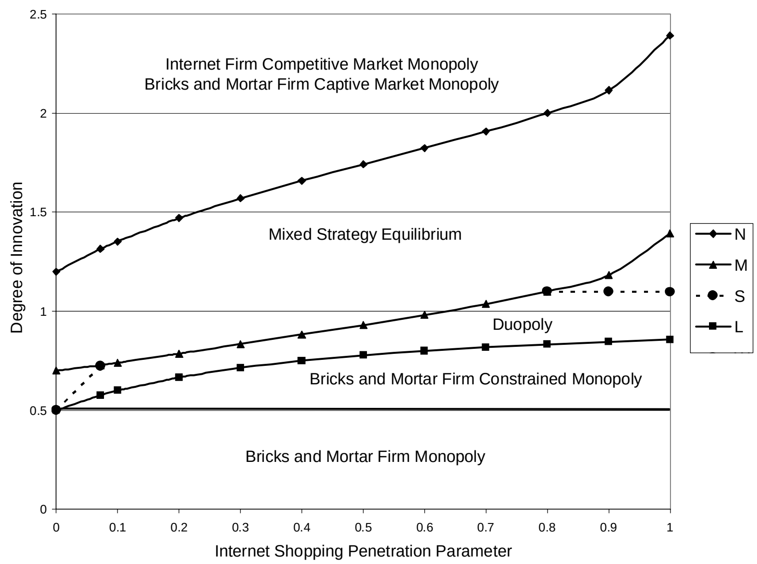

Figure 1: Market Structure Map for Niche Appeal of 1.2 as Internet Shopping Penetration Parameter and Degree of Innovation Vary

We now hold the niche appeal and degree of innovation constant. Moving to the right horizontally reveals the consequences of increasing the penetration parameter. In all market outcomes except the bricks and mortar firm monopoly, the offline firm faces increased competition and smaller profits, as expected. However, we shall also see that the Internet firm does not necessarily see profits increase monotonically. For example, Figure 2 shows a case in which, if the degree of innovation is between 0.5 and 0.8, then increasing penetration from an initial level of 0 first increases duopoly profits to the online firm, but then decreases them once the strategic threshold, S, is crossed.

At small levels of penetration, the rate at which the online firm's profits increase can be quite high, particularly if represented as a percentage increase per percentage increase in penetration. In this zone, the offline firm obtains high profits from the captive market and virtually cedes the competitive market to the online firm. However, as penetration increases, the captive market is diminished in size and the offline firm finds it attractive to compete in the competitive market. Once the strategic threshold is crossed, each increase in penetration induces a sufficiently large price reduction from the offline firm that the online firm loses out overall. For degrees of innovation in the 0.5 to 0.6 range, the online firm's profits even disappear entirely for high values of penetration. In general, enthusiasm for the online firm's prospects of success may occur when penetration is low. However, as Figure 2 clearly shows, as penetration increases, this enthusiasm may be unwarranted. To warrant continued enthusiasm, the Internet firm must be sufficiently innovative, which happens through a combination of the Internet firm lowering unit costs and making the online shopping experience highly valued by potential customers.

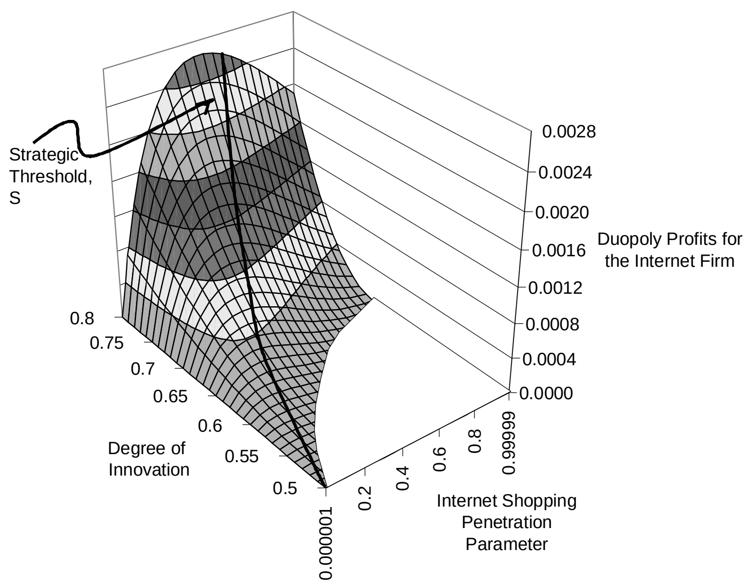

Figure 2: Firm I's Duopoly Profits as a Function of Penetration and Degree of Innovation for Niche Appeal of 1.9, Showing the Profit Crest at the Strategic Threshold, S

We also examine the effects of changing the niche appeal. An increase (decrease) in niche appeal means the potential customer segment with positive reservation prices for the online product decreases (increases). In other words, as niche appeal increases, fewer people are willing to buy the online product at any price, thus it appeals to more of a niche. For the Internet firm, increasing or decreasing niche appeal can be beneficial, depending on the values of penetration and the degree of innovation. We show (Theorems 7 and 8) that if the Internet firm has a degree of innovation sufficiently below the strategic threshold S, it can increase its niche appeal, causing its market to decrease, yet increase profits. It is essentially acting as a monopolist in its niche, giving it pricing power. Conversely, by decreasing its niche appeal and and thereby trying to sell to a larger market, the Internet firm increases competition, reducing prices. The result for the Internet firm is always a larger market, but profit growth requires the degree of innovation to be above S.

In §2, we review some related literature, while in §3, we present our model. The Nash equilibria are presented in §4. Our results and discussion of those results are in §5 and §6. We conclude in §7.

2 Related Literature

Schmidt and Porteus (2000) (S/P) develop the linear reservation price model that is the underlying framework of our model. They demonstrate a connection between their model and the more traditional approach to vertically differentiated products (see Gabszewicz and Thisse 1979 and Shaked and Sutton 1982). They also discuss the interpretation of linear reservation price curves as linear monopolistic demand curves and illustrate their derivation from the use of conjoint analysis. They use this model in the context of product development, characterize the two firm Nash equilibria and develop the concept of the degree of innovation. Our model differs from theirs in that we consider a different setting, with distinctive managerial implications, and we introduce the Internet shopping penetration parameter. This parameter leads to the existence of two linked markets, and changes the solution in important ways. For example, we find regions in which the unique Nash equilibrium consists of a mixed strategy for one of the firms, whereas all their equilibria consist of pure strategies.

Our model is similar to the spatial duopoly models of product differentiation such as the linear and circular city models (Hotelling 1929, Salop 1979). For example, in both, the firms can determine who will buy each product by examining the surplus each consumer receives, given the prices set by the firms. In a city model, the attributes that a consumer values can be boiled down to being equivalent to a (single dimensional) distance. Customers who are "close" to one firm may not be close to the other, which implies that each firm may start with a niche of customers who strongly prefer its product and therefore constitute a weak form of a captive market. Our model differs in that a (possibly large) fraction of the customers constitute a strong form of a captive market for the bricks and mortar firm, while the remainder of the customers form the competitive market in which those who value one product the most also value the other product the most, which can lead to cutthroat competition in that market.

Swaminathan and Tayur (2003) give an excellent survey of the literature on supply chains operating in Internet channels. We mention a few of the closely related papers here. Cattani et al. (2004) model two decisions that bricks and mortar firms face as they expand into electronic commerce – the optimal price on the new channel and whether the new channel should act as a competitor to the bricks and mortar channel. They also establish a unique Nash equilibrium when the two channels are operated by separate firms, using a probabilistic form of a city model with two relevant distances.

Balasubramanian (1998) models competition between a direct marketer, such as an e-tailer or a catalog firm, with conventional retailers. Conventional retailers are evenly spaced around a circle, with consumers located uniformly around the circle. Consumers face a distance cost for the conventional retailers, while the location of the direct marketer is not a factor. The reservation prices are assumed high enough that all consumers will make a purchase. Consumer demand is assumed to be inelastic, and all firms face the same marginal cost. Each firm faces a fixed cost if it operates and consumers incur a cost (disutility) when purchasing directly. This represents the negatives of direct purchases such as harder returns, waiting for the product, etc. Our model differs from his in several ways: (1) the differentiating attribute between the two products is not necessarily equivalent to a distance; (2) the marginal costs of the two firms can differ; and (3) consumers may have heterogeneous positive or negative utilities from the e-commerce experience.

Chiang et al. (2003) examine Stackelberg price competition between a retailer and a manufacturer offering a direct channel. Consumers have heterogeneous valuations but the product has unifornly less value to all consumers when purchased online. They find that the manufacturer may offer a direct channel to mitigate double marginalization. Key differences between our model and theirs are that all consumers choose between the two channels in their model, while we introduce the Internet shopping penetration parameter, and we assume that consumers may find the online channel more or less valuable than the offline channel, while they assume it is always less valuable.

