

# Optimal Pricing Scheme for Information Services

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## Abstract

This paper examines which, among three commonly used pricing schemes: the flat-fee, pure usage-based and the two-part tariff pricing, is optimal for a monopolist providing information services. Our analysis suggests that under zero marginal costs and monitoring costs, when customers are homogeneous or when customers have heterogeneous marginal willingness to pay (which corresponds to different downward sloping demand curves), flat-fee pricing and two-part tariff pricing always achieve the same profit level, and dominate usage based pricing. However, when customers are characterized by heterogeneous maximum consumption levels (or usage levels), the two part tariff pricing is the most profitable among the three. We also examine how sensitive the optimal pricing scheme is to marginal costs and monitoring costs. Our analysis shows that when marginal cost is below a certain value, the flat fee pricing is the optimal scheme regardless how large or how small monitoring cost is (as long as it's positive) when customers are homogeneous or have heterogeneous marginal willingness to pay. But as monitoring cost becomes zero, the two-part tariff will also become one of the optimal pricing schemes.

## 1. Introduction

The advance of the Internet and other telecommunication networks has made many new information services available. For example, a number of software applications services are provided by online Application Service Providers (ASP) over the Web or private networks for businesses or end users. Online discounted brokerage is also an example of information services which end users could get access to over the Web. Short Messaging Services (SMS) and WAP services are among the most popular services for mobile users. Moreover, as carriers roll out their 2.5G and 3G networks, a host of other new services are being offered-- from mobile e-mail to multimedia messaging services (MMS)-- that represent valuable new revenue opportunities (Maier, 2002). However, these information service providers generally face one difficulty in providing the services, that is, how to price and bill these information services (Maier, 2002), which are characterized by negligible marginal, distribution, and monitoring cost.

In this paper we examine the issue of pricing information services. In particular, we are interested in knowing which among the three most popular pricing schemes used in practice: flat fee, usage-based and two-part tariffs, is the best for a monopolist providing information services.

While some researchers believe that reduction in monitoring cost or distribution cost makes usage-based pricing a relatively more attractive option (Choi, Stahl and Whinston, 1997; Metcalfe, 1997), some argue that negligible marginal production cost makes flat-fee pricing more profitable (Fishburn, Odlyzko and Siders, 1997). There is so far no clear guideline about when the information service providers should adopt flat rate pricing and when pure usage-based pricing (without a subscription fee) or even a two-part tariff (usage-based pricing plus a subscription fee) is more profitable. Many information service providers have struggled to find best ways to price their services and bill their customers, and this is reflected from the non-

agreed-upon pricing schemes offered by different information service providers. For example, Verizon Wireless, which rolled out its 3G Express Network in late January, 2002 has chosen a flat fee pricing scheme, while AT&T Wireless has implemented a usage-based pricing scheme that bill for the amount of data a customer uses. Yet another scheme adopted by NTT DoCoMo's I-mode service in Japan charges users a \$2.50 monthly fee, plus 25 cents per data packet (one packet is equivalent to 128 bytes of data), is a two-part tariff scheme. Outsourcing of IT services uses both fixed-fee pricing in some cases, and usage-based pricing with or without subscription fees in some other cases (Gopal et al., 2001). For example, the newly signed contracts between American Express Co. and IBM, and convenience-store chain 7-Eleven Inc. and EDS are both based on usage-based pricing with some fixed fees, i.e., the two-part tariff pricing (Greenemeier, 2002).

While there has been increasing interest on how to price *information goods* (Bakos and Brynjolfsson, 1999; Chuang and Sirbu, 1999; Varian, 2000), much of the work either does not address information services, or is only indirectly applicable to such cases. For example, the advantages of pure bundling in Bakos and Brynjolfsson (1999) result from reduction in the variance for customers' valuations for a bundle of *different* information goods. This modeling technique cannot be applied to information services, since each unit of information services is essentially identical,<sup>1</sup> and therefore, we cannot expect variance to be reduced through aggregation of *identical* units. We use a framework in which both buyers and sellers of information goods optimize their net values in order to determine which pricing scheme works best under different conditions.

Recent works that are related to information service pricing include Fishburn, Odlyzko and

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<sup>1</sup> Thus they will have perfect correlation across units in customers' valuations.

Siders (1997), Essegaiier, Gupta and Zhang (2002) and Sundararajan (2002). Our paper is complementary to these papers. Sundararajan (2002) considers a fixed-fee and a nonlinear usage-based pricing schemes. But the focus of this study is different from ours in this paper. He shows that in the presence of contract administration costs, such as monitoring cost for usage based pricing, a monopolist can improve its profits by offering fixed-fee pricing in addition to a usage-based contract. However, while this study has suggested that a firm could improve its profits by adopting a mix of pricing schemes, the results are based on a utility function that has to satisfy the Spence-Mirrlees single-crossing property, which allows a firm to possibly segment customers through their self-selection profitably. When this property does not hold, it is not clear whether adopting multiple schemes will still be profit improving. In addition, this paper does not offer a direct guideline about which pricing scheme is most profitable when the firm could only opt for one pricing scheme, the major focus of our paper. There are several situations where firms may favor to adopt one pricing scheme only. For example, when a *new* information service is just being provided, the firm may prefer to adopt only one pricing scheme to keep the marketing simple; easier administration and management may also make the firm may prefer one pricing scheme only. Fishburn, Odlyzko and Siders (1997) compare the flat fee and the usage-based pricing and show that a flat fee is better than a metered rate for a monopolist offering information services on the Internet. However, they have simplified the problem with some very restrictive assumptions. For example, they assume that consumers choose the quantity of service to buy and stick to it *before* examining the available prices. It is not clear whether their results could be generalized to more general demand functions, i.e., downward sloping demand function. And finally, both of these works do *not* consider the two-part tariff pricing, which is popular both in theory and practice; we examine this case also. Essegaiier, Gupta and Zhang's (2002) also

consider the two-part tariff pricing together with flat fee and usage-based pricing. As in Fishburn, Odlyzko and Siders (1997), they assume that consumer usage is inelastic to price change. Moreover, they assume that both heavy and light users have the same total reservation price for the service, which may be doubtful as users usually have quite different and diminishing marginal utility for each unit of service they consume.<sup>2</sup> It is also questionable to assume that marginal cost is zero when service provider has capacity constraint (another problem with capacity constraint is the possible queuing problems, which is not discussed in their paper).

