BMGT 332
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The Craft of Decision Making

- Focus of book
  - Analysis and consequences of decisions
  - Data analysis
  - Model building
  - Connections to numerous related fields
- Features of book
  - Non-technical
  - Sophisticated
  - Case studies
  - No single correct answer

Goals of Course

- Convey an understanding of the field of OR (operational or operations research)
- Convey an appreciation for what an OR analyst does
- Teach students that real-world OR consulting projects do not have a single correct answer
    - Issues are not clear-cut
    - Data are ambiguous
    - Some information may be missing
    - There may be multiple objectives
- It is not our intent to train OR theoreticians in this course

Decisions

- Why analyze decisions? Why be quantitative?
  - Complexity
  - Trial and error takes too long or is impractical
  - Wrong decisions can be costly

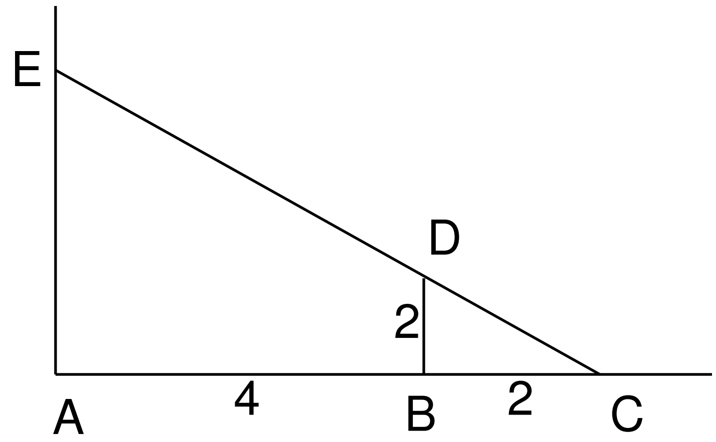
- Examples of decisions
  - Choosing where to live
  - Selecting a new Business School Dean
  - Selecting a college
  - Deciding whether to send a child to private school
  - Selecting a summer job
  - Locating a facility

Decisions

- Additional examples of decisions
  - Deciding how much to sell small business for
  - IOC's decision to offer 2012 Olympic Games to London
  - Plan for balancing the Federal budget
  - Truman's decision to bomb Hiroshima
  - Obama's choice of Biden as running mate

What are the Assumptions?

- Example 1
  - A man 2 meters tall is standing 4 meters from a lamppost. He observes that his shadow is 2 meters long. What is the height of the lamppost?
  - AC = 6, BC = 2, BD = 2, AE = ?



- $\frac{AE}{AC} = \frac{BD}{BC} \Rightarrow AE = 6$

Assumptions

- Underlying assumptions
  - The lamppost is vertical
  - The man is vertical
  - The ground is straight and horizontal
  - The man is standing full height, not sagging
  - He is not wearing a hat nor is he wearing shoes
  - The light is at the top of the lamppost
  - There is no other lamp nearby also casting a shadow
- Example 2
  - Name a former president of the United States who is not buried within the U.S.A.
  - What is the assumption that most people make?

Assumptions

- Example 3
  - Minor injuries in coal mines increased significantly last January
  - Consultant was called in to determine why
  - Reason—very attractive nurse began working around Christmas
  - Moral—you have to ask questions of people involved

- Example 4—Syringe Trouble
  - The Regional Health Authority (RHA) provides disposable syringes to a large area
  - The RHA buys thousands each month from 6 makers of syringes
  - The RHA buys approximately the same amount from each maker
  - Syringes are delivered to storehouses in batches of 3000
  - Syringes can be faulty

Assumptions

- Example 4 (continued)
  - In general, batches are high quality, but "freak" batches do exist

	To storehouse					
From maker	Gallop	Horestead	Inman	Jones	Killick	Long
Abel	X		X	X		
Baker		X				X
Charles	X				X	X
Donaldson	X			X		
Ellerman			X			X
Fineway		X	X		X	

Suppliers and storehouses. A cross denotes a supplier.

Testing Assumptions

- Sequence of events
  - The RHA Director of Medical Relations (DMR) informs the Director of Purchasing (DP) of a possible problem with the syringes
  - The DP suggests that the doctors and nurses may be mistreating the syringes
  - The DMR is not convinced and advises the DP to investigate
  - The DP hires an OR analyst
  - The analyst discovers that the complaints come from regions supplied by Gallop, Killick, and Long storehouses
  - The analyst observes that one maker (Charles) supplies all three of the questionable storehouses
  - The simplest explanation would be to focus on what happens at Charles
  - The analyst finds nothing unusual about the production process at Charles
  - The raw materials used to make syringes at Charles are fine

Testing Assumptions

- Next, the analyst looks into the prescribed inspection scheme and the one used at Charles
- Prescribed scheme: a random sample of 25 syringes will be taken from every batch and each syringe in the sample will be tested. A batch will be passed on to a supplier only when a sample shows no defectives at all.
- Impact: Suppose 1% defective in each batch. Then out of every 100 batches, about 78 will pass the test. That is, there is a 78% probability that a batch with 1% defective will pass the test. Do you know the probability distribution that we used?

  Binomial: $$\binom{25}{0}(.01)^{0}(.99)^{25} = .78$$

- Inspection scheme at Charles: When a defective syringe is identified in the sample, instead of rejecting the batch, another random sample is taken from the same batch. The result is that batches are never rejected. Eureka!
- The moral: Don't assume that everyone is on the same wavelength

Decisions and the Scientific Method

- Elements of a Decision
  - The range of choice
  - The consequences of each of these choices
  - The objective(s) involved

- Problems of Interest
  - There is no easily available, acceptable, and valid unit of measure
  - The range of choice of courses of action is uncertain, or, if known, too large to be able to consider each of them
  - The consequences of these choices are uncertain
  - There is more than one objective or even, perhaps, no agreed upon objective(s)

Decision Making

- Measurement
  - Measurement involves a view of the world
  - Different measures are often linked to different objectives
  - Example—cost of a car journey
    - Average cost per mile of driving the car
    - Marginal cost per mile
    - Opportunity cost
  - Example—tax increase
    - Current dollars vs. real dollars

Decision Making

- Multiple choice
  - Objectives
  - Constraints
  - Feasible solutions
  - Best solutions
  - Linear programming
  - Example—the diet problem
    - Satisfy minimum daily requirements
    - Minimize cost of food
  - Nonlinear programming

Decision Making

- Uncertainty
  - Uncertainty is the nature of the universe
  - Uncertainty is measured by probability
  - Variability is measured by statistical measures such as variance
  - Types of probabilities
    - Based on history—number of defective parts produced per batch
    - Based on mathematics—the birthday problem
    - Based on belief—likelihood that hurricane will reach land
- Objectives
  - Objectives often evolve and emerge
  - Hidden objectives
  - Multiple objectives for a group

Mathematical Models

- Models and Science
  - Models are often mathematical
  - They are needed to deal with problem complexity
  - Models help us simplify
  - The purpose is to understand that which we model
  - Examples
    - Newton's law of gravitation
    - Economic models
    - Movement of stars and planets
      - Greeks (explanatory)
      - Babylonian (description)

Mathematical Models

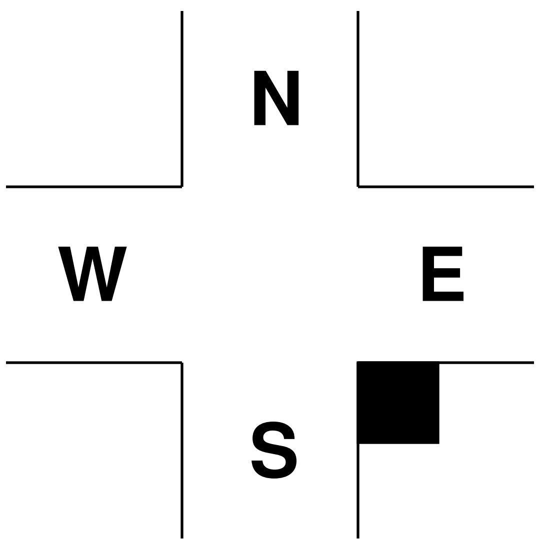
- Gas Station Sales
  - Marketing managers were asked by a team of analysts to list most important factors
  - These included traffic flow, number of pumps, number of attendants, etc.
  - Economists formed the descriptive model

$$S = a_1 x_1 + a_2 x_2 + \ldots + a_{24} x_{24}$$

  - Model could estimate sales to within ± 50%
  - Reaction: the more variables you need to describe something, the less you know about it
  - Next, the analysts visited the stations
  - All stations were at road junctions

Mathematical Models

- Gas Station Sales (continued)
  - The team observed cars passing through the junction at each station
  - 90% of sales were from cars following paths S to E, E to S, or S to N
  - Now they had a descriptive model with only three variables
  - Of the 16 possible routes through the junction, these three required the least additional time for a gas station stop
  - Thus, they had an explanatory model



Mathematical Models

- Explanatory vs. Descriptive Models
  - Explanatory models are much more valuable than descriptive models
  - With a descriptive model, one must ask why does it make sense
  - Example
    - Birth rate in Scandinavia is positively correlated with the number of storks in summer
    - Is there a cause and effect relation? No
    - Prosperity over summer => abundant supply of crops and harvest => storks stay longer before migrating
    - Also, prosperity => birth rate increases
    - Thus, birth rate and stork count are both related to prosperity

Mathematical Models

- Variables
  - Controllable variables
  - Uncontrollable variables
    - Controllable by someone else
      - Neutral
      - Friendly
      - Malevolent
    - Natural (rainfall or harvest size)
    - Disaster (hurricane)
  - Example
- Problem components need to be carefully examined
  - Variables
  - Constraints
  - Objectives

Mathematical Models

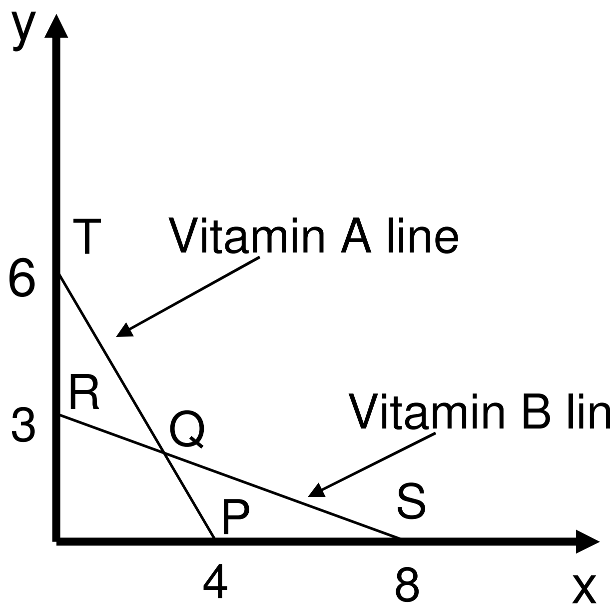
- The Analyst
  - Should visit and observe what is going on
  - Should go to the source of the data

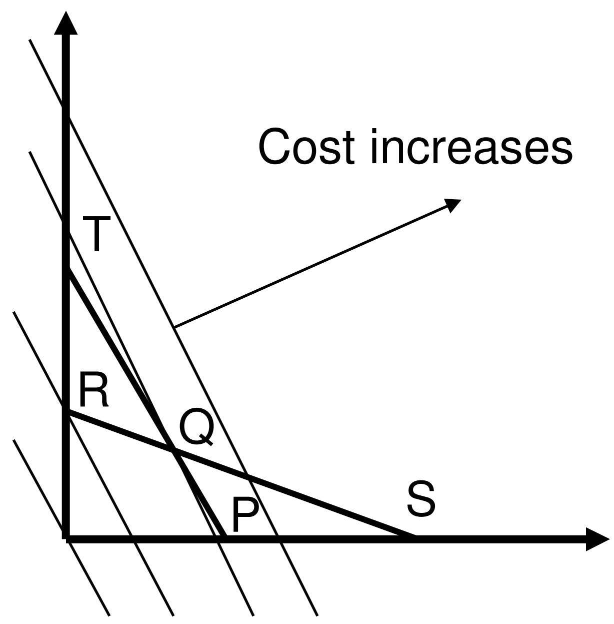
- Appendix 1
  - Two foods (x and y)
  - One unit of x contains 6 units of vitamin A, 3 units of vitamin B
  - One unit of y contains 4 units of vitamin A, 8 units of vitamin B
  - Each unit of x costs $50
  - Each unit of y costs $30
  - Mixture of x and y must have at least 24 units of vitamin A and 24 units of vitamin B
  - Seek to minimize cost

Linear Programming

Linear Program

Minimize	$60x + 30y$
Subject to	$6x + 4y \geq 24$
	$3x + 8y \geq 24$
	$x, y \geq 0$





Mathematical Models

- Linear Programming continued
  - Corner point solution
  - Simplex method
  - Should go to the source of the data
- Appendix 2 (The Birthday Problem)
  - Ignore leap years
  - Person number one has a birthday
  - The probability that person number two has a different birthday is 364/365
  - The probability that person number three has a different birthday is 363/365
  - The probability (P) that 26 people have different birthdays is

$$P = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \ldots \times \frac{365 - 26 + 1}{365}$$

  - The probability that at least two persons share the same birthday is 1 – P
  - It takes some calculation, but 1 – P > .50

Data-Driven Decision Analysis

- Question Assumptions
  - Go to the source
  - What is the scale of measurement
  - Example I
    - King Charles II
    - Why does a dead fish weigh more than when it was alive?
  - Example II
    - France surrenders in 1940
    - Germany occupied ports on west coast of France
    - Submarines were sent to intercept supplies from U.S. and Canada to Britain
    - At that time, submarines traveled on the surface for speed, submerged for action

Example from World War II

- British RAF search for German subs using visual inspection and early radar
- Depth charges were pre-set to explode at certain depth
- Initially, depth setting was about 100 feet
- Few subs were being destroyed
- The OR section of the RAF decided to collect data to find out why
- They discovered that the setting of the depth charge should be reduced from 100 feet
- It was reduced to 50 feet, then 33 feet, and finally to 25 feet

Example from World War II

- The results were impressive

Period	Total no. attacks	Sunk (%)	Seriously damaged (%)
Sep. 1939-June 1941	215	1	4
July 1941-Dec. 1941	127	2	13
Jan. 1942-June 1942	79	4	19
July 1942-Dec. 1942	346	7	9

- The moral: data collection and analysis led to positive action

Data Analysis

- The Pastry Man's Tale
  - Background
    - Mr. Patrick sells meat pies to bakery shops in north England
    - He has four salesmen – w, x, y, and z
    - Salesmen compete with one another
    - Mr. Patrick sets up a contest
      - 40-day test period
      - 4 bakeries – A, B, C, and D
      - Each salesman will call on each bakery 3 times, 12 visits in all
      - No collusion allowed
      - Who is the best salesman?
    - First look at the data
      - Raw data is shown in Table 1 (page 43)

Data Analysis

Salesman	Ackerman	Breadmaster	Collins	Doughboy	Total
W	46	128	138	177	489
X	87	104	154	236	581
Y	41	68	207	234	550
Z	27	106	90	102	325
Total	201	406	589	749	1945

- * Summary data is shown above
- * X is the clear winner
- * Z is a total loser
- * Our initial reaction is to award a prize to X and to fire Z

- A second look at the data
  - * The longer since the last visit from a salesman, the more pies can be sold
  - * We, therefore, need to take into account the elapsed time since the last visit