Viswanathan (2005) captures competition between an online, an offline, and a hybrid firm using a variant of a circular city model. Similar to our model, each firm sells the same product but, unlike ours, the firms sell to homogeneous customers dispersed around two unit circles representing the online and offline channels. The distance between the consumer and firm represents channel misfit costs, which correspond to the purely negative costs of shopping on a particular channel for a particular consumer in our model. He assumes no benefits as we do. He further assumes that the online and offline channels have a different, fixed, number of potential customers, while the hybrid firm may sell to either group of potential customers. This is similar to our penetration parameter, but he does not explore changes in the outcomes as the market sizes vary. He finds that network externalities and switching costs increase price competition and may lead to situations where a monopoly by the online firm emerges. In our model, changes in the degree of innovation may lead to a monopoly by the online firm, caused by underlying changes in product costs, a firm's service, and consumer perception.

Lal and Sarvary (1999) look at price competition in a differentiated duopoly with a focus on customer information search. They break product attributes into digital (i.e., price, etc.) and nondigital attributes (i.e., feel, fit, etc.). Our model differs because 1) our physical products are identical, and 2) we assume the costs and benefits of shopping on the two channels, our differentiating factors, can be estimated accurately by consumers without search. Contrary to their results, we show that when Internet penetration is high, price competition increases, unless the online channel is much more innovative, leading to a high level of differentiation between the channels.

3 Model

Imagine a consumer who has decided to buy a single product that is available from only two firms, one that sells online and the other from a conventional (offline, bricks and mortar) outlet. Each potential consumer has two reservation prices for the product, one indicating the most s/he is willing to pay for the product from firm B (the bricks and mortar firm), and the other from firm I (the Internet firm). As discussed earlier, these reservation prices may differ because the shopping experience and related costs may differ. The two firms compete in a single period by offering a single substitute product at possibly different prices, denoted by P_I and P_B, respectively. Both firms are committed to being in the market.

We assume each potential purchase would be made by a distinct (potential) consumer, and that B and I refer not only to the firm, but to the channel and product offered by that firm. Following S/P, we sort the reservation prices for product B from largest to smallest, creating a sequencing of potential consumers, which we call customer types and treat as a continuous variable, z. The resulting curve, assumed to be linear, would represent the downward sloping demand curve for product B if firm B had a monopoly. It is convenient to interpret customer types as representing how time-starved they are, with the most time-starved customers valuing product B the most.

It is then plausible to assume, as we do, that the reservation price curve for product I, using the same sequencing of customers, is also downward sloping. We assume that the most time-starved customers value product I more than product B, and that this perceived premium in value (difference in reservation prices) decreases in the customer type. This assumption is partially supported by Wolfinbarger and Gilly's (2001) finding that time-starved consumers are likely to be online shoppers and the assertion of Ratchford et al. (2003) that customers with high time costs are willing to pay more to reduce search costs. For analytical tractability, we also assume that this premium decreases linearly, so that the reservation price curve for product I is linear and decreases faster than the curve for product B. An example appears in Figure 3.

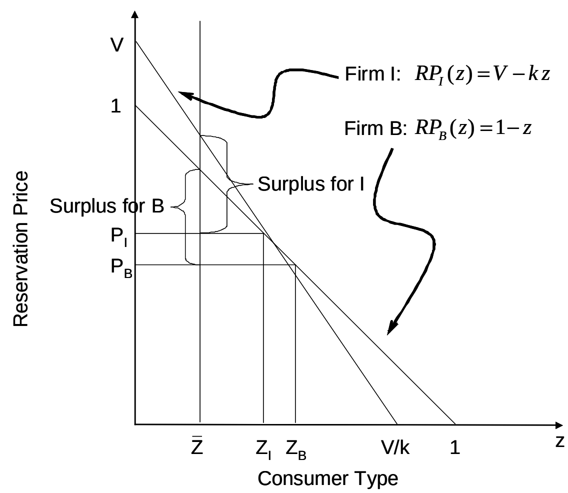

Figure 3: Example 1

It is convenient for expositional purposes to assume temporarily that all potential consumers consider purchasing both products. We normalize the prices and quantities so that the reservation price curve for firm B is of the form $RP_B(z) = 1 - z$ for $0 \leq z \leq 1$ and that for firm I is of the form $RP_I(z) = V - kz$, where $V \geq 1$ and $k > 1$. (Terms are summarized in Table 1.) Think of the horizontal axis z as representing both the customer type and a (normalized) representation of the potential total purchase quantity. For example, customer type $z = 0$ values both products more than all other customers, product B at 1 and product I at V. The surplus that customer type z would obtain by buying is $S_B(z) := RP_B(z) - P_B$ if from B, and $S_I(z) := RP_I(z) - P_I$ if buying from I. Let $Z_B := 1 - P_B$ and $Z_I := (V - P_I)/k$, which we call the *targets* for firms B and I, respectively. Note that Z_B would be B's (normalized) level of unit sales (quantity sold) if it were the only firm offering the product, but generally exceeds B's actual unit sales when it has competition. It is convenient to conduct our analysis in terms of the firms' respective targets, rather than their unit prices, although of course, they are equivalent. The next section reveals that the analysis hinges on three cases, which are based on the possible relationships between Z_B and Z_I. However, an ordering of the targets does not necessarily correspond to an opposite ordering of the prices. For example, it is possible to have $Z_B > Z_I$ and $P_B > P_I$ simultaneously. In short, although our analysis is done in terms of targets, it is important to remember that the competition is in prices, not quantities.

Table 1: Key Terms

Term	**Interpretation**
z	Consumer type, $0 \leq z \leq 1$
V	Maximum (normalized) reservation price for product I, $V \geq 1$
k	Niche appeal of the Internet product; or its normalized negative slope, $k > 1$
Z_i	Target for firm i; the sales if it were the only firm
$\bar{Z}$	Consumer with equal surplus for both products; $\bar{Z} = (kZ_I - Z_B)\,/\,(k-1)$
q	Internet shopping penetration, $0 < q < 1$
C_i	Unit cost of product i, $i \in \{B, I\}$
D	Degree of Innovation; $D = (V - C_I)\,/\,(1 - C_B)$

Let $\bar{Z} := (kZ_I - Z_B)/(k-1)$, which gives the consumer type that is indifferent between buying from the two firms, ignoring whether the consumer type will buy at all: $S_B(\bar{Z}) = S_I(\bar{Z})$. It is straightforward to see that if $0 \leq z \leq \bar{Z}$, then customer type z has a higher surplus for product I and therefore prefers to buy from firm I rather than firm B. If furthermore $z \leq Z_I$, then customer type z prefers buying from I than not buying, and therefore will buy from I. Similarly, if $\bar{Z} \leq z \leq Z_B$, then z will buy from firm B. Otherwise, z will not buy. (For convenience, a customer of type $z \in \left\{0, Z_I, Z_B, \bar{Z}\right\}$ is technically allowed to buy both products or buy both a product and buy nothing, because such a customer represents an infinitesimal volume.) The parameter k is the ratio of the slopes of the respective reservation price curves and is called the *niche appeal* of product I. An example of the reservation price curves, the targets, and the resulting surpluses for each firm is shown in Figure 3. In this example, the customers with higher surplus for product I are in the interval $\left[0, \bar{Z}\right]$, while those with higher surplus for product B are in the interval $\left[\bar{Z}, Z_B\right]$.

We now introduce $q \in (0, 1)$, the *Internet shopping penetration parameter*, which denotes the deterministic fraction of customers who base their purchase decision on both prices. In particular, for arbitrary $0 \leq a < b$ corresponding to $b - a$ potential customers, $q(b - a)$ of these have access to the Internet and will buy on the channel that gives them the highest positive surplus. The remaining $(1 - q)(b - a)$ potential customers consider purchasing only from firm B. In this paper, we limit our results to $q \in (0, 1)$. When penetration equals zero, firm I cannot sell anything. When penetration is complete, the results are covered by those in S/P and are interpreted in our context in Druehl (2003). We therefore have the following single period, dual market model: Each firm sets its unit price, which is left unchanged throughout the period, for its product. In the *captive market*, consisting of the fraction $1 - q$ of the customers, B is purchased by all who obtain a positive surplus from doing so. In the *competitive market*, consisting of the fraction q of the customers, sales are still made only to customers with a positive surplus, but each firm only sells to those who have the higher surplus from its product. Note that we assume firm B must offer its product at the same price in both markets, and that the amounts sold are deterministic consequences of the prices. We represent both markets in one figure, such as in Example 1, shown in Figure 3. (See Druehl 2003 for how to represent the two markets separately and, of course, obtain the same results as what follows.)