Overall, our analysis suggests that under zero marginal and monitoring costs, when customers are homogeneous or when customers have heterogeneous marginal willingness to pay (which corresponds to different downward sloping demand curves), flat-fee pricing and two-part tariff pricing always achieve the same profit level, and are strictly better than the usage-based pricing. However, when customers are characterized by heterogeneous maximum consumption levels, two-part tariff pricing is the most profitable among the three. We also examine how sensitive the optimal pricing scheme is to marginal costs and monitoring costs when customers are homogeneous or when customers have heterogeneous marginal willingness to pay. Our analysis shows that when marginal cost is below a certain value, the flat fee pricing is the optimal scheme regardless how large or how small monitoring cost is (as long as it is positive) when customers are homogeneous or have heterogeneous marginal willingness to pay. But as monitoring cost becomes zero, the two-part tariff becomes one of the optimal pricing schemes.

The paper is organized as follows: In section 2, we provide the general model for the market for information services. Section 3 reports on the analysis of different pricing schemes and when each of them is most profitable. Section 4 outlines some model extensions. We provide

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<sup>2</sup> Although they try to extend their model near the end of the paper by using the same unit reservation price for the two consumer segments, this assumption still fails to reflect the truth that users usually have quite different and diminishing marginal utility for each unit of service they consume.

concluding remarks in section 5.

## 2. Market Model for Information Services

We examine the optimal pricing scheme for an information service provider who sells one kind of information service (such as voice communication service or data transmission service) to consumers. We consider three pricing schemes: pure flat fee, pure usage-based and two-part tariff pricing. The information service provider chooses which pricing scheme to adopt and the price(s) to offer. Consumers then make decisions about whether to join the plan, and how much to consume given the pricing scheme and prices set by the information service provider.

Since information services usually experience some peak-hours and some non-peak hours, we assume that consumers may have different utility functions in peak hours and non-peak hours. As a result, information service providers may charge different prices for the two time segments when using usage-based pricing. In addition, given limited time and attention, we assume that consumers face certain upper bounds in consuming the services. For example, given that there are only 24 hours in a day, consumers can't consume the service for more than 24 hours a day.

### ***2.1. Consumers Optimization Problem***

Given the pricing scheme (flat rate, usage-based, or two-part tariff) and price(s) set by the information service provider, consumer  $i$  will decide whether or not she wants to join the service program and her consumption level of the service in both peak hours and non-peak hours to *maximize her total net utility*.

*Given Parameters:*

$P$ : the subscription fee for the consumer to join the program  
 $P_X$ : the unit price of the service set by the provider in peak hours  
 $P_Y$ : the unit price of the service set by the provider in non-peak hours  
 $U_i(X_i, Y_i)$ : the utility function of consumer  $i$  at the consumption level of  $X_i$  in peak hours and  $Y_i$  in non-peak hours  
 $\bar{X}_i$ : consumer  $i$ 's maximum consumption level of the service in peak hours  
 $\bar{Y}_i$ : consumer  $i$ 's maximum consumption level of the service in non-peak hours

*Decision Variables:*

$X_i$ : consumer  $i$ 's consumption level of the service in peak hours  
 $Y_i$ : consumer  $i$ 's consumption level of the service in non-peak hours  
 $Z_i$ : the decision variable which is 1 if consumer  $i$  chooses to join the program and 0 otherwise

*Consumers Optimization Problem:*

$$\text{Max}_{X_i, Y_i, Z_i} U_i(X_i, Y_i) - P_X X_i - P_Y Y_i - P Z_i \quad (1)$$

s.t.

$$X_i \leq \bar{X}_i Z_i \quad (2)$$

$$Y_i \leq \bar{Y}_i Z_i \quad (3)$$

$$U_i(X_i, Y_i) - P_X X_i - P_Y Y_i - P Z_i \geq 0 \quad (4) \quad (\text{the Individual Rationality constraints})$$

$$Z_i = 0 \text{ or } 1 \quad (5)$$

The objective function (1) is to maximize the consumer surplus given the price(s) set up by the information service provider. In our model, we do not consider the initialization cost for the consumer to join the program, such as the purchase of 3G mobile devices in the 3G wireless service scenarios for two reasons. First, when we consider the long-run relationship between the supplier and consumers, this kind of one-time-expense may not be as important as the monthly usage fee and the subscription fee. Further, this one time fee does not affect the optimization problem, and it can be absorbed by  $U_i(X_i, Y_i)$ . Note also that there is no parameter in this model

that indicates the pricing scheme adopted by the information service provider. Rather than using additional parameter to indicate the pricing mechanism, the pricing scheme chosen actually is reflected by the values of  $P_X$ ,  $P_Y$ , and  $P$ . For example, when  $P_X$  and  $P_Y$  are both zero and  $P$  is positive, it is the pure flat rate pricing; when  $P_X$  and  $P_Y$  are positive and  $P$  is zero, it is the pure usage-based pricing; and when  $P_X$ ,  $P_Y$ , and  $P$  are all positive, it is the two-part tariff pricing. Additionally, in this paper, we majorly consider the simple and most commonly adopted usage-based and two-part tariff pricing in which the unit price of the service is constant and doesn't change with the consumer's consumption level. For example, almost all, if not all, residential long distance voice communication service (with or without a monthly fee) and wireless data transmission service have a constant unit price.

Given  $P_X$ ,  $P_Y$ , and  $P$ , consumer  $i$  will decide if she wants to join the program. If she decides not to join by choosing  $Z_i = 0$ , constraint (2) and (3) will enforce her consumption level  $X_i$  and  $Y_i$  to be zero, and her total utility and cost are both zero. On the other hand, if she decides to join the program by choosing  $Z_i = 1$ , she then has to decide her optimal consumption level  $X_i$  and  $Y_i$ , which cannot exceed her upper bounds  $\bar{X}_i$  and  $\bar{Y}_i$ , as enforced by constraint (2) and (3). Also note that the consumption level  $X_i$  and  $Y_i$  here could be the consumption time, such as in the voice communication service, the traffic volume, such as in the data transmission service, or number of uses/accesses, such as in the application services, or number of messages sent in SMS/MMS services.

## **2.2. The Supplier's Optimization Problem**

Given the optimization problem faced by the consumers, the information service provider

will decide what pricing scheme to adopt so as to maximize its total profit. We assume that marginal cost, i.e., marginal production cost for provider one more unit of the service to the customer, and monitoring cost, i.e., marginal administration cost or monitoring cost for one unit of the service in usage-based pricing,<sup>3</sup> are both negligible, i.e., zero. We'll discuss this assumption in Section 4.