Data Analysis

Values of Q/t													
	Ackerman			Breadmaster			Collins			Doughboy			Overall average
W	5.5	4.8	5.5	9.7	10.5	9.0	16.0	14.2	16.5	18.0	19.2	21.3	12.5
X	4.8	4.8	4.9	9.0	9.7	9.3	14.2	13.5	14.0	20.0	19.4	20.0	12.0
Y	6.0	4.7	5.2	9.3	11.0	9.0	14.0	15.5	14.4	18.5	20.8	18.7	12.3
Z	8.0	9.0	10.0	10.7	15.0	13.5	17.5	17.5	20.0	23.5	27.0	28.0	16.6

* Table 3 (page 44) indicates the timing of the visits to each of the bakeries over the 40-day period
* Let t be the number of days since the last visit by any salesman
* Let Q/t be the "normalized" or "adjusted" quantity delivered
* Using t and Q/t, we can construct Table 4 (page 46)
* A summary of orders received per day since last call is shown above
* Based on the Q/t measure, Z is the winner

Logic and Common Sense

- Big Picture
  - Common sense can let you down
  - Logic relies on assumptions
  - Don't lose track of the assumptions
- The Unfair Sample
  - Airforce application
  - Armor protection needed for fighter bombers
  - The more armor, the lower the bomb load
  - Where to put armor
  - Examine data—returning bombers
  - Note location of bullet holes and damage

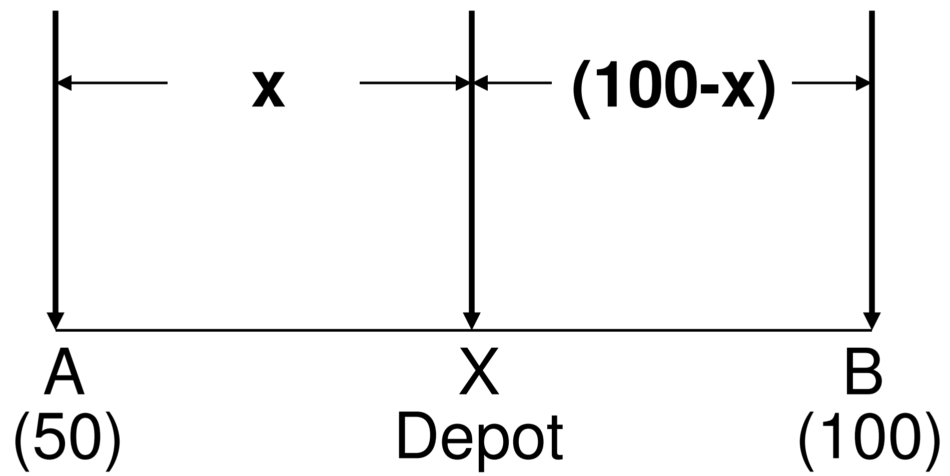
Airforce Application

- Cover parts of aircraft fuselage where bullet holes were found?
- On the other hand, returning planes survived
- Where were non-returning planes hit?
- If only these were in the sample
- Maybe armor should go where there were <u>fewer</u> holes
  - Why?

- The Perils of Intuition
  - If common sense fails us on simple problems…
  - We need to locate a depot
  - Two customers -- A and B
  - Transport cost = amount x distance x c
  - Where should the depot be located

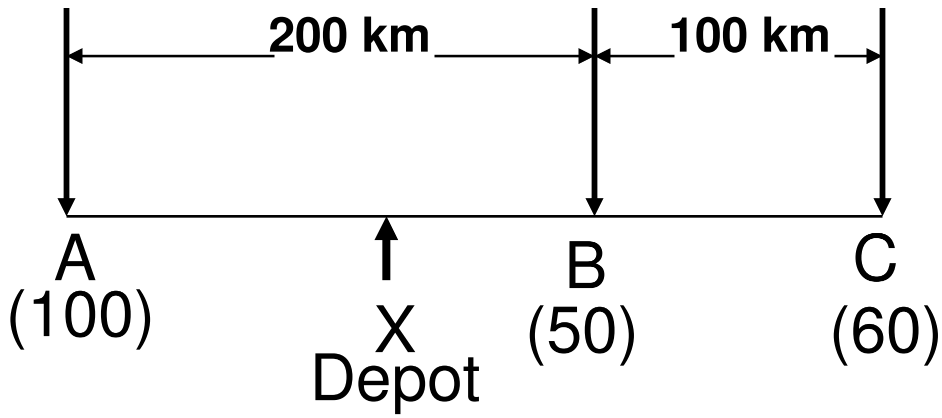
Location of Depot

- A and B are 100 km apart
- Intuition tempts us to find the center of gravity
- This is 2/3 of the way from A to B
- Suppose the depot is located here
- Transport cost = (2/3)(100)(50)c + (1/3)(100)(100)c = 6666.7c
- On the other hand, locate the depot at B
- Transport cost = (100)(50)c = 5000c



Location of Depot

- If we move the depot one km towards A, the new cost will be (50)(99)c + 100c = 5050c
- If we move a second km towards A, the cost will be (50)(98)c + 100(2)c = 5100c
- No matter how far apart A and B are, the optimal location is at the customer with the larger demand
- If both customers have the same demand, then all positions between A and B have the same cost
- Now consider three customers in a line as below
- Where to locate the depot



Location of Depot

- Tentatively locate the depot at B
- Transport cost = (100)(200)c + (100)(60)c = 26,000c
- If we move the depot one km towards A, the new cost will be (100)(199)c + 50c + 60(101)c = 26,010c
- If we move the depot one km towards, C, the new cost will be 100(201)c + 50c + 60(99)c = 26,090c
- B is the best location
- The general case can be solved using a map, string, and weights as shown on page 54
- Key point: The center of gravity assumption was plausible, but wrong

The Transportation Problem

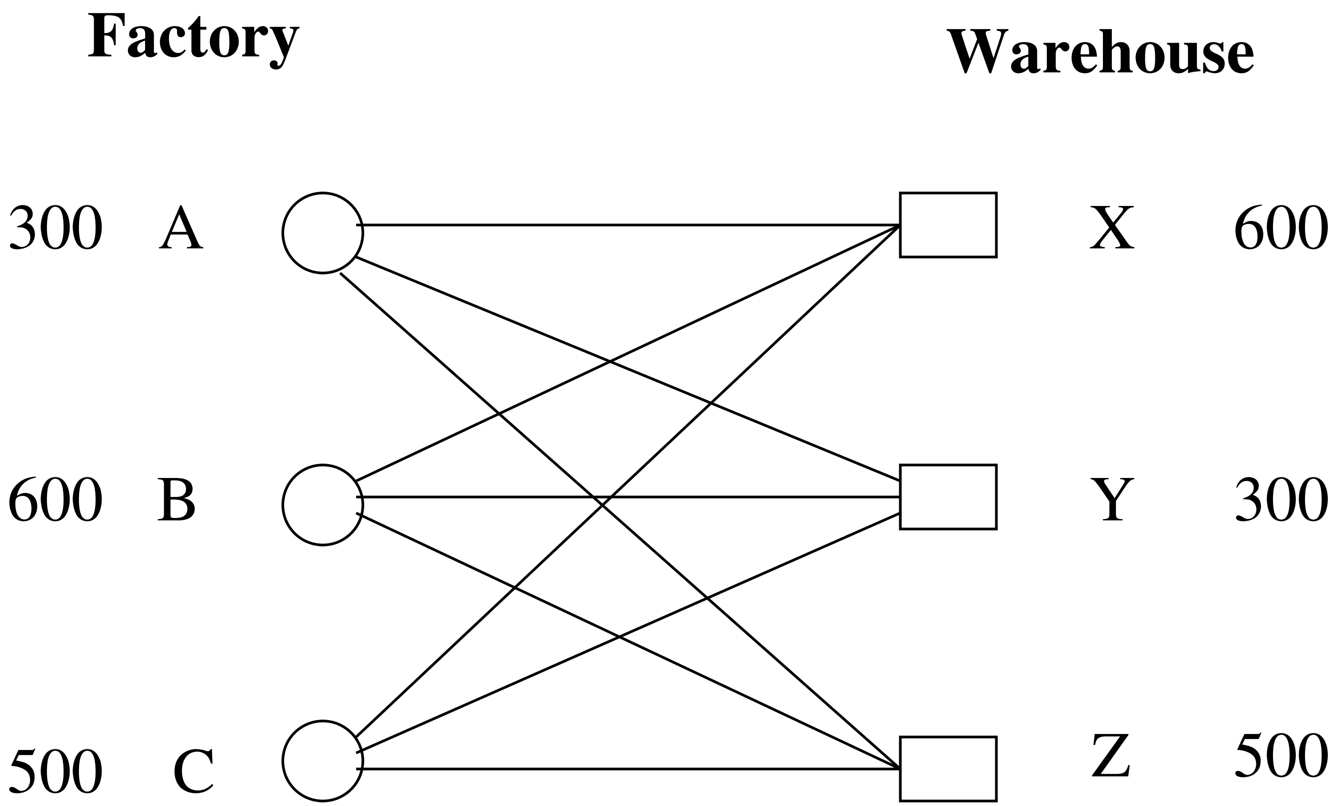
- Don't be myopic
  - Logic can sometimes be used to reduce the number of alternative choices, but be careful
  - Consider the transportation problem below

Warehouse

Factory	**x**	**y**	**z**	**Supply**
A	0	4	1	300
B	1	6	3	600
C	3	7	6	500
Demand	600	300	500	1400

  - Mathematical representation follows

The Transportation Problem



Mathematical Formulation

Minimize 0 flow(A, x) + 4 flow(A, y) + 1 flow(A, z) + 1 flow(B, x)
+ 6 flow(B, y) + 3 flow(B, z) + 3 flow (C, x) + 7 flow(C, y)
+ 6 flow(C, z)

subject to
flow(A, x) + flow(A, y) + flow(A, z) = 300
flow(B, x) + flow(B, y) + flow(B, z) = 600
flow(C, x) + flow(C, y) + flow(C, z) = 500
flow(A, x) + flow(B, x) + flow(C, x) = 600
flow(A, y) + flow(B, y) + flow(C, y) = 300
flow(A, z) + flow(B, z) + flow(C, z) = 500
all flows are $\geq$ 0

- The above formulation is a linear program

The Transportation Problem

- Observation 1. The route from A to x has zero cost
- Observation 2. The route from C to y has the highest cost
- Combining these, don't use route (C, y) and use route (A, x) as much as possible
- Use the seven remaining routes as shown next

Warehouse

Factory	x	y	z	Supply
A	0 300	4 0	1 0	300
B	1	6	3	600
C	3	7 don't use	6	500
Demand	600	300	500	1400

Warehouse

Factory	x	y	z	Supply
A	0 / 300	4 / 0	1 / 0	300
B	1 / 300	6 / 300	3 / 0	600
C	3	7 (X)	6	500
Demand	600	300	500	1400

Warehouse

Factory	x	y	z	Supply
A	0 / 300	4 / 0	1 / 0	300
B	1 / 300	6 / 300	3 / 0	600
C	3 / 0	7 (X)	6 / 500	500
Demand	600	300	500	1400

The Transportation Problem

- Total cost = 5100
- But, a much better solution exists
- It can be found using a transportation algorithm
- Best solution is shown below
- Total cost = 4000

Warehouse

Factory	**x**	**y**	**z**	**Supply**
A	0 / 0	4 / 0	1 / 300	300
B	1 / 400	6 / 0	3 / 200	600
C	3 / 200	7 / 300	6 / 0	500
Demand	600	300	500	1400

- Note 1. Nothing goes along the zero cost route
- Note 2. Most expensive route is used as much as possible

Waiting Lines Everywhere

- A Little Bit of Queuing Theory
  - The mathematical theory of waiting lines
  - Service time varies from customer to customer
  - Random arrivals
  - Each arrival is a potential demand for service
  - Examples
    - Post office
    - Bank
    - Doctor's office
    - Aircraft queuing to take off
    - Ambulance service
    - Cable TV repair
    - Telephone help line for Dell, HP, etc.

Queuing Theory

- How much time does a person spend in the system?
- Waiting time plus service time
  - Customer wants minimum time in system
  - Server wants to be constantly busy
  - Conflicting goals
- Time in system depends on time between arrivals and service time
- If customers arrive faster than they can be served, the wait will grow until it becomes infinite (in theory)
- In practice, customers won't join a very long queue

Queuing Theory

- Mathematically, the average number of customers in the system can be shown to be $C = A / (S - A)$

  where S = average number of people served per unit of time
  A = average number of arrivals per unit of time, and
  $S > A$

- For example, if $A = 10$ and $S = 12$, then $C = 5$
- When $S = A$, are supply and demand in balance?
  - No
  - Queue length becomes infinite
  - Illustration 1
    - Taxi queue at an airport
    - 1000 customers per day
    - 1000 taxi arrivals per day

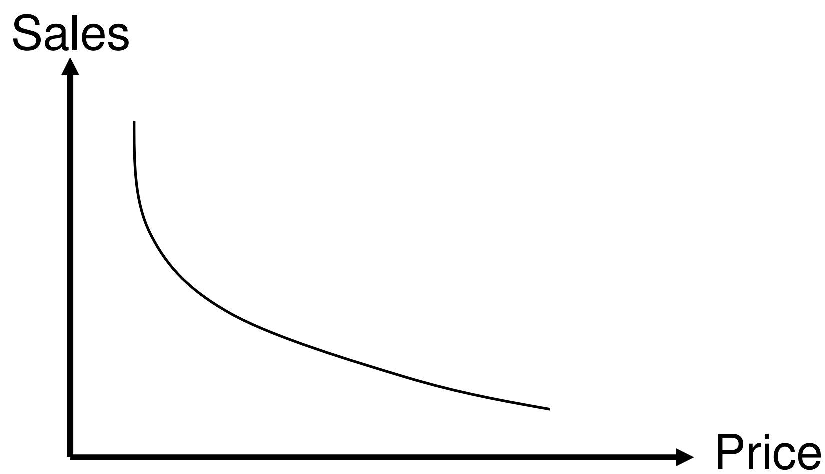
Queuing Theory

  - Some taxis arrive, find no one waiting, and leave
  - During peak, more customers arrive than taxis
  - Useful taxi arrivals per day < customers per day

- Illustration 2
  - Job shop environment
  - Orders requiring different man-hours of work
  - These orders arrive at random
  - If the number of man-hours of work available each week = the number of man hours of work needed on the jobs ordered, then overtime will be required
  - During regular hours, workers will sometimes be idle

Price vs. Sales

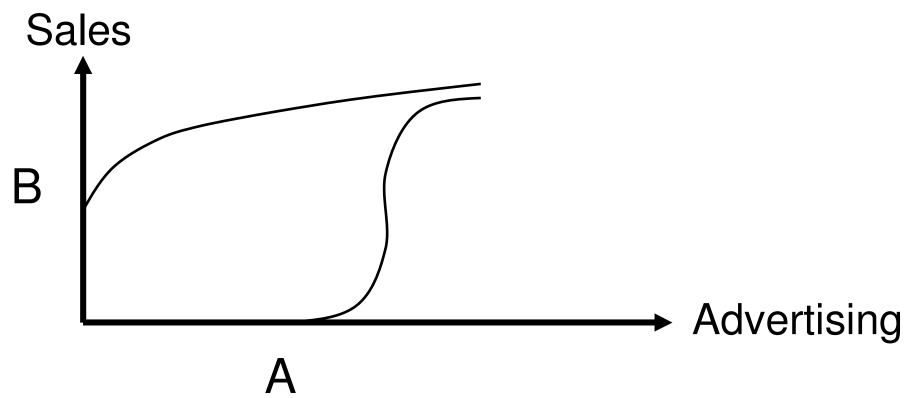
- Marketing models
  - Price elasticity: as price is increased, sales will decrease