The firms are assumed to incur a constant, but possibly unequal, (normalized) marginal cost for each unit of product sold. Therefore, all relevant costs, such as production, marketing and fulfillment costs, are assumed to be proportional to volume. Firm I incurs unit cost C_I, $0 \leq C_I < V$, and B incurs unit cost C_B, $0 \leq C_B < 1$, for each unit sold. Components of our solutions will be proportional to $1 - C_B$, which is an artifact of the reservation price curve normalization we chose to employ. That is, instead of normalizing so that the maximum reservation price for firm B's product is 1, we could have normalized so that the difference between the maximum reservation price and B's unit cost would be 1. Then, the term $1 - C_B$ would disappear from our solutions.

The market clearing mechanism is as follows: (1) the firms (simultaneously) post their respective unit prices, which correspond to their target quantities; (2) deterministic demand is revealed, based on reservation prices and penetration; and (3) production or purchase occurs to order. Hence, there is no excess demand nor any unsold stock.

The *degree of innovation*, defined here by $D := (V - C_I)/(1 - C_B)$, is a relative measure of the innovation of the product sold by I to that of the one sold by B. In contrast to S/P, our measure is independent of the niche appeal, facilitating understanding of the consequences of changes in the niche appeal. Our degree of innovation can be interpreted as the ratio of the monopoly margins: If firm I had a monopoly, its price would be $(V + C_I)/2$ and its margin would therefore be the difference between that and its unit cost, namely $(V - C_I)/2$. Similarly, if B had a monopoly, its unit margin would be $(1 - C_B)/2$. If $D > 1$, then we say that product I is more innovative. Which product is more innovative boils down to which would have the highest margin in a monopoly. In this setting, D is determined by factors already discussed, such as marketing, cost, convenience, and service execution, because the physical product is the same. For example, D can be high if the direct overhead and delivery costs are lower for I than for B. One-click shopping, a service innovation, could be valued by all consumers, raise V, and therefore contribute to a higher level of D. The advantage of defining D is that the Nash equilibria can be found as explicit functions of it, k, and q.

4 Nash Equilibria

To determine the Nash equilibria, we first present the firms' profit expressions. Recall that after normalization, there are q customers in the competitive market and $1 - q$ customers in firm B's captive market. In the captive market, B sells $(1 - q)Z_B$ units and I sells nothing.

To determine the unit sales (and profits) in the competitive market, it is convenient to define three cases: Case I: $Z_B \leq Z_I$, Case B: $Z_I < Z_B < kZ_I$ and Case BD: $kZ_I \leq Z_B$, where Z_I and Z_B are required to be positive. Case I, which is illustrated in Frame A of Figure 4, is equivalent to $Z_I \leq \bar{Z}$, which means that firm I sells qZ_I units and firm B sells nothing (in the competitive market). In cases B and BD, $Z_B > Z_I$. In these cases, $Z_I > \bar{Z}$, which means that B sells a positive quantity in the competitive market. In case B, which is the case illustrated in Figure 3, $\bar{Z} > 0$, so I sells $q\bar{Z}$ and B sells $q(Z_B - \bar{Z})$. In case BD, which is illustrated in Frame B of Figure 4, $\bar{Z} \leq 0$, so I sells nothing and B sells qZ_B. In this case, B dominates both markets, selling a total of Z_B units.

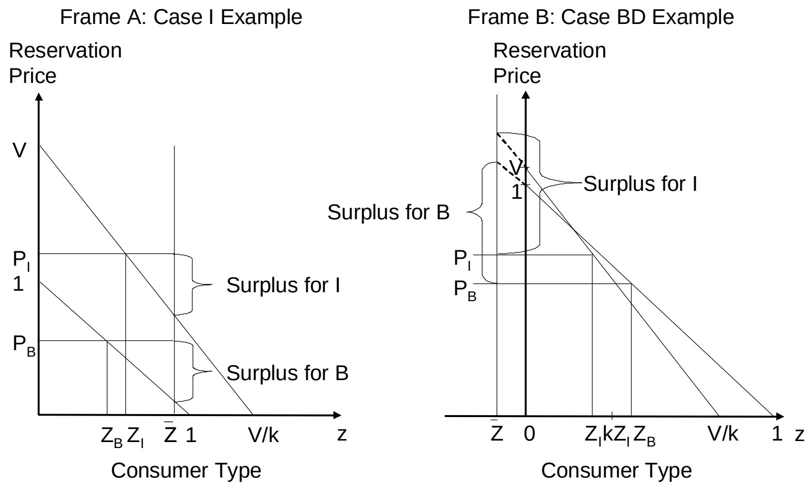

Figure 4: Examples of Case I, $Z_B \leq Z_I$, and Case BD, $kZ_I \leq Z_B$

Adopting the convention of specifying the case with a superscript and the firm with a subscript, we present the profit expressions for each firm and case in Table 2. For example, in case B (Figure 3), firm B sells $q(Z_B - \bar{Z})$ and $(1-q)Z_B$ in the competitive and captive markets, respectively, each at a margin of $P_B - C_B = 1 - Z_B - C_B$, leading to a total return of $(1 - Z_B - C_B)(Z_B - q\bar{Z})$.

Table 2: Firm Profits for Each Case

Case	**I's Profit**	**B's Profit**
I	$\pi_I^I(Z_I, q, k) = (V - kZ_I - C_I)Z_I q$	$\pi_B^I(Z_B, q) = (1 - Z_B - C_B)(1-q)Z_B$
B	$\pi_I^B(Z_I \vert Z_B, q, k) = (V - kZ_I - C_I)q\bar{Z}$	$\pi_B^B(Z_B \vert Z_I, q) = (1 - Z_B - C_B)\left(Z_B - q\bar{Z}\right)$
BD	$\pi_I^{BD}(Z_I) = 0$	$\pi_B^{BD}(Z_B) = (1 - Z_B - C_B)Z_B$

To find the Nash equilibria, we develop the optimal response functions for each firm, then partition all possibilities into a finite number of cases, determine the unique Nash equilibrium within the interior of each, and show that at any nonempty intersection of cases, the Nash equilibria for each case coincide. Druehl and Porteus (2005) show that, for each specification of parameters, there exists a unique Nash equilibrium, which is given explicitly. Notation is defined in the Appendix in Tables 3 and 4, and we also show the optimal response function development in the Appendix. Firm B's optimal response differs depending on the level of k, the niche appeal. In particular, we say that product I has *high niche appeal* if $k > \frac{1}{2}(2 - q + \sqrt{q})$, and has *low niche appeal* otherwise. We present and interpret the equilibrium results for the high niche appeal case first. The parameters

L, M, and N specify, along with $\frac{1}{2}$, the boundaries between the different market structure regions. (Recall this notation is defined in Table 3 in the Appendix.) The parameters and the resulting market structure regions are illustrated in Figure 1, for $k = 1.2$. In this, the high niche appeal case, it follows that $\frac{1}{2} < L < M < N$.

Theorem 1 *If $k > \frac{1}{2}(2 - q + \sqrt{q})$, then the unique Nash equilibrium prices, quantities, and profits are as follows.*

	Firm B Monopoly	*Firm B Constrained Monopoly*
Conditions	$0 < D \leq \frac{1}{2}$	$\frac{1}{2} \leq D \leq L$
Targets	$Z_B = Z_B^I$ *and* $Z_I = Z_I^{MC}$.	$Z_B = Z_B^{MC}$ *and* $Z_I = Z_I^{MC}$.
Quantities	$q_B = Z_B^I$ *and* $q_I = 0$.	$q_B = Z_B^{MC}$ *and* $q_I = 0$.
Prices	$P_B = (1 + C_B)/2$ *and* $P_I = C_I$.	$P_B = 1 - Z_B^{MC}$ *and* $P_I = C_I$.
Profits	$\pi_B = (1 - C_B)^2/4$ *and* $\pi_I = 0$.	$\pi_B = (1 - C_B)^2(1 - D)D$ *and* $\pi_I = 0$.

	Duopoly	*Captive Market Monopoly / Competitive Market Monopoly*
Conditions	$L \leq D < M$	$N \leq D$
Targets	$Z_B = Z_B^B$ *and* $Z_I = Z_I^B$.	$Z_B = Z_B^I$ *and* $Z_I = Z_I^I$.
Quantities	$q_B = \frac{(1 - C_B)\beta(2\beta - q - qD)}{(k-1)(4\beta - q)}$ *and* $q_I = \frac{q(1 - C_B)((2\beta - q)D - \beta)}{(k-1)(4\beta - q)}$.	$q_B = (1 - q)Z_B^I$ *and* $q_I = qZ_I^I$.
Prices	$P_B = 1 - Z_B^B$ *and* $P_I = V - kZ_I^B$.	$P_B = \frac{1 + C_B}{2}$ *and* $P_I = \frac{V + C_I}{2}$.
Profits	$\pi_B = \frac{(1 - C_B)^2\beta(2\beta - q - qD)^2}{(k-1)(4\beta - q)^2}$ *and* $\pi_I = \frac{(1 - C_B)^2 q(\beta - D(2\beta - q))^2}{(k-1)(4\beta - q)^2}$.	$\pi_B = \frac{(1 - q)(1 - C_B)^2}{4}$ *and* $\pi_I = \frac{qD^2(1 - C_B)^2}{4k}$.