*Given Parameters:*

$X_i^* = X_i(P_X, P_Y, P)$ : consumer  $i$ 's consumption level of the service in peak hours  
 $Y_i^* = Y_i(P_X, P_Y, P)$ : consumer  $i$ 's consumption level of the service in non-peak hours  
 $Z_i^* = Z_i(P_X, P_Y, P)$ : consumer  $i$ 's decision variable regarding participation  
 $U_i(X_i, Y_i)$ : the utility function of consumer  $i$  at the consumption level of  $X_i$  in peak hours and  $Y_i$  in non-peak hours  
 $\bar{X}_i$ : consumer  $i$ 's maximum consumption level of the service in peak hours  
 $\bar{Y}_i$ : consumer  $i$ 's maximum consumption level of the service in non-peak hours

*Decision Variables:*

$P$ : the subscription fee for the consumer to join the program  
 $P_X$ : the unit price of the service set by the provider in peak hours  
 $P_Y$ : the unit price of the service set by the provider in non-peak hours

*The Supplier's Optimization Problem:*

$$\text{Max}_{P, P_X, P_Y} \sum_i (P_X X_i^* + P_Y Y_i^* + P Z_i^*) \quad (6)$$

where  $(X_i^*, Y_i^*, Z_i^*) = \text{argmax } U_i(X_i, Y_i) - P_X X_i - P_Y Y_i - P Z_i$

s.t.

$$X_i \leq \bar{X}_i Z_i$$

$$Y_i \leq \bar{Y}_i Z_i$$

$$U_i(X_i, Y_i) - P_X X_i - P_Y Y_i - P Z_i \geq 0$$

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<sup>3</sup> Since there is no need to monitor customer usage level in flat-fee pricing, we assume that flat-fee pricing does not incur monitoring cost in the analyses throughout.

$$Z_i = 0 \text{ or } 1$$

The objective function (6) is to maximize the total profit given the optimization problems faced by the consumers. Note that we do not consider the initialization fixed cost of providing the service to each consumer as it's not as important if we consider the long-run relationship between the supplier and consumers. In addition, we assume the service provider has enough capacity, so that the marginal cost of providing the service is zero.<sup>4</sup> Based on this model, we can find the most profitable pricing scheme and price(s) to charge the consumers given the consumers it faces.

### 3. Analysis

#### ***3.1. The Base Case: Homogeneous Consumers***

As the first case, we consider homogeneous consumers in the market with the same utility function and the same upper bounds  $\bar{X}$  and  $\bar{Y}$  on the consumption level in peak hours and non-peak hours, respectively. For analytical convenience, we adopt the frequently used Cobb-Douglas utility function,  $U(X, Y) = a \log X + b \log Y$ , with one minor modification.<sup>5</sup>

$$U(X, Y) = a \log(X + 1) + b \log(Y + 1) \quad (7)$$

With this modification, when the consumption level is zero, consumers will get zero utility rather than negative infinite utility. Note that this utility function is increasing and strictly concave in consumption level and that  $X$  and  $Y$  are substitutes in that one could substitute each

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<sup>4</sup> Given any capacity, we can assume that the marginal cost within capacity is zero, a firm only faces large “marginal cost” when it needs to increase capacity, but this is actually another investment decision that needs to be made by a firm, rather than marginal cost of the service, because with new larger capacity, marginal cost goes to zero again.

<sup>5</sup> Log denotes natural log here.

other to get the same utility. We adopt this specific utility function for two reasons: it not only greatly simplifies our derivations but also allow us to explore how the homogeneity (in this section) and heterogeneity (in Section 3.2) of consumer utility functions (with diminishing marginal utility property) affect a firm's choice of pricing structure. Working on the general form of utility function  $U_i(X_i, Y_i)$  would have made our analysis much less tractable and transparent without any apparent promise for new insights.

With this specific utility function, each consumer will then face the following optimization problem:

*Consumers Optimization Problem:*

$$\text{Max}_{X,Y,Z} a \log(X+1) + b \log(Y+1) - P_X X - P_Y Y - PZ \quad (8)$$

s.t.

$$X \leq \bar{X} Z \quad (9)$$

$$Y \leq \bar{Y} Z \quad (10)$$

$$a \log(X+1) + b \log(Y+1) - P_X X - P_Y Y - PZ \geq 0 \quad (11)$$

$$Z = 0 \text{ or } 1 \quad (12)$$

Given consumers' optimization problem above, the information service provider tries to solve the following optimization problem:

*The Supplier's Optimization Problem:*

$$\text{Max}_{P, P_X, P_Y} \sum_i (P_X X^* + P_Y Y^* + PZ^*) \quad (13)$$

where  $(X^*, Y^*, Z^*) = \text{argmax} a \log(X+1) + b \log(Y+1) - P_X X - P_Y Y - PZ$

s.t.

$$X \leq \bar{X} Z$$

$$Y \leq \bar{Y} Z$$

$$a \log(X + 1) + b \log(Y + 1) - P_X X - P_Y Y - PZ \geq 0$$

$$Z = 0 \text{ or } 1$$

**Proposition 1** *When all consumers in the market are homogeneous and have utility function given by (7), the pure flat rate pricing and the two-part tariff pricing yield the same profit, which is strictly higher than the pure usage-based pricing.*

**Proof:**

We first make the following observations: First, since all consumers are assumed homogeneous, all consumers will make the same join-or-not decision and the service provider either serve all of them or serve none of them. In order to maximize the profit, the service provider will make sure that all consumers want to join the program. Second, since the major pricing mechanisms we are studying in this paper are pure flat rate, pure usage-based, and the two-part tariff pricing, we can do the analysis separately and see what is the best profit the service provider can get by each pricing plan.

1) If the service provider uses the pure flat rate pricing by setting  $P_X = 0$ ,  $P_Y = 0$ , and  $P > 0$ :

It is clear that given this pricing plan, the consumers will fully utilize the service by choosing the consumption level  $X = \bar{X}$  and  $Y = \bar{Y}$  with the maximum utility the consumers can get  $a \log(\bar{X} + 1) + b \log(\bar{Y} + 1)$ . It is then obvious that the maximum flat rate the service provider can charge is  $a \log(\bar{X} + 1) + b \log(\bar{Y} + 1)$ , with maximum profit:  $\sum_i [a \log(\bar{X} + 1) + b \log(\bar{Y} + 1)]$ .