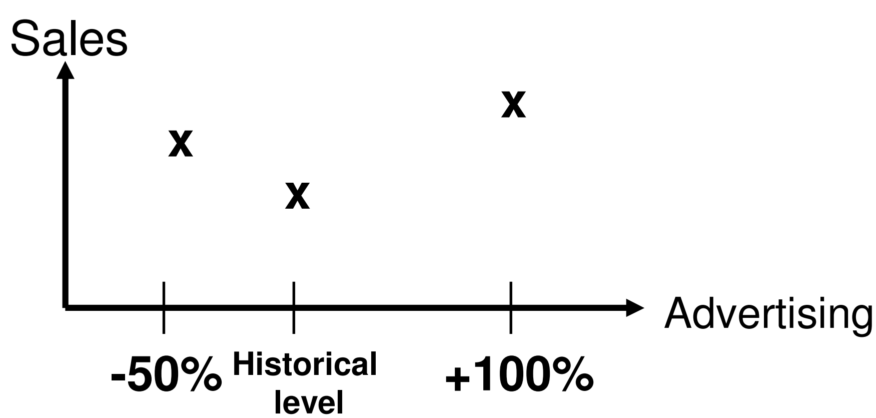
  - Is this always true?
  - Sales can decline at low prices because customers assume the quality is poor
  - Is this irrational?
  - Diversion: When can a queue attract arrivals?
  - Sales vs. advertising

Sales vs. Advertising

- Basic assumption is that sales will increase with advertising expenditures until a saturation level is reached
- Hypothetical models are shown below



Experimental results are shown next



Sales vs. Advertising

* Conclusion: We can no longer assume that sales never decrease as advertising increases
* Customers may get fed up with ad
* Different customers may get fed up more quickly than others (see page 61)
* Sometimes aggregating the data can obscure what is really going on

- Sometimes Problems are not so Complex
  * The chocolate bar problem
  * Each break adds 1 to the number of pieces
  * You start with one piece
  * In the end, you want 18 pieces
  * Therefore, 17 breaks are required

Use Common Sense

- Think before you leap (Anecdote)
  - Major city wished to reduce the number of cars driven by commuters
  - Proposal
    - Toll for each car, plus
    - Fee for each passenger
  - Enormous data collection task
  - Consultant called upon
  - Consultant replied immediately
    - No analysis necessary
    - The proposal would encourage drivers to reduce the number of passengers and hence increase the number of cars

Use Common Sense

- Better proposal: charge drivers for the number of empty seats
  - * Encourage car pooling
  - * Reduce the size of cars

➢ Moral: think <u>before</u>, <u>during</u>, and <u>after</u> data collection

- Focus on key variables or factors
  - ➢ Sensitive
  - ➢ Robust

Berwyn Bank Case Study

- Involves inventory of cash held at branch banks
  - You don't want to run short of cash
  - Holding cash is expensive
    - Lending $\leq$ M x liquid resources
    - Liquid resources doesn't include cash held at Branch Banks
- Banks collect cash periodically to reduce inventory of cash
- Berwyn Bank contacts Tom Tryer, consultant
- Details
  - 4 branches
  - Each receives more cash than it pays out
  - Security people collect the cash at stated intervals and bring it to Berwyn's money store
  - Two types of cost involved

Berwyn Bank

  - Head office charges 6 pounds/day for every 10,000 pounds of cash on hand at end of business day (not weekends)
  - Security firm charges 200 pounds/day for a van plus 100 pounds per branch visited (assume the money store is far away)
- Tradeoff
  - Use van rarely – high inventory cost, low travel cost
  - Use van daily – low inventory cost, high travel cost
- How often should van be used?
- Net cash receipts are shown for each of the four branches in Table 1 (page 67)
- Security firm wants its schedule of visits to branches to be somewhat random

Berwyn Bank

- Stage 1: Initial discussion
  - Consider a single branch, A
  - One unit of cash = 10,000 pounds
  - Cost of holding cash = 6 pounds per unit/day
- Stage 2: First analysis of branch A
  - Cost of holding cash at A grows at 42 pounds/day
  - Cost of collecting cash is 200 + 100 = 300 pounds/collection
  - Review Table 2 on page 69
  - Problem is to minimize {21(n+1) + 300/n}
    - Take first derivative
    - Set equal to zero
    - Obtain n = $\sqrt{\frac{300}{21}}$ = 3.78 (say 4 days)

Berwyn Bank

- Review Figure 1 on page 69
- Review Table 3 on page 70
- Review Table 4 on page 70
- Assume r x 10,000 = the net cash gain/day
- For branch A, r = 7
- Assume an n – day cycle
- Average holding cost/day = 3r (n+1)
- Average cost of collection/day = (300/n)
- For Branch A, the minimum is at n = 4
- This is a discrete alternative to the calculus approach

Berwyn Bank

- Stage 3: Extension
  - Treat the other three branches in the same way
  - Obtain n, for each, such that total cost is minimized
  - Results

Branch	r	Best n	Cost/day
A	7	4	180
B	4	5	132
C	2	7 or 8	91
D	1	9, 10, or 11	63

  - Review Table 5 on page 71

Berwyn Bank

- Stage 4: Alternative Schedules

  - Strategy 1

    - Treat each branch separately (more or less)
    - Let A, B, C, and D have collection intervals of 4, 5, 8, and 10 days
    - Why do we select 8 and 10 for C and D?
    - On day 20, A, B, and D will be served
    - On days 8, 16, 24, and 32, A and C will be served
    - On days 10 and 30, B and D will be served
    - On day 40, A, B, C, and D will be served
    - Total cost now needs to be computed

Berwyn Bank

* Consider a 40-day period
* Why 40 days?
* Treating the four branches independently, the daily costs would be 180, 132, 91, and 63
* The sum would be 466
* But on days when more than one collection takes place, the cost is less
* On day 20, the amount saved is 3(300) – 500 = 400
* On days 8, 16, 24, and 32, the amount saved is 2(300) – 400 = 200
* On days 10 and 30, the amount saved is 2(300) – 400 = 200
* On day 40, the amount saved is 4(300) – 600 = 600

Berwyn Bank

  * Total reduction in costs is 400 + 4(200) + 2(200) + 600 = 2200
  * Cost reduction/day = 2200/40 = 55
  * Total cost becomes 411/day

- Strategy 2
  - Visit all branches together
  - In this case, r = 14 (see Table 1)
  - Cost collection trip = 600
  - Average holding cost per day = 42(n+1)
  - Average collection cost per day = 600/n
  - Total cost = 42(n+1) + 600/n
    * Take first derivative
    * Set equal to zero
    * Solve for n

Berwyn Bank

- $n = \sqrt{\frac{600}{42}} \cong 3.78$ (or 4 days) – minimizes total cost
- In general, $n = \sqrt{\frac{c}{3r}}$
- See Table 6 for discrete analysis
- Observe that Strategy 2 beats Strategy 1 for n = 3, 4, 5, or 6

	Total Cost / day
Strategy 1	411
Strategy 2	
n = 3	368
n = 4	360
n = 5	372
n = 6	394

Berwyn Bank

- Strategy 3
  - Visit big branches (A and B) together
  - Visit small branches (C and D) together
  - For A and B, r = 11, c = 400, n = $\sqrt{\frac{400}{33}} \cong 3.48$
  - From Table 7, we see n = 3 or 4
  - Daily costs would be 265
  - For C and D, r = 3, c = 400, n = $\sqrt{\frac{400}{9}} \cong 6.67$
  - From Table 8, we see n = 6 or 7

Berwyn Bank

- Daily costs would be 129 or 130
- Suppose we let n = 3 for A and B and n = 6 for C and D
- Why?
- Taken separately, daily costs would be 265 + 130 = 395
- But on day 6, both big branches and small branches would be visited
- Amount saved over the six days would be 2(400) – 600 = 200
- Amount saved daily would be 200 / 6 = 33
- Actual daily costs = 395 – 33 = 362

➢Stage 5: Summing Up

- Strategy 1. Treat all branches separately
  - Total cost of 411 pounds/day

Berwyn Bank

- Strategy 2. Treat all branches together
  - Total cost of 360 pounds/day
- Strategy 3. Combine A and B and combine C and D
  - Total cost of 362 pounds/day

Stage 6. Randomization

- Daily cost = 3r(n+1) + c/n
- Optimal solution is $n^* = \sqrt{c/3r}$
- Minimal resulting cost = $3r + 2\sqrt{3rc}$
- Suppose we use kn* instead of n*, where k is a fraction
- Ratio of cost (kn*) to cost (n*) =

$$\frac{\left(k + \frac{1}{k}\right)\sqrt{3rc} + 3r}{2\sqrt{3rc} + 3r}$$

Berwyn Bank

- Ignore 3r and 3rc for the sake of simplicity
- The ratio then becomes $\frac{1}{2}\left(k+\frac{1}{k}\right)$

k	Approximate Ratio
0.5	1.25
0.6	1.13
0.7	1.06
0.8	1.03
0.9	1.01
1.0	1.00
1.1	1.00
1.2	1.02
1.3	1.03
1.4	1.06
1.5	1.07

Berwyn Bank

- Key point: a large change in the number of days in a cycle has a small impact on resulting daily cost
- The total cost curve is relatively flat (see Figure 2)
- Now back to the cost of randomization
- Suppose all branches are visited on the same day
- The cycle time, n, can vary
- It can be randomly selected from 3, 4, and 5 days
- Daily costs will, therefore, vary between 360 and 372
- The cost of randomizing is relatively small
- Key observation: we have obtained several good solutions to this problem using different strategies
- The objective function must be very flat

Describing a Problem

- A system is a collection of entities
  - Each entity impacts at least one other
  - Each entity is impacted by at least one other
  - All entities are "connected"
  - See figure on page 87
- Difficulties in analyzing systems
  - We cannot carry out laboratory experiments
  - We cannot deduce cause and effect relationships from observations
  - It is hard to draw boundaries around subsystems

Systems Analysis

- The Analyst
  - Analyst is invited to “solve” a “problem”
  - Analyst and client will not share the same perspective
  - Analyst and client have different biases, prejudices, and experiences
  - Story of old lady, third floor, heart attack
  - Problem “definition” will depend on who is the client
  - Problem description is a better term
  - But, remember that problem description may change during the duration of the project

Systems Analysis

- The Analyst (cont.)
  - Rivett likes soft systems methodology (SSM)
  - SSM does not view the problem description as a given
  - Rather, it is open for discussion
  - The analyst must recognize the temptation to describe the problem in such a way that she can be a major contributor to its solution

The Happy Hamburger Company

- The normal distribution
  - The basis of statistical theory is the normal distribution
  - But, not every distribution is normal
  - Plot the data, whenever possible
  - Anecdote: The Rocket Attack on London
    - Rockets fell on London in 1944
    - Distribution of hits was a bivariate normal
    - Center of distribution was central London
    - German spies were captured and forced to send incorrect locations of hits back to Germany

Distribution Deception

- A fabricated bivariate normal distribution of hits with center north of London was transmitted
- The information was received and the launchers were adjusted
- From then on, most of the rockets fell south of London
- Key point: the fact that the transmitted distribution was bivariate normal lent credibility to the message

- Happy Hamburger Case
  - Mr. Evans buys approximately 42000 kilos of beef substitute for Happy Hamburger
  - Suppliers (A, B, C, D) offer a batch of substitute of given weight and price by phone
  - No bargaining takes place
  - If the offer price is not above Evans' maximum price, he accepts the offer

Happy Hamburger

- The system has been at steady state
- Mr. Evans is asked to report to the managing director
- A special promotion will require an extra 3000 kilos of substitute per month and will yield a net profit increase of 9000 pounds
- Can Mr. Evans buy the extra amount at less than 9000 pounds?
- The managing director needs an answer in an hour or two
- Examine Table 1 on page 102
- Evans' maximum price seems to be 2.20 pounds per kilo
- From Table 1, we can construct Table 2 (see page 103)
- From Table 2, we can construct Figure 1 (see page 102)
- A discrete version of Figure 1 is presented in Figure 2 (see page 104)

Happy Hamburger

- Analysis
  - Guess the missing right end of the histogram
  - The role of symmetry
  - Mr. Evans' guesses are shown below

Price per kilo	Kilos available
2.30	3300
2.40	1500
2.50	400

  - Extra cost of an additional 3000 kilos would be 3000 x 2.30 = 6900 pounds
  - Numerous errors in "price per kilo" starting on page 101

Happy Hamburger

- Is there a relationship between price per kilo and the lot size?
- Examine Table 3 on page 105
- Focus on rightmost column
- Are the different suppliers the same with respect to prices and lot sizes?
- See bottom row of Table 3
- It shows that the four suppliers sell at remarkably similar average costs
- How would you use Table 1 to answer to last question?

Uncertainty

- Introduction
  - Uncertainty is everywhere
  - Each of us reacts to uncertainty in different ways
    - Risk prone
    - Risk averse
  - Impact is pervasive
    - Investments
    - Lifestyle
    - Career choice
  - Worth mentioning
    - Entrepreneurs
    - *Born to Rebel* by Frank Sulloway

Uncertainty

- Measurement of uncertainty
  - Uncertainty is measured by probability (likelihood)
  - Practical examples
    - Stock markets
    - Gambling
    - Insurance
    - Weather forecasting
- Measurement of probability
  - Mathematical probability
    - Toss of a coin
    - Throw of a die
    - Spin of a roulette wheel
    - If a total of *n* equally likely occurrences includes *m* equally likely ways in which a particular event may occur, the probability of that event is *m/n*

Probability

- The probability of a particular event is the ratio of the number of favorable occurrences to the total number of possible occurrences
- Examples
  * The probability of throwing a 4 with a die is 1/6
  * The probability of drawing a card higher than a 5 (ace is high) from a full deck is 36/52
  * The probability of throwing heads twice in a row is 1/4
- If p = *m/n* is the probability of success, then q = 1 – p is the probability of failure
- Probabilities are between 0 and 1
- To say that the odds against an event are 7 to 2 means the probability of its occurrence is 2/9

Probability

- Frequency in the long run
  - If we throw a symmetric die 600 times, we expect to obtain each score approximately 100 times
  - Historically, we can estimate the probability of 10 or more admissions per night to the emergency room of a given hospital
- Subjective or prior probabilities
  - Non-repeatable situations or situations that have not yet occurred (no historical data)
  - Based on belief
  - Odds against the Yankees winning four straight after losing the first two games to Atlanta in the 1996 World Series
  - What is the likelihood that you will get accepted by Wharton's MBA program?
  - What is the likelihood that you will get married before the age of 25?

Probability

- Measurement and validation

  - It can be difficult to measure probabilities with even moderate accuracy
  - Suppose a bank lends $50,000 to a small business
  - How can it estimate the likelihood of default?
  - Suppose many such small businesses default?
  - As with measurement, validation of subjective probabilities can be extremely difficult
  - In some cases, you can estimate the probabilities using (at least two) very different approaches
  - In other cases, you do the best that you can

A Paradox

- The Perils of Averaging
  - Consider the following two choices
    - * Take part in a lottery in which you can
      - Win zero with probability 1/2
      - Win \$1m with probability 1/4
      - Win \$2m with probability 1/8
      - Win \$4m with probability 1/16
      - Win \$8m with probability 1/32, etc., etc.
    - * Receive \$2m with certainty
    - * The expected value of the lottery is

$$0\left(\frac{1}{2}\right)+1\left(\frac{1}{4}\right)+2\left(\frac{1}{8}\right)+4\left(\frac{1}{16}\right)+\ldots=$$

$$0+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\ldots= \text{ infinity}$$

    - * Most people would choose the sure \$2m
    - * Why?

