Customer Base Valuation in a Contractual Setting: The Perils of Ignoring Heterogeneity

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It is increasingly apparent that the financial value of a firm depends on off-balance-sheet intangible assets. In this article, the authors focus on the most critical aspect of a firm: its customers. Specifically, they demonstrate how valuing customers makes it feasible to value firms, including high-growth firms with negative earnings. The authors define the value of a customer as the expected sum of discounted future earnings. They demonstrate their valuation method by using publicly available data for five firms. They find that a 1% improvement in retention, margin, or acquisition cost improves firm value by 5%, 1%, and .1%, respectively. They also find that a 1% improvement in retention has almost five times greater impact on firm value than a 1% change in discount rate or cost of capital. The results show that the linking of marketing concepts to shareholder value is both possible and insightful.

Valuing Customers

THE CUSTOMER LIFETIME VALUE CONCEPT AND ITS CONTRIBUTION TO CORPORATE VALUATION

*by Hans H. Bauer, Maik Hammerschmidt and Matthias Braehler**

ABSTRACT

The shareholder value and the customer lifetime value approach are conceptually and methodically analogous. Both concepts calculate the value of a particular decision unit by discounting the forecasted net cash flows by the risk-adjusted cost of capital. However, virtually no scholarly attention has been devoted to the question if any of the components of the shareholder value could be determined in a more market-oriented way using individual customer lifetime values. Therefore, the main objective of this paper is to systematically explore the contribution of both concepts to the field of corporate valuation.

At first we present a comprehensive calculation method for estimating both the individual lifetime value of a customer and the customer equity. After a critical examination of the shareholder value concept, a synthesis of both value approaches allowing for a disaggregated and more realistic corporate valuation will be presented.

Hypothetical Contractual Setting

Number of active customers each year by year-of-acquisition cohort:

2001	2002	2003	2004	2005
10,000	6,334	4,367	3,264	2,604
	15,000	9,501	6,551	4,896
		17,500	11,085	7,642
			19,000	12,035
				20,500
10,000	21,334	31,368	39,900	47,677

Hypothetical Contractual Setting

Assume

- Each contract is annual, starting on January 1 and expiring at 11:59pm on December 31.
- An average net cashflow of \$100/year.
- A 10% discount rate

What is the expected residual value of the customer base at December 31, 2005?

$$47{,}677 \times \sum_{t=1}^{\infty} \$100 \times 0.681^t/(1+0.1)^{t-1} = \$8{,}528{,}000$$

Annual Retention Rates by Cohort

2001	2002	2003	2004	2005
--	0.63	0.69	0.75	0.80
	--	0.63	0.69	0.75
		--	0.63	0.69
			--	0.63
				--
--	0.63	0.65	0.67	0.68

Why Do Retention Rates Increase Over Time?

Individual-level time dynamics (e.g., increasing loyalty as the customer gains more experience with the firm).

vs.

A sorting effect in a heterogeneous population.

The Role of Heterogeneity

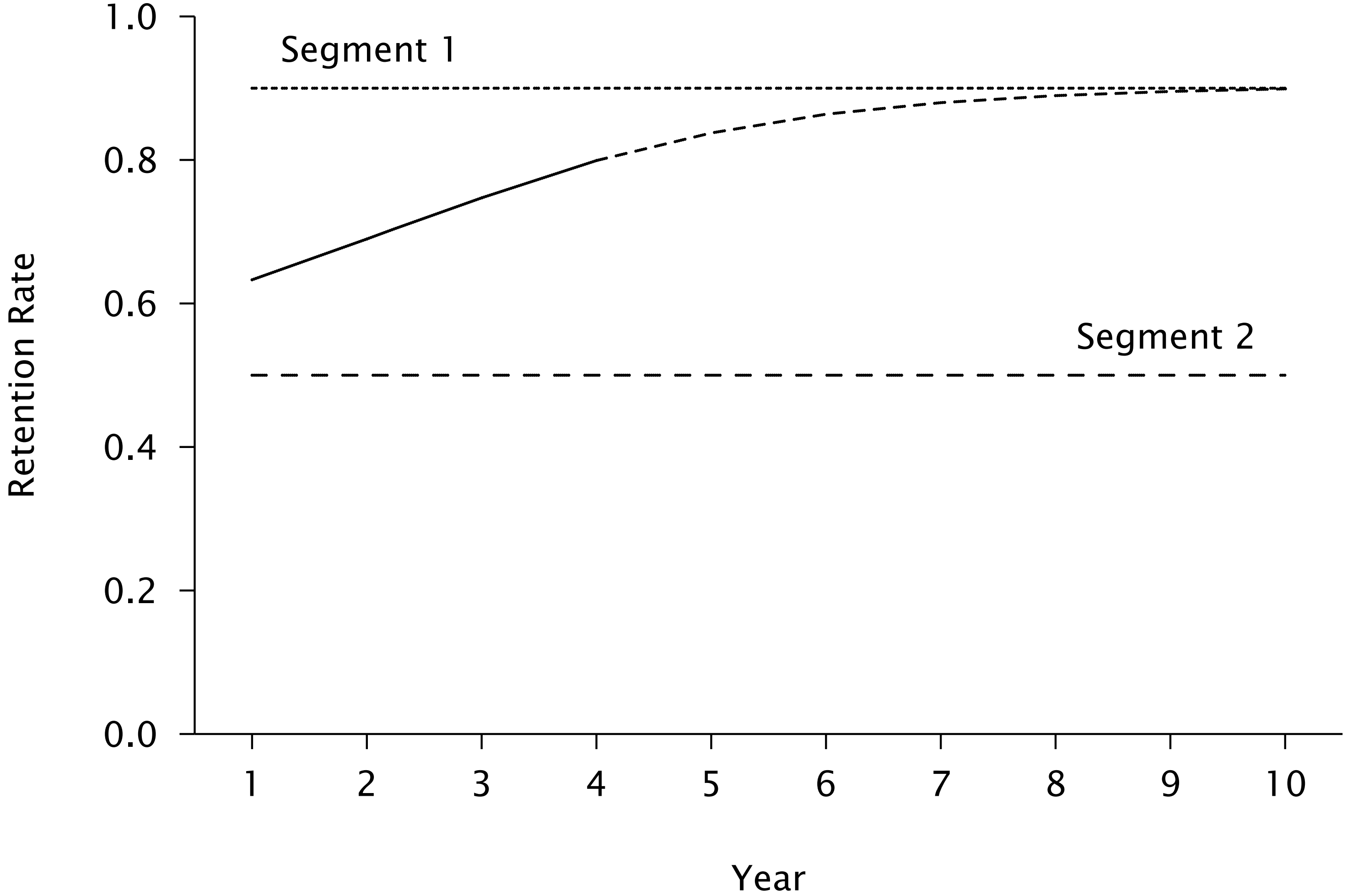
Suppose we track a cohort of 10,000 customers, comprising two underlying segments:

- Segment 1 comprises one-third of the customers, each with a time-invariant annual retention probability of 0.9.
- Segment 2 comprises two-thirds of the customers, each with a time-invariant annual retention probability of 0.5.

The Role of Heterogeneity

	# Active Customers			r_t		
Year	Seg 1	Seg 2	Total	Seg 1	Seg 2	Total
1	3,333	6,667	10,000			
2	3,000	3,334	6,334	0.900	0.500	0.633
3	2,700	1,667	4,367	0.900	0.500	0.689
4	2,430	834	3,264	0.900	0.500	0.747
5	2,187	417	2,604	0.900	0.500	0.798

The Role of Heterogeneity



ERLV of an Active 2001 Cohort Member

- If this person belongs to segment 1:

$$\begin{aligned} ERLV &= \sum_{t=1}^{\infty} 100 \times \frac{0.9^t}{(1+0.1)^{t-1}} \\ &= \$495 \end{aligned}$$

- If this person belongs to segment 2:

$$\begin{aligned} ERLV &= \sum_{t=1}^{\infty} 100 \times \frac{0.5^t}{(1+0.1)^{t-1}} \\ &= \$92 \end{aligned}$$

ERLV of an Active 2001 Cohort Member

According to Bayes' theorem, the probability that this person belongs to segment 1 is

$$\frac{P(\text{renewed contract four times} \mid \text{segment 1}) \times P(\text{segment 1})}{P(\text{renewed contract four times})}$$

$$= \frac{0.9^4 \times 0.333}{0.9^4 \times 0.333 + 0.5^4 \times 0.667}$$

$$= 0.84$$

$$\Longrightarrow \mathit{ERLV} = 0.84 \times \$495 + (1 - 0.84) \times \$92 = \$430$$

Valuing the Existing Customer Base

Recognizing the underlying segments:

Cohort	# Active in 2005	P(seg. 1)	ERLV
2001	2,604	0.840	$430
2002	4,896	0.745	$392
2003	7,642	0.618	$341
2004	12,035	0.474	$283
2005	20,500	0.333	$226

Total expected residual value = $13,683,000

Valuing the Existing Customer Base

Cohort	Total RV	Underestimation
Naïve	$8,528,000	38%
Segment (model)	$13,683,000	

Exploring the Magnitude of the Error

- Systematically vary heterogeneity in retention rates
- First need to specify a flexible model of contract duration

A Discrete-Time Model for Contract Duration

1. An individual remains a customer of the firm with constant retention probability $1 - \theta$

   $\longrightarrow$ the duration of the customer's relationship with the firm is characterized by the (shifted) geometric distribution:

$$S(t \mid \theta) = (1 - \theta)^t, \quad t = 1, 2, 3, \ldots$$

2. Heterogeneity in θ is captured by a beta distribution with pdf

$$f(\theta \mid \alpha, \beta) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha, \beta)}.$$

A Discrete-Time Model for Contract Duration

- The aggregate survivor function is

$$S(t \mid \alpha, \beta) = \int_0^1 S(t \mid \theta) f(\theta \mid \alpha, \beta)\, d\theta$$
$$= \frac{B(\alpha, \beta + t)}{B(\alpha, \beta)}, \quad t = 1, 2, \ldots$$

- Standing at the end of period n, just prior to the point in time at which the customer makes her contract renewal decision,

$$ERLV = \sum_{t=n}^{\infty} m \frac{S(t \mid \alpha, \beta)}{S(n-1 \mid \alpha, \beta)} \left(\frac{1}{1+d}\right)^{t-n}$$

Impact of Heterogeneity on Error

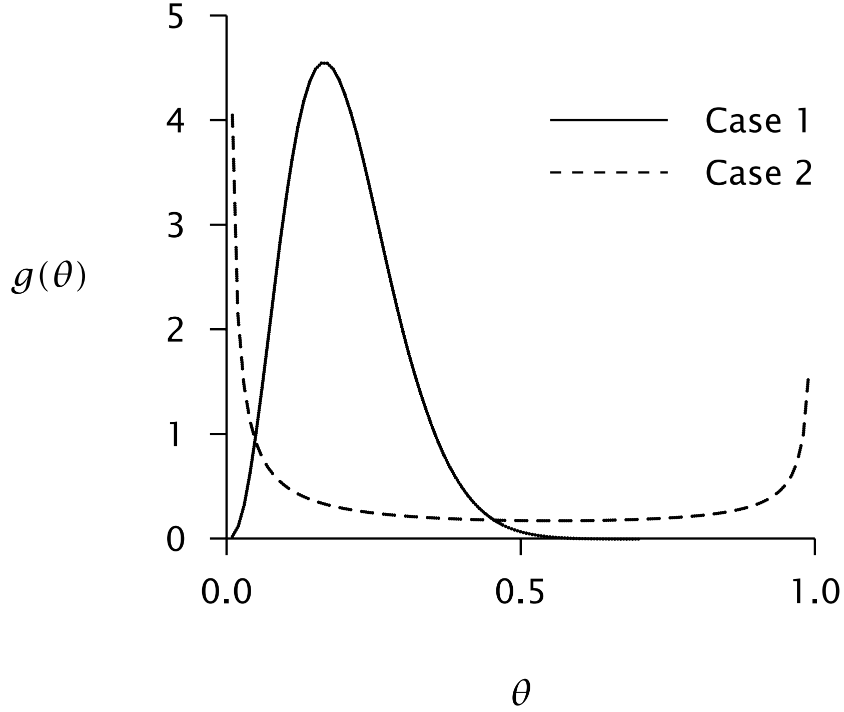
- Assume the following arrival of new customers:

2001	2002	2003	2004	2005
10,000	15,000	17,500	19,000	20,500

- Assume $m = \$1$ and a 10% discount rate.

- For given values of α and β, determine the error associated with computing the residual value of the existing customer base using the naïve approach (a constant aggregate retention rate) compared with the "correct" model-based approach.

Two Scenarios



Case	α	β	$E(\theta)$	$S(1)$	$S(2)$	$S(3)$	$S(4)$
1	3.80	15.20	0.20	0.800	0.684	0.531	0.439
2	0.067	0.267	0.20	0.800	0.760	0.738	0.724

Number of Active Customers: Case 1

2001	2002	2003	2004	2005	n	ERLV
10,000	8,000	6,480	5,307	4,391	5	$3.84
	15,000	12,000	9,720	7,961	4	$3.72
		17,500	14,000	11,340	3	$3.59
			19,000	15,200	2	$3.45
				20,500	1	$3.31
10,000	23,000	35,980	48,027	59,392		

Aggregate 04–05 retention rate = 38,892/48,027 = 0.81

Impact of Heterogeneity on Error: Case 1

$$\text{Naïve valuation} = 59{,}392 \times \sum_{t=1}^{\infty} \frac{0.81^t}{(1+0.1)^{t-1}}$$
$$= \$182{,}292$$

$$\begin{aligned}\text{Correct valuation} &= 4{,}391 \times \$3.84 + 7{,}961 \times \$3.72 \\ &\quad + 11{,}340 \times \$3.59 + 15{,}200 \times \$3.45 \\ &\quad + 20{,}500 \times \$3.31 \\ &= \$207{,}438\end{aligned}$$

Naïve underestimates correct by 12%.

Number of Active Customers: Case 2

2001	2002	2003	2004	2005	n	ERLV
10,000	8,000	7,600	7,383	7,235	5	$10.19
	15000	12000	11,400	11,074	4	$10.06
		17500	14000	13,300	3	$9.86
			19000	15200	2	$9.46
				20500	1	$7.68
10,000	23,000	37,100	51,783	67,309		

Aggregate 04–05 retention rate = 46,809/51,783 = 0.90

Impact of Heterogeneity on Error: Case 2

$$\text{Naïve valuation} = 67{,}309 \times \sum_{t=1}^{\infty} \frac{0.9^t}{(1+0.1)^{t-1}}$$

$$= \$341{,}402$$

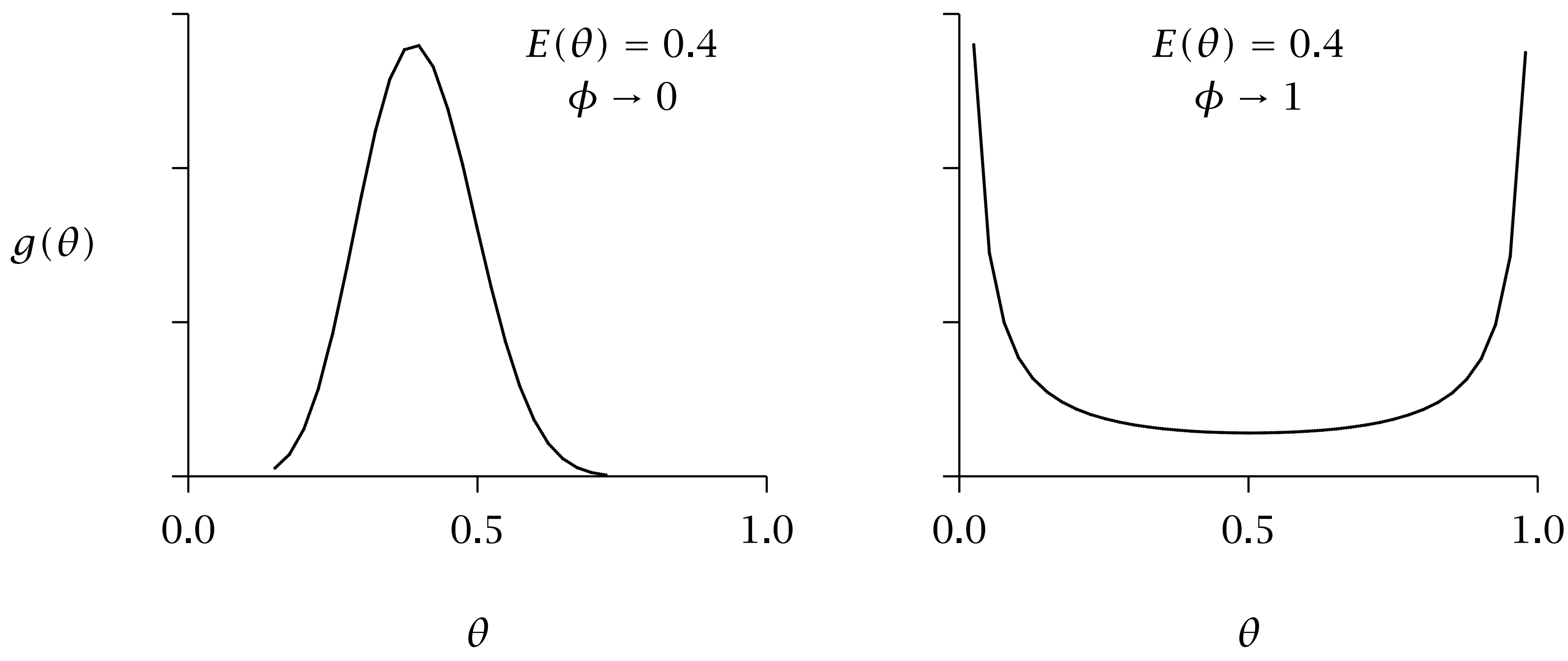
$$\text{Correct valuation} = 7{,}235 \times \$10.19 + 11{,}074 \times \$10.06 + 13{,}300 \times \$9.86 + 15{,}200 \times \$9.46 + 20{,}500 \times \$7.68$$

$$= \$617{,}536$$

Naïve underestimates correct by 45%.

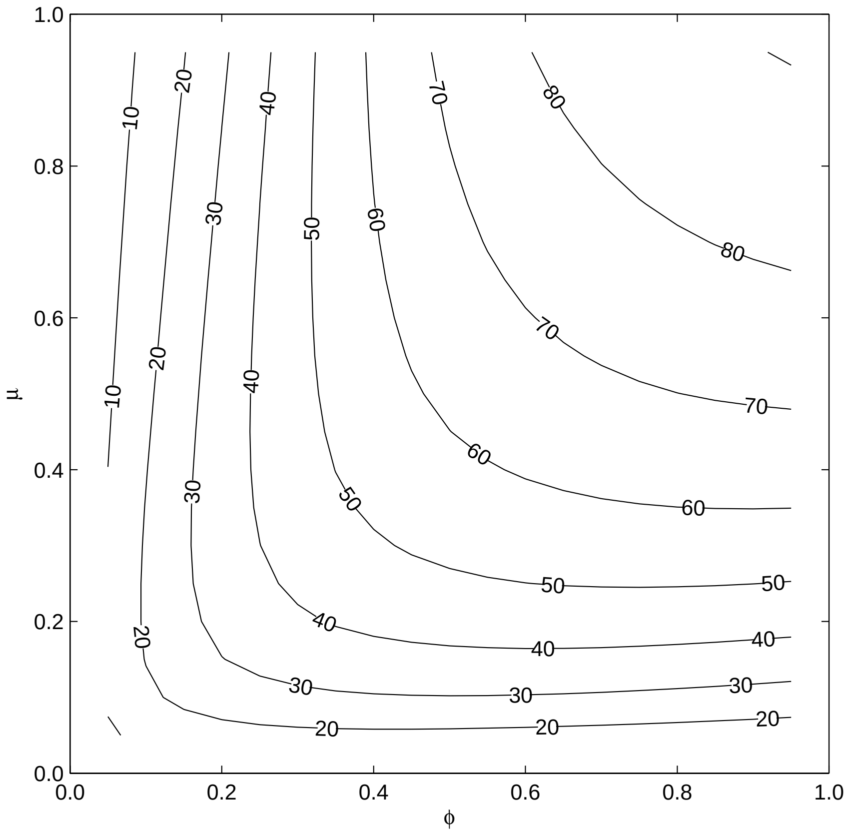
Interpreting the Beta Distribution Parameters

$$\text{mean } \mu = \frac{\alpha}{\alpha + \beta} \text{ and polarization index } \phi = \frac{1}{\alpha + \beta + 1}$$



Error as a Function of μ and ϕ

For a fine grid of points in the (μ, ϕ) space, we determine the corresponding values of (α, β) and compute % underestimation:



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Valuing Customers

Retention Elasticities as a Function of μ and ϕ

We determine the retention elasticity for the values of α and β associated with each point on the (μ, ϕ) unit square: