

**The Impact of Metaheuristics on Solving the Vehicle Routing Problem:
Algorithms, Problem Sets, and Computational Results**

by

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Focus of Paper

- Briefly review the history of the VRP (pre-1990)
- Talk about recent approaches (metaheuristics)
- Discuss our compilation of recent computational results from the literature
- Make some observations regarding this literature
- Solve some larger VRPs
- Discuss some future directions in vehicle routing research

Introduction to the Standard VRP

- Single depot
- Each vehicle departs from and returns to the same depot
- Known customer locations and demands
- Each customer is visited once
- Homogeneous fleet of vehicles
- Customer demand must be fully satisfied
- Capacity constraints
- Minimize the total distance traveled by the fleet
- Route-length constraints

Issues not Addressed in this Paper

- VRP variants

 - Multiple depots

 - Time windows

 - Backhauls

 - Period routing

 - Mixed Fleet

 -
 -
 -

- Number of vehicles in the solution

A Brief History of Vehicle Routing

1950s The VRP is formulated as an IP

Small problems (10 to 20 customers) are “solved”

1960s Early route-building heuristics (e.g., Clarke & Wright) are proposed

2-opt and 3-opt are applied to the VRP (Christofides & Eilon)

Problems with 30 to 100 customers are “solved”

1970s A number of two-phase heuristics are proposed (e.g., Gillett & Miller)

Computational efficiency becomes an issue (e.g., Golden, Magnanti & Nguyen)

Larger problems (100 to 1,000 customers) are “solved”

Some problems with 25 to 30 customers can be solved using optimal methods

1980s Mathematical programming-based procedures are proposed (e.g., Fisher & Jaikumar)

Interactive (man-machine) heuristics are developed (e.g., Cullen, Jarvis & Ratliff)

Some problems with approximately 50 customers can be solved using optimal methods

1990s Metaheuristics are applied to the VRP

Some problems with 50 to 100 customers can be solved using optimal methods (Fisher)

Metaheuristics

- Simulated annealing (1983)
- Deterministic annealing (1990)
 - Threshold accepting
 - Record-to-record travel
 - Great deluge algorithm
- Genetic algorithms (mid-1980s)
- Neural networks (1985)
- Tabu search (1986)

Test Problems from the Literature

- There are 14 benchmark problems
- All problems are described in a 1979 article by Christofides, Mingozzi, and Toth
- They range in size from 50 to 199 nodes
- Seven of the problems have route-length constraints, as well as capacity constraints
- Ten have randomly distributed customers
- In four problems, customers appear in clusters
- In the last ten years, the focus has been on quality of solution

How Large are the VRPs that MS/OR Needs to Solve?

- In a 1986 *EJOR* article, we solve real-world problems with 368 and 650 deliveries per day
- In a 1987 *OR* article, we discuss the application of vehicle routing software to a Mid-Atlantic Coca-Cola problem with more than 1200 customers serviced daily
- The average number of customers visited daily in soft drink and beer distribution is approximately 600
- In sanitation (node) routing, the number of sites visited daily is often between 200 and 1000
- In general, companies with routing problems that involve fewer than 200 stops per day have difficulty justifying the purchase of routing software
- Most real-world applications of vehicle routing software involve at least 200 stops per day
- Bottom line: Heuristics remain the only practical option

Why is the Cost of Running Vehicle Routing Software so High?

- Data
 - street database
 - address match customers

- Customization and consulting
 - special constraints
 - interface with billing system

- Internal project management

- Total cost might range from 3 to 6 times the cost of software

Some Recent Approaches to the VRP

Neural Networks
Ghaziri

Genetic Algorithms
Van Breedam

Simulated Annealing
Robusté et al.
Alfa
Osman
Van Breedam

Tabu Search
Taillard
Semet and Taillard
Gendreau et al.
Rochat and Taillard
Rego and Roucairol
Xu and Kelly
Van Breedam

Observations from the Recent Literature

□ Overall Performance

- ◆ NN approaches are not competitive
- ◆ SA has been found to outperform GA
- ◆ TS has been found to outperform SA
- ◆ TS seems to work best when infeasible intermediate solutions are allowed via penalty functions

□ Computational Experimentation

- ◆ Recent papers use real distances between customers
- ◆ In general, solution quality has been emphasized at the expense of running time
- ◆ TS procedures perform well, but involve a large number of parameters
- ◆ Parameter values are adjusted in an ad hoc way
- ◆ Some TS procedures have been parallelized

Observations from the Recent Literature (continued)

□ Computational Reporting

◆ Two ways to report results

☞ Some authors only report best results

☞ Other authors report single pass results, as well as best results

◆ Four ways to report running times

☞ No running time

☞ Time to obtain the best solution

☞ Time to obtain a solution within 1%, 5% of the best-known solution

☞ Total computation time

◆ Some authors include the routes of their best solutions

Tabu Search VRPs

Summary of Solutions to VRPs from CMT: Tabu Search Procedures

<i>Pr</i>	<i>n</i>	<i>G91</i>	<i>T92</i>	<i>T93</i>	<i>OTSF93</i>	<i>OTSB93</i>	<i>GSP94</i>	<i>GB94</i>	<i>RT95</i>	<i>RRS95</i>	<i>RRP95</i>	<i>XK96</i>
1	50	524.61	524.61	524.61	524.61	524.61	524.61	524.61		524.61	524.61	524.61
2	75	836.37	835.32	835.26	844	844	835.77	835.32		837.50	835.32	835.26
3	100	826.14	828.98	826.14	838	835	829.45	826.14		827.53	827.53	826.14
4	150	1034.90	1029.64	1028.42	1044.35	1052	1036.16	1031.07		1054.29	1044.35	1029.56
5	199	1329.29	1300.89	1298.79	1334.16	1354	1322.65	1311.35	1291.45	1338.49	1334.55	1298.58
6	50	555.43	555.43	555.43	555.44	555.44	555.43	555.43		555.43	555.43	
7	75	913.23	909.67	909.68	911	913	913.23	909.68		909.68	909.68	
8	100	865.94	865.94	865.94	878	866.75	865.94	865.94		868.29	866.75	
9	150	1189.79	1164.24	1162.55	1184	1188	1177.76	1162.89		1178.84	1164.12	
10	199	1421.88	1403.21	1397.94	1441	1422	1418.51	1404.75	1395.85	1420.84	1420.84	
11	120	1043.94	1073.05	1042.11	1043	1042.11	1073.47	1042.11		1043.54	1042.11	1042.11
12	100	822.85	819.56	819.56	819.59	819.59	819.56	819.56		819.56	819.56	819.56
13	120	1551.63	1550.15	1541.14	1545.98	1547	1573.81	1545.93		1550.17	1550.17	
14	100	866.37	866.37	866.37	866.35	866.35	866.37	866.37		866.53	866.37	

Bold indicates best-known solution. Problems 1 and 12 are optimal solutions.

Tabu Search Procedures

G91 Gendreau et al. June 1991 working paper

T92 Taillard's March 1992 working paper

T93 Taillard's 1993 Networks article

OTSF93 is Osman's tabu search algorithm, first-best-admissible strategy, 1993 Annals of OR

OTSB93 is Osman's tabu search algorithm, best-admissible strategy, 1993 Annals of OR

GSP94 Gendreau et al. 1994 Management Science article – single pass of Taburoute algorithm

GB94 Gendreau et al. 1994 Management Science article – best solution in multiple passes of Taburoute

RT95 Rochat and Taillard's 1995 Journal of Heuristics article

RRS95 Rego and Roucairol's sequential algorithm in Meta-Heuristics: Theory & Applications

RRP95 Rego and Roucairol's parallel algorithm in Meta-Heuristics: Theory & Applications

XK96 Xu and Kelly's 1996 Transportation Science article

Tabu Search VRPs – Percent

Summary of Tabu Search Solutions to VRPs from CMT: Percent Above Best-known Solution

Pr	n	G91	T92	T93	OTSF93	OTSB93	GSP94	GB94	RT95	RRS95	RRP95	XK96
1	50	0	0	0	0	0	0	0		0	0	0
2	75	0.133	0.007	0	1.046	1.046	0.061	0.007		0.268	0.007	0
3	100	0	0.344	0	1.436	1.072	0.401	0		0.168	0.168	0
4	150	0.63	0.119	0	1.549	2.293	0.753	0.258		2.516	1.549	0.111
5	199	2.93	0.731	0.568	3.307	4.843	2.416	1.541	0	3.642	3.337	0.552
6	50	0	0	0	0	0	0	0		0	0	
7	75	0.39	0	0	0.145	0.365	0.39	0		0	0	
8	100	0	0	0	1.393	0.094	0	0		0.271	0.094	
9	150	2.343	0.145	0	1.845	2.189	1.308	0.029		1.401	0.135	
10	199	1.865	0.527	0.149	3.235	1.873	1.623	0.638	0	1.79	1.79	
11	120	0.176	2.969	0	0.085	0	3.009	0		0.137	0	0
12	100	0.401	0	0	0.004	0.004	0	0		0	0	0
13	120	0.681	0.585	0	0.314	0.38	2.119	0.311		0.586	0.586	
14	100	0	0	0	0	0	0	0		0.018	0	
Avg		0.682	0.388	0.051	1.026	1.011	0.863	0.199		0.771	0.548	0.095

Bold indicates best-known solution. Problems 1 and 12 are optimal solutions.

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Procedure	Average	High
T93	0.051	0.568
XK96	0.095	0.552
GB94	0.199	1.541
T92	0.388	2.969
RRP95	0.548	3.337
G91	0.682	2.343
RRS95	0.771	3.642
GSP94	0.863	3.009
OTSB93	1.011	4.843
OTSF93	1.026	3.307

SA and NN VRPs

Summary of Solutions to VRPs from CMT: Simulated Annealing and Neural Networks

<i>Pr</i>	<i>n</i>	<i>OSA93</i>	<i>HGH95</i>	<i>HGS95</i>
1	50	528	545.1	539.2
2	75	838.62		
3	100	829.18	911.2	893.4
4	150	1058	1133.2	1084.7
5	199	1376	1421.5	1401.4
6	50	555.44	567.3	561.3
7	75	909.68		
8	100	866.75	902.4	894.3
9	150	1164.12	1304.5	1264
10	199	1417.85	1604.3	1580.4
11	120	1176	1113.4	1102.4
12	100	826	826.6	825.1
13	120	1545.98	1645.8	1608.5
14	100	890	881.5	879.8

Bold indicates best-known solution. Problems 1 and 12 are optimal solutions.

Simulated Annealing Procedures

OSA93 Osman's simulated annealing procedure, 1993 Annals of OR

Neural Network Procedures

HGH95 Ghaziri's hierarchical deformable net algorithm in Meta-heuristics: Theory & Applications

HGS95 Ghaziri's supervised hierarchical deformable net algorithm in Meta-heuristics: Theory & Applications

SA and NN VRPs -- Percent

Summary of Solutions to VRPs from CMT: Simulated Annealing and Neural Networks

<i>Pr</i>	<i>n</i>	<i>OSA93</i>	<i>HGH95</i>	<i>HGS95</i>
1	50	0.646	3.906	2.781
2	75	0.402		
3	100	0.368	10.296	8.141
4	150	2.876	10.188	5.472
5	199	6.547	10.07	8.514
6	50	0	2.135	1.055
7	75	0		
8	100	0.094	4.211	3.275
9	150	0.135	12.21	8.727
10	199	1.576	14.933	13.221
11	120	12.848	6.841	5.785
12	100	0.786	0.859	0.676
13	120	0.314	6.791	4.371
14	100	2.727	1.746	1.55
Avg		2.094	7.016	5.297

<i>Procedure</i>	<i>Average</i>	<i>High</i>
OSA93	2.094	12.848
HGS95	5.297	13.221
HGH95	7.016	14.933

Bold indicates best-known solution. Problems 1 and 12 are optimal solutions.

Simulated Annealing Procedures

OSA93 Osman's simulated annealing procedure, 1993 Annals of OR

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HGS95 Ghaziri's supervised hierarchical deformable net algorithm in Meta-heuristics: Theory & Applications

CMT Problems -- Running Times

"We do not give CPU times ... because it is very difficult to give them as our algorithm does not find the best-known solution of every problem at each run."

Running Times (minutes)

<i>Pr</i>	<i>n</i>	<i>OSA93</i>	<i>OTSF93</i>	<i>OTSB93</i>	<i>GSP94</i>	<i>XK96</i>
1	50	2.79	1.90	1.12	6	29.92
2	75	107.24	2.98	1.18	53.8	48.80
3	100	155.56	25.72	11.25	18.4	71.93
4	150	83.54	59.33	51.25	58.8	149.90
5	199	38.64	54.10	32.88	90.9	272.52
6	50	56.84	2.88	2.34	13.5	
7	75	10.44	17.62	3.38	54.6	
8	100	15.95	49.97	20.00	25.6	
9	150	1405.02	79.26	40.73	71	
10	199	95.13	76.02	55.17	99.8	
11	120	5.26	24.09	23.31	22.2	91.23
12	100	10.53	14.87	6.79	16	56.61
13	120	127.04	47.23	22.38	59.2	
14	100	5.09	19.59	92.98	65.7	
Avg		151.36	33.97	26.05	46.82	102.99

XK96 Xu and Kelly, total time on a DEC ALPHA Workstation (DEC OSF/1 v3.0)

OSA93, OTSF93, OTSB93 Osman SA and TS procedure, CPU to end, Vax 8600 computer

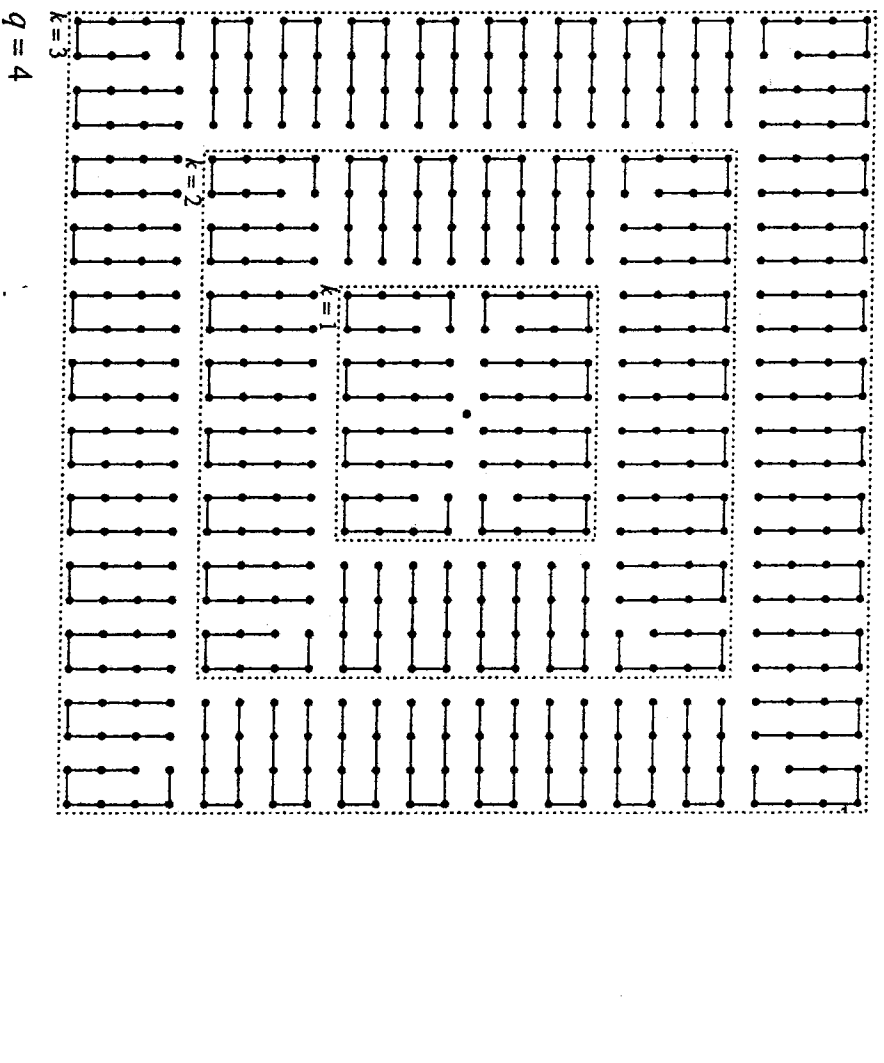
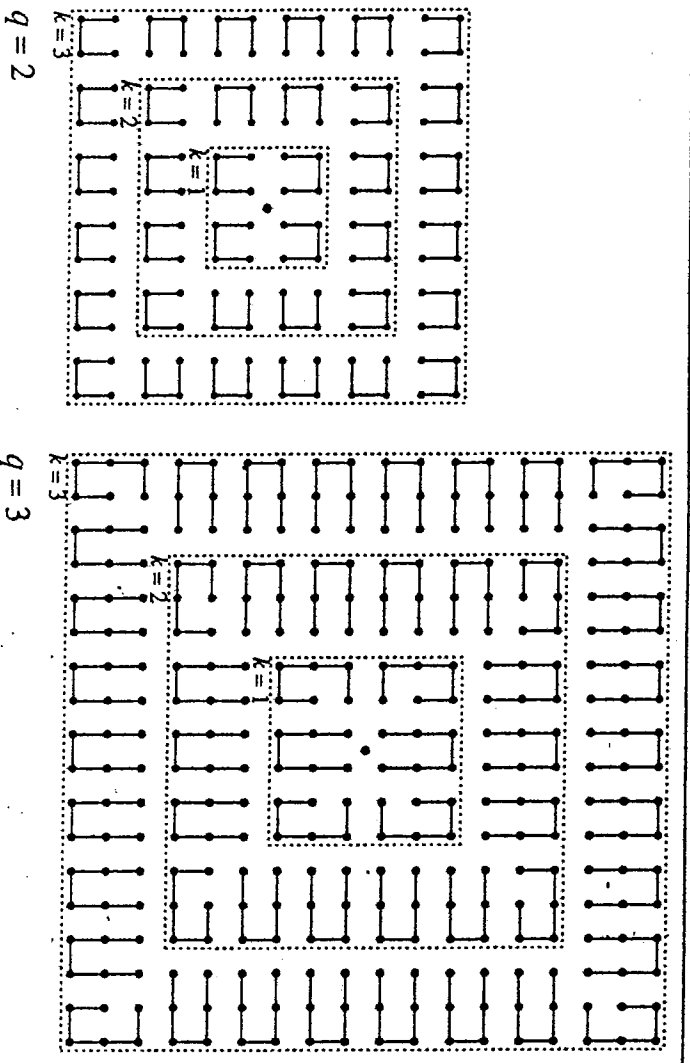
GSP94 Gendreau et al., total computation time, Silicon Graphics workstation,

36 MHz, 5.7Mflops

Some Larger Vehicle Routing Problems

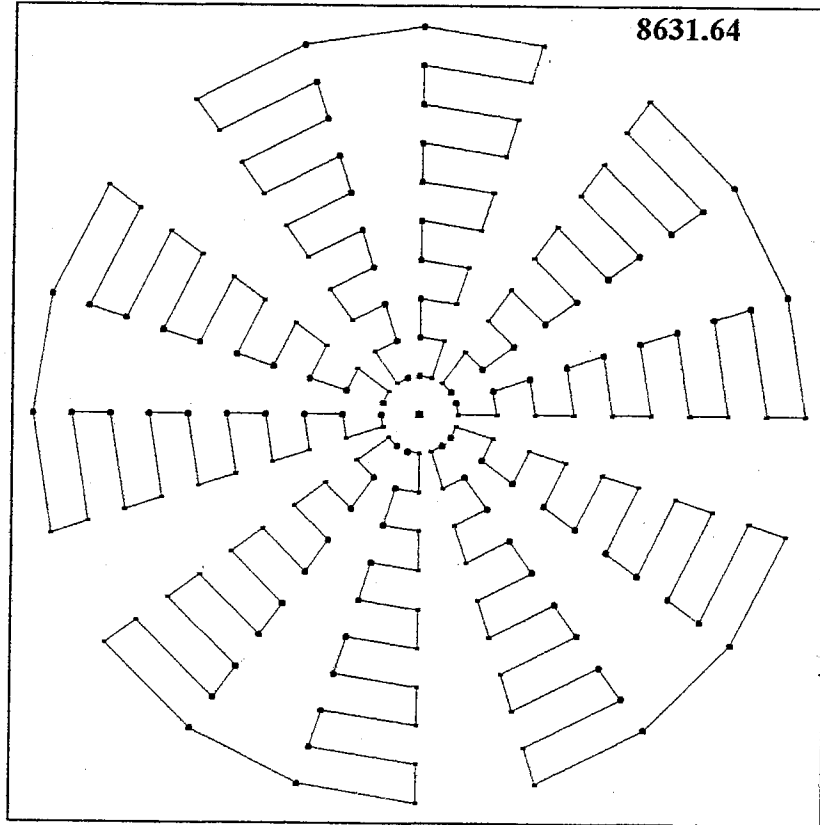
- ❑ Authors construct families of problems
- ❑ These problems are highly symmetric
- ❑ One can estimate good solutions by inspection

Grid Problems of Taillard (144, 324, and 576 cities) with Pseudo-optimum Solutions

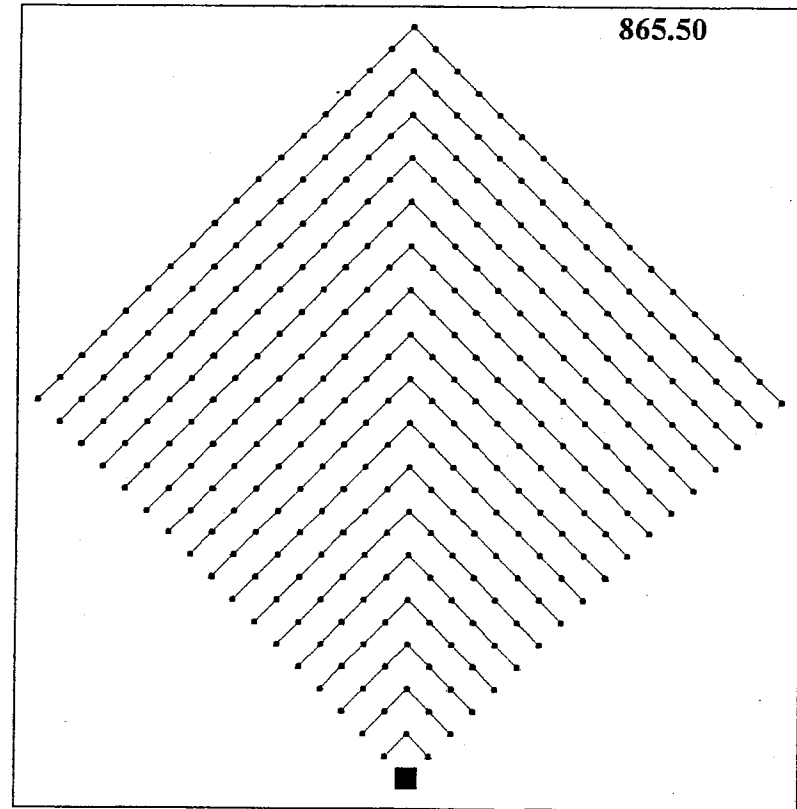


Examples of Large-scale Vehicle Routing Problems

Estimate Solution to LSVRP No. 5

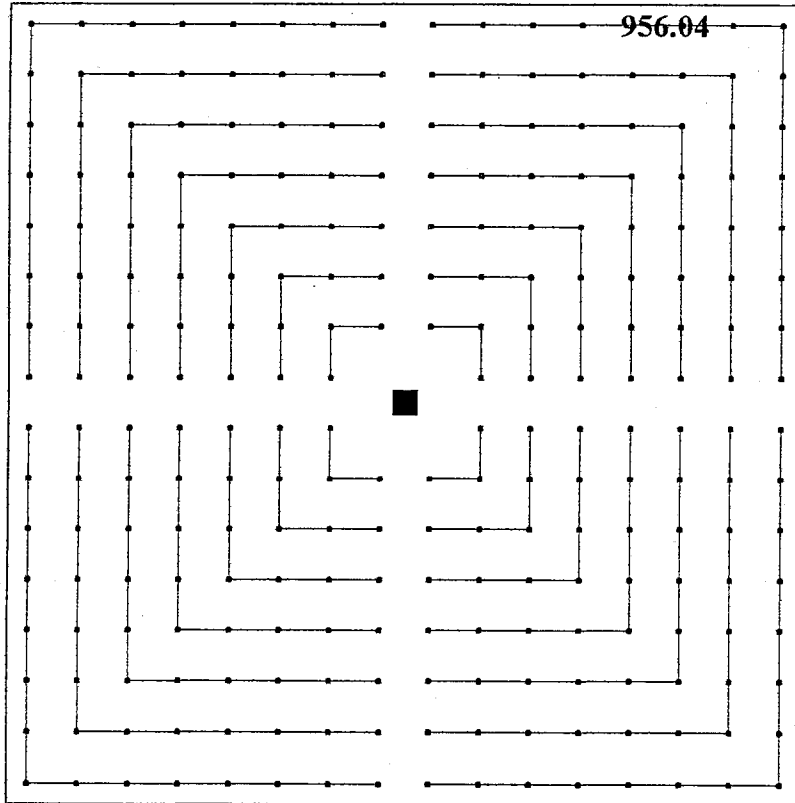


Estimate Solution to LSVRP No. 10

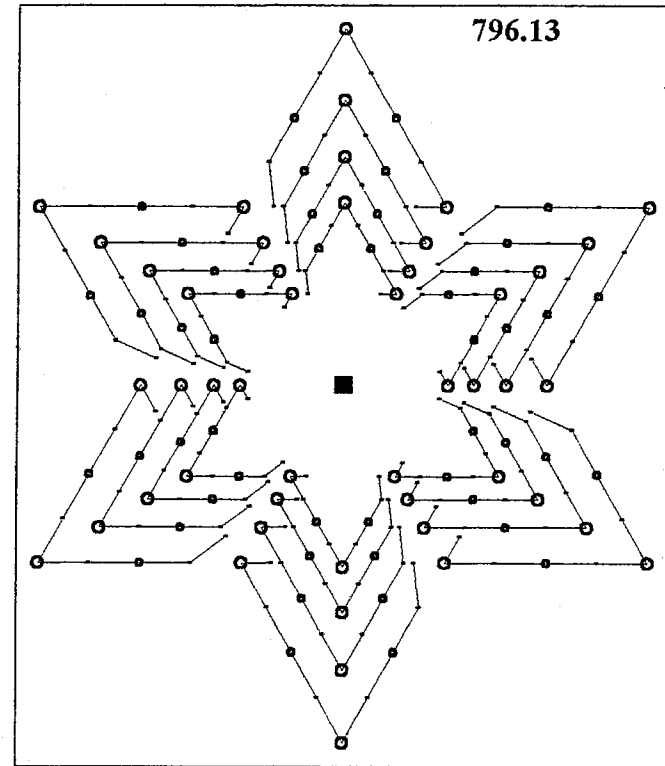


Examples of Large-scale Vehicle Routing Problems (continued)

Estimate Solution to LSVRP No. 13



Estimate Solution to LSVRP No. 17



Projected Running Times for Large-scale VRPs

<u>Pr</u>	<u>n</u>	<u>Actual Total Time (minutes)</u>	
		<u>GSP94*</u>	<u>XK96**</u>
1	50	6.0	29.92
2	75	53.8	48.80
3	100	18.4	71.93
4	150	58.8	149.90
5	199	90.9	272.52
11	120	22.2	91.23
12	100	16.0	56.61

* Silicon Graphics Workstation, 36 MHz, 5.7 Mflops

** DEC ALPHA Workstation (DEC OSF/1 v 3.0)
roughly five times as fast as an SG Workstation

GSP94

Estimated Total Time = $0.001784 n^{2.025}$ $R^2 = 0.95$ (with second point removed)

XK96

Estimated Total Time = $0.049688 n^{1.591}$ $R^2 = 0.95$

<u>n</u>	<u>Projected Total Time (mins)</u>	
	<u>GSP94</u>	<u>XK96†</u>
250	128.0	324.6
300	185.2	433.9
350	253.0	554.5
400	331.6	685.7
450	420.9	827.0
500	521.0	977.9

† Actual times were 480 to 2880 mins on large-scale VRPs