Paradox Resolved

- We choose $2m for the same reason that most people take out insurance
  - We are willing to pay to convert an uncertainty to a certainty
  - In part, this reflects our attitude toward risk

- Combining districts
  - You are an experienced senior salesman and your company has just reorganized its sales districts
  - Old districts A and B have been combined to form new district I
  - Old districts C and D have been combined to form new district II
  - Because you are a senior salesman, you can choose between districts I and II
  - You focus on the most important statistic: the percentage of total sales calls that lead to sales

Combining Districts

	District			
	A	B	C	D
Successful Calls	500	2300	400	4400
Total Calls	2500	2500	2500	5000
Percentage Successful	20	92	16	88

* You find that the percentage of successful calls was higher in district A than in district C
* Also, the percentage of successful calls was higher in district B than in district C
* Therefore, you conclude that district I will have the higher percentage of successful calls
* When the districts are merged, the table below emerges

	District	
	I	II
Successful Calls	2800	4800
Total Calls	5000	7500
Percentage Successful	56	64

* What happened?

Risk

* Rationality and Consistency
    - Behavior in the face of uncertainty is very personal
    - There are no “correct” answers, only consistent answers
    - Attitudes towards risk will change as resources increase
    - It is important that organizations should have an internally consistent attitude towards risk
    - The area of risk assessment is extremely complex
    - For example, consider the FAA infant restraint-seat rule

Getting a Lift Up

- Objectives and goals constantly change
  - Different parts of an organization
  - Individuals and groups have their own hidden agenda
  - Analyst is a change agent
- Expect the objectives to change during a study
- Anecdote on the optimal size of a parish
  - Look at two examples

Parishes	Total Costs	Availability for Worshippers
Many	High	High
Few	Low	Low

Going Up?

- Consult the literature on distribution management
  - Does the probability of church attendance follow the inverse square law?
  - No, over half the worshippers did not attend nearest church
  - Style of service and vicar were more important than distance
- The key factor was not the cost of "purchasing" the product, but the nature of the product

- The Problem with Elevators
  - Patrick House is a 22-floor office building
  - Owners are sensitive to complaints from tenants
  - However, owners don't want to spend much money
  - The owners have received many complaints regarding the length of time occupants have to wait for the elevator

Going Up?

- The floors are occupied as shown in Table 1
- Table 2 summarizes responses to a short questionnaire
- Thomas and MGR have leases that are up for renewal next year
- These complaints are looked at very carefully
- Three alternatives are proposed
  - Increase the power of the elevator motors
  - Stagger the arrival and departure times of occupants so that peak loading is reduced
  - Limit the stopping place of some of the lifts so that less time is spent in loading and unloading
- First alternative is dismissed due to the associated cost
- Tenants object to second and third alternatives
- The student analyst has an idea

Up, Up, and Away

- Idea is based on the notion of setting objectives
- Objective here is not to reduce waiting times, but to reduce complaints
- Student analyst proposes a series of large mirrors on each floor near elevators
- Owners install the mirrors and occupants are much happier
- Moral: keep your focus on the objective

Deterministic Problems

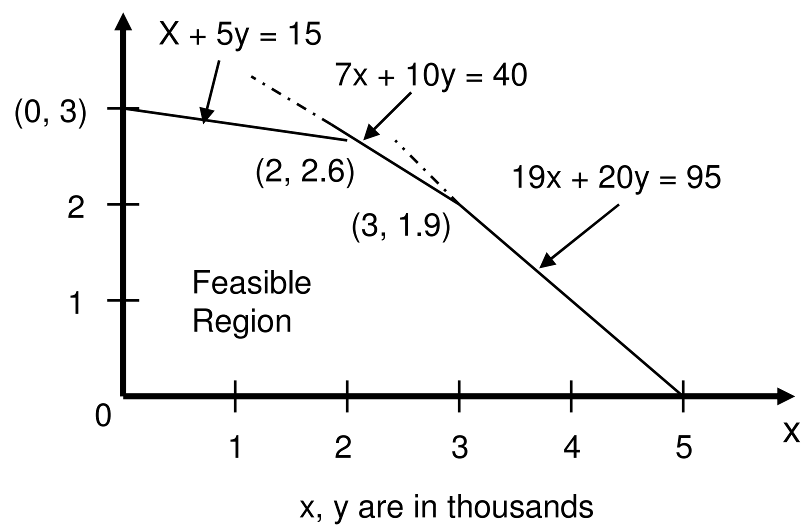
- Three elements in any management analysis
  - Units of quantitative measurement
  - Probability
  - Effect of time
- When uncertainty is small, we have a (nearly) deterministic problem
- Four examples
  - Example 1: a simple linear program
  - Example 2: a more complex linear program
  - Example 3: risk analysis
  - Example 4: decisions over time

A Linear Program

- A simple linear program
    - A factory manufactures two products—widgets and plaps
    - 1 widget requires 1 bong, 7 doodles, and 19 scams
    - 1 plap requires 5 bongs, 10 doodles, and 20 scams
    - $2 profit per widget made
    - $3 profit per plap made
    - 15,000 bongs, 40,000 doodles, and 95,000 scams are available
    - Demand for widgets and plaps is very large
    - Seek to maximize profit
    - How many widgets and plaps should be manufactured?
    - What will your maximum profit be?

A Linear Program

- Let x = # of widgets made
- Let y = # of plaps made
- Constraint 1: $x + 5y \le 15{,}000$
- Constraint 2: $7x + 10y \le 40{,}000$
- Constraint 3: $19x + 20y \le 95{,}000$
- Objective function: $2x + 3y$
- See figure below



A Linear Program

- The optimal solution must be at a vertex or extreme point of the feasible region

- There are five such points – (0, 0), (0, 3k), (2k, 2.6k), (3k, 1.9k), and (5k, 0)

- Optimal solution: x = 2000, y = 2600

- Total profit will be 2(2000) + 3(2600) = $11,800

A Large Linear Program

- A more complex linear program (see figure on page 131)
  - Transport coal from 20 mines to 10 washeries to 5 ovens
  - 200 road links between mines and washeries
  - 200 rail links between mines and washeries
  - 50 road links between washeries and ovens
  - 50 rail links between washeries and ovens
  - Each link has an associated cost per ton
  - Goal is to ship coal from mines to ovens at minimum cost, without violating constraints
  - The constraints are listed next
    - The production at each of the 20 mines (20 constraints)
    - The max amount that can be shipped from each mine by road (20 constraints)

A Large Linear Program

- The max amount that can be shipped from each mine by rail (20 constraints)
- The max amount shipped on each link from mines to washeries (500 constraints)
- The max amount that can be received (by road and rail) and sent (by road and rail) at the washeries (40 constraints)
- The throughput capacity at the washeries (10 constraints)
- The max amount that can be received by road and by rail at the coke ovens (10 constraints)
- The demand at the ovens (5 constraints)

➢ 625 constraints in all

➢ As far as real world LPs are concerned, this is small

➢ LP is a deterministic technique

Simulation

- Risk analysis
  - The Monte Carlo method
    - Simulate random numbers
    - Early example: The Buffon needle experiment to estimate pi
  - What is the return on investment?
    - Simulate the investment process
    - Keep score
    - Repeat many times
  - Results

Range to return	Frequency of return in the range
30% +	0.05
26% to 30%	0.10
22% to 26%	0.20
18% to 22%	0.30
14% to 18%	0.18
10% to 14%	0.11
6% to 10%	0.06
Average approximately 20%	

Critical/Longest Paths

- Critical Path Analysis
  - A project can be separated into numerous tasks
  - Each task requires a given number of hours to complete
  - Consider the network on page 135
  - Precedence relations
    - L cannot start until G and J are completed
    - I cannot start until F is completed
  - Critical path analysis identifies the sequence of tasks that leads to the minimum completion time for the overall project
  - Find the longest path from "start" to "finish"
  - Longest path is C, E, F, J, L
  - Duration is 24 hours

PERT

- Why do we seek the longest path?
- Note that the discussion in the book is wrong
- Now suppose task durations are not known with certainty
- We may be given probability distributions on task durations
- The goal is to compute the expected duration of the project and the variance of the project duration
- This is known as PERT

- Continuous time
  - How do we compare different time streams of money?
  - For example, see the table on page 136
  - If the interest rate is r, then a dollars today is worth
    - a $(1 + r)$ in one year
    - a $(1 + r)^2$ in two years
    - a $(1 + r)^3$ in three years

Net Present Value

- Alternatively, a_1 next year is worth $a_1 / (1 + r)$ today
- a_2, in two years, is worth $a_2 / (1 + r)^2$ today
- A cash flow of $a_1, a_2, a_3, \ldots, a_{10}$ over the next 10 years is worth $\sum_{i=1}^{10} \frac{a_i}{(1+r)^i}$ today
- The above is the present value of the cash flow
- If the initial investment which gives rise to the cash flow is I, then $\sum_{i=1}^{10} \frac{a_i}{(1+r)^i} - I$ is the net present value of the cash flow

A Network Model

- A finance-related Linear Program
  - Fred has $2200 to invest over the next five years
  - At the beginning of each year, he can invest money in one or two-year time deposits
  - The bank pays 8% interest on one-year time deposits
  - The bank pays 17% total on two-year time deposits
  - Also, three-year certificates will be offered starting at the beginning of the second year
  - These certificates will return 27% total
  - Fred reinvests his available money every year
  - Formulate a linear program to maximize his total cash on hand at the end of the fifth year
  - Represent the linear program as a network

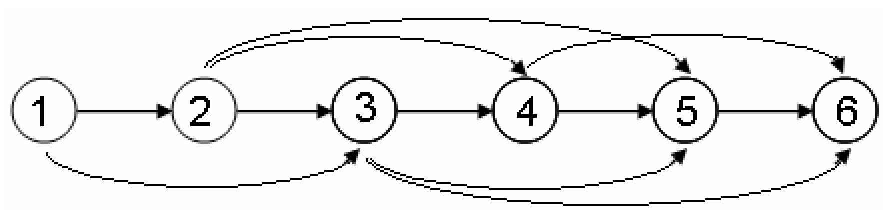
A Network Model

- Linear Program

Max	$1.27X_{33}+$	$1.17X_{42}+$	$1.08X_{51}$		
s. t.	$X_{11}+$	X_{12}		=	2200
	$X_{21}+$	$X_{22}+$	X_{23}	=	$1.08X_{11}$
	$X_{31}+$	$X_{32}+$	X_{33}	=	$1.17X_{12}+ 1.08X_{21}$
	$X_{41}+$	X_{42}		=	$1.17X_{22}+ 1.08X_{31}$
	X_{51}			=	$1.27X_{23}+ 1.17X_{32}+ 1.08X_{41}$
			X_{ij}	$\geq$	0

where X_{ij} = amount invested in period i for j years

- Network representation



Tattie Fabrix

- Introduction
  - This case uses probabilities in a fundamental way
  - It is based on actual studies in the textile industry in the U.S. and U.K.
  - Anecdote (a good forecast is one that works)
- Tattie Fabrix: Background
  - Dinkie Fabrix (DF) is a major manufacturer of a wide range of fabrics
  - Tattie Fabrix (TF) is a small subdivision
  - TF markets fabrics in the fashion market
  - TF's fabric sales are seasonal

Tattie Fabrix

- Season lasts 20 weeks
- Fabrics are designed before the season begins
- Samples are made by DF
- In response to trade buyers, DF prepares a supply of various lines
- During the season, orders are received week by week
- TF requests DF each week to manufacture given amounts of various lines
- Due to business demands, DF allows TF to place orders up to the 12th week of the season
- The sales manager at TF, Mr. Markup, is judged on profitability
- Mr. Markup purchases each line from the production arm at a given price per 100 pieces
- Suppose he buys one line at $100 per 100 pieces
- He may sell at $200 per 100 pieces
- As a last resort, he can sell back to DF at $70 per 100 pieces

Tattie Fabrix

- Consultant, Caroline Addup, meets with Mr. Markup
- Mr. Markup said he would buy at least 1000 pieces of each line at start of season
- Mr. Markup needs help in deciding on how much to order of each line in week 12 for the rest of the season
- There were ten lines last season as well as this one
- Table 1 (on page 145) shows the sales orders for last season for each of 10 lines
- Table 2 (on page 146) shows the total sales to date this season, remaining stock, profit, and loss for each of 10 lines after week 10

Tattie Fabrix

- How much of each line should be produced from one week to the next?
- If we produce nothing, we cannot realize a profit
- If we produce too much, we lose money
- If we make 100 pieces of a line and they sell, we gain P
- If they do not sell, we gain L (where L is negative)
- If p is the probability of selling, then the expected profit is

$$pP + (1 - p)L$$

- We need to estimate p each week

Tattie Fabrix

- Will total season's sales on a line exceed the sales already made, plus the stock on hand, plus the extra 100 pieces?
- To answer, we need to estimate p, given the available data
- Table 3 shows total sales by week – lines are aggregated
- Table 4 recasts Table 3 in terms of cumulative sales percentages
- Figure 1 represents Table 4 graphically
- How do we use this information to estimate p?
- Assume this season's sales build up as they did last year
- Suppose sales after 8 weeks total 36,000
- 36,000/total = 35%/100% ⇨
  Estimated annual sales = (36,000 x 100)/35 = 103,000

Tattie Fabrix

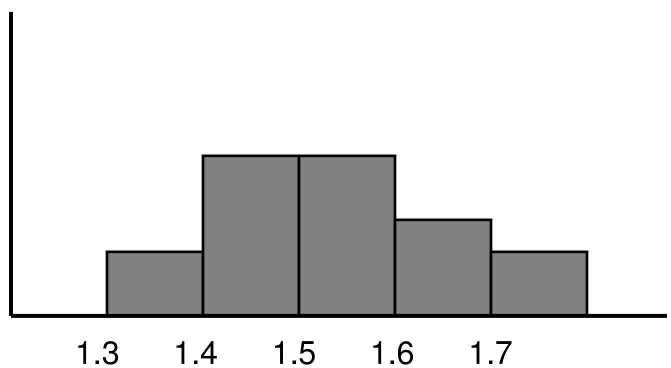
- Focus on a given line, after 12 weeks
- Suppose we have received orders for 5000
- Suppose stock on hand is 3000
- To order another batch of 100 pieces, we must believe that total season's sales ≥ 8100 (note that 8100/5000 = 1.62)
- Or total season's sales ≥ 1.62 x the 12 week total sales
- How likely is this?
- Imagine cumulating down each column of Table 1
- Then divide the column total by the cumulative sum in row k

Tattie Fabrix

- Table 5 consists of these “weekly multipliers”
- For example, focus on row 12 of this table

	Multipliers (week 12)									
Line	1	2	3	4	5	6	7	8	9	10
	1.64	1.45	1.70	1.55	1.49	1.57	1.53	1.44	1.64	1.37

- View the above multipliers as a random sample from the distribution of multipliers that we are likely to observe
- Next, we build a histogram



Tattie Fabrix

- From the histogram, we can construct a continuous curve such as Figure 2 (page 150)
- The probability of exceeding a particular multiplier is given by p
- Now review Table 6 on page 150
- After 12 weeks, this season, consult Table 7 (on page 151)
- Take line 1
- Relevant multiplier is 7100/5000 = 1.42
- What is the probability of exceeding this multiplier?
- From Table 6, p = .93
- Expected profit = .93(100) - .07(30) = 91
- Should we order an additional 100 pieces?
- Relevant multiplier is 7200/5000 = 1.44

Tattie Fabrix

- What is the probability of exceeding this multiplier?
- From Table 6, p = .87
- Expected profit = .87(100) - .13(30) = 83
- Analysis continues until Table 8 emerges
- Table 9 generalizes this approach to all 10 lines
- To obtain the appropriate order quantity per line, add the (positive) column entries
- Suppose we are only allowed to produce 100 additional pieces
- Line 2 is best
- Why?