	Mixed Strategy Equilibrium
Conditions	$M \leq D < N$
Targets	$Z_B = \begin{cases} Q_B^B(Z_B^{EH}) & \text{a fraction } \alpha_H \text{ of the time} \\ Z_B^I & \text{a fraction } 1-\alpha_H \text{ of the time} \end{cases}$ and $Z_I = Z_B^{EH}$.
Quantities	$q_B = \begin{cases} (1-C_B)\gamma/(2(k-1)) & \text{a fraction } \alpha_H \text{ of the time} \\ (1-q)Z_B^I & \text{a fraction } 1-\alpha_H \text{ of the time} \end{cases}$ and $q_I = \begin{cases} \dfrac{(1-C_B)((2\beta-q)\gamma-2(k-1)\beta)}{2(k-1)\beta} & \text{a fraction } \alpha_H \text{ of the time} \\ qZ_B^{EH} \text{ a fraction } 1-\alpha_H \text{ of the time.} \end{cases}$
Prices	$P_B = \begin{cases} 1-Q_B^B(Z_B^{EH}) & \text{a fraction } \alpha_H \text{ of the time} \\ (1+C_B)/2 & \text{a fraction } 1-\alpha_H \text{ of the time} \end{cases}$ and $P_I = V - kZ_B^{EH}$.
Profits	$\pi_B = (1-q)(1-C_B)^2/4$ and $\pi_I = \begin{cases} \dfrac{(1-C_B)^2(\beta-\gamma-qD)((2\beta-q)\gamma-2(k-1)\beta)}{2q(k-1)\beta} & \text{a fraction } \alpha_H \text{ of the time} \\ (1-C_B)^2\dfrac{(\gamma-\beta)(\beta-\gamma-qD)}{kq} & \text{a fraction } 1-\alpha_H \text{ of the time.} \end{cases}$

For $D \leq \frac{1}{2}$, the equilibrium falls within case BD: Firm I's product is not compelling to consumers and/or B has lower costs, naturally giving the market to B. Firm B sets its monopoly price and I offers its product at cost, but still is unable to sell any. For $\frac{1}{2} \leq D \leq L$, the equilibrium again falls within case BD. However, I's product, although still inferior to B's, is sufficiently innovative that B's monopoly price is no longer in equilibrium. Instead, B has a constrained monopoly, with I again selling at cost and still selling none. B earns less now than it would if I's product were less innovative (i.e., a lower D).

For $L \leq D < M$, the equilibrium falls within case B, yielding a pure strategy duopoly in the competitive market. For $M \leq D < N$, there is a mixed strategy equilibrium, where I chooses a target greater than that corresponding to its monopoly price such that B is indifferent between the captive market monopoly and the duopoly. This equilibrium derives from a discontinuity in B's optimal response function. There is more competition within this range of D than within the subsequent one where $D \geq N$ and I has a monopoly in the competitive market (case I). In this latter range, B only sells in the captive market and therefore prices at its monopoly price. I's product is so compelling or so low cost that all consumers who want to buy the product and are able to do so over the Internet buy it online.

We now present the Nash equilibrium for the low niche appeal case, $k \leq \frac{1}{2}(2-q+\sqrt{q})$. The main difference between this result and the previous one is that the duopoly in pure strategies occurs only in the high niche appeal case. The boundaries between the relevant regions for D are also simpler in this case.

Theorem 2 *If $k \leq \frac{1}{2}(2 - q + \sqrt{q})$, then the unique Nash equilibrium prices, quantities and profits are as follows.*

	Firm B Monopoly	*Firm B Constrained Monopoly*	*Competitive / Captive Market Monopoly*
Conditions	$0 < D \leq \frac{1}{2}$	$\frac{1}{2} \leq D < (1+\sqrt{q})/2$	$1+\sqrt{q} \leq D$
Targets	$Z_B = Z_B^I$ *and* $Z_I = Z_I^{MC}$.	$Z_B = Z_B^{MC}$ *and* $Z_I = Z_I^{MC}$.	$Z_B = Z_B^I$ *and* $Z_I = Z_I^I$.
Quantities	$q_B = Z_B^I$ *and* $q_I = 0$.	$q_B = Z_B^{MC}$ *and* $q_I = 0$.	$q_B = (1-q)Z_B^I$ *and* $q_I = qZ_I^I$.
Prices	$P_B = \frac{1+C_B}{2}$ *and* $P_I = C_I$.	$P_B = 1 - Z_B^{MC}$ *and* $P_I = C_I$.	$P_B = \frac{1+C_B}{2}$ *and* $P_I = \frac{V+C_I}{2}$.
Profits	$\pi_B = \frac{(1-C_B)^2}{4}$ *and* $\pi_I = 0$.	$\pi_B = (1-C_B)^2(1-D)D$ *and* $\pi_I = 0$.	$\pi_B = \frac{(1-q)(1-C_B)^2}{4}$ *and* $\pi_I = \frac{qD^2(1-C_B)^2}{4k}$.

	Mixed Strategy Equilibrium
Conditions	$(1+\sqrt{q})/2 \leq D < 1+\sqrt{q}$
Targets	$Z_B = \begin{cases} kZ_B^{EL} & \text{a fraction } \alpha_L \text{ of the time} \\ Z_B^I & \text{a fraction } 1-\alpha_L \text{ of the time} \end{cases}$ *and* $Z_I = Z_B^{EL}$.
Quantities	$q_B = \begin{cases} kZ_B^{EL} & \text{a fraction } \alpha_L \text{ of the time} \\ (1-q)Z_B^I & \text{a fraction } 1-\alpha_L \text{ of the time} \end{cases}$ *and* $q_I = \begin{cases} 0 & \text{a fraction } \alpha_L \text{ of the time} \\ qZ_B^{EL} & \text{a fraction } 1-\alpha_L \text{ of the time.} \end{cases}$
Prices	$P_B = \begin{cases} 1-kZ_B^{EL} & \text{a fraction } \alpha_L \text{ of the time} \\ \frac{1+C_B}{2} & \text{a fraction } 1-\alpha_L \text{ of the time} \end{cases}$ *and* $P_I = V - kZ_B^{EL}$.
Profits	$\pi_B = \frac{(1-q)(1-C_B)^2}{4}$ *and* $\pi_I = \begin{cases} 0 & \text{a fraction } \alpha_L \text{ of the time} \\ \frac{(1-C_B)^2(1+\sqrt{q})\,q\,(2D-1-\sqrt{q})}{4k} & \text{a fraction } 1-\alpha_L \text{ of the time.} \end{cases}$

5 Comparative Statics for the High Niche Appeal Case

It is useful to examine how the competitive consequences, including prices, quantities, and profits, depend on the three parameters of our model. In particular, it is plausible that this price competition will be repeated over time for some specific products, with the only substantive change being in the parameter values. For example, more customers are likely to have access to the Internet over time, increasing q, the penetration parameter. Of course, a large scale security scare, such as credit card fraud, would likely reduce q. A cost reduction for units sold over the Internet would increase D, the degree of innovation. The addition of features that would appeal primarily to customers who value the product the most would likely increase both k, the niche appeal, and D.

We provide comparative statics only for the case of high niche appeal, $k > \frac{1}{2}(2 - q + \sqrt{q})$. Technically, this case is not that restrictive, as it will always hold if $k > 1.125$. Furthermore, it would seem to be an appropriate fit for any product whose "best" customers (those who value it the most) are time-starved and, therefore, value the online purchase option. Furthermore, our analysis is suggestive of what might be done in the low niche appeal case.

A market structure map, as in Figure 1, in which one parameter is held constant ($k = 1.2$ in this case), and the other two are varied, is a convenient way to view the competition and draw insights. If we assume that k will not change over time, and assume a trajectory of both q and D over time, we can plot that trajectory on Figure 1 and estimate when the market structure will change and in what way. That much can be accomplished solely using Theorem 1. However, we want to say more about how the equilibrium changes, such as how the respective profits and sales change, on such a trajectory. The roles of the boundary functions L, M, and N are already clear from Theorem 1. We now formally introduce $S = S(q,k)$, defined in the Appendix, which we call the *strategic value* of D. The justification for this label will come shortly. We first characterize L, M, N and S as functions of the parameters. (The proofs for all parts can be found in Druehl and Porteus 2005 and are straightforward applications of first and second partial derivatives.)

Lemma 3 *(a) $L(q,k)$ is increasing and concave in q for each k.*

(b) $L(q,k)$ is decreasing and convex in k for each q.

(c) $M(q,k)$ is convex in k for each q.

(d) $N(q,k)$ is increasing in q for each k.

(e) $N(q,k)$ is decreasing in k if and only if $1 < k \leq \frac{1}{2}(2 - q + \sqrt{q})$ and increasing in k if and only if $k > \frac{1}{2}(2 - q + \sqrt{q})$, for each q.

(f) $S(q,k)$ is increasing in q if and only if $0 \leq q \leq 4(k-1)$ and decreasing in q if and only if $q > 4(k-1)$ for each k.

(g) $S(q,k)$ is increasing in k if and only if $1 < k \leq 1 + q/4$ and decreasing in k if and only if $k > 1 + q/4$ for each q.

Our next results apply to the case of $L(q,k) \le D < M(q,k)$ in which a pure strategy duopoly holds.

Lemma 4 *Suppose that $L(q,k) \le D < M(q,k)$.*

(a) Firm I's profits are convex and increasing in D.

(b) Firm B's profits are convex and decreasing in D.

(c) Both firms' prices decrease in q.

(d) Both firms' prices increase in k.

As D increases, (by part (a)) the marginal return to firm I increases. We discuss the managerial consequences of this phenomenon in §6. At the same time, B is being hurt, but at a decreasing rate. As q increases, prices fall, essentially because of the increased competition. As k increases, we have the opposite effect: A smaller group (niche) of potential customers finds product I attractive, which effectively reduces competition, causing prices to increase.

Despite the fact that each firm's price falls as penetration increases, the quantity sold by each firm (or equivalently, market) may increase or decrease. Two new functions, $T = T(q,k)$ and $U = U(q,k)$, defined in Table 4 in the Appendix, further subdivide the duopoly region in terms of how the equilibrium unit sales change as a function of q. (It is straightforward to show that $L(q,k) \le T(q,k) \le U(q,k) \le M(q,k)$.)

Theorem 5 *Suppose $L(q,k) \le D < M(q,k)$. For each k,*

	As q Increases:	
Range of D	*I's Unit Sales*	*B's Unit Sales*
$D \le T(q,k)$	*Decrease*	*Increase*
$T(q,k) \le D \le U(q,k)$	*Increase*	*Increase*
$U(q,k) \le D$	*Increase*	*Decrease*

As q increases, prices will decrease (Lemma 4) and unit sales to each firm can either increase or decrease (Theorem 5). Our next result shows how the firms' profits change.

Theorem 6 *Suppose $L(q,k) \le D < M(q,k)$.*

(a) Firm B's profits are decreasing in q.

(b) Firm I's profits are decreasing in q if $L(q,k) \le D \le S(q,k)$, and are increasing in q if $L(q,k) \le S(q,k) \le D < M(q,k)$.

In short, firm B is always hurt when q increases. However, as illustrated earlier in Figure 2, and now verified by part (b), I is hurt (when q increases) if D is below S and is helped if $D \geq S$. This justifies calling S the strategic value of D. It is straightforward to show that $S(q,k) \geq T(q,k)$ and, if $k \leq 1 + q/2$, then $S(q,k) \geq U(q,k)$.

Next we turn to the effects of k. We saw in Lemma 4(d) that both firms' prices increase in k. Our next two results show that S, T and U are again key cutoffs in determining how unit sales and profits change as a function of k.

Theorem 7 *Suppose $L(q,k) \leq D < M(q,k)$. For each q,*

	As k Increases:	
Range of D	*I's Unit Sales*	*B's Unit Sales*
$D \leq T(q,k)$	*Increase*	*Decrease*
$T(q,k) \leq D \leq U(q,k)$	*Decrease*	*Decrease*
$U(q,k) \leq D$	*Decrease*	*Increase*

Theorem 8 *Suppose $L(q,k) \leq D < M(q,k)$.*

(a) Firm B's profits are increasing in k for each q.

(b) Firm I's profits are increasing in k if $L(q,k) \leq D \leq S(q,k)$ and are decreasing in k if $L(q,k) \leq S(q,k) \leq D \leq M(q,k)$ for each q.

Theorems 7 and 8 show that increasing k has the same effect as decreasing q (in the pure strategy duopoly region). In particular, if $D < S$, then firm I's profits increase in k and decrease in q. In this case, the online product is not particularly innovative. If k increases, then the online product appeals to a smaller market niche, leading B to raise its price and profits, and I to raise its price to the extent that its total profit increases. A similar thing happens as q decreases: Firm B faces less competition and increases its price, while I can raise its price and increase its profits (as shown in Figure 2). In both cases, firm I can increase its profits (thereby allowing B to increase its profits too) by being less threatening. We therefore say that firm B is the stronger firm and I is the weaker firm when $D < S$.

If $D > S$, then the opposite is true: I's profits increase if either k is decreased or q is increased. In this case, the online product is sufficiently innovative that both these changes increase competition, lower prices, and threaten firm B with lower profits. If q increases, B faces more competition and both firms lower their prices, but I is able to increase its profits because its product is sufficiently strong that its price decrease is more than compensated by its increase in sales. A similar thing happens if k decreases: The online product appeals to a larger market niche, leading both B and I to lower their prices. I's profits will again increase, because its price decrease is more than compensated by its increase in market. We therefore say that B is the weaker firm and I is the stronger firm when $D > S$.

Figure 2 gives a three dimensional view of this effect, for the case of $k = 1.9$ being held constant. For values of D between 0.5 and 0.8, firm I's profits initially increase in q and then decrease. If I finds itself in the range where profits are decreasing in q, it must increase D and/or k to maintain its profit level as q continues to increase. Figure 1 held k constant and varied q and D. Figure 5 holds $q = 0.9$ constant, and varies k and D. For example, if we hold $q = 0.9$ and $D = 1$ constant and increase k, moving horizontally to the right, we see that I's profits will increase up until about $k = 1.82$, because $D < S$ in this interval, and I's profits will decrease thereafter.

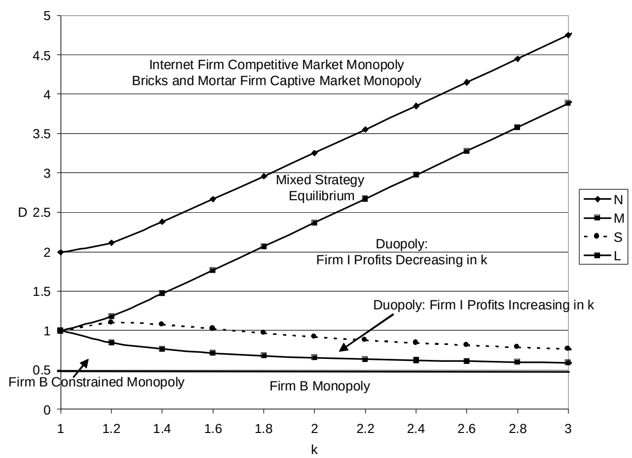

Figure 5: Market Structure for $q = 0.9$ as k and D Vary

We now briefly examine profits in the double monopoly, which occurs when $D \geq N(q, k)$.

Theorem 9 *Suppose that $D \geq N(q, k)$.*

(a) Firm I's profits are increasing, and firm B's profits are decreasing, in q.

(b) Firm I's profits are increasing in, and firm B's profits are independent of, D.

(c) Firm I's profits are decreasing in, and firm B's profits are independent of, k.

When the online firm has a monopoly, increasing q or D is beneficial, while an increase in k, targeting a niche, is detrimental.

6 Managerial Insights

We provide managerial insights within our environment. Our model is a simplified version of reality, and, hence, may not directly apply. For example, in practice, there will be many more than just one online firm and one offline firm. (We summarize some of the other assumptions in §7.) However, useful insights can be drawn and we provide parallels to what is empirically observed in online versus offline competition.

As discussed in the Introduction, our model offers a possible rationale for the unwarranted enthusiasm seen for online firms in the beginning of the Internet boom. Because of the interaction between D, q, and S, the Internet firm's profits may actually decrease as more people shop online. In short, just because the online firm starts off well does not mean it will remain profitable as q increases. When we consider fixed costs, the high failure rates of e-businesses become obvious.