2) If the service provider uses the pure usage-based pricing by setting  $P_X > 0$ ,  $P_Y > 0$ , and  $P = 0$ :

Taking first-order conditions for optimality of consumer's optimization problem yield:

$$\frac{a}{X^*+1} = P_X \Rightarrow X^* = \frac{a}{P_X} - 1$$

$$\frac{b}{Y^*+1} = P_Y \Rightarrow Y^* = \frac{b}{P_Y} - 1$$

*Suppliers Optimization Problem* becomes:

$$\text{Max} \sum_i (P_X X^* + P_Y Y^*) = \text{Max} \sum_i (a - P_X + b - P_Y)$$

It is clear that to maximize the equation above, the supplier will have to minimize  $P_X$  and  $P_Y$ . From FOC above, we know that as  $P_X$  and  $P_Y$  decrease,  $X^*$  and  $Y^*$  will increase. But since  $X$  and  $Y$  are bounded,  $X^*$  and  $Y^*$  will eventually become  $\bar{X}$  and  $\bar{Y}$ . In other words, the best  $P_X$  and

$P_Y$  will be  $P_X = \frac{a}{\bar{X}+1}$  and  $P_Y = \frac{b}{\bar{Y}+1}$ , with maximum profit:  $\sum_i (a - \frac{a}{\bar{X}+1} + b - \frac{b}{\bar{Y}+1}) =$

$$\sum_i [a(1 - \frac{1}{\bar{X}+1}) + b(1 - \frac{1}{\bar{Y}+1})].$$

3) If the service provider uses the two-part tariff pricing by setting  $P_X > 0$ ,  $P_Y > 0$ , and  $P > 0$ :

Again, the first-order conditions for optimality of consumer's optimization problem are:

$$\frac{a}{X^*+1} = P_X \Rightarrow X^* = \frac{a}{P_X} - 1$$

$$\frac{b}{Y^*+1} = P_Y \Rightarrow Y^* = \frac{b}{P_Y} - 1$$

*Suppliers Optimization Problem* becomes:

$$\text{Max} \sum_i (P_X X^* + P_Y Y^* + P) = \text{Max} \sum_i (a - P_X + b - P_Y + P)$$

Likewise, it is clear that to maximize the equation above, the supplier will have to minimize  $P_X$  and  $P_Y$ . From FOC above, we know that as  $P_X$  and  $P_Y$  decrease,  $X^*$  and  $Y^*$  will

increase. But since  $X$  and  $Y$  are bounded,  $X^*$  and  $Y^*$  will eventually become  $\bar{X}$  and  $\bar{Y}$ . In other words, the best  $P_X$  and  $P_Y$  will be  $P_X = \frac{a}{\bar{X}+1}$  and  $P_Y = \frac{b}{\bar{Y}+1}$ . The maximum subscription fee  $P$  the supplier can charge is then the difference between the maximum utility the consumers can get,  $a \log(\bar{X}+1) + b \log(\bar{Y}+1)$ , and the payment for their usage,  $(a - \frac{a}{\bar{X}+1} + b - \frac{b}{\bar{Y}+1})$ . Therefore, the maximum profit achievable by the service provider is  $\sum_i [a \log(\bar{X}+1) + b \log(\bar{Y}+1)]$ , the same as in the case when the service provider adopts the flat rate pricing mechanism.

Note that since  $\log(\bar{X}+1) > (1 - \frac{1}{\bar{X}+1})$  and  $\log(\bar{Y}+1) > (1 - \frac{1}{\bar{Y}+1})$  for all  $\bar{X}, \bar{Y} > 0$ , we have  $a \log(\bar{X}+1) + b \log(\bar{Y}+1) > [a(1 - \frac{1}{\bar{X}+1}) + b(1 - \frac{1}{\bar{Y}+1})]$ . That is, the pure flat rate pricing and the two-part tariff pricing are strictly better than the pure usage-based pricing from the service provider's profit maximization point of view. **QED.**

### 3.2. *Heterogeneous Customers*

In previous analysis, we have shown that flat fee and two-part tariff is more profitable than the pure usage-based pricing. However, the assumption of homogeneous consumers may be somewhat restrictive, so we relax this assumption by considering different types of heterogeneous customers. Following Jain et al. (1999), we examine two-sets of customer segmentation: "high-end" and "low-end" in terms of willingness-to-pay (Section 3.2.1), and "heavy" and "light" in terms of level of usage (Section 3.2.2). We further assume that it's the firm's interest to serve both segments in each case; otherwise, the problem is reduced to that

considered in Section 3.1. We further assume that the information service provider cannot discriminate between these two consumer segments. This assumption is reasonable since it is usually hard for the service provider to tell which segment the consumers belong to. Note that if the information service provider can discriminate these two types of consumers, the problem again becomes that considered in Section 3.1, and the information service provider can simply offer different flat rate pricing or two-part tariff pricing to different consumer segments.

### ***3.2.1 Heterogeneous customers: the high-end customers and the low-end customers***

For simplicity, we will call the high-end “business” consumers, and the low-end “personal” consumers. We suppose there are  $m$  business consumers ( $i=1$ ) and  $n$  personal consumers ( $i=2$ ). To study how heterogeneous willingness to pay affects a firm’s pricing scheme, we assume each consumer in both segments has the same upper bounds  $\bar{X}$  and  $\bar{Y}$  in peak hours and non-peak hours, and  $a_1 > a_2$ ,  $b_1 > b_2$ .

*Consumers Optimization Problem:*

$$\text{Max}_{X,Y,Z} a_i \log(X_i + 1) + b_i \log(Y_i + 1) - P_X X_i - P_Y Y_i - P Z_i \quad (14)$$

s.t.

$$X_i \leq \bar{X} Z_i \quad (15)$$

$$Y_i \leq \bar{Y} Z_i \quad (16)$$

$$a_i \log(X_i + 1) + b_i \log(Y_i + 1) - P_X X_i - P_Y Y_i - P Z_i \geq 0 \quad (17)$$

$$Z_i = 0 \text{ or } 1 \quad (18)$$

*The Supplier’s Optimization Problem:*

$$\text{Max}_{P_X, P_Y} m(P_X X_1^* + P_Y Y_1^* + P Z_1^*) + n(P_X X_2^* + P_Y Y_2^* + P Z_2^*) \quad (19)$$

where  $(X_i^*, Y_i^*, Z_i^*) = \text{argmax } a_i \log(X_i + 1) + b_i \log(Y_i + 1) - P_X X_i - P_Y Y_i - P Z_i$

s.t.

$$X_i \leq \bar{X} Z_i$$

$$Y_i \leq \bar{Y} Z_i$$

$$a_i \log(X_i + 1) + b_i \log(Y_i + 1) - P_X X_i - P_Y Y_i - P Z_i \geq 0$$

$$Z_i = 0 \text{ or } 1$$

**Lemma 1:** when  $a_1 < \frac{m+n}{m} a_2$  and  $b_1 < \frac{m+n}{m} b_2$ ,<sup>6</sup> if the service provider uses the pure flat rate,

the price charged will be  $a_2 \log(\bar{X} + 1) + b_2 \log(\bar{Y} + 1)$ , and the maximum profit achievable will

be:  $(m+n)[a_2 \log(\bar{X} + 1) + b_2 \log(\bar{Y} + 1)]$ .

**Proof :**

It is clear that given  $P_X = 0$ ,  $P_Y = 0$ , and  $P > 0$ , if a consumer chooses to join the program, she will fully utilize the service by choosing the consumption level  $X_1 = \bar{X}$ ,  $Y_1 = \bar{Y}$  or  $X_2 = \bar{X}$ ,  $Y_2 = \bar{Y}$ . Given this, it is obvious that the service provider can charge each business consumer no more than  $a_1 \log(\bar{X} + 1) + b_1 \log(\bar{Y} + 1)$ , and each personal consumer no more than  $a_2 \log(\bar{X} + 1) + b_2 \log(\bar{Y} + 1)$ . It can be easily shown that if  $a_1 < \frac{m+n}{m} a_2$  and  $b_1 < \frac{m+n}{m} b_2$ , the

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<sup>6</sup> These conditions correspond to the case that it's more profitable for the firm to serve both segments.

service provider will charge  $a_2 \log(\bar{X} + 1) + b_2 \log(\bar{Y} + 1)$  and serve both business and personal consumers with the maximum profit achievable  $(m + n)[a_2 \log(\bar{X} + 1) + b_2 \log(\bar{Y} + 1)]$ . **QED.**

**Lemma 2 :** If the service provider uses the pure usage-based pricing, when  $m\bar{X} > n$ ,<sup>7</sup> the optimal price in the peak hours is  $P_X = \frac{a_1}{\bar{X} + 1}$ ; when  $m\bar{Y} > n$ , the optimal non-peak hour price is

$P_Y = \frac{b_1}{\bar{Y} + 1}$ . The maximum profit is:  $m(\frac{a_1\bar{X}}{\bar{X} + 1} + \frac{b_1\bar{Y}}{\bar{Y} + 1}) + n(a_2 - \frac{a_1}{\bar{X} + 1} + b_2 - \frac{b_1}{\bar{Y} + 1})$ ; otherwise, the

optimal prices are given by  $P_X = \frac{a_2}{\bar{X} + 1}$  and  $P_Y = \frac{b_2}{\bar{Y} + 1}$  with profit :  $(m + n)(\frac{a_2\bar{X}}{\bar{X} + 1} + \frac{b_2\bar{Y}}{\bar{Y} + 1})$ .

**Proof :**

When  $P_X > 0$ ,  $P_Y > 0$ , and  $P = 0$ , the first-order conditions for optimality of business/personal consumer optimization problem yield:

$$\frac{a_1}{X_1^* + 1} = P_X \Rightarrow X_1^* = \frac{a_1}{P_X} - 1 \quad (20)$$

$$\frac{b_1}{Y_1^* + 1} = P_Y \Rightarrow Y_1^* = \frac{b_1}{P_Y} - 1 \quad (21)$$

$$\frac{a_2}{X_2^* + 1} = P_X \Rightarrow X_2^* = \frac{a_2}{P_X} - 1 \quad (22)$$

$$\frac{b_2}{Y_2^* + 1} = P_Y \Rightarrow Y_2^* = \frac{b_2}{P_Y} - 1 \quad (23)$$

*The Supplier's Optimization Problem becomes:*

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<sup>7</sup> If we normalize  $\bar{X}$  to be 1, this condition means  $m > n$ .

$$\text{Max } m(P_X X_1^* + P_Y Y_1^*) + n(P_X X_2^* + P_Y Y_2^*) = \text{Max } m(a_1 - P_X + b_1 - P_Y) + n(a_2 - P_X + b_2 - P_Y)$$

To maximize the equation above, the supplier will have to minimize  $P_X$  and  $P_Y$ . From (20)-(23), we know that as  $P_X$  and  $P_Y$  decrease,  $X_1^*$ ,  $X_2^*$ ,  $Y_1^*$  and  $Y_2^*$  will increase. But since  $X_1$ ,  $X_2$ ,  $Y_1$  and  $Y_2$  are bounded (constraints (15) and (16)),  $X_1^*$ ,  $X_2^*$ ,  $Y_1^*$  and  $Y_2^*$  cannot exceed  $\bar{X}$  and  $\bar{Y}$  respectively, and this suggests that as price goes down further, no increase in demand can be expected. To derive the optimal prices, we consider the peak-hour problem first. The peak-hour demand curves of the business and personal consumers (constraints (20) and (22)) are shown in Figure 1 ( $D_1$  and  $D_2$ ).

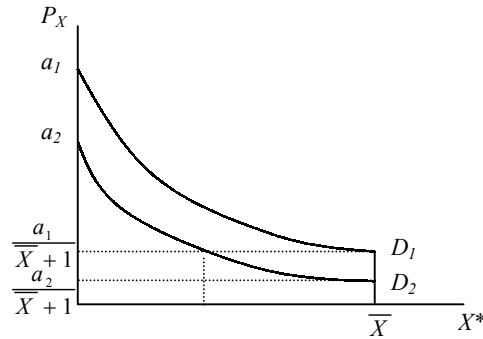


Figure 1. The peak-hour demand curves of the business and personal consumers

The supplier's optimization problem is:

$$\text{Max } m(P_X X_1^*) + n(P_X X_2^*) = \text{Max } m(a_1 - P_X) + n(a_2 - P_X).$$

To maximize this equation, the supplier will have to minimize  $P_X$  and therefore the best price  $P_X$  cannot be larger than  $\frac{a_1}{\bar{X}+1}$ . On the other hand, if the supplier sets the price below

$\frac{a_2}{\bar{X}+1}$ , the profit is not optimal since  $X_1^*$  and  $X_2^*$  cannot exceed  $\bar{X}$  and user demand won't

increase as the price decreases. Hence, the best price  $P_X$  must be somewhere between  $\frac{a_1}{\bar{X}+1}$  and

$\frac{a_2}{\bar{X}+1}$ . When the price is in this interval, the demand of the business consumer is fixed at  $\bar{X}$

while the demand of the personal consumer keeps on increasing as the price goes down. Thus, we have:

$$\text{Max } m(P_X X_1^*) + n(P_X X_2^*) = \text{Max } m(P_X \bar{X}) + n(a_2 - P_X) = \text{Max } na_2 + (m\bar{X} - n)P_X.$$

When  $m\bar{X} > n$ , the best price  $P_X$  in this interval is therefore  $\frac{a_1}{\bar{X}+1}$ , otherwise,  $P_X = \frac{a_2}{\bar{X}+1}$ .

Similar analysis can be done on the non-peak-hour problem and we can get the best

price  $P_Y = \frac{b_1}{\bar{Y}+1}$  when  $m\bar{Y} > n$ , or  $P_Y = \frac{b_2}{\bar{Y}+1}$  otherwise. **QED.**

**Lemma 3:** If the service provider uses the two-part tariff pricing, when  $a_1 < \frac{m+n}{m}a_2$  and

$b_1 < \frac{m+n}{m}b_2$ , optimal  $P_X$  can be set anywhere between 0 and  $\frac{a_2}{\bar{X}+1}$ ,  $P_Y$  can be set anywhere

between 0 and  $\frac{b_2}{\bar{Y}+1}$ , and  $P = a_2 \log(\bar{X}+1) + b_2 \log(\bar{Y}+1) - P_X \bar{X} - P_Y \bar{Y}$  with the maximum

profit achievable:  $(m+n)[a_2 \log(\bar{X}+1) + b_2 \log(\bar{Y}+1)]$ .

**Proof :**

When  $P_X > 0$ ,  $P_Y > 0$ , and  $P > 0$ , the first-order conditions for the business/personal consumer optimization problem yield (20)-(23).

Likewise, we use the divide-and-conquer technique to do the analysis. As before, we consider only the peak-hour problem here and the joint problem with non-peak-hour consideration can be solved in a similar way. Equations (20) and (22) are the peak-hour demand

curves of the business and personal consumers ( $D_1$  and  $D_2$  in Figure. 2.).

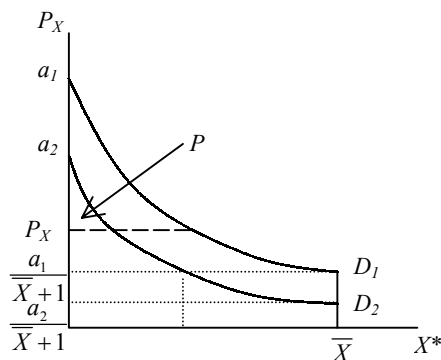


Figure. 2. The best subscription fee is equal to the consumer surplus of the personal consumers

First, we make the following observation: no matter what usage price  $P_X$  the supplier sets for the service, the best subscription fee  $P$  it can charge the consumers is the consumer surplus of the personal consumers (the triangle area under  $D_2$  and above  $P_X$ ).<sup>8</sup> Any subscription fee more than this will let the supplier lose all of the personal consumers.

Note that given these demand functions, if the supplier set the usage price  $P_X \geq \frac{a_1}{X+1}$ , the supplier's optimization problem will be:

$$\begin{aligned} \text{Max } m(P_X X_1^*) + n(P_X X_2^*) + (m+n)P &= \text{Max } m(a_1 - P_X) + n(a_2 - P_X) + (m+n) \int_0^{\frac{a_2}{P_X}-1} \left( \frac{a_2}{X_2+1} - P_X \right) dX_2 \\ &= \text{Max } m(a_1 - a_2) + (m+n)a_2 \log\left(\frac{a_2}{P_X}\right) \end{aligned}$$

To maximize this equation the supplier will have to minimize  $P_X$  and therefore the best price

<sup>8</sup> Due to our assumption that it's more profitable for the firm to serve both market segments, and the conditions for

this to be true are:  $a_1 < \frac{m+n}{m} a_2$  and  $b_1 < \frac{m+n}{m} b_2$ .

$P_x$  in this interval is  $\frac{a_1}{\bar{X}+1}$  with the maximum profit achievable

$$m(a_1 - a_2) + (m+n)a_2 \log \frac{a_2}{a_1} (\bar{X} + 1).$$

Second, if the supplier set the usage price  $\frac{a_2}{\bar{X}+1} \leq P_x \leq \frac{a_1}{\bar{X}+1}$ , the supplier's optimization problem will be:

$$\begin{aligned} \text{Max } m(P_x X_1^*) + n(P_x X_2^*) + (m+n)P &= \text{Max } m(P_x \bar{X}) + n(a_2 - P_x) + (m+n) \int_0^{\frac{a_2}{P_x}-1} \left( \frac{a_2}{X_2+1} - P_x \right) dX_2 \\ &= \text{Max } m(\bar{X}+1)P_x - ma_2 + (m+n)a_2 \log \left( \frac{a_2}{P_x} \right) \end{aligned}$$

The best price  $P_x$  in this interval is  $\frac{a_2}{\bar{X}+1}$  with the maximum profit achievable  $(m+n)a_2 \log(\bar{X}+1)$ . Note that this profit is larger than  $m(a_1 - a_2) + (m+n)a_2 \log \frac{a_2}{a_1} (\bar{X} + 1)$  (the profit we can get from another boundary point  $\frac{a_1}{\bar{X}+1}$ ).

Third, if the supplier set the usage price  $0 \leq P_x \leq \frac{a_2}{\bar{X}+1}$ , the supplier's optimization problem will be:

$$\begin{aligned} \text{Max } m(P_x X_1^*) + n(P_x X_2^*) + (m+n)P &= \text{Max } m(P_x \bar{X}) + n(P_x \bar{X}) + (m+n) \int_0^{\bar{X}} \left( \frac{a_2}{X_2+1} - P_x \right) dX_2 \\ &= \text{Max } (m+n)a_2 \log(\bar{X}+1) \end{aligned}$$

It is clear that to maximize this equation the supplier can set the price anywhere between 0

and  $\frac{a_2}{\bar{X}+1}$  with the maximum profit achievable  $(m+n)a_2 \log(\bar{X}+1)$ .

Given the above analysis, we know that the supplier can set the optimal price  $P_X$  anywhere between 0 and  $\frac{a_2}{\bar{X}+1}$ , and  $P_Y$  anywhere between 0 and  $\frac{b_2}{\bar{Y}+1}$ , and subscription fee  $P$  equal to the consumer surplus of the personal consumers, with the maximum profit achievable  $(m+n)[a_2 \log(\bar{X}+1) + b_2 \log(\bar{Y}+1)]$ . **QED.**

**Proposition 2** *When there are two types of consumers characterized by heterogeneous willingness to pay in the market, the flat-fee pricing and the two-part tariff pricing yield the same profit, which is higher than the pure usage-based pricing.*

**Proof :**

Dirctely from Lemma 1-3, we already get the maximum profits achievable when the service provider adopts each of the pricing mechanism: pure flat rate, pure usage-based and the two-part tariff pricing. It is not hard to show that for all  $\bar{X} \geq 0$  and  $\bar{Y} \geq 0$ ,  $m[a_2 \log(\bar{X}+1) + b_2 \log(\bar{Y}+1)] + n[a_2 \log(\bar{X}+1) + b_2 \log(\bar{Y}+1)] > m(a_1 - \frac{a_1}{\bar{X}+1} + b_1 - \frac{b_1}{\bar{Y}+1}) + n(a_2 - \frac{a_1}{\bar{X}+1} + b_2 - \frac{b_1}{\bar{Y}+1})$ . Therefore we can say the pure flat rate pricing and the two-part tariff pricing are strictly better than the pure usage-based pricing from the service provider's profit maximization point of view. **QED.**

Note that the conclusions from this subsection is exactly the same as that derived in

Proposition 1, that is, the flat-fee pricing and the two-part tariff always yield the same profit, and that the usage-based pricing is strictly dominated. While these results are established under one or two segments of customers, they could be generalized to a continuous type of customers (as shown as Proposition 5 in Appendix).