Tattie Fabrix

- Suppose we are only allowed to produce 500 additional piece
- Line 2: 300 pieces

  Line 4: 200 pieces

- Underlying assumptions
  - The cost and profit figures make sense
  - The build-up of sales week by week is similar by year and homogeneous from line to line
  - The logic of the build-up of M against p is acceptable

Decision Making in Health Care

- Three examples
  - Red Cross Bloodmobiles
  - Cardiac Surgery Line Capacity
  - PACU Boarding

Go With the Flow:
Improving Red Cross Bloodmobiles Using Simulation Analysis

Prepared for BMGT 332

Outline of Study

- The Red Cross worried that long waiting lines and the time to donate blood might affect donors' willingness to repeat
- In response, we developed a computer simulation model to study customer service and productivity issues for Red Cross bloodmobiles
- We tested several strategies to alleviate this problem
- Initial implementation experience indicated positive results

Background

- The American Red Cross collects over 6 million units of blood per year in the U.S.
- There are 52 blood services regions
- There are over 400 fixed and mobile collection sites
- Mobile sites are in business, school, and community locations or in modified buses or trucks
- About 80% of Red Cross blood is collected at mobile sites

More Background I

- Donation time is said to be one hour, but is often 1½ to 2 hours
- Arrival at blood drives is random
- Donor scheduling (i.e., appointments) is largely avoided by the Red Cross
- The belief is that imposing appointments will alienate donors
- A key factor that has increased donation time is AIDS and hepatitis

More Background II

- AIDS has affected the donation process in two ways
  - Donor screening procedures have become more rigorous
  - Staff must take additional precautions
- Red Cross blood centers have limited budgets
- There is a severe shortage of nurses nationwide

Project Motivation

- The Red Cross relies heavily on repeat donors
- Donors are volunteers
- The Red Cross, therefore, wants satisfied (happy) donors
- They seek to minimize time spent in line and at the donation site
- Blood drive sponsors also want to minimize donation time
- If a drive sponsor is dissatisfied, The Red Cross may not be invited back

System Description

- See Figure 1 for the seven steps
- Figure 2 shows a typical physical set-up for a six-bed drive
- This set-up is common when 50 to 75 donors are expected in a five to six-hour period
- Significant delays occur in registration, taking vital signs, obtaining donor's health history, and in the donor room

The Blood Collection Model

- We have a typical queuing system
  - Donor arrivals are random
  - Servers are limited
  - Handful of decision points
- We used the six-bed unit as a basis for our model
- We were able to obtain data from historical records

Blood Donor Arrivals

- We examined the operations records for 76 blood drives
- We then modeled arrivals as a nonstationary Poisson process
- Three dominant patterns emerged
- See Figure 3

Service Times

- We collected service times for each of the major steps in the blood donation process
- We fit probability distributions to the observed data for each step
- We used a chi-square goodness of fit test
- We chose parameters using maximum likelihood estimation
- The results are summarized in Table 1

Model Development and Testing I

- We developed the blood collection model using GPSS/PC on an IBM PS/2 Model 60 computer
- We debugged, verified, and validated the model
- The Red Cross confirmed that it was intuitively valid
- We performed a variety of sensitivity analyses

Model Development and Testing II

- The results indicated that waiting and transit times were not overly sensitive to any one step in the process
- Increasing throughput at any one point (by adding servers or reducing service time) would have little beneficial impact
- Waiting time would simply increase at the next step

Model Development and Testing III

- Increasing throughput at the last constraining step (the donor room) would produce some benefit
- But, adding servers here would be costly in terms of personnel and space
- These tests indicated that any modifications had to balance the throughputs at the various steps to avoid bottlenecks

Modeling Analysis I

- We saw three possibilities for changing the collection process
  - Combine some or all of the donor screening steps into a single functional work station
  - Abandon the three-bed unit concept in the donor room in favor of having two phlebotomists share responsibility for 6, 7, or 8 beds
  - Develop formal work rules for floating staff who would assist in screening <u>and</u> in the donor room

Modeling Analysis II

- The first alternative would
  - Result in reduced service time since some tasks could be performed simultaneously
  - Make available more servers
  - Reduce the psychological cost of waiting
- This alternative obtains a 5% reduction in mean transit time and a 12% reduction in mean waiting time

Model Analysis III

- The second alternative would increase the likelihood that a phlebotomist is available to start or disconnect a donor
- This reduces the time a donor spends on a bed
- This alternative obtains a 13% reduction in mean transit time and a 51% reduction in mean waiting time

Model Analysis IV

- We did not model the third alternative by itself
- Rather, we modeled the three alternatives in various combinations
- Four scenarios are compared against the control scenario in Table 2
- Time saved (in minutes) over the control scenario is shown in Table 3

Implementation of Results I

- We conducted field trials of the strategies developed
- We modified one of the promising scenarios due to limited staff availability (see Figure 4)
- We collected detailed time data
- We surveyed donors to get their impressions
- We tried the new scenario on five blood drives

Implementation of Results II

- The new scenario was fine-tuned on the first and second blood drives
- We collected data only on the last three of the five drives
- The detailed results are shown in Table 4
- In the first two drives (at Duke and Lundy), mean transit times were much improved
- In the Easco drive, more donors arrived than expected

Implementation of Results III

- On the customer satisfaction side, the results were also positive
- Of repeat donors, 62% felt the donation process was shorter
- 73% felt that waiting time was reduced
- For specific comments, see page 11
- Within a year or two, 20% of Red Cross regions had implemented at least some of our recommendations

Conclusions

- Simulation was used to identify strategies to make the blood donation process easier on donors
  - Decrease donor waiting times
  - Decrease donor transit times
  - Improve the queuing environment
- In the future, the Red Cross will need to also develop an effective donor scheduling system
- The Red Cross considered this study to be a major success

Maximizing Cardiac Surgery Throughput at a Major Hospital

by

Carter Price, *University of Maryland*

Timothy Babineau, *University of Maryland Medical Center*

Bruce Golden, *University of Maryland*

Bartley Griffith, *University of Maryland Medical Center*

Edward Wasil, *American University*

Presented at ICHSS 2008

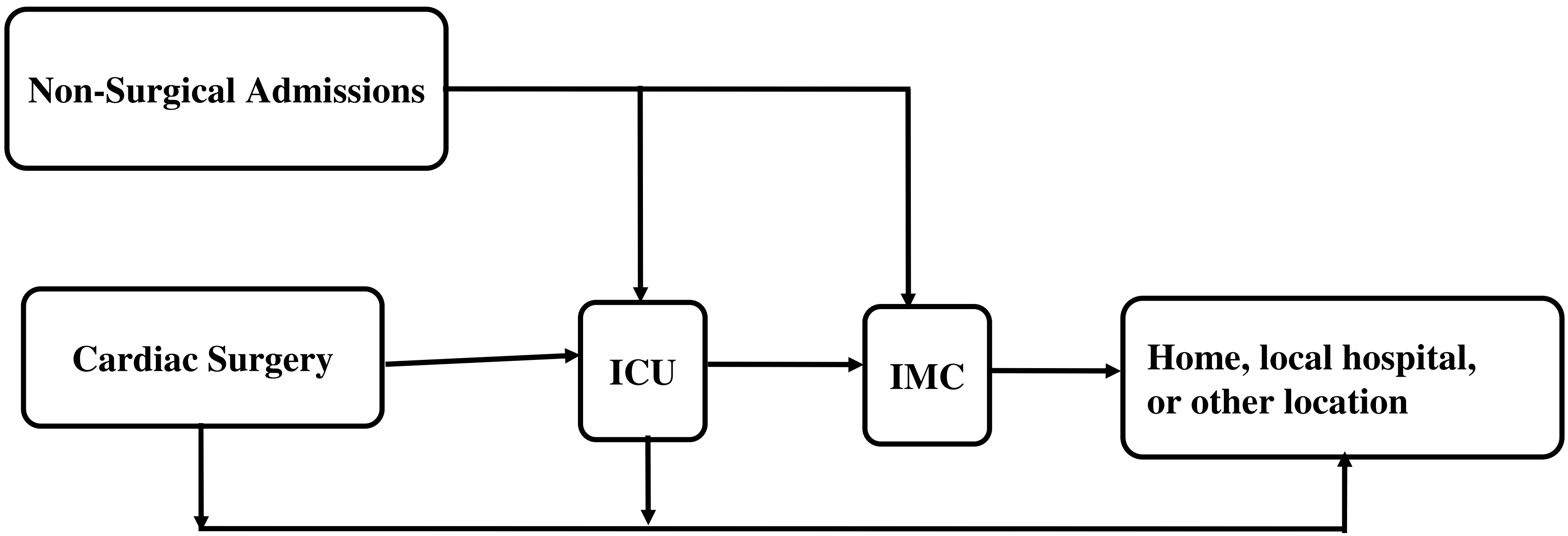
Problem Statement

- The Cardiac Surgery service line at the UMMC has 30 beds that are split between the intensive care unit (ICU) and the intermediate care unit (IMC)
- Total yearly capacity is 365 x 30 = 10,950 bed days
- From 7/1/05 to 6/30/06, there were 9,613 bed days used
- The service line is expected to grow at a rate of 13% --FY07 utilization will be at 99.2% of capacity

Problem Statement--continued

- At the time of the study, there were 11 ICU beds and 18 IMC beds
- One bed was not in use because of insufficient staffing
- **Key Question**: What is the best mix of ICU and IMC beds?

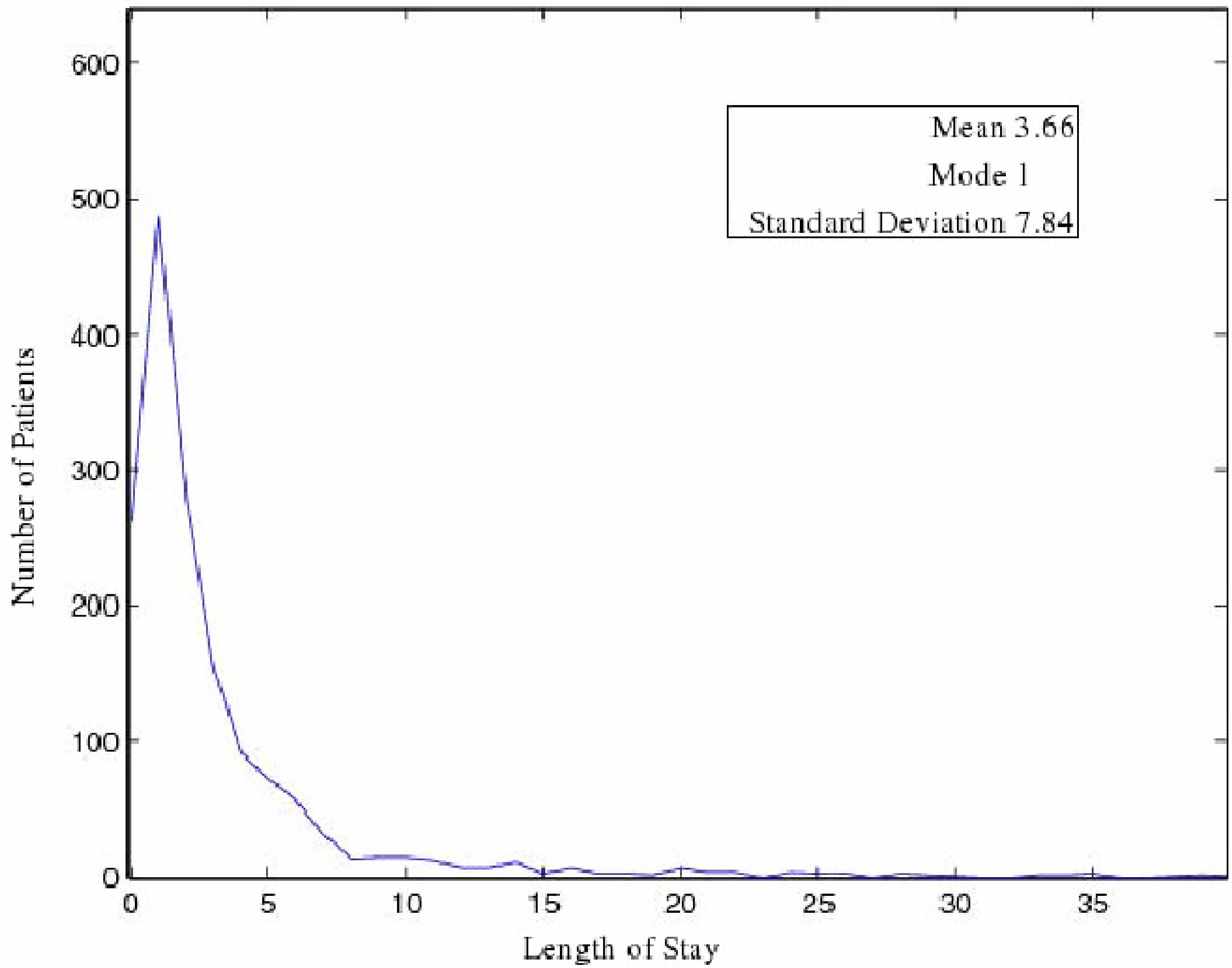
Flow of Patients



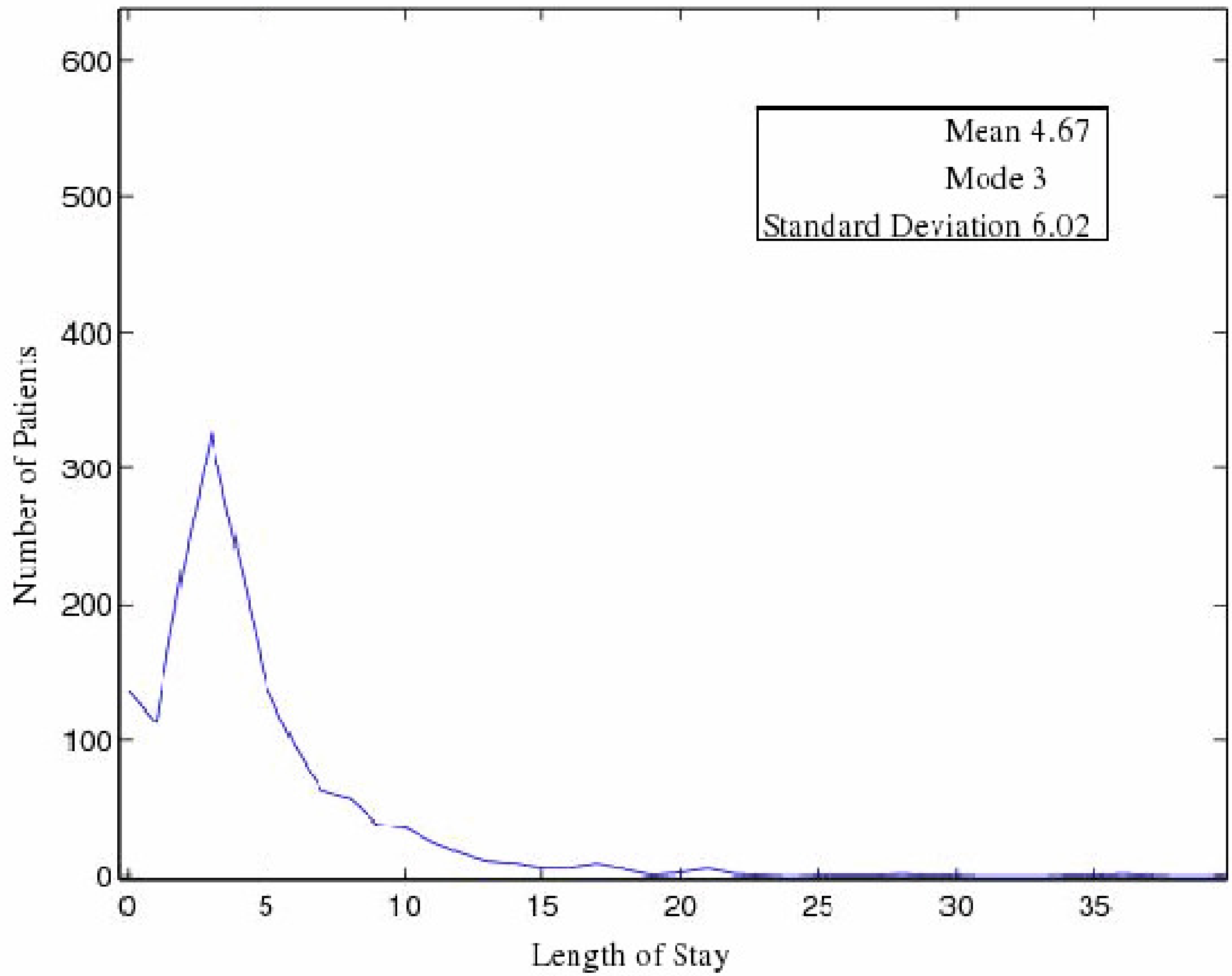
Data Set

- The data set contained detailed information about the length of stay for every cardiac surgery patient from FY05 and FY06
- 1,675 patients had 1,725 operations and spent more than 17,000 days in the hospital
- 83 patients did not spend time in cardiac surgery post-operative units
- On average, each patient in post-operative care had 1.085 operations

ICU Length of Stay



IMC Length of Stay



Methodology

- We used the data to perform a simulation of different mixes of ICU and IMC beds
- Blocking occurred when the IMC was full
- Initially, we assumed that the amount of time spent blocked in the ICU does not effect time in IMC
- We wanted to determine the maximum throughput, so a patient was admitted to the ICU whenever there was an open bed

Methodology--continued

- There were two different scenarios
  - Scenario 1: looked at maximizing throughput using all 30 available beds
  - Scenario 2: maintained the current staffing level of 80 nurses
- Each case in each scenario was simulated 999 times
- To approximate steady-state conditions, we simulated a 13 week period with a warm-up period of 13 weeks

ICU Throughput--Scenario 1

Bed Mix	11 ICU 18 IMC	12 ICU 18 IMC	13 ICU 17 IMC	14 ICU 16 IMC	15 ICU 15 IMC
Mean	230.04	248.31	262.88	270.20	268.39
Standard Deviation	15.44	14.66	13.17	12.03	11.40
Minimum	175	202	213	227	227
Bottom 5%	204	224	240	250	249
Median	230	248	264	271	268
Top 5%	255	272	283	290	288
Maximum	278	291	299	305	300

% Blocked--Scenario 1

Bed Mix	11 ICU 18 IMC	12 ICU 18 IMC	13 ICU 17 IMC	14 ICU 16 IMC	15 ICU 15 IMC
Mean	0.33	0.69	2.04	**4.44**	**8.43**
Standard Deviation	0.37	0.60	1.20	1.88	2.63
Minimum	0.00	0.00	0.00	0.38	1.51
Bottom 5%	0.00	0.03	0.45	1.65	4.16
Median	0.19	0.55	1.79	**4.21**	**8.30**
Top 5%	1.06	1.86	4.25	7.80	12.86
Maximum	2.55	3.75	8.76	11.23	16.10

Blocking

- Throughput is significantly effected when the system is in the blocked state more than 4% of the time, on average.
- It is counter-intuitive that changing an IMC bed to and ICU bed would reduce the throughput (a patient can spend his IMC time recovering in an ICU bed)
- We changed the model so that every day a patient spends blocked in an ICU bed, one fewer day was spent in the IMC.

ICU Throughput--Scenario 1

Bed Mix	11 ICU 18 IMC	12 ICU 18 IMC	13 ICU 17 IMC	14 ICU 16 IMC	15 ICU 15 IMC
Mean	285.62	317.85	337.72	367.97	375.77
Standard Deviation	35.14	30.26	32.83	35.12	40.42
Minimum	222	229	254	272	262
Bottom 25%	265	299	312	346	351
Median	282	318	343	372	382
Top 25%	305	334	360	390	406
Maximum	360	390	425	446	447

Patients Blocked--Scenario 1

Bed Mix	11 ICU 18 IMC	12 ICU 18 IMC	13 ICU 17 IMC	14 ICU 16 IMC	15 ICU 15 IMC
Mean	8.40	11.73	17.60	25.16	29.96
Standard Deviation	5.24	5.20	5.89	6.04	6.90
Minimum	1.35	1.43	3.85	11.29	14.12
Bottom 25%	4.61	7.72	13.58	20.76	26.38
Median	7.91	11.39	17.68	25.29	30.46
Top 25%	11.76	15.46	22.13	29.34	35.19
Maximum	20.28	22.56	32.39	39.01	51.69

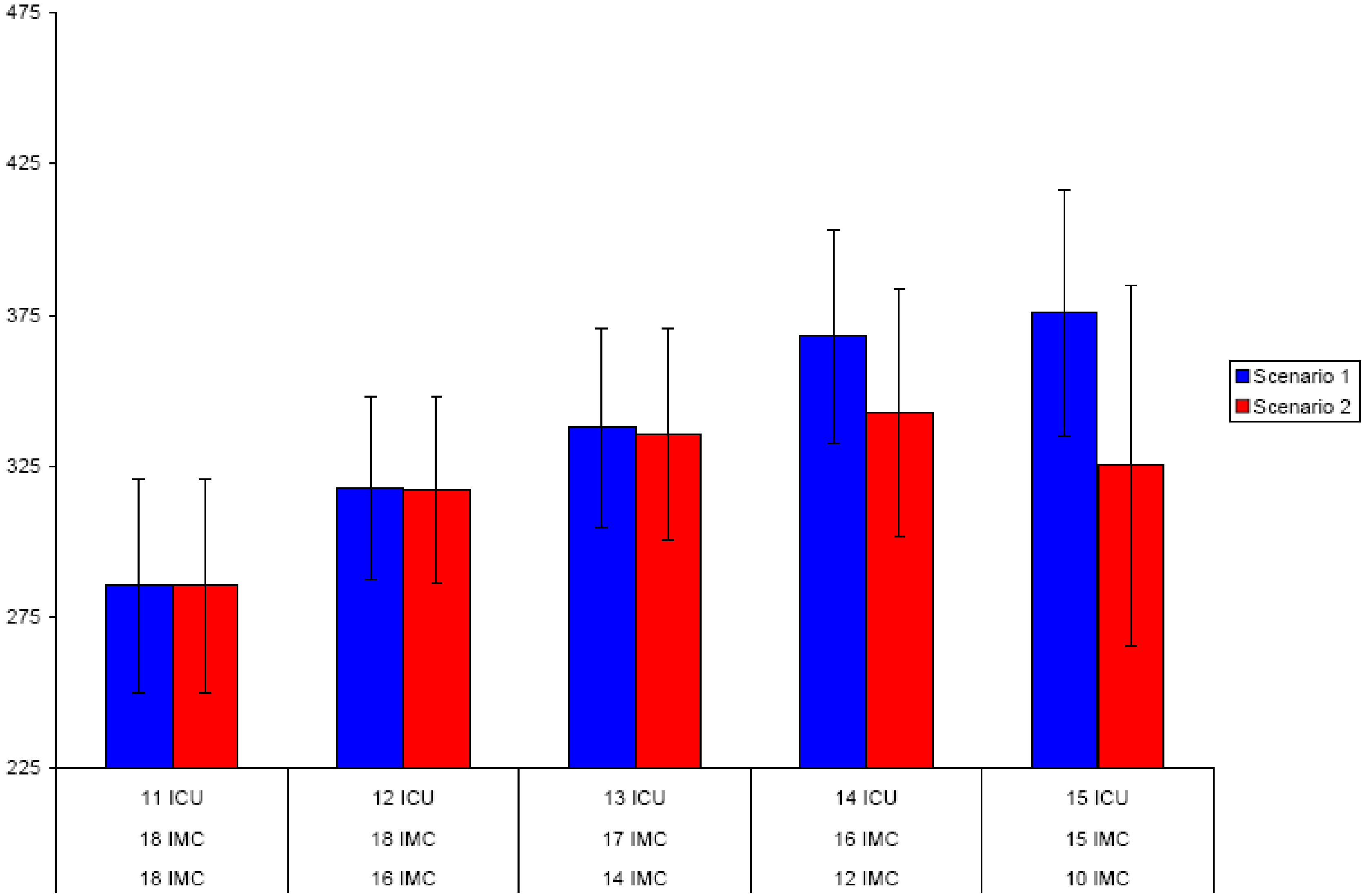
ICU Throughput--Scenario 2

Bed Mix	11 ICU 18 IMC	12 ICU 16 IMC	13 ICU 14 IMC	14 ICU 12 IMC	15 ICU 10 IMC
Mean	285.62	317.38	335.77	342.83	325.35
Standard Deviation	35.14	30.68	34.85	40.93	59.53
Minimum	222	232	244	246	237
Bottom 25%	265	295	310	318	275
Median	282	318	338	348	315
Top 25%	305	334	360	370	368
Maximum	360	387	409	429	439

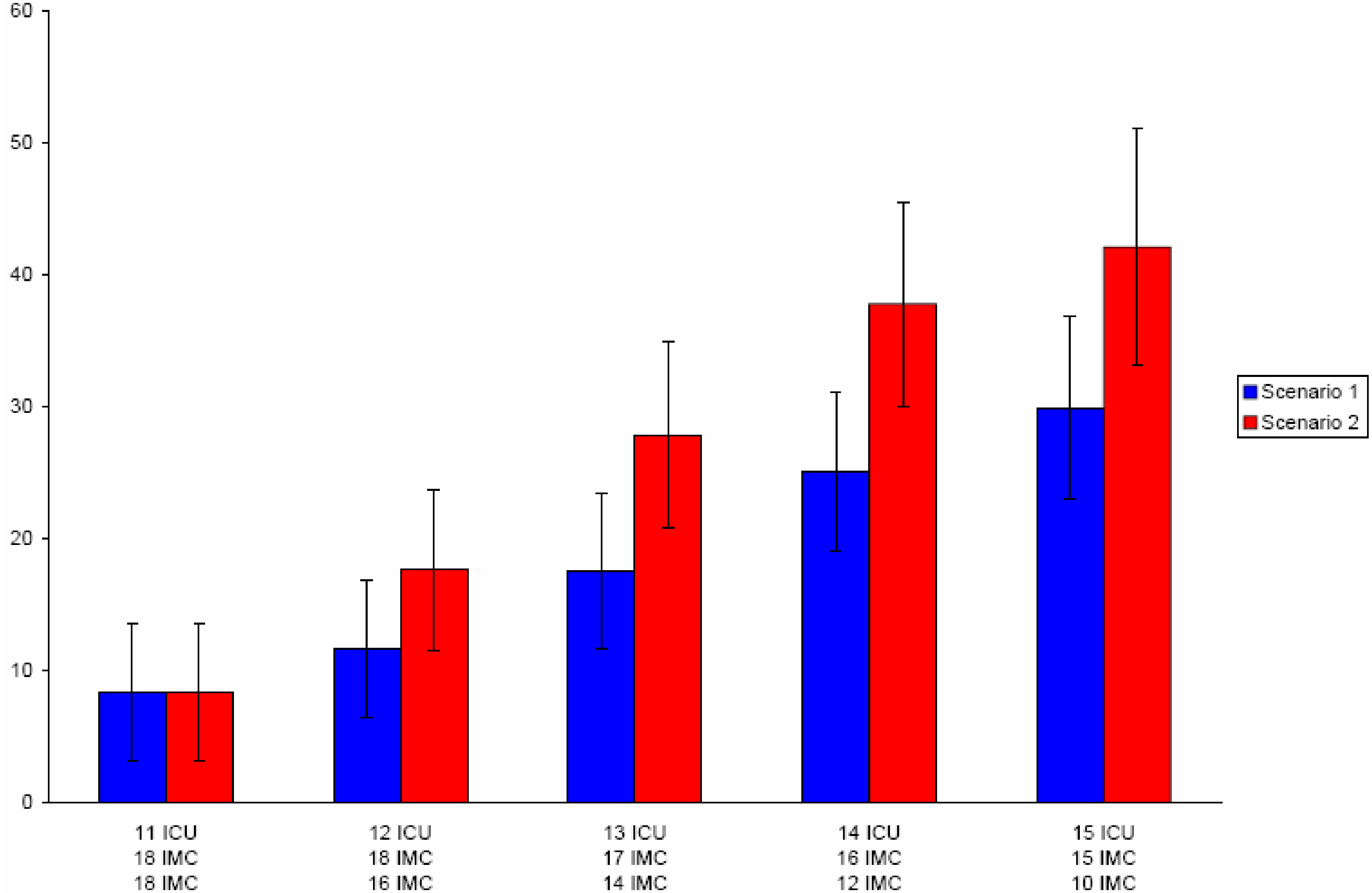
Patients Blocked—Scenario 2

Bed Mix	11 ICU 18 IMC	12 ICU 16 IMC	13 ICU 14 IMC	14 ICU 12 IMC	15 ICU 10 IMC
Mean	8.40	17.69	27.89	37.38	42.12
Standard Deviation	5.24	6.08	7.01	7.73	9.00
Minimum	1.35	3.88	9.47	15.04	24.89
Bottom 25%	4.61	13.40	22.89	30.99	32.60
Median	7.91	17.28	27.95	38.78	40.19
Top 25%	11.76	22.12	32.12	43.17	49.09
Maximum	20.28	31.66	44.94	55.44	62.71

ICU Throughput



Patients Blocked



Results--Scenario 1

- The 15/15 bed mix enabled a total volume increase of 31.57%
- Each cardiac surgery provides a net income of roughly $20,000
- Each nurse costs roughly $100,000
- The 15/15 bed mix yields an annual increase in profit of as much as 90 x 4 x $20,000 – 8 x $100,000 = $6.21 million

Results--Scenario 2

- The 14/12 bed mix enabled a total volume increase of 20.03%
- Each cardiac surgery provides a net income of roughly \$20,000
- Staffing levels are constant, so there is no additional cost for nurses
- The 14/12 bed mix yields an annual increase in profit of as much as 57 x 4 x \$20,000=\$4.58 million