Our analysis may give guidance to the online firm in the decision of whether to focus on a niche or mass market. If the online firm focuses on its niche, those who value the product the most, then it may offer special services and information that such people are apt to find valuable. For our purposes, we consider focusing on the niche market as increasing both V and k. This makes the product more attractive to those in the niche, those who value the product the most, and makes it less attractive to most other potential customers. Alternatively, the firm could appeal to the mass market, which we consider to be decreasing both V and k. As discussed in §5 and in particular in Theorem 8, if $D < S$, then k should be increased to increase the niche appeal in order to benefit the online firm. If $D > S$, then the online firm should focus on the mass market by decreasing k.

6.1 Pivotal Role of Degree of Innovation

The degree of innovation plays a pivotal role in our analysis. Figures 1 and 6 both reveal that if D is large enough, specifically greater than the strategic value S, then the online firm's profits will continue to increase over time (as q continues to increase). From Theorems 5 and 6, we find that for a low value of D where $L(q,k) \leq D \leq T(q,k)$, I's market and profits will decrease as q increases. I's product is not compelling enough to consumers or low cost enough to take advantage of penetration increases. If D is higher so that $T(q,k) \leq D \leq S(q,k)$, then penetration increases will lead to market growth only. For relatively higher values of D, specifically $S(q,k) \leq D \leq M(q,k)$, I experiences both market and profit growth as q increases. In Figure 1, only a small portion of the duopoly region is located above $S(q,k)$. Thus firm I will be mostly limited to growing its market while competing in the duopoly as penetration increases. Fortunately, this is not always the case. Figure 6 shows an example where firm I can grow profits as q increases for most of the duopoly region. By taking actions that would sufficiently increase D, the online firm can strategically place itself into a region of market and profit growth.

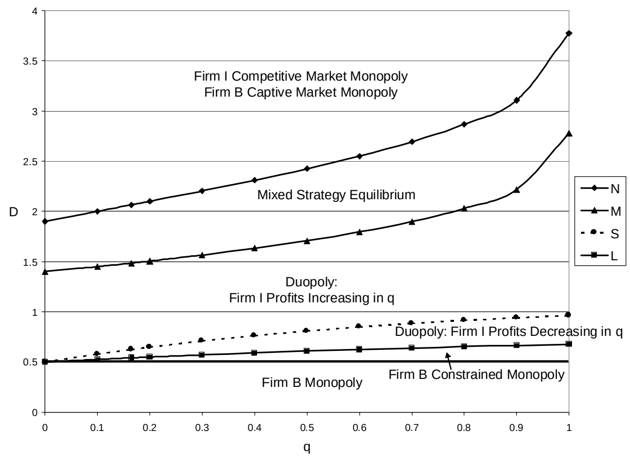

Figure 6: Market Structure for $k = 1.9$ as q and D Vary

There are many other ways in which the key role of D can be seen. Figure 1 shows that it must be nearly 2.4 to ensure that the online firm can dominate the competitive market when $k = 1.2$ and almost all consumers consider both channels. That is, the online firm's monopoly margin must be 2.4 times that of the offline firm. In most practical settings, that is vastly more than a slight innovation. In short, the online product must be vastly superior. The requirement is even higher in Figure 6, where D must be more than about 3.7. It is straightforward to see that the ratio of competitive market monopoly profits to firm B monopoly profits is qD^2/k (from Theorem 1). Thus, in the case shown in Figure 6, the offline firm will eventually go out of business and the online profits will approach $D^2/k = 7.2$ times what the offline firm started with (as q approaches one). The presence of the online firm and the resulting competition increases the size of the market, resulting in this large gain in profits over the initial market.

Lemma 4 gives compelling evidence of the key role played by D. It says that if the firms are competing in the pure strategy duopoly region, then online profits are convex increasing in D. That is, in contrast to the usual reasoning in microeconomics leading to decreasing marginal returns, the marginal return from an increased degree of innovation *increases*! This is a little like compound interest: The return from two small improvements is more than the sum of the returns from each.

Thus, it makes sense to discuss some of the ways in which D might increase. Note that the online firm's profits are not strictly increasing in D in every region: They are zero for all low values of D, when the offline firm has a monopoly or a constrained monopoly.

An obvious way in which D will increase is to lower the unit cost of providing the product online. The bad news is that much of the cost advantage that the online firm might have in providing the product comes from lower overhead, rather than variable cost. For example, the online firm need not have expensive retail space, including showrooms and nearby stockrooms. Instead, a single stock point (allowing inventory pooling) can serve customers who would otherwise require numerous retail outlets. Conversely, the cost of picking a unit from storage and shipping usually involves a higher variable cost than an offline product. Nevertheless, some of the overhead cost reductions may convert into lower variable costs.

Another way in which D increases is for the reservation price for the online product to shift upward. This happens if customers value their time more and thereby assign higher value to the time they would take to search for the product, get to the offline outlet, and wait, if necessary, for it to be restocked, should it be out of stock. This also can happen if the online brand is recognized as signaling higher quality than the offline brand.

6.2 Early Adoption Pricing

Figures 1 and 6 show that if D is reasonably high and q is near zero, the market structure would be a double monopoly. If q starts near zero at the advent of the online shopping era, then the few customers who buy online then can be considered to be the early adopters. In our model, this phenomenon occurs not because these customers are willing to pay more to be the first to obtain something (this would inflate their reservation prices for both channels), but from their high reservation prices for buying online (they may want to be the first to buy online). The offline firm is not threatened by the new technology and continues to charge monopoly prices. However, as can be seen in Figure 1 as q increases through 0.2 for the case of $D = 1.5$, if q continues to increase over time, the online firm can become enough of a threat to the offline firm that a mixed strategy competitive equilibrium arises, and prices drop. Empirical research (Bailey 1998a,b, Brynjolfsson and Smith 1999) on Internet pricing for books and CDs supports such an early adoption pricing pattern. Online prices were higher in early years and decreased as more consumers shopped online.

6.3 Practical Examples

It is interesting to discuss some practical examples to get a sense how our model connects with reality. Our discussion will assume that our model applies in each environment, without repeating our assumptions each time.

Online brokerage is an interesting example. The variable cost of transacting a trade is much less for the online firm, as little or no human intervention, including commissions, is required. This argues that D is high, which is supported by the fact that online brokers, such as Charles Schwab, were, in the late 1990s, increasingly offering services similar to those of the full service brokers, but at a much lower cost and a much lower price. Indeed, online trading was estimated to be 20% of the market in 1998 and growing to 49% in 2000 (Spiro 1999). Merrill Lynch, facing an 85% compression in its trades (Spiro 1999), realized that it could no longer charge premium prices for a commoditized trading service and changed its business model to start charging for advice and money management instead (Altschuler et al. 2002). The fact that the online brokers were not generally making profits at that time suggests that they were incurring significant fixed overhead costs. Some recent refinements to the basic online brokerage model, such as higher fees for low volume traders, may be designed to ameliorate that problem. The traditional telephone brokerage firm is destined for obscurity.

NetFlix introduced an online subscription DVD rental business, in which customers pay a monthly fee to have a certain number, such as three, of DVDs in their possession at any time. Newly requested DVDs are mailed upon receiving old DVDs in the mail. Some customers find it appealing that if they are slow to return their DVDs by mail, they simply have to wait longer to get new ones, rather than pay punitive late fees. We interpret the fact that Blockbuster and Wal-Mart responded by offering trial versions of subscription rental businesses and Blockbuster significantly changed its penalty structure as evidence that NetFlix poses a significant threat, which in our model would mean that D is high. In this case, we are seeing a different service model, one that may be particularly well-suited to the Internet.

Purchasing diamonds online appeals only to a particular niche of customers. Blue Nile has taken advantage of such a niche. By offering services such as courier delivery and educational information, combined with reasonable prices, it targets men looking for information so they can feel comfortable buying diamond jewelry. Customers who trust the information provided will value the service, while others may only trust an in-store personal interaction with a sales person. If this primarily increases V and leaves k unchanged, then D increases and Blue Nile is positioned to be very profitable as q increases over time. If k also increases, then Blue Nile would remain niche-focused.

The Wine Buyer sends e-mails to potential customers on highly rated wines that it offers on sale. It provides reviews and ratings by independent sources, such as Wine Spectator and The Wine Advocate. Time-starved enjoyers of a good glass of wine may value this service, while others may consider it to be a nuisance. We interpret this approach as focusing on the niche market. However, our model is not high fidelity in this regard because we assume there is a specific product being sold by both firms, rather than a product category, as in this example. Our model is silent on product choice from a category.

Toys R Us's inability to fulfill online orders for the holiday season in 1999 likely convinced some people to avoid Toys R Us online completely. If this primarily increased k and left V unchanged, the niche who found the online product attractive was reduced in size. However, the mishap likely reduced consumer's valuation of the service, resulting in a lower V as well, thus further reducing the likelihood this niche would buy online.