### 3.2.2 *Heterogeneous customers: the high-demand customers and the low-demand customers*

In this subsection, we consider how heterogeneous maximum consumption level might affect a firm's pricing choice. Again, we assume two types of customers, the high-demand customers (type 1) with maximum consumption level at  $\bar{X}_1$  and  $\bar{Y}_1$  and the low-demand customers (type 2) with maximum consumption level at  $\bar{X}_2$  and  $\bar{Y}_2$ , where  $\bar{X}_1 > \bar{X}_2$  and  $\bar{Y}_1 > \bar{Y}_2$ . As before, there are  $m$  type 1 customers and  $n$  type 2 customers with  $a_1 = a_2 = a$  and  $b_1 = b_2 = b$ .

**Proposition 3** *When there are two types of consumers characterized by heterogeneous maximum consumption levels, the two-part tariff always dominates the flat-fee pricing and the usage-based pricing.*

#### **Proof Sketch:**

The optimal price(s) and maximum profit under each scheme are characterized in the following:

- Under the flat fee scheme :

$$P = a \log(\bar{X}_2 + 1) + b \log(\bar{Y}_2 + 1), \text{ with maximum profit:}$$

$$(m + n)[a \log(\bar{X}_2 + 1) + b \log(\bar{Y}_2 + 1)].$$

- Under the pure usage-based pricing:

when  $n\bar{X}_2 \geq m$ ,  $P_X = \frac{a}{\bar{X}_2 + 1}$  and  $P_Y = \frac{b}{\bar{Y}_2 + 1}$ , with profit  $= (m + n) \left( \frac{a\bar{X}_2}{\bar{X}_2 + 1} + \frac{b\bar{Y}_2}{\bar{Y}_2 + 1} \right)$

when  $n\bar{X}_2 < m$ ,  $P_X = \frac{a}{\bar{X}_1 + 1}$  and  $P_Y = \frac{b}{\bar{Y}_1 + 1}$ , with profit  $= m \left( \frac{a\bar{X}_1}{\bar{X}_1 + 1} + \frac{b\bar{Y}_1}{\bar{Y}_1 + 1} \right) + n \left( \frac{a\bar{X}_2}{\bar{X}_1 + 1} + \frac{b\bar{Y}_2}{\bar{Y}_1 + 1} \right)$

- Under the two-part tariff pricing:

$P_X = \frac{a}{\bar{X}_1 + 1}$ ,  $P_Y = \frac{b}{\bar{Y}_1 + 1}$  and the subscription fee:  $P = a \log(\bar{X}_2 + 1) + b \log(\bar{Y}_2 + 1) -$

$\left( \frac{a\bar{X}_2}{\bar{X}_1 + 1} + \frac{b\bar{Y}_2}{\bar{Y}_1 + 1} \right)$ . Therefore, the maximum profit achievable by the service provider

is  $m \left( a \frac{\bar{X}_1 - \bar{X}_2}{\bar{X}_1 + 1} + b \frac{\bar{Y}_1 - \bar{Y}_2}{\bar{Y}_1 + 1} \right) + (m + n) (a \log(\bar{X}_2 + 1) + b \log(\bar{Y}_2 + 1))$ , which is greater than that can be

achieved in the flat fee pricing and the usage-based pricing. **QED.**

#### 4. Model Extension—Marginal Cost and Monitoring Cost

In previous analyses, we assume that marginal cost (denoted by  $c$ ) and monitoring cost (denoted by  $t$ ) are both zero. In this section, we relax this assumption and examine how marginal cost and monitoring cost affect optimal pricing scheme. In general, positive marginal cost is expected to make the flat fee pricing less attractive and favor two part tariff pricing and usage based pricing, while positive monitoring cost tend to make the two-part tariff pricing or usage-based pricing less desirable than the flat fee pricing since there's no need to incur monitoring expenses in the flat-fee pricing. The optimal pricing scheme thus depends on the tradeoff between these two costs. However, under the case where customers are homogeneous or are characterized by heterogeneous willingness to pay, the optimal scheme becomes insensitive to these two costs when  $c$  is below a certain value, in particular, holding  $c$  constant, reducing

monitoring cost does not make the two-part tariff pricing a better choice than the flat fee pricing.

**Proposition 4** *Under the case where customers are homogeneous or are characterized by heterogeneous willingness to pay, when  $c \leq \min\{\frac{a}{(\bar{X}+1)^2}, \frac{b}{(\bar{Y}+1)^2}\}$ , the flat fee pricing dominates the two-part tariff pricing and the usage based pricing.*

**Proof sketch:**

The proof here is based on homogeneous customers for simplicity. However, the same analysis can be done on heterogeneous customers case, the results remain the same, but only the condition on  $c$  changes.

As before, we analyze the peak hour problem, and the joint problem with non-peak hour consideration can be solved in the same manner.

With positive marginal cost and monitoring cost, the profit from each customer (denoted by  $\pi$ ) for the firm under each pricing scheme is given by:

- Under the flat-fee pricing:  $\pi = P - c\bar{X}$ , and to maximize the profit,  $P$  will be set at  $a \log(\bar{X} + 1)$ , with profit achievable:  $a \log(\bar{X} + 1) - c\bar{X}$ .

- Under the usage-based pricing:  $\pi = (P_X - c - t)X(P_X)$ , where  $X(P_X)$  is the demand function characterized by  $X(P_X) = \frac{a}{P_X} - 1$  from the first order condition in customer's

optimization problem. The profit-maximization per-use price can be shown to be:

$$P_X^* = \max\left\{\frac{a}{\bar{X}+1}, \sqrt{a(c+t)}\right\} \text{ from the first order condition } \frac{\partial \pi}{\partial P} = 0, \text{ and the maximum profit can}$$

be shown to be:  $(P_X^* - c - t)X(P_X^*) = X(P_X^*) \cdot P_X^* - X(P_X^*) \cdot (c + t)$ .