Financial Results

Scenario 1

Bed Mix	12 ICU/ 18 IMC	13 ICU/ 17 IMC	14 ICU/ 16 IMC	15 ICU/ 15 IMC
Change in Mean	32.24	52.11	82.35	90.16
% Change	11.29	18.24	28.83	31.57
Change in Profit	$2,178,888.04	$3,568,591.01	$5,788,392.99	$6,212,551.41

Scenario 2

Bed Mix	12 ICU/ 16 IMC	13 ICU/ 14 IMC	14 ICU/ 12 IMC	15 ICU/ 10 IMC
Change in Mean	31.72	50.16	57.22	39.73
% Change	11.11	17.56	20.03	13.91
Change in Profit	$2,537,699.92	$4,012,551.41	$4,577,303.88	$3,178,492.00

Conclusions

- Currently, the hospital uses 13 to 14 ICU beds and 16 to 17 IMC beds depending on the immediate staff availability
- Simulation can help administrators optimize resource levels under a variety of constraints
- This work can be reproduce in other service lines and at other hospitals with similar results

Reducing PACU Boarding by Altering the Block Schedule

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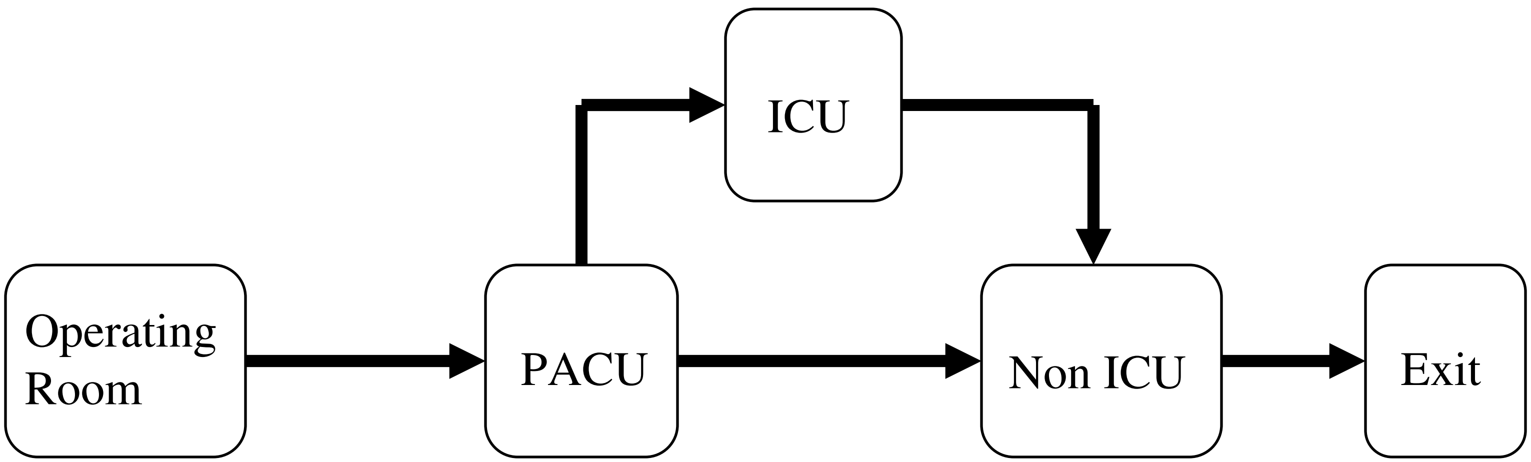
Presented at MSOM Conference, College Park, MD, June 2008

Problem Statement

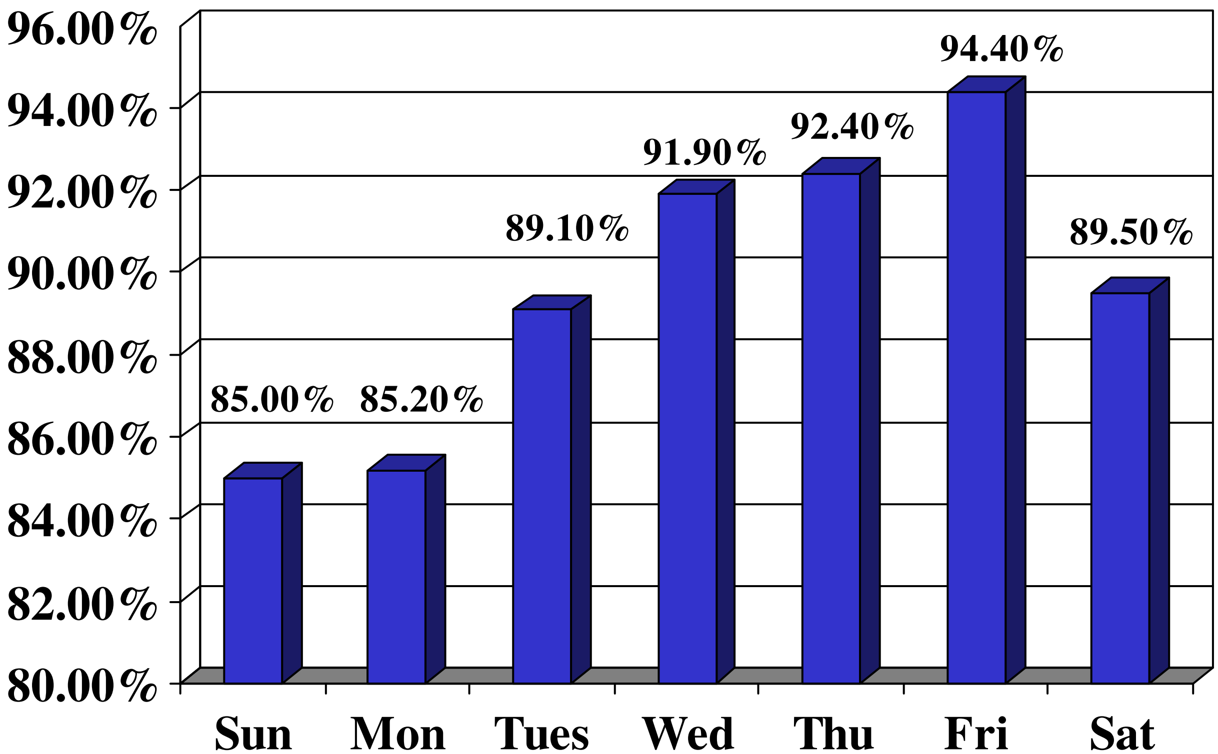
- The current structure of the block schedule does not consider the downstream effects of inpatient census
- Because of differences in service line case volumes per block, patient acuity, and post-op LOS, the current scheduling approach creates artificial variability that impacts inpatient census
- This artificial variability contributes to spikes in the inpatient census resulting in overnight boarders in the PACU

Patient Flow

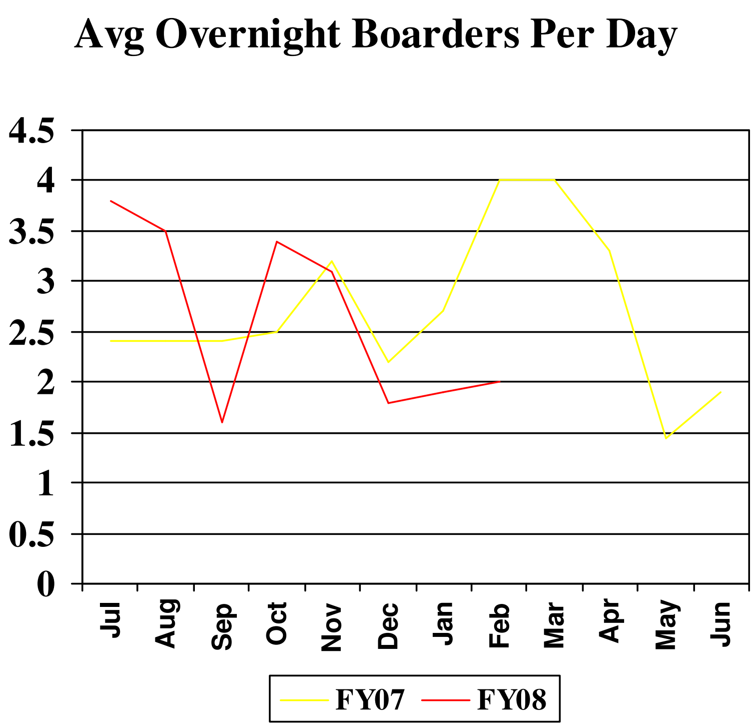


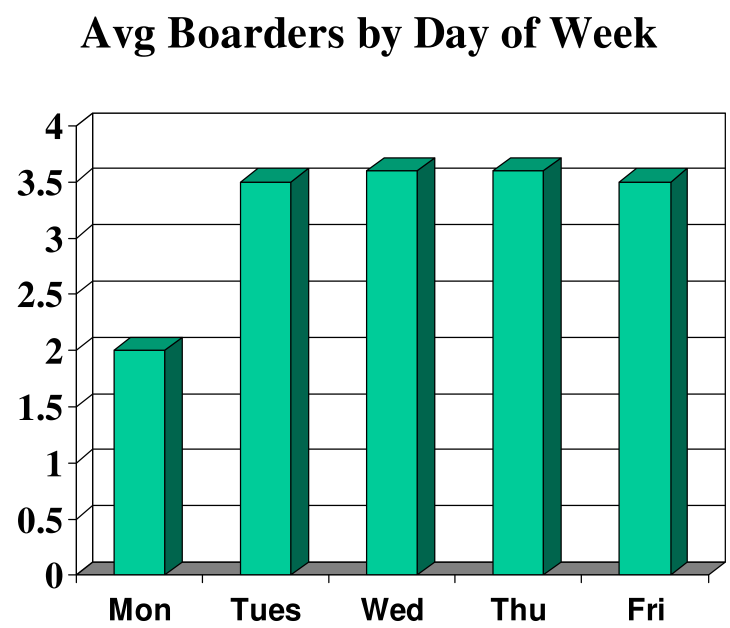
Surgical Staffed Bed Occupancy

FY 08 Jul-Feb

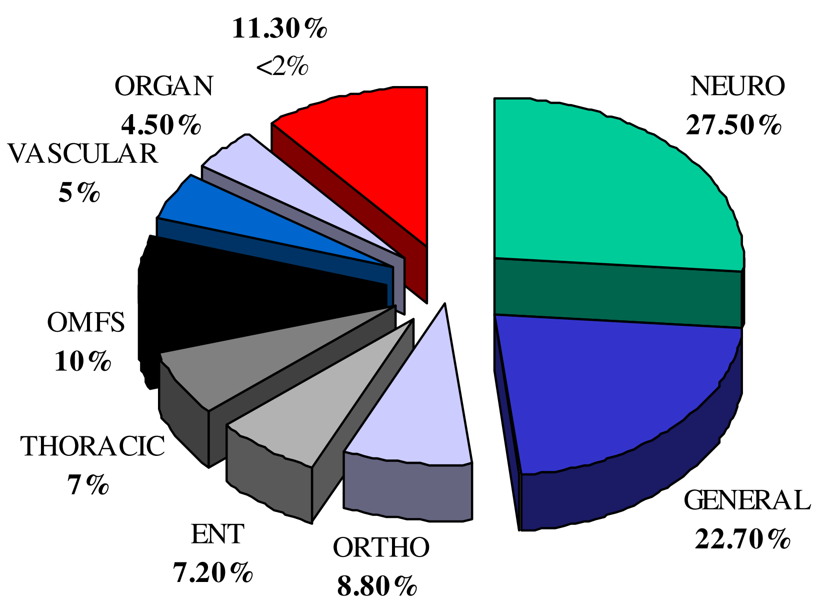


PACU Boarders

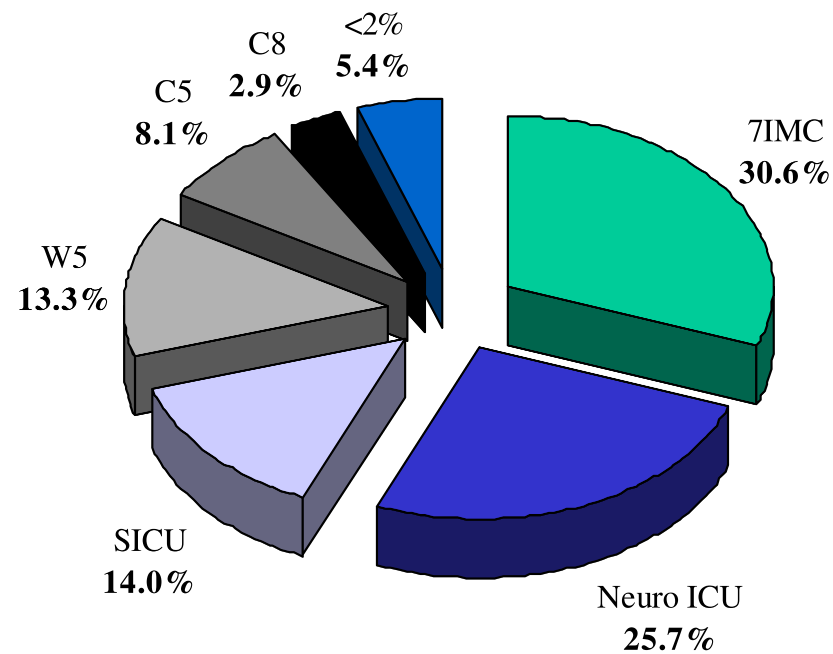




PACU Boarder Detail



SERVICE



UNIT

Objective of Study

- Understand current block allocation methodology and census variability via rigorous analysis of historic operating room and surgical inpatient volume data
- Develop a model linking the schedule to the census
- Create a load balancing schema for the surgical block schedule
- Validate model through computer simulation

Approach

- First, we clustered the surgical service lines by length of stay and cases per block (as measured by average case duration)
- Next, we constructed a mixed integer programming model (MIP) to match the flow of patients into the ICU with the expected discharges of patients from the ICU
- Finally, we tested different scheduling approaches using a simulation experiment

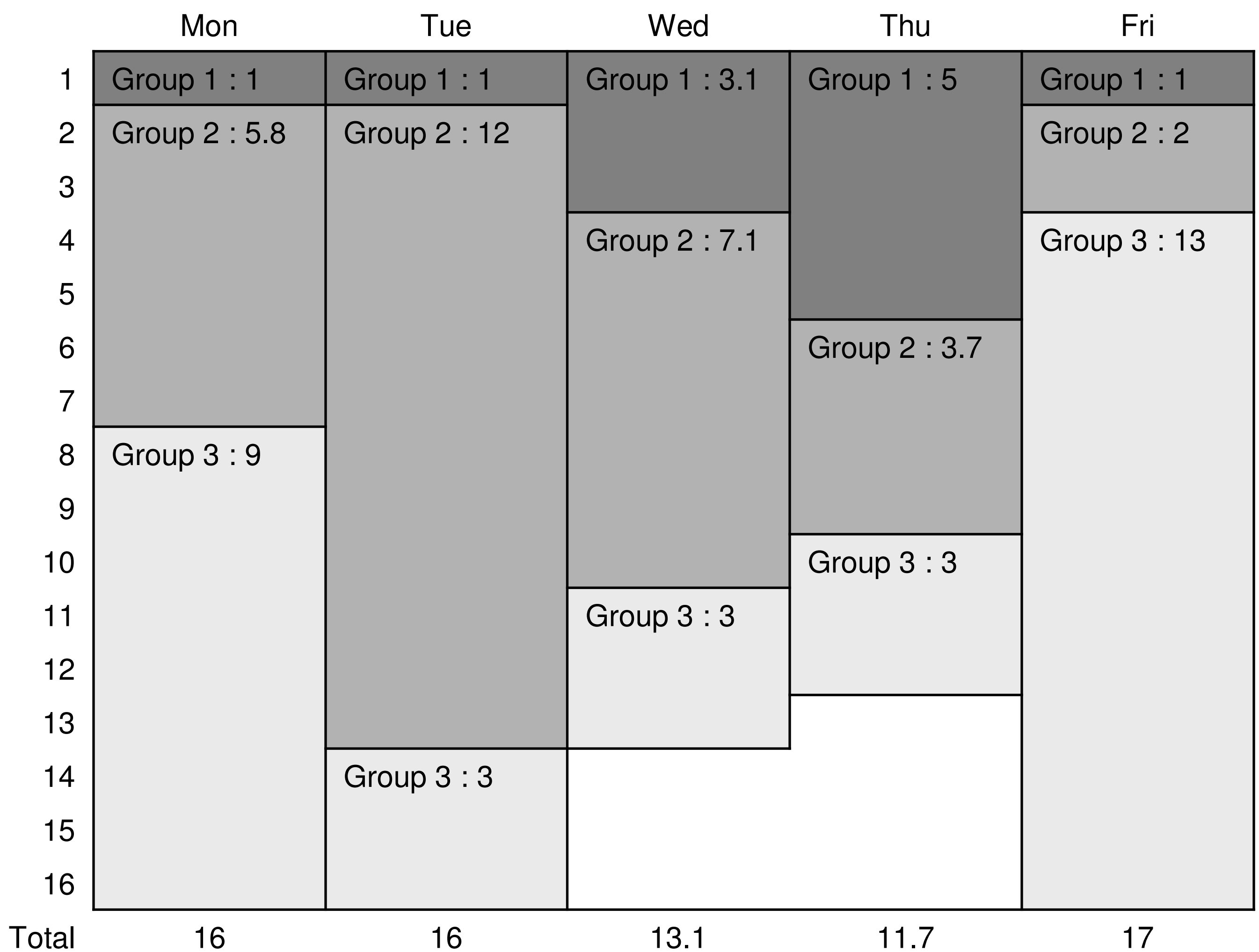
Groupings

- Service Lines were grouped by volume (cases performed in one room in one day) and post-operative length of stay
  - Group 1: Gynecology, Ophthalmology, and Urology (high volume, short LOS)
  - Group 2: General, Oral, Otolaryngology, Plastic, and Vascular (medium volume, medium LOS)
  - Group 3: Neurosurgery, Oncology, Organ Transplant, Orthopedics, and Thoracic (low volume, long LOS)

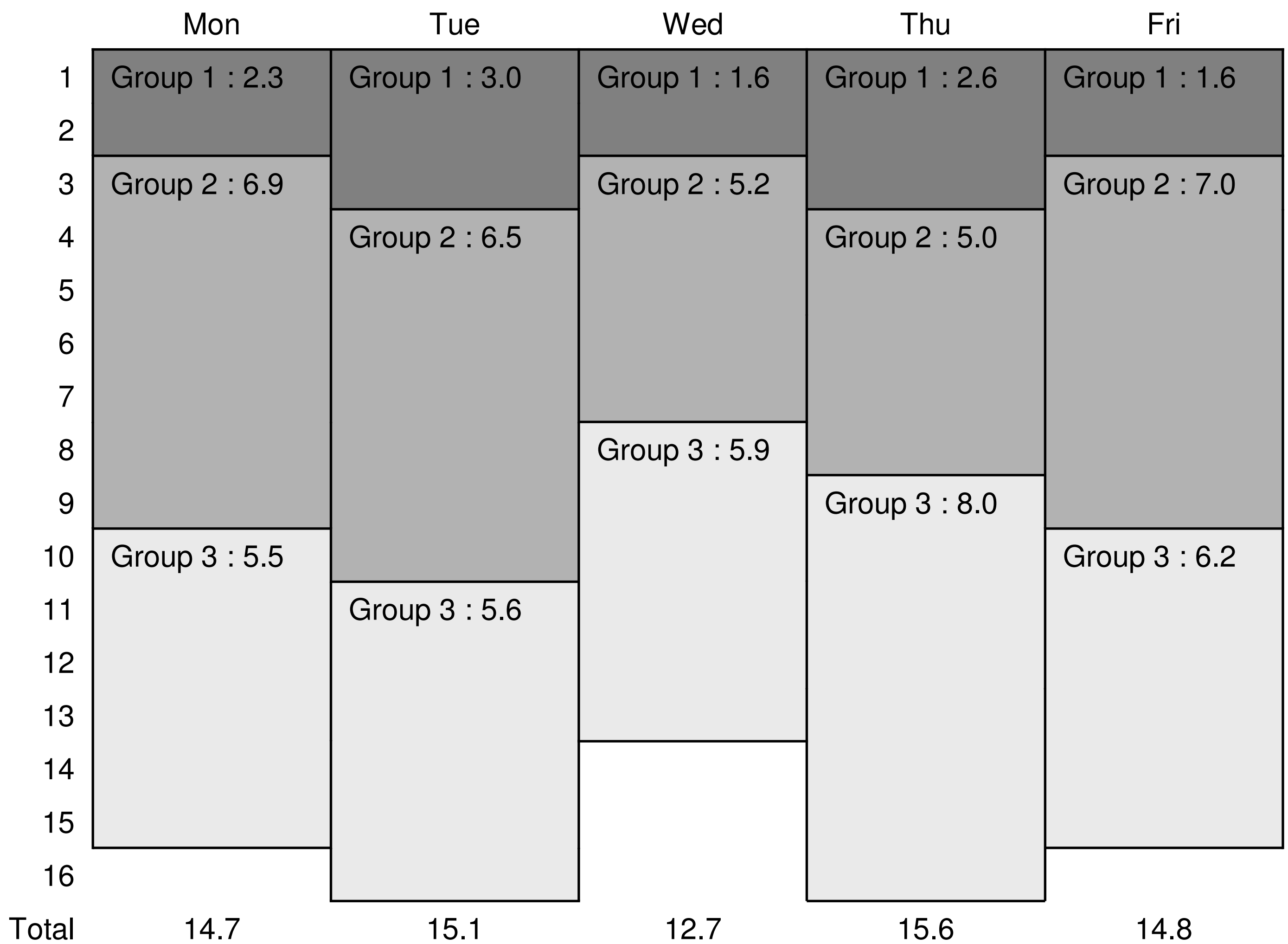
Model

- Redistribute service block time without altering total block time allocation
- Match ICU patients arrival with ICU patients departure from unit
- Cap total blocks per day and each group’s blocks per day based upon current allocation

IP Schedule


Mon
Tue
Wed
Thu
Fri
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
Total
Group 1 : 1
Group 2 : 5.8
Group 3 : 9
Group 1 : 1
Group 2 : 12
Group 3 : 3
Group 1 : 3.1
Group 2 : 7.1
Group 3 : 3
Group 1 : 5
Group 2 : 3.7
Group 3 : 3
Group 1 : 1
Group 2 : 2
Group 3 : 13
16
16
13.1
11.7
17

Current Schedule



Rules of Thumb

- Meet minimum daily demand
- Split Group 1 time between Wed and Thu
- Maximize Group 2 time on Tue
- Split Group 2 time between Mon, Wed, and Thu
- Split Group 3 time between Mon and Fri

Rules of Thumb (1 of 5)

- Meet minimum daily demand
- Based on the current block schedule certain service lines (Oto, Neuro, Ortho, Gen, Uro) receive at least one block every day of the week

	Mon	Tue	Wed	Thu	Fri	Total
Group 1	1	1	1	1	1	5
Group 2	2	2	2	2	2	10
Group 3	4	4	4	4	4	20
Total	7	7	7	7	7	35

Rules of Thumb (2 of 5)

- Split Group 1 time between Wed and Thu
- Wednesday and Thursday are the "heavy" days for Group 1 in the IP

	Mon	Tue	Wed	Thu	Fri	Total
Group 1	1	1	4.1	4	1	11.1
Group 2	2	2	2	2	2	10
Group 3	4	4	4	4	4	20
Total	7	7	10.1	10	7	41.1

Rules of Thumb (3 of 5)

- Maximize Group 2 time on Tue
- The IP puts the most blocks for Group 2 on Tuesdays

	Mon	Tue	Wed	Thu	Fri	Total
Group 1	1	1	4.1	4	1	11.1
Group 2	2	9	2	2	2	17
Group 3	4	4	4	4	4	20
Total	7	14	10.1	10	7	48.1

Rules of Thumb (4 of 5)

- Split Group 2 time between Mon, Wed, and Thu
- Monday, Wednesday, and Thursday had more than the minimum number of blocks in the IP

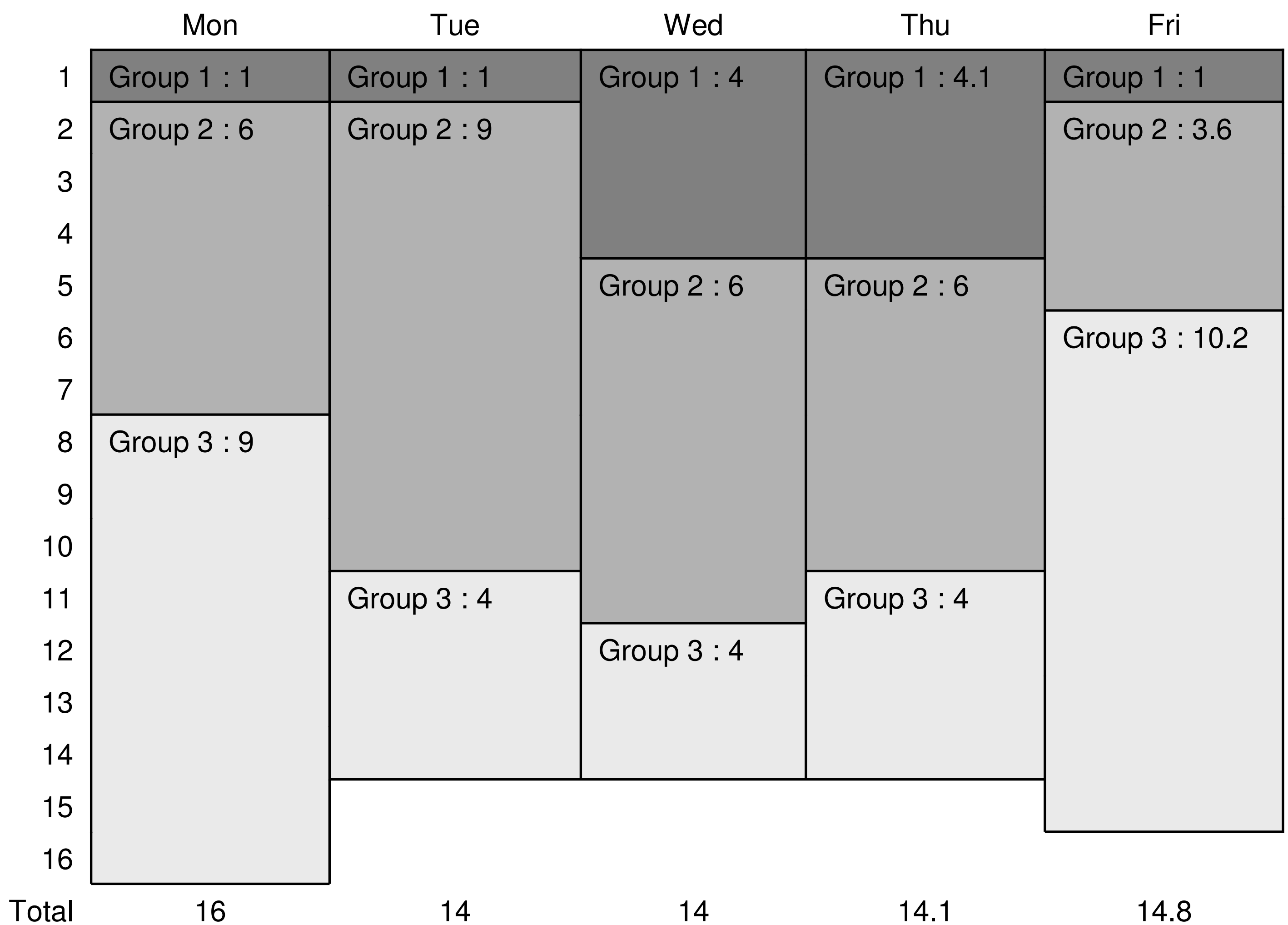
	Mon	Tue	Wed	Thu	Fri	Total
Group 1	1	1	4.1	4	1	11.1
Group 2	6	9	6	6	3.6	30.6
Group 3	4	4	4	4	4	20
Total	11	14	14.1	14	8.6	61.7

Rules of Thumb (5 of 5)

- Split Group 3 time between Mon and Fri
- Mon and Fri had most of the blocks for Group 3
- Wed gets an extra block because the total demand for Group 3 must be met

	Mon	Tue	Wed	Thu	Fri	Total
Group 1	1	1	4.1	4	1	11.1
Group 2	6	9	6	6	3.6	30.6
Group 3	9	4	4	4	10.2	31.2
Total	16	14	14.1	14	14.8	72.9

Rules of Thumb Schedule



Perturbation Schedule

- There were concerns about totally replacing the current block schedule
- We looked into making a few swaps to the current schedule based on the rules of thumb
- Swap 1: a Grp 1 on Mon with a Grp 3 on Wed
- Swap 2: a Grp 2 on Mon with a Grp 3 on Tues
- Swap 3: 2 Grp 2 on Fri with 2 Grp 3 on Thu

Perturbation Schedule (cont.)

- How could this be done?
- Swap 1: a Grp 1 on Mon with a Grp 3 on Wed
  - GYN on Mon (19) with Thoracic on Wed (23)
- Swap 2: a Grp 2 on Mon with a Grp 3 on Tues
  - General on Mon with Thoracic on Tues
- Swap 3: 2 Grp 2 on Fri with 2 Grp 3 on Thu
  - General on Fri (18) with Thoracic on Thu (23)
  - Oncology on Fri (16) with Oto on Thu (16)

Simulation Tests

- Five schedules tested: Current, Even, MIP, Rules of Thumb, and Perturbed
- 5 week warm up period
- 10 weeks of data collection
- 10,000 runs for each schedule
- Estimated daily boarders using simple formula: Boarders_i=Max(ICUCensus_i – ICUCapacity_i,0)

Simulation Results

	Average Boarders per Day			Census	
	5%	Mean	95%	Mean	St. Dev.
Historical	3.36	4.67	6.06	30.93	11.70
Even	3.20	4.50	5.89	30.94	11.37
Perturbed	3.07	4.32	5.69	30.93	11.11
MIP	2.70	4.00	5.50	30.94	10.76
Rules	2.73	4.02	5.46	30.94	10.65

ICU Capacity = 31
(SICU and Neuro ICU)

Efficiency

Schedule	Swaps	Weekly Reduction	Standard Deviation	Efficiency
Perturbed	4	2.45	11.11	0.61
Rules	16	4.55	10.65	0.28
MIP	33	4.69	10.76	0.14

Financial Implications

- On an annual basis, there would be 4.69 * 50 = 234.5 fewer boarders
- If 234.5 additional surgeries were performed, the hospital could have 234.5*$15,000 = $3,517,500 in additional revenue

Conclusions

- The simulation results indicate that the Rules of Thumb schedule, IP schedule, and Perturbed schedule reduce the number of boarders and the variation in the census
- The Perturbed schedule gets about half of the total benefit with only four block exchanges