More consumers are turning to online grocery shopping: 4.5 million households shopped for groceries online in 2000, and Forrester predicted that 8.2 million households would in 2004 (Rubin 2001). Webvan, a well-known but now defunct e-grocer, was designed with a mass market in mind, with a specialized cost-efficient infrastructure requiring high volume to work well, and it failed because it was unable to attract enough volume. It seems unlikely that unit costs of providing groceries online can be much lower than conventional groceries without such a specialized infrastructure. Thus, D cannot get very high, and we therefore expect this to remain a niche market, with both V and k high.

7 Summary, Limitations, and Extensions

Using the degree of innovation, we characterize the market structure for the dual market competition and discuss the pivotal role of the degree of innovation. A higher degree of innovation always increases profits for the Internet firm (and dramatically so because profits are convex increasing in the degree of innovation) and decreases them for the bricks and mortar firm. Even within a duopoly, there is reason for the firms to consider the degree of innovation. As the niche appeal or penetration increases, $S(q, k)$ is the single crucial value of the degree of innovation that determines whether or not the Internet firm will increase its market and/or profits. From the market structure maps, we observe that even when all consumers are willing to shop online, there still *may* be a role for both Internet and bricks and mortar firms for most values of D. Only when there are extreme differences in the maximum possible margins, resulting in a very low or very high value of D, will a monopoly for either firm emerge. Indeed, for the firms to maintain D in the duopoly region requires competencies in cost management, service execution, and marketing. In order to increase its highest reservation price (V for firm I), a firm needs to increase consumers' perception of value and improve the execution of the service. Operational improvements that will affect costs are numerous, including supply chain improvements, inventory efficiency, and manufacturing productivity.

Moreover, we show that, at odds with one's expectations, as penetration increases, the Internet firm's profits may decrease, all else equal. When the online firm is not worse off in terms of profits, the market structure changes from a less competitive market structure to a more competitive one as penetration increases. An increase in Internet shopping penetration alone is not enough to improve the online firm's market position. This helps to explain how at the beginning of the Internet boom, some online firms looked promising, but quickly failed.

We recognize that our model is only an approximation of reality, even if the market had only these two channels. Instead of observing price on both channels, a consumer may look on the web to check the price on the Internet channel, and use that information to decide whether to make the effort to determine the price (and availability) on the traditional channel. Another consumer may go directly to the traditional outlet, without consulting the Internet first, and, finding the item in stock at a certain price, may decide whether to buy immediately, assuming the surplus is positive. We have assumed linear, downward sloping reservation price curves with specific parameter ranges, $V \geq 1$ and $k > 1$. An obvious extension is to examine other parameter ranges. It would be possible, albeit cumbersome, to generalize our model to piecewise linear reservation price curves, possibly with different q-values for different segments. It would be interesting to make our model probabilistic, such as assuming q is the probability that a consumer type will compare prices on the two channels. It would also be interesting to incorporate fixed costs into the analysis, so that an online firm could achieve dominance by driving the bricks and mortar firm's profits down below its fixed costs, forcing it out of business.

A Appendix

A.1 Development of Optimal Response Functions

It is straightforward to determine the optimal response function for each firm: For each possible competitor target, the firm knows what its return would be, as a function of its response if that response puts it into each of the three cases. That return is a concave function of its response in each case, so it is easy to find the best response for each achievable case. Finally, the firm picks the best achievable case. The results are given in the next three theorems, whose proofs can be found in Druehl and Porteus (2005). Notation is also defined in Tables 3 and 4 and is interpreted shortly after first being used. For brevity, we omit the dependencies on k and q in this section. For example, $Z_I^I(k)$ is written as Z_I^I.

Theorem 10 *The optimal response functions are as follows.*

Firm I

$$Z_I^*(Z_B) = \begin{cases} Z_I^I & \text{if } 0 \le Z_B \le Z_I^I \\ Z_B & \text{if } Z_I^I \le Z_B \le Z_B^{IB} \\ Q_I^B(Z_B) & \text{if } Z_B^{IB} \le Z_B \le Z_B^{MC} \\ Z_I^{MC} & \text{if } Z_B^{MC} \le Z_B. \end{cases}$$

Firm B

If $k > \frac{1}{2}(2 - q + \sqrt{q})$	If $k \le \frac{1}{2}(2 - q + \sqrt{q})$
$Z_B^*(Z_I) = \begin{cases} Z_B^I & \text{if } 0 \le Z_I \le Z_B^I/k \\ kZ_I & \text{if } Z_B^I/k \le Z_I \le Z_I^{BBD} \\ Q_B^B(Z_I) & \text{if } Z_I^{BBD} \le Z_I \le Z_B^{EH} \\ Z_B^I & \text{if } Z_B^{EH} \le Z_I. \end{cases}$	$Z_B^*(Z_I) = \begin{cases} Z_B^I & \text{if } 0 \le Z_I \le Z_B^I/k \\ kZ_I & \text{if } Z_B^I/k \le Z_I \le Z_B^{EL} \\ Z_B^I & \text{if } Z_B^{EL} \le Z_I. \end{cases}$

The quantity Z_I^I is firm I's monopoly target (corresponding to its monopoly price in the competitive market), which it will select if firm B selects a smaller target, corresponding to a relatively high price. $Q_I^B(\cdot)$ gives firm I's optimal unconstrained target, as a function of B's target, assuming that it receives the returns specified in case B. Z_B^{IB} and Z_B^{MC} give the lower and upper limits, respectively, to Z_B such that this response meets the requirements of case B, namely that $Q_I^B(Z_B) < Z_B < kQ_I^B(Z_B)$. When firm B's target is below Z_B^{IB} yet above Z_I^I, it is optimal for firm I to select the boundary point between cases I and B, namely to match firm B's target. At the other extreme, if firm B targets above Z_B^{MC}, then firm I targets Z_I^{MC}, which corresponds to its pricing at marginal cost.

A full specification of firm B's optimal response function is a little trickier because one of the possibilities when the niche appeal is high disappears when the niche appeal is low. First consider the high niche appeal case. The quantity Z_B^I is firm B's target, corresponding to its monopoly price, which is its unconstrained optimal price in both cases I and BD. That price is feasible for case BD if $kZ_I \leq Z_B^I$, which explains the first line. The function $Q_B^B(\cdot)$ is firm B's unconstrained optimal response function in case B. For $Z_I \leq Z_I^{BBD}$, $\bar{Z}$ will be negative, so there will be no consumers who prefer product I, resulting in case BD. When I's target is between Z_B^I/k and Z_I^{BBD}, it is optimal for firm B to choose the boundary point between cases B and BD, namely kZ_I, which explains the second line. If $Z_I \geq Z_I^{BBD}$, then firm B can pick to be in either case B or I. The quantity Z_B^{EH} is the point at which the unconstrained optimal returns in cases B and I are equal: $\pi_B^I(Z_B^I) = \pi_B^B(Q_B^B(Z_B^{EH})|Z_B^{EH})$. The proof shows that there exists a positive interval within which both the case B and I unconstrained optimal solutions are feasible and Z_B^{EH} falls within this interval. This explains the third and fourth lines. It is straightforward to see that firm B's optimal response function is continuous except at Z_B^{EH}, where there are two distinct optimal responses, $Q_B^B(Z_B^{EH})$ and Z_B^I. Indeed, B's optimal response increases in Z_I up until Z_B^{EH}, where it makes a discontinuous drop. This discontinuity leads to the existence of a mixed strategy Nash equilibrium.

For the low niche appeal case, it is never optimal for firm B to respond in case B. Firm B is unable to compete profitably in a duopoly (case B) and prefers to either dominate the market (case BD) when I sets a small target (and a high price) or to cede the competitive market to I and focus on the captive market (case I). Similar to the high niche appeal situation, there is a range of I's target where B's response for two cases, I and BD now, are feasible. At I's target Z_B^{EL}, B's optimal response changes.