- Under the two-part tariff pricing:  $\pi = (P_X - c - t)X(P_X) + P$ , again  $X(P_X)$  is the demand

function characterized by  $X(P_X) = \frac{a}{P_X} - 1$  from the first order condition in customer's

optimization problem. According to the first order condition  $\frac{\partial \pi}{\partial P} = 0$ , we can derive the profit-

maximization per-use price:  $P_X^* = \max\left\{\frac{a}{\bar{X} + 1}, \sqrt{a(c + t)}\right\}$ , and  $P$ , the subscription fee, will be set

at:  $a \log(X(P_X^*) + 1) - X(P_X^*) \cdot P_X^*$ , i.e., to fully extract customer surplus. The maximum profit

is thus equal to:  $a \log(X(P_X^*) + 1) - X(P_X^*) \cdot (c + t)$ .

We can show that when  $c \leq \frac{a}{(\bar{X} + 1)^2}$ , flat fee pricing always dominates the two-part tariff

pricing and the usage based pricing no matter how large or how small the monitoring cost is (as

long as it is positive), and when monitoring cost,  $t$ , goes down to zero, the two-part tariff pricing

will derive the same profit as the flat fee pricing. Similarly, we can show that in non-peak hours,

when  $c \leq \frac{b}{(\bar{Y} + 1)^2}$ , flat fee pricing always dominates the two-part tariff pricing and the usage

based pricing no matter how large or how small the monitoring cost is (as long as it is positive).

**QED.**

This result is very interesting because it suggests that as marginal cost goes down (but not necessarily zero), flat fee pricing becomes the optimal scheme even though that monitoring cost

may goes down at a larger scale. A direct implication of this Proposition is that as marginal costs

and monitoring costs are both lowered with the advance of information technology or with

evolution of electronic markets, flat-fee pricing will become more attractive for information

service providers. However, when monitoring cost becomes negligible, two-part tariff pricing

will become as attractive.

## 5. Discussion and Conclusions

The main objective of this paper is to offer a guideline for information service providers about what pricing scheme is most profitable and what price(s) it should charge. This issue about how to price information services has become increasingly more important as marginal “production” and monitoring costs for information services are being reduced with the advance of modern information technology, and more and more information services are being offered with the evolution of the Internet.

Many researchers have suggested that zero marginal cost will favor the flat-fee pricing scheme. Overall, our analysis shows that when customers are homogeneous or have heterogeneous marginal willingness to pay, if marginal cost is below a certain value, flat-fee pricing is indeed more attractive. However, as processing power keeps increasing and monitoring costs become negligible, the two-part tariff pricing becomes as attractive.

Our analysis also shows that under zero marginal cost *and* zero monitoring cost, when customers are fairly homogeneous, the usage-based pricing is strictly dominated by the flat fee and two-part tariff pricing schemes, with the latter two always achieving the same profit level. The same results sustain when customers have heterogeneous marginal willingness to pay (which corresponds to different downward sloping demand curves). However, when customers are characterized by heterogeneous maximum consumption levels, the two-part tariff pricing dominates both the flat-fee pricing and the usage-based pricing.

### **Appendix:**

**Proposition 5** *When consumers are characterized by heterogeneous willingness to pay in the*

market, the flat-fee pricing and the two-part tariff pricing always yield the same profit, which is higher than the pure usage-based pricing.

**Proof sketch:**

Assume customer  $i$  has type:  $a_i$  and  $b_i$ , which follow the uniform distribution, i.e.,  $a_i, b_i \sim U[0,1]$ . As before, we solve the peak hour problem, and the joint problem with non-peak hour consideration can be solved with the similar procedure.

- Under the flat-fee pricing:

Given any flat fee,  $P$ , charged by the firm, we can determine the marginal customer,  $a_0$ , who will sign up by  $a_0 \log(\bar{X} + 1) = P$ , with profit  $= (1 - a_0)P$ . To maximize the profit, the firm will set  $P = \frac{1}{2} \log(\bar{X} + 1)$ , with  $a_0 = \frac{1}{2}$ , i.e., customers with type higher than  $\frac{1}{2}$  will join the service.

The maximum profit under this scheme is thus  $(1 - a_0)P = (1 - \frac{1}{2})\frac{1}{2} \log(\bar{X} + 1) = \frac{1}{4} \log(\bar{X} + 1)$ .

- Under the pure usage-based pricing:

As before, the demand function is characterized by:  $X_i = \frac{a_i}{P_x} - 1 \leq \bar{X}$ , that is,

when  $P_x \geq \frac{a_i}{\bar{X} + 1}$ , we have  $X_i \leq \bar{X}$ , and when  $P_x < \frac{a_i}{\bar{X} + 1}$ , we have  $X_i = \bar{X}$ . And given this

demand function, a customer with type higher than  $P_x(\bar{X} + 1)$  will consume  $\bar{X}$ .

The firm will choose  $P_x$  that maximizes its profit:  $\int_0^{P_x(\bar{X}+1)} X_i P_x da + (1 - P_x(\bar{X} + 1))\bar{X}P_x$ .

Solving this optimization problem, we get  $P_x = \frac{\bar{X}}{(\bar{X} + 1)^2}$ , with profit:  $\frac{1}{2} \left( \frac{\bar{X}}{\bar{X} + 1} \right)^2$ , which can be

shown to be lower than  $\frac{1}{4} \log(\bar{X} + 1)$ , the profit in the flat-fee pricing.

- Under the two-part tariff pricing:

The firm will choose  $P_X$  and the subscription fee,  $P$ , to maximize its profit. Note that given any  $P_X$  and  $P$ , the marginal customer,  $a_0$ , who will sign up the service is determined by:

$a_0 \log(X(a_0) + 1) - P - X(a_0)P_X = 0$ , which can be simplified to  $P = a_0 \log\left(\frac{a_0}{P_X}\right) - (a_0 - P_X)$ . The

profit given  $P_X$  and  $P$  is:  $\int_{a_0}^{P_X(\bar{X}+1)} X_i P_X da + (1 - P_X(\bar{X} + 1))\bar{X}P_X + (1 - a_0)P$ . To maximize the

profit, the firm will set  $P_X = \frac{1}{2} \frac{1}{(\bar{X} + 1)}$  and  $P = \frac{1}{2} \log(\bar{X} + 1) - \frac{1}{2} \frac{\bar{X}}{\bar{X} + 1}$ , with maximum profit:

$\frac{1}{4} \log(\bar{X} + 1)$ , which is exactly the same as what can be achieved in the flat-fee pricing scheme.

**QED.**

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