A.2 Summary of Notation

Table 3: Notation

Notation	Interpretation	Definition
Z_B^I	B's target corresponding to setting price at the monopoly level	$\frac{1-C_B}{2}$
$Z_I^{MC} = Z_I^{MC}(k)$	I's target corresponding to setting price equal to marginal cost	$\frac{D(1-C_B)}{k}$
Z_B^{MC}	B's target beyond which firm I's price in case B drops below cost	$D(1-C_B)$
$\beta = \beta(q,k)$	Useful local parameter	$k-1+q$
$\gamma = \gamma(q,k)$	Useful local parameter	$\sqrt{(\beta - kq)\beta}$
$Z_B^B = Z_B^B(q,k)$	Firm B's unconstrained target in case B	$\frac{(1-C_B)(2\beta+qD)}{4\beta-q}$
$Z_I^B = Z_I^B(q,k)$	Firm I's unconstrained target in case B	$\frac{(1-C_B)(2D+1)\beta}{k(3\beta+k-1)}$
$Z_I^I(k)$	I's target corresponding to setting price at the monopoly level	$\frac{D(1-C_B)}{2k}$
$Q_B^B(Z_I,q,k)$	Solution to $\nabla\pi_B^B(Z_B \mid Z_I)=0$	$Z_B^I + \frac{kqZ_I}{2\beta}$
$Z_B^{EH} = Z_B^{EH}(q,k)$	Value of Z_I at which B is indifferent to responding in case I or case B, i.e., $\pi_B^I(Z_B^I) = \pi_B^B(Q_B^B(Z_B^E) \mid Z_B^E)$	$\frac{(1-C_B)(\beta-\gamma)}{kq}$
$Z_B^{EL} = Z_B^{EL}(q,k)$	Value of Z_I at which B is indifferent between case I and case BD when he is constrained, i.e., $\pi_B^I(Z_B^I) = \pi_B^{BD}(kZ_I)$	$\frac{(1-C_B)(1+\sqrt{q})}{2k}$
$Q_I^B(Z_B,k)$	Solution to $\nabla\pi_I^B(Z_I \mid Z_B)=0$	$Z_I^I + \frac{Z_B}{2k}$
$L = L(q,k)$	High niche appeal case boundary between constrained monopoly and duopoly	$\frac{\beta}{2\beta-q}$
$M = M(q,k)$	High niche appeal case boundary between duopoly and mixed strategy equilibrium	$\frac{2\beta-q}{q} - \frac{(4\beta-q)\gamma}{2q\beta}$
$N = N(q,k)$	High niche appeal case boundary between mixed strategy equilibrium and double monopoly	$\frac{2(\beta-\gamma)}{q}$

Table 4: Additional Notation

Notation	Interpretation	Definition
$Z_B^{IB} = Z_B^{IB}(k)$	The lower limit of Z_B such that I's response falls into case B	$\frac{D(1-C_B)}{2k-1}$
$Z_I^{BBD} = Z_I^{BBD}(q,k)$	The lower limit of Z_I such that B's response falls into case B	$\frac{(1-C_B)\beta}{k(2\beta-q)}$
$\alpha_H = \alpha_H(q,k)$	The fraction of the time that B chooses to play its case B response in the high niche appeal case	$\frac{2(k-1)\beta[2\beta-\gamma)-qD]}{\beta q(2D+k)-(4\beta-kq)(\beta-\gamma)}$
$\alpha_L = \alpha_L(q,k)$	The fraction of the time that B chooses to play its case B response in the low niche appeal case	$\frac{2(k-1)\beta\left(1+\sqrt{q}-D\right)}{2D+(1+\sqrt{q})(k-2)}$
$T = T(q,k)$	A cutoff point for D, determining whether quantity sold increases or decreases as either q or k increase	$\frac{(2\beta-q)(2\beta+q)}{8(k-1)^2+8q(k-1)+3q^2}$
$U = U(q,k)$	A cutoff point for D, determining whether quantity sold increases or decreases as either q or k increase	$\max\left[M(q,k), \frac{6(k-1)^2+8q(k-1)+3q^2}{(2\beta-q)(2\beta+q)}\right]$
$S = S(q,k)$	The strategic value of D	$\max\left\{L(q,k), \min\left[M(q,k), \frac{4(k-1)^2+9(k-1)q+3q^2}{8(k-1)^2+6(k-1)q+3q^2}\right]\right\}$

A.3 Proofs

Proof of Lemma 4. Suppose $L(q,k) \leq D < M(q,k)$. (a) Taking the derivative of I's profits with respect to D, we find $\partial\pi_I/\partial D \geq 0$ for $D \geq L(q,k)$. Taking the second derivative with respect to D, we find it is positive. Thus I's profits are increasing and convex in D. (b) Taking the derivative of B's profits with respect to D, we find $\partial\pi_B/\partial D \leq 0$ for $D \leq (2\beta - q)/q$, which is greater than $M(q,k)$. Taking the second derivative with respect to D, we find it is positive. Thus B's profits are decreasing and convex in D. (c) I chooses $P_I(Z_B, Z_I, q, k) = V - kZ_I^B(q,k)$ and B chooses $P_B(Z_B, Z_I, q, k) = 1 - Z_B^B(q,k)$. Then, $\partial(V - kZ_B^I(q,k))/\partial q \leq 0$ and $\partial(1 - Z_B^B(q,k))/\partial q \leq 0$. (d) Taking the derivatives with respect to k of the prices in (c), we find they are positive.■

Proof of Theorem 5. Suppose $L(q,k) \leq D < M(q,k)$. I's quantity sold is $q\bar{Z}$ and B's quantity sold is $Z_B^B(q,k) - q\bar{Z}$. We find that $\partial(q\bar{Z})/\partial q \geq 0$ if $D \geq T(q,k)$. Similarly, we obtain $\partial(Z_B^B(q,k) - q\bar{Z})/\partial q \geq 0$ if $D \leq U(q,k)$. Because $T(q,k) \leq U(q,k)$, it is straightforward to show the results in the theorem.■

Proof of Theorem 6. Suppose $L(q,k) \leq D < M(q,k)$. (a) $\partial\pi_B^B(Z_B^B(q,k), Z_I^B(q,k), q, k)/\partial q \leq 0$ for $D \leq (2\beta - q)/q$, which is greater than $M(q,k)$. Thus B's duopoly profits are decreasing in q. (b) $\partial\pi_I^B(Z_B^B(q,k), Z_I^B(q,k), q, k)/\partial q = (1-C_B)^2\left((2\beta-q)D-\beta\right)/\left((k-1)(4\beta-q)^3\right)$ $\times\left[4(2D-1)(k-1)^2+3(2D-3)(k-1)q+3(D-1)q^2\right]$. The term $((2\beta-q)D-\beta)$ is positive if $D \geq L(q,k)$. The term $4(2D-1)(k-1)^2 + 3(2D-3)(k-1)q + 3(D-1)q^2$ is positive if $D \geq S(q,k)$ and negative if $D \leq S(q,k)$. Because $S(q,k) \geq L(q,k)$, if $L(q,k) \leq D \leq S(q,k)$, then I's duopoly profits are decreasing in q. If $L(q,k) \leq S(q,k) \leq D \leq M(q,k)$, then I's duopoly profits are increasing in q.■

Proof of Theorem 7. Suppose $L(q,k) \leq D < M(q,k)$. I's quantity sold is $q\bar{Z}$ and B's quantity sold is $Z_B^B(q,k) - q\bar{Z}$. We find that $\partial(q\bar{Z})/\partial k \geq 0$ if $D \leq T(q,k)$. Similarly, we obtain $\partial(Z_B^B(q,k) - q\bar{Z})/\partial k \geq 0$ if $D \geq U(q,k)$. Because $T(q,k) \leq U(q,k)$, it is straightforward to show the results in the theorem.■

Proof of Theorem 8. Suppose $L(q,k) \leq D < M(q,k)$. (a) $\partial\pi_B^B(Z_B^B(q,k), Z_I^B(q,k), q, k)/\partial k$ ≥ 0 for $L(q,k) \leq D < M(q,k)$. Thus, profits increase in k. (b) $\partial\pi_I^B(Z_B^B(q,k), Z_I^B(q,k), q, k)/\partial k =$ $-(1-C_B)^2 q((2D-1)(k-1)+q(D-1))\times\left[4(2D-1)(k-1)^2+3(2D-3)(k-1)q+3(D-1)q^2\right]$ $/\left((k-1)^2(4\beta-q)^3\right)$. The term $(2D-1)(k-1)+q(D-1)$ is positive if $D \geq L(q,k)$. The term $-(4(2D-1)(k-1)^2+3(2D-3)(k-1)q+3(D-1)q^2)$ is positive if $D \leq S(q,k)$. The remaining terms are all positive. Thus if $D \leq S(q,k)$, profits increase in k.■

Proof of Theorem 9. Suppose $D \geq M(q,k)$. (a) $\partial\pi_I^I(q,k)/\partial q = 0$ and $\partial\pi_B^I(q)/\partial q < 0$. (b) $\partial\pi_I^I\left(Z_B^I, Z_I^I\right)/\partial D$ is minimized at $D = N(q,k)$, the closest feasible point. The second partial is strictly positive so it is convex in D. As it is convex and minimized at its lower boundary, it is increasing in D. B's captive monopoly profits do not depend on D, hence $\partial\pi_B^I\left(Z_B^I\right)/\partial D = 0$. (c) $\partial\pi_I^I\left(Z_B^I, Z_I^I\right)/\partial k < 0$. B's captive monopoly profits do not depend on k, hence $\partial\pi_B^I\left(Z_B^I\right)/\partial k = 0$.■